Yantai University@Summer School

Lectures on Direct Detection of Light Dark Matter

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Contents

Motivation of Light Dark Matter

Historical Perspective

Understanding the Electroweak Sector

- **Discovery of Radioactivity** (1890s)
- **Fermi Scale Identified** (1930s)
- **Non-Abelian Gauge Theory** (1950s)
- **Higgs Mechanism** (1960s)
- **W/Z Bosons Discovered** (1970s)
- **Higgs Discovered** (2010s)

Each step required revolutionary theoretical/experimental leaps

$t \sim 100\text{years}$

Gordan Krnjaic, Brookhaven Forum 2017

Yesterday Once More

Understanding the Dark Sector?

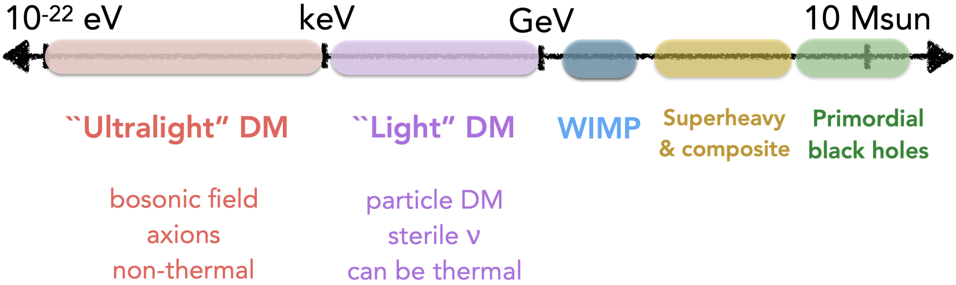
- Discovery of missing mass (1930s)
- Rotation curves (1970s)
- Precision CMB measurements (1990s)
- Dark Matter Discovery? (2030s)?

Discovery Crisis

No clear target for non-gravitational contact → Landscape of dark matter scales

Mass Scale of Dark Matter

Figure from talk by Tongyan Lin at Summer Institute 2019, Korea

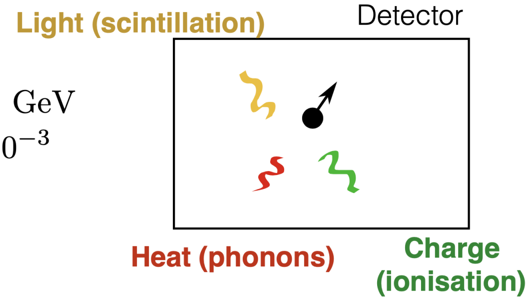


- **Bad news:** DM-SM interactions are not obligatory. If nature is unkind, we may never know the right scale.
- **Good news:** Most discoverable DM candidates are in thermal equilibrium with us in the early universe. → WIMP + Light DM

Direct Detection of WIMP

- Search for collisions of invisible particles with atomic nuclei → Design driver: big exposure
- Coherent elastic scattering → Big idea: Scatter coherently off all the nucleons in a nucleus: $R \sim A^2$ enhancement
- Expected low-energy of recoiling nucleus (with maximum of a few tens of keV) → Predicted signature: recoil induced ionization and scintillation

$$m_\chi \gtrsim 1 \text{ GeV}$$
$$v \sim 10^{-3}$$



Direct WIMP Detection Experiments Worldwide

Numerous underground laboratories

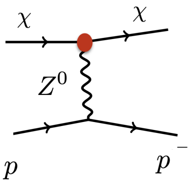
Go underground to shield detector from cosmic rays and their decay products

Direct WIMP Detection Experiments Worldwide

Variety of techniques and dedicated experiments

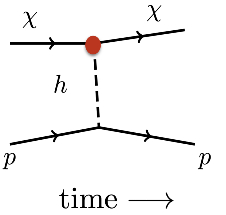
Use only radiopure materials and fabrication techniques

Classifying WIMP Interactions



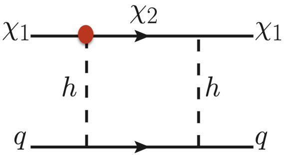
Z Exchange

$$\sigma_p \sim 10^{-39}\,\mathrm{cm}^2$$



Higgs Exchange

$$\sigma_p \sim 10^{-45}\mathrm{cm}^2$$



$$\Delta \equiv m_2 - m_1$$

Inelastic coupling
EW loop

$$\sigma_p \sim 10^{-47}\mathrm{cm}^2$$

Very different at low energy, despite high energy similarities

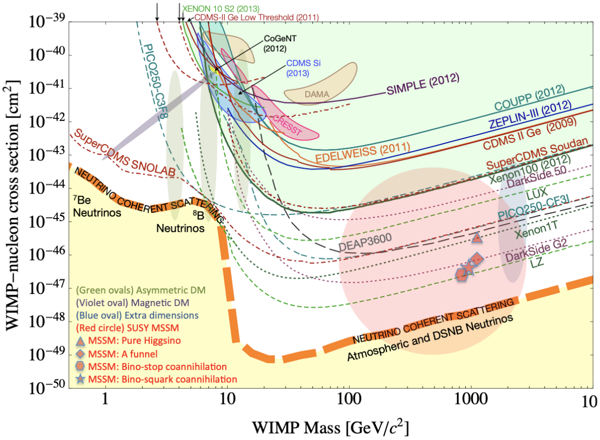
WIMP Milstones

Cushman et al. arxiv:1310.8327

Elastic Z exchange →

Higgs exchange →

1-Loop EW box →



Neutrino floor is coming for WIMP!

WIMP Search Status

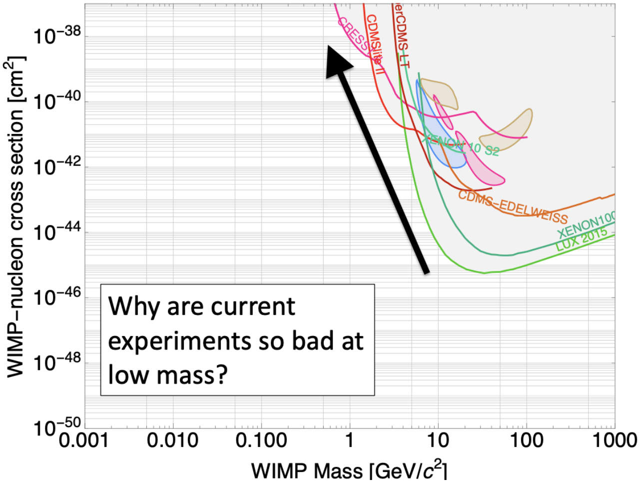
Figure from talk by Haibo Yu at CAU BSM workshop

“上穷碧落下黄泉，两处茫茫皆不见。”白居易《长恨歌》

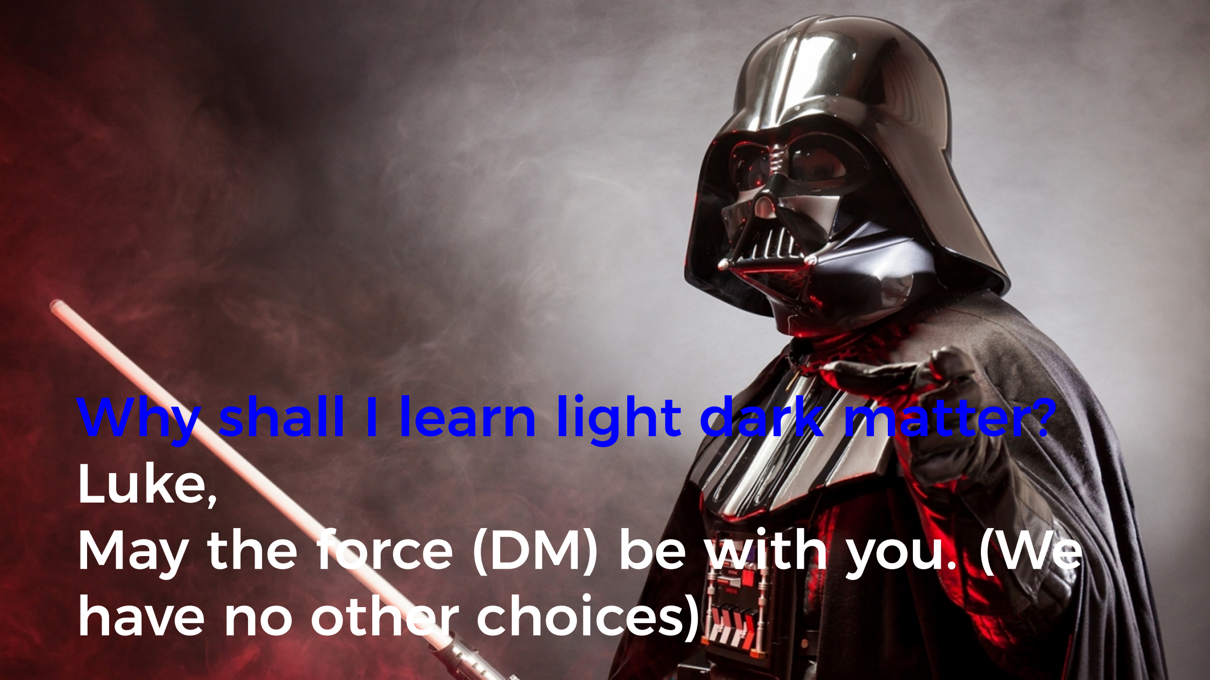
He exhausted all avenues in heaven and the nether world,
… he could not bring her existence to light.

A Song of Immortal Regret, Bai Juyi (772-846)

Opportunity or Crisis
Is Light dark matter possible target?



There is huge room for light dark matter detection → Can we go lower in DM mass?



Why shall I learn light dark matter?

Luke,

May the force (DM) be with you. (We have no other choices)

Why is nucleus bad at light dark matter?

Kinematic No-go Theorem

When dark matter is lighter than $1\mathrm{GeV}$, it resulting recoil energy is smaller than threshold 1keV

Prove that there is inefficient energy transfer from DM to nucleus → How to increase recoil energy

$$E_{\mathrm{NR}} = \frac{q^2}{2m_N} \leq \frac{2\mu_{\chi N}^2 v^2}{m_N} \simeq 1\mathrm{eV} \times \left(\frac{m_\chi}{100\mathrm{MeV}}\right)^2 \left(\frac{20\mathrm{GeV}}{m_N}\right) \quad \mathrm{vs} \quad E_{\mathrm{DM}} \sim \frac{1}{2} m_\chi v_\chi^2$$

Best nuclear recoil threshold is currently $E_R > 30\mathrm{eV}$ (CRESST-III) with DM reach of $m_\chi > 160\ \mathrm{MeV}$.

The kinematics of DM scattering against free nuclei is inefficient, and it does not always describe target response accurately.

Strategies for detecting nuclear recoils from Light DM

- Decreasing the heat threshold of detector - new experimental search.
  See Sec 3 and Sec 5
- Increasing the charge signal - Migdal effect.
  See Sec. 6
- Depositing the whole kinetic energy - DM absorption, Inelastic DM.
  See Sec. ??
- Add kinetic energy to light dark matter through exotic sources or processes - Accelerated DM.
  See Sec. 7

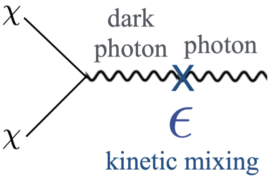
Light Dark Matter Models

What is Light Dark Matter

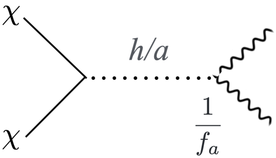
$m = \mathrm{keV} - \mathrm{GeV}$

- Light dark matter needs new forces, otherwise it would be overproduced without such mediator
- Light dark matter has portal to Standard Model

vector portal



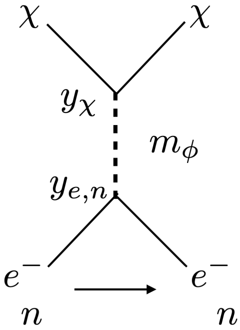
Higgs or axion portal



Model Building for Consistent Production

Vast options and constraints which can be found in Shaofeng and Zuowei's lecture

Overview of Scalar Mediator



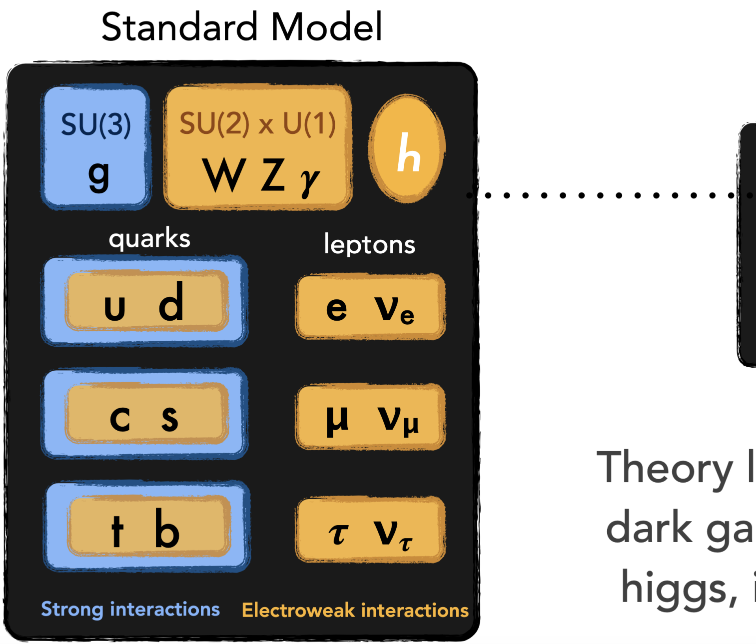
For the moment, consider all scalars

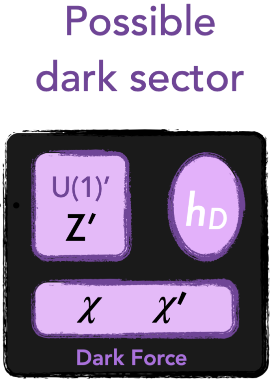
$$\mathcal{L} \supset -\frac{1}{2}m_\chi^2\chi^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}y_\chi m_\chi \phi \chi^2$$

Scattering in direct detection:

$$\bar{\sigma}_e \equiv \frac{y_\chi^2 y_e^2}{4\pi}\frac{\mu_{\chi e}^2}{(m_\phi^2 + \alpha^2 m_e^2)^2} \qquad q_{\text{typical}} \simeq \alpha m_e$$

$$\sigma_n \equiv \frac{y_n^2 y_\chi^2}{4\pi}\frac{\mu_{\chi n}^2}{(m_\phi^2 + v_{DM}^2 m_\chi^2)^2} \qquad q_{\text{typical}} \simeq m_\chi v_{DM}$$





Theory landscape includes dark gauge forces, flavor, higgs, inelastic DM, etc.

Outline

Outline

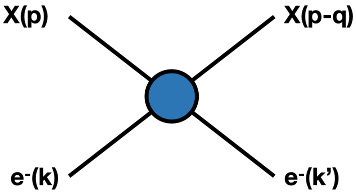
Theory of Dark Matter-Electron Scattering and Electronic Excitation

Outline

Why Electrons?

Kinematics: Just replace m_N by m_e, we can obtain a much larger electron recoil energy!

$$E_i = m_\chi + m_e + \frac{1}{2} m_\chi v^2 + E_{e,1}$$
$$E_f = m_\chi + m_e + \frac{\left| m_\chi \vec{v} - \vec{q} \right|^2}{2 m_\chi} + E_{e,2}$$

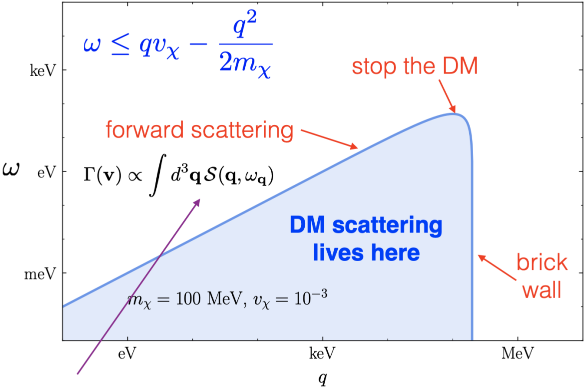


From energy conservation $E_i = E_f$ (momentum?), we obtain

$$\Delta E_{1 \to 2} = -\frac{q^2}{2 m_\chi} + q v \cos \theta_{qv}$$

Kinematic Matching

$$\Delta E = \frac{1}{2m_\chi}\left((m_\chi v)^2 - (m_\chi \boldsymbol{v} - \boldsymbol{q})^2\right) \leq vq - \frac{q^2}{2m_\chi}$$



Goal: maximize the response function inside the DM parabola

Zeroth-order Consideration

typical momentum transfer

typical size of the momentum transfer is set by the **electron's** momentum not DM.

$$q_{\text{typ}} \simeq m_e v_e \sim Z_{\text{eff}} \alpha m_e$$

typical energy transfer

in principle, all of the DM's kinetic energy is transferred to electron

$$\Delta E_{e,\text{typ}} \simeq q_{\text{typ}} v \sim 4 \text{ eV}$$

Minimal Mass

How to estimate which dark matter mass our sensitivity breaks down?

strategry

use energy and momentum conservation to derive it

- Initial dark matter energy $E_\chi = \frac{1}{2} m_\chi v_\chi^2$
- Minimal ionization energy E_{nl} (Binding energy)
- $E_\chi \geq E_{nl}$ and $v_\chi \lesssim v_{\rm esc} + v_{\rm E}$

Result: lowest bound to have ionization

$$m_\chi \gtrsim 250 {\rm keV} \times \left(\frac{E_{nl}}{1 {\rm eV}} \right)$$

Different target material can probe different mass range of light DM

General Formula for Free Electron

If dark matter scatters with free electron, it is just a conventional $2 \to 2$ scattering process with cross section to be

$$\sigma v_{\text{free}} = \frac{1}{4E'_\chi E'_e} \int \frac{d^3q}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \frac{1}{4E_\chi E_e} (2\pi)^4 \delta\left(E_i - E_f\right) \delta^3\left(\vec{k} + \vec{q} - \vec{k}'\right) \left|\overline{\mathcal{M}_{\text{free}}(\vec{q})}\right|^2$$

- momentum transfer effect is absorbed in dark matter form factor $F_{\text{DM}}(q)$. It does not mean dark matter is composite particle

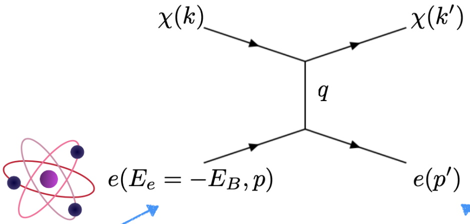
$$\overline{|\mathcal{M}_{\text{free}}(\vec{q})|^2} \equiv \overline{|\mathcal{M}_{\text{free}}\left(\alpha m_e\right)|^2} \times |F_{\text{DM}}(q)|^2$$

- constant cross section is thus defined

$$\overline{\sigma}_e \equiv \frac{\mu^2_{\chi e} \overline{|\mathcal{M}_{\text{free}}\left(\alpha m_e\right)|^2}}{16\pi m^2_\chi m^2_e}$$

Dark matter-Real electron scattering

Figure from talk by McCabe in Sixteenth Marcel Grossmann Meeting



initial electron is not a momentum eigenstate (not a plane wave)

ionised electron propagates in the Coulomb field of the ionised atom (not a plane wave)

Difference between Free Electron and Bound Electron
Different Wave-Function

for free electrons

$$\left\langle \chi_{\vec{p}-\vec{q}}, e_{\vec{k}'} \left| H_{\text{int}} \right| \chi_{\vec{p}}, e_{\vec{k}} \right\rangle = C\mathcal{M}_{\text{free}}(\vec{q}) \times (2\pi)^3 \delta^3 \left(\vec{k} - \vec{q} - \vec{k}' \right)$$

The wave-functions for electrons are just plane wave.

for bound electrons

$$\left\langle \chi_{\vec{p}-\vec{q}}, e_2 \left| H_{\text{int}} \right| \chi_{\vec{p}}, e_1 \right\rangle = C\mathcal{M}_{\textbf{free}} (\vec{q}) \int \frac{V d^3 k}{(2\pi)^3} \widetilde{\psi}_2^*(\vec{k} + \vec{q}) \widetilde{\psi}_1(\vec{k})$$

Final and initial electrons are not plane waves but to be solved by schrodinger equation. Challenge: we need to calculate bound/unbound states

Transition Probablity

$$\left| f_{1\to 2}(\vec{q}) \right|^2 = \left| \int \frac{d^3 k}{(2\pi)^3} \widetilde{\psi}_2^* \left(\vec{k}' \right) \widetilde{\psi}_1(\vec{k}) \right|^2$$

Momentum conservation is now replaced by wave-function

General Formula for Bound Electron

In terms of dark matter form factor and electron transition probability, cross-section is rewritten

$$\sigma v_{1\to 2} = \frac{\overline{\sigma}_e}{\mu_{\chi e}^2} V \int \frac{d^3q}{4\pi} \frac{d^3k'}{(2\pi)^3} \delta\left(\Delta E_{1\to 2} + \frac{q^2}{2m_\chi} - qv\cos\theta_{qv}\right) \times |F_{\mathrm{DM}}(q)|^2 \, |f_{1\to 2}(\vec{q})|^2$$

- If only one final electron state, $V = 1$ and phase space d^3k', d^3q.
- Kinematics is respected by delta-function.
- Dark matter form factor $F_{\mathrm{DM}}(q)$ captures momentum transfer for specific dark matter model.
- Transition probability captures of electron response after scattering

Deal with Phase Space

- Electron recoil energy $E_e = k'^2/2m_e$

ionized electron phase space $= \sum_{l'm'} \int \frac{k'^2 dk'}{(2\pi)^3} = \frac{1}{2} \sum_{l'm'} \int \frac{k'^3 d\ln E_e}{(2\pi)^3}$

- We assume the potential is spherically symmetric and we ionize a full atomic shell Why ignore the $d\phi_k$ therefore, sum over all initial and final angular momentum variables

$$\sigma v_{\text{ion}} = \frac{\overline{\sigma}_e}{\mu_{\chi e}^2} \sum_{n'l'm'} \int \frac{d^3q}{8\pi} \frac{k'^3 d\ln E_e}{(2\pi)^3} \delta\left(\Delta E_{i\to k'l'm'} + \frac{q^2}{2m_\chi} - qv\cos\theta_{qv}\right) |F_{\text{DM}}(q)|^2 \, |f_{i\to k'l'm'}(\vec{q})|^2$$

Why using E_e

We want to have a similar behavior with DM-nucleus scattering

Differential Cross-Section over Electron Recoils

Evaluate the energy conservation δ-function, and $q_{\max}$ and $q_{\min}$?

$$\frac{\mathrm{d}\langle\sigma v\rangle}{\mathrm{d}\ln E_e} = \frac{\overline{\sigma}_e}{8\mu_{\chi e}^2}\int_{q_{\min}}^{q_{\max}} q\,\mathrm{d}q\left|f_{\mathrm{ion}}(k',q)\right|^2\left|F_{\mathrm{DM}}(q)\right|^2\eta(v_{\min})$$

We do not know where DM comes from → Astrophysics Uncertainty

Need to perform a velocity distribution integral to get statistical result → **Average**

$$\eta\left(v_{\min}\right) = \int_{v_{\min}}\frac{\mathrm{d}^3 v}{v}f_{\mathrm{MB}}(v)$$

- f_{MB} is Maxwell-Boltzmann distribution $f_{\mathrm{MB}} = \frac{1}{N_{\mathrm{esc}}}\left(\frac{3}{2\pi\sigma_v^2}\right)^{3/2} e^{-3\mathbf{v}^2/2\sigma_v^2}$
- $v_{\min}$ is the minimal velocity for ionziation and $q_{\min}$, $q_{\max}$ are determined by kinematics

After introducing the d^3v, the energy conservation function δ is now replaced! Why and How?

Signal Rate

Bridge to connect theory and experiment

Event rate = DM flux $\times$ particle physics $\times$ detector response

$$R = N_T \frac{\rho_\chi}{m_\chi} \int_{E_{e,cut}} \mathrm{d} \ln E_e \frac{\mathrm{d}\langle \sigma v \rangle}{\mathrm{d} \ln E_e}$$

Experiment prefers events rather than cross-section

R = number of events/time/volume

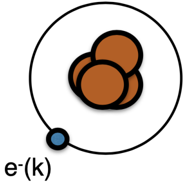
- N_T is the number of target atoms $\rightarrow$ material dependent
- $\rho_\chi = 0.4 \mathrm{GeV}/\mathrm{cm}^3$ is the local DM density
- $\mathrm{R} \times \mathrm{Exposure} = \mathrm{Events}$

Simplest Target: Isolated Atom

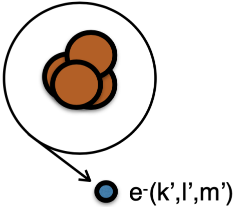
There is no many-body correlation

Typical atom: Hydrogen, Xenon, and Argon

$$\Delta E_B \sim 10\text{eV}, \quad m_\chi > 2.5\text{MeV}$$



initial state



final state

Ionization Factor for Isolated Atom

Absorb phase space of electron into ionization factor

$$|f_{\text{ion}}(k', q)|^2 = \frac{2k'^3}{(2\pi)^3} \sum_{n'l'm'} \left| \int d^3x \psi^*_{k'l'm'}(\vec{x}) \psi_i(\vec{x}) e^{i\vec{q}\cdot\vec{x}} \right|^2$$

- **Simplified version: outgoing electron is free plane wave, initial electron is part of a spherically symmetric atom with full shells. See Essig or Ran Ding**

$$\left|f^i_{ion}(k', q)\right|^2 = \frac{k'^2}{4\pi^3 q} \int_{k'-q}^{k'+q} k\, dk\, |\chi_{nl}(k)|^2$$

- **More realistic version: solve radial Schrödinger equation for the exact unbound wavefunctions, using the effective potential extracted from the bounded wavefunctions. See Timon Emken or Zheng-Liang Liang, Lei Wu**

Ionization Factor for Isolated Atom

- relevant quantity is transition probability

$$f_{1\to 2}(\mathbf{q}) = \int \mathrm{d}^3 x \psi^*_{k'\ell' m'}(\mathbf{x}) e^{i\mathbf{x}\cdot\mathbf{q}} \psi_{n\ell m}(\mathbf{x})$$

- Expressed the initial and final state electron wave functions in terms of spherical coordinates

$$\psi_{n\ell m}(\mathbf{x}) = R_{n\ell}(r) Y_\ell^m(\theta, \phi)$$

- Thus transition probability is function of scalar product of radial wave function

$$f_{1\to 2}(\mathbf{q}) = \int \mathrm{d}^3 x R^*_{k'\ell'}(r) Y_{\ell'}^{m'*}(\theta,\phi) R_{n\ell}(r) Y_\ell^m(\theta,\phi) \times 4\pi \sum_{L=0}^{\infty} i^L j_L(qr) \sum_{M=-L}^{+L} Y_L^{M*}\left(\theta_q, \phi_q\right) Y_L^M(\theta,\phi)$$

$$= 4\pi \sum_{L=0}^{\infty} i^L \sum_{M=-L}^{L} I_1(q) Y_L^{M*}\left(\theta_q,\phi_q\right) \int \mathrm{d}\Omega Y_{\ell'}^{m'*}(\theta,\phi) Y_\ell^m(\theta,\phi) Y_L^M(\theta,\phi)$$

Radial Part and Angular Part

For angular part: the integral over three spherical harmonics can be re-written in terms of the Wigner $3j$ symbols

$$f_{1\to 2}(\mathbf{q}) = \sqrt{4\pi} \sum_{L=|\ell-\ell'|}^{\ell+\ell'} i^L I_1(q) \sum_{M=-L}^{+L} Y_L^{M*}\left(\theta_q, \phi_q\right) (-1)^{m'} \sqrt{(2\ell+1)\left(2\ell'+1\right)(2L+1)} \times \begin{pmatrix} \ell & \ell' & L \\ 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} \ell & \ell' & L \\ m & -m' & M \end{pmatrix}$$

The orthogonality of Wigner $3j$ symbols, allows us to sum over the L' and M'

$$\sum_{m=-\ell}^{\ell} \sum_{m'=-\ell'}^{\ell'} \left|f_{1\to 2}(\mathbf{q})\right|^2 = 4\pi \sum_L I_1(q)^2 \sum_M Y_L^{M*}\left(\theta_q, \phi_q\right)$$

Radial part: the core is wavefunction

$I_1(q) \equiv \int \mathrm{d}r r^2 R_{k'\ell'}^*(r) R_{n\ell}(r) j_L(qr)$

Initial and Final State Wave Functions

Initial state wave-function is Roothaan-Hartree-Fock (RHF) ground state wave function

It is just a linear combination of Slater-type orbitals

$$R_{n\ell}(r) = a_0^{-3/2} \sum_j C_{j\ell n} \frac{\left(2Z_{j\ell}\right)^{n'_{j\ell}+1/2}}{\sqrt{\left(2n'_{j\ell}\right)!}} \left(\frac{r}{a_0}\right)^{n'_{j\ell}-1} \exp\left(-Z_{j\ell}\frac{r}{a_0}\right)$$

Final state wave function is similar with hydrogen wave function except energy is positive and spectra is continuum

It is solved by the Schrodinger equation with a hydrogenic potential $-Z_{eff}/r$

$$R_{k'\ell'}(r) = \frac{(2\pi)^{3/2}}{\sqrt{V}} (2k'r)^{\ell'} \frac{\sqrt{\frac{2}{\pi}} \left|\Gamma\left(\ell' + 1 - \frac{iZ_{\text{eff}}}{k'a_0}\right)\right| e^{\frac{\pi Z_{\text{eff}}}{2k'a_0}}}{(2\ell'+1)!} e^{-ik'r} {}_1F_1\left(\ell' + 1 + \frac{iZ_{\text{eff}}}{k'a_0}, 2\ell'+2, 2ik'r\right)$$

Scattering Kinematics

In terms of energy conservation

$$\mathbf{v}\cdot\mathbf{q} = \Delta E_{1\to 2} + \frac{q^2}{2m_\chi}$$

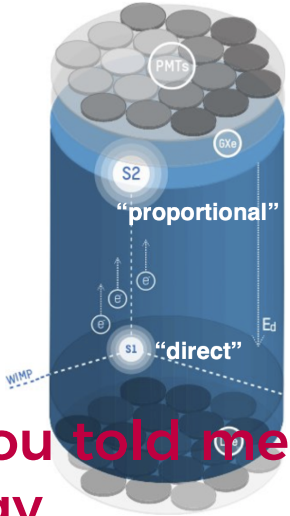
- **Minimal velocity is obtained by setting** $\cos\theta_{qv} = 1$

$$v_{\min}(k', q) = \frac{E_B + k'^2/(2m_e)}{q} + \frac{q}{2m_\chi}$$

- **Taking** $\cos\theta_{qv} = 1$, $E_e = 0$ **and** $v = v_{\max}$, **the range of** q **is**

$$\begin{aligned} q_{\min} &= m_\chi v_{\max} - \sqrt{m_\chi^2 v_{\max}^2 - 2m_\chi E_B} \\ &= \frac{E_B}{v_{\max}}, \qquad \text{for } m_\chi \to \infty \\ q_{\max} &= m_\chi v_{\max} + \sqrt{m_\chi^2 v_{\max}^2 - 2m_\chi E_B} \end{aligned}$$

a XENON detector



https://commons.wikimedia.org/wiki/File:2phaseTPC_b.jpg

i.e. XENON10, XENON100, XENON1T, LUX

DM-electron scattering

=

S2 only signal

measures **P**hoto**E**lectrons

PE, you told me, we measure recoil energy

*can also do this with LAr detectors like DarkSide

Real Event Rate

Frome Recoil energy E_e to PE

$$\frac{\mathrm{d}R_{\mathrm{ion}}}{\mathrm{dS2}} = \int d\ln E_e \epsilon(\mathrm{S2}) P\left(\mathrm{S2} \mid \Delta E_e\right) \frac{\mathrm{d}R_{\mathrm{ion}}}{\mathrm{d}\ln E_e}$$

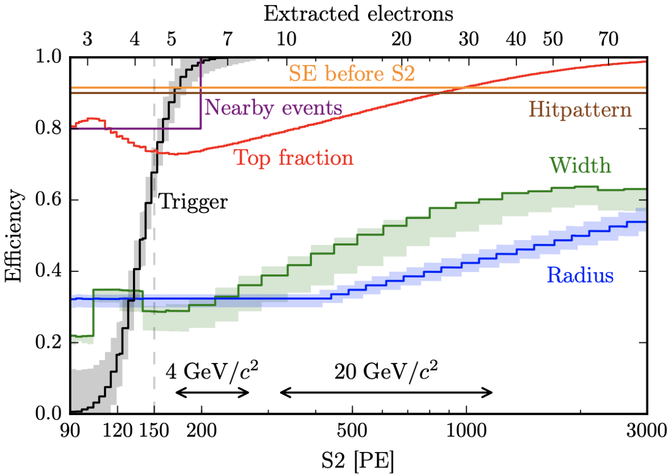
- $\epsilon(\mathrm{S2})$ **is the detector efficiency,** $S2 = \mathrm{PE}$
- **The probability function** P **that converts energy transfer into the photoelectron (PE) in S2**
- $\Delta E_e = E_e + E_{nl}$

True Signal Rate

We can compare our signal rate $dR/dS2$ **to data directly to obtain exclusion limit**

Detector Efficiency

Take XENON1T as example



Probability Function $P\left(\mathrm{S2} \mid \Delta E_e\right)$

$$P\left(\mathrm{S2} \mid \Delta E_e\right)=\sum_{n_e^{\mathrm{s}}, n_e} P\left(\mathrm{S2} \mid n_e^{\mathrm{s}}\right) \cdot P\left(n_e^{\mathrm{s}} \mid n_e\right) \cdot P\left(n_e \mid \langle n_e\rangle\right)$$

- $P\left(n_e \mid \langle n_e\rangle\right)$ **is the number of electrons escaping the interaction point, which follows a binomial distribution**

$$P\left(n_e \mid \langle n_e\rangle\right)=\operatorname{binom}\left(n_e \mid N_Q, f_e\right)=C_{N_Q}^{n_e} f_e^{n_e}\left(1-f_e\right)^{N_Q-n_e}, N_Q=\Delta E_e / 13.8 \mathrm{eV}$$

- $P\left(n_e^{\mathrm{s}} \mid n_e\right)=80\%$ **is possibility of electrons surviving the drift in Xenon1T**
- **PE transformation probability** $P\left(\mathrm{S2} \mid n_e^{\mathrm{s}}\right)$ **is Gaussian distribution**

$$P\left(\mathrm{S2} \mid n_e^{\mathrm{s}}\right)=\operatorname{Gauss}\left(\mathrm{S2} \mid g_2 n_e^{\mathrm{s}}, \sigma_{\mathrm{S2}}\right)$$

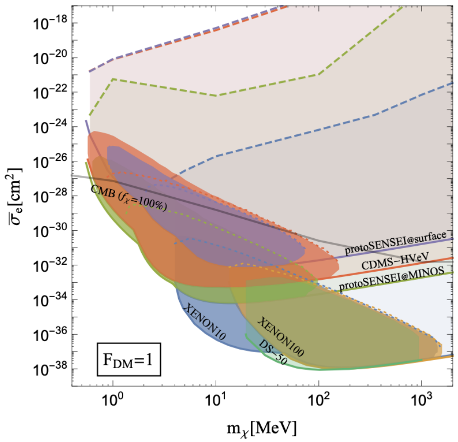
XENON10 and XENON1T Data

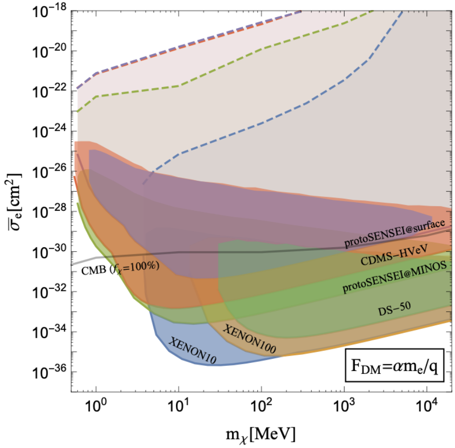
XENON10	
bin [S2]	obs. events
[14,41)	126
[41,68)	60
[68,95)	12
[95,122)	3
[122,149)	2
[149,176)	0
[176,203)	2

XENON1T	
bin [S2]	obs. events
[150,200)	8
[200,250)	7
[250,300)	2
[300,350)	1
-	-
-	-
-	-

Find Limits

Signal + Backgrounds < Number of observed events





Upper bound comes from the Earth attenuation effect. See Sec 8

Brief Summary on DM Induced Electron Ionizations

- Complication: Target electrons are bound states.
- Electrons are not in a momentum eigenstates
- Example: Ionization spectrum for isolated atom:

$$\frac{\mathrm{d}R_{\mathrm{ion}}}{\mathrm{d}E_e} = \frac{1}{m_N}\frac{\rho_\chi}{m_\chi}\sum_{nl}\frac{\left\langle \mathrm{d}\sigma_{\mathrm{ion}}^{nl} v\right\rangle}{\mathrm{d}E_e}$$

$$\frac{\mathrm{d}\left\langle \sigma_{\mathrm{ion}}^{nl} v\right\rangle}{\mathrm{d}E_e} = \frac{\sigma_e}{8\mu_{\chi e}^2 E_e}\int \mathrm{d}q q \left|F_{\mathrm{DM}}(q)\right|^2 \left|f_{\mathrm{ion}}^{nl}\left(k', q\right)\right|^2 \eta\left(v_{\min}\left(\Delta E_e, q\right)\right)$$

- Predictions require the precise evaluation of an ionization form factor.
- There is still theoretical uncertainty in the evaluation of the ionization form factors. See 1904.07127.
- For crystals, this requires methods from condensed matter physics. form factors.

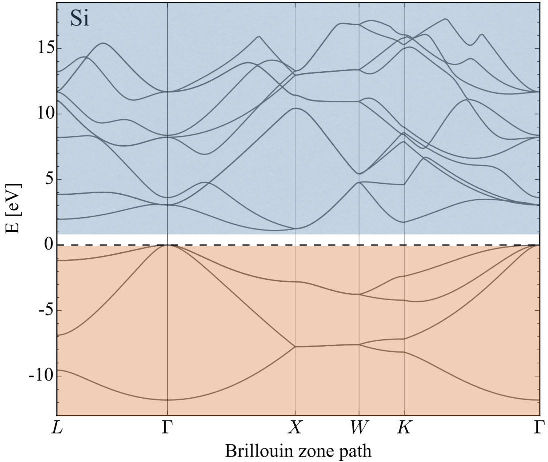
Challenge for Isolated Atom

- detector specific backgrounds i.e. e^- gets trapped in liquid-gas interface and is later released
  Need a better detector setup
- ionization energy $(12.1\mathrm{eV})$ limits DM mass reach to few MeV
  Find a material with smaller ionization energy

Better Target: Crystal

semiconductors: $E_B \sim 1\mathrm{eV}$

Semiconductor Target



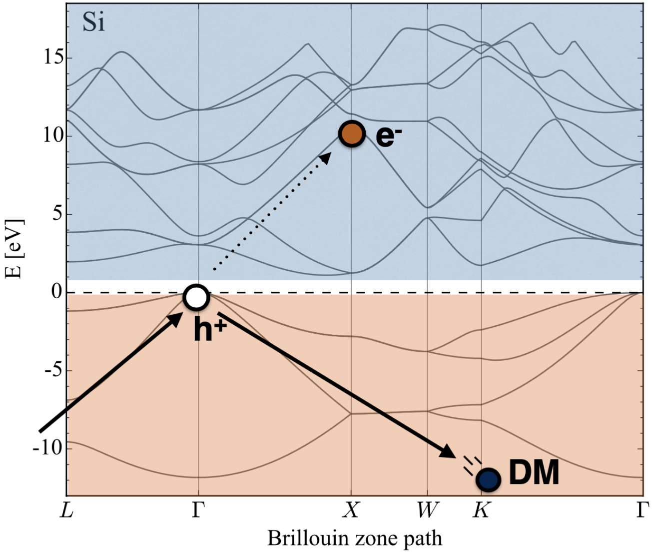
empty conduction band

band gap

filled valence band

Essig, Fernandez-Serra, Mardon, Soto, TTY [1509.01598] JHEP 1605 (2016) 046

Semiconductor Process

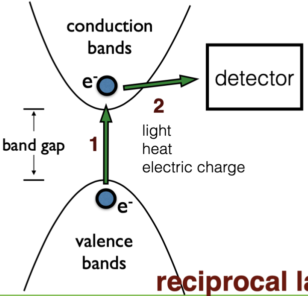


apply an electric field and extract the electron(s) "ionization"

e^-/h^+ recombine to produce photon(s) "scintillation"

Transition Probability For Semiconductor

solid



reciprocal lattice vector

$$\left|f_{i\to i'}(\vec{q},\vec{k})\right|^2 = \left|\sum_G \psi^*_{i'}(\vec{k}+\vec{G}+\vec{q})\psi_i(\vec{k}+\vec{G})\right|^2$$

electrons in a solid are part of a complicated, many-body system

Wavefunction in Crystal

Each possible electron level is labelled by a band index i and a wavevector $\vec{k}$ in the first Brillouin Zone (BZ). Due to the periodicity of the potential, the wavefunctions are

$$\psi_{i\vec{k}}(\vec{x}) = \frac{1}{\sqrt{V}} \sum_{\vec{G}} u_i(\vec{k} + \vec{G}) e^{i(\vec{k}+\vec{G})\cdot\vec{x}}$$

-

Differential Rate for Crystal

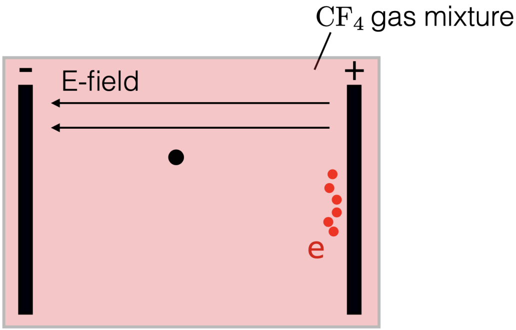
$$\frac{dR_{\textbf{crystal}}}{d\ln E_e} = \frac{\rho_\chi}{m_\chi} N_{\textbf{cell}}\, \bar{\sigma}_e \alpha \times \frac{m_e^2}{\mu_{\chi e}^2} \int d\ln q \left(\frac{E_e}{q} \eta\left(v_{\min}\left(q, E_e\right)\right)\right) F_{\mathrm{DM}}(q)^2 \left| f_{\textbf{crystal}}\left(q, E_e\right)\right|^2$$

Outline

Directional Experiment

Definition

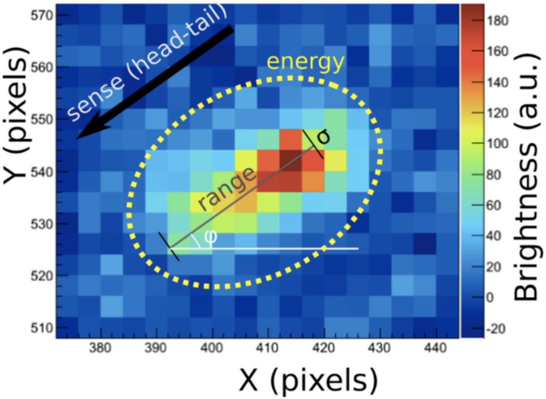
Most mature technology for recoil direction is the gaseous Time Projection Chamber (TPC): DRIFT, MIMAC, DMTPC, NEWAGE



Get x and y from distributions of electrons hitting anode
Get z of track from timing of electrons hitting anode

A Signal Example

Deaconu et al. (DMTPC, 2015)

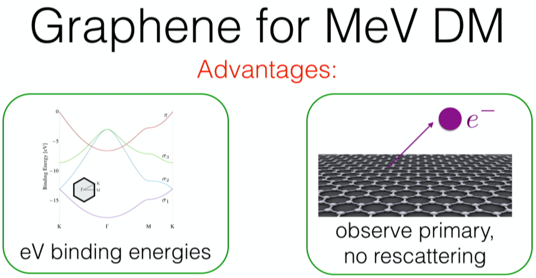


- Finite angular resolution $\Delta\theta \sim 20^\circ - 80^\circ$
- May not get full 3-D track information

(Directional) Detection of Dark Matter with Graphene

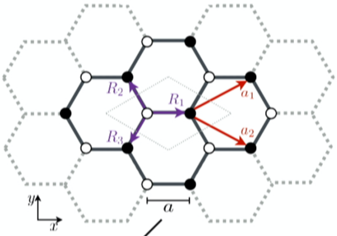
Why Graphene?

Two-dimensional materials such as graphene sheets can serve as excellent detectors for dark matter (DM) with couplings to electrons. The ionization energy of graphene is eV, making it sensitive to DM as light as an MeV, and the ejected electron may be detected without rescattering in the target, preserving directional information.



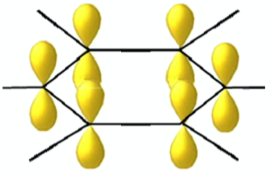
Is there interaction between dark matter and nucleus?

1-atom thick lattice of carbon

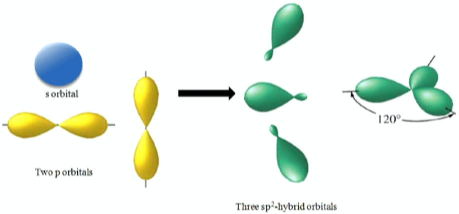


$$0.142\ \mathrm{nm} \simeq \frac{2\pi}{8.7\ \mathrm{keV}}$$

4 valence electrons per atom:



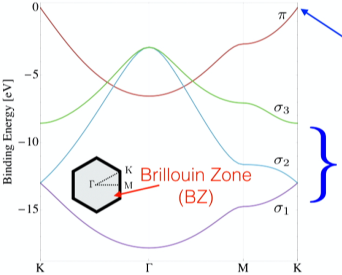
π electrons



σ electrons

Electron Binding Energies

Binding energy depends on lattice momentum l



Dirac point: massless charge carriers, FQHE, all "interesting" properties

can dominate DM scattering cross section, need to treat these too!

No effect from Dirac cone. Bad news!

Cross Sections
Almost the same as DM-Electron in Isolated Atom

Graphene

$$\sigma(l)v = \frac{\overline{\sigma}_e}{\mu_{e\chi}^2} \int \frac{d^3k_f}{(2\pi)^3} \frac{d^3q}{4\pi} |F_{\rm DM}(q)|^2 \left|\widetilde{\Psi}_i\left(\ell, \mathbf{q} - \mathbf{k}_f\right)\right|^2 \delta\left(\frac{k_f^2}{2m_e} + E_i(\ell) + \Phi + \frac{q^2}{2m_\chi} - \mathbf{q}\cdot\mathbf{v}\right)$$

Xenon

$$\sigma v_{1\to 2} = \frac{\overline{\sigma}_e}{\mu_{\chi e}^2} V \int \frac{d^3q}{4\pi} \frac{d^3k'}{(2\pi)^3} \delta\left(\Delta E_{1\to 2} + \frac{q^2}{2m_\chi} - qv\cos\theta_{qv}\right) \times |F_{\rm DM}(q)|^2 \left|f_{1\to 2}(\vec{q})\right|^2$$

- **In Graphene, σ is function of lattice, which needs additional summation**
- **k' and k_f both stand for out-going electron momentum**
- **Kahn's treatment does not consider electron initial momentum**
- **Transition probability is the same. The difference is the electron wave-function in graphene rather than xenon**
- **working function Φ is inevitable in graphene**

Electron Wave-Functions in Graphene
Tight-binding method

Take π-band as example

$$\Psi_\pi(\ell, \mathbf{r}) \approx \mathcal{N}_\ell \left(\phi_{2\mathrm{p}_z}(\mathbf{r}) + e^{i\varphi_\ell} \sum_{j=1}^{3} e^{i\ell \cdot \mathbf{R}_j} \phi_{2\mathrm{p}_z}\left(\mathbf{r} - \mathbf{R}_j\right) \right)$$

- lattice momentum $\ell = (l_x, l_y)$ in the Brillouin zone
- $\mathcal{N}_\ell$ is normalization constant. R_j are the nearest-neighbor vectors. φ_ℓ is l-dependent phase
- Hydrogenic orbital for the $2p_z$ wavefunction of carbon

$$\phi_{2\mathrm{p}_z}(\mathbf{r}) = \mathcal{N} a_0^{-3/2} \frac{r}{a_0} e^{-Z_{\mathrm{eff}} r / 2a_0} \cos\theta$$

Directionally Averaged Event Rate

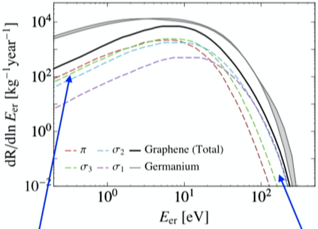
Two more summation

Sum over bands, integrate over BZ and DM velocity $g(v) = f_{\rm MB}/v$

$$R = 2 \sum_{i=\pi,\sigma_{1,2,3}} \frac{\rho_\chi}{m_\chi} N_{\rm C} A_{\rm uc} \int_{\rm BZ} \frac{d^2\ell}{(2\pi)^2} d^3 v\, g(\mathbf{v}) \sigma_i(\ell) v$$

Energy spectrum

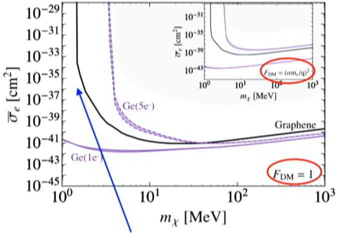
(F_{DM} = 1, 100 MeV DM)



Binding energy dominates

Momentum tail dominates

Reach



work function sets threshold

Directional Detection

Key observation: ionized electron kinematics highly correlated with initial DM direction

$$\widetilde{\phi}\left(\mathbf{q}-\mathbf{k}_f\right) \sim \frac{1}{\left(a_0^2\left|\mathbf{q}-\mathbf{k}_f\right|^2+(Z_{\mathrm{eff}}/2)^2\right)^{l+1}}$$

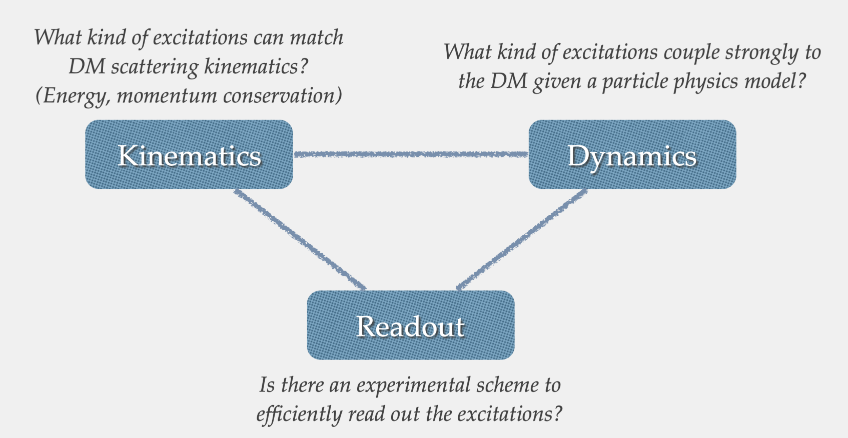
- $\mathbf{k}_f || \mathbf{q}$ corresponds to maximal transition probability
-

Dirac Material

Multi-Channel Direct Detection: Unified Description

Key words when pursuing new ideas

From Zhengkang "Kevin" Zhang at IMPU



Material-Independent Method

$$R \sim \int d^3\mathbf{v}\, f(\mathbf{v}) \int d^3\mathbf{q}\, F^2(\mathbf{q}) S(\mathbf{q}, \omega_{\mathbf{q}})$$

DM properties   Material properties

General framework that works for **any** many-body system

- **Kinematic matching: identify new detection channels, especially in AMO/CM systems**
- **Theoretical framework for multi- channel direct detection (rate calculation from first principles)**

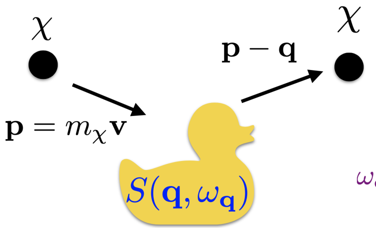
General Derivation

Assume spin-independent (SI) interactions. For given DM mass and incoming velocity,

$$\Gamma = \int \frac{d^3q}{(2\pi)^3} |\mathcal{M}|^2 S(\boldsymbol{q}, \omega)\Big|_{\omega = \boldsymbol{q}\cdot\boldsymbol{v} - \frac{q^2}{2m_\chi}}$$

- $\mathcal{M}$: particle-level $\chi\psi \to \chi\psi$ matrix element (ψ is SM particle).
- $S(\boldsymbol{q}, \omega)$: dynamic structure factor (target response to an energy-momentum transfer)
- For any target system, the rate can be calculated from first principles by
  - Identifying accessible final states (low energy d.o.f.).
  - Quantizing number density operators in the appropriate Hilbert space.

Response Functions = Dynamic Structure Factor



does the target have
an eigenstate at $\omega_\mathbf{q}$?

Energy deposited by DM:

$$\omega_\mathbf{q} = \frac{\mathbf{p}^2}{2m_\chi} - \frac{(\mathbf{p}-\mathbf{q})^2}{2m_\chi} = \mathbf{q}\cdot\mathbf{v} - \frac{q^2}{2m_\chi}$$

Require energy conservation,
but **not** momentum conservation

$$S(\boldsymbol{q}, \omega) = \frac{1}{V}\sum_f \left|\langle f \,|\mathcal{F}_T(\boldsymbol{q})|\, i\rangle\right|^2 2\pi\delta\left(E_f - E_i - \omega\right)$$

$$\mathcal{F}_T(\boldsymbol{q}) = \frac{f_p(\boldsymbol{q})\widetilde{n}_p(-\boldsymbol{q}) + f_n(\boldsymbol{q})\widetilde{n}_n(-\boldsymbol{q}) + f_e(\boldsymbol{q})\widetilde{n}_e(-\boldsymbol{q})}{f_\psi^0}$$

Dynamic structure factor for Nuclear recoils

reduces to nuclear recoils:

- $2\pi/V$ remains always, and redfines $1/V = \rho_T/m_N$.
- nucleus is only excited not ionized $|i\rangle = |f\rangle = |N\rangle$
- $\mathcal{F}_T = \exp(iqr_\alpha)$, where α stands for single nucleon in nucleus.

$$S(\boldsymbol{q}, \omega) = 2\pi \frac{\rho_T}{m_N} \frac{f_N^2}{f_n^2} F_N^2 \delta\left(\omega - \frac{q^2}{2m_N}\right)$$

$$f_N \equiv f_p Z + f_n (A - Z)$$

$$F_N = \frac{3j_1(qr_n)}{qr_n} e^{-(qs)^2/2} = \langle N| \frac{1}{A} \sum_{\alpha=1}^{A} \exp(iqr_\alpha) |N\rangle$$

$$r_n \simeq 1.14 A^{1/3} \mathrm{fm},\ s \simeq 0.9 \mathrm{fm}$$

Dynamic structure factor for electron recoils

- $2\pi/V$ remains always.
- $|f\rangle$ stands for unbounded electron with arbitrary momentum k, $|i\rangle$ stands for bound electron
- $\delta(E_f - E_i - \omega)$

$$
\begin{aligned}
S(q,\omega) &= \frac{2\pi}{V}\frac{d^3k}{(2\pi)^3}\delta(E_f - E_i - \omega)|\langle f|\exp(iq\cdot r_e)|i\rangle|^2 \\
&= \frac{2\pi}{V}\frac{d^3k}{(2\pi)^3}\delta(E_f - E_i - \omega)\int d^3r|\langle f|r\rangle\langle r|\exp(iqr_e)|i\rangle|^2 \\
&= \frac{2\pi}{V}\frac{d^3k}{(2\pi)^3}\delta(E_f - E_i - \omega)\int d^3r\psi_k^*(r)\psi_i(r)\exp(iqr)
\end{aligned}
$$

Dynamic structure factor for phonon recoils

Left for next section

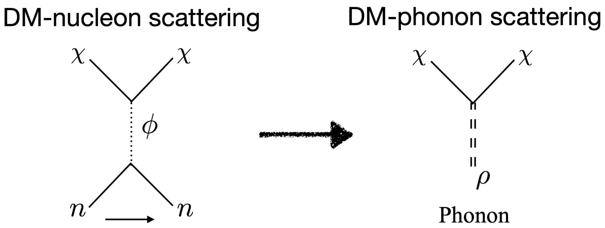
Theory of Dark Matter Induced Phonon Excitation

Why Phonons?
Compared with Electron

Common Sense: Two most common elementary excitations in solid state materials: electrons and phonons.

It exists. Thus we study it!

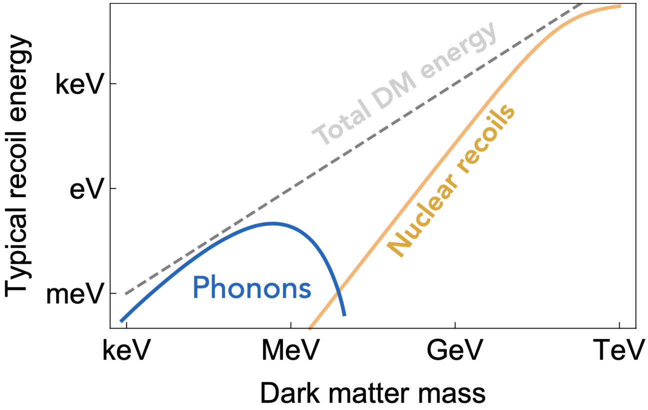
- $q \gg \mathcal{O}(1-10)\text{keV}$ $\rightarrow$ **recoil against individual nuclei**
- $q \ll \mathcal{O}(1-10)\text{keV}$ $\rightarrow$ **excite phonons (lattice/fluid vibrations)**



Kinematics

Different target mass

Kinematics of phonon excitation is suited to $\sim 10\mathrm{keV} - \mathrm{MeV}$ dark matter

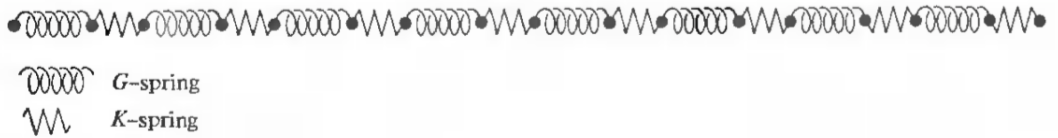


Phonons in crystals

- Consider a 1D lattice:

$$U^{\text{harm}} = \frac{1}{2} K \sum_n [u(na) - u([n+1]a)]^2$$

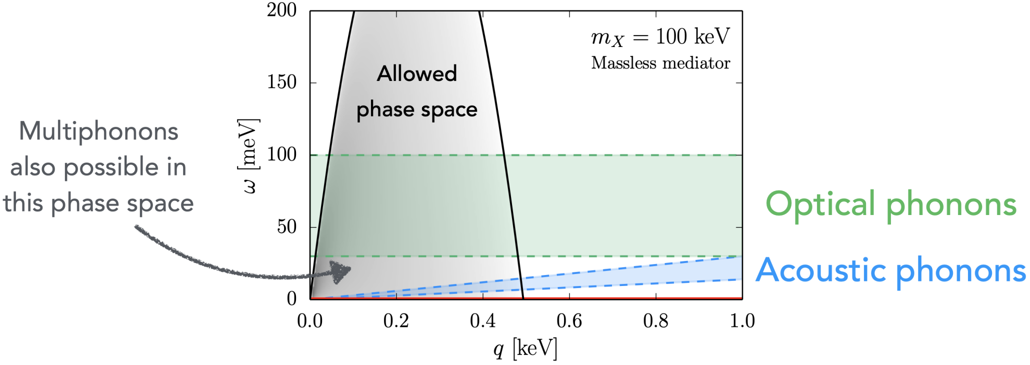
- Diagonalize the Hamiltonian → canonical oscillation modes: acoustic phonons
- Now suppose there are two inequivalent atoms in the primitive cell



Dispersion
Two Different Phonons

Energy deposition

$$\omega = \mathbf{q} \cdot \mathbf{v}_i - \frac{\mathbf{q}^2}{2m_\chi}$$



Phonons from DM scattering

$$S(\boldsymbol{q}, \omega) = \frac{\pi}{\Omega} \sum_{\nu} \frac{1}{\omega_{\nu,\boldsymbol{k}}} \left| \sum_{j} \frac{e^{-W_j(\boldsymbol{q})}}{\sqrt{m_j}} e^{i\boldsymbol{G}\cdot\boldsymbol{x}_j^0} \left(\boldsymbol{Y}_j \cdot \boldsymbol{\epsilon}^*_{\nu,\boldsymbol{k},j} \right) \right|^2 \delta\left(\omega - \omega_{\nu,\boldsymbol{k}}\right)$$

- W_j is Debye-Waller factor ~ 0
- Form factor for spin-dependent interaction to excite a phonon in branch ν with momentum q
- Dark matter that couples to nucleon number excites acoustic phonons most easily
- Dark matter that couples to electric charge (such as freeze-in benchmark) excites optical phonons in polar materials

Dynamic Structure Factor

It encodes the response of target under dark matter scattering

Scattering off a cold target in ground state

$$S(\mathbf{q},\omega) \equiv \frac{1}{N}\sum_{\lambda_f}\left|\sum_J g_J \left\langle \lambda_f \left| e^{-i\mathbf{q}\cdot\mathbf{r}_J} \right| 0 \right\rangle\right|^2 \delta\left(E_{\lambda_f}-\omega\right)$$

- Phonon comes into play through positions of ions

$$\mathbf{r}_J(t) = \mathbf{r}_J^0 + \mathbf{u}_J(t)$$

- Quantized displacement field $\mathbf{u}_J(t) \sim \sum_{\mathbf{q}} \frac{1}{\sqrt{2NM_J\omega_{\mathbf{q}}}}\left(\hat{a}_{\mathbf{q}}^\dagger \mathbf{e}_{\mathbf{q}}^* e^{i\omega_{\mathbf{q}}t} + \mathrm{h.c.}\right)$
- Expansion in $q^2/m_N\omega$ and in anharmonic phonon interactions

$$S(\mathbf{q},\omega) = (0-\mathrm{phonon}) + (1-\mathrm{phonon}) + (2-\mathrm{phonon}) + \cdots$$

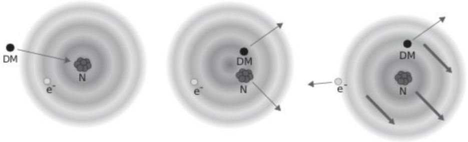
Theory of Dark Matter-Atom Interaction: Migdal Effect

Outline

The Migdal Effect

Definition

Ionization mechanism proposed by A.B.Migdal in 1941: emission of an atomic electron when the respective nucleus suddenly acquires a given velocity



Concept: the electron eigenstates for the moving nucleus:

$$\left|\Phi'_{ec}\right\rangle = e^{-im_e \sum_i \mathbf{v}\cdot\hat{\mathbf{x}}_i} \left|\Phi_{ec}\right\rangle$$

are not orthogonal to those for the initial nucleus at rest, therefore the transition probability between ground and ionized electron states $\mathcal{P} = \left|\left\langle \Phi^*_{ec} \mid \Phi'_{ec} \right\rangle\right|^2$ could be non-zero.

Migdal History

Publications by Arkady Migdal: http://www.itp.ac.ru/en/persons/migdal-arkady-beinusovich/

136. A. Migdal, *Ionizatsiya atomov pri α- i β-raspade*, ZhETF, 11, 207-212 (1941) [A. Migdal, *Ionization of atoms accompanying α- and β-decay*, J. Phys. Acad. Sci. USSR 4(1-6), 449-453 (1941)].

137. A. Migdal, J. Pomeranchuk, *Note on the Ends of the Mesotron Tracks Observed in an Expansion Chamber*, Phys. Rev. 57(10), 934-934 (1940), Scopus: 2-s2.0-36149008739.

138. A. Migdal, I. Pomeranchuk, *O kontse treka mezotrona v kamere Vil'sona*, Dokl. AN SSSR, 27, 652-653 (1940) [A. Migdal, J. Pomeranchuk, *Note on the ends of the mesotron tracks observed in an expansion chamber*, C. R. (Dokl.) Acad. Sci. URSS, n. Ser. 27, 652-653 (1940)].

139. A. Migdal, *Rasseyanie ionov v paramagnetikakh*, ZhETF, 10, 5 (1940).

140. A. Migdal, *Ionizatsiya atomov pri yadernykh reaktsiyakh*, ZhETF, 9, 1163-1165 (1939).

Migdal Effect

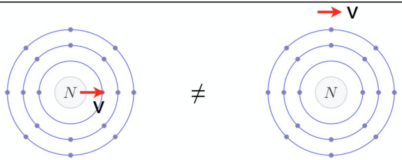
Ionization of an Atom in Nuclear Reactions

In nuclear collisions involving large energy transfer there must occur ionization of the recoil atoms. If the velocity acquired by the nucleus is not too large, then it can carry its electrons off with it, and ionization takes place only in the outer, weakly bound shells. For large velocities, on the other hand, the nucleus recoils right out of its electronic shells instead of carrying them with it.

We shall calculate the probability of ionization when a neutron collides with the nucleus (Migdal 1939). The duration of the neutron-nucleus collision is of order $\tau \sim R/v$, where R is the nuclear radius and v the neutron velocity. This time is much less than the electronic periods τ_{el}, so that the electron wave function is practically unchanged over the duration of the collision.

A.B. Migdal
"Qualitative Methods in Quantum Theory"
Advanced Book Classics CRC Press, 2000

Conventionally the recoiling nucleus is treated as a recoiling neutral atom however as described by Migdal in reality it takes some time for the electrons to catch up … and this may lead to atomic ionisation !



A long time ago, Migdal (1941, 1977) gave a rather simple formula for this ionisation probability. The transition probability P_i from an initial electron state i to a final electron state f is given by

$$P_i = |\langle \varphi_f(\boldsymbol{r})| \exp[\mathrm{i}(m_e/\hbar)\boldsymbol{v}_f \cdot \boldsymbol{r}]|\varphi_i(\boldsymbol{r})\rangle|^2$$

G. Baur, F. Rosel and D. Trautmann
"Ionisation induced by neutrons"
J. Phys. B: At Mol. Phys. 16 (1983) L419-L423

Formulation of Migdal Effect in Dark Matter

Migdal effect calculations is reformulated by Ibe et al. with ionization probabilities for atoms and recoil energies relevant to dark matter searches:

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Migdal effect in dark matter direct detection experiments

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Relation between Migdal and DM-electron Scattering

- **The transition probability from electronic state to electronic state $| i >$ is proportional to $| f >$ in DM-electron scattering,**

$$|\langle f|e^{i\boldsymbol{q}\cdot x}|i\rangle|^2$$

where $\boldsymbol{q}$ is the momentum lost by the dark matter particle.

- **The transition probability from electronic state to electronic state $| i\rangle$ is proportional to $| f\rangle$ in Migdal,**

$$|\langle f|e^{i\boldsymbol{q_e}\cdot x}|i\rangle|^2$$

where $q_e \sim (m_e/m_N)q$

Similarity helps!

Migdal Effect in Isolated Atoms

- **Bound, initial state of the electron:** $| n, l\rangle$
  n: **Principal quantum number,** l: **Orbital quantum number**
- **Positive energy final state in a continuum:** $|p_e, l'\rangle$
  p_e: **Final momentum,** l': **Final angular momentum,** E_e: **Final energy**

$$|\langle p_e, l'| \, e^{i\boldsymbol{q}_e \cdot x} |n, l\rangle|^2 = \frac{1}{2\pi} \frac{dp_{nl \to p_e l'}}{dE_e} = \frac{1}{4} \left| f_{nl}^{\text{ion}} \, (p_e, q_e) \right|^2$$

Ionization factor appears again

The Ionization Form Factor

Migdal Cross-Section

Factorization like soft photon emission

$$\frac{d^2\sigma_{n,l}}{dE_R\,dE_e} = \frac{d\sigma}{dE_R} \times \frac{1}{2\pi}\frac{dp_{n,l\to E_e}}{dE_e}$$

- First term is DM-nucleus elastic cross-section
- Second term is ionization factor
- In order to compare with electron scattering, exchange the integration order between d^3v and $dE_R = q\,dq/2m_N$

$$\frac{d\langle\sigma_{n,l}v\rangle}{d\ln E_e} = \frac{\bar{\sigma}_n}{8\mu_n^2}\left[f_p Z + f_n(A-Z)\right]^2 \int dq\,\Big[q\,|F_N(q)|^2 \times |F_{\rm DM}(q)|^2 \left|f_{nl}^{\text{ion}}(p_e,q_e)\right|^2 \eta\left(v_{\min}\left(q,\Delta E_{n,l}\right)\right)\Big]$$

Outline

Observation of the Migdal Effect

Migdal effect would make xenon experiments competitive to search for sub-GeV DM particle. However, the Migdal effect has not been confirmed experimentally yet.

- Rutherford Appleton Laboratory (RAL) in UK is currently developing an experiment to confirm the Migdal effect, funded by STFC (Xenon Future R&D project): MIGDAL experiment
- Besides RAL, other collaborating institutes are Imperial College (UK), LIP, CERN (Switzerland) and ICRR (Japan)
- Create a dedicated environment for an unambiguous observation with suppressed background
- Detect clearly the effect with energies available from using high flux n-generators creating high energy nuclear recoils

Collaboration in MIGDAL

CERN (GDD) F. Brunbauer, F. Garcia (HIP), E. Oliveri, L. Ropelewski, L. Scharenberg, R. Veenhof

Coimbra-LIP E. Asamar, I. Lopes, F. Neves, V. Solovov

Imperial College London H. Araujo, J. Borg, T. Marley, M. Nakhostin, T. Sumner,

King's College London C. McCabe

STFC (ISIS) C. Cazzaniga, C. Frost, M. Kastriotou; **(PPD)** S. Balashov, C. Brew, M. Van der Grinten, A. Khazov, P. Majewski; **(TD)** Project Engineer

Royal Holloway University of London A. Kaboth

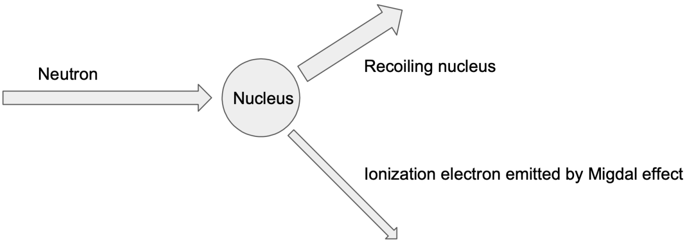
University of New Mexico D. Loomba

University of Oxford H. Kraus

University of Sheffield V. Kudryavtsev

Concept of Experiment

use neutrons to induce Migdal effect in atoms of a gas target contained in a tracking detector. Therefore signal consists of two tracks that start from the same vertex



Experimental Goal

Observation of two simultaneously created tracks of the ionization electron and the nuclear recoil originating from the same vertex

Ambitious Plan For the Experiment

Phase I (High profile with clear high-impact deliverables)

- Proof of principle of detecting low-energy NR and ER in low-pressure O-TPC
- Exposure to high intensity beam of neutrons and first ever observation of the effect

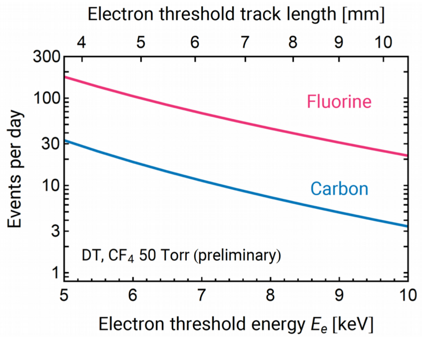
Phase II

- Study of the effect in various elements (gaseous mixtures) relevant to DM
- Study towards a first observation of the effect in LXe

Migdal Experiment Setup

- **Neutron source**: deuterium-tritium (DT) generator, producing 14.1 MeV neutrons at a rate of 10^{10} Hz
- **Target gas**: carbon tetrafluoride (CF_4) at low pressure (50torr) in order to have visible Migal electron tracks

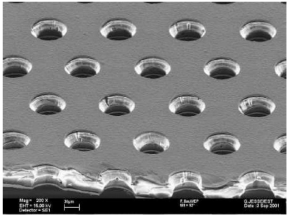
In a $10 \times 10 \times 10 \times 5\ \text{cm}^3$ active volume the rate of neutron scatter events is about $60\ \text{Hz}$, and the predicted signal rate is about $200\ \text{events/day}$

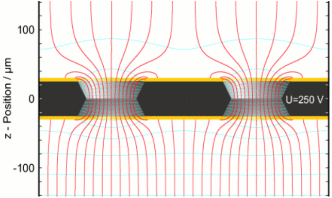


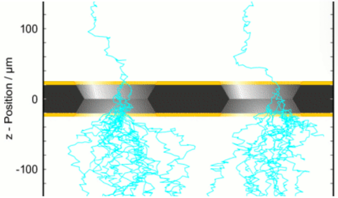
Migdal Detector

Tracking detector: optical time projection chamber (OTPC) that consist of:

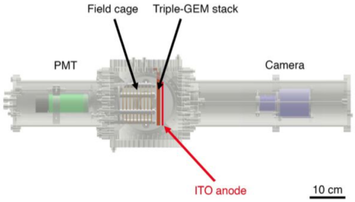
- A cathode
- Two consecutive gaseous electron multipliers (GEMs)
- An indium tin oxide (ITO) anode
- A camera

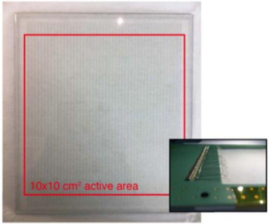




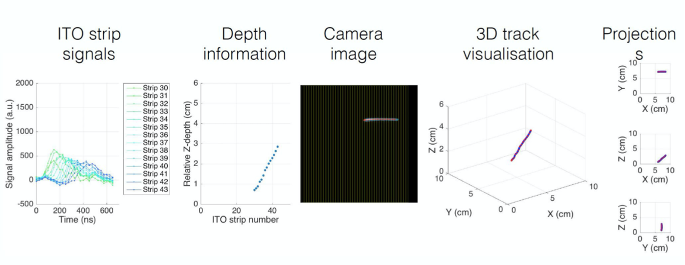


O-TPC at CERN (from F. Brunbauer)



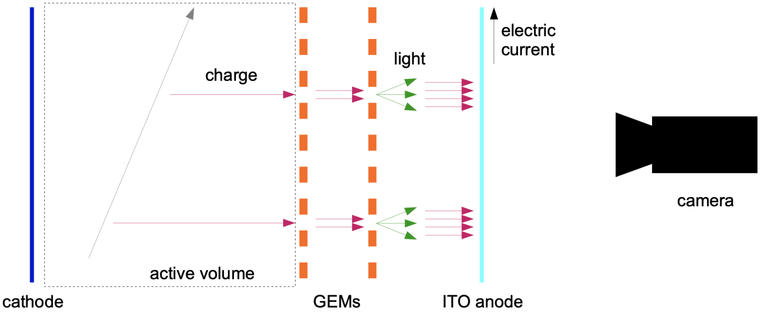


3D track reconstruction
in Ar/CF4 (80/20)
at 100 Torr

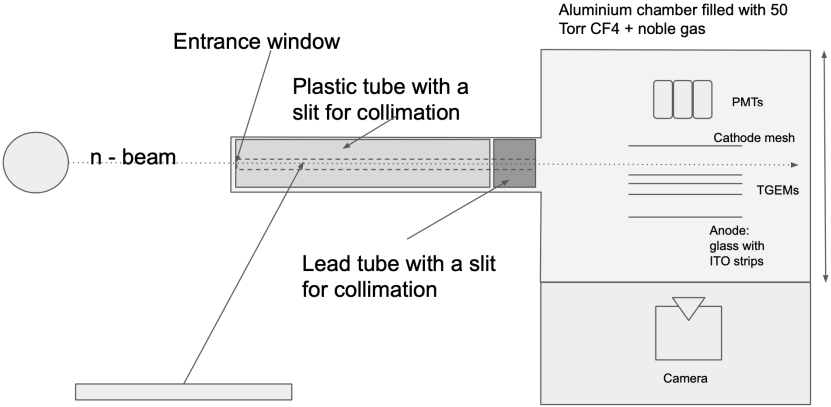


Detector Procedure-1

- Particles produce ionization that drifts to GEMs, and then electron multiplication produces scintillation light from CF_4
- Scintillation light is captured by the camera, and the amplified charge is collected at the ITO anode



Detector Procedure-2



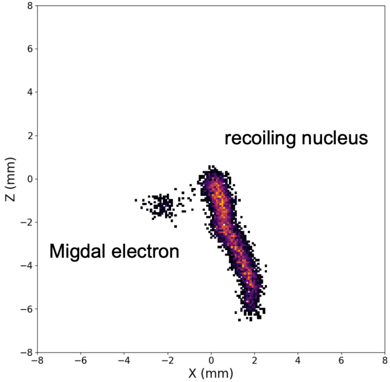
Slit in plastic and lead: 60 mm x 3 mm

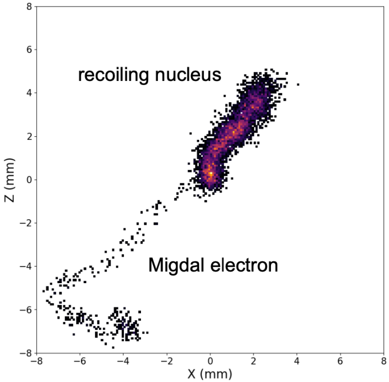
TGEM size: 100 mm x 100 mm

Number of stages to be optimised

Migdal Signal

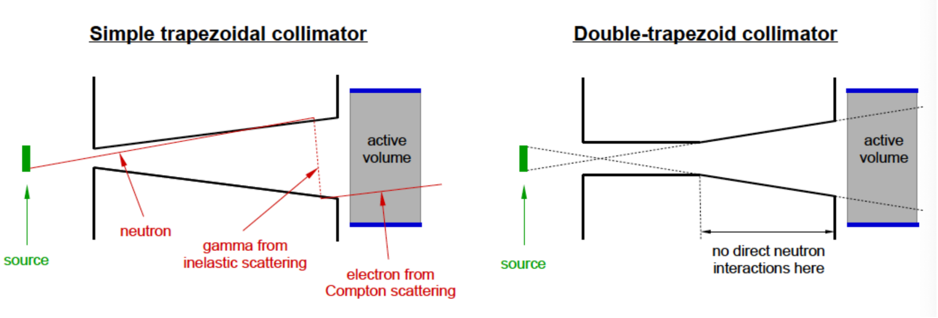
The image from scintillation light is a 2D projection of the event tracks, while the timing of the charge collected at the ITO anode provides depth information





Collimator

- DT generator produces neutrons isotropically → Experiment requires setup to define a neutron beam focused on the active OTPC volume: frond shield and collimator
- Optimization of front shield and collimator has been an important contribution from LIP-Coimbra
- **Front shield:** $70\ \mathrm{cm}$ **iron** $+\ 20\ \mathrm{cm}$ **borated polyethylene** $+10\ \mathrm{cm}$ **lead**
- Collimator: double-trapezoid tunnel with copper walls



Current Status

Conducting an extensive study to identified potential sources of backgrounds
The only relevant background source identified to date is random track coincidences, estimating $30\ \mathrm{events/day}$

- MIGDAL experiment expects to be able to confirm Migdal effect in CF_4 after 1 month of data taking
- Current status: starting construction, planning to operate before the end of 2021
- If confirmed, Migdal effect will be also studied in argon and xenon (using $Ar + CF_4$ and $Xe + CF_4$ mixtures), and in a lower. energy regime (using deuterium-deuterium neutrons)

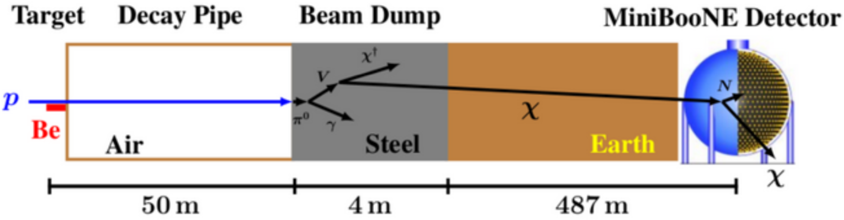
Challenge

- **Looking for rare event**: understanding of background and event topologies
- **Background from high rate of neutrons**: shielding and material choice
- **O-TPC operation at low pressure**: use of selected TGEMs and charge readout, operate at low diffusion and high light yield for accurate 3D track reconstruction
- **High event rate**: low noise camera with high rate readout capability
- **Data storage**: data transfer, temporary and permanent storage

Theory of Accelerated Dark Matter

Collider Experiments

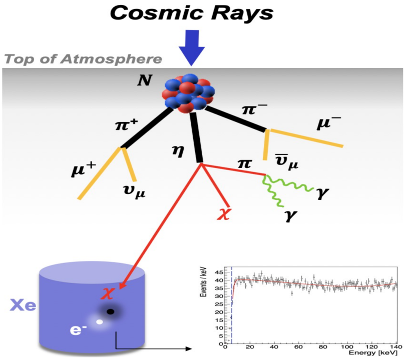
In traditional beam dump experiments like MiniBooNE, the accelerated protons are directed at a fixed target with a detector collinear with the beam.



Using the Atmoshere

In our scenario, the protons come from the cosmic rays, the flux of which is measured by balloon experiments like AMS.

Atmospheric DM (ADM)
The Atmosphere is a Beam Dump: Alvey etc



interstellar medium
or
atmosphere on Earth ✓

Inelastic Scattering

$$p+N \rightarrow Mesons \rightarrow ADM$$

We will consider then the interactions in the atmosphere and use the fat that the mesons produced have a non-zero probability to decay into dark matter particles

The Process

Two processes are related in our consideration:

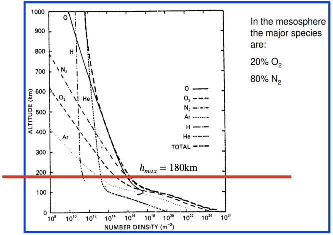
$$p + p \to \pi^0 \to \gamma + (A' \to \chi + \chi)$$

and

$$p + p \to \eta^0 \to \pi^0 + (S \to \chi + \chi)$$

In CRs pions π are produced 10 times more often than η-mesons, but its branching ratio into invisible is 10^4 times more constrained.

Step-1: Differential cosmic ray flux
function of kinetic energy and height



We start with the proton flux as measured by AMS, and simulate the collisions with nitrogen in the atmosphere using CRMC package.

- Cosmic-ray flux is attenuated by height as the cross section is practically constant over energies

$$\frac{\mathrm{d}\phi_p\left(T_p, h\right)}{\mathrm{d}T_p} = y_p(h)\frac{\mathrm{d}\phi_p\left(T_p, h_{\max}\right)}{\mathrm{d}T_p}$$

- Suppression factor is determined

$$\frac{\mathrm{d}y_p(h)}{\mathrm{d}h} = \sigma_{pN} n_N(h) y_p(h)$$

Step-2:Attenuation of dark matter flux

Earth-Scattering Effect in Direct Detection

Recall Direct Detection Formula

Take DM-Nucleus scattering as example

$$\frac{\mathrm{d}R}{\mathrm{d}E_R} = \frac{\rho_\chi}{m_\chi m_A} \int_{v_{\min}}^{\infty} v f(\mathbf{v}) \frac{\mathrm{d}\sigma}{\mathrm{d}E_R} \mathrm{d}^3\mathbf{v}$$

Astrophysics

Particle and nuclear physics

In addition, include all particles with enough speed to excite recoil of energy E_R:

$$v_{\min} = \sqrt{\frac{m_N E_R}{2\mu_{\chi N}^2}}$$

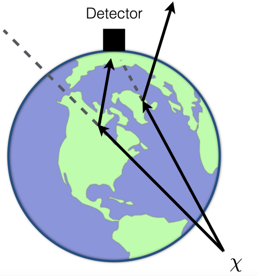
Astrophysics of DM: Velocity Distribution

Standard Halo Model (SHM) is typically assumed: isotropic. spherically symmetric distribution of particles with $\rho(r) \sim r^{-2}$. Leads to a Maxwell-Boltzmann (MB) distribution in the lab frame:

$$f_0(\mathbf{v}) = \left(2\pi\sigma_v^2\right)^{-3/2} \exp\left[-\frac{(\mathbf{v}-\mathbf{v}_\mathrm{e})^2}{2\sigma_v^2}\right] \Theta\left(|\mathbf{v}-\mathbf{v}_\mathrm{e}| - v_\mathrm{esc}\right)$$

- $\mathbf{v}_e$ **- Earth's velocity i.e.** $v_e \sim 220 - 250\ \mathrm{km\ s^{-1}}$
- $\sigma_v \sim 155 - 175\ \mathrm{km\ s^{-1}}$
- $v_\mathrm{esc} = 533^{+54}_{-41}\ \mathrm{km\ s^{-1}}$

Direct Detection of DM on Earth

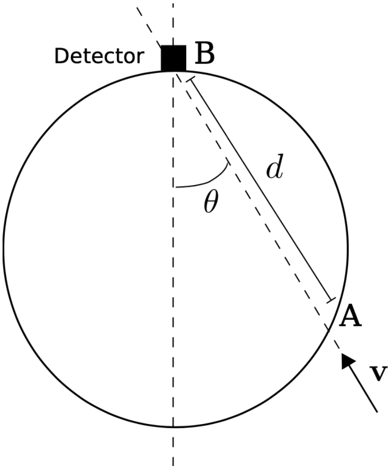


- If un-scattered (free) DM: $f_0(\mathbf{v})$
- Previous calculations usually only consider DM attenuation: $f(\mathbf{v}) \to f_0(\mathbf{v}) - f_A(\mathbf{v})$ i.e. attenuation of flux
- Consider deflection to make DM towards detector: $f(\mathbf{v}) \to f_0(\mathbf{v}) + f_D(\mathbf{v})$ i.e. deflection of flux

Total DM velocity distribution: $\tilde{f}(\mathbf{v}) = f_0(\mathbf{v}) - f_A(\mathbf{v}) + f_D(\mathbf{v})$

altered flux, daily modulation, directionality

Attenuation



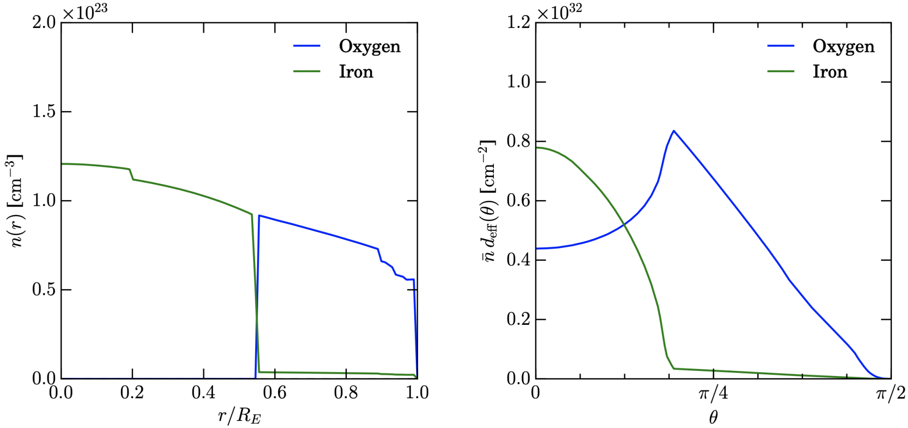
- incoming dark matter wind - $\mathbf{v} = (v, \cos\theta, \phi)$
- mean free path - $\bar{\lambda}_i(v)^{-1} = \bar{n}_i \sigma(v)$
- effective earth-crossing distance - $d_{\mathrm{eff},i} = \frac{1}{\bar{n}_i} \int_{\mathrm{AB}} n_i(\mathbf{r}) \mathrm{d}l$

Formula

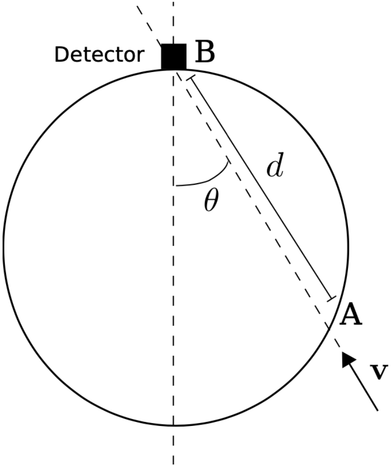
$$f_0(\mathbf{v}) - f_A(\mathbf{v}) = f_0(\mathbf{v}) \exp\left[-\sum_i^{\text{species}} \frac{d_{\mathrm{eff},i}(\cos\theta)}{\bar{\lambda}_i(v)} \right]$$

Effective Earth-crossing distance

Sum over 8 **most abundant elements in the Earth:** $\mathrm{O, Si, Mg, Fe, Ca, Na, S, Al}$. **Most scattering comes from Oxygen (in the mantle) and Iron (in the core).**



Kinematics of Deflection



- incoming dark matter wind - $\mathbf{v} = (v, \cos\theta, \phi)$
- mean free path - $\bar{\lambda}_i(v)^{-1} = \bar{n}_i \sigma(v)$
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Formula

$$f_0(\mathbf{v}) - f_A(\mathbf{v}) = f_0(\mathbf{v}) \exp\left[-\sum_i^{\text{species}} \frac{d_{\mathrm{eff},i}(\cos\theta)}{\bar{\lambda}_i(v)}\right]$$

Underground Scattering of DM particles

- If DM can scatter in a detector, they would also do so in the Earth
- Underground DM-nucleus scattering have two consequences:
  - spatial re-distribution of DM particles inside the Earth
  - deceleration of the DM particles
- For stronger DM-nucleus interactions, these two effects could influence the outcome of a DM detection experiment severely
  - daily/dirunal modulation of the signal rate
  - loss of sensitivity to strongly interacting DM

The Fundamental Random Processes

- **Initial Conditions**: Where does the particle start?
- **Free Distance**: Where does the particle scatter?
- **Target**: What does the particle scatter on?
- **Scattering Angle**: How does the particle scatter?

Repeat steps 2-4

These random processes/variables have to be simulated/sampled many times

Free Distance

- The CDF (CDF[dist, x] gives the cumulative distribution function for the distribution dist evaluated at x.) is given by

$$P(L) = 1 - \exp\left(-\int_0^L \frac{dx}{\lambda(x, v)}\right), \quad integratealongthepath$$

where we have used the mean free path,

$$\lambda(\mathbf{x}, v)^{-1} = \sum_i \lambda_i(\mathbf{x}, v)^{-1} \equiv \sum_i n_i(\mathbf{x})\sigma_{\chi i}$$

- To sample a specific free distance, we need a uniform random number ξ and solve

$$P(L) = \xi \in \mathcal{U}_{[0,1]} \to L = P^{-1}(\xi), \quad \text{inverse transform sampling}$$

- Can be complicated for (dis-)continuous changes of the medium

Scattering Angle

- The PDF (PDF[dist, x] gives the probability density function for the distribution dist evaluated at x.) of the scattering angle $\cos\theta$ is given by

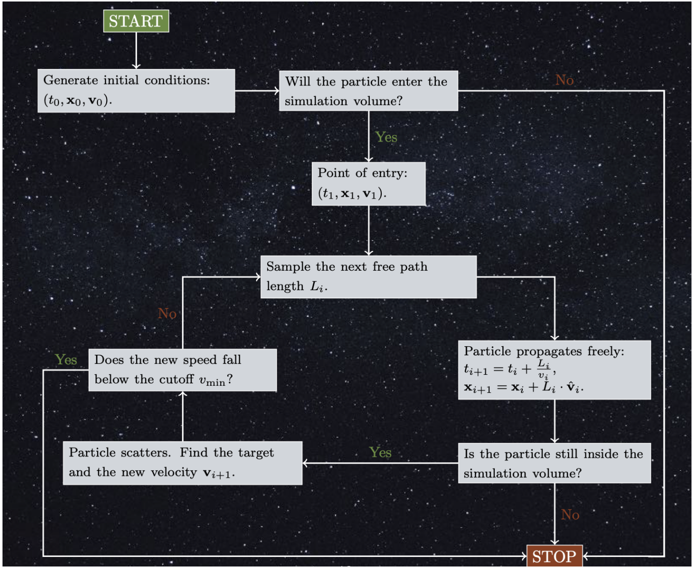
$$f_\theta(\cos\theta) = \frac{1}{\sigma_N}\frac{\mathrm{d}\sigma_N}{\mathrm{d}\cos\theta} = \frac{E_R^{\max}}{2\sigma_N}\frac{\mathrm{d}\sigma_N}{\mathrm{d}E_R},$$ particle physics input

  For isotropic contact interactions, this is simply $f_\theta(\cos\theta) = \frac{1}{2}$. For other interaction type, it can be more complicated

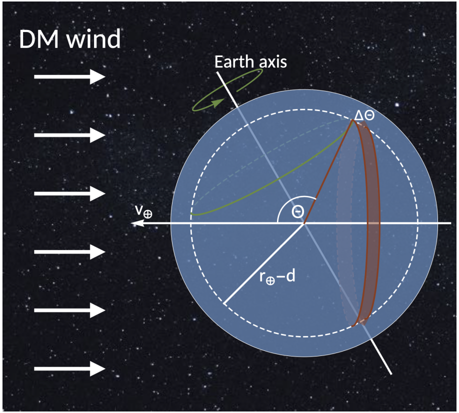
- This angles fixes the new DM velocity after a scattering

$$\mathbf{v}'_\chi \approx \frac{m_T v_\chi \mathbf{n} + m_x \mathbf{v}_\chi}{m_T + m_\chi} \quad (\mathbf{v}_T \approx \mathbf{0}),$$ Kinematics

Monte Carlo Simulation Algorithm



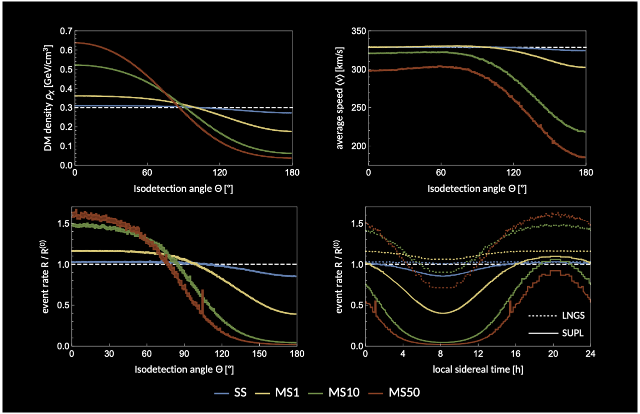
Isodetection Angle



- **How deep is a laboratory in Earth's shadow?**

DM Distribution inside the Earth

Diurnal Modulation Results



$m_\chi = 500\text{MeV}$

SS: $\sigma_p = 0.5\text{pb}$, **MS1:** $\sigma_p = 04.3\text{pb}$, **MS10:** $\sigma_p = 42.5\text{pb}$, **MS50:** $\sigma_p = 300\text{pb}$,

Direct Detection Constraints on Strongly Interacting DM

- DM detectors are typically underground to shield off background sources from the detector.
- Above some high critical cross-section, the overburden (Earth crust/atmosphere) shield off DM particles itself.
- Terrestrial experiments lose sensitivity to DM above this critical value
- Rare-event techniques are absolutely crucial: Importance Sampling, Importance Splitting.

The Dark Photon Model

- Extend the SM by a DM particle and a $U(1)$ gauge group with kinetic mixing

$$\mathcal{L}_D = \bar{\chi}\left(i\gamma^\mu D_\mu - m_\chi\right)\chi + \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + m_{A'}^2 A'_\mu A^\mu + \varepsilon F_{\mu\nu}F'^{\mu\nu}$$

- For kinetic mixing with the photon, the DM couples to electric charge

$$\frac{\mathrm{d}\sigma_N}{\mathrm{d}q^2} = \frac{\sigma_p}{4\mu_{\chi p}^2 v_\chi^2} F_{\mathrm{D}M}(q)^2 F_N(q)^2 Z^2$$

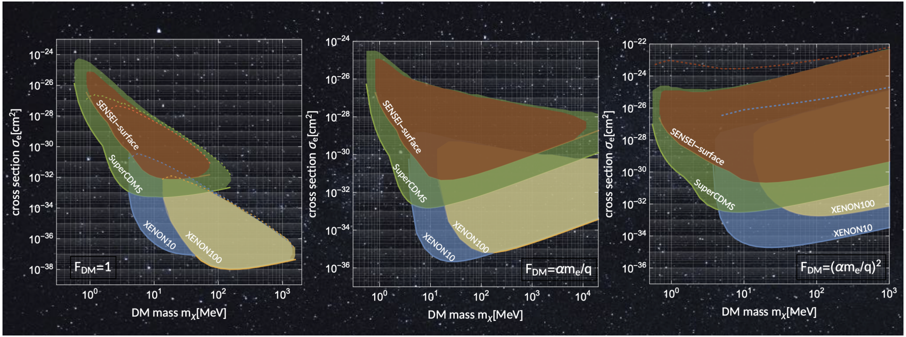
- Hierarchy between the DM-proton and DM-electron cross section

$$\frac{\sigma_p}{\sigma_e} = \left(\frac{\mu_{\chi p}}{\mu_{\chi e}}\right)^2$$

- DM form factor

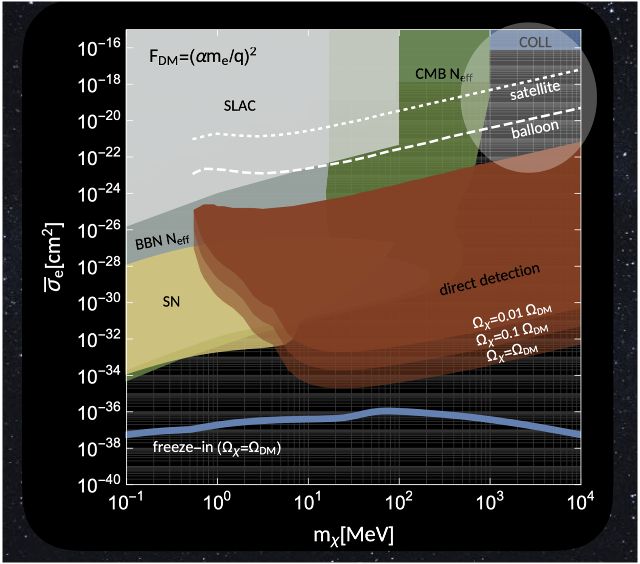
$$F_{\mathrm{DM}}(q) = \begin{cases} 1, & \text{for heavy mediator} \\ \frac{q_{\mathrm{ref}}}{q}, & \text{for ED interaction} \\ \left(\frac{q_{\mathrm{ref}}}{q}\right)^2, & \text{for light mediator} \end{cases}$$

Constraints on DM-Electron Scattering



Above these constraints, only direct detection experiments at high altitudes could probe strongly interacting DM.

Is there open parameter space for light mediators?



Open Science: DAMASCUS

The code is public: http://github.com/temken/

- DaMaSCUS is a MC simulator of dark matter particles as they move through the Earth and scatter on terrestrial nuclei.
- It allows to compute the local distortions of the DM density and velocity distribution caused by collisions with nuclei.
- The distorted distribution functions and redistributed densities are used to give precise estimates of time-dependent signal rates for direct detection experiments and diurnal modulations.
- A full, realistic model of the Earth is implemented as well as the Earth's time-dependent velocity and orientation in the galactic frame.
- DaMaSCUS is written in C++ and fully parallelized (MPI).

Getting Started

Dependencies: There are the dependencies of DaMaSCUS

- libconfig: To handle the input configuration files we use the textitlibconfig library.
  http://www.hyperrealm.com/libconfig/
- Eigen: DaMaSCUS relies heavily on this linear algebra C++ library
  http://eigen.tuxfamily.org/
- openMPI: For the parallelization we implemented our code using the open Message Passing Interface.
  https://www.open-mpi.org

The DaMaSCUS code is available at: https://github.com/temken/DaMaSCUS/

To download it via git simply run

```
git clone https://github.com/temken/DaMaSCUS/
```

Folder Structure

You will now find the following folders in your destination directory:

- /bin/: After successful compilation this folder contains two executables as well as the configuration file.
- /build/: This folder contains all object files. Both the object files and the executables in /bin/ are deleted via
  make clean
- /data/: Once a simulation run is performed, the generated data will be stored here
- /include/: The DaMaSCUS header files are stored here. Necessary 3rd party libraries can also be placed here.
- /plots/: To visualize the results, created by the analysis module we include the small Mathematica package DaMaSCUStoolbox and an example notebook creating and saving plots.
- /results/: The analysis module saves its results and histograms here.
- /src/: All the source code files of the two DaMaSCUS modules can be found here.

Installation

DaMaSCUS consists of two more ore less independent modules:

- DaMaSCUS-Simulator: Simulates the dark matter trajectories and generates the raw data.
- DaMaSCUS-Analyzer: Analyzes the raw data and calculates e.g. velocity histograms or detection rates.

The code is compiled using the Makefile. You might have to adjust the first lines

```
#Compiler and compiler flags
CXX := mpic++
CXXFLAGS := -Wall -std=c++11
LIB := -lconfig++
INC := -I include
(...)
```

to your local settings. Next to install DaMaSCUS and compile the code simply run Make

Using DaMaSCUS
Work Flow

To simulate dark matter trajectories and analyze the generated data DaMaSCUS has a clear work flow.

- Adjust your input parameter (such as the dark matter mass) inside the configuration file and assign a simulation ID to identify this simulation run.
- Run the simulation module to generate the raw data.
- Run the analysis module to process the data.
- The results can be plotted e.g. with the induced Mathematica notebook

Configuration File

You can find an example file at /bin/DaMaSCUS.cfg.
Here you adjust all the input parameter for the next DaMaSCUS run. We go through it block by block

The Idea of Solar Reflection of DM

- direct detection experiments can only probe DM masses above

$$m_\chi^{\min} = \frac{m_N}{\sqrt{\frac{2m_N}{E_R^{\text{thr}}}}v_{\max} - 1}$$

- Experimentalists have pushed this mass down by decreasing the target mass and the recoil threshold, e.g. the CRESST collaboration
- What if some process could increase the maximum DM speed somehow?

$$v_{\max} > v_{\text{esc}}^{(\text{gal})} + v_\oplus$$

- Elastic scatterings on hot solar targets inside the Sun could do just that.
H. An, M. Pospelov, J. Pradler, A. Ritz, Phys. Rev. Lett. 120 (2018), 141801

DM Scattering in the Sun

- **Scattering rate in a spherical shell of the Sun:**

$dS = d\Gamma$ **passing rate** $\times$ $dP_{\rm scat}$ **scattering probability** $P_{\rm shell}$ **prob to reach**

- **The three facts are given by**

$$\mathrm{d}\Gamma = 4\pi R^2\ \mathrm{d}\Phi_\chi = \pi n_\chi f_\chi(u) \frac{\mathrm{d}u\ \mathrm{d}J^2}{u}$$

$$\mathrm{d}P_{\textsf{scat}} = \frac{\mathrm{d}l}{w} \times \Omega(r, w)$$

$$P_{\textsf{shell}}(r) = P_{\textsf{surv}}(r, R_\odot)\left[1 + P_{\textsf{surv}}(r_p, r)^2\right]\Theta\left(w\left(u, r_p\right) r_p - J\right)$$

As opposed to Gould's formalism, this scattering rate applies not just to the single scattering regime.

A. Gould, Astrophys.J.321, (1987),571;Astrophys.J.321, (1987),560; Astrophys.J.388, (1992),338

Detection of Reflected Dark Matter

- DM flux of solar reflection

$$\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}v\ \mathrm{d}r} = \frac{\mathrm{d}S}{\mathrm{d}v\ \mathrm{d}r}\left\langle P_{\text{leave}}(v,r)\right\rangle_{J2}$$

- red-shift to the Earth's location

$$\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}u} = \int_0^{R_\odot} \mathrm{d}r\, \frac{\mathrm{d}\mathcal{R}}{\mathrm{d}v\ \mathrm{d}r}\,\frac{\mathrm{d}v}{\mathrm{d}u}\ \Big|_{v=\sqrt{u^2+v_{\mathrm{esc}}(r)^2}}$$

- Include into recoil spectrum

$$\frac{\mathrm{d}R}{\mathrm{d}E_R} = \frac{1}{m_N}\int_{u_{\min}(E_R)}^{\infty} du\left[\frac{\rho_\chi}{m_\chi} u f_\oplus(u) + \frac{1}{4\pi\ell^2}\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}u}\right]\frac{\mathrm{d}\sigma_N}{dE_R}$$

- Compute constraints as usual.
- Higher exposures probe lower masses
- Results are conservative (single scattering)

MC simulations

MC Simulations of DM Particles in the Sun

Extend the Earth simulations in four ways:

- Solar model: density, composition, temperature
- Gravity: Solve the equations of motion numerically
- Temperature: thermal motion of the solar targets
- Initial conditions: Generalize generation of ICs to account for gravitational focussing