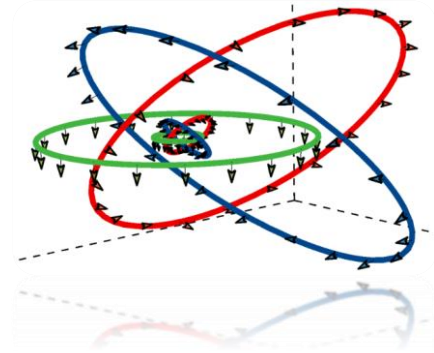


Synthetic topological Yang-Mills vacua in ultracold atomic Bose Einstein condensates



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- Introduction
- Topology of $SU(2)$ gauge field: Winding
- Simulating topological $SU(2)$ vacua
- Experimental realization using a BEC
- Outlook

What's a vacuum ?



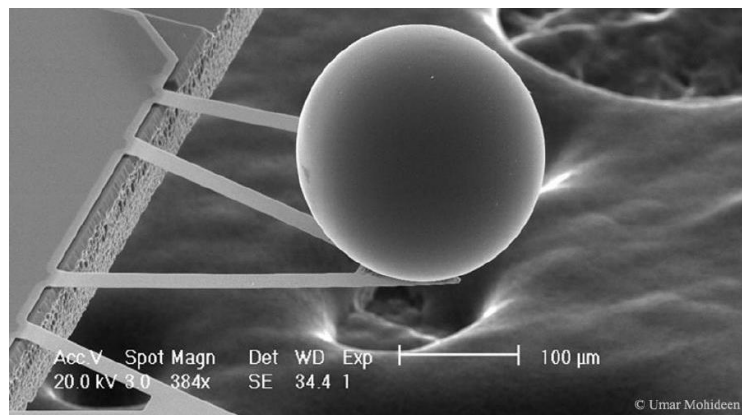
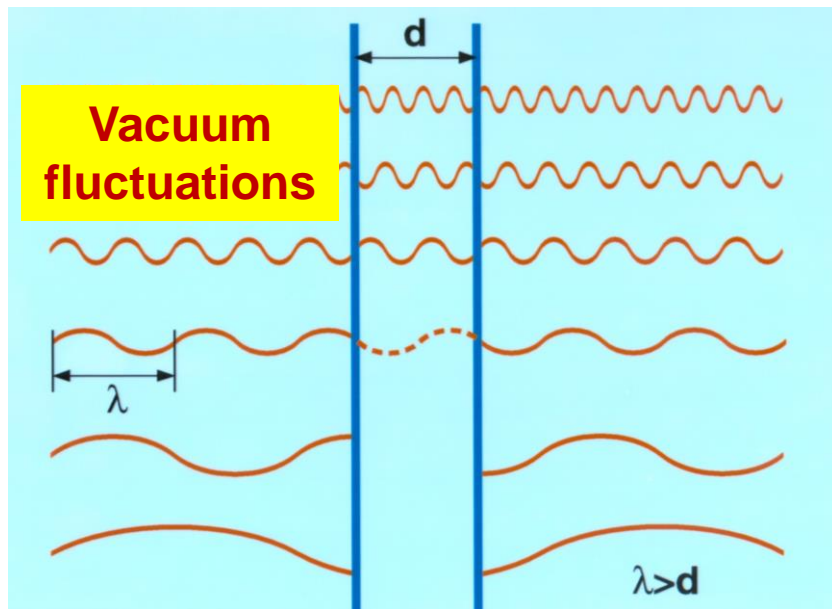
empty ??

Vacuum is **NOT** empty

lin

QED Vac: Local Effect

Casimir Effects



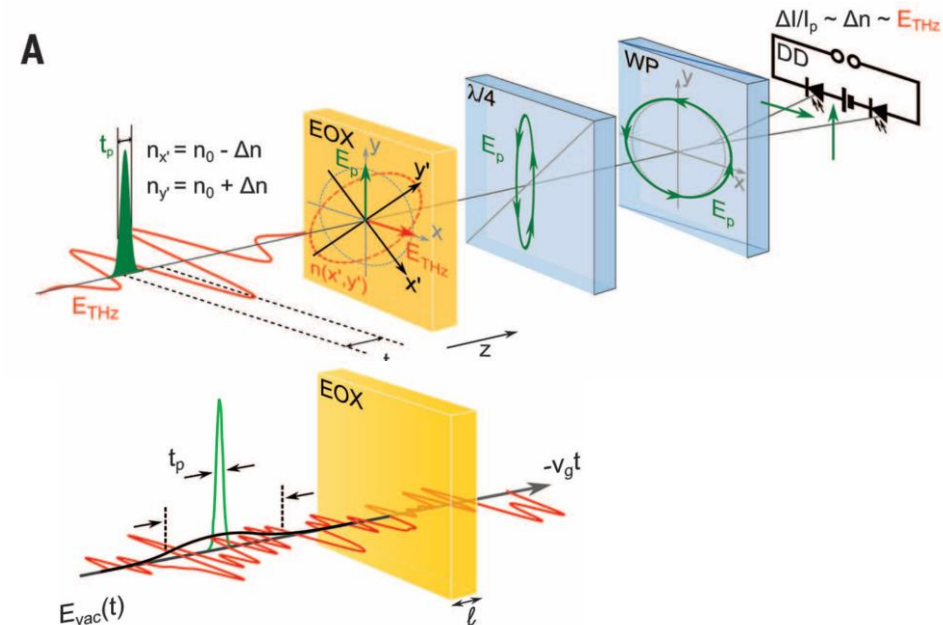
Pic: Nothing But Net | by Brian Koberlein

QUANTUM OPTICS

Direct sampling of electric-field vacuum fluctuations

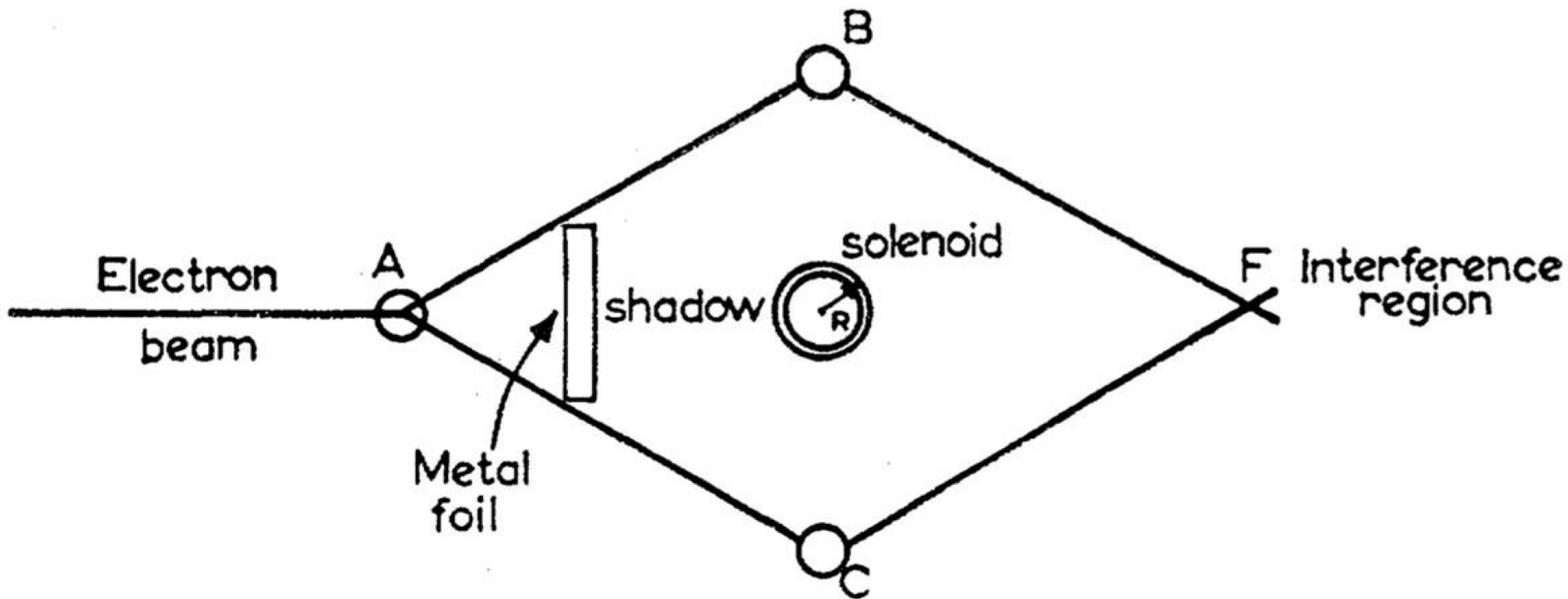
C. Riek, D. V. Seletskiy, A. S. Moskalenko, J. F. Schmidt, P. Krauspe, S. Eckart, S. Eggert, G. Burkard, A. Leitenstorfer*

The ground state of quantum systems is characterized by zero-point motion. This motion, in the form of vacuum fluctuations, is generally considered to be an elusive phenomenon that manifests itself only indirectly. Here, we report direct detection of the vacuum fluctuations of electromagnetic radiation in free space. The ground-state electric-field variance is inversely proportional to the four-dimensional space-time volume, which we sampled electro-optically with tightly focused laser pulses lasting a few femtoseconds. Subcycle



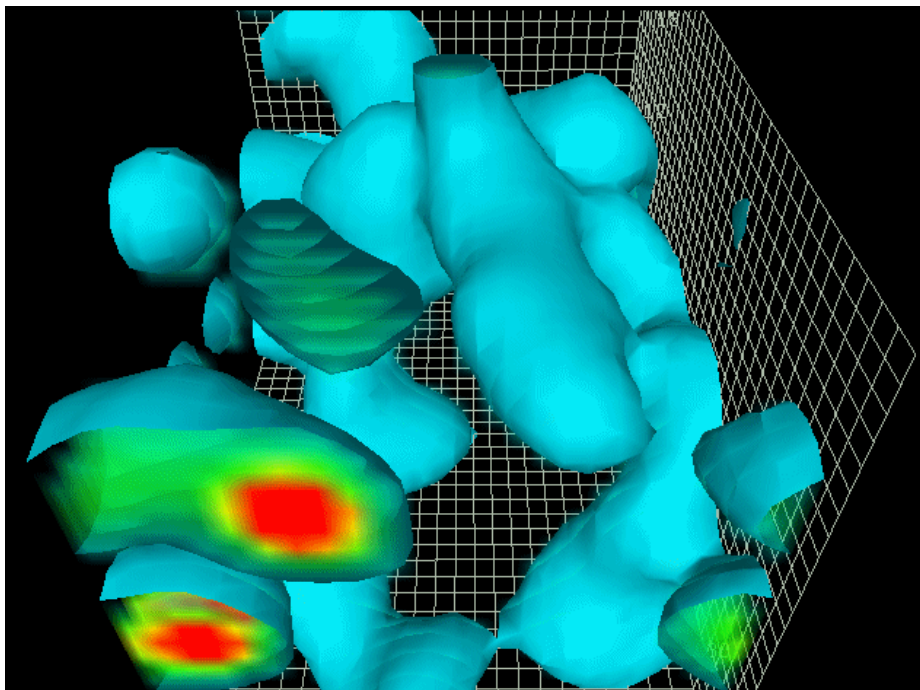
Science 350.420 (2015)

QED Vac: Global Effect

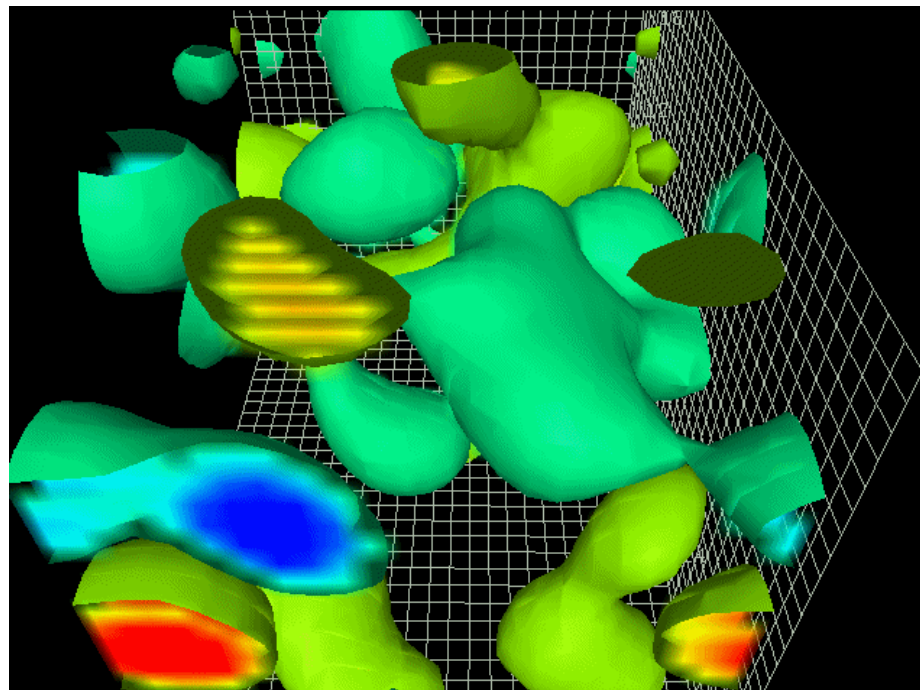


electron interference: Electromagnetic waves show **global geometric effects**

Action Density



Topological Charge Density



Vacuum structure matters !

Geometry of non-perturbative vacuum

- **Electromagnetic vacuum**
 - ✓ space-time “stratified” structure is charge-neutral;
 - ✓ can be in a *non-deformed state*;
 - ✓ delocalized zero-point fluctuations fill up the whole space-time
- **“Weak” vacuum (Higgs condensate)**
 - ✓ space-time “stratified” structure is spontaneously deformed;
 - ✓ layers are “weakly” charged;
 - ✓ deformations (shifts) are regular and *continuous*;
 - ✓ is *classically determined* and zero-point fluctuations is only slightly disturb it
- **“Strong” or QCD vacuum (Quark-Gluon condensate)**
 - ✓ space-time “stratified” structure is *spontaneously deformed*;
 - ✓ layers carry different “color” charges;
 - ✓ deformations are localized and determined totally by quantum effects;
 - ✓ such a structure is not classically determined

Physical Vacuum is the quantum superposition of substructures (vacuum condensates) constantly transforming one into another

Properties of matter are totally determined by properties of vacuum structures! 11

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EM waves: Vector potential matters

Maxwell Eqs: Coupled **Field** Eqs.

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

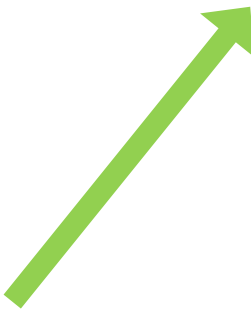
Potentials: Scalar and Vector

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0$$

$$\Leftrightarrow \mathbf{E} = -\nabla \Phi - \frac{\partial \mathbf{A}}{\partial t}$$

How to
determine
(\mathbf{A}, Φ)



Gauge choices: **Gauge transformation**

Field (\mathbf{B}, \mathbf{E}) are gauge invariant

$$\mathbf{A} \rightarrow \mathbf{A}' = \mathbf{A} + \nabla \Lambda$$

$$\Phi \rightarrow \Phi' = \Phi - \frac{\partial \Lambda}{\partial t}$$

e.g. Coulomb gauge: $\nabla \cdot \mathbf{A} = 0$

also transverse/radiation gauge

Lorentz condition: $\nabla \cdot \mathbf{A} + \frac{1}{c^2} \frac{\partial \Phi}{\partial t} = 0$

Classical: local field strength (\mathbf{E}, \mathbf{B}) + charge+ Maxwell Eq.

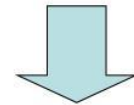
Quantum: nonlocal $\varphi_D = \frac{e}{\hbar c} \oint \mathcal{A} dr$ (Dirac Phase) e.g. **AB effect**

Gauge Symmetry

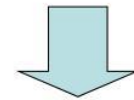
Noether theorem



All Gauge theories are based upon charge conservation.



The continuous symmetry that leads, by Noether's Theorem, to charge conservation is called Local Gauge Invariance



Local Gauge Invariance defines the full structure of electrodynamics

$$\frac{dL}{ds} = 0 \quad \Rightarrow$$

$$\frac{d}{dt} \left[\frac{\partial L}{\partial \dot{x}} \frac{dx}{ds} \right] = 0 \quad \Rightarrow$$

$$\mathbf{C} = \frac{\partial L}{\partial \dot{x}} \frac{dx}{ds} = \text{常数}$$

U(1) Group: **Abelian**

EM waves



Potentials (A, Φ):

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\nabla\Phi - \frac{\partial\mathbf{A}}{\partial t}$$

Observable:

Gauge invariant

In Minkowski space: 4-vector $(t, x, \dots) = (r_0, r_1, \dots)$

QM under **local GT**: $U = \exp(i\phi(r_\alpha)) \in U(1)$

$$\psi \rightarrow \exp(i\phi)\psi \text{ while } \mathbf{A} \rightarrow \mathbf{U}\mathbf{A}\mathbf{U}^{-1} + i\mathbf{U}^{-1}\partial_\alpha\mathbf{U}$$

$$\mathcal{A}^\alpha \rightarrow \mathcal{A}^\alpha - \partial_\alpha\phi, \mathcal{F}^{\alpha\beta} = \partial_\alpha\mathcal{A}^\beta - \partial_\beta\mathcal{A}^\alpha$$

$$\text{Vacuum: } \mathcal{F}^{\alpha\beta} = \partial_\alpha\mathcal{A}^\beta - \partial_\beta\mathcal{A}^\alpha = 0 \Rightarrow \mathcal{A}^\beta = 0$$

$$\varphi_D = \frac{e}{\hbar c} \oint \mathcal{A} d\mathbf{r} = \frac{e}{\hbar c} \int \mathcal{B} ds = \frac{e}{\hbar c} \Phi \text{ (Observable)}$$

T.T.Wu & Ch.N.Yang [PRD1975]: EM theory \Leftrightarrow **gauge invariance** of φ_D

U(1) determines the **EM properties**

U(n > 1) Group: Non-Abelian



Multi-component wavefunctions $\vec{\psi}$ ($n \geq 2$) $\Leftrightarrow \hat{A}$ gauge potential

Gauge transformation group $U(n)$: Non-Abelian

Gauge potential \hat{A} : local $\hat{U}(\mu) \in U(n)$

$$\vec{\psi} \rightarrow \hat{U}\vec{\psi}, \quad \hat{A} \rightarrow \hat{U}^{-1}\hat{A}\hat{U} + i\hat{U}^{-1}\partial_{\mu}\hat{U}$$

$$\mathcal{F}^{\alpha\beta} = \partial_{\alpha}\mathcal{A}^{\beta} - \partial_{\beta}\mathcal{A}^{\alpha} - ig[\mathcal{A}^{\alpha}, \mathcal{A}^{\beta}]$$

In certain initial frame: $\hat{A}(\mu) = \mathbf{0} \rightarrow$ Pure gauge potential: $\hat{A} = ig^{-1}\hat{U}^{-1}\partial_{\mu}\hat{U}$

Zero Field strength (vacuum): $\mathcal{F}^{\alpha\beta} = 0$ (no force)

Non-Abelian $\hat{U} \rightarrow \varphi_B = \hat{P} \oint \hat{A}d\mu \neq 0$ (Wilson-Loop)

Topology of Vacuum

Yang-Mills Field – Non-Abelian gauge field $\hat{U}(\vec{x}) \in U(n)$

$\vec{\psi}(\vec{x}) \rightarrow \hat{U}(\vec{x})\vec{\psi}(\vec{x}) \rightarrow$ Pure gauge: $\hat{A} = ig^{-1}\hat{U}^{-1}\partial_\mu\hat{U} \rightarrow L = \frac{1}{4}Tr(F_{uv}F^{uv}) = 0$

Ground state of Yang-Mills Gauge Field \rightarrow Vacuum

Topological invariant (winding number) $w = \frac{1}{24\pi^2} \int d^3x \epsilon_{ijk} Tr(A_i A_j A_k)$

e.g. $\hat{U}(\vec{x}) \in SU(2)$

$$\hat{U}^{(1)}(\vec{x}) = \frac{\vec{x}^2 - \eta^2}{\vec{x}^2 + \eta^2} + \frac{2i\eta\vec{\sigma} \cdot \vec{x}}{\vec{x}^2 + \eta^2}$$

$\hat{U}^{(n)}(\vec{x}) = [\hat{U}^{(1)}(\vec{x})]^n \rightarrow w = n$, w -fold degenerate vacua

Emergent Synthetic Gauge Field

Schrödinger equation for the motion of an atom:

$$i\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2}{2m} \nabla^2 + \hat{H}_s(\vec{r}) \right) \psi$$

$$\hat{H}_s(\vec{r}) = \mu_B g_S \hat{\mathbf{B}}(\vec{r}) \cdot \hat{\mathbf{S}} + \mu_N g_I \hat{\mathbf{B}}(\vec{r}) \cdot \hat{\mathbf{I}} + \alpha_{hf} \hat{\mathbf{J}} \cdot \hat{\mathbf{I}}$$

introduce a unitary matrix $\hat{\mathbf{U}}(\vec{r}) \rightarrow \hat{\mathbf{U}}^\dagger(\vec{r}) \hat{H}_s(\vec{r}) \hat{\mathbf{U}}(\vec{r}) = \hat{\Lambda}(\vec{r})$ and $\tilde{\psi} = \hat{\mathbf{U}}^\dagger \psi$

$$i\hbar \frac{\partial \tilde{\psi}}{\partial t} = \left(-\frac{1}{2m} (-i\hbar \nabla - \hat{\mathbf{A}}(\vec{r}))^2 + \hat{\Lambda}(\vec{r}) \right) \tilde{\psi}$$

where $\hat{\mathbf{A}} = i\hbar \hat{\mathbf{U}}^\dagger \nabla \hat{\mathbf{U}} \Leftrightarrow \hat{\mathbf{A}} = i g^{-1} \hat{\mathbf{U}}^{-1} \partial_\mu \hat{\mathbf{U}}$

Emergent synthetic gauge field $\hat{\mathbf{A}}(\vec{r})$ is determined by $\hat{\mathbf{U}}(\vec{r})$

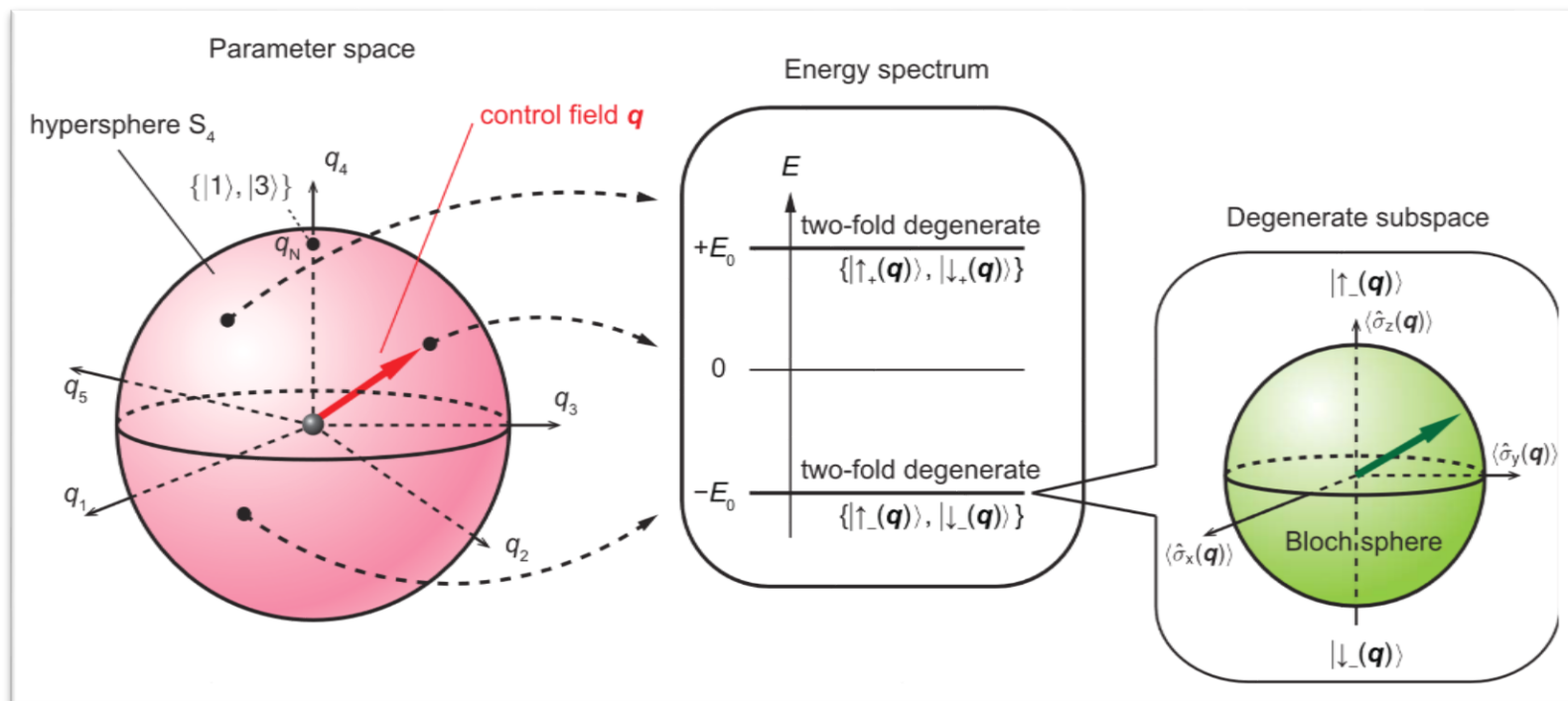
Artificial gauge field in cold atoms

$\hat{H} = \hat{H}(\boldsymbol{\mu})$ depends on parameter $\boldsymbol{\mu}$: eigenvector adiabatically moves in $\boldsymbol{\mu}$ -space

Berry connection: $\hat{A} = \hat{A}(\boldsymbol{\mu})$ is **nontrivial** when space contains **degenerate point**

Berry phase $\varphi_B \neq 0$ (Observable): $\varphi_B = \int \hat{A} d\boldsymbol{\mu}$, topological effects

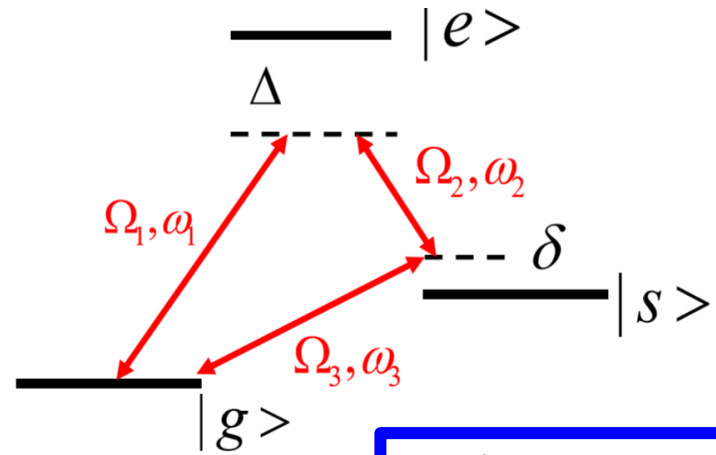
Berry curvature \hat{F} (Observable): $\mathcal{F}^{\alpha\beta} = \partial_\alpha \mathcal{A}^\beta - \partial_\beta \mathcal{A}^\alpha$, non-zero **dynamical effects**



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Theoretical Scheme

$$H = \begin{pmatrix} 0 & \Omega_3^* & \Omega_1^* \\ \Omega_3 & -\alpha\delta & \Omega_2^* \\ \Omega_1 & \Omega_2 & \alpha\Delta \end{pmatrix} \quad \alpha = \text{sign}(r - \eta)$$



$$\Omega_3 = \sqrt{\frac{\delta}{\Delta}} \Omega_1 e^{-i\frac{\pi}{2}} = -i\Omega_1 \sqrt{\frac{\delta}{\Delta}} \quad (\delta, \Delta > 0) \ll 1 \text{ Hz}$$

Unitary matrix
 $U = (|\psi_+\rangle, |\psi_-\rangle)$

For $\Delta \gg \delta, \Omega$ (Large detuning)

$$H \rightarrow H_r = \begin{pmatrix} \frac{|\Omega_1|^2}{-\Delta} & \frac{\Omega_1^* \Omega_2}{-\Delta} + i\Omega_1^* \sqrt{\frac{\delta}{\Delta}} \\ \frac{\Omega_1 \Omega_2}{-\Delta} - i\Omega_1 \sqrt{\frac{\delta}{\Delta}} & \frac{|\Omega_2|^2}{-\Delta} - \delta \end{pmatrix}$$

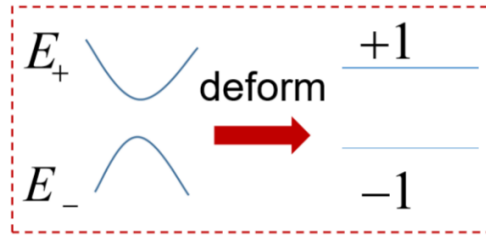
$$H_r |\psi_n\rangle = E_n |\psi_n\rangle$$

$$E_+ = 0, \quad E_- = -\alpha\delta - \frac{\Omega^2}{\alpha\Delta}$$

$$\Omega = \sqrt{|\Omega_1|^2 + |\Omega_2|^2}$$

$w =$ Hopf index χ

$$H = U(r) \begin{pmatrix} E_+ & 0 \\ 0 & E_- \end{pmatrix} U(r)^{-1}$$



Flattened Hamiltonian

$$\begin{aligned} \tilde{H} &= \Pi_+ - \Pi_- \\ &= |+\rangle\langle+| - |-\rangle\langle-| \\ &= U(r) \sigma^3 U(r)^{-1} \end{aligned}$$

Winding number:

$$w = \frac{1}{24\pi^2} \int dr \varepsilon^{ijk} \text{tr} (U^{-1} \partial_i U U^{-1} \partial_j U U^{-1} \partial_k U)$$

Equivalent \longleftrightarrow

Hopf index:

$$\chi = -\frac{1}{4\pi^2} \int d^3 r \varepsilon_{ijk} a_i \partial_j a_k$$

Berry connection: $a_\mu = \langle - | \partial_\mu | - \rangle$

$$f_{uv} = \partial_u a_v - \partial_v a_u$$

Topological quantum matter with cold atoms

D.W. Zhang, **Y. Q. Zhu**, Y.X.Zhao, S. L. Zhu, Adv. Phys. 67(4), 253-402(2018).

$$\chi = -\frac{1}{4\pi^2} \int d^3 r \varepsilon_{ijk} a_i \partial_j a_k = -\frac{1}{4\pi^2} \int d^3 r \vec{a}(\vec{r}) \cdot \vec{f}(\vec{r})$$

Detection $f_{uv}(\vec{r})$

f_{uv} gauge invariant, a_u gauge dependent: $\vec{f}(\vec{r}) = \nabla \times \vec{a}(\vec{r})$

$$\chi = -\frac{1}{4\pi^2} \int d^3r \epsilon_{ijk} a_i \partial_j a_k = \int d^3r \vec{a}(\vec{r}) \cdot \vec{f}(\vec{r})$$

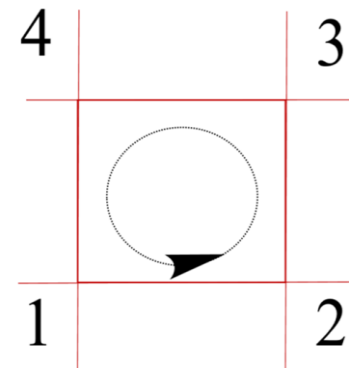
$$a_\mu = \langle \psi(\vec{r}) | \partial_\mu \psi(\vec{r}) \rangle = (\langle \psi(\vec{r}) | \psi(\vec{r} + \delta\vec{\mu}) \rangle - 1) / \delta\vec{\mu}, \delta\vec{\mu} \rightarrow 0$$

$$U(1) \text{ link: } U_\mu(\vec{r}) = \langle \psi(\vec{x}) | \psi(\vec{x} + \delta\mu) \rangle \approx e^{a_\mu \delta\mu} \rightarrow a_\mu \delta\mu = \ln U_\mu(\vec{r})$$

$$\text{Check: } C(\vec{r}) = \delta x \delta y f_{xy} = \ln \text{Tr}(\rho(\vec{r} + \delta\vec{x}) \rho(\vec{r} + \delta\vec{x} + \delta\vec{y}) \rho(\vec{r} + \delta\vec{y}) \rho(\vec{r}))$$

$$\Rightarrow f_{xy} = C(\vec{r}) / \delta x \delta y \text{ With } \rho(\vec{r}) = |\psi(\vec{r})\rangle \langle \psi(\vec{r})|$$

Gauge invariants



Detection $a(\vec{r})$?

$\vec{a}(\vec{r})$ gauge dependent \rightarrow Find $\vec{a}(\vec{r}_0)$ from $\vec{f}(\vec{r}_0)$?

Choose Coulomb gauge

$$\nabla \cdot a = 0$$

$$\because f = \nabla \times a$$

$$\therefore \nabla \times f = \nabla(\nabla \cdot a) - \nabla^2 a$$

Introduce effective current density

$$j = -\nabla^2 a$$

$$a_\mu(r) = \int_{V'} \frac{\nabla \times f(r') dV'}{4\pi|r-r'|}$$

$$j_\mu = (\nabla \times f)^\mu \\ = \partial_\nu f^\lambda - \partial_\lambda f^\nu$$

Review the magnetostatic equation in electrodynamics.

$$\vec{B} = \nabla \times \vec{A}, \quad \nabla \cdot \vec{A} = 0$$

$$\nabla \times \vec{B} = \mu \vec{J}$$

$$= \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

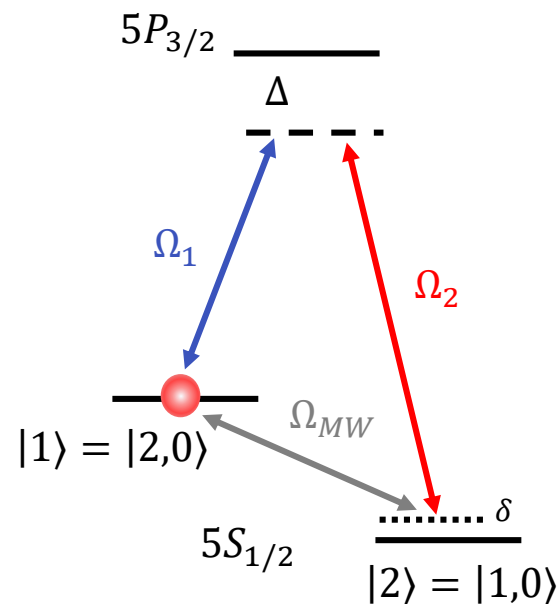
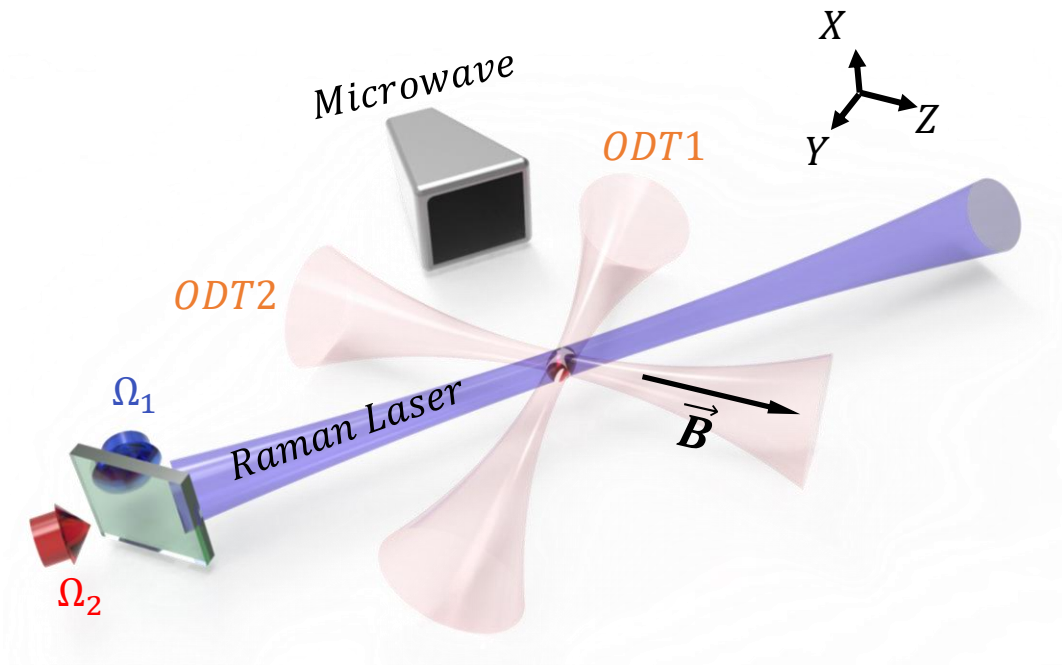
$$= -\nabla^2 \vec{A}$$

$$\vec{A}(r) = \frac{\mu}{4\pi} \int_V \frac{\vec{J}(r') dV'}{|r-r'|}$$

$$\approx \sum_i \frac{\mu \vec{J}(r') \delta V}{4\pi r_i}$$

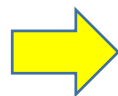
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Experimental Setup



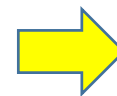
Initial state preparation

$$|F = 2, m_F = 0\rangle$$



Adiabatic state preparation

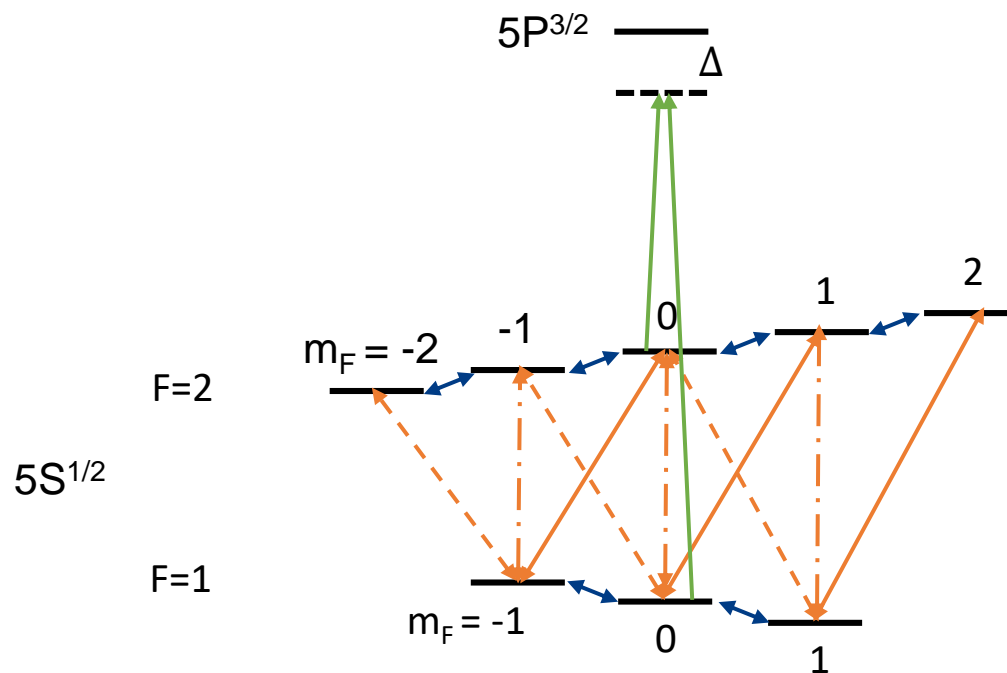
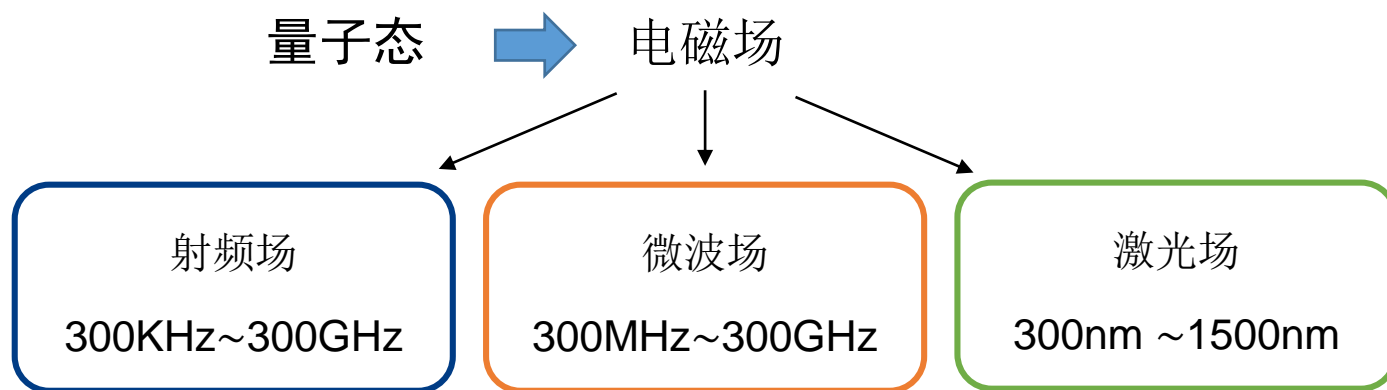
$$|\Psi_{\pm}\rangle = \begin{bmatrix} \cos(\theta/2) \\ \pm \sin(\theta/2) e^{i\varphi} \end{bmatrix}$$



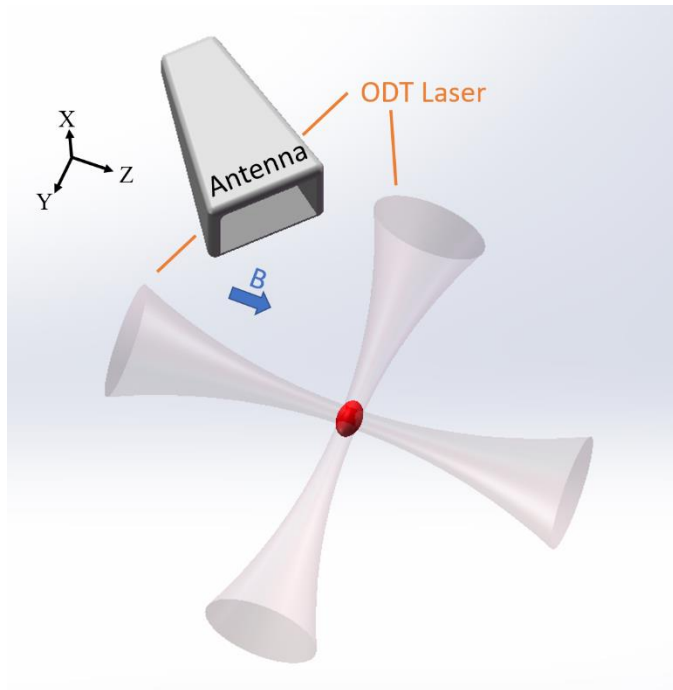
Quantum state tomography

$$\hat{\rho} = \begin{pmatrix} \rho_{00} & \rho_{10} \\ \rho_{01} & \rho_{11} \end{pmatrix}$$

Quantum state manipulation

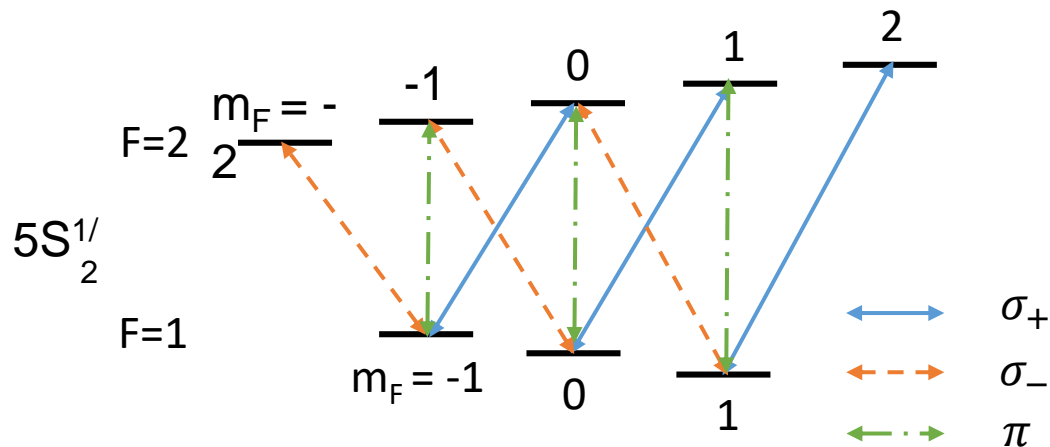
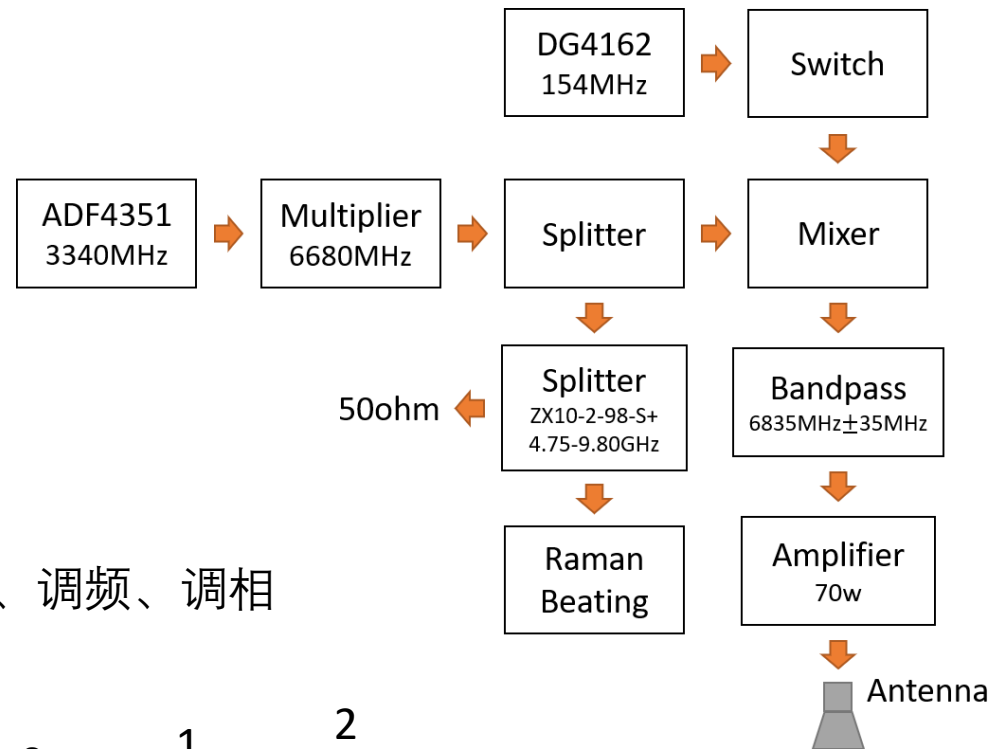


Microwave and RF

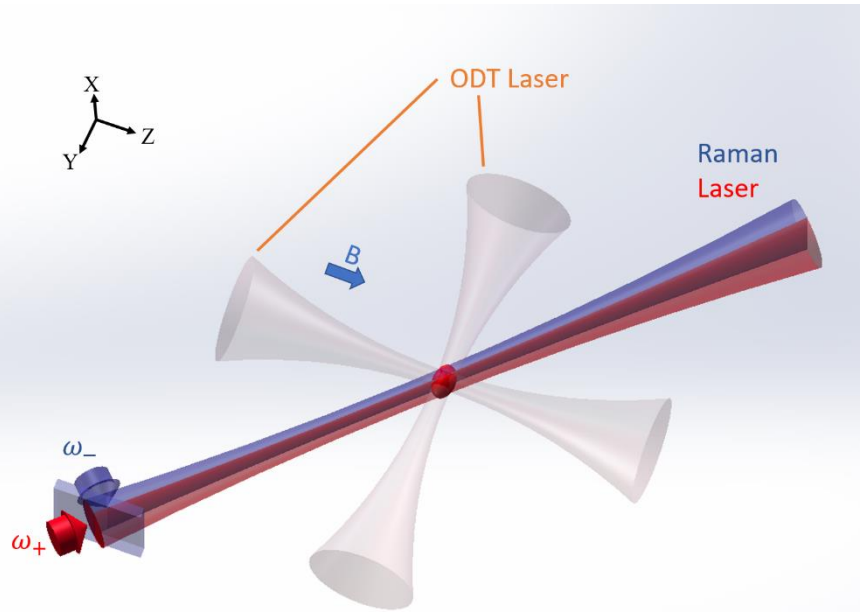


微波:

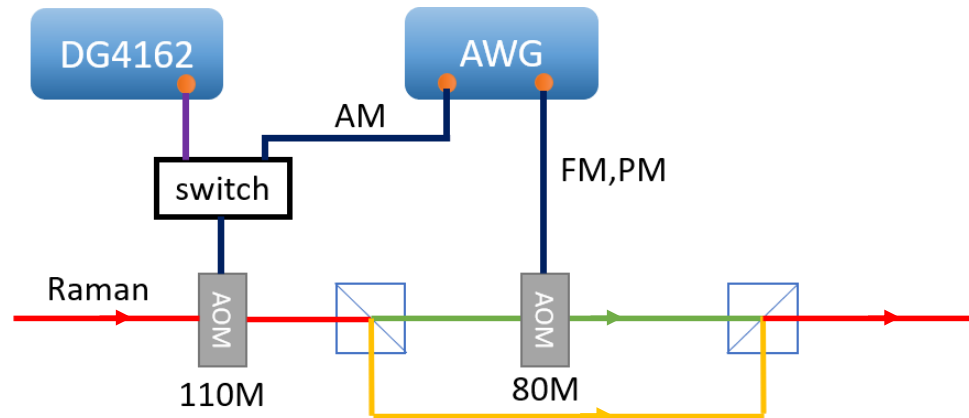
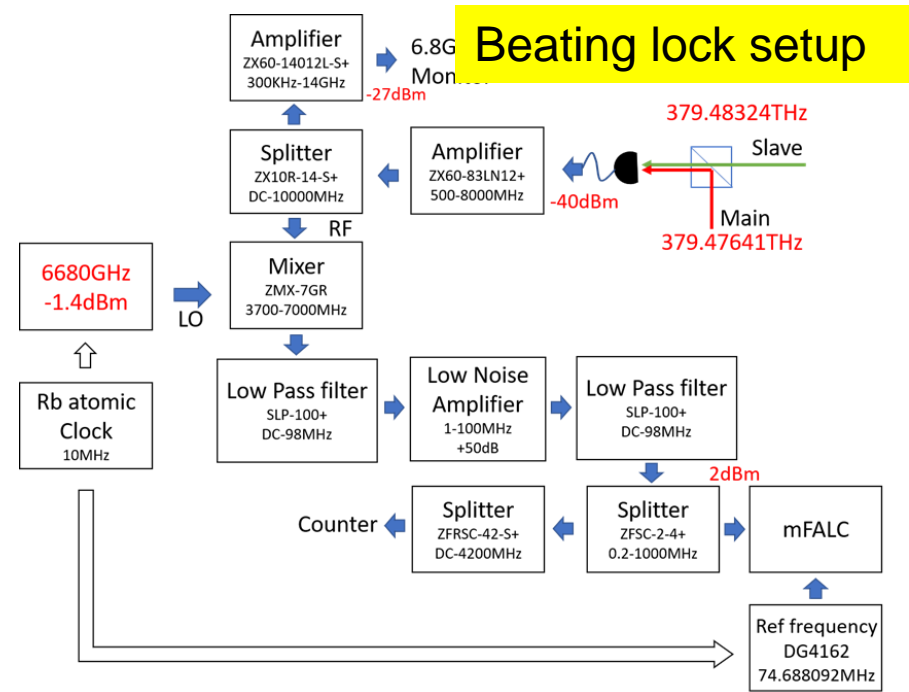
调幅、调频、调相



Raman Laser



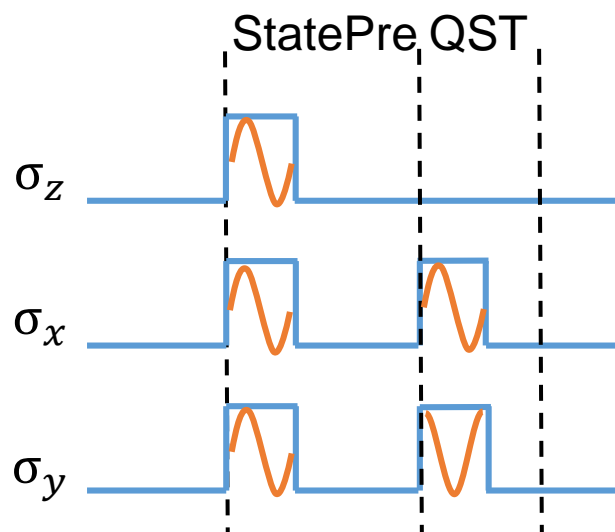
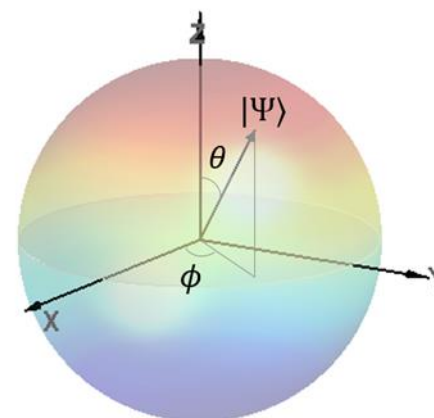
Raman laser: intensity, phase, frequency



Quantum state tomography

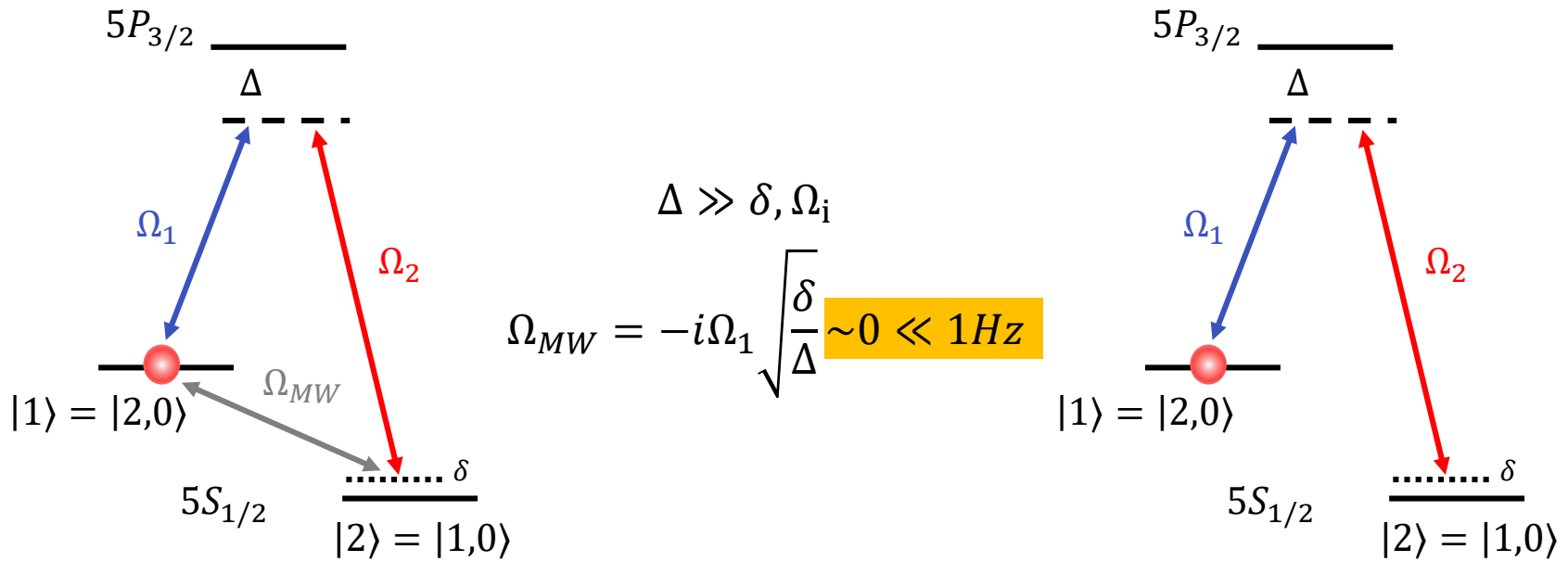
$$\begin{aligned}\hat{\rho} &= |\Psi\rangle\langle\Psi| \\ &= \begin{pmatrix} \rho_{00} & \rho_{10} \\ \rho_{01} & \rho_{11} \end{pmatrix} \\ &= \frac{1}{2}\hat{\sigma}_0 + \frac{1}{2}(v_x\hat{\sigma}_x + v_y\hat{\sigma}_y + v_z\hat{\sigma}_z)\end{aligned}$$

$$|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle$$



- 最大似然估计→符合物理性质的最有可能的密度矩阵

Energy level scheme



$$H = U(r) \begin{pmatrix} E_+ & 0 \\ 0 & E_- \end{pmatrix} U(r)^{-1}$$

$$E_+ = 0, E_- = -\alpha\delta - \frac{\Omega^2}{\alpha\Delta} \ll E_{excited} \sim \Delta$$

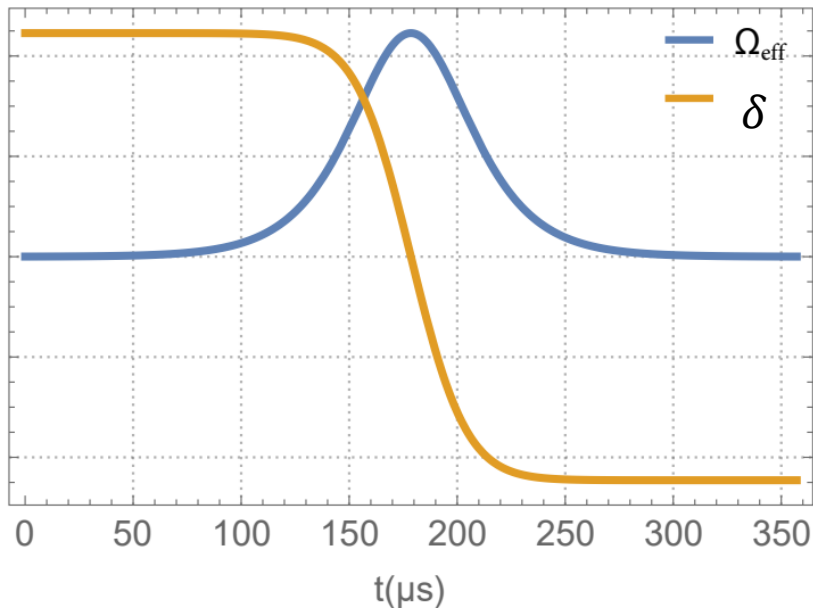
$$H = \frac{\hbar}{2} \begin{bmatrix} \delta_{exp} & \Omega e^{-i\varphi_{exp}} \\ \Omega e^{i\varphi_{exp}} & -\delta_{exp} \end{bmatrix}$$

$$E_{\pm} = \pm \frac{\hbar}{2} \sqrt{\Omega^2 + \delta_{exp}^2} \ll E_{excited} \sim \Delta$$

Quantum state preparation

$$|\Psi(0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \text{adiabatic} \rightarrow \begin{pmatrix} \cos \frac{\theta}{2} e^{-i\phi} \\ \sin \frac{\theta}{2} \end{pmatrix}$$

$$\hat{H} = \frac{\hbar}{2} \begin{pmatrix} \delta & \Omega e^{-i\phi} \\ \Omega e^{i\phi} & -\delta \end{pmatrix} = \frac{\hbar}{2} \Omega_0 \begin{pmatrix} \cos \theta & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix}$$



Single photon detuning:

$$\Delta = -3904 \text{GHz}$$

Raman power:

$$P_1 = 45 \text{mW}, P_2 = 53 \text{mW}$$

Raman size:

$$r_1 = 375 \mu\text{m}, r_2 = 150 \mu\text{m}$$

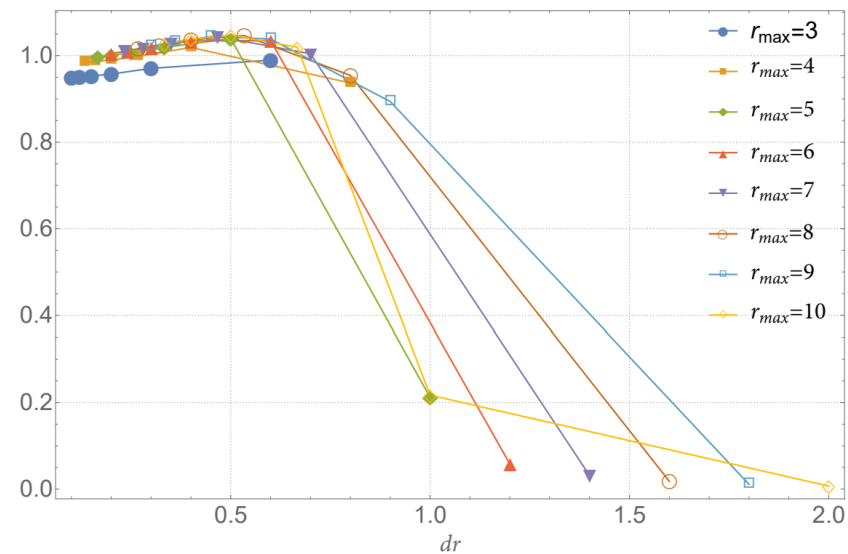
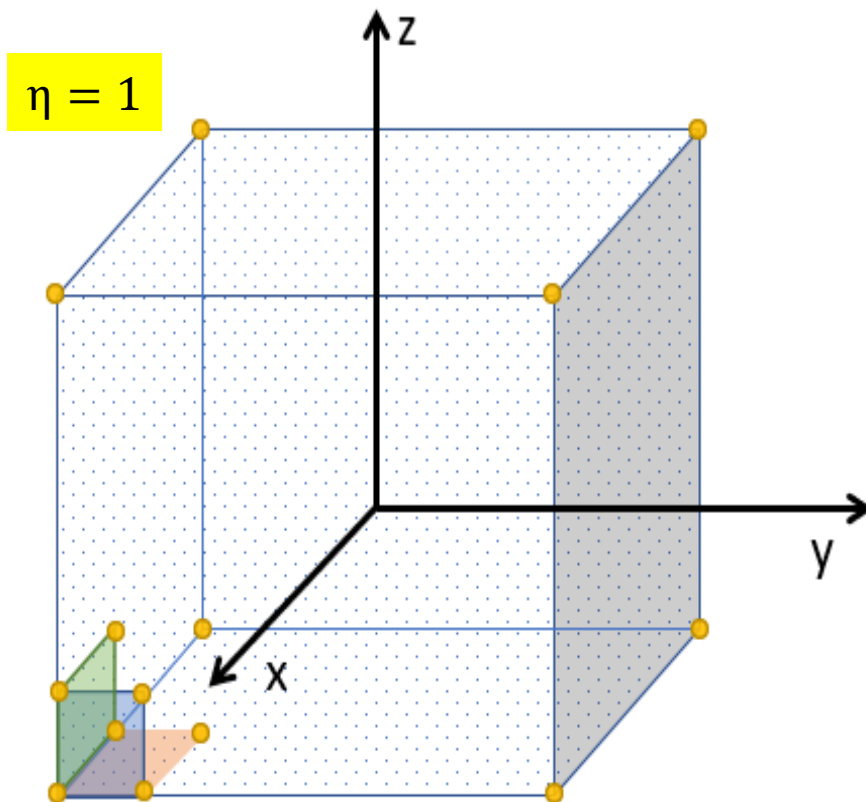
Controlled parameter: $\theta = \arctan(\Omega/\delta)$

pulse: $\Omega t = 20\pi \sim 350 \mu\text{s}$

Grid size

$$U^{(1)}(\vec{x}) = \frac{\vec{x}^2 - \eta^2}{\vec{x}^2 + \eta^2} - \frac{2i\eta\vec{\sigma} \cdot \vec{x}}{\vec{x}^2 + \eta^2}$$

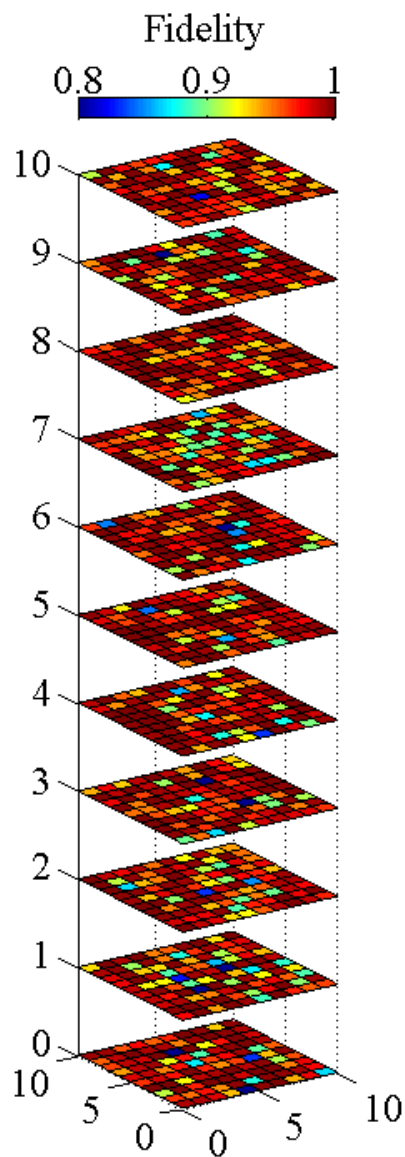
$$\chi = -\frac{1}{4\pi^2} \int d^3r \vec{a}(\vec{r}) \cdot \vec{f}(\vec{r})$$



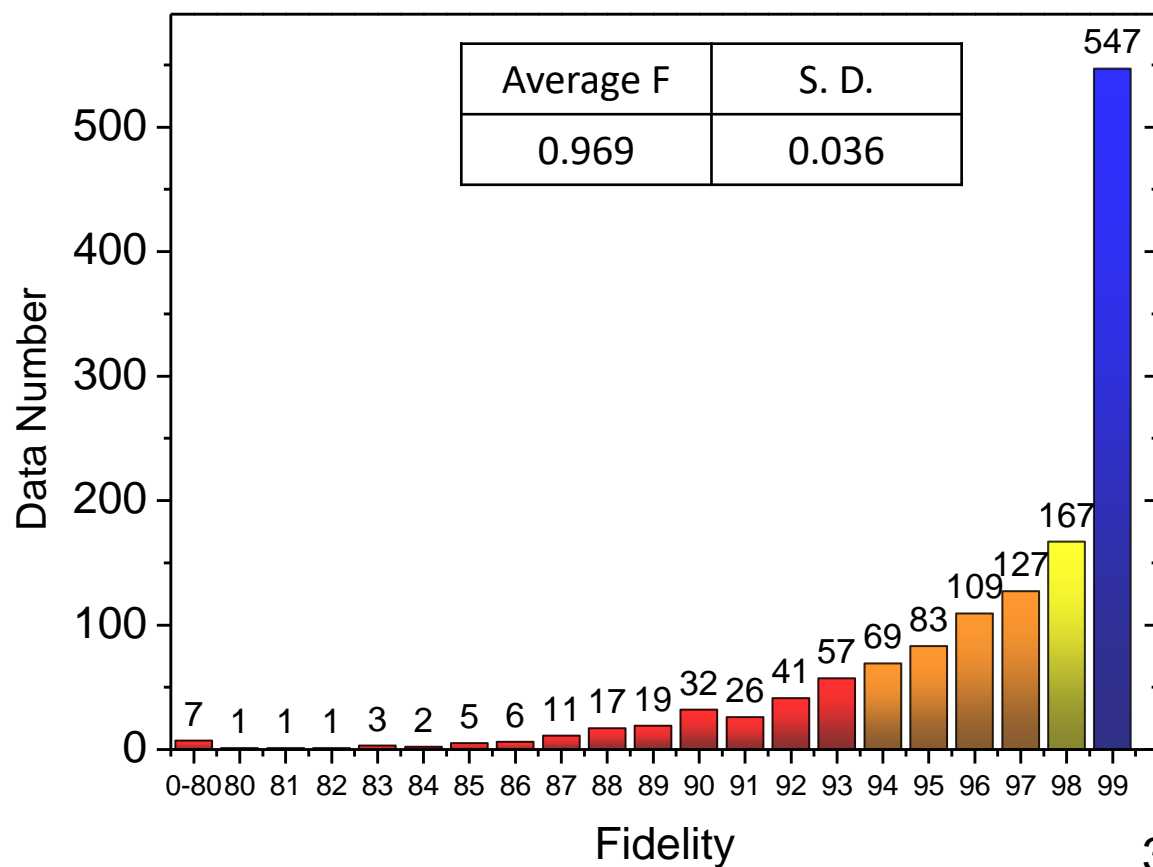
Grid size: 10x10x10

$$L = [-3.3, 3.3]$$

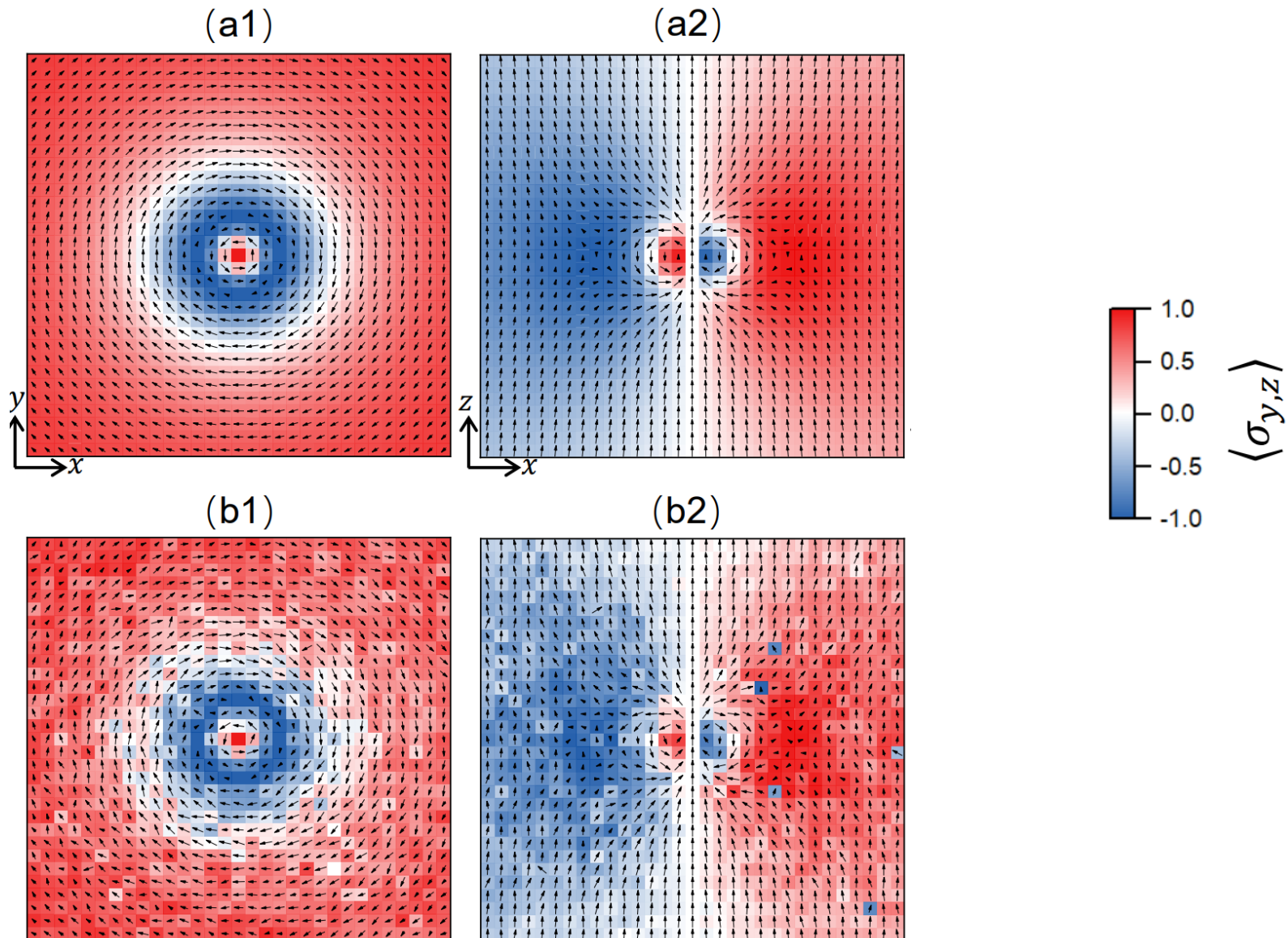
Quantum state fidelity



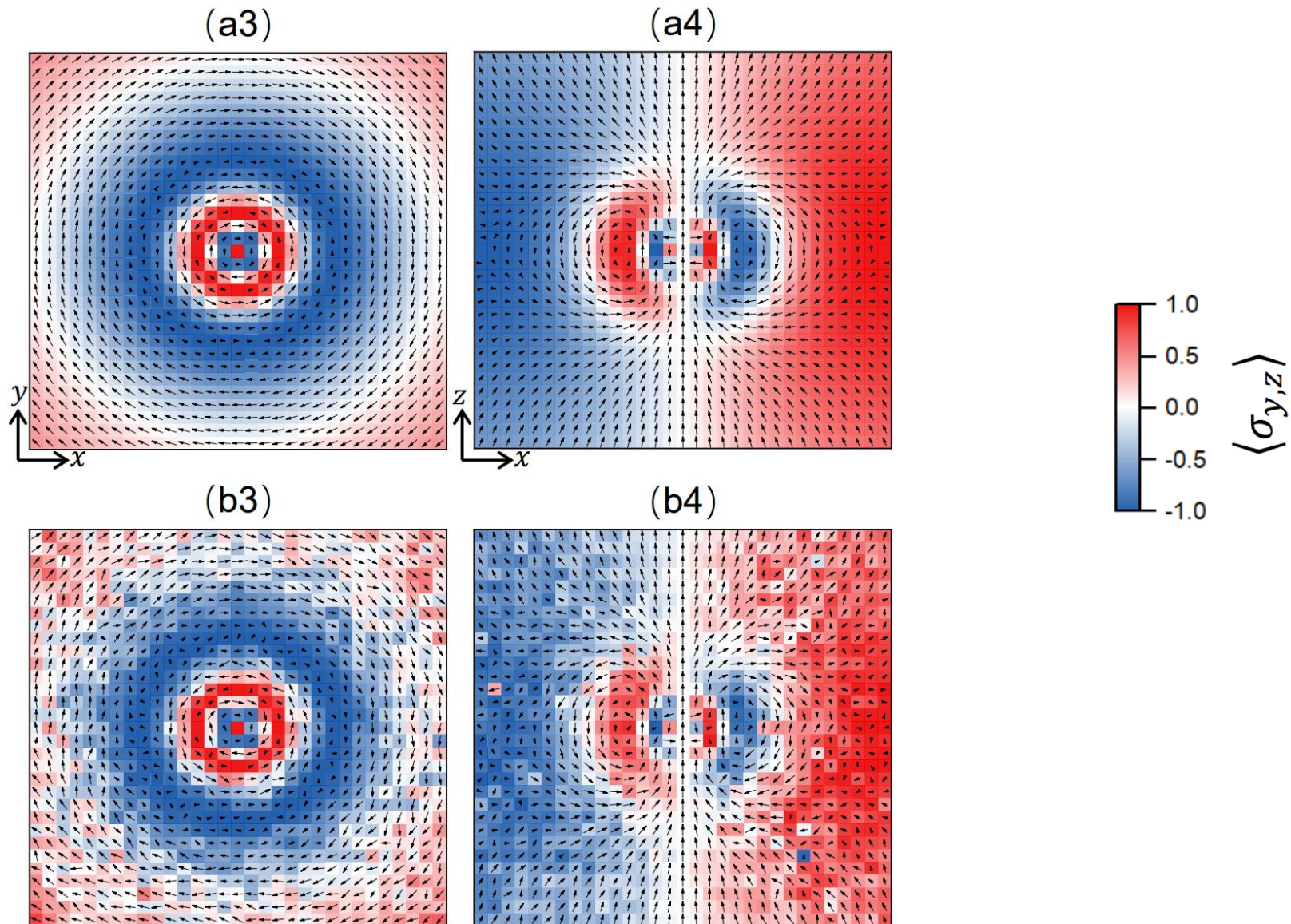
	Theory	Experiment	Using Data F>95%	Using Data F>99%
χ	0.986	0.912	0.955	0.980



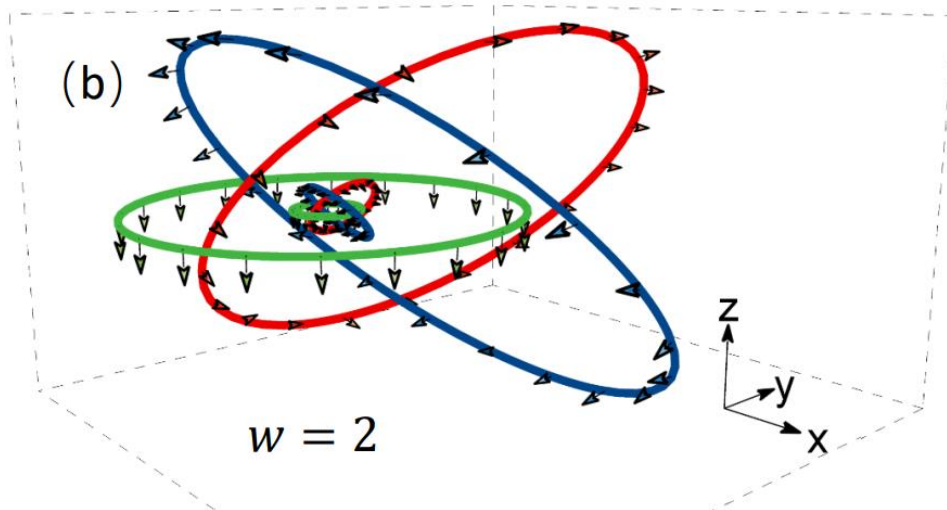
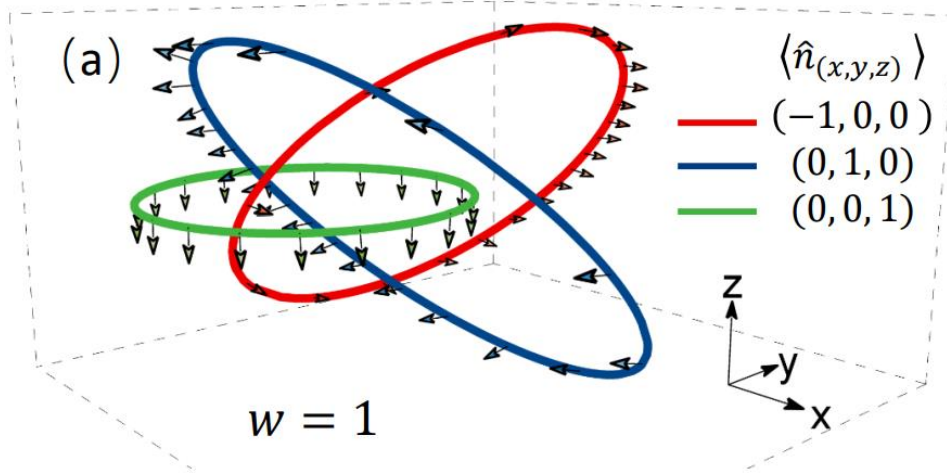
Spin Texture of $w = 1$



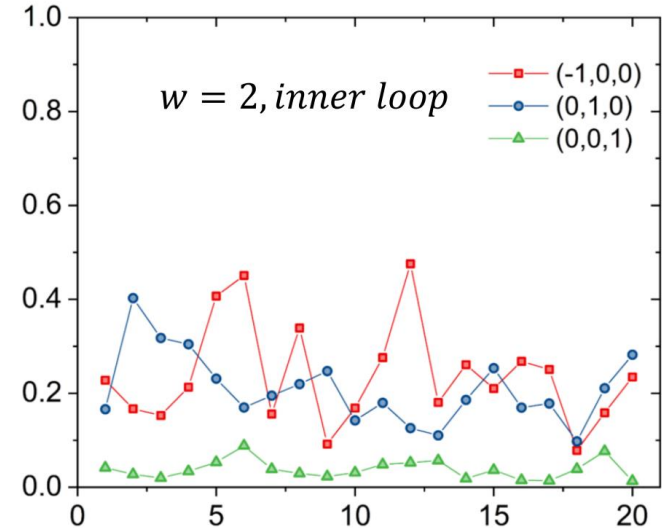
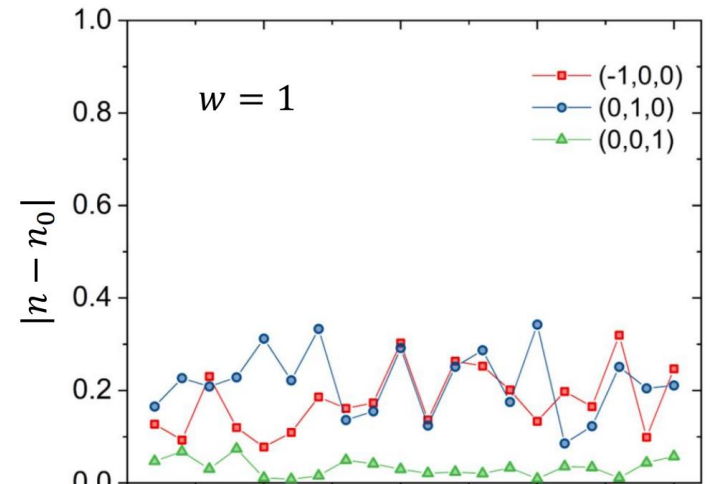
Spin Texture of $w = 2$



Hopf Link

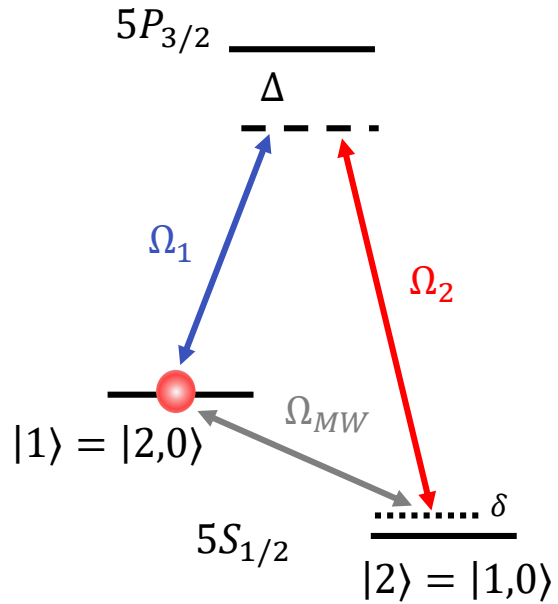


Spin Vector Error



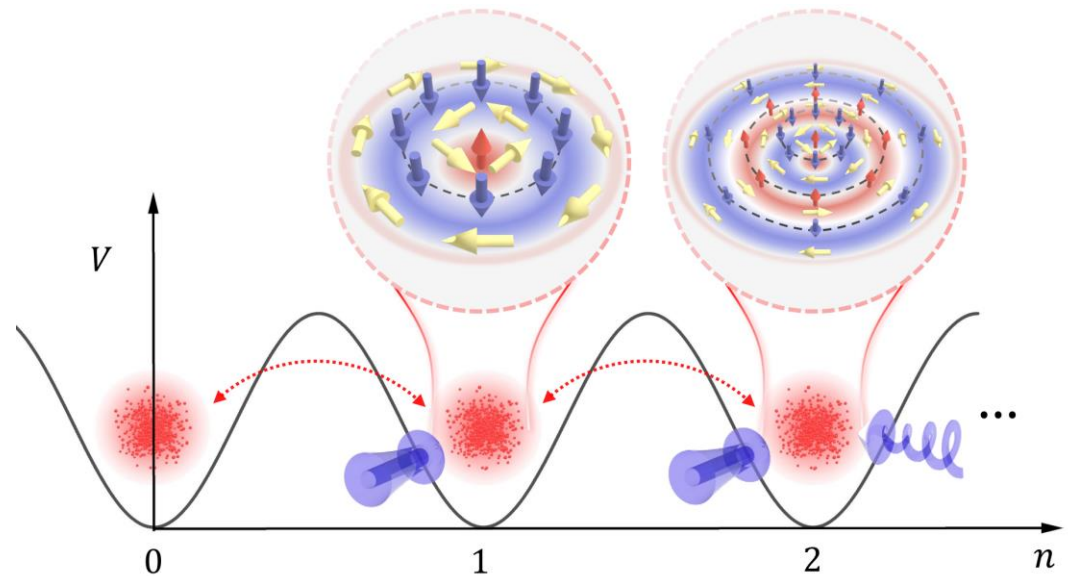
- Introduction
- Topology of $SU(2)$ gauge field: Winding
- Simulating topological $SU(2)$ vacua
- Experimental realization using a BEC
- Outlook

Real Space Non-Abelian Vacua



$$H_{exp}(\mathbf{r}) = \hbar \begin{pmatrix} 0 & 0 & \Omega_1^* \\ 0 & -\alpha\delta & \Omega_2^* \\ \Omega_1 & \Omega_2 & \alpha\Delta \end{pmatrix}$$

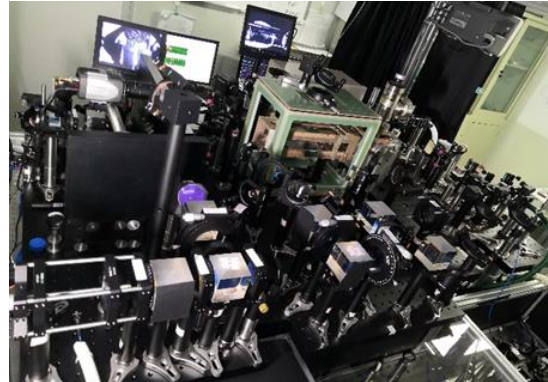
Vacuum tunneling- Instantons



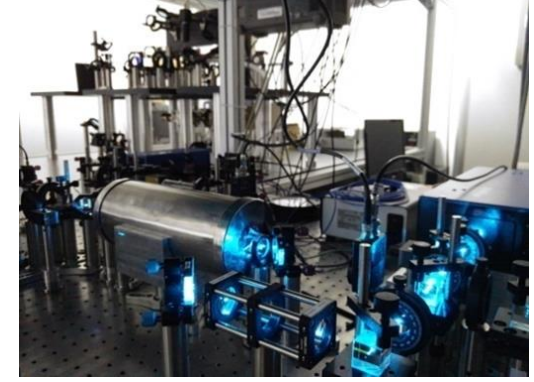
Platforms @SCNU



Quantum control



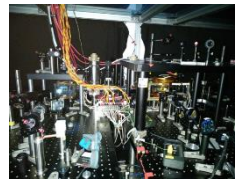
Quantum Memory



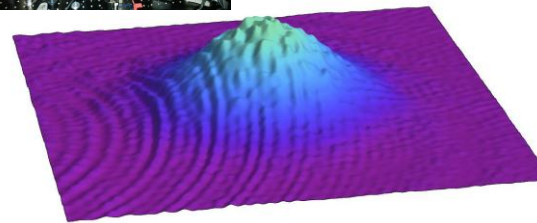
Cold Rydberg atom



Hybrid cold system



2019-July-24
Rb BEC



Rb BEC: SOC,OL



2021-Dec-31
Yb BEC

Yb Quantum gas

Team and Funding

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Prof. Yan-Xiong Du

Dr. Qing-Xian Lv、 Zhen-Tao Liang

PostDoc position opening

