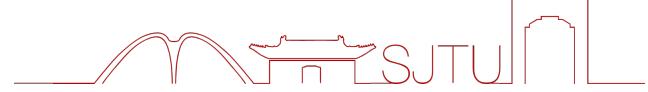




上海交通大学
SHANGHAI JIAO TONG UNIVERSITY



精确计算与味物理 提纲

王伟

上海交通大学

2022.7.5

饮水思源 · 爱国荣校



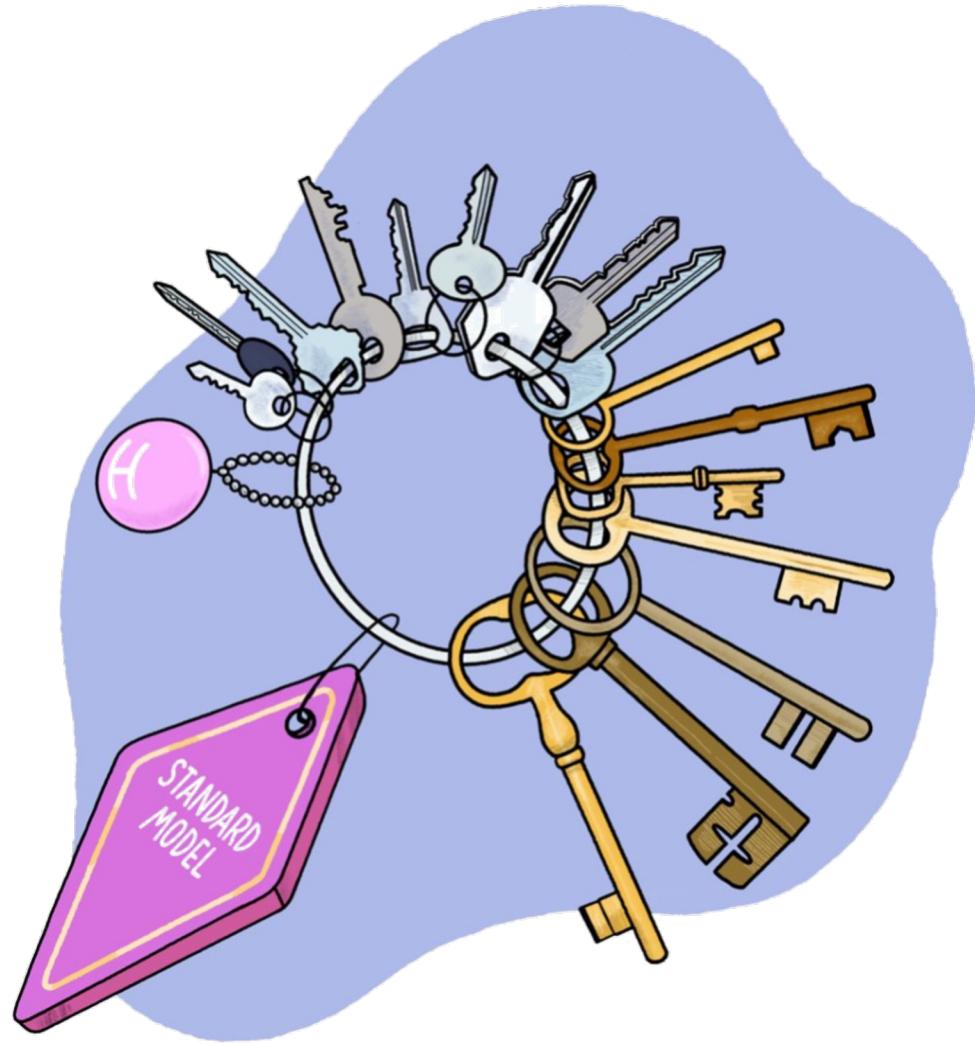
Outline



- ✓ 标准模型
- ✓ 重味夸克
- ✓ 弱衰变
- ✓ CKM +CP violation
- ✓ New Physics

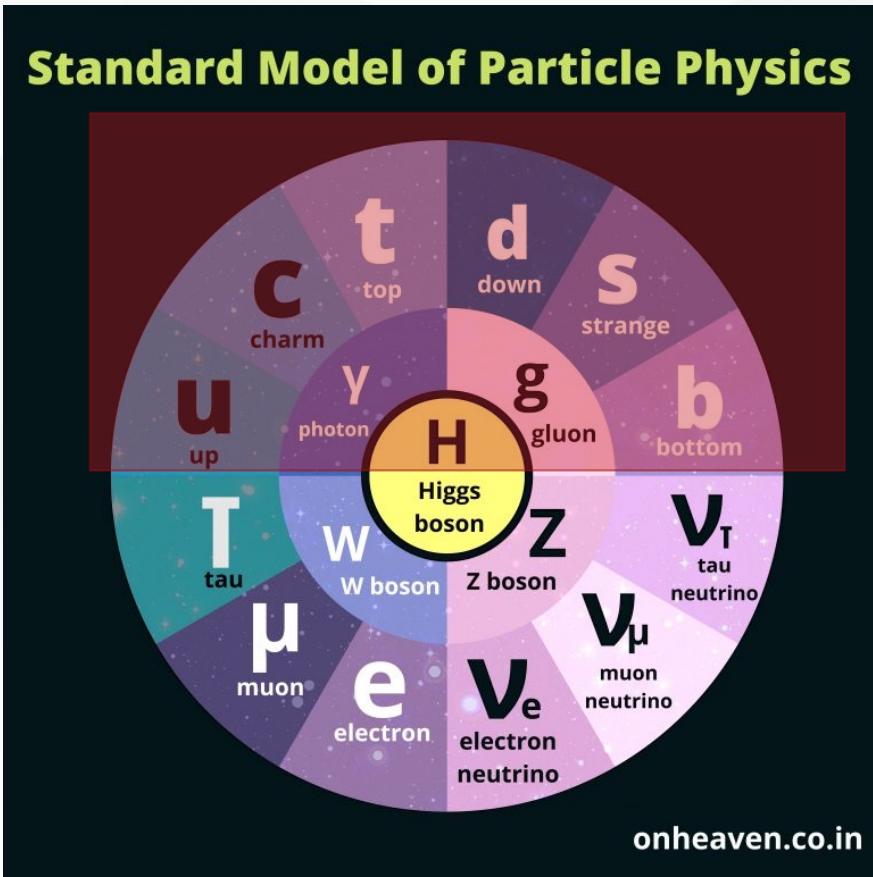
- ✓ Lattice QCD
- ✓ Ξ_c 半轻衰变

标准模型

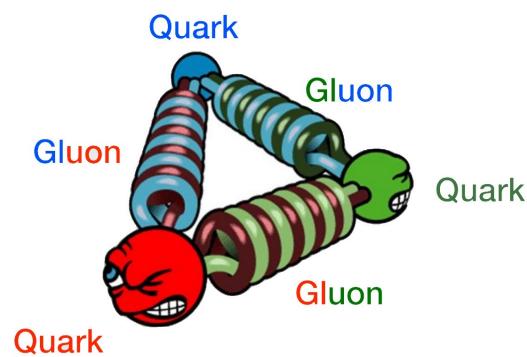




The Standard Model (SM)

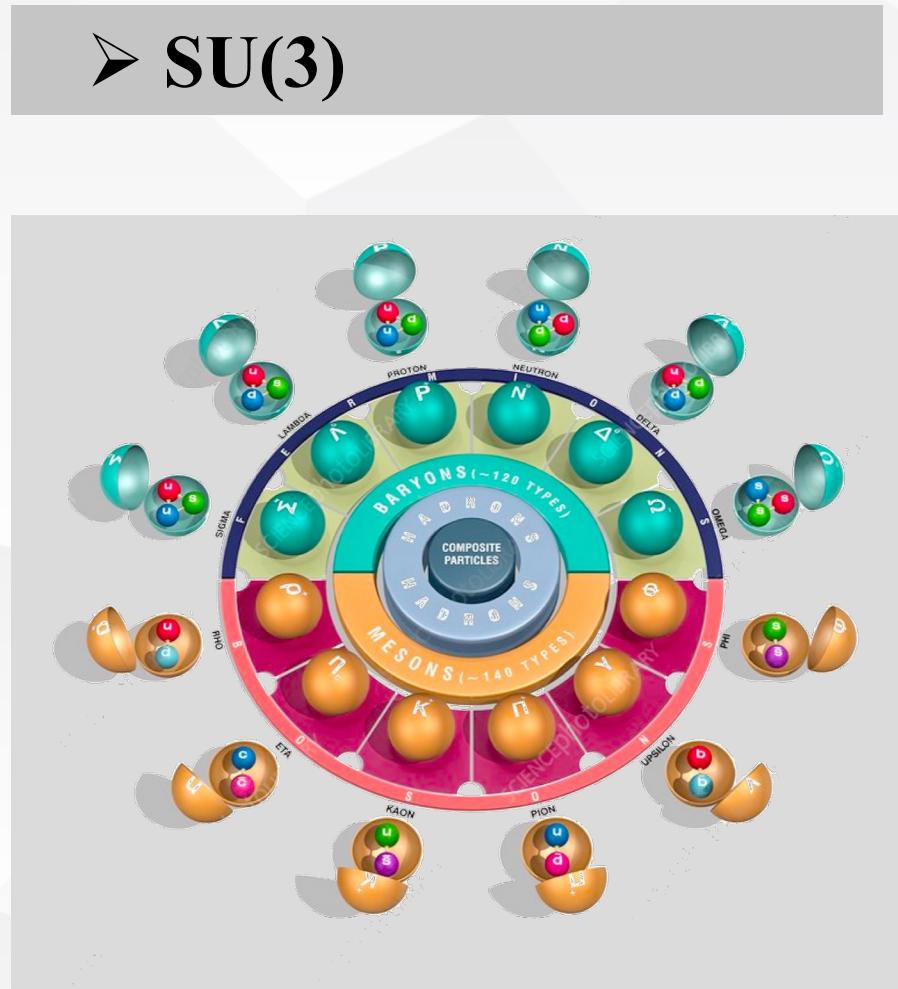
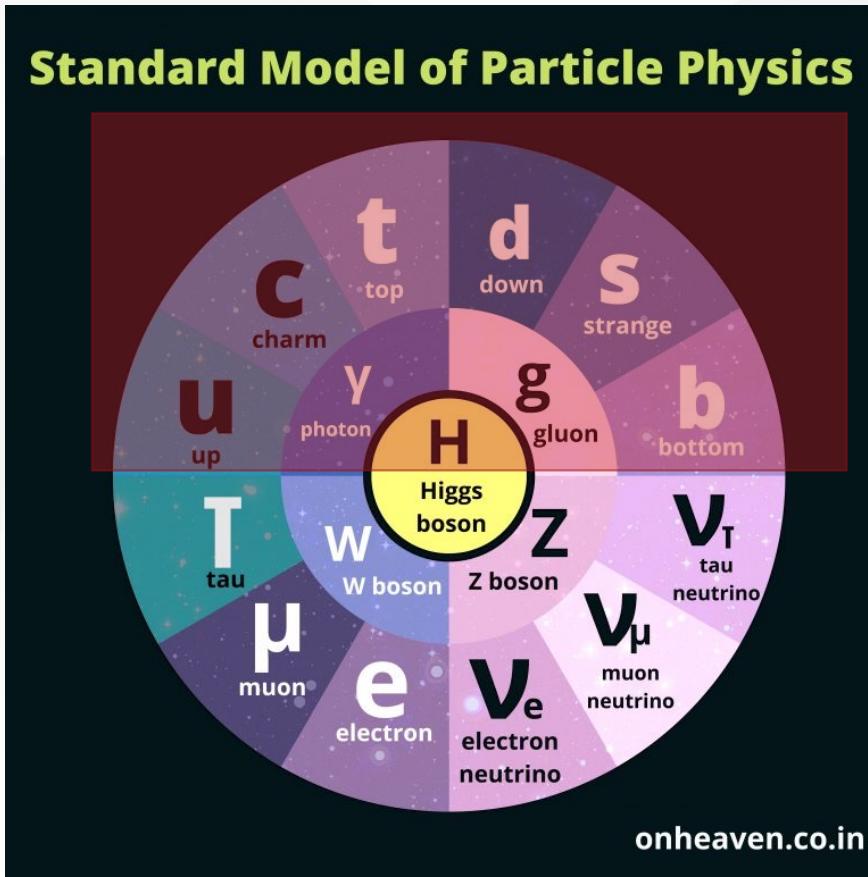


➤ SU(3)



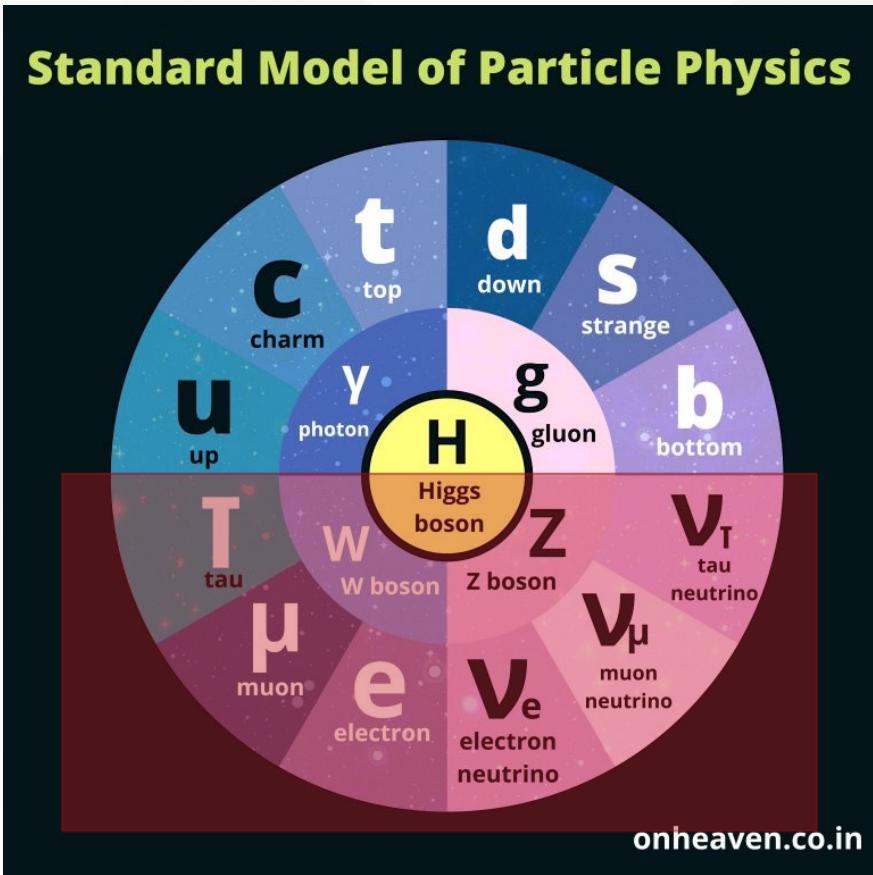


The Standard Model (SM)



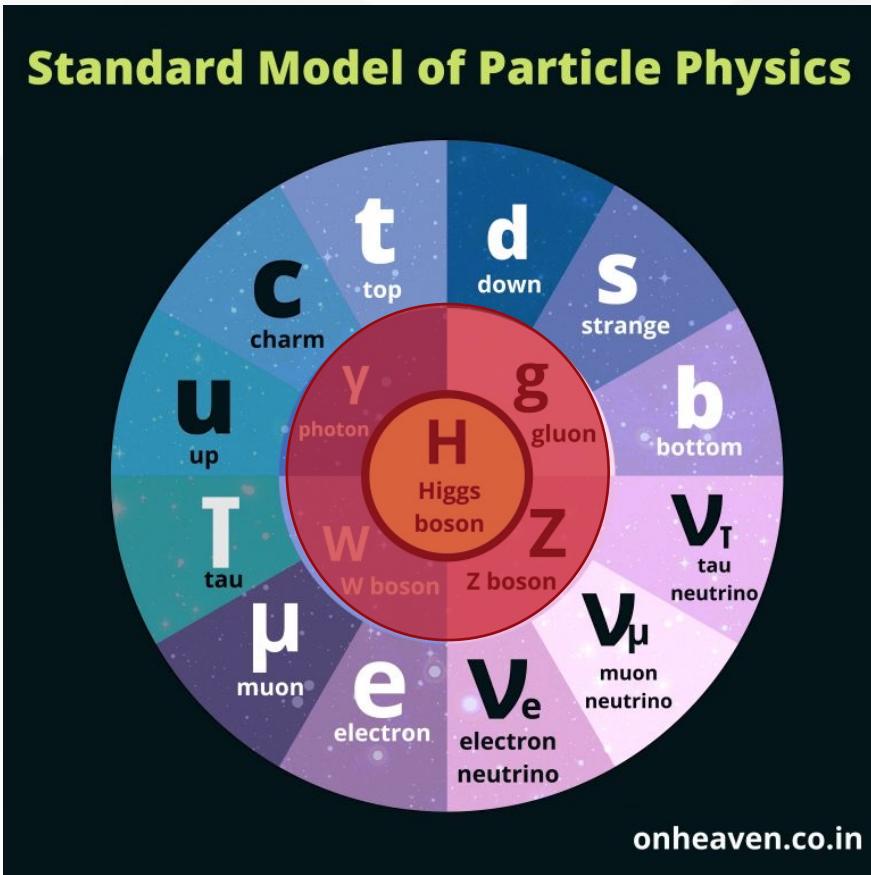


The Standard Model (SM)

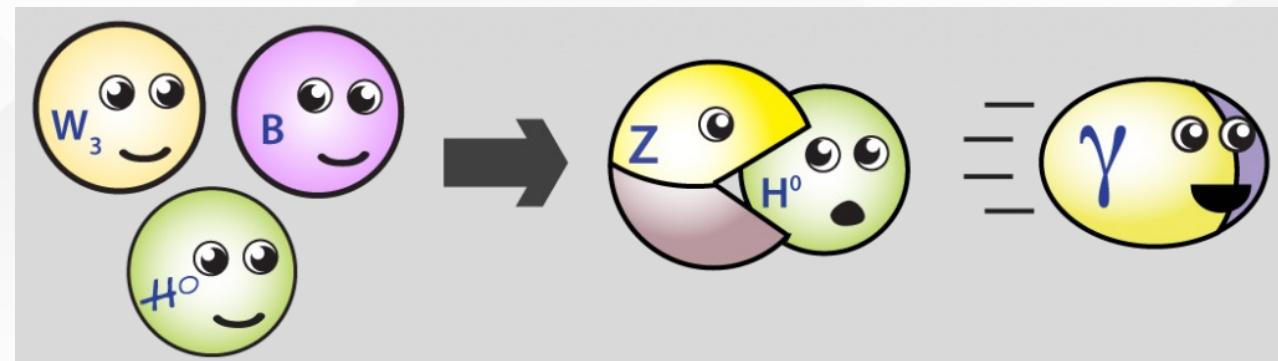


- $SU(2) \times U(1)$
- Fermion:
 - ✓ Quark
 - ✓ Lepton

The Standard Model (SM)



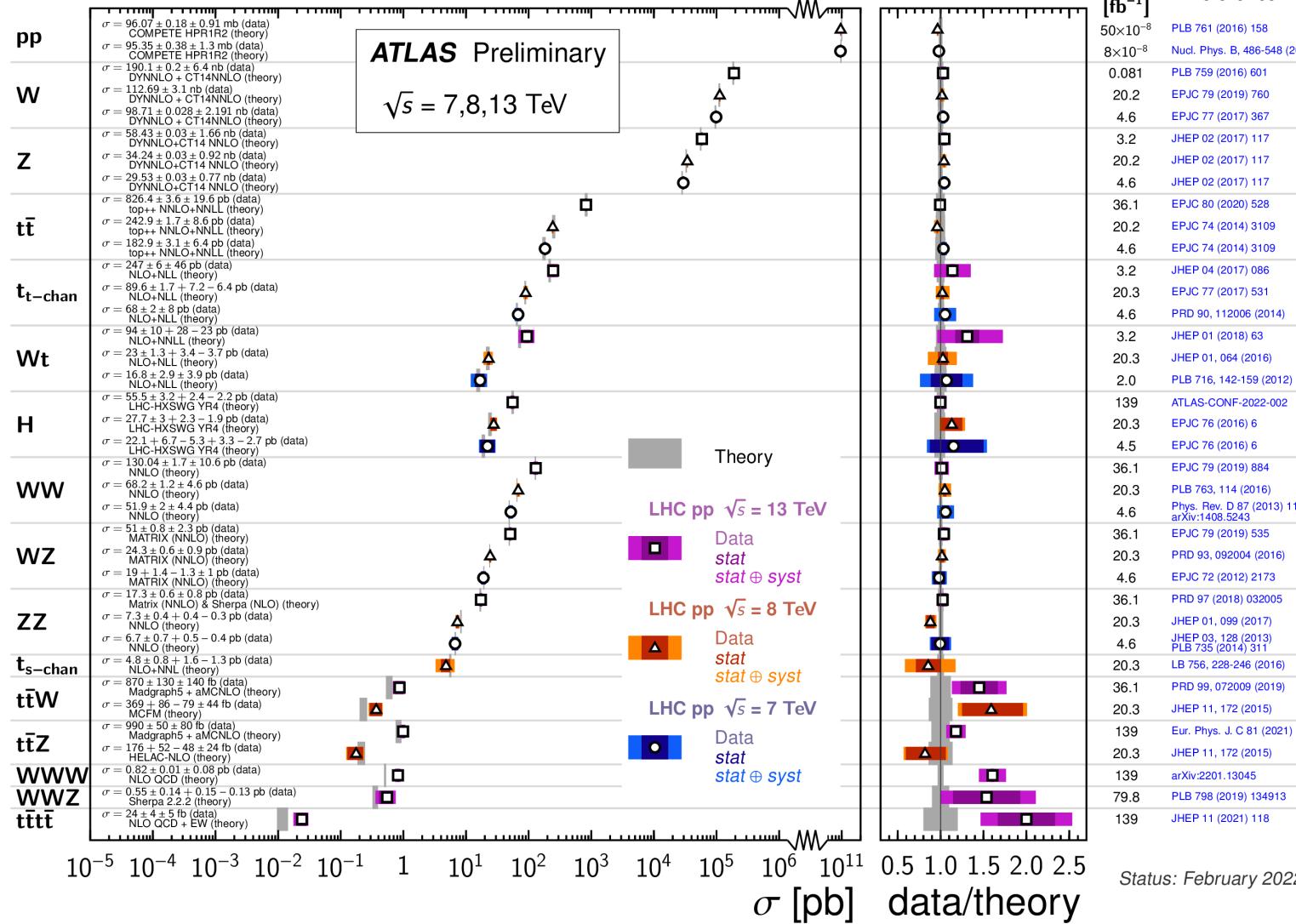
- $SU(2) \times U(1)$
- Fermion:
 - ✓ Quark
 - ✓ Lepton



Success of SM

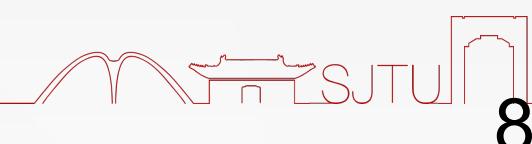


Standard Model Total Production Cross Section Measurements



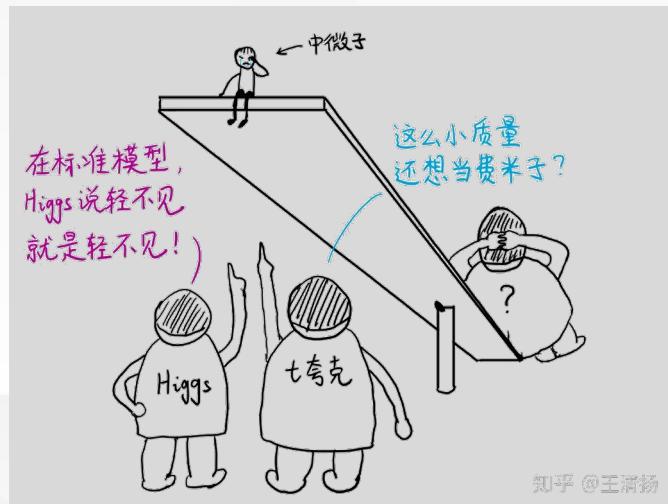
Success:

- ✓ Electroweak
- ✓ QCD

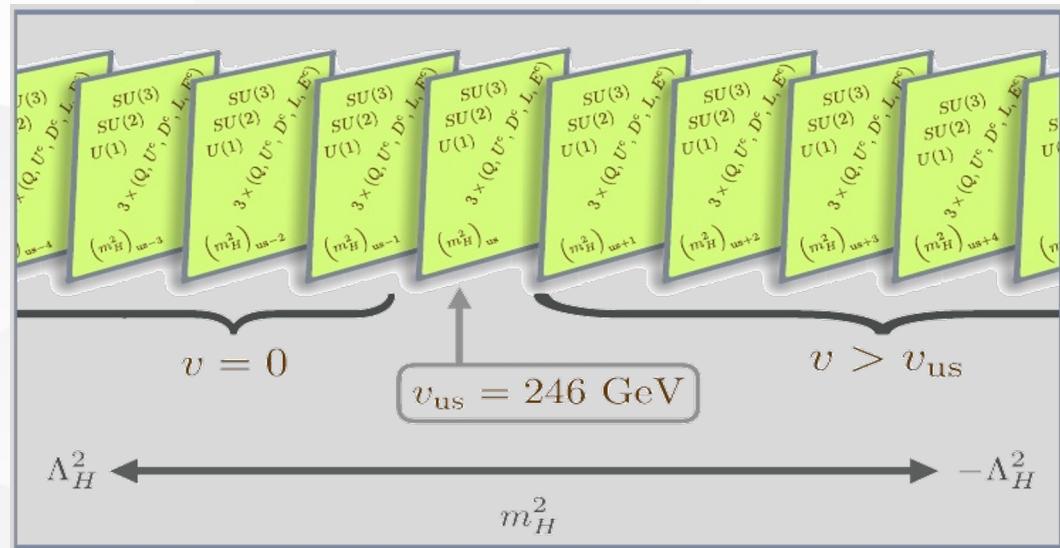




- ✓ Neutrino: Mass, Dirac or Majorana Fermion
- ✓ Dark Matter: 27%
- ✓ Dark Energy



- ✓ Hierarchy problem?
- ✓ Landau pole vs Triviality
- ✓ ...



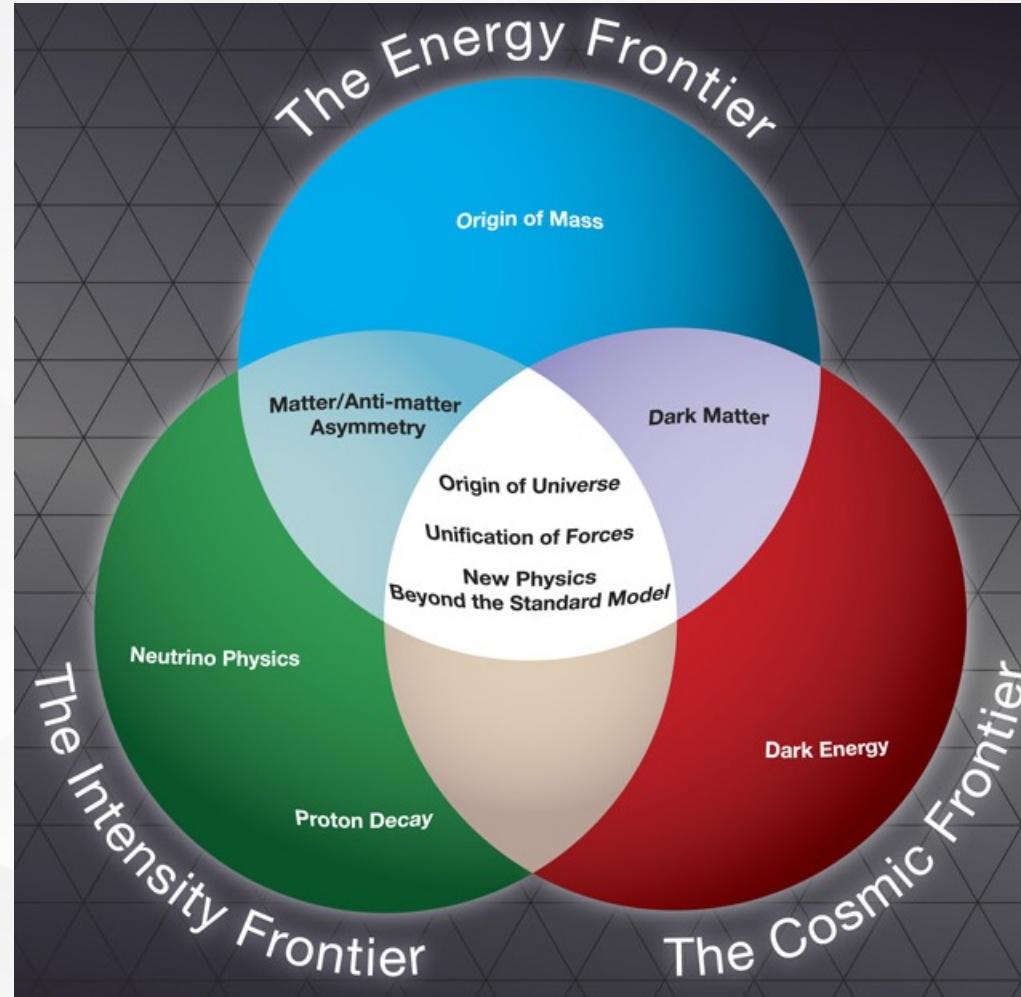


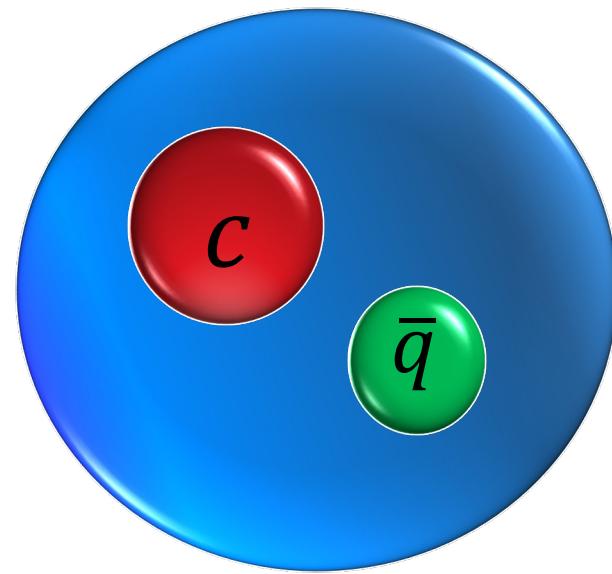
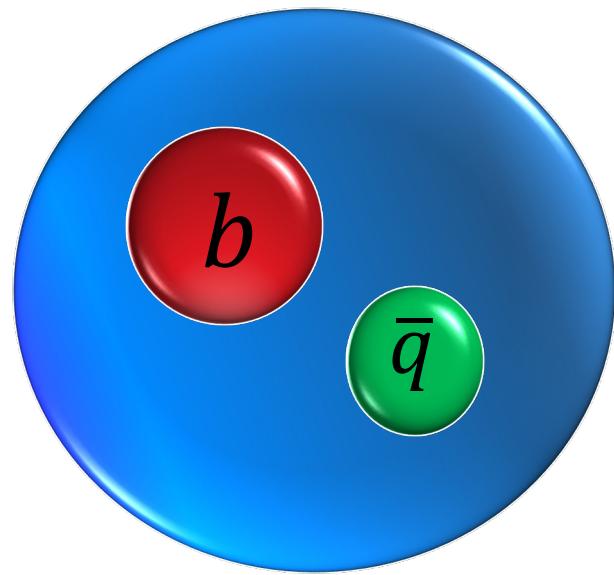
Beyond SM: Three Frontiers



Direct search:
LHC

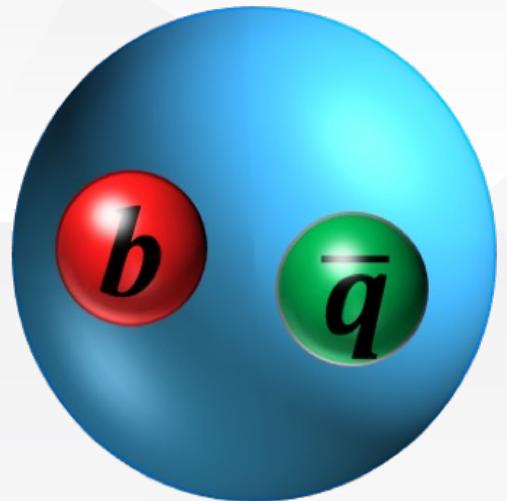
indirect search:
 $g-2$
Heavy quark
Tau/charm factory







Heavy Quark Physics



Heavy quark:
✓ Bottom
✓ Charm

u

$$I(J^P) = \frac{1}{2}(\frac{1}{2}+)$$

$$m_u = 2.16^{+0.49}_{-0.26} \text{ MeV} \quad \text{Charge} = \frac{2}{3} e \quad I_z = +\frac{1}{2}$$
$$m_u/m_d = 0.47^{+0.06}_{-0.07}$$

d

$$I(J^P) = \frac{1}{2}(\frac{1}{2}+)$$

$$m_d = 4.67^{+0.48}_{-0.17} \text{ MeV} \quad \text{Charge} = -\frac{1}{3} e \quad I_z = -\frac{1}{2}$$
$$m_s/m_d = 17-22$$
$$\bar{m} = (m_u+m_d)/2 = 3.45^{+0.55}_{-0.15} \text{ MeV}$$

s

$$I(J^P) = 0(\frac{1}{2}+)$$

$$m_s = 93^{+11}_{-5} \text{ MeV} \quad \text{Charge} = -\frac{1}{3} e \quad \text{Strangeness} = -1$$
$$m_s / ((m_u + m_d)/2) = 27.3^{+0.7}_{-1.3}$$

c

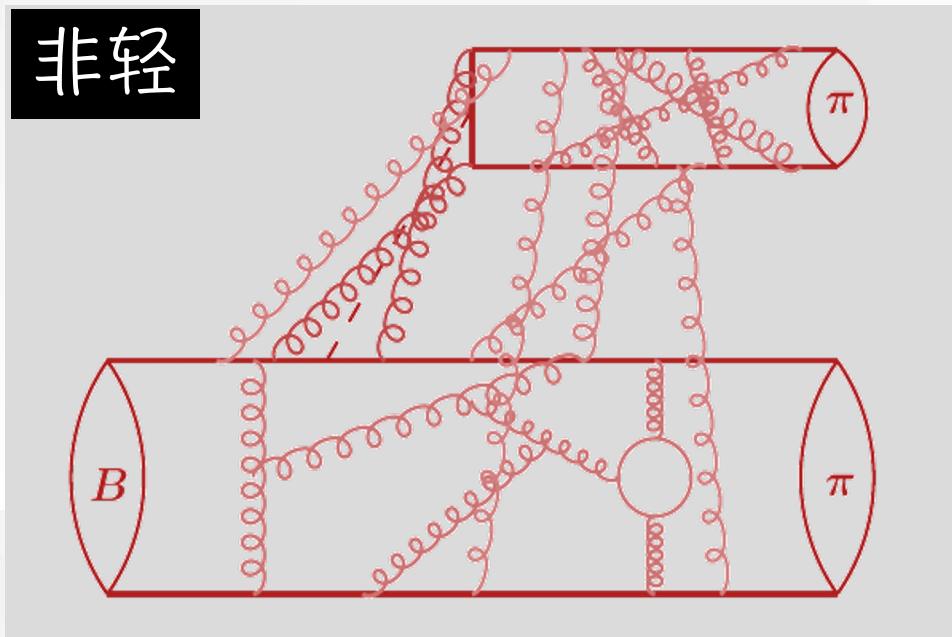
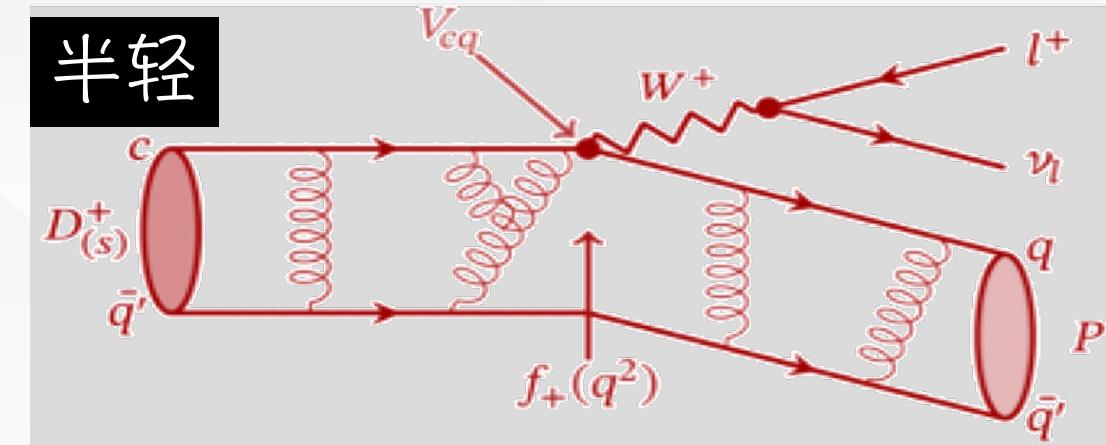
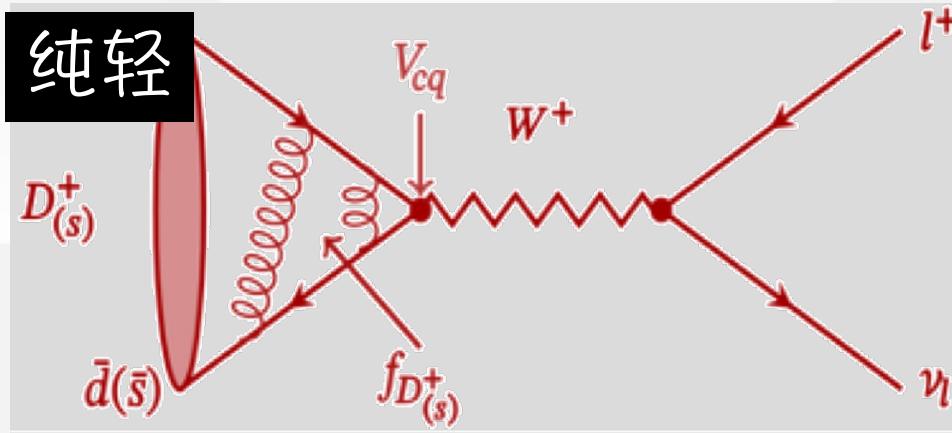
$$I(J^P) = 0(\frac{1}{2}+)$$

$$m_c = 1.27 \pm 0.02 \text{ GeV} \quad \text{Charge} = \frac{2}{3} e \quad \text{Charm} = +1$$
$$m_c/m_s = 11.72 \pm 0.25$$
$$m_b/m_c = 4.577 \pm 0.008$$
$$m_b - m_c = 3.45 \pm 0.05 \text{ GeV}$$

b

$$I(J^P) = 0(\frac{1}{2}+)$$

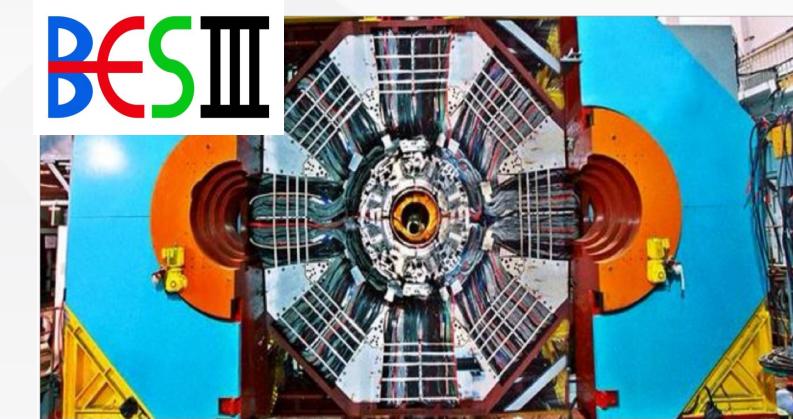
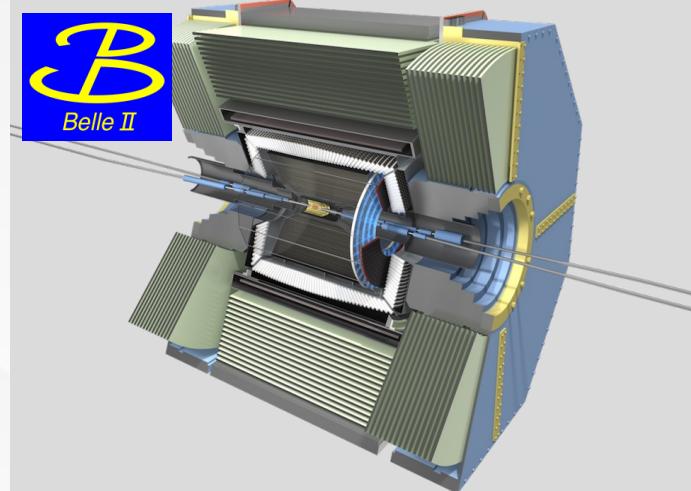
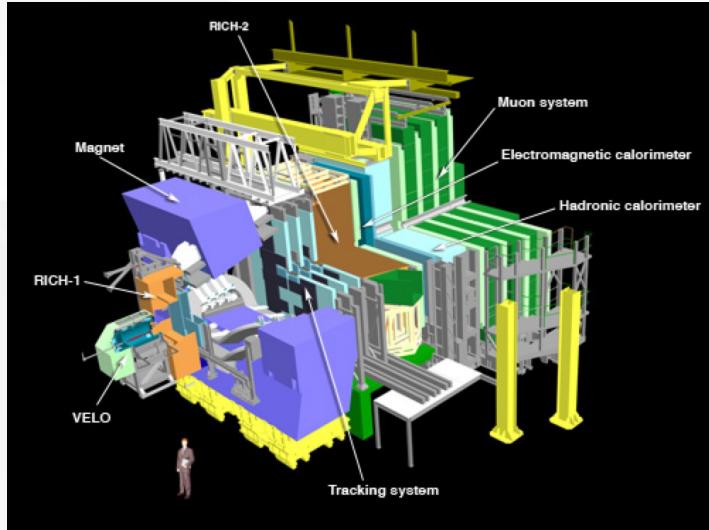
$$m_b = 4.18^{+0.03}_{-0.02} \text{ GeV} \quad \text{Charge} = -\frac{1}{3} e \quad \text{Bottom} = -1$$
$$m_b = 4.10^{+0.03}_{-0.02} \text{ GeV} \quad \text{Charge} = -\frac{1}{3} e \quad \text{Bottom} = -1$$



- 衰变类型**
- ✓ 纯轻 : 末态**都是**轻子
 - ✓ 半轻 : 末态**部分**轻子
 - ✓ 非轻 : 末态**都是**强子
 - ✓ 纯轻辐射 : 光子+轻子
 - ✓ 半轻辐射 : 光子+轻子+强子
 - ✓ 辐射 : 光子+强子

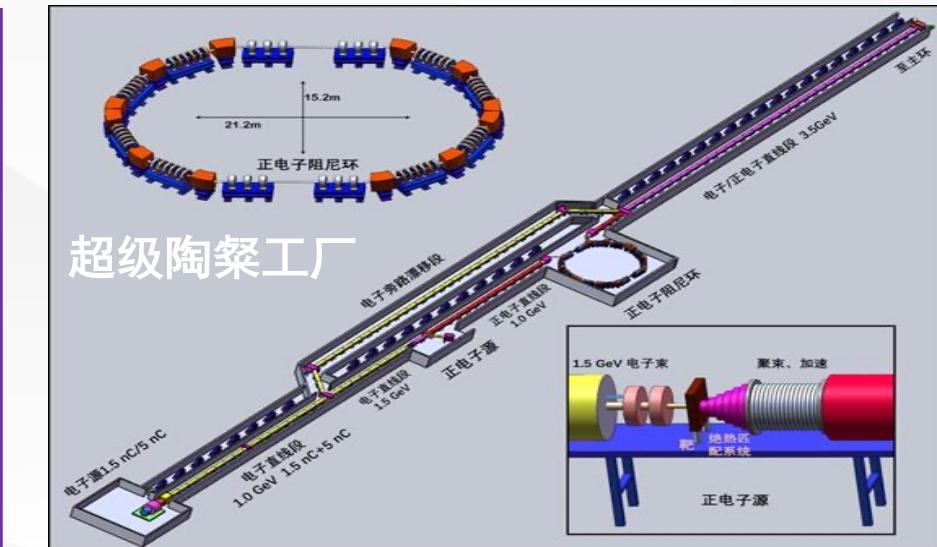


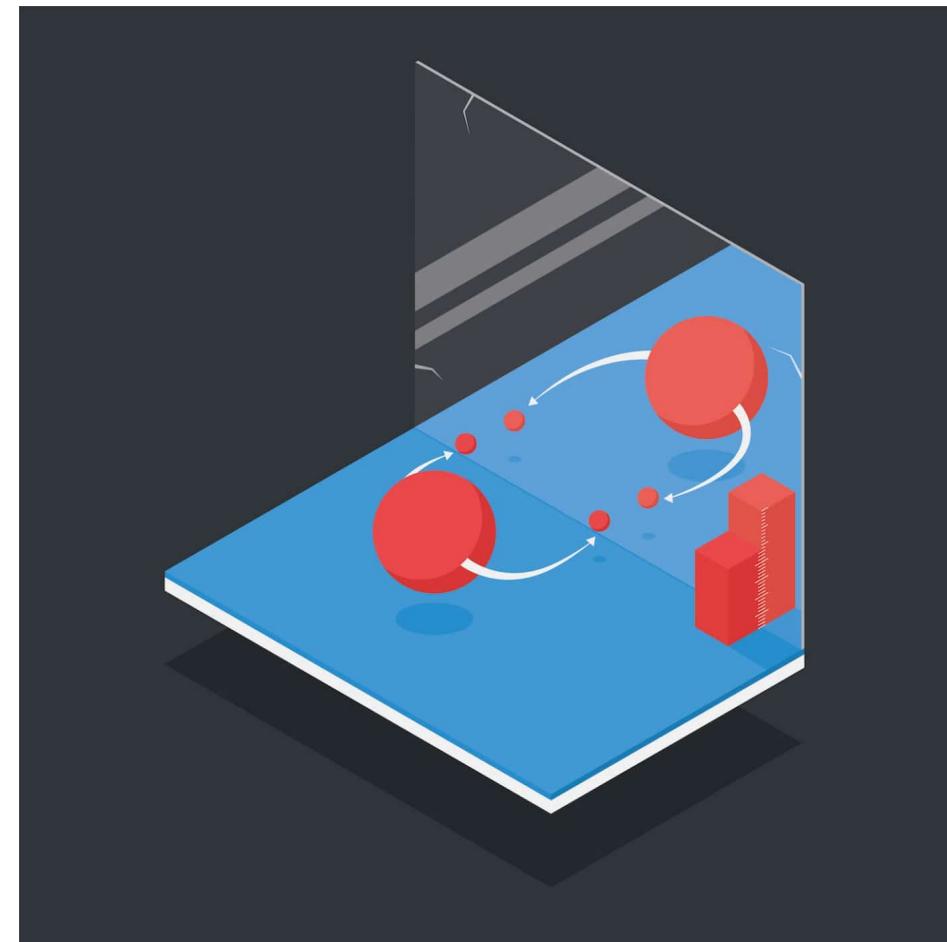
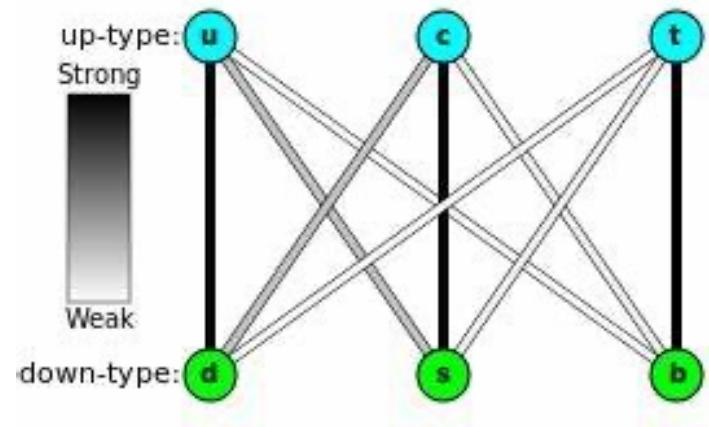
重味物理实验



- 2010年前: Babar, Belle
- 正在运行: LHCb, BESIII, Belle-II
- 计划建造: 超级陶粲工厂

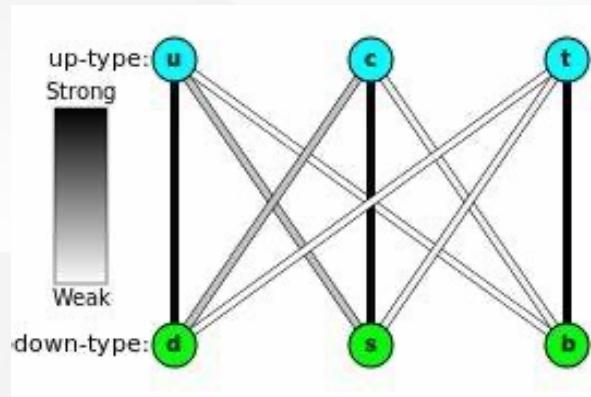
专门从事重味物理研究的大科学装置







Heavy Quark Physics: CKM矩阵元的测量



- 弱作用本征态 \neq 质量本征态
- 三代夸克之间的混合矩阵有三个混合角和一个相位

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$



Heavy Quark Physics: CKM矩阵元的测量



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

First row:

$$\begin{aligned} |V_{ud}| &= 0.97370 \pm 0.00014 \\ |V_{us}| &= 0.2245 \pm 0.0008 \\ |V_{ub}| &= 0.00382 \pm 0.00024 \end{aligned}$$

10^{-4} accuracy

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985 \pm 0.0006$$

Second row:

$$\begin{aligned} |V_{cd}| &= 0.221 \pm 0.004 \\ |V_{cs}| &= 0.987 \pm 0.011 \\ |V_{cb}| &= 0.0410 \pm 0.0014 \end{aligned}$$

10^{-2} accuracy

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.025 \pm 0.022$$

Heavy Quark Physics: CKM矩阵元的测量

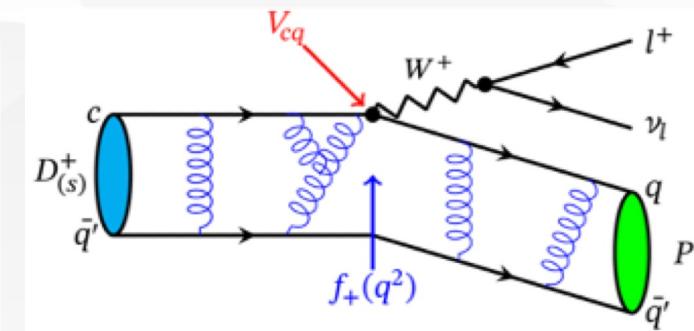
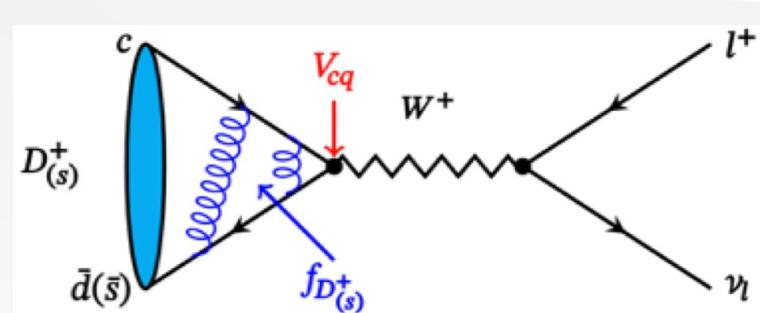


$$|V_{cd}| = 0.221 \pm 0.004$$

$$|V_{cs}| = 0.987 \pm 0.011$$

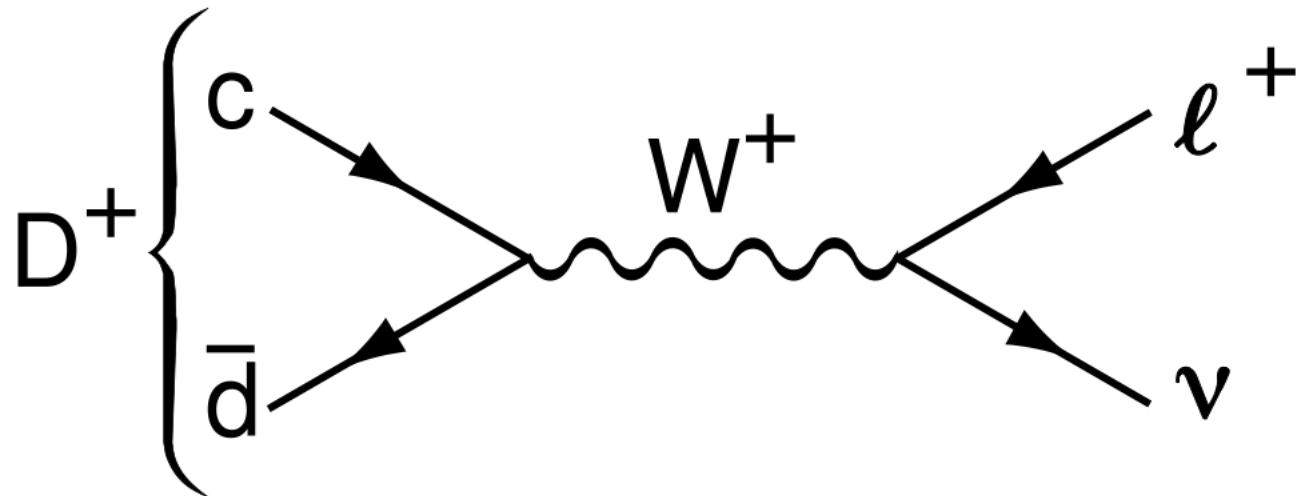
$$|V_{cb}| = 0.0410 \pm 0.0014$$

PDG2021



determinations of $|V_{cs}|$ can be obtained using the PDG values for the mass and lifetime of the D_s , the masses of the leptons, and $f_{D_s} = (249.9 \pm 0.5)$ MeV [14]. The average of these determinations gives $|V_{cs}| = 0.992 \pm 0.012$, where the error is dominated by the experimental uncertainty. In semileptonic D decays, lattice QCD calculations of the $D \rightarrow K l \nu$ form factor are available [14]. Using $f_+^{DK}(0) = 0.765 \pm 0.031$ and the average [24] of CLEO-c [28], Belle [29], *BABAR* [48], and recent BESIII [26, 49] measurements of $D \rightarrow K l \nu$ decays, one obtains $|V_{cs}| = 0.939 \pm 0.038$, where the dominant uncertainty is from the theoretical calculation of the form factor. Averaging the determinations from leptonic and semileptonic decays, we find

$$|V_{cs}| = 0.987 \pm 0.011 . \quad (12.10)$$



$$\Gamma^{(0)}(P \rightarrow \ell\nu) = \frac{G_F^2}{8\pi} f_P^2 m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2 |V_{q_1 q_2}|^2 . \quad (72.1)$$

Here M_P is the P mass, m_ℓ is the ℓ mass, $V_{q_1 q_2}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element between the quarks $q_1 \bar{q}_2$ in P , and G_F is the Fermi coupling constant. The decay constant f_P is proportional to the matrix element of the axial current between the one- P -meson state and the vacuum:

$$\langle 0 | \bar{q}_1 \gamma_\mu \gamma_5 q_2 | P(p) \rangle = i p_\mu f_P , \quad (72.2)$$

$$\langle K^+(p_2) | \bar{u} \gamma_\mu b | \bar{B}_s(p_B) \rangle = F_+^{\bar{B}_s \rightarrow K}(q^2) \left(P_\mu - \frac{m_{B_s}^2 - m_K^2}{q^2} q_\mu \right) + F_0^{\bar{B}_s \rightarrow K}(q^2) \frac{m_{B_s}^2 - m_K^2}{q^2} q_\mu,$$

With the form factors defined in eq. (2.2), we evaluate the $B_s \rightarrow K$ matrix elements as

$$H_0 = \frac{G_F}{\sqrt{2}} V_{ub} \frac{\sqrt{\lambda}}{\sqrt{q^2}} F_1^{B_s \rightarrow K}(q^2), \quad H_t = \frac{G_F}{\sqrt{2}} V_{ub} \frac{m_{B_s}^2 - m_K^2}{\sqrt{q^2}} F_0^{B_s \rightarrow K}(q^2), \quad (3.1)$$

and obtain the differential decay rate for $\bar{B}_s^0 \rightarrow K^+ \ell^- \bar{\nu}_\ell$ as a function of q^2 and θ_l

$$\frac{d^2 \Gamma(\bar{B}_s \rightarrow K^+ \ell^- \bar{\nu}_\ell)}{dq^2 d \cos \theta_l} = \frac{1}{512\pi^3 m_{B_s}^3} \sqrt{\lambda} 4q^2 \beta_l^2 [\sin^2 \theta_l |H_0|^2 + \hat{m}_l^2 |H_t + H_0 \cos \theta_l|^2], \quad (3.2)$$

where $\beta_l = 1 - \hat{m}_l^2$, $\hat{m}_i = m_i / \sqrt{q^2}$ and $\lambda = \lambda(m_{B_s}^2, q^2, m_K^2)$ is the Källen function

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ca). \quad (3.3)$$

Here, θ_l is defined as the polar angle of the lepton momentum relative to the moving direction of the B_s -meson in the q^2 rest frame. Integrating over the polar angle, the differential partial width in q^2 is given by

$$\frac{d\Gamma(\bar{B}_s \rightarrow K^+ \ell^- \bar{\nu}_\ell)}{dq^2} = \frac{1}{128\pi^3 m_{B_s}^3} \sqrt{\lambda} q^2 \beta_l^2 \left[\frac{4}{3} |H_0|^2 + 2\hat{m}_l^2 |H_t|^2 + \frac{2}{3} \hat{m}_l^2 |H_0|^2 \right]. \quad (3.4)$$

One can also explore the q^2 -dependent ratio

$$\mathcal{R}_K^{\tau/\mu}(q^2) = \frac{d\Gamma(\bar{B}_s \rightarrow K^+ \tau^- \bar{\nu}_\tau)/dq^2}{d\Gamma(\bar{B}_s \rightarrow K^+ \mu^- \bar{\nu}_\mu)/dq^2}, \quad (3.5)$$

and its integrated form:

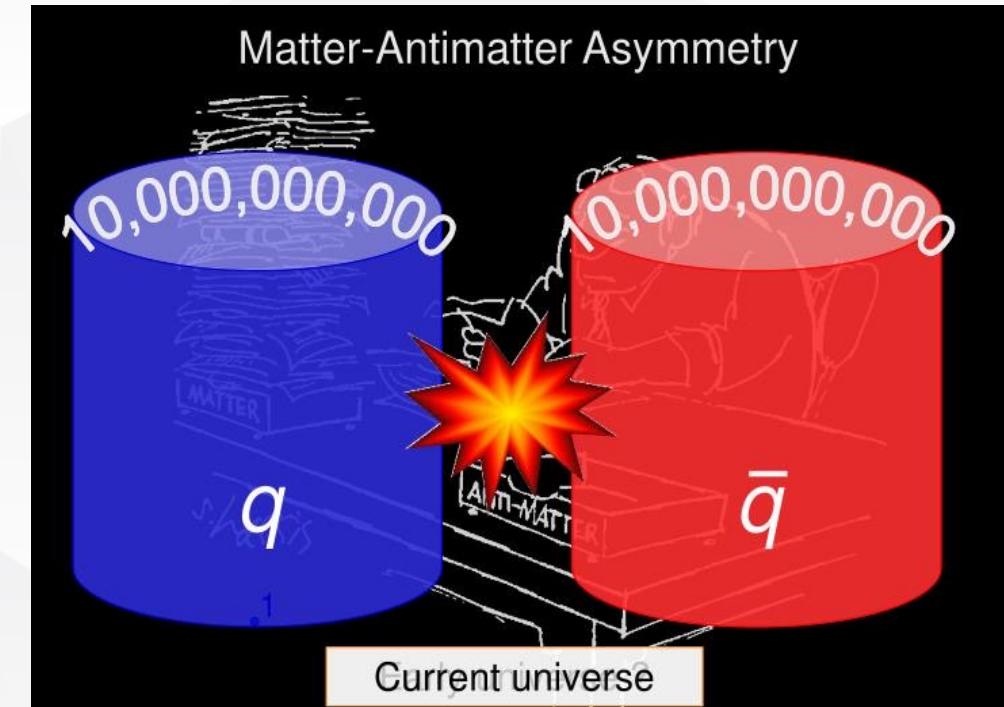
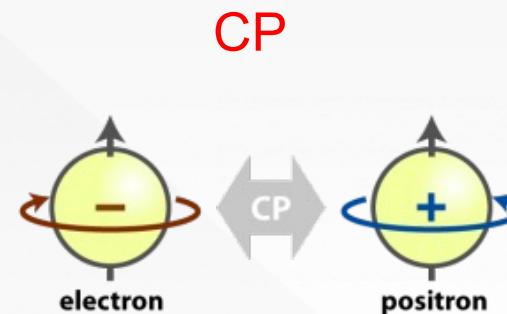
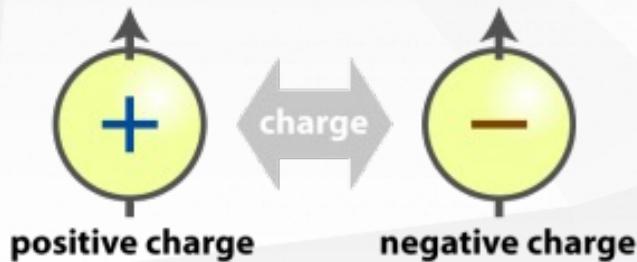
$$R_K^{\tau/\mu} = \frac{\Gamma(\bar{B}_s \rightarrow K^+ \tau^- \bar{\nu}_\tau)}{\Gamma(\bar{B}_s \rightarrow K^+ \mu^- \bar{\nu}_\mu)}, \quad (3.6)$$



Heavy Quark Physics: CP破坏



C:物质—反物质变换





Heavy Quark Physics: CP破坏

$$\mathcal{A} = |A_1|e^{i\delta_1}e^{i\phi_1} + |A_2|e^{i\delta_2}e^{i\phi_2},$$

$$\overline{\mathcal{A}} = |\overline{A}_1|e^{i\delta_1}e^{-i\phi_1} + |\overline{A}_2|e^{i\delta_2}e^{-i\phi_2},$$

$$\mathcal{A} = |A_1|e^{i\delta_1}e^{i\phi_1} (1 + re^{i\delta}e^{i\phi})$$

$$r = |A_2|/|A_1|, \phi = \phi_2 - \phi_1, \delta = \delta_2 - \delta_1$$



Heavy Quark Physics: CP破坏

$$\begin{aligned}
 a_{\text{CP}} &= \frac{|\mathcal{A}|^2 - |\overline{\mathcal{A}}|^2}{|\mathcal{A}|^2 + |\overline{\mathcal{A}}|^2} \\
 &= \frac{|A_1|^2|1 + re^{i\delta}e^{i\phi}|^2 - |A_1|^2|1 + re^{i\delta}e^{-i\phi}|^2}{|A_1|^2|1 + re^{i\delta}e^{i\phi}|^2 + |A_1|^2|1 + re^{i\delta}e^{-i\phi}|^2} \\
 &= \frac{2r \cos(\delta + \phi) - 2r \cos(\delta - \phi)}{2(1 + r^2 + r \cos(\delta + \phi) + r \cos(\delta - \phi))} \\
 &= \frac{2r \sin \delta \sin \phi}{1 + r^2 + 2r \cos \delta \cos \phi}
 \end{aligned}$$

$$\mathcal{A} = |A_1| e^{i\delta_1} e^{i\phi_1} (1 + r e^{i\delta} e^{i\phi})$$

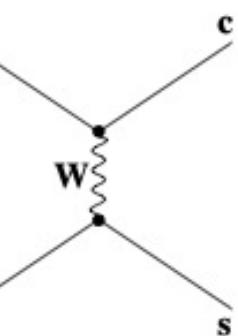
#0, if $\delta \neq 0$ and $\phi \neq 0$

- 振幅至少有两项
- 且弱相位不同
- 干涉造成CP破坏

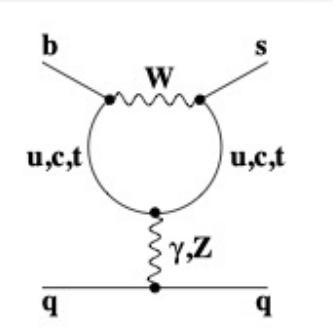
$$\begin{pmatrix}
 c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
 -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
 s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
 \end{pmatrix}$$



Heavy Quark Physics: CP破坏



V_{ub} V_{ud}^*



V_{tb} V_{td}^*

Heavy Quark Physics: CKM中的复相位



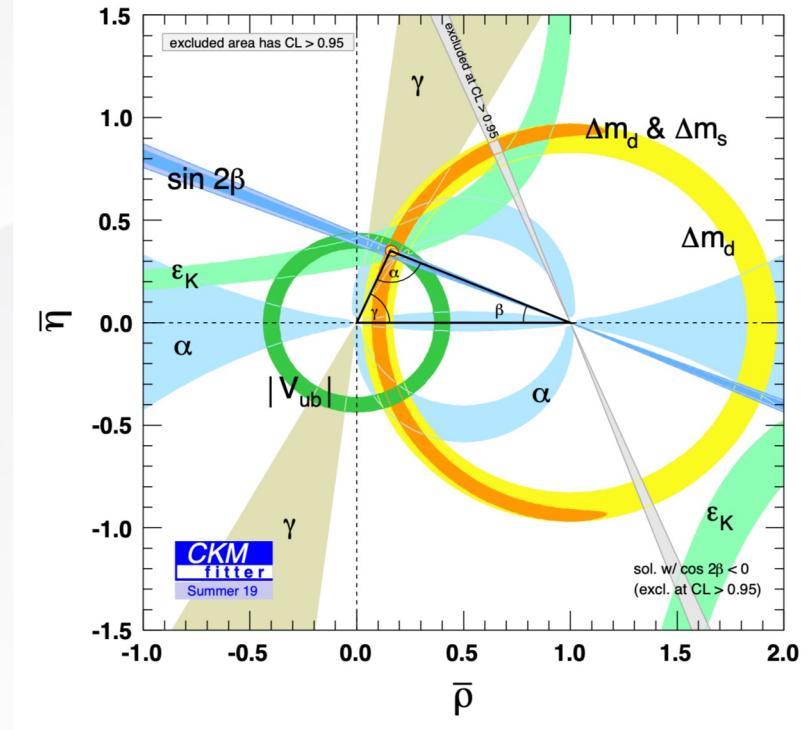
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



$$VV^\dagger = 1 \rightarrow \sum_k V_{ik} V_{jk}^* = \delta_{ij}$$

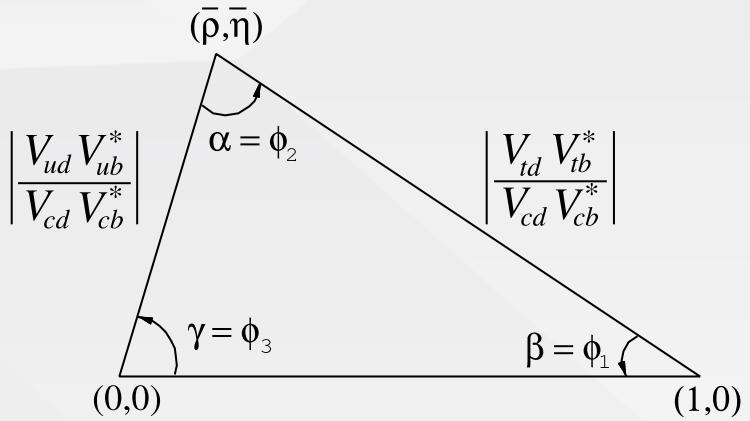


$$V_{ub} V_{ud}^* + V_{cb} V_{cd}^* + V_{tb} V_{td}^* = 0$$



Heavy Quark Physics:

$$V_{ub}V_{ud}^* + V_{cb}V_{cd}^* + V_{tb}V_{td}^* = 0$$



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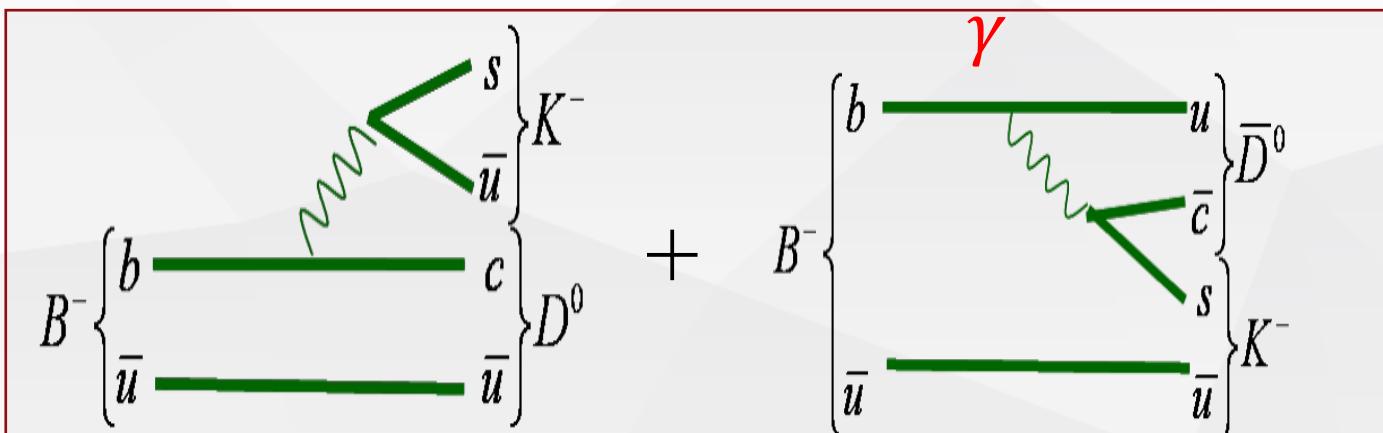
$$\alpha = (84.9^{+5.1}_{-4.5})^\circ$$

$$\beta = (22.2 \pm 0.7)^\circ$$

$$\gamma = (72.1^{+4.1}_{-4.5})^\circ$$

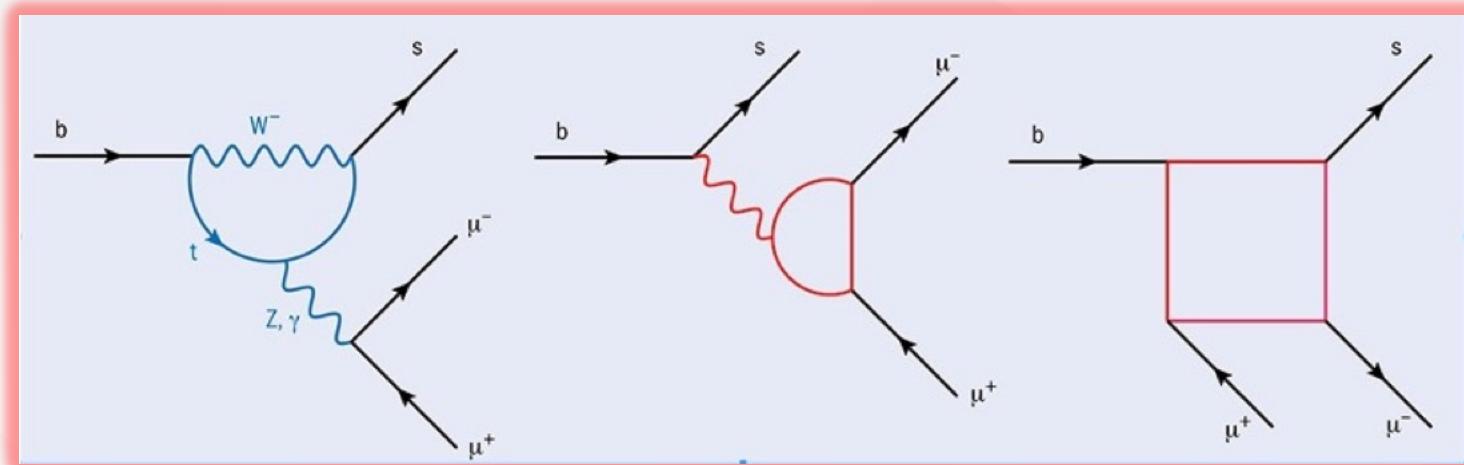
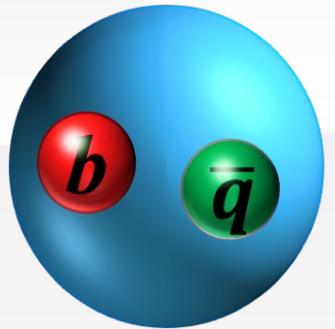
$$\alpha + \beta + \gamma = (179^{+7}_{-6})^\circ$$

$$\sin(2\beta) = 0.699 \pm 0.017$$

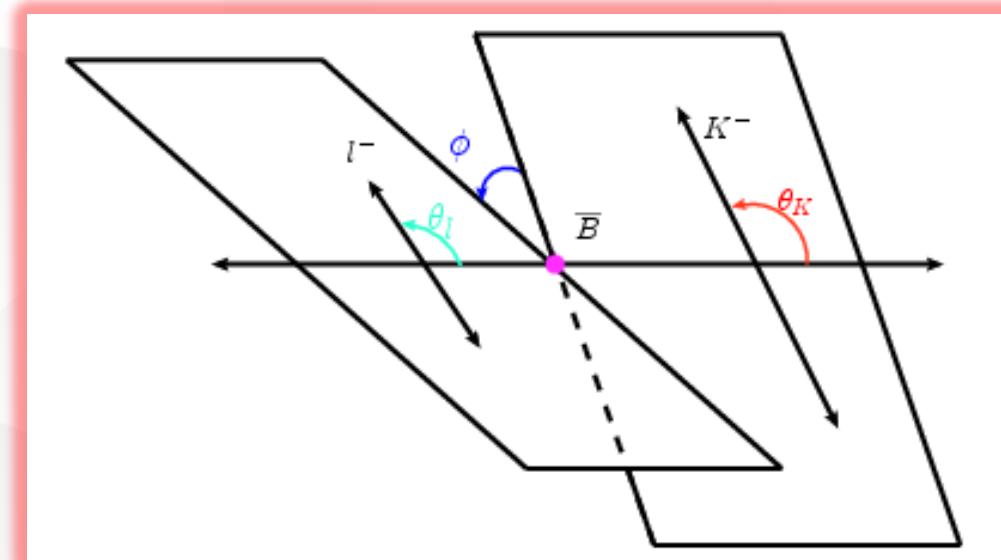




Indirect Search and Heavy Quark Physics



$$B \rightarrow K^* l^+ l^- \rightarrow K \pi l^+ l^-$$





Indirect Search and Heavy Quark Physics



Test of lepton universality with $B^0 \rightarrow K^{*0} \ell^+ \ell^-$ decays

LHCb Collaboration • R. Aaij (CERN) et al. (May 16, 2017)

Published in: *JHEP* 08 (2017) 055 • e-Print: 1705.05802 [hep-ex]

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Angular analysis of the $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decay using 3 fb^{-1} of integrated luminosity

LHCb Collaboration • Roel Aaij (CERN) et al. (Dec 14, 2015)

Published in: *JHEP* 02 (2016) 104 • e-Print: 1512.04442 [hep-ex]

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Measurement of the Differential Branching Fraction and Forward-Backward Asymmetry for $B \rightarrow K^{(*)} \ell^+ \ell^-$

Belle Collaboration • J.-T. Wei (Taiwan, Natl. Taiwan U.) et al. (Apr, 2009)

Published in: *Phys.Rev.Lett.* 103 (2009) 171801 • e-Print: 0904.0770 [hep-ex]

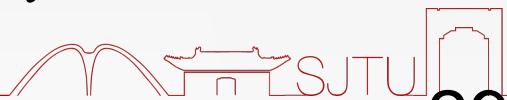
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Lepton Flavor Universality

Angular Analysis and P'_5

Forward-backward Asymmetry



§2.2.1 算符乘积展开，基本思想

人们在研究 β 衰变的四费米子相互作用时就已经开始使用低能有效哈密顿量的研究方法。在那里，人们将标准模型中的中间玻色子积分掉（积掉重场）进而得到有效的哈密顿量，使研究计算变得简单。同样，在 B 物理领域，也出现了很多的重场，例如 W^\pm 、 Z^0 以及顶夸克，而这些重的自由度均不出现在初末态中，并且这些重的场的能标 $\mathcal{O}(m_{W,Z})$ 远远大于 B 介子的质量能标 $\mathcal{O}(m_b)$ 。为了方便并且基本上不影响结果，我们便积分掉这些重场，用一些只含有 B 介子衰变初末态自由度的合适的定域低能有效相互作用来描述 B 介子衰变。

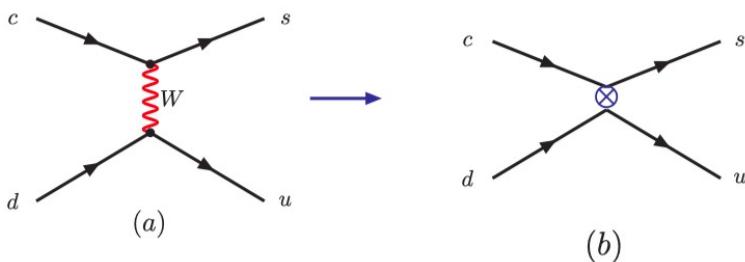


图 2.3 粒子衰变过程 $c \rightarrow sud\bar{d}$ 的树图级费曼图。左图对应“完整理论”，右图对应“有效理论”。

完整理论的树图级的振幅:

$$\begin{aligned} \mathcal{A}_{full} &= \bar{s} \cdot \frac{ig}{2\sqrt{2}} \gamma_\mu (1 - \gamma_5) V_{cs}^* \cdot c \cdot \frac{-ig_{\mu\nu}}{k^2 - M_W^2} \cdot \bar{u} \cdot \frac{ig}{2\sqrt{2}} \gamma_\nu (1 - \gamma_5) V_{ud} \cdot d \\ &= i \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \frac{m_W^2}{k^2 - m_W^2} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} \\ &= -i \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \left(1 + \frac{k^2}{M_W^2} + \dots \right) (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} \\ &= -i \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} + \mathcal{O}\left(\frac{k^2}{m_W^2}\right), \end{aligned} \quad (2.44)$$

其中 $G_F = 1.16637 \times 10^{-5} GeV^{-2}$ 是费米耦合常数， $(\bar{s}c)_{V-A} \equiv \bar{s} \gamma_\mu (1 - \gamma_5) c$, $G_F/\sqrt{2} = g^2/(8M_W^2)$ 。对于 D, B 介子衰变，由于通过 W 玻色子传递的动量比 m_W 要小很多，即 $k^2/m_W^2 \ll 1$ 。那么在领头阶， $\mathcal{O}(k^2/m_W^2)$ 量级的项可以安全地忽略掉。这样，低能有效的哈密顿量可以写为：

$$\mathcal{H}_{eff} \equiv i \mathcal{A}_{full} = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} + \text{高量纲项} \quad (2.45)$$

在上式中的高量纲算符通常带有偏导数项，相应的项是 $\mathcal{O}(\frac{k^2}{m_W^2})$ 压低的。在以后的计算中，只保留量纲为五或者是六的有效算符，更高量纲的算符都忽略。

通过这个例子我们可以知道，两个带电的流算符的乘积可以展开为一系列的定域算符之和，每一个算符的贡献的大小可以用有效的耦合常数— Wilson 系数来表征。量纲高的算符，相应的耦合常数通常会被一个大的质量幂次压低。以上是算符乘积展开的基本思想。



上面是只考虑树图的结果。如果考虑单圈 QCD 修正的影响，那么可以把 \mathcal{H}_{eff} 改写为：

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* [C_1(\mu) O_1 + C_2(\mu) O_2], \quad (2.46)$$

其中

$$O_2 = (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A}, \quad (2.47)$$

$$O_1 = (\bar{s}_\alpha c_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A} \quad (2.48)$$

可以看出：

(1) 除了色单态的算符 O_2 以外，还有一个新的算符 O_1 出现。二者具有相同的味道形式，但是颜色结构不一样。如果考虑到两个色单态弱流之间的单胶子交换图（见图 -2.4），即考虑 $SU(3)_c$ 群生成元矩阵 T_{ij}^a 所满足的恒等式：

$$T_{\alpha\beta}^a T_{\gamma\delta}^a = \frac{1}{2} \left[\delta_{\alpha\delta} \delta_{\gamma\beta} - \frac{1}{N_c} \delta_{\alpha\beta} \delta_{\gamma\delta} \right] \quad (2.49)$$

那么在两个色单态弱流之间就会产生“色混合”，导致算符 O_1 的出现。上式右端的第一项是“色允许”的项，第二项是“色压低”的项。请注意：这里是按照 Buras 的约定 [8] 来定义 O_2 （色单态）和 O_1 。在有些文献中是反过来的。

§2.2.2 由完整理论到有效理论的匹配: Wilson 系数

在本小节, 我们通过文献 [8] 中给出的一个具体的例子: $c \rightarrow s u \bar{d}$ 带电流过程, 来解释如何做完整理论到有效理论的匹配, 进而导出 Wilson 系数。完整理论指的是所有粒子都以动力学自由度形式出现, 有效理论指的是将 W 等重粒子积分掉以后的结果。

根据前面的计算, 我们知道, 所谓匹配就是要求在次领头阶水平下式成立:

$$\mathcal{A}_{full} \equiv \mathcal{A}_{eff} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} [C_1(\mu) < O_1 > + C_2(\mu) < O_2 >]. \quad (2.5)$$

“匹配”过程可以分为以下三步来完成。

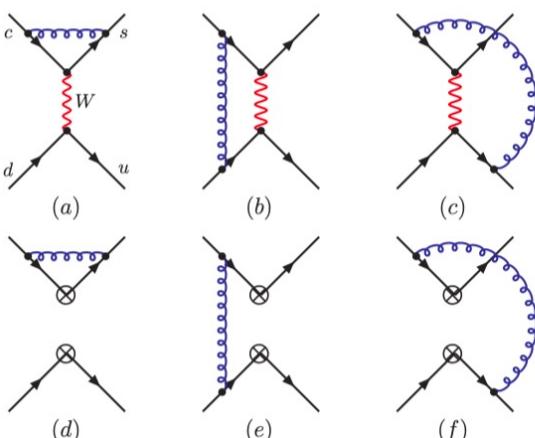


图 2.4 在完整理论中对流 - 流树图的单圈 QCD 修正的典型费曼图 (a,b,c)。在有效理论中对应的费曼图 (d,e,f)。

第一步: 在单圈图水平计算完整振幅 \mathcal{A}_{full} 。

对图 2.4 中第一行的 (a,b,c) 和与其同类 (在任意两条夸克线之间连一条胶子线, 外线夸克均为轻夸克) 的 QCD 单圈费曼图做解析计算, 得到在 $\mathcal{O}(\alpha_s)$ 阶的完整振幅 \mathcal{A}_{full} ($m_i^2 = 0, p^2 < 0$) [8] :

$$\begin{aligned} \mathcal{A}_{full} = & \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left\{ \left[1 + 2C_F \frac{\alpha_s}{4\pi} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{-p^2} \right) \right] S_2 + \frac{3}{N_c} \frac{\alpha_s}{4\pi} \ln \frac{m_W^2}{-p^2} S_2 \right. \\ & \left. - 3 \frac{\alpha_s}{4\pi} \ln \frac{m_W^2}{-p^2} S_1 \right\}, \end{aligned} \quad (2.51)$$

其中 $S_{1,2} \equiv < O_{1,2} >_{tree} = O_{1,2}$, 是树图阶的矩阵元 $O_{1,2}$ 。

如果对夸克场做重整化,

$$\Psi_q^{(0)} = \sqrt{Z_q} \Psi_q, \quad (2.52)$$

那么 \mathcal{A}_{full} 表达式中的发散部分将被抵消掉, 即有

$$\begin{aligned} \mathcal{A}_{full} = & \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left\{ \left[1 + 2C_F \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} \right] S_2 + \frac{3}{N_c} \frac{\alpha_s}{4\pi} \ln \frac{m_W^2}{-p^2} S_2 \right. \\ & \left. - 3 \frac{\alpha_s}{4\pi} \ln \frac{m_W^2}{-p^2} S_1 \right\}. \end{aligned} \quad (2.53)$$

在计算中, 我们使用了如下的约定和近似:

1. 使用 Feynman 规范 ($\xi = 1$), 取外线夸克零质量 ($m_i^2 = 0$), $p^2 < 0$;
2. 采用 “large logarithmic approximation”: 即忽略那些正比于 α_s 的项, 只保留 $\alpha_s \cdot \log$ 项。



第二步：在单圈图水平计算振幅 \mathcal{A}_{eff} 。

采用同样的约定和近似，对图 -2.4 中第二行的费曼图和其它相关 QCD 单圈费曼图做解析计算，得到没有重整化的流 - 流算符 $O_{1,2}$ 的矩阵元 $\langle O_1 \rangle^{(0)}$ 和 $\langle O_2 \rangle^{(0)}$ 。考虑没有 QCD 修正的矩阵元 $S_{1,2}$ ，并且忽略外腿的自能修正，可以得到

$$\begin{aligned} \langle O_1 \rangle^{(0)} &= \left[1 + 2C_F \frac{\alpha_s}{4\pi} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{-p^2} \right) \right] S_1 + \frac{3}{N_c} \frac{\alpha_s}{4\pi} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{-p^2} \right) S_1 \\ &\quad - 3 \frac{\alpha_s}{4\pi} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{-p^2} \right) S_2, \end{aligned} \quad (2.54)$$

$$\begin{aligned} \langle O_2 \rangle^{(0)} &= \left[1 + 2C_F \frac{\alpha_s}{4\pi} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{-p^2} \right) \right] S_2 + \frac{3}{N_c} \frac{\alpha_s}{4\pi} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{-p^2} \right) S_2 \\ &\quad - 3 \frac{\alpha_s}{4\pi} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{-p^2} \right) S_1. \end{aligned} \quad (2.55)$$

在 (2.54-2.55) 式的第一项中的发散项 ($1/\epsilon$ 项) 同样可以通过夸克场的重整化去掉。但与 (2.51) 式的 \mathcal{A}_{full} 不同，(2.54-2.55) 式中第二和第三项的发散部分，即使做了夸克场的重整化也消不掉。

为了消除这些发散，我们需要引入算符的重整化：

$$O_i^{(0)} = Z_{ij} O_j, \quad i, j = (1, 2). \quad (2.56)$$

显然，这里的重整化常数 Z 是一个 2×2 的矩阵。这样，没有重整化的截腿 Green 函数 $\langle O_{1,2} \rangle^{(0)}$ 和重整化的截腿 Green 函数 $\langle O_{1,2} \rangle$ 之间的关系是

$$\langle O_i \rangle^{(0)} = Z_q^{-2} Z_{ij} \langle O_j \rangle. \quad (2.57)$$

这样，我们用 Z_q^{-2} 抵消掉 (2.54-2.55) 式中第一项的发散项 ($1/\epsilon$ 项)，用 Z_{ij} 抵消掉剩余的发散项。那么，在 \overline{MS} 方案下，有

$$\hat{Z} = 1 + \frac{\alpha_s}{4\pi} \cdot \frac{1}{\epsilon} \begin{pmatrix} 3/N & -3 \\ -3 & 3/N \end{pmatrix}. \quad (2.58)$$

因此，重整化的矩阵元为

$$\langle O_1 \rangle = \left(1 + 2C_F \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} \right) S_1 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} S_1 - 3 \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} S_2, \quad (2.59)$$

$$\langle O_2 \rangle = \left(1 + 2C_F \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} \right) S_2 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} S_2 - 3 \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} S_1. \quad (2.60)$$



第三步：通过 "mathching", 抽出 Wilson 系数 $C_i(\mu)$ 。

将重整化的 $\langle O_i \rangle$ 表达式带入 (2.50) 式，并和 (2.53) 式比较，就可以抽出 Wilson 系数 $C_{1,2}$:

$$C_1(\mu) = -3 \frac{\alpha_s}{4\pi} \ln \frac{m_W^2}{\mu^2}, \quad C_2(\mu) = 1 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \frac{m_W^2}{\mu^2}. \quad (2.61)$$

可以看出，在做矩阵元的重整化时导致原始算符 O_1 和 O_2 之间的混合。为了研究算符系统 (O_1, O_2) 的重整化群性质，我们定义一组新基底 O_\pm :

$$O_\pm = \frac{O_2 \pm O_1}{2}, \quad C_\pm = C_2 \pm C_1. \quad (2.62)$$

此时， O_+ 和 O_- 在重整化时是相互独立的，没有混合:

$$O_\pm^{(0)} = Z_\pm O_\pm, \quad (2.63)$$

其中

$$Z_\pm = 1 + \frac{\alpha_s}{4\pi} \frac{1}{\epsilon} \left(\mp 3 \frac{N \mp 1}{N} \right). \quad (2.64)$$

在新的算符基底 (O_+, O_-) 下，振幅表达式为:

$$\mathcal{A} \equiv \mathcal{A}_+ + \mathcal{A}_- = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} [C_+(\mu) \langle O_+(\mu) \rangle + C_-(\mu) \langle O_-(\mu) \rangle], \quad (2.65)$$

其中

$$C_\pm(\mu) = 1 + \left(\frac{3}{N} \mp 3 \right) \frac{\alpha_s}{4\pi} \ln \frac{m_W^2}{\mu^2}, \quad (2.66)$$

$$\langle O_\pm(\mu) \rangle = \left(1 + 2C_F \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} \right) S_\pm + \left(\frac{3}{N} \mp 3 \right) \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} S_\pm. \quad (2.67)$$

由上面二式可以看出：

- 采用算符乘积展开的方法，可以把短程贡献（Wilson 系数 $C_i(\mu)$ ）与长程贡献（算符矩阵元 $\langle O_\pm(\mu) \rangle$ ）分离开来，此即“因子化”（factorization）。直观来看，因子化呈现出下面的结构

$$\left(1 + \alpha_s G \ln \frac{m_W^2}{-p^2} \right) \doteq \left(1 + \alpha_s G \ln \frac{m_W^2}{\mu^2} \right) \cdot \left(1 + \alpha_s G \ln \frac{\mu^2}{-p^2} \right) \quad (2.68)$$

其中 $-p^2 < \mu^2 \ll m_W^2$ 。当然，在“微扰区”和“非微扰区”之间并没有明确的界限。一般认为在 $\mu > 1\text{GeV}$ 的区域，“微扰贡献”起主要作用，Wilson 系数的重整化群演化是可信的。

- Wilson 系数 $C_\pm(\mu)$ 是定义在高能标“可微扰”区间 $[\mu, m_W]$ ；更低能区 $[-p^2, \mu]$ 的贡献则被分离到矩阵元 $\langle O_\pm(\mu) \rangle$ 中。重整化能标 μ 扮演的是因子化能标的角色。



§2.2.3 算符乘积展开：有效哈密顿量

除了上面讨论的带电流树图作用外，圈图级的味道改变中性流过程，例如企鹅图过程也是非常重要的。在考虑了树图和单圈图 QCD 和电弱贡献以后，对夸克水平的三体衰变 $b \rightarrow sV^* \rightarrow sq'\bar{q}'$ ($q' \in \{u, d, s\}$)，其有效哈密顿 \mathcal{H}_{eff} [8] 可以写为

$$\begin{aligned} \mathcal{H}_{eff} = & \frac{G_F}{\sqrt{2}} \left\{ \sum_{i=1}^2 C_i(\mu) [V_{ub} V_{us}^* O_i^u(\mu) + V_{cb} V_{cs}^* O_i^c(\mu)] - V_{tb} V_{ts}^* \sum_{j=3}^{10} C_j(\mu) O_j(\mu) \right. \\ & \left. - V_{tb} V_{ts}^* [C_{7\gamma}(\mu) O_{7\gamma}(\mu) + C_{8g}(\mu) O_{8g}(\mu)] \right\}. \end{aligned} \quad (2.69)$$

上式中的算符 O 可以分为四类¹

1. 由图 -2.5a 产生的流 - 流算符

$$\begin{aligned} O_1^u &= (\bar{s}_\alpha u_\beta)_{V-A} (\bar{u}_\beta b_\alpha)_{V-A}, \quad O_2^u = (\bar{s}u)_{V-A} (\bar{u}b)_{V-A}, \\ O_1^c &= (\bar{s}_\alpha c_\beta)_{V-A} (\bar{c}_\beta b_\alpha)_{V-A}, \quad O_2^c = (\bar{s}c)_{V-A} (\bar{c}b)_{V-A}, \end{aligned} \quad (2.70)$$

其中 α, β 是 $SU(3)_c$ 色指标。

2. 由图 -2.5b 产生的 QCD 企鹅算符

$$O_3 = (\bar{s}b)_{V-A} \sum_q (\bar{q}q)_{V-A}, \quad O_4 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A}, \quad (2.71)$$

$$O_5 = (\bar{s}b)_{V-A} \sum_q (\bar{q}q)_{V+A}, \quad O_6 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A}, \quad (2.72)$$

其中 $q \in \{u, d, s, c, b\}$ 。算符的前半部分来自于图 -2.5b 的上半部分，因为仅出现 W 玻色子和夸克的耦合，因而是纯 $V - A$ 形式的耦合。算符的后半部分来自于图 -2.5b 的下半部分，即胶子和夸克的耦合，是纯 V 形式的耦合，可以分解为 $V - A$ 和 $V + A$ 两部分，因而出现 $O_3 - O_6$ 算符。

3. 由图 -2.5c 产生的 QED 企鹅算符

$$O_7 = \frac{3}{2} (\bar{s}b)_{V-A} \sum_q e_q (\bar{q}q)_{V+A}, \quad O_8 = \frac{3}{2} (\bar{s}_\alpha b_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V+A}, \quad (2.73)$$

$$O_9 = \frac{3}{2} (\bar{s}b)_{V-A} \sum_q e_q (\bar{q}q)_{V-A}, \quad O_{10} = \frac{3}{2} (\bar{s}_\alpha b_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V-A}, \quad (2.74)$$

其中 e_q 是夸克 q 的电荷 ($q \in \{u, d, s, c, b\}$)。算符的前半部分与 QCD 算符相同。算符的后半部分来自于图 -2.5c 的下半部分，即纯 V 形式的 $\gamma q\bar{q}$ 耦合， $Zq\bar{q}$ 耦合是 $V - A$ 和 $V + A$ 耦合的混合型耦合，总体可以分解为 $V - A$ 和 $V + A$ 两部分。

¹这里我们给出 $b \rightarrow s$ 跃迁算符 O 的表达式。对 $b \rightarrow d$ 跃迁，把表达式中的 s 换成 d 即可。



4. 由图 2.5d 产生的电磁偶算符和色磁偶算符

$$O_{7\gamma} = \frac{e}{4\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} R b_\alpha F_{\mu\nu}, \quad (2.75)$$

$$O_{8g} = \frac{g_s}{4\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} R T_{\alpha\beta}^a b_\beta G_{\mu\nu}^a. \quad (2.76)$$

另外两类常见的箱图算符和半轻子衰变算符为 [8]

1. 由图 2.5e 产生的 $\Delta S = 2, \Delta B = 2$ 箱图算符

$$O(\Delta S = 2) = (\bar{s}d)_{V-A} (\bar{s}d)_{V-A}, \quad O(\Delta B = 2) = (\bar{b}d)_{V-A} (\bar{b}d)_{V-A}, \quad (2.77)$$

2. 由图 2.5f 产生的半轻子衰变算符

$$\begin{aligned} O_{9V} &= (\bar{s}b)_{V-A} (\bar{\mu}\mu)_V, \quad O_{10A} = (\bar{b}s)_{V-A} (\bar{\mu}\mu)_A, \\ O_{\nu\bar{\nu}} &= (\bar{s}b)_{V-A} (\bar{\nu}\nu)_{V-A}, \quad O_{\bar{\mu}\mu} = (\bar{s}b)_{V-A} (\bar{\mu}\mu)_{V-A}, \end{aligned} \quad (2.78)$$

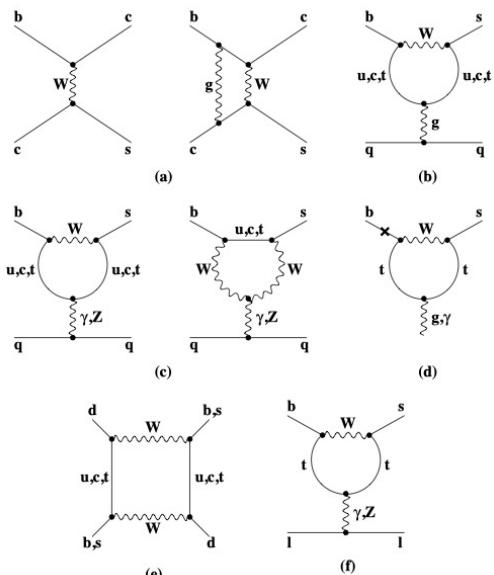
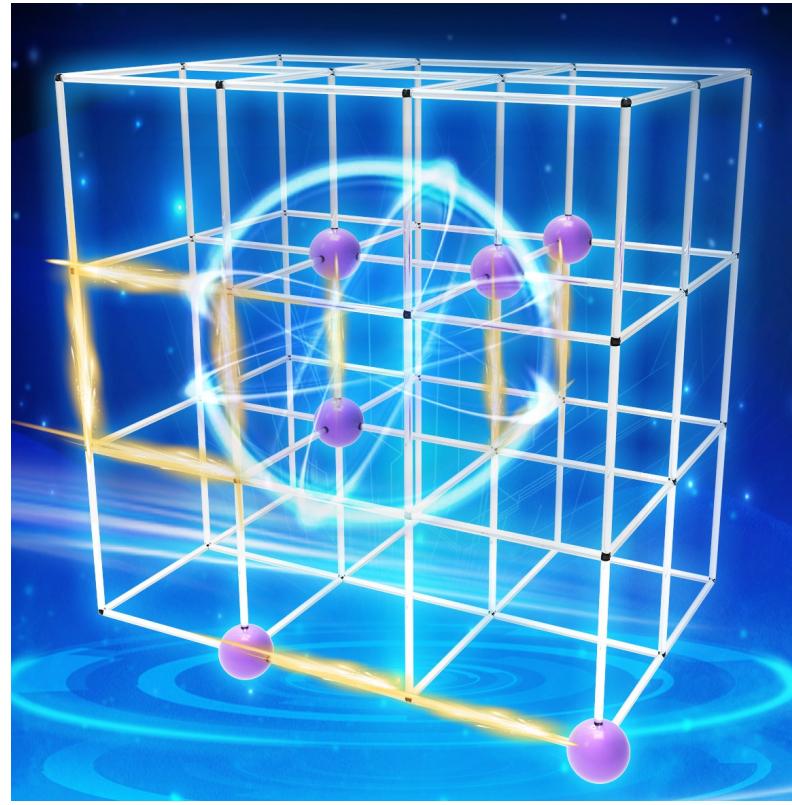
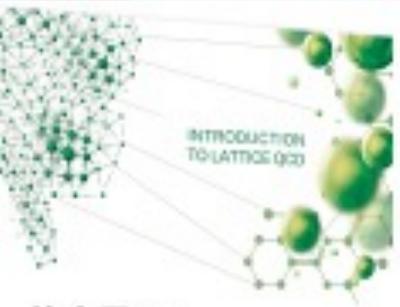


图 2.5 与算符 O_i 相对应的典型费曼图. 其中图 -(d) 的“ \times ”号表示质量插入, 亦即保留与 m_b 成正比的项。





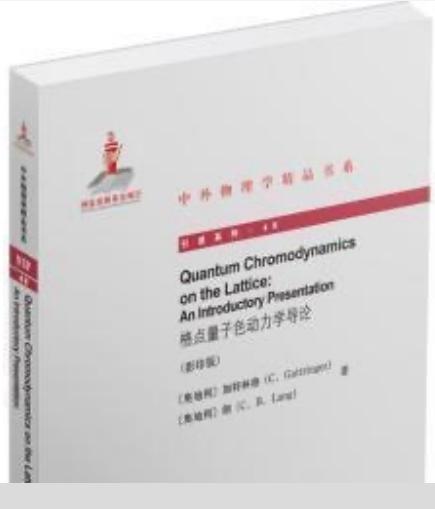
Lattice QCD: some references



格点量子
色动力学导论

刘川

格点量子色动力学
导论



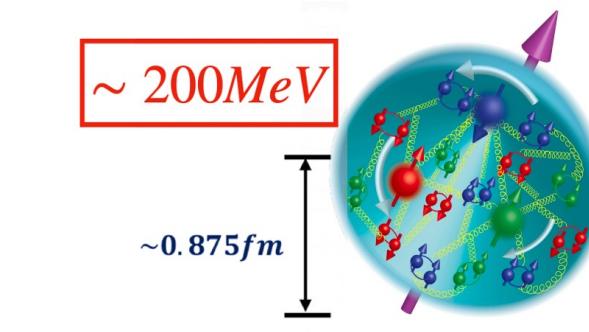
C. Gattringer, C. Lang
QCD on the Lattice



黄涛、王伟等
量子色动力学专题
第二章 陈莹 格点QCD

- ✓ T. DeGrand and C. DeTar ,Lattice methods for Quantum Chromodynamics
- ✓ H.J. Rothe, Lattice gauge theories: An Introduction
- ✓ J. Smit Introduction to quantum fields on a lattice: A robust mate

➤ The force that binds quarks to form protons and neutrons is described by QCD

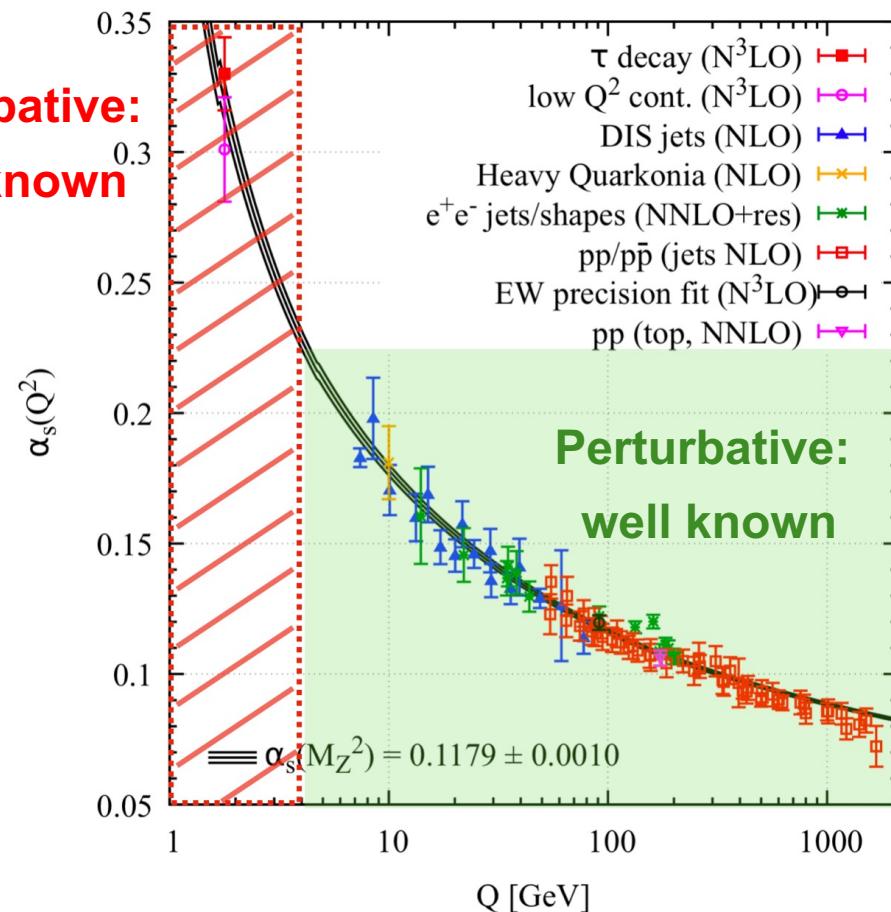


**Strength of interaction reflected by
the strong coupling constant α_s ;**

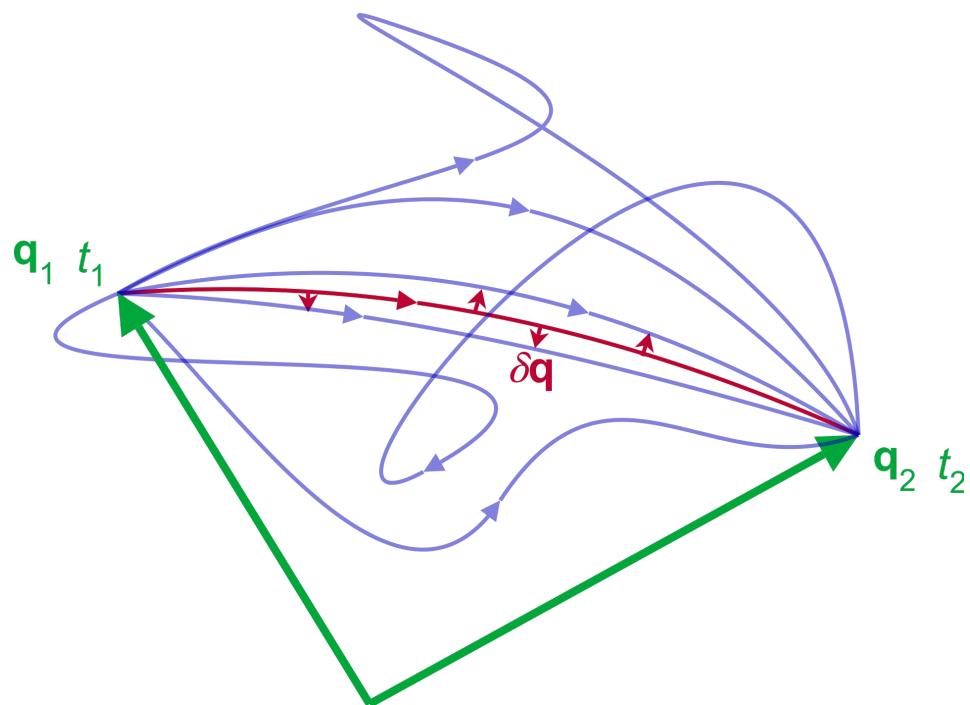
- **≈ 200 MeV: perturbative theory failed;**
- **Nonperturbative methods:**

Lattice QCD, ChPT, QCDSR

**Nonperturbative:
not well known**



Lattice QCD



最小作用量原理



$$\delta \int_{t_1}^{t_2} L(\mathbf{q}, \dot{\mathbf{q}}, t) dt = 0.$$

牛顿第二定律

麦克斯韦方程

薛定谔方程

克莱因-戈登方程

狄拉克方程

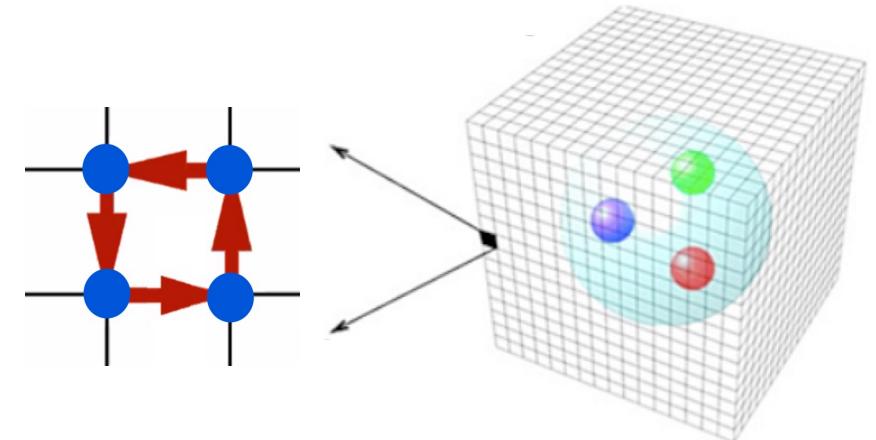
...

- Numerical simulation in discretized 4D Euclidean space-time;

- Lattice QCD: action

$$S_E^{\text{latt}} = - \sum_{\square} \frac{6}{g^2} \text{Re} \operatorname{tr}_N(U_{\square, \mu\nu}) - \sum_q \bar{q}(D_{\mu}^{\text{latt}} \gamma_{\mu} + am_q)q$$

Wilson gauge action Lattice fermion action



- Correlation functions:

$$\begin{aligned} \langle \mathcal{O}(U, q, \bar{q}) \rangle &= \frac{\int [\mathcal{D}\mathcal{U}] \prod_q [\mathcal{D}q_q] [\mathcal{D}\bar{q}_q] e^{-S_E^{\text{latt}}} \mathcal{O}(U, q, \bar{q})}{\int [\mathcal{D}\mathcal{U}] \prod_q [\mathcal{D}q_q] [\mathcal{D}\bar{q}_q] e^{-S_E^{\text{latt}}}} \\ &= \frac{\int [\mathcal{D}\mathcal{U}] e^{-S_{\text{glue}}^{\text{latt}}} \prod_q \det(D_{\mu}^{\text{latt}} \gamma_{\mu} + am_q) \tilde{\mathcal{O}}(U)}{\int [\mathcal{D}\mathcal{U}] e^{-S_{\text{glue}}^{\text{latt}}} \prod_q \det(D_{\mu}^{\text{latt}} \gamma_{\mu} + am_q)} \end{aligned}$$

➤ Monte Carlo:

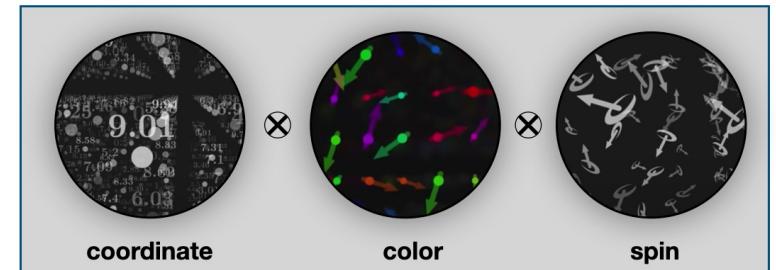
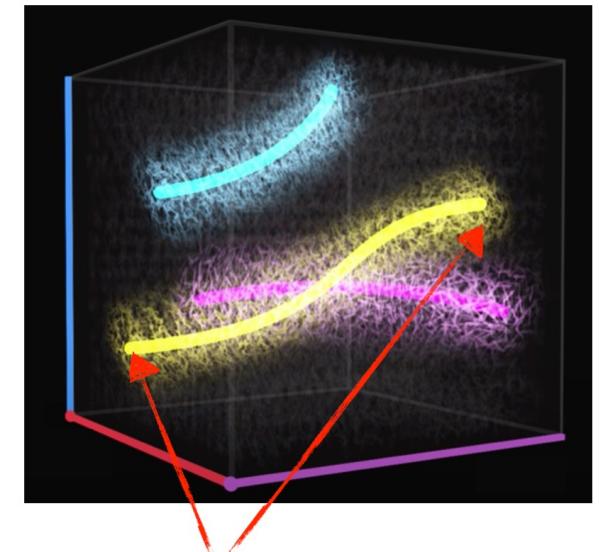
- The integration is performed for all link variables: $n_s^3 \times n_t \times N_{\text{color}} \times N_{\text{spin}}$
- Importance sampling:

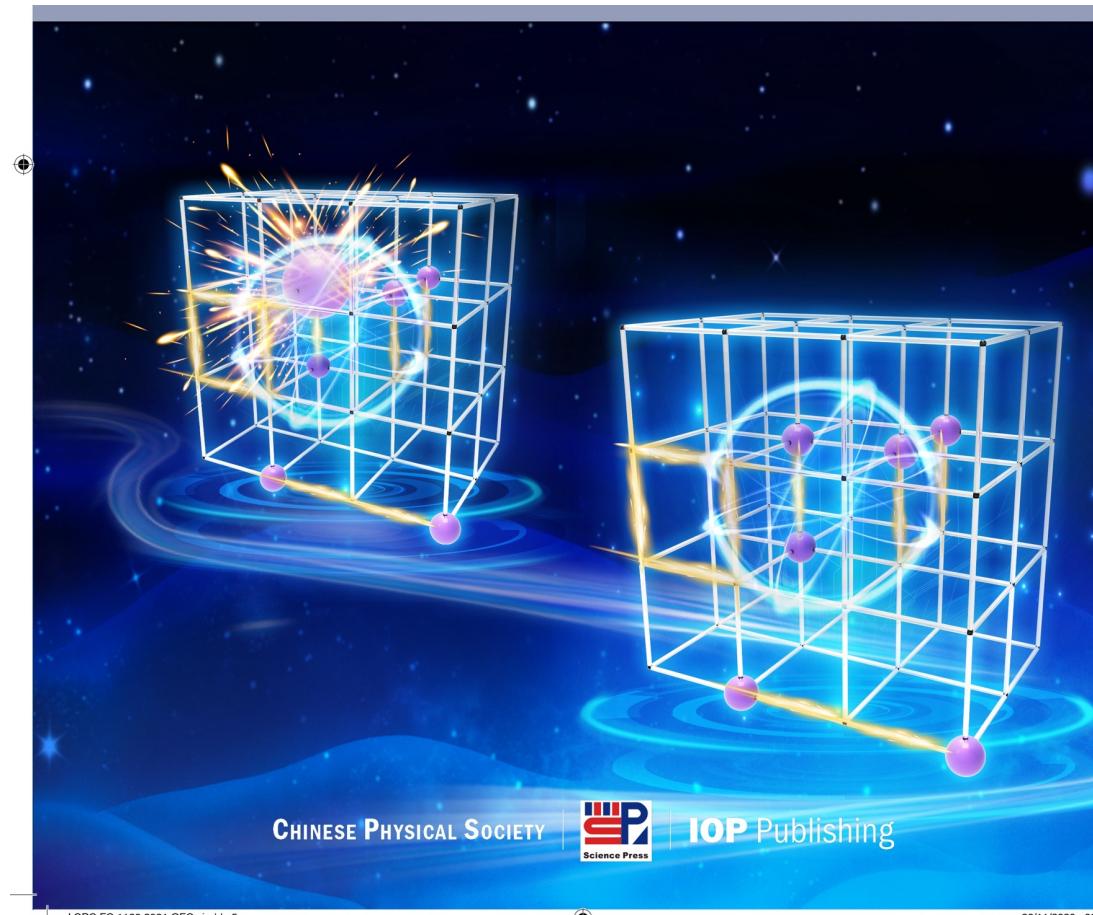
$$e^{-S_{\text{glue}}^{\text{latt}}(U)} \prod_q \det(D_{\mu}^{\text{latt}}(U) \gamma_{\mu} + am_q)$$

- Therefore

$$\langle \mathcal{O}(U, q, \bar{q}) \rangle = \frac{1}{N_{\text{conf}}} \sum_{k=1}^{N_{\text{conf}}} \tilde{\mathcal{O}}(U^{(k)})$$

➤ Have achieved great successes in calculating hadron masses, decay constants, α_s , form factors and so on.





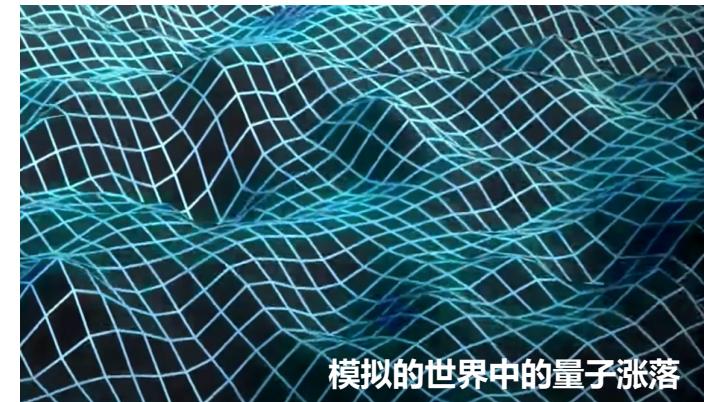
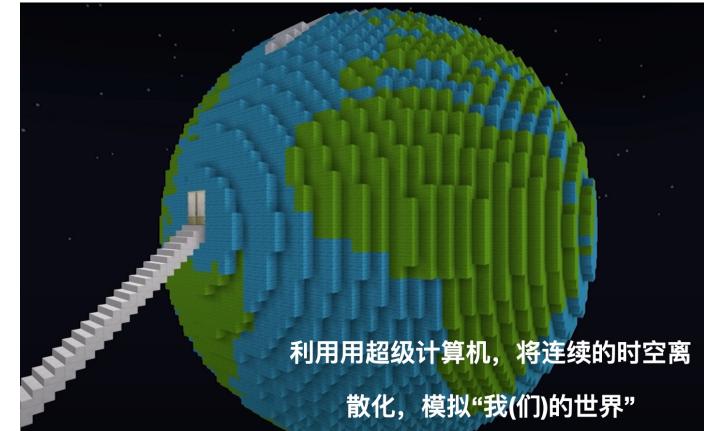
Chin.Phys.C 46 (2022) 7, 011002

Lattice QCD

- 格点量子色动力学第一性原理出发的理论；
- 利用超级计算机数值计算路径积分，实现**强相互作用**的模拟；

➤ 蒙特卡洛算法（Monte Carlo）：

- 对所有可能的规范场路径 U 的积分，
积分维度约 $10^8 \sim 10^{10}$ 维！
- **重点采样：生产马尔可夫链--> 组态**，
标志着模拟的微观世界的量子涨落





Lattice QCD: new configurations



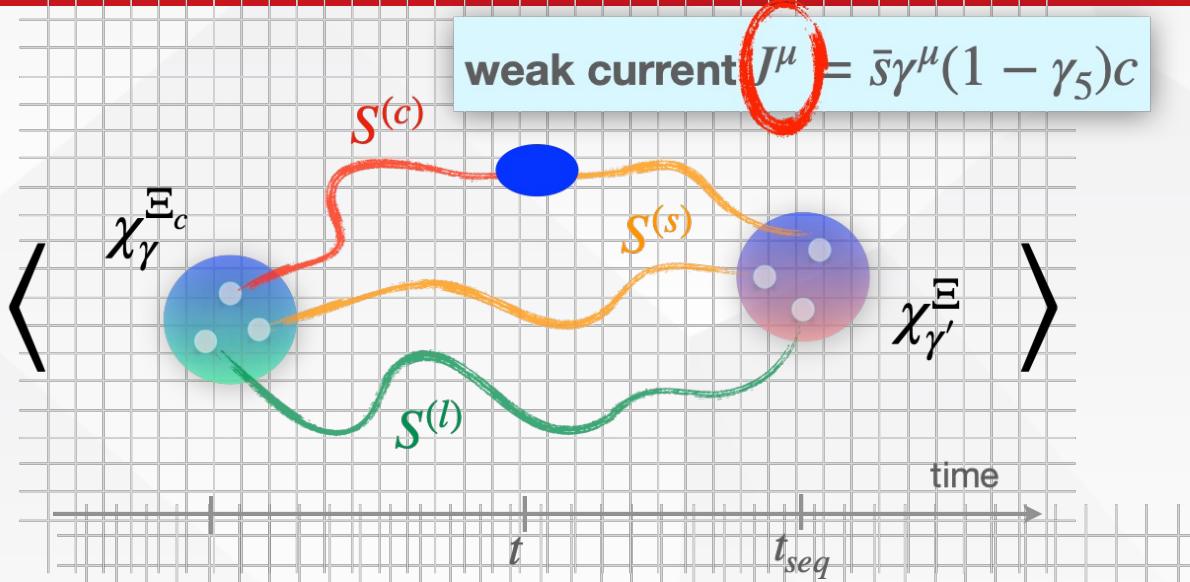
Lattice size	Lattice spacing	Pion mass	N cfg
24x72	0.108fm	280-290MeV	2000+
32x96	0.08fm	280-300MeV	1000+
48x144	0.055fm	280MeV	producing
48x96	0.108fm	200MeV	producing
48x96	0.108fm	140MeV	prepare

Liuming Liu, Peng Sun, Wei Sun, Yibo Yang

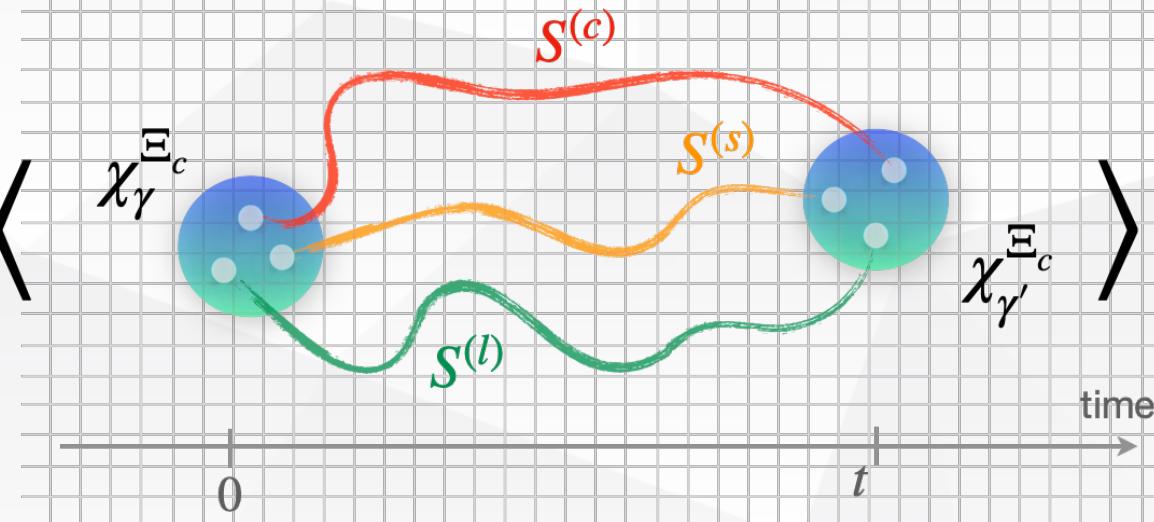
Form factor on the LQCD



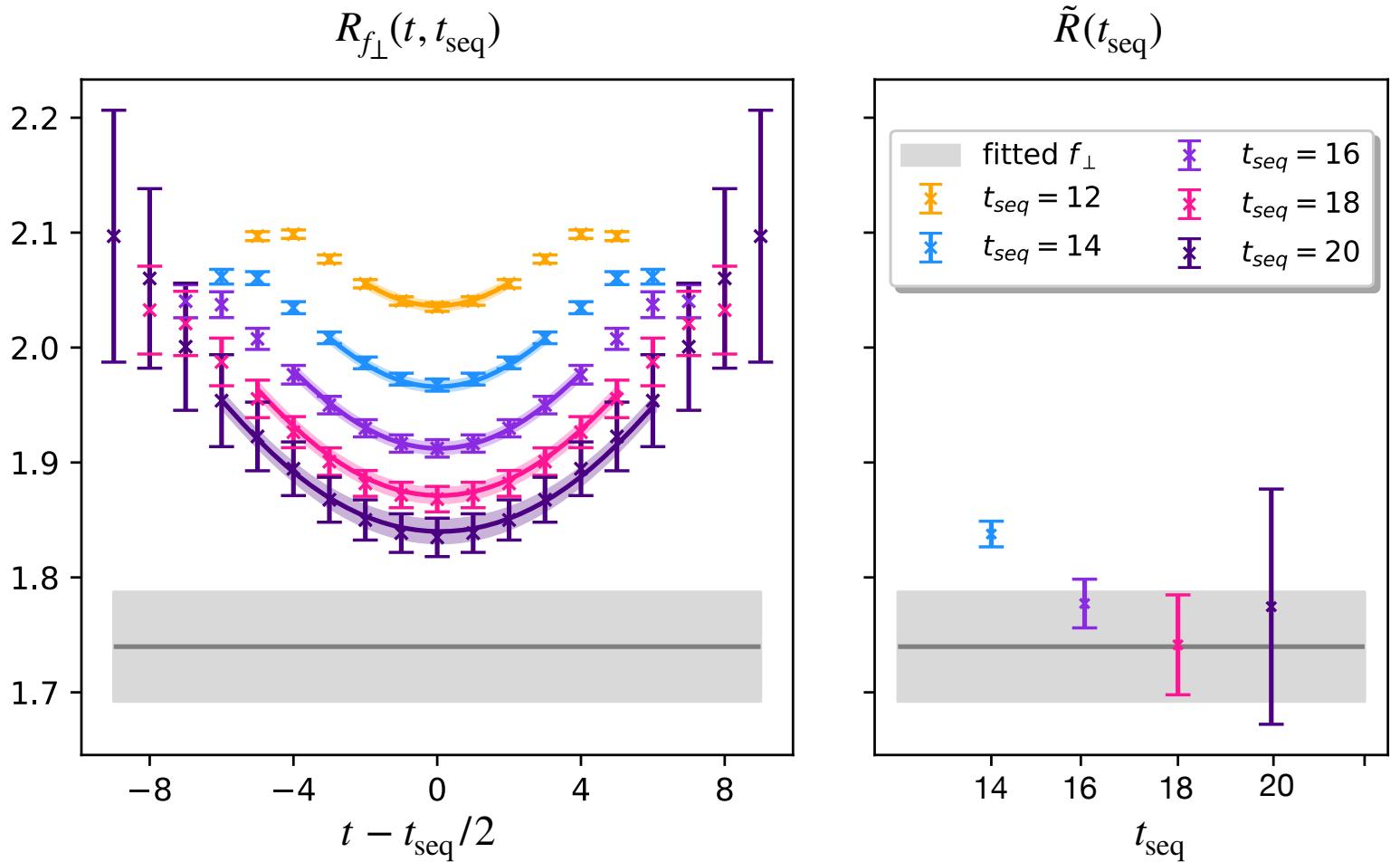
$$C_3(q^2, t, t_{seq}) \sim T^{\gamma\gamma'} \langle$$



$$C_2^{E_c}(t) \sim I^{\gamma\gamma'} \langle$$



Form factor on the LQCD



$$R_{f_\perp} \equiv \frac{R_V(\gamma_5 \gamma^x, \gamma^y)}{4m_{\Xi_c} N_z \hat{p}} = f_\perp \left(\frac{\left(1 + c_1 e^{-\Delta E_1 t} + c_2 e^{-\Delta E_2 (t_{\text{seq}} - t)} \right) \left(1 + c_1 e^{-\Delta E_1 (t_{\text{seq}} - t)} + c_2 e^{-\Delta E_2 t} \right)}{(1 + d_1 e^{-\Delta E_1 t_{\text{seq}}}) (1 + d_2 e^{-\Delta E_2 t_{\text{seq}}})} \right)^{1/2}$$

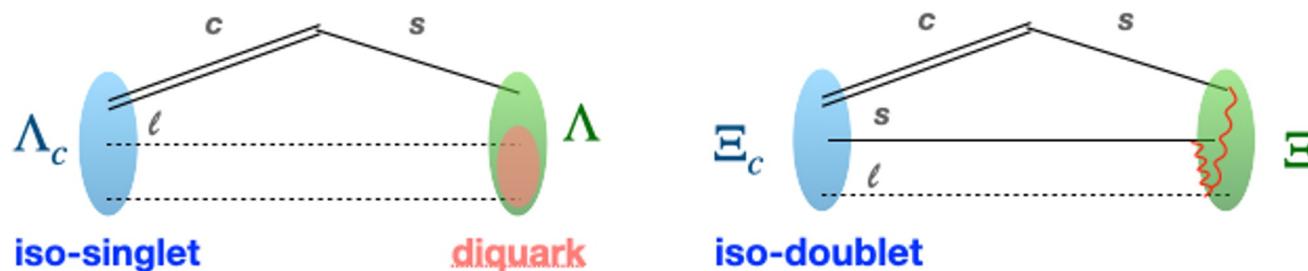


Ξ_c Decays



- Ξ_c contains more versatile decay modes

- $\Xi_c \rightarrow \Xi$ contain different QCD dynamics with $\Lambda_c \rightarrow \Lambda$;



- A different pattern between inclusive and exclusive decays of Λ_c and D :

$$\frac{\mathcal{B}(\Lambda_c^+ \rightarrow X e^+ \nu_e)}{\mathcal{B}(\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e)} = (3.95 \pm 0.34 \pm 0.09) \%$$

~ 1

$$\frac{\mathcal{B}(D^0 \rightarrow X e^+ \nu_e)}{\mathcal{B}(D^0 \rightarrow K^- e^+ \nu_e)} = (6.49 \pm 0.11) \%$$

~ 2

M. Ablikim et al. [BESIII], PRL121, 251801 (2018)

- Importance for the experimental researches of heavy baryons:

-Studies of doubly-charmed baryon Ξ_{cc}^{++} decay

R. Aaij et al. [LHCb], PRL121, 162002 (2018)

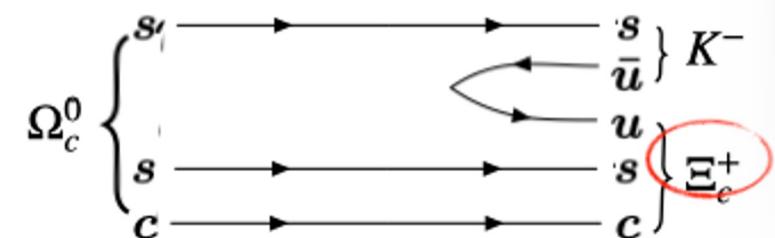
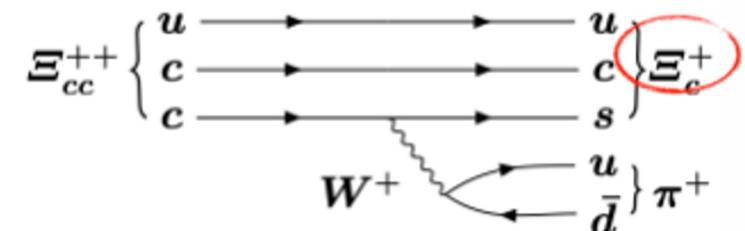
-Precision measurement of the lifetime of Ξ_b^0

R. Aaij et al. [LHCb], PRL113, 032001 (2014)

-Discovery of new exotic hadron candidates Ω_c^{11}

R. Aaij et al. [LHCb], PRL118, 182001 (2017)

.....





Ξ_c Decays



✓ Experimental

Belle $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50) \%$

*Y. B. Li et al. [Belle],
arXiv:2103.06496 [hep-ex].*

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu) = (1.71 \pm 0.17 \pm 0.13 \pm 0.50) \%$$

ALICE $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (2.43 \pm 0.25 \pm 0.35 \pm 0.72) \%$

*J. Zhu on behalf of the
ALICE collaboration, PoS
ICHEP2020 (2021) 524.*

✓ Theoretical

QCD SR $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (3.4 \pm 1.7) \%$ *Z. X. Zhao, arXiv:2103.09436 [hep-ph].*

LF QM $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (3.49 \pm 0.95) \%$ *C. Q. Geng et al, arXiv:2012.04147 [hep-ph].*

LCSR $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (2.4^{+0.9}_{-1.0}) \%$ *Y. L. Liu et al, J. Phys. G 37, 115010 (2010).*

Lattice

?

Ξ_c Decays on the Lattice



$\beta = \frac{10}{g^2}$	$L^3 \times T$	a	c_{sw}	κ_l	m_π	κ_s	m_{η_s}	
s108	6.20	$24^3 \times 72$	0.108	1.161	-0.2770	290	0.1330	640
s080	6.41	$32^3 \times 96$	0.080	1.141	-0.2295	300	0.1318	650

Zhang, Hua, et.al., 2103.07064, Chinese Physics C

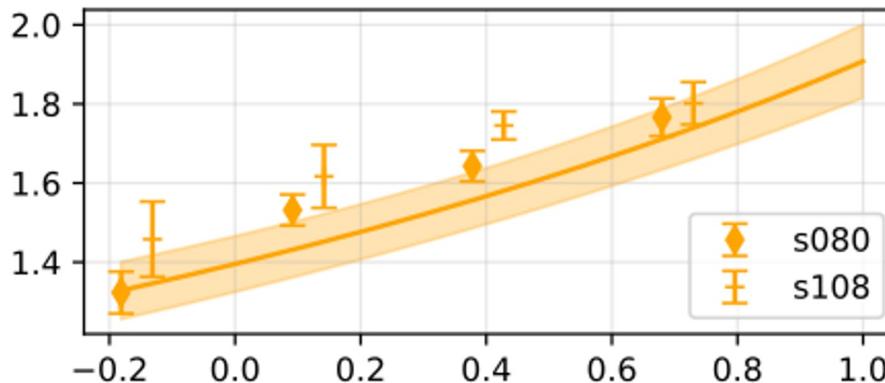
Ξ_c Decays on the Lattice



- Extrapolate to the **continuum limit** (shaded regions);
- **z -expansion parametrization** to obtain the **q^2 -distribution**:

$$f(q^2) = \frac{1}{1 - q^2 / (m_{\text{pole}}^f)^2} \sum_{n=0}^{n_{\text{max}}} (c_n^f + d_n^f a^2) [z(q^2)]^n$$

- Use **D_s meson pole mass** for $m_{\text{pole}}^{f_\perp}$, ...
- Consider the **discretization effects** by estimating the d_n^f terms.



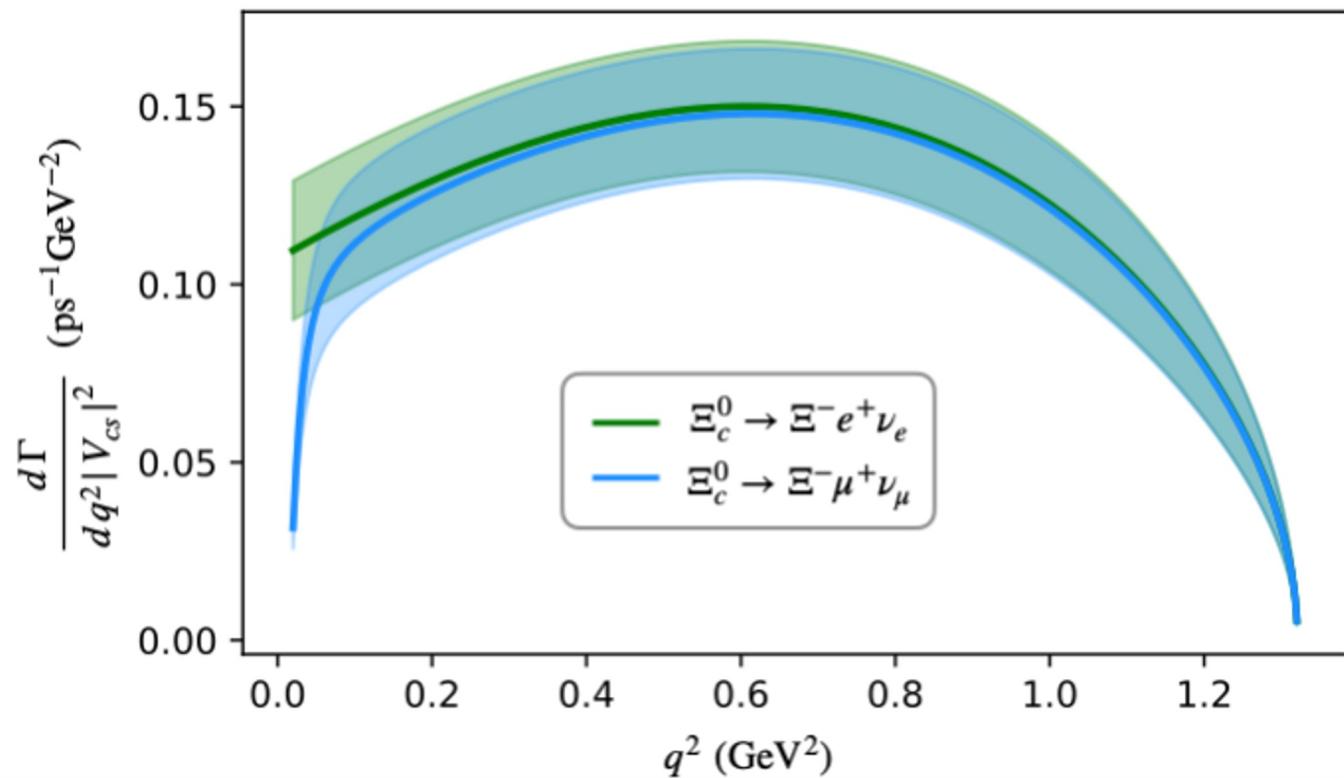
Fit results for the z -expansion parameters

	c_0	c_1	c_2
f_\perp	1.51 ± 0.09	-1.88 ± 1.21	1.71 ± 0.49
f_0	0.64 ± 0.09	-1.83 ± 1.22	0.56 ± 0.51
f_+	0.77 ± 0.07	-4.09 ± 1.18	0.35 ± 0.49
g_\perp	0.56 ± 0.07	-0.35 ± 1.26	0.15 ± 0.29
g_0	0.63 ± 0.07	-1.37 ± 1.36	0.15 ± 0.29
g_+	0.56 ± 0.08	0.00 ± 1.38	0.14 ± 0.29

Ξ_c Decays on the Lattice



-The differential decay widths of $\Xi_c^0 \rightarrow \Xi^- l^+ \nu_l$:





Ξ_c Decays on the Lattice



-Predicted decay branching fractions:

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = 2.38(0.30)(0.32)(0.07) \%$$

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu) = 2.29(0.29)(0.30)(0.06) \%$$

$$\mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e) = 7.18(0.90)(0.96)(0.20) \%$$

$$\mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 \mu^+ \nu_\mu) = 6.91(0.87)(0.91)(0.19) \%$$

- Statistical errors
- Systematic errors from continuum extrapolation
- Systematic errors from renormalization

$$(2.38 \pm 0.44) \%$$

-Compare with PDG, experiment and theory:

PDG $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (1.8 \pm 1.2) \%$



Belle $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50) \%$



ALICE $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (2.43 \pm 0.25 \pm 0.35 \pm 0.72) \%$



QCD SR $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (3.4 \pm 1.7) \%$



LFQM $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (3.49 \pm 0.95) \%$



LCSR $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (2.4^{+0.9}_{-1.0}) \%$



Fitted well with all
data (within $1-\sigma$) !



Ξ_c Decays on the Lattice



-From Belle measurements:

Y. B. Li et al. [Belle], arXiv:2103.06496 [hep-ex].

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50)\%$$

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu) = (1.71 \pm 0.17 \pm 0.13 \pm 0.50)\%$$

$$|V_{cs}| = 0.834 \pm (0.051)_{\text{stat.}} \pm (0.056)_{\text{syst.}} \pm (0.127)_{\text{exp.}}$$

Theo. error ~ 8.9% Exp. error ~ 15.2%

-From ALICE measurements:

J. Zhu, PoS ICHEP2020 (2021) 524.

$$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (2.43 \pm 0.25 \pm 0.35 \pm 0.72)\%$$

$$|V_{cs}| = 0.983 \pm (0.060)_{\text{stat.}} \pm (0.065)_{\text{syst.}} \pm (0.167)_{\text{exp.}}$$

Exp. error ~ 17.0%

-Compare with PDG result:

$$|V_{cs}| = 0.987 \pm 0.011$$

$$|V_{cs}| = 0.939 \pm 0.038 \quad D \rightarrow K \ell \bar{\nu}$$

From the uncertainty of $\Xi_c^0 \rightarrow \Xi^- \pi^+$

Theoretical uncertainties:

- total ~ 8.9%
- statistical ~ 6.1%
- systematic from extrapolation ~ 6.5%
- systematic from renormalization ~ 1.5%

Experimental uncertainties:

- Belle ~ 15.2%
- ALICE ~ 17.0%



Charmed Baryon Decays on the Lattice: Prospect

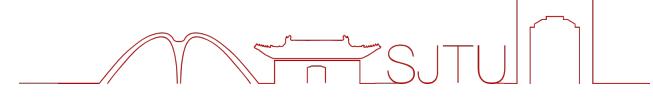


- More precise:
 - Continuum limit: one more lattice spacing;
 - Physical pion mass: 290MeV/300MeV → 140MeV;
 - A finer grid to obtain a better resolution of q^2 -dependence;
- More general:
 - Generalized analysis of semileptonic weak decays of singly charmed baryons (Λ_c^+ , $\Xi_c^{0,+}$, Ω_c^0);
 - Doubly charmed baryon decays;
 -



上海交通大学

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Backup



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