Quantum Simulation: from Quantum Matter to Gauge Theory

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Quantum Simulation of the PXP Model Two Realizations of the PXP Model: 1. Rydberg Atom Array 2. Lattice Gauge Simulator Physics in the PXP Model 1. Quantum Thermalization 2. Confinement-Deconfinement Transition 3. Quantum Spin Liquid How gauge theory picture helps understand quantum matter?

Two Realizations of the PXP Model

 $\hat{H} = \Omega \sum P_{i-1} \hat{S}_i^x P_{i+1} - \Delta \hat{n}_i$ i

Rydberg Atom

A Rydberg atom is an excited atom with one or more electrons that have a very high principal quantum number



Rydberg Atom Arrays



Platform for Quantum Simulation and Quantum Computation

Rydberg Blockade

 $\hat{H} = \sum_{i} \left(\Omega \hat{S}_{i}^{x} - \Delta \hat{n}_{i} \right) + \sum_{ij} V_{ij} \hat{n}_{i} \hat{n}_{j}$





Rydberg Blockade

$$\hat{H} = \sum_{i} \left(\Omega \hat{S}_{i}^{x} - \Delta \hat{n}_{i} \right) + \sum_{ij} V_{ij} \hat{n}_{i} \hat{n}_{j}$$
$$V_{ij} = V \quad \text{for} \quad j = i \pm 1$$
$$V_{ij} = 0 \quad \text{otherwise}$$

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 $\mathcal{M} = \prod_{i} (1 - n_i n_{i+1})$ Prevent



PXP Hamiltonian

$$\begin{split} \hat{H} &= \sum_{i} \left(\Omega \hat{S}_{i}^{x} - \Delta \hat{n}_{i} \right) + \sum_{ij} V_{ij} \hat{n}_{i} \hat{n}_{j} \\ V_{ij} &= V \quad \text{for} \quad j = i \pm 1 \\ V_{ij} &= 0 \quad \text{otherwise} \\ \mathcal{M} &= \prod_{i} (1 - n_{i} n_{i+1}) \quad \text{Prevent} \quad \bullet \quad \bullet \\ \mathcal{M} \hat{H} \mathcal{M} \quad \bullet \quad \hat{H} &= \Omega \sum_{i} P_{i-1} \hat{S}_{i}^{x} P_{i+1} - \Delta \hat{n}_{i} \\ P_{i} &= 1 - n_{i} \end{split}$$

Quantum Simulation with Rydberg Atoms Array





2)

l





Physical charge



 $|\downarrow\rangle$

te Matter Site

Gauge Site

Gauge Site

 \uparrow

$$\hat{H}_{\text{eff}} = \sum_{l} \left[-\frac{\tilde{J}}{2} \hat{S}_{l,l+1}^{+} \hat{b}_{l} \hat{b}_{l+1} + \text{h.c.} + m \hat{n}_{l} \right]$$

Lattice Schwinger Model

Local Conserved Quantity

$$\hat{H}_{\text{eff}} = \sum_{l} \left[-\frac{\tilde{J}}{2} \hat{S}_{l,l+1}^{+} \hat{b}_{l} \hat{b}_{l+1} + \text{h.c.} + m\hat{n}_{l} \right]$$

Local Gauge Symmetry:

$$\hat{b}_l \rightarrow e^{i\theta_l}\hat{b}_l$$

$$\hat{S}_{l,l+1}^+ \to e^{-i\theta_l} \hat{S}_{l,l+1}^+$$

$$\hat{S}^+_{l-1,l} \to e^{-i\theta_l} \hat{S}^+_{l-1,l}$$

Conservation Quantity:

$$G_l = S_{l-1,l}^z + S_{l,l+1}^z + n_l$$

Local Conserved Quantity

$$\hat{H}_{\text{eff}} = \sum_{l} \left[-\frac{\tilde{J}}{2} \hat{S}_{l,l+1}^{+} \hat{b}_{l} \hat{b}_{l+1} + \text{h.c.} + m \hat{n}_{l} \right]$$

Conservation Quantity:

$$G_l = S_{l-1,l}^z + S_{l,l+1}^z + n_l$$

Gauss's Law: $\nabla \cdot \mathbf{E} = \rho$

$$E_{l-1,l} = (-1)^l S_{l-1,l}^z \qquad Q_l = (-1)^l n_l$$

$$E_{l,l+1} - E_{l-1,l} = Q_l$$

Local Conserved Quantity

$$\hat{H}_{\text{eff}} = \sum_{l} \left[-\frac{\tilde{J}}{2} \hat{S}_{l,l+1}^{+} \hat{b}_{l} \hat{b}_{l+1} + \text{h.c.} + m\hat{n}_{l} \right]$$

$$G_l = S_{l-1,l}^z + S_{l,l+1}^z + n_l$$





Lattice Gauge Realization of the PXP Model

$$\hat{H}_{\text{eff}} = \sum_{l} \left[-\frac{\tilde{J}}{2} \hat{S}_{l,l+1}^{+} \hat{b}_{l} \hat{b}_{l+1} + \text{h.c.} + m\hat{n}_{l} \right]$$

Gauge charge

No need to read the matter site





Lattice Gauge Realization of the PXP Model

$$\hat{H}_{\text{eff}} = \sum_{l} \left[-\frac{\tilde{J}}{2} \hat{S}_{l,l+1}^{+} \hat{b}_l \hat{b}_{l+1} + \text{h.c.} + m\hat{n}_l \right]$$

Gauge charge



Connected by spin flip



Lattice Gauge Realization of the PXP Model

$$\hat{H}_{\text{eff}} = \sum_{l} \left[-\frac{\tilde{J}}{2} \hat{S}_{l,l+1}^{+} \hat{b}_{l} \hat{b}_{l+1} + \text{h.c.} + m \hat{n}_{l} \right]$$

Gauge charge

$$\hat{H} = \sum_{l} \left[\hat{P}_{l-1,l} \left(-\frac{\tilde{J}}{2} \hat{S}_{l,l+1}^{+} + \text{h.c.} \right) \hat{P}_{l+1,l+2} - m(S_{l-1,l}^{z} + S_{l,l+1}^{z}) \right]$$

Gauge Conservation

Matter Field Mass



Rydberg Blockade



Physics in the PXP Model

1. Quantum Thermalization

Loss of initial state memory

 $|\Psi
angle$



 T, μ, \ldots

Quantum wave function

Unitary evolution

Thermal equilibrium

 $\langle \hat{\mathcal{O}} \rangle_{\infty} = \langle \rho_{eq}(E) \hat{\mathcal{O}} \rangle$

Local observable

Equilibrium Density Matrix of the Whole System

Sufficient Long Time Evolution

Loss of initial state memory



Munich Group, Science 2015

Eigen-state thermalization

$$\begin{split} |\Psi(0)\rangle &= \sum_{\alpha} a_{\alpha} |\alpha\rangle & \qquad |\Psi(t)\rangle = \sum_{\alpha} a_{\alpha} e^{-iE_{\alpha}t} |\alpha\rangle \\ \langle \hat{\mathcal{O}} \rangle_{\infty} &= \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} \langle \Psi(t) | \hat{\mathcal{O}} | \Psi(t) \rangle dt = \sum_{\alpha} |a_{\alpha}|^{2} \langle \alpha | \hat{\mathcal{O}} | \alpha \rangle \end{split}$$

no Eigodicity; all eigenstates are thermal

$$\langle \alpha | \hat{\mathcal{O}} | \alpha \rangle = \langle \rho_{\rm mc}(E_{\alpha}) \hat{\mathcal{O}} \rangle$$

a function of energy only

 α



Volume-law of entanglement entropy



Harvard Group, Science, 2016



HZ, Ultracold Atomic Physics

Quantum Many-Body Scars

ARTICLE

doi:10.1038/nature24622

Probing many-body dynamics on a 51-atom quantum simulator

Nature 2017

Hannes Bernien¹, Sylvain Schwartz^{1,2}, Alexander Keesling¹, Harry Levine¹, Ahmed Omran¹, Hannes Pichler^{1,3}, Soonwon Choi¹, Alexander S. Zibrov¹, Manuel Endres⁴, Markus Greiner¹, Vladan Vuletić² & Mikhail D. Lukin¹



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uantum Many-Body

Scars

Nature 2017



Why This is Unusual?

ARTICLE

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physics Many-Body Scar in PXP Hamiltonian

Weak ergodicity breaking from quantum many-body scars

Nature Physics 2018

C. J. Turner¹, A. A. Michailidis^{1,2}, D. A. Abanin³, M. Serbyn² and Z. Papić¹*



Many-Body Scar in PXP Hamiltonian

Significantly low entanglement entropy



Many-Body Scar in PXP Hamiltonian

Z2 state cannot thermalize





Phase Diagram of Lattice Schwinger Model

$$\hat{H}_{\text{eff}} = \sum_{l} \left[-\frac{\tilde{J}}{2} \hat{S}_{l,l+1}^{+} \hat{b}_{l} \hat{b}_{l+1} + \text{h.c.} + m\hat{n}_{l} \right]$$

 $m \to \infty$

 $m \to -\infty$

Matter sites occupied

 $\begin{array}{c|c} & & & & & \\ \hline \\ |\uparrow\rangle & & |\downarrow\rangle & & |\uparrow\rangle & & |\downarrow\rangle & & |\uparrow\rangle & & |\downarrow\rangle \\ \hline \\ & & & & & \\ \hline \\ |\downarrow\rangle & & |\uparrow\rangle & & |\downarrow\rangle & & |\uparrow\rangle & & |\downarrow\rangle & & |\uparrow\rangle \end{array}$

Matter sites vacuum

m

Z2 Symmetry Breaking Ising Transition
Phase Diagram of Lattice Schwinger Model

 $\left|\downarrow\right\rangle$

Matter sites occupied



2N)



Matter sites vacuum



USTC Nature 2020



Driving Scars to Critical



Prediction of Ising CFT



Prediction of Ising CFT





PXP Model

Transverse Field Ising Model

Free Fermion States



Thermalization and Criticality





Experimental Observations



Interrelated Thermalization and Quantum Criticality in a Lattice Gauge Simulator

arXiv: 2210.17032



Precise Determine the Quantum Critical Point



Precise Determine the Thermalization Value

From Ground State to Excitations



From Ground State to Excitations

From Ground State to Excitations

Ferromagnetic state of dressed spins



$$\hat{\eta}_i^{\dagger}|\mathrm{GS}\rangle =$$

 $\hat{\eta}_i^{\dagger} \left(u \right| \Uparrow \rangle_i + v | \Downarrow \rangle_i \right) = -v | \Uparrow \rangle_i + u | \Downarrow \rangle_i$

Spin wave







Quantum Phase Transition

$$\hat{H} = \Omega \sum_{i} P_{i-1} \hat{S}_i^x P_{i+1} - \Delta \hat{n}_i$$



Quantum phase transition

Physics in the PXP Model

1. Quantum Thermalization

2. Confinement-Deconfinement Transition

Topological Angle



What is Missing ?







Topological Angle



Probe Confinement-Deconfinement



Confinement-Deconfinement Transition



Confinement-Deconfinement in the PXP Model

$$\hat{A}_{l} = -\hat{S}_{l-1,l}^{z} - \hat{S}_{l,l+1}^{z} + \bar{S}_{l-1,l}^{z} + \bar{S}_{l,l+1}^{z}$$

$$\mathcal{G}(r,t) = \sum_{l} \langle \Psi(t) | \hat{A}_{l} \hat{A}_{l+r} | \Psi(t) \rangle$$



Physics in the PXP Model

- Quantum Thermalization
 Confinement-Deconfinement Transition
- 3. Quantum Spin Liquid

Quantum Spin Liquid with Rydberg Atom Arrays

RESEARCH

TOPOLOGICAL MATTER

Probing topological spin liquids on a programmable quantum simulator

G. Semeghini¹, H. Levine¹, A. Keesling^{1,2}, S. Ebadi¹, T. T. Wang¹, D. Bluvstein¹, R. Verresen¹, H. Pichler^{3,4}, M. Kalinowski¹, R. Samajdar¹, A. Omran^{1,2}, S. Sachdev^{1,5}, A. Vishwanath^{1*}, M. Greiner^{1*}, V. Vuletić^{6*}, M. D. Lukin^{1*}





Rydberg Blockade



Rydberg Blockade







Quantum Spin Liquid in Rydberg Atom Arrays

RESEARCH

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Science 2021





Spin Liquid: A quantum superposition of exponentially many spin configurations.



Advantage of Quantum Simulator: Direct Probe Topological Order (= Long Rang Coherence)





Number Parity of Rydberg Atoms along the Loop

Advantage of Quantum Simulator: Direct Probe Topological Order (= Long Rang Coherence)



Coherence between Different Configurations







Quantum Many-Body State of Spin Liquid

$$\hat{H}_{\text{LGT}} = \sum_{\langle ij \rangle} -\Omega \left(\hat{S}_{\langle ij \rangle}^{-} \hat{f}_{i}^{\dagger} \hat{f}_{j}^{\dagger} + \text{h.c.} \right) - \Delta \hat{n}_{\langle ij \rangle}$$
$$|\text{QSL}\rangle = \frac{1}{\mathcal{N}} \prod_{\langle ij \rangle} \left(u + v \hat{f}_{i}^{\dagger} \hat{f}_{j}^{\dagger} \hat{S}_{\langle ij \rangle}^{-} \right) |\text{vac}\rangle$$
$$|\text{vac}\rangle = |0\rangle \otimes |\uparrow\rangle^{\otimes N}$$

Very efficient way to represent superposition

Recover two limits of positive and negative detuning

Automatically satisfy the constraint

BCS wave function possesses Z2 topological order

Quantum Many-Body State of Spin Liquid v Soability 0.0 8.0

$$|\text{QSL}\rangle = \frac{1}{\mathcal{N}} \prod_{\langle ij \rangle} \left(u + v \hat{f}_i^{\dagger} \hat{f}_j^{\dagger} \hat{S}_{\langle ij \rangle}^{-} \right) |\text{vac}\rangle$$
$$|\text{vac}\rangle = |0\rangle$$

Comparing with exact diagonalization



Wave function overlap

Energy difference

-2



Evidence of Quantum Spin Liquid




without any fitting parameter

 $\Delta X^2 \equiv \langle \hat{X}_{\text{close}} \rangle - \langle \hat{X}_{\text{open}} \rangle^2$

Yanting Cheng, Chengshu Li and HZ, arXiv 2021



 $\tilde{t} \approx 8 2 I^2 / I \approx 70 \, \text{Hz}$

References and Collaborators

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- Yanting Cheng, Chengshu Li and HZ, arXiv 2021
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Thank you very much for your attention !