暗物质

experimental evidence ("Dark Matter Problem")

1. galaxy (cluster) rotation curve

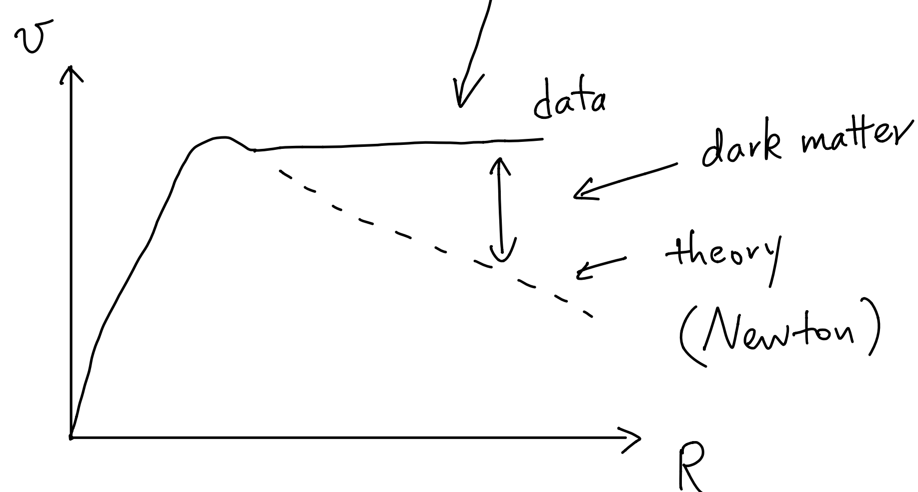
   1933 Fritz Zwicky → "dark matter"

   Coma Cluster

   1970 Rubin & Ford (optical)

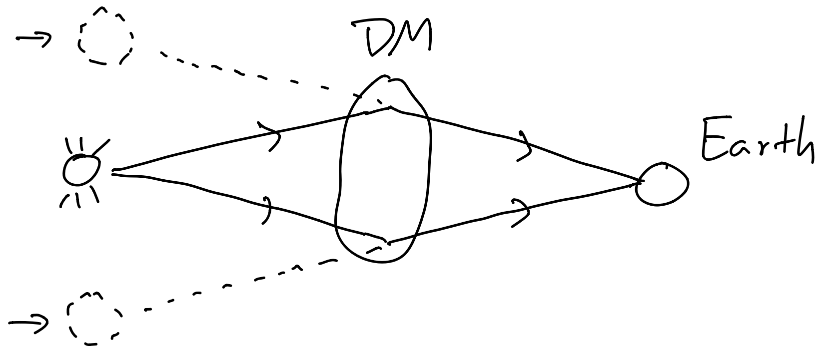
   galaxy → "flattening"

   21 cm M31

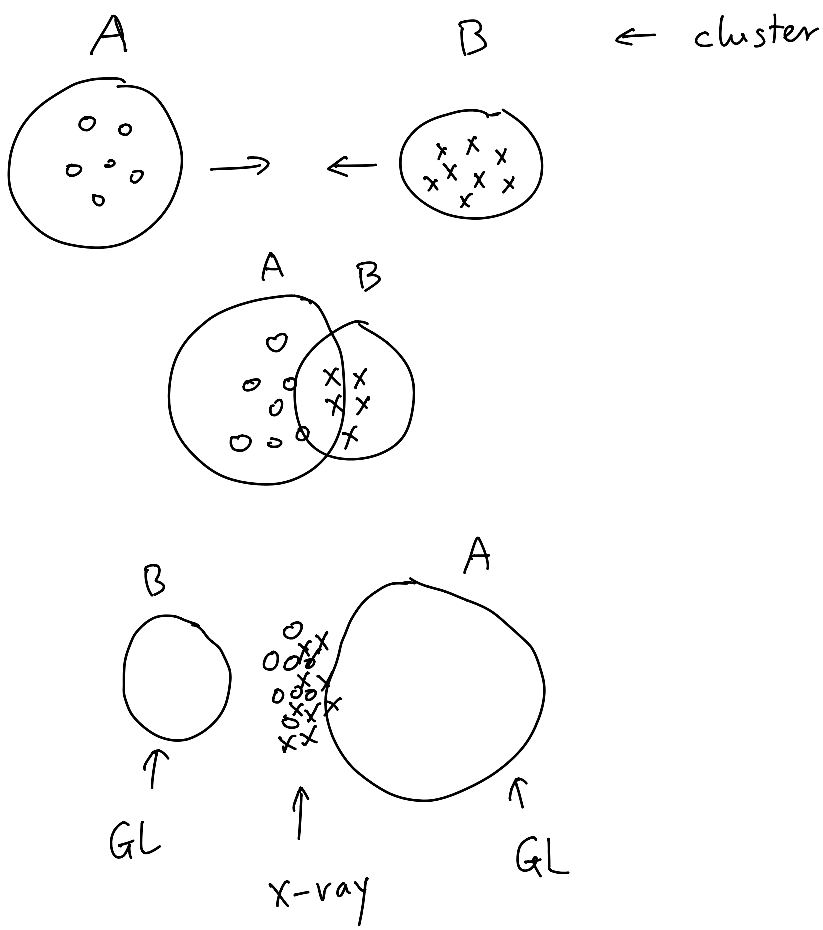


"Mond"

2 gravitational lensing $\equiv$ GL



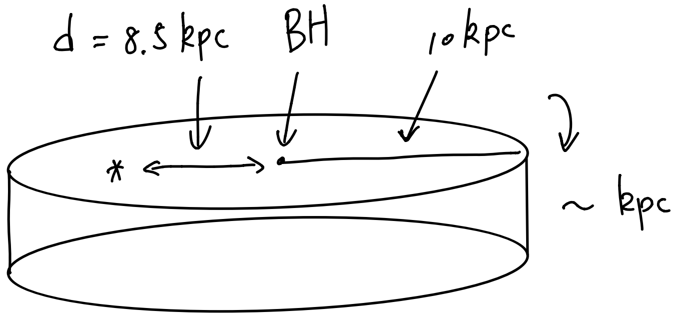
Bullet cluster

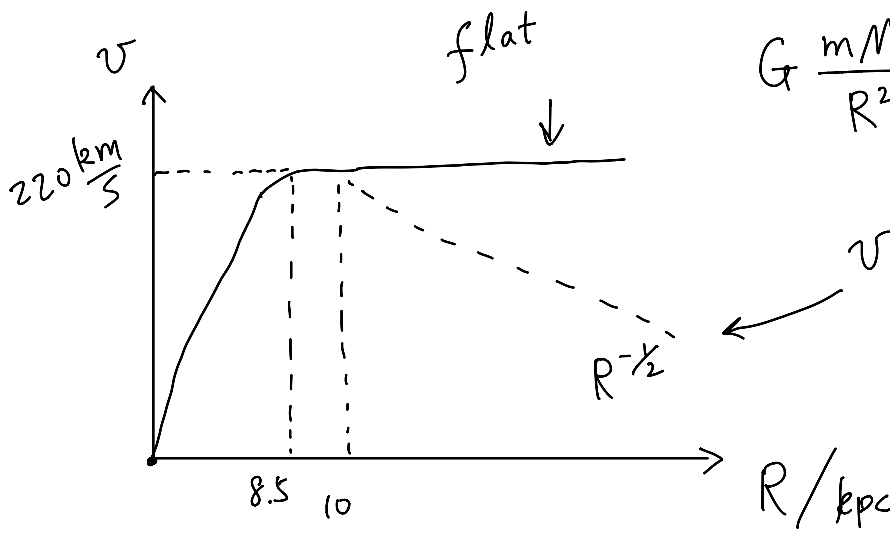


③ CMB = cosmic microwave backgroud ③

④ LSS = Large scale structure

MW galaxy





$$G\frac{mM}{R^2} = m\frac{v^2}{R}$$

$$v = \sqrt{\frac{GM}{R}}$$

$$M(R) \sim R \Rightarrow \rho_{DM} \sim \frac{M}{V} \sim R^{-2} \Leftrightarrow v$$

N-body simulation $\rightarrow \rho_{DM}$

NFW profile

$\swarrow$ DMID

$$\rho(r) \sim \frac{1}{r/r_s \, (1 + r/r_s)^2}$$

Exercise: Use $V_0 = 220\,\mathrm{km/s}$ & $d = 8.5\,\mathrm{kpc}$ to determine the local DM density $\rho_\odot^{DM}$

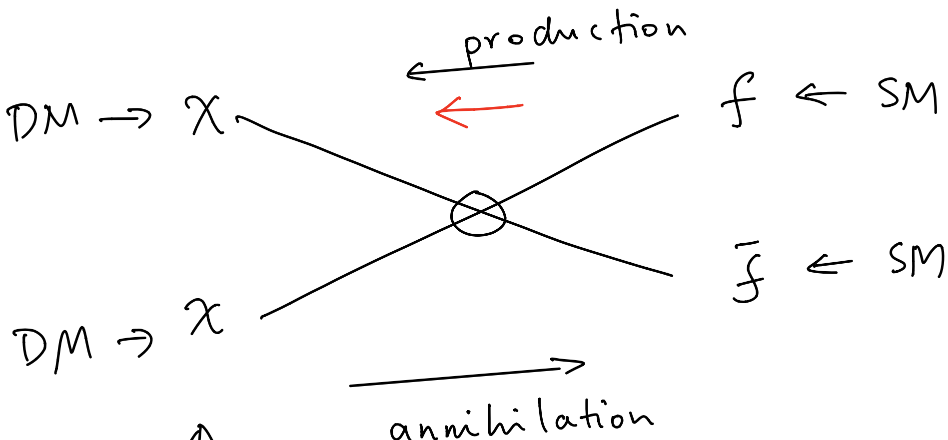
DM genesis

1. thermal freeze out
   decouple from the SM thermal bath
2. freeze in
3. asymmetric DM
4. axion $\Leftarrow$ misalignment
5. PBH
6. Forbidden DM, SIMP, ...

DM thermal freeze out

thermal bath : $\begin{cases} \text{SM particles} \\ \text{BSM} \quad \text{"} \quad \leftarrow \text{DM} \end{cases}$

1 Early Universe ($T \gg m$)

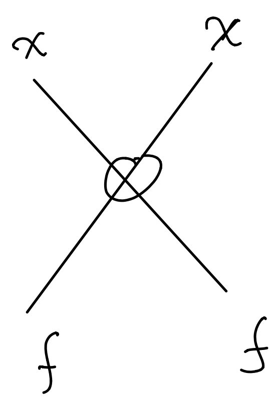


$\uparrow$

$\boxed{\text{equillibrium w/ thermal bath}}$

$\updownarrow$

σ



② Universe expand $\rightarrow T \downarrow$

$\rightarrow n_\chi \downarrow \rightarrow$ freeze out

cold DM : $T \ll m_\chi$

$$\boxed{\begin{aligned} T \gg m_\chi &: \quad n_\chi \sim T^3 \\ T \ll m_\chi &: \quad n_\chi \sim (m_\chi T)^{3/2} e^{-m_\chi/T} \end{aligned}}$$

Relativistic

energy density

$$\rho = \left[1, \frac{7}{8}\right] \frac{\pi^2}{30} g T^4$$

$\uparrow$ B $\downarrow$ $\quad$ $\uparrow$ F $\downarrow$

$$n = \left[1, \frac{3}{4}\right] \frac{\zeta(3)}{\pi^2} g T^3 \qquad \zeta(3) \approx 1.2$$

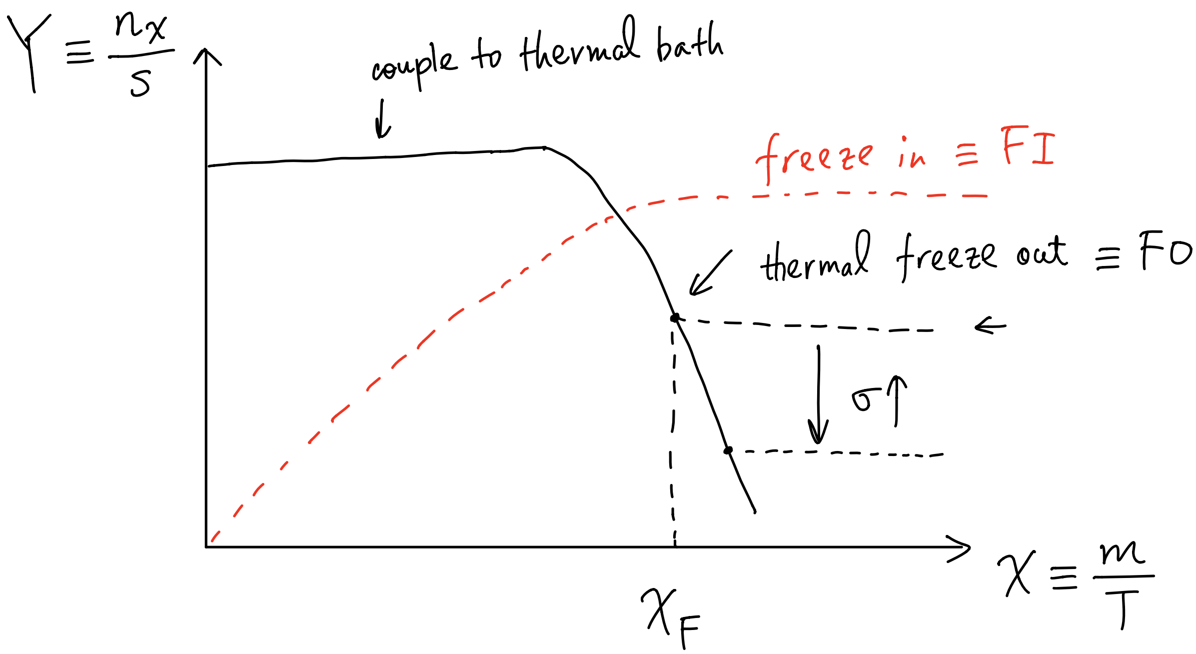
NR $\quad n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} \; ; \; \rho = mn$

entropy density $\quad p = \frac{1}{3}\rho$

$$s = \frac{\rho + p}{T} \xrightarrow{\text{Radiation}} \frac{4}{3}\frac{\rho}{T}$$

$$s = \frac{2\pi^2}{45}\left\{\sum_B g_B T_B^3 + \frac{7}{8}\sum_F g_F T_F^3\right\} \equiv \frac{2\pi^2}{45} g_{*s} T^3$$

$+ NR$



FO: $\sigma\uparrow \quad x_F\uparrow \quad T_F\downarrow \quad n_\chi\downarrow \quad \Rightarrow \quad n_\chi \sim \frac{1}{\sigma}$

FI: $\sigma\uparrow \quad n_\chi\uparrow \quad \Rightarrow \quad n_\chi \sim \sigma$

$$\Gamma_x \sim H$$

annihilate rate per DM particle

$$\Gamma_x = n_x \sigma v_x$$

$$H \cong 1.66 \sqrt{g_*} \frac{T^2}{M_{pl}}$$

freeze-out time : $\Gamma = H$

<u>Ex</u> neutrino decoupling

$$\left.\begin{aligned} &\Gamma \sim H \\ &\Gamma \sim n \sigma v \sim T^3 \left[G_F^2 T^2 \right] \\ &H \sim T^2 M_{pl}^{-1} \end{aligned}\right\}$$

$$T \sim \left[G_F^{-2} M_{pl}^{-1} \right]^{1/3} \sim \text{MeV}$$

Boltzmann Eq.

Ref: Kolb & Turner "Early Universe"

Process $\quad \chi + a + b + \cdots \to i + j + \cdots$

$$\dot{n}_\chi + 3Hn_\chi = -\int d\Pi_\chi \, d\Pi_a \, d\Pi_b \cdots d\Pi_i \, d\Pi_j \cdots$$
$$(2\pi)^4 \delta^4(p) \, |M|^2 \left[f_\chi f_a f_b \cdots - f_i f_j \cdots \right]$$

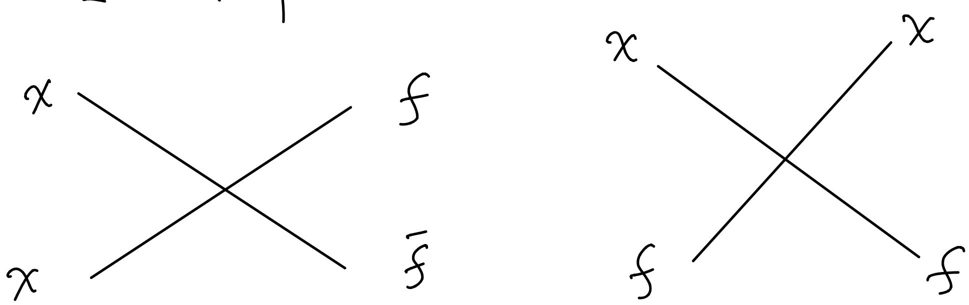
$|M|^2$ averaged over both initial & final states

$$d\Pi_i = \frac{g_i}{(2\pi)^3} \frac{d^3 p_i}{2E_i}$$

g_i = internal dof

if RHS $= 0$, $\quad n_\chi \sim a^{-3} \to$ conservation of # of particles in a comoving volume

A $2 \to 2$ process $(A + B \to C + D)$ 10



$$\dot{n}_A + 3Hn_A = -\int d\Pi_A\, d\Pi_B\, d\Pi_C\, d\Pi_D\, (2\pi)^4 \delta^4(p) |\mathcal{M}|^2 [f_A f_B - f_C f_D]$$

$$\sigma(\chi\chi \to f\bar{f}) = \frac{1}{2E_A 2E_B} \frac{1}{|v_A - v_B|} \int d\Pi_C\, d\Pi_D$$

$$* \ (2\pi)^4 \delta^4(p) \overline{|\mathcal{M}|^2} \frac{1}{g_C g_D}$$

$$\dot{n}_A + 3Hn_A = -\int d\Pi_A\, d\Pi_B \left[4E_A E_B \sigma v_{rel} \right] [f_A f_B - f_C f_D]$$

$$f_C = e^{-E_C/T}$$

$$f_D = e^{-E_D/T}$$

$$f_C f_D = \exp\left[-\frac{E_C + E_D}{T}\right] = \exp\left[-\frac{E_A + E_B}{T}\right] = f_A^{EQ} f_B^{EQ}$$

$$\dot{n}_A + 3Hn_A = -g_A g_B \int \frac{d^3p_A}{(2\pi)^3} \frac{d^3p_B}{(2\pi)^3} \sigma v_{rel} \left[f_A f_B - f_A^{EQ} f_B^{EQ} \right]$$

$$\langle \sigma v \rangle \equiv \frac{g_A g_B \int \frac{d^3p_A}{(2\pi)^3} \frac{d^3p_B}{(2\pi)^3} f_A^{EQ} f_B^{EQ} \left(\sigma v_{rel} \right)}{g_A g_B \int \frac{d^3p_A}{(2\pi)^3} \frac{d^3p_B}{(2\pi)^3} f_A^{EQ} f_B^{EQ}} \equiv \frac{D}{n_A^{EQ} n_B^{EQ}}$$

$$\boxed{\dot{n}_A + 3Hn_A = - \langle \sigma v \rangle \left[n_A n_B - n_A^{EQ} n_B^{EQ} \right]}$$

$\chi + \chi \to f + \bar{f}$

$A = B = \chi$ (self-conjugate DM) $\left\{ \begin{array}{l} \text{Mojorana DM} \\ \text{real scalar DM} \end{array} \right.$

$$\boxed{\dot{n}_\chi + 3Hn_\chi = -\langle \sigma v \rangle \left[n_\chi^2 - (n_\chi^{EQ})^2 \right]}$$

$\uparrow$ $\chi\chi \to f\bar{f}$ $\qquad$ $\uparrow$ $f\bar{f} \to \chi\chi$

Why $\Gamma = H$ sets freeze-out ?

$$3H \quad \longleftrightarrow \quad \sigma v\, n_\chi = \Gamma$$

annihilation rate per DM particle

Define $Y \equiv \dfrac{n_\chi}{s}$ w/ s = entropy density

$$\dot{Y} = \frac{\dot{n}_\chi}{s} + (-1)\frac{n_\chi}{s^2}\dot{s} = \frac{\dot{n}_\chi}{s} - \frac{n_\chi}{s^2}\left(-3\frac{\dot{a}}{a}s\right)$$

$$s a^3 = \text{constant} \Rightarrow \dot{s}a^3 + s\,3a^2\dot{a} = 0$$

$$\dot{Y} = \frac{\dot{n}_\chi}{s} + 3H\frac{n_\chi}{s} \Rightarrow s\dot{Y} = \dot{n}_\chi + 3H n_\chi$$

$$s\dot{Y} = -\langle \sigma v\rangle \left[n_\chi^2 - (n_\chi^{EQ})^2 \right]$$

Define $x \equiv \dfrac{m_\chi}{T}$ $\qquad H = 1.66\sqrt{g_*}\,\dfrac{T^2}{M_{pl}}$

Radiation dominated era $\qquad 2tH = 1$

$$\frac{dY}{dx} = \frac{-x}{H(m)s} \langle \sigma v \rangle \left[n_\chi^2 - (n_\chi^{EQ})^2 \right]$$

$$H(m) = 1.66 \sqrt{g_*} \frac{m_\chi^2}{M_{pl}}$$

$$\boxed{\frac{dY}{dx} = \frac{-xs}{H(m)} \langle \sigma v \rangle \left[Y^2 - Y_{EQ}^2 \right]}$$

Compare $\Gamma = n\sigma v$ & $H \rightarrow$ freeze out

$$\Gamma = n_\chi \sigma v = n_\chi \sigma_0 \quad \leftarrow \text{s-wave}$$

$$= g_\chi m_\chi^3 x^{-3/2} \frac{1}{\sqrt{2\pi}^3} e^{-x} \sigma_0 \qquad x \equiv \frac{m_\chi}{T}$$

$$H = 1.66 \sqrt{g_*} m_\chi^2 \frac{1}{M_{pl}} x^{-2}$$

$$\Gamma = H \Rightarrow$$

$$g_\chi m_\chi^3 x_f^{-3/2} \frac{1}{\sqrt{2\pi}^3} e^{-x_f} \sigma_0 = 1.66 \sqrt{g_*} m_\chi^2 \frac{1}{M_{pl}} x_f^{-2}$$

$$x_f = \ln \left[\frac{1}{1.66 \sqrt{2\pi}^3} \frac{g_\chi}{\sqrt{g_*}} m_\chi M_{pl} \sigma_0 x_f^{\frac{1}{2}} \right]$$

$$x_f \approx 20-25 \qquad n_\chi(x_f) = \frac{H(x_f)}{\sigma_0}$$

$$\frac{n_\chi(x_f)}{s(x_f)} = \frac{n_\chi(x_{today})}{s(x_{today})}$$

$$s = \frac{2\pi^2}{45} g_{*s} T^3 \qquad T_0 \cong 2.7\,K \qquad g_{*s} \cong 3.91$$

dark matter relic density

$$\Omega_\chi h^2 \equiv \frac{\rho_\chi}{\rho_c} h^2 = \frac{n_\chi m_\chi}{\rho_c} h^2$$

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

$$\Omega_\chi h^2 \cong 0.1 \frac{pb}{\sigma_0}$$

$$\boxed{\sigma_0 = 1\ pb}$$

$$\frac{dY}{Y^2} = \frac{-xs}{H(m)} \langle \sigma v \rangle \, dx$$

$$\boxed{\frac{1}{Y(x)} = \frac{1}{Y(x_f)} + \sqrt{\frac{\pi}{45}} \frac{g_{*s}}{\sqrt{g_*}} m_\chi M_{pl} \int_{x_f}^{x} \frac{\langle \sigma v \rangle}{x^2} dx}$$

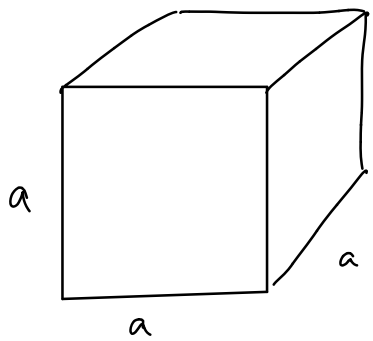
Griest & Seckel 1991

1. co-annihilation

   ($\tilde{\tau}$, $\tilde{g}$, $\tilde{t}$ ...)

2. threshold

3. resonance



$$V = a^3$$

$$N_\chi = n_\chi a^3$$

$$\chi + \chi \leftrightarrow f + \bar{f}$$

$$\frac{dN_\chi}{dt} = -\Gamma N_\chi \frac{2}{2} \qquad \Gamma = n_\chi \sigma v$$

$$\frac{d(na^3)}{dt} = -n_\chi^2 \sigma v\, a^3$$

$$\dot{n}_\chi a^3 + 3a^2 \dot{a}\, n_\chi = -a^3 n_\chi^2 \sigma v$$

$$\boxed{\dot{n}_\chi + 3H n_\chi = -\sigma v \left[n_\chi^2 - (n_\chi^{EQ})^2 \right]}$$

$$H = \frac{\dot{a}}{a}$$

Majorana DM $\quad \sigma v \sim pb \sim 3 * 10^{-26} \frac{cm^3}{s}$

Dirac DM $\quad \sigma v \sim 2\,pb$

$$n_{DM} = n_\chi + n_{\bar{\chi}}$$

Freeze-in

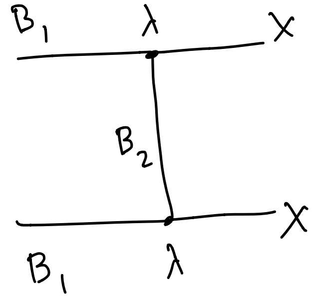
Ex : $\lambda \; \chi \; B_1 \; B_2$

χ ← DM

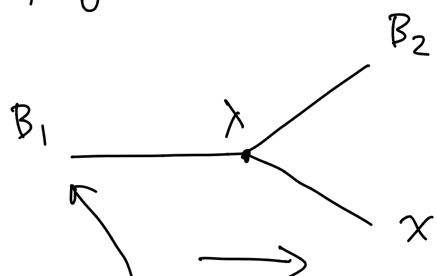
λ ← small coupling

B_1, B_2 ← Bath particles

$B_1 \to B_2 + \chi$



long-lived particle ←



$$\frac{dY}{dx} = \frac{x}{H(m)s} \int d\Pi_1 \, d\Pi_2 \, d\Pi_\chi \, (2\pi)^4 \delta^4(p) |\mathcal{M}|^2 \left[f_1 - \underbrace{f_2 f_\chi}_{=0} \right]$$

initial # density of $\chi \to 0$

$$\frac{dY}{dx} = \frac{x}{H(m)s} \underbrace{\int d\Pi_1 \, \underbrace{d\Pi_2 \, d\Pi_\chi \, (2\pi)^4 \delta^4(p) |\mathcal{M}|^2}_{\Gamma(B_1 \to B_2 + \chi) \equiv \Gamma_1} f_1}_{\frac{1}{2\pi^2} g_1 m_1^2 \Gamma_1 T K_1\left(\frac{m_1}{T}\right)}$$

$$\begin{cases} \Omega_\chi h^2 \cong 0.1 \\ \Omega_\chi h^2 \cong 10^{27} \dfrac{g_1}{g_{*S}\sqrt{g_*}} \dfrac{m_\chi \Gamma_1}{m_1^2} \end{cases}$$

$$\lambda \cong 10^{-13} \sqrt{\frac{M_1}{m_\chi} \frac{g_*^{3/4}(m_1)}{g_{bath}^{1/2}}}$$