Application of Quantum Computing in (High Energy) Nuclear Theory

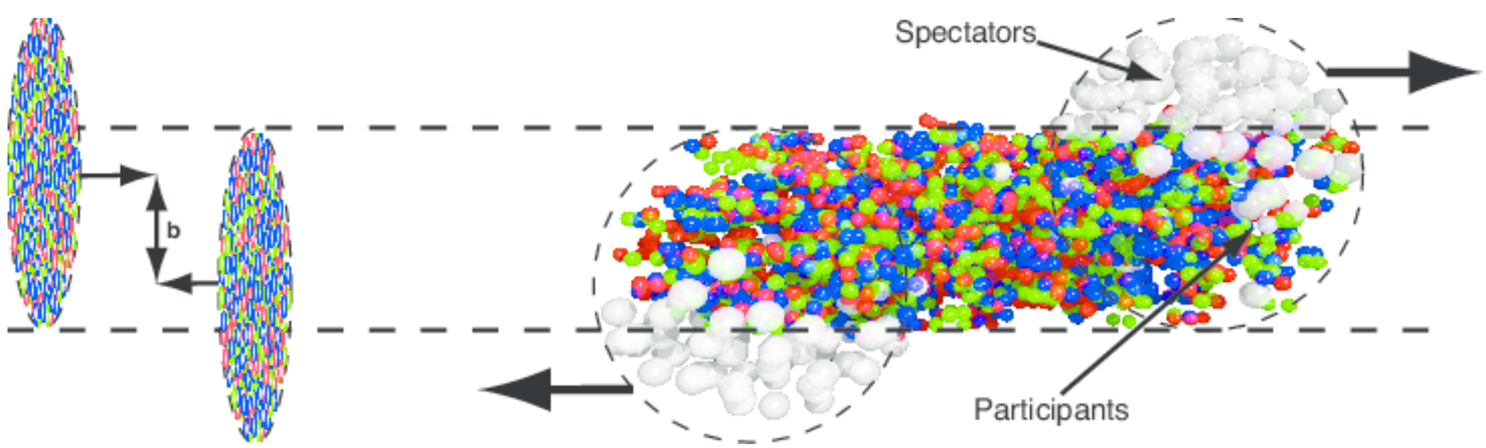
Xiaojun Yao
University of Washington

量子计算与高能核物理交叉前沿讲习班
Organized by 华南师范大学

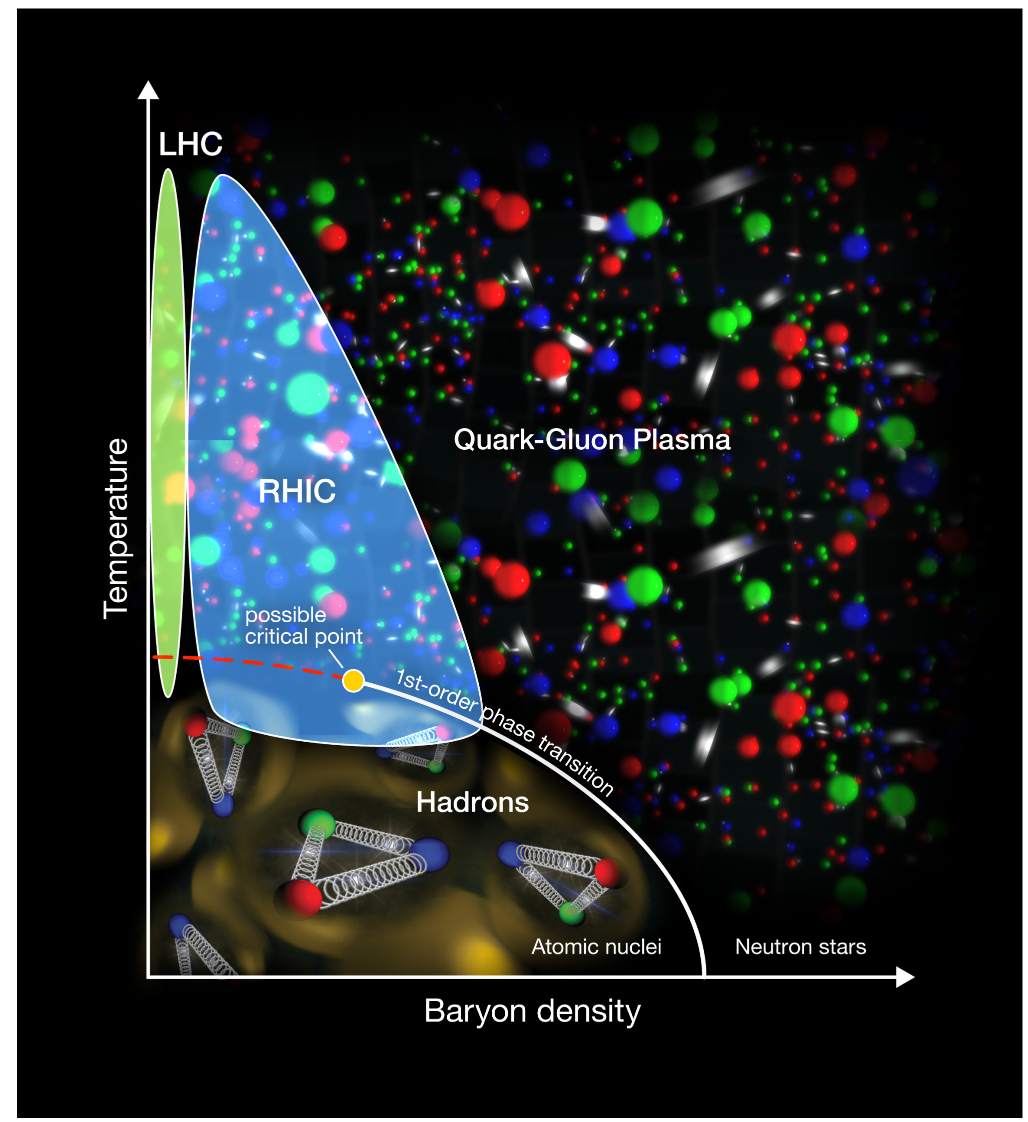
November 21, 2022

Some Aspects of High Energy Nuclear Physics

- Relativistic heavy ion collisions and quark-gluon plasma (QGP)
  - Search for the critical point in phase diagram of nuclear matter
  - Study properties of QGP using various probes

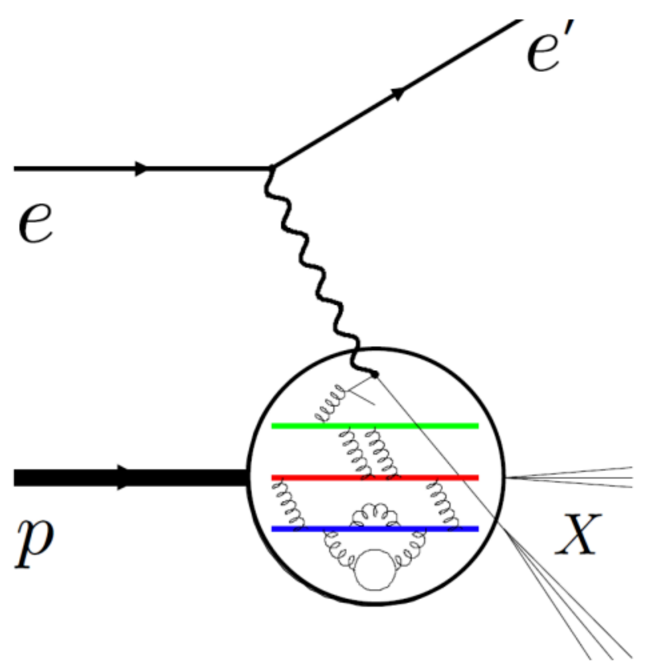


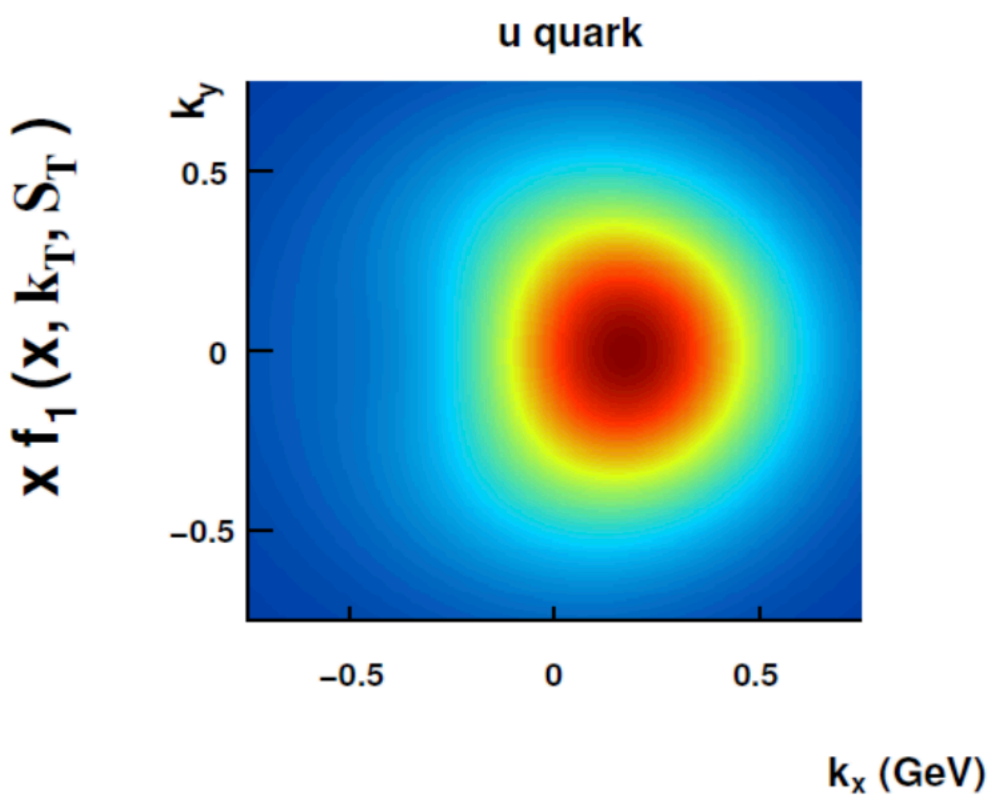
See lectures by Professors 王群，庄鹏飞



Some Aspects of High Energy Nuclear Physics

- Future electron-ion collider
  - Study proton/nucleus structure (transverse momentum dependent parton distribution function (TMD PDF), etc)
  - Study properties of cold nuclear matter (such as heavy nucleus)





See lectures by Professor 周剑

Why Quantum Computing

- At high energy/temperature, perturbative calculations are applicable
- At low energy/temperature, nonperturbative methods such as Euclidean lattice QCD methods and holography are useful: e.g. parton distribution function of proton, QCD matter crossover at zero baryon chemical potential, transport coefficients (e.g. heavy quark diffusion and jet quenching),

See lectures by Professors 丁亨通，杨一玻

- Challenges of Euclidean lattice QCD, sign problem (signal-to-noise ratio): fermion at finite chemical potential (QCD phase diagram at non-zero chemical potential), real-time observables (e.g. fragmentation function)
- Use quantum computers to simulate quantum systems

"Simulating physics with computers" R. P. Feynman, 1982

- Quantum devices are developing quickly: superconducting circuits, trapped ions, Rydberg atoms, photonic system, etc.
- Quantum computing v.s. quantum simulation —> digital v.s. analog

See lectures by Professors 周正威，翟荟

Contents

- Part 1: digital quantum simulation for open quantum systems
  - Heavy quarks and quarkonia in QGP as open quantum systems
  - Simple example: U(1) gauge theory in 1+1 dimension (Schwinger model)
- Part 2: digital quantum simulation for jet quenching in nuclear environments
  - Light-front Hamiltonian of QCD
  - Landau-Pomeranchuk-Migdal effect in gluon radiation

I. Open Quantum Systems for Heavy Quarks and Quarkonia in QGP

Open Quantum System

- Total system = subsystem + environment: $H = H_S + H_E + H_I$

$$\rho(t=0) = \rho_S \otimes \rho_E$$

Subsystem & environment

(Heavy quark pair & QGP)

→ **Unitary evolution** / **Time reversible** →

$$U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)$$

Subsystem & environment

↓ **Trace out (integrate out) environment** ↓

$$\rho_S(t=0)$$

Subsystem

→ **Non-unitary** / **Time irreversible** →

$$\mathrm{Tr}_E\Big[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\Big]$$

- Has been widely used to deepen our understanding of quarkonium transport inside QGP

Review: XY, 2102.01736

Towards Lindblad Equation: Two Limits

Subsystem: non-unitary, time-irreversible evolution

$$\mathrm{Tr}_E\Big[U(t,0)(\rho_S\otimes\rho_E)U^\dagger(t,0)\Big]$$

Markovian (weak coupling)

Quantum optical limit (low T)

Quantum Brownian motion (high T)

Lindblad equation

Lindblad equation

$$\frac{d\rho_S(t)}{dt} = -i\big[H_{S,\mathrm{eff}},\rho_S(t)\big] + \sum_n \Big(L_n\rho_S(t)L_n^\dagger - \frac{1}{2}\{L_n^\dagger L_n,\rho_S(t)\}\Big)$$

Wigner transform (smearing for positivity)

Semiclassical (gradient expansion)

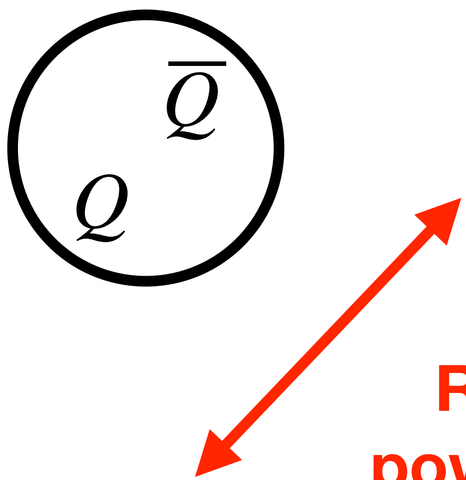
Boltzmann equation

Review: XY, 2102.01736

Langevin/Fokker-Planck equation

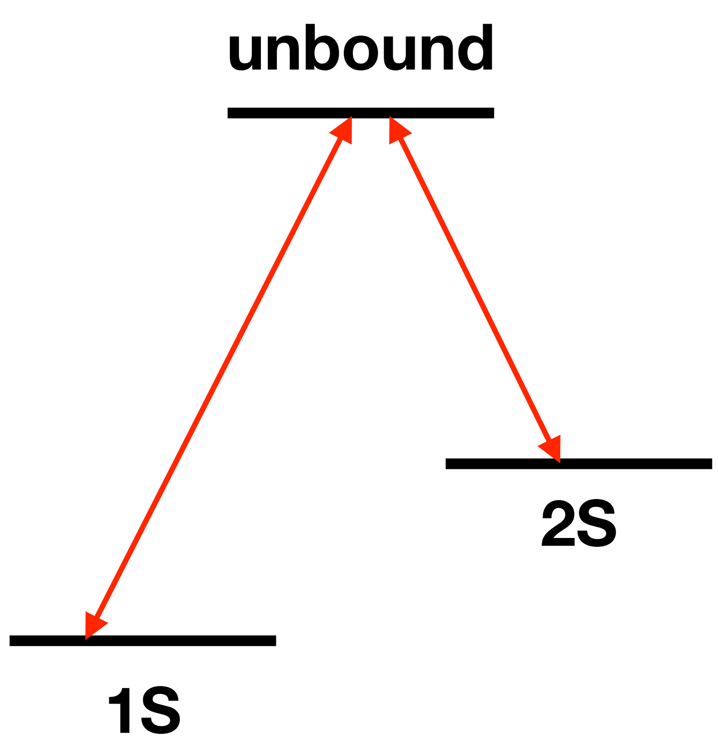
Physical Pictures of Two Limits

- Quantum optical limit (low T)

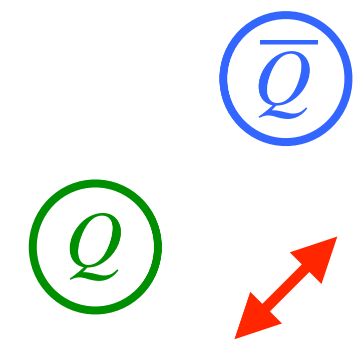


Resolving power of QGP

Transitions between levels

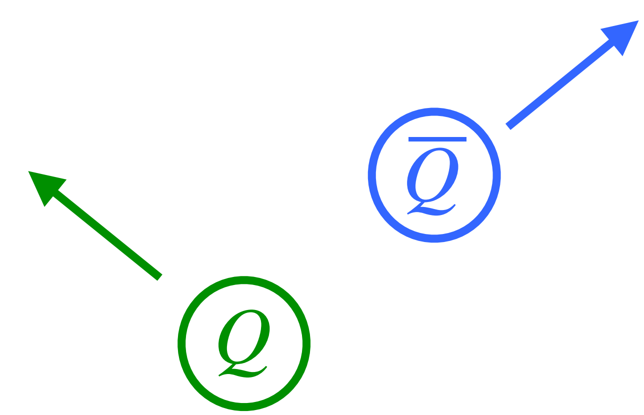


- Quantum Brownian motion (high T)



Resolving power of QGP

Diffusion of heavy Q pair



Wavefunction decoherence
—> dissociation

Towards Lindblad Equation: Two Limits

Subsystem: non-unitary, time-irreversible evolution

$$\mathrm{Tr}_E\left[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\right]$$

Markovian (weak coupling)

Quantum optical limit (low T) → **Lindblad equation**

Quantum Brownian motion (high T) → **Lindblad equation**

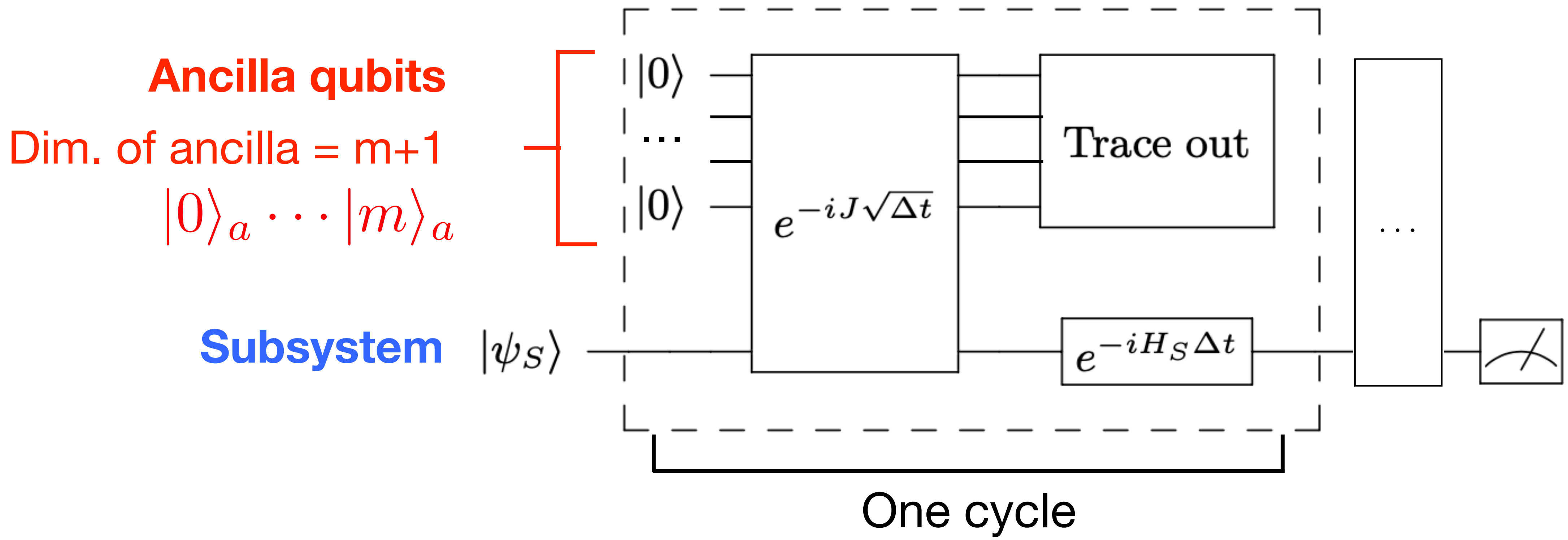
$$\frac{d\rho_S(t)}{dt} = -i\left[H_{S,\mathrm{eff}}, \rho_S(t)\right] + \sum_n \left(L_n \rho_S(t) L_n^\dagger - \frac{1}{2}\{L_n^\dagger L_n, \rho_S(t)\}\right)$$

- Solving Lindblad equation for many heavy quark pairs can be difficult, since the size of the Hilbert space grows exponentially. Can quantum computers help?
- Understand time evolution beyond Lindblad —> non-Markovian dynamics

Quantum Simulation of Lindblad Equation

- **Lindblad evolution = unitary evolution of subsystem coupled with ancilla with ancilla traced out**

$$\frac{d\rho_S}{dt} = -i\left[H_S, \rho_S\right] + \sum_{j=1}^{m} \left(L_j \rho_S L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho_S\} \right)$$



Reproduce Lindblad equation if expanded to linear order in Δt

$$J = \begin{pmatrix} 0 & L_1^\dagger & \dots & L_m^\dagger \\ L_1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ L_m & 0 & \dots & 0 \end{pmatrix} \qquad \rho(0) = |0\rangle_a \langle 0|_a \otimes \rho_S(0) = \begin{pmatrix} \rho_S(0) & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}$$

We need to represent a gauge theory on a quantum computer

Example: Schwinger Model

- **U(1) gauge theory in 1+1D** $\mathcal{L} = \overline{\psi}\big(iD^\mu\gamma_\mu - m\big)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$

$$\psi = \begin{pmatrix}\psi_u \\ \psi_d\end{pmatrix} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad E = F^{10}$$

$$D_\mu = \partial_\mu - ieA_\mu \qquad \gamma^0 = \sigma_z,\ \gamma^1 = i\sigma_y,\ \gamma^0\gamma^1 = \sigma_x$$

- **Equation of motion from Euler-Lagrangian equation**

$$0 = \frac{\partial\mathcal{L}}{\partial A_0} - \partial_1\frac{\partial\mathcal{L}}{\partial(\partial_1 A_0)} = e\overline{\psi}\gamma^0\psi + \partial_1 F^{10} = e\psi^\dagger\psi + \partial_1 E$$

- **Canonical quantization in light-cone axial gauge** $A_0 = 0$

$$\Pi_\psi = \frac{\partial\mathcal{L}}{\partial(\partial^0\psi)} = \overline{\psi}\,i\gamma^0 = i\psi^\dagger,\ \Pi_A = \frac{\partial\mathcal{L}}{\partial(\partial^0 A^1)} = -E$$

$$\{\psi(t,x), \Pi_\psi(t,y)\} = i\delta(x-y),\ [A^1(t,x), \Pi_A(t,y)] = i\delta(x-y)$$

See e.g. appendix of 2106.08394

Example: Schwinger Model

- **Hamiltonian density of Schwinger model**

$$\mathcal{H} = \Pi_\psi \partial^0 \psi + \Pi_A \partial^0 A^1 - \mathcal{L} = -i\overline{\psi}\gamma^1(\partial_1 + ieA^1)\psi + m\overline{\psi}\psi + \frac{1}{2}E^2$$

- **Discretize spatial dimension $x = na$**

$$a\sum_n \Big(- i\psi^\dagger(n)\sigma_x \frac{\psi(n+1) - \psi(n-1)}{2a} + m\psi^\dagger(n)\sigma_z\psi(n) - e\psi^\dagger(n)\sigma_x A(n)\frac{\psi(n+1)+\psi(n-1)}{2}\Big) + \mathcal{O}(a^3)$$

Define new variable

$$\chi(n) = \sqrt{a}\,(\sigma_x)^n\psi(n), \;\; \chi^\dagger(n) = \sqrt{a}\,\psi^\dagger(n)(\sigma_x)^n$$

$$\frac{1}{2a}\sum_n \Big(- i\chi^\dagger(n)\chi(n+1) - ae\chi^\dagger(n)A(n)\chi(n+1) + i\chi^\dagger(n)\chi(n-1)$$
$$- ae\chi^\dagger(n)A(n)\chi(n-1) + 2ma(-1)^n\chi^\dagger(n)\sigma_z\chi(n) + \mathcal{O}(a^2)\Big)$$

See e.g. appendix of 2106.08394

Example: Schwinger Model

- **Introduce Wilson lines**

$$U(z,y) = \mathcal{P}\exp\left(-ie\int_y^z dx A^1(x)\right) = \mathcal{P}\exp\left(ie\int_y^z dx A_1(x)\right)$$

$$= \sum_{n=0}^{\infty} \frac{(ie)^n}{n!} \int_y^z dx_1 \int_y^z dx_2 \cdots \int_y^z dx_n \mathcal{P}\big(A_1(x_1)A_1(x_2)\cdots A_1(x_n)\big)$$

One can show $\big[E(y), U(z,y)\big] = eU(z,y)$

$$\chi^\dagger(n)U(n,n+1)\chi(n+1) = \chi^\dagger(n)\chi(n+1) - iae\chi^\dagger(n)A(n)\chi(n+1) + \mathcal{O}(a^2)$$

$$\chi^\dagger(n)U(n,n-1)\chi(n-1) = \chi^\dagger(n)\chi(n-1) + iae\chi^\dagger(n)A(n)\chi(n-1) + \mathcal{O}(a^2)$$

The fermion part of the Hamiltonian can be written as

$$\frac{1}{2a}\sum_n \Big(-i\chi_u^\dagger(n)U(n,n+1)\chi_u(n+1) + i\chi_u^\dagger(n)U(n,n-1)\chi_u(n-1) + 2ma(-1)^n\chi_u^\dagger(n)\chi_u(n)$$

$$- i\chi_d^\dagger(n)U(n,n+1)\chi_d(n+1) + i\chi_d^\dagger(n)U(n,n-1)\chi_d(n-1) - 2ma(-1)^n\chi_d^\dagger(n)\chi_d(n)\Big)$$

χ_u and χ_d behave the same way!

See e.g. appendix of 2106.08394

Example: Schwinger Model

- **Dropping χ_d and apply Jordan-Wigner transform (local fermion field—>non-local spin product)**

$$\chi_u(n) \to \left(\prod_{m<n} -i\sigma_z(m)\right)\sigma^-(n)\,, \quad \chi_u^\dagger(n) \to \sigma^+(n)\left(\prod_{m<n} +i\sigma_z(m)\right)$$

- **For gauge sector** $[E(n), U(n+1,n)] = eU(n+1,n)$

Recall quantum ladder system

$$E(n)|\ell_n\rangle = e\ell_n|\ell_n\rangle$$

$$U(n\pm 1,n)|\ell_n\rangle = |\ell_n \pm 1\rangle$$

$$E(n) \to e\ell_n\,,\ U(n,n-1) \to L^+_{n-1}\,,\ U(n,n+1) \to L^-_n$$

- **Finally obtain the discretized Hamiltonian for Schwinger model**

$$H_S = \frac{1}{2a}\sum_n \Big(\sigma^+(n)L_n^-\sigma^-(n+1) + \sigma^+(n)L^+_{n-1}\sigma^-(n-1)\Big)$$

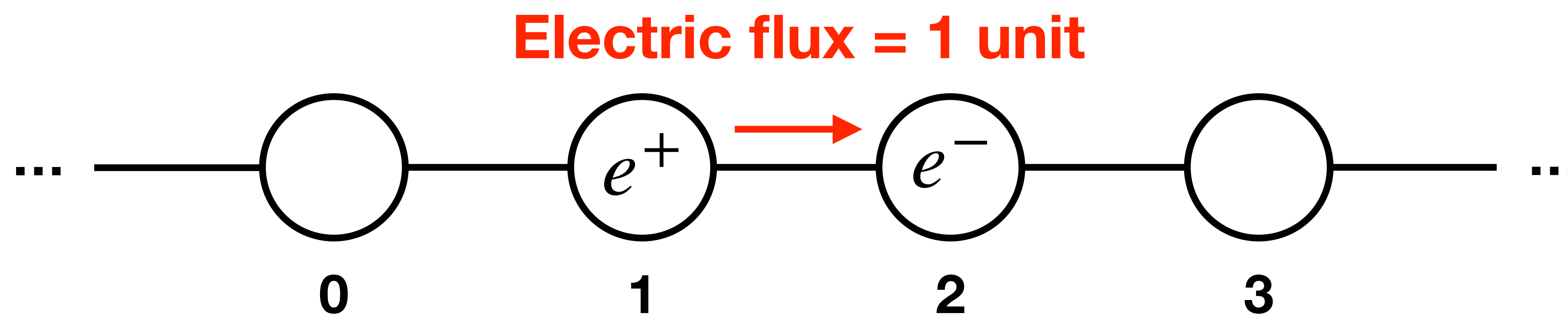
$$+\frac{m}{2}\sum_n(-1)^n\big(\sigma_z(n)+1\big) + \frac{ae^2}{2}\sum_n \ell_n^2$$

See e.g. appendix of 2106.08394

States in Schwinger Model

- **Fermion sector—>spins, gauge sector—>ladders, truncated**

Periodic b.c.



Even sites: fermion (spin-up for occupied)
odd sites: anti-fermion (spin-down for occupied)

- **Impose Gauss law for physical states**

$$e\psi^\dagger\psi + \partial_1 E = 0 \xrightarrow{\text{discretized, stagger fermion, Jordan-Wigner transform}}$$

$$E(n+1) - E(n) = -e\sigma^+(n)\sigma^-(n) - e\frac{(-1)^n - 1}{2}$$

- **Focus on states with specific momentum and symmetry to reduce size**

k=0 state: first find states that are equivalent under cyclic permutation
then take symmetrized linear combination (trivial Fourier transform)

Positive parity: reflection w.r.t. a chosen site

N. Klco, et al, 1803.03326

Schwinger Model with Two Physical Sites (Four Fermion Sites)

- **Hamiltonian matrix**

$$H_S^{\mathbf{k}=\mathbf{0},+} = \begin{pmatrix} -2m & \frac{1}{a} & 0 & 0 & 0 \\ \frac{1}{a} & \frac{ae^2}{2} & \frac{1}{\sqrt{2}a} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}a} & ae^2 + 2m & \frac{1}{\sqrt{2}a} & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}a} & \frac{3ae^2}{2} & \frac{1}{\sqrt{2}a} \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}a} & 2ae^2 - 2m \end{pmatrix}$$

- **Some operators**

$$\hat{A}_{E^2} = \frac{e^2}{2N}\sum_n \ell_n^2$$

$$\hat{A}_{N_{e^+e^-}} = \sum_{n,\,\text{even}} \sigma^+(n)\sigma^-(n)$$

$\longrightarrow$

$$\hat{A}_{E^2}^{\mathbf{k}=\mathbf{0},+} = \frac{e^2}{4}\text{diag}(0,1,2,3,4)$$

$$\hat{A}_{N_{e^+e^-}}^{\mathbf{k}=\mathbf{0},+} = \text{diag}(0,1,2,1,0)$$

See e.g. appendix of 2106.08394

Schwinger Model Coupled w/ Thermal Scalars

- **Hamiltonians** $H = H_S + H_E + H_I$

$$H_E = \int dx \left[\frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{4!}\lambda\phi^4 \right]$$

$$H_I = \lambda \int dx\, \phi(x)\overline{\psi}(x)\psi(x) = \int dx\, O_E(x) O_S(x)$$

- **Lindblad equation in quantum Brownian motion limit**

$$\frac{d\rho_S(t)}{dt} = -i\big[H_S, \rho_S(t)\big] + L\rho_S(t)L^\dagger - \frac{1}{2}\big\{L^\dagger L, \rho_S(t)\big\}$$

- **Focus on states with zero momentum and positive parity**

Only one Lindblad operator: $L = \sqrt{2aND(k_0 = 0, k = 0)}\Big(O_S - \frac{1}{4T}\big[H_S, O_S\big]\Big)$

$$O_S^{\alpha\beta} = \frac{1}{aN_f}\sum_n \Big\langle k = 0, \alpha \Big| \frac{(-1)^n(\sigma_z(n) + 1)}{2} \Big| k = 0, \beta \Big\rangle$$

How to Covert Hamiltonian Evolution into Quantum Circuits

- **Pauli decomposition**

$$H = \sum_{\mu_1,\mu_2,\cdots\mu_n} a_{\mu_1\mu_2\cdots\mu_n}\sigma_1^{\mu_1}\otimes\sigma_2^{\mu_2}\otimes\cdots\otimes\sigma_n^{\mu_n}$$

$$a_{\mu_1\mu_2\cdots\mu_n} = \frac{1}{2^n}\mathrm{Tr}\left[H\left(\sigma_1^{\mu_1}\otimes\sigma_2^{\mu_2}\otimes\cdots\otimes\sigma_n^{\mu_n}\right)\right]$$

- **Trotterization**

$$e^{-iH\Delta t} = e^{\mathcal{O}((\Delta t)^2)}\prod_{\mu_1,\mu_2,\cdots\mu_n} e^{-i\Delta t\, a_{\mu_1\mu_2\cdots\mu_n}\sigma_1^{\mu_1}\otimes\sigma_2^{\mu_2}\otimes\cdots\otimes\sigma_n^{\mu_n}}$$

- **Only need to consider implementing**

$$e^{-i\theta\,\sigma_1^{\mu_1}\otimes\sigma_2^{\mu_2}\otimes\cdots\otimes\sigma_n^{\mu_n}}$$

See e.g. section 4 of 2205.07902

How to Covert Hamiltonian Evolution into Quantum Circuits

- **Covert $\sigma_{x,y}$ into σ_z**

$$h_i\, e^{-i\theta\sigma_i^x}\, h_i = e^{-i\theta\sigma_i^z}$$

Hadamard gate

$$h = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

$$(R_x)_i\, e^{-i\theta\sigma_i^y}\, (R_x^\dagger)_i = e^{-i\theta\sigma_i^z}$$

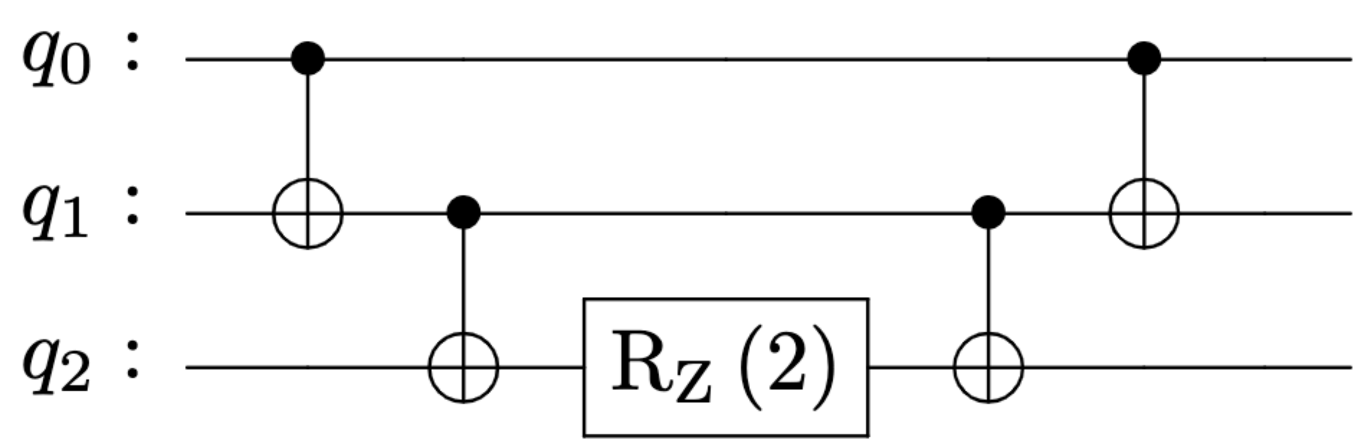
$$R_x = S^\dagger h\, S^\dagger$$

S-gate $\quad S = \begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}$

- **Only need to consider tensor products of 1 and σ_z**

$$e^{-i\theta\sigma_1^z\otimes\sigma_2^z\otimes\sigma_3^z}$$

$$R_z(\theta, j) = \mathrm{diag}(e^{-i\theta/2}, e^{+i\theta/2})$$

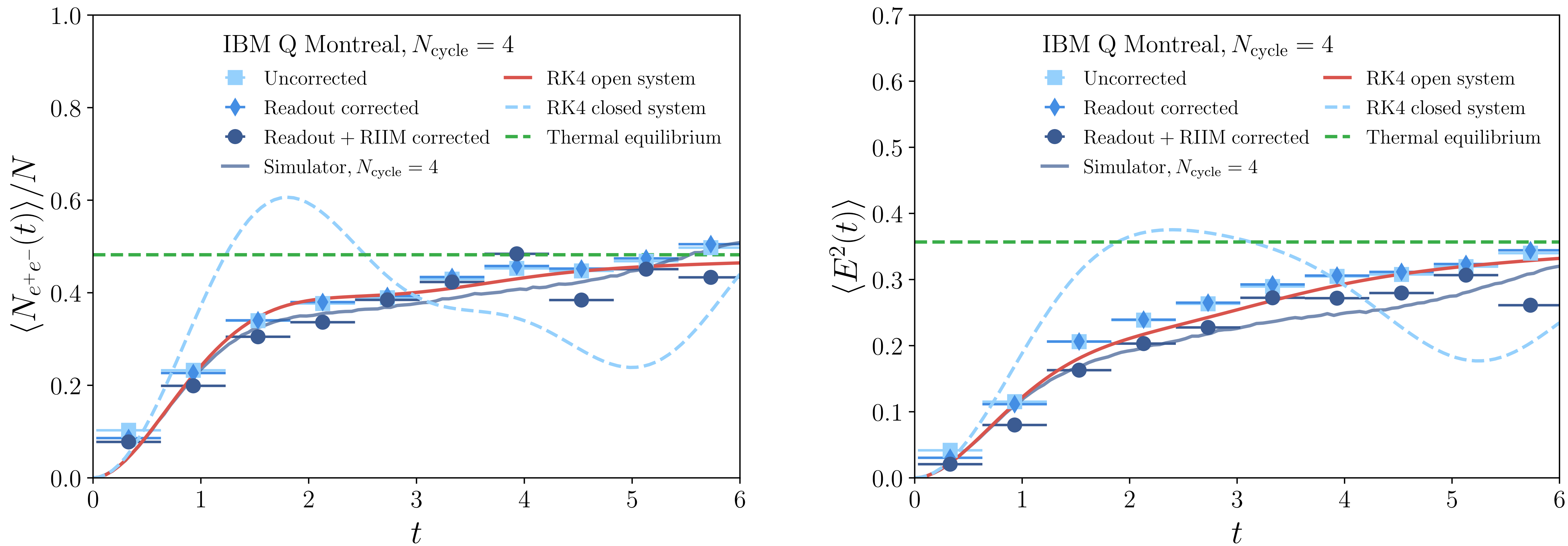


See e.g. section 4 of 2205.07902

Results from Real Quantum Devices

$N = 2$ spatial sites (4 fermion sites)

$$e = \frac{1}{a},\ m = \frac{0.1}{a},\ \beta = 0.1a,\ a = 1$$



Possible to run more cycles to reach closer to equilibrium

Provide a way to prepare a thermal state approximately

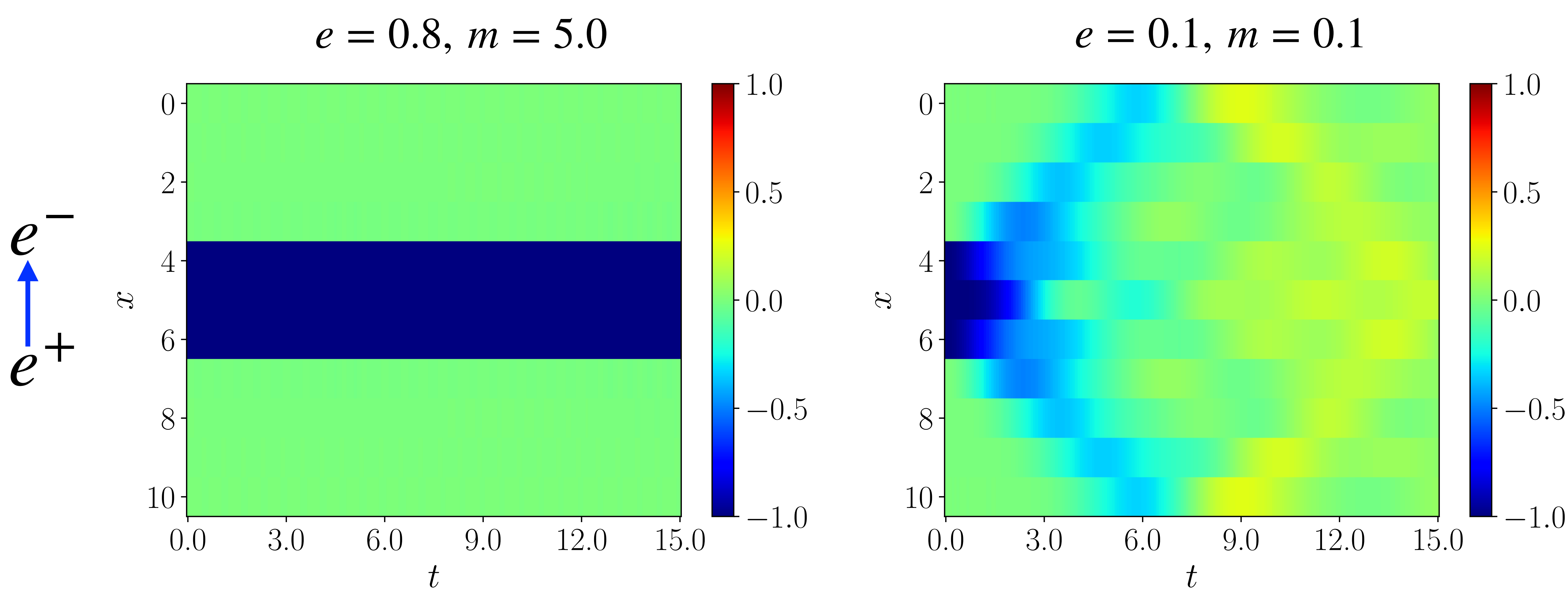
W.A.de Jong, K.Lee, J.Mulligan, M.Ploskon
F.Ringer and XY, 2106.08394

String Breaking in Vacuum

- **Closed Schwinger model with open boundary condition**

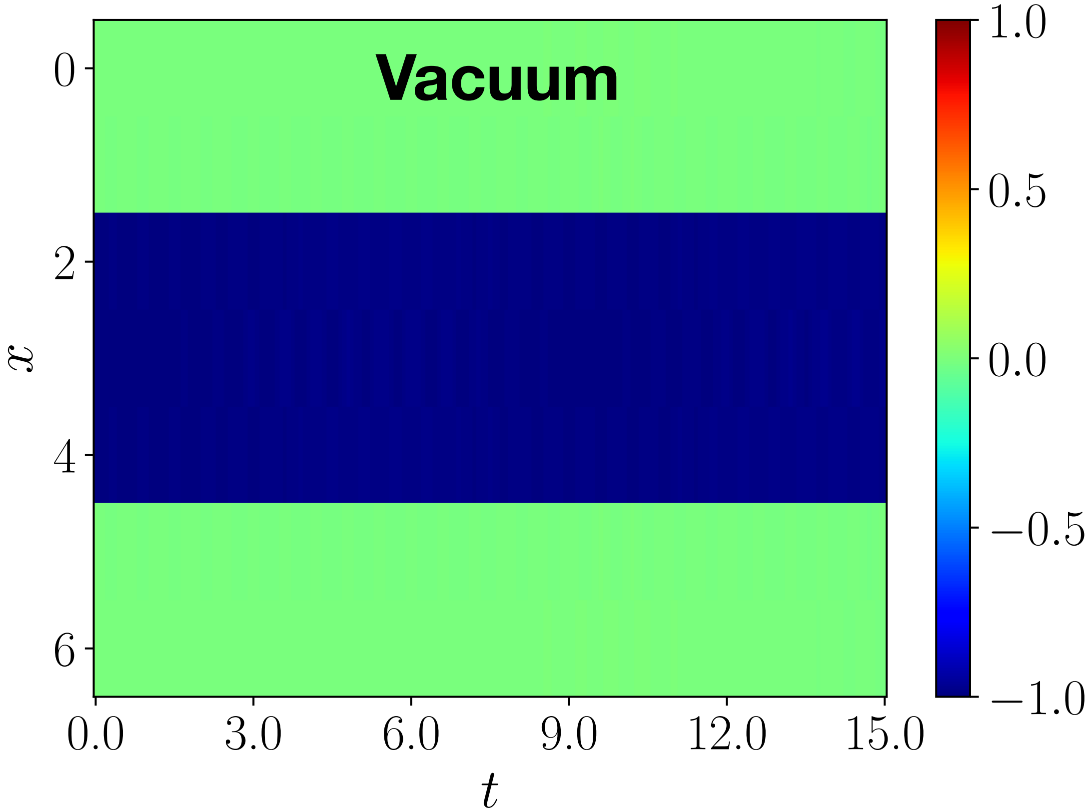
Initialize a pair of electron and positron, time evolve

Calculate electric flux at each site (with vacuum contribution subtracted)



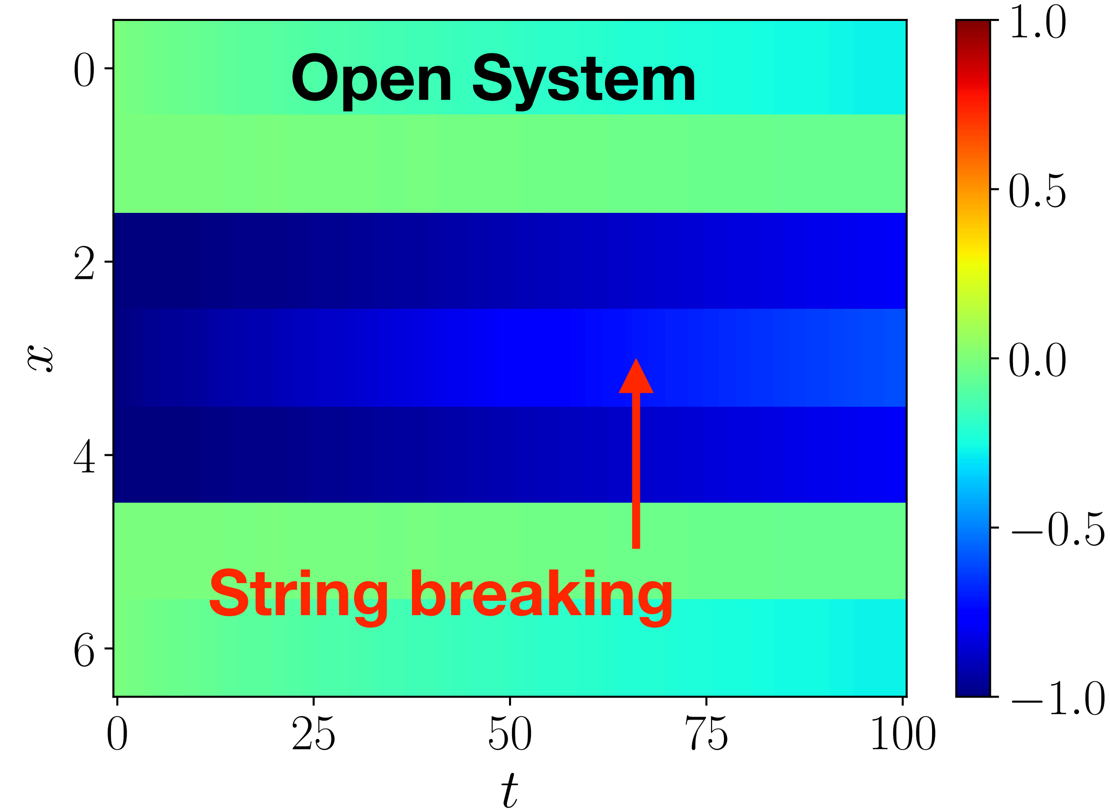
See also e.g. G. Magnifico, et al, 1909.04821

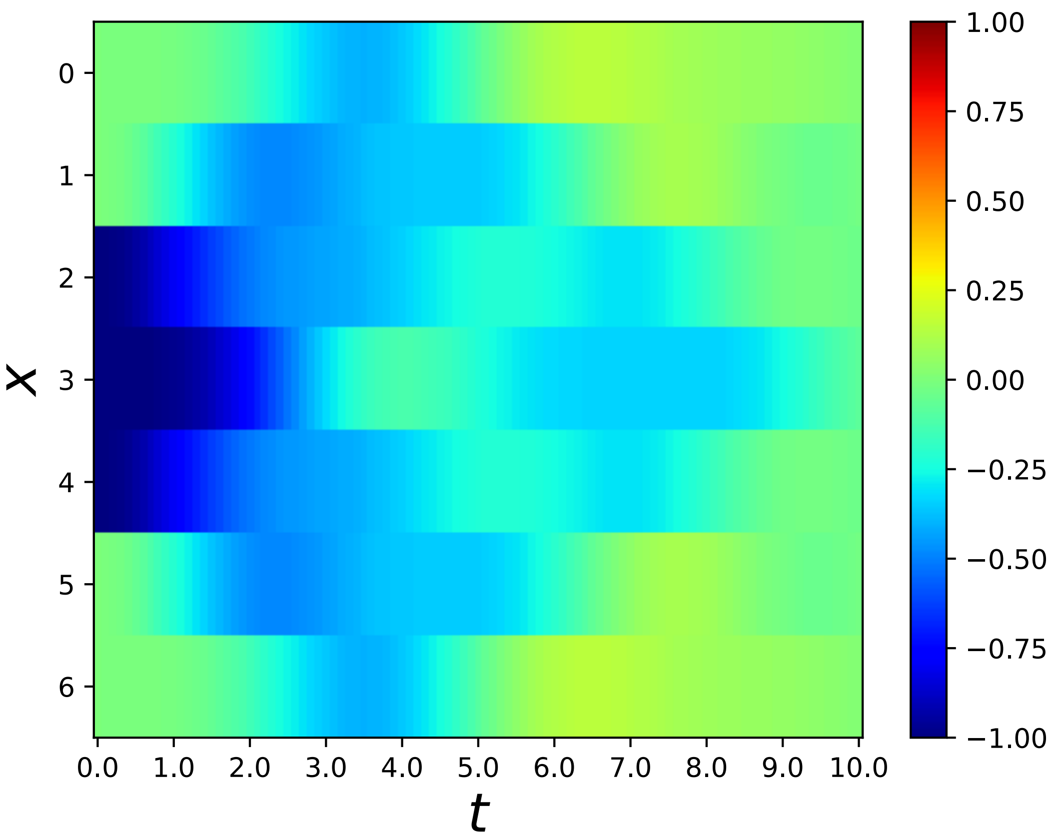
String Breaking/Reconnection in Medium



$e = 0.8,\ m = 5.0$

$\beta = 0.1$



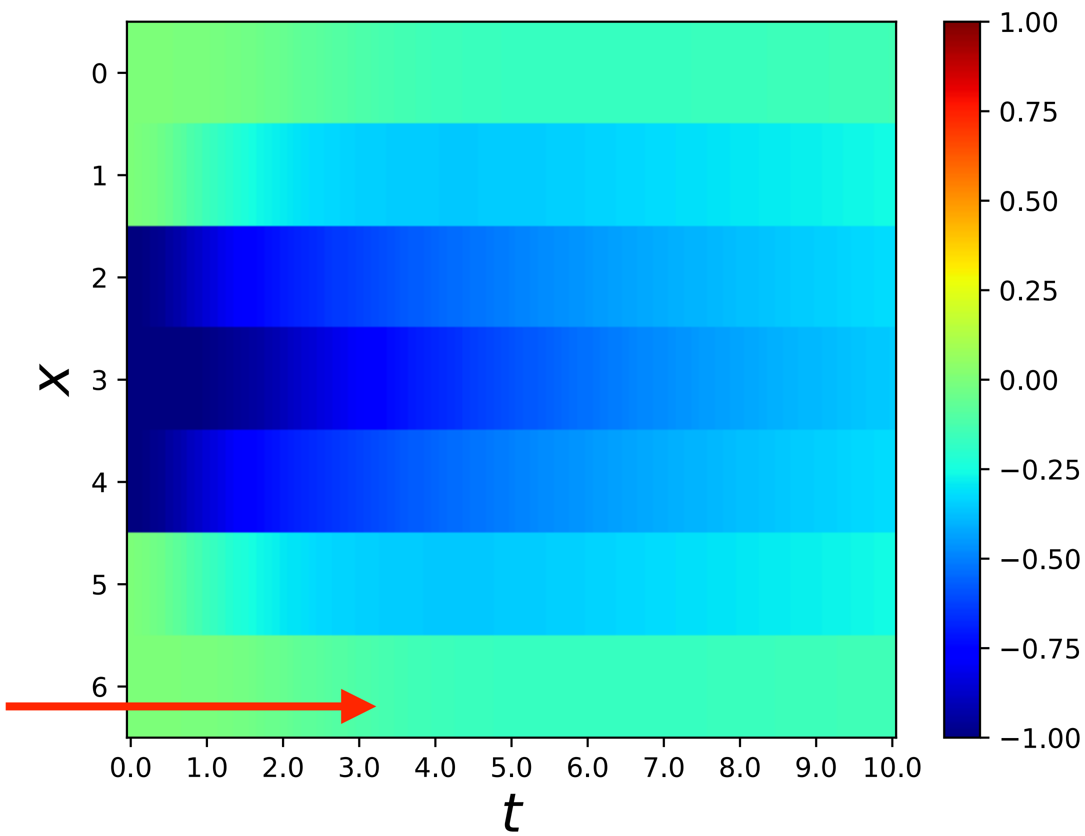


$e = 0.36$

$m = -0.17$

$\beta = 0.5$

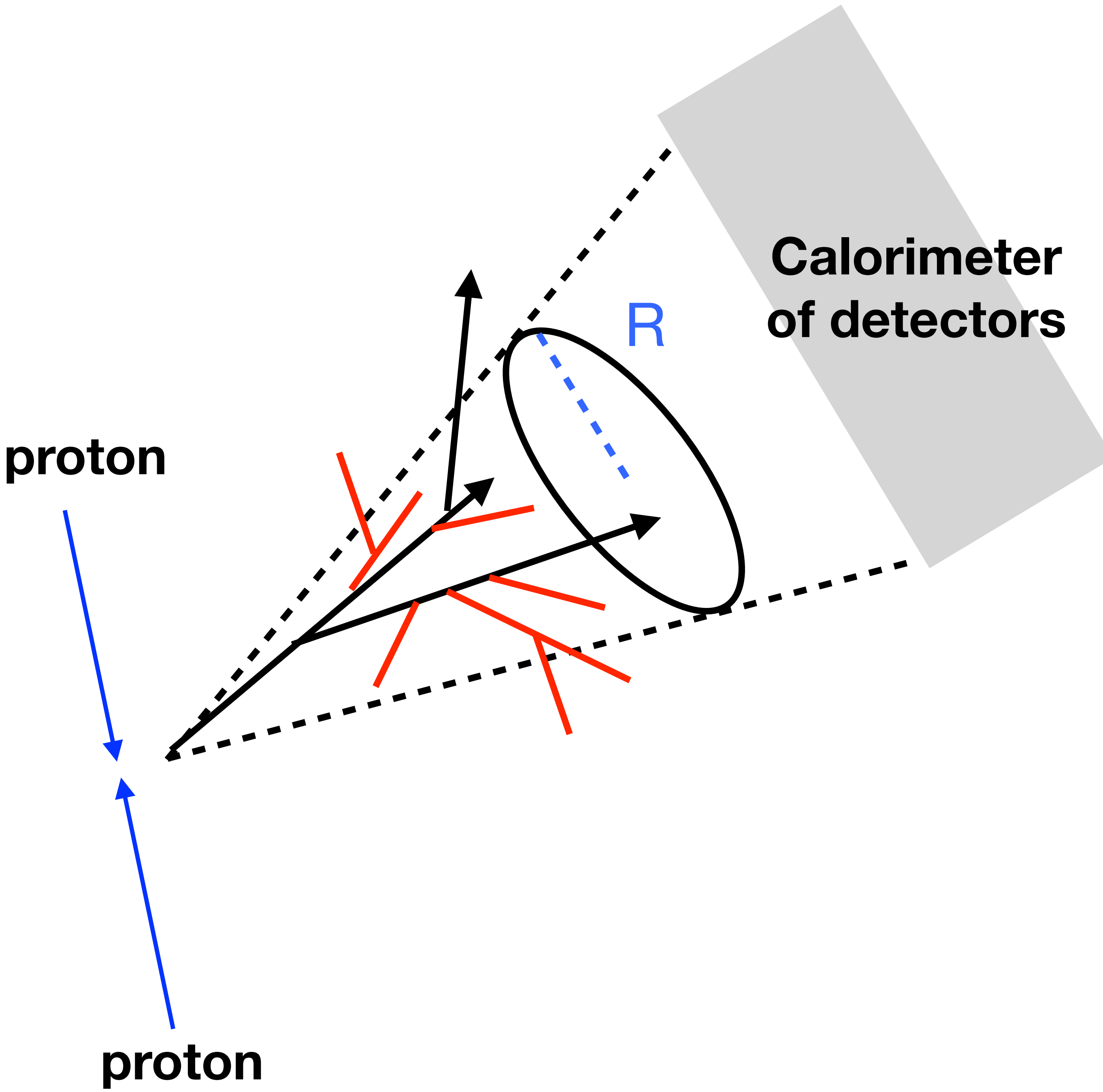
Reconnection

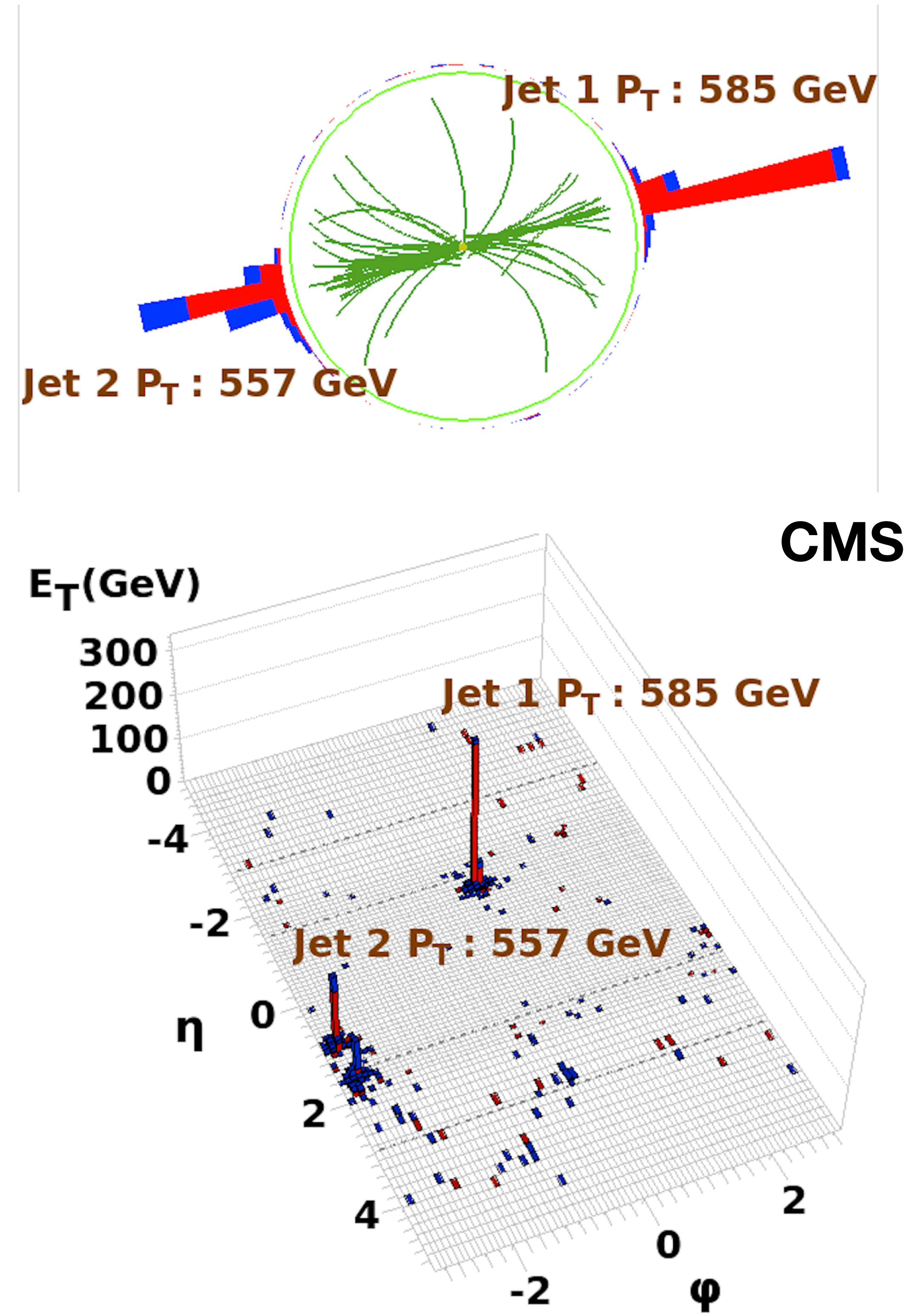


II. Quantum Simulation of Jet Quenching

Jets in High-Energy Collisions

- **Jets**: collimated spray of particles + finding algorithm (e.g. anti-kT)





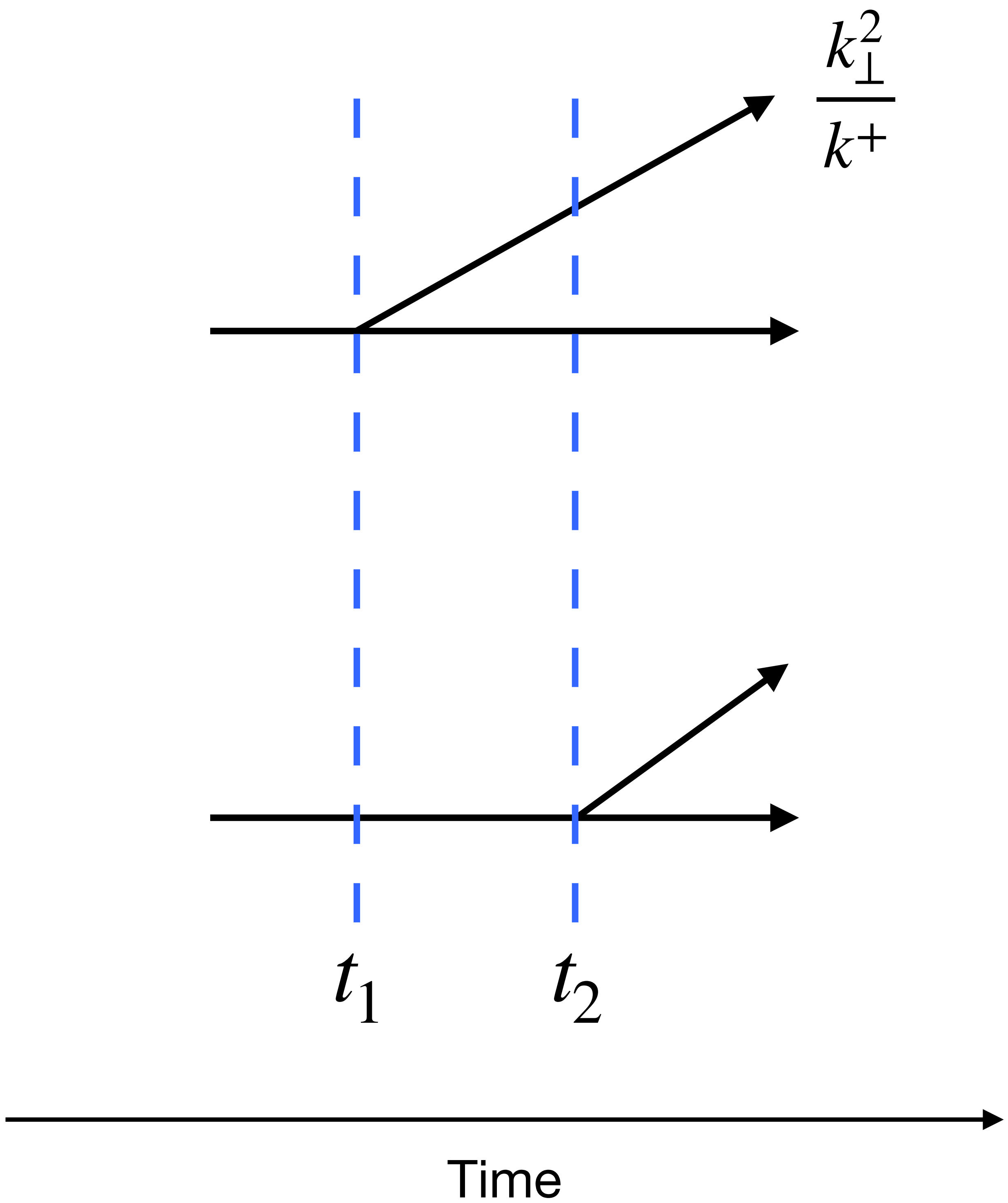
Jets in High Energy Collisions

- Jet can be used as probe of QGP
- Measurements on jet quenching/jet substructure modification <— jet energy loss, medium response, selection bias
- Jet energy loss mechanism
  - Perturbative picture: radiation with medium soft kicks —> Landau-Pomeranchuk-Migdal (LPM) effect —> suppression of radiation
  - Nonperturbative picture: AdS/CFT
  - Mixed: perturbative radiation + nonperturbative soft kicks —> LPM

Quantum interference is important

Jet Radiation in Vacuum

- **Perturbative treatment of jet evolution**

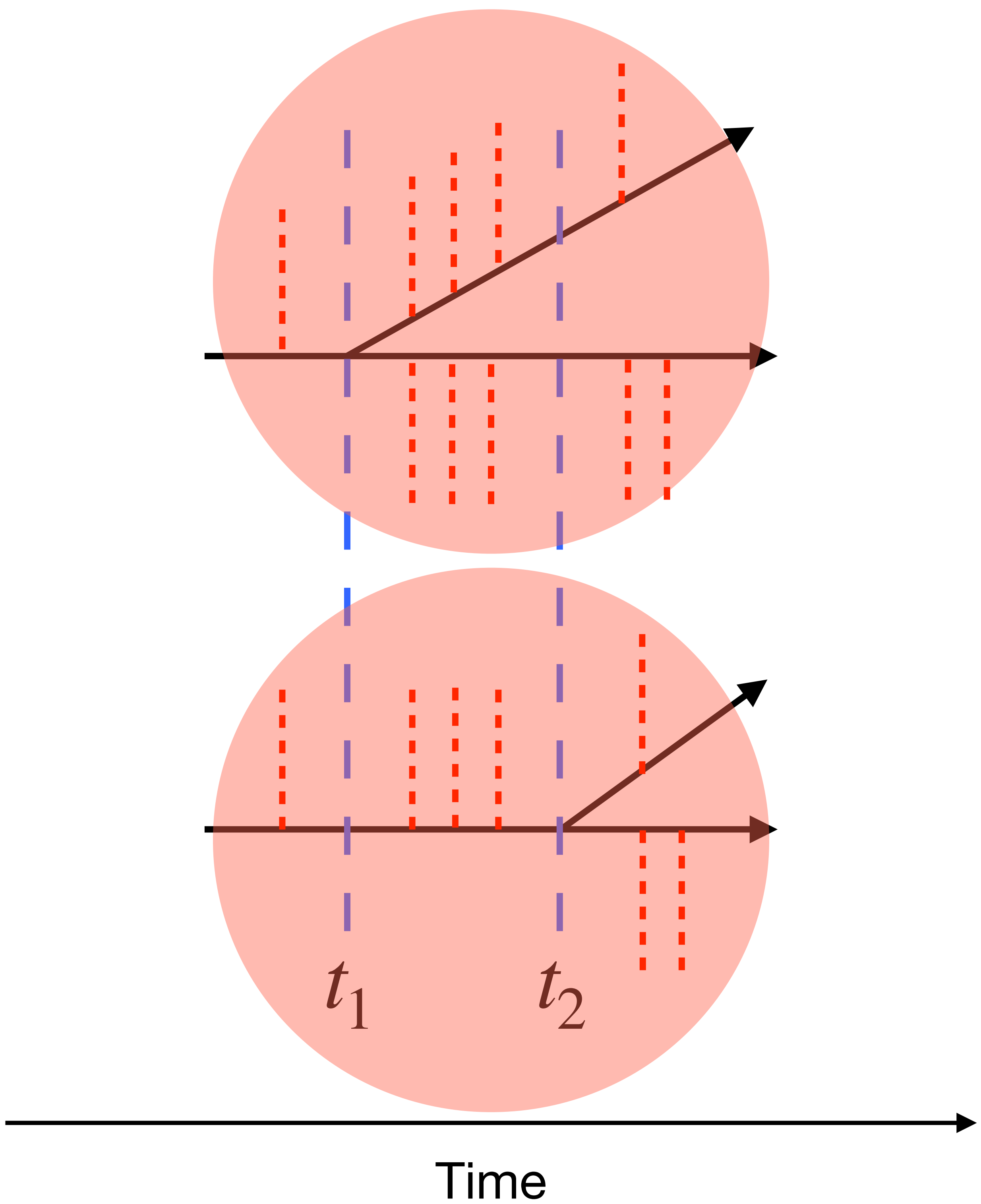


In vacuum: radiations happening at different times contribute coherently

$$\left|1 + e^{i\frac{k_\perp^2}{k^+}(t_2 - t_1)}\right|^2$$
$$= 2 + 2\cos\frac{k_\perp^2}{k^+}(t_2 - t_1)$$

Jet Radiation in Medium: LPM Effect

- **Landau-Pomeranchuk-Migdal (LPM) effect**



In medium: radiations happening at different times lose coherence

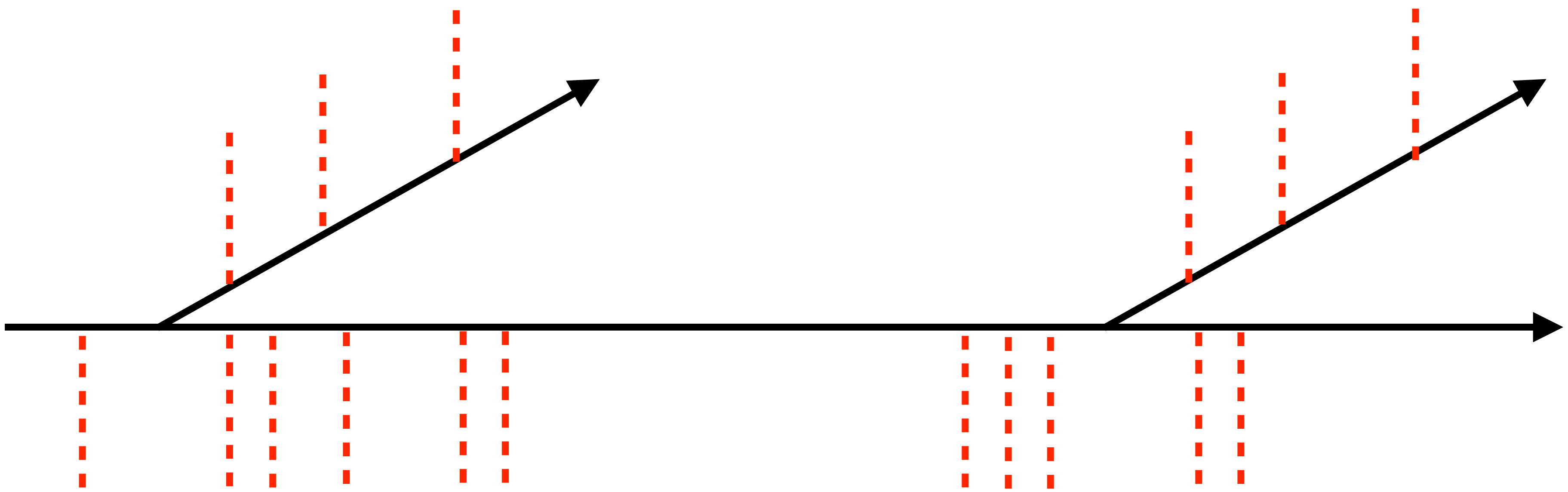
The phase of the radiated parton changes randomly between t_1 and t_2

$$\left|1 + e^{i\frac{k_\perp^2}{k^+}(t_2 - t_1)}\right|^2 \xrightarrow{\text{average } k_\perp} 2$$

Gyulassy, Wang
BDMPS-Z
Wiedermann
AMY
and many more

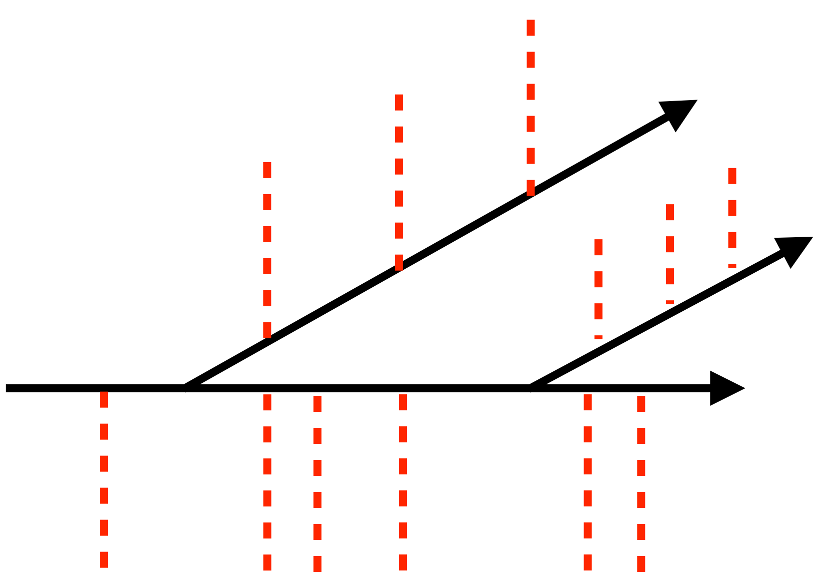
Two Jet Radiations in Medium

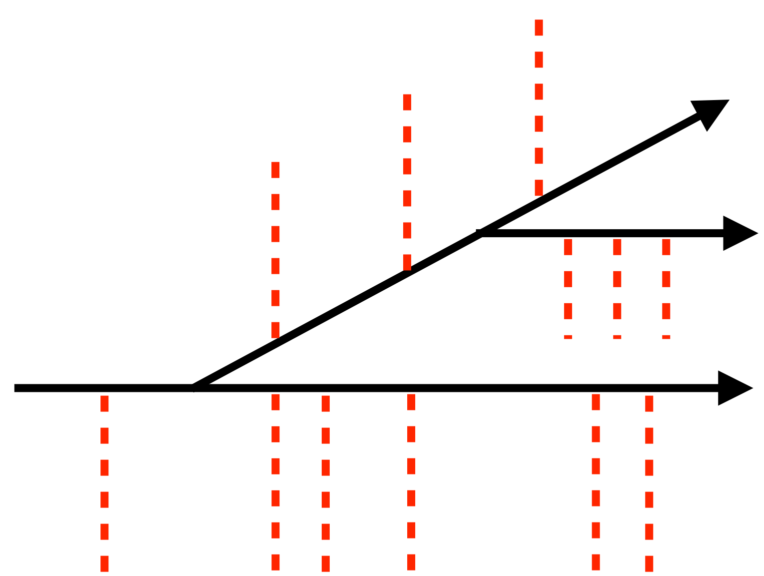
- **Two independent radiations: treat each independently**



Time separation between two radiations >> formation time

- **Two radiations with overlap formation time, coherent treatment**





Arnold, Gorda, Iqbal

Analysis more complicated

No studies on three radiations or beyond

Light-Front Hamiltonian Dynamics

- **Time evolution**

$$2i\frac{\partial}{\partial x^+}|\Psi\rangle = H|\Psi\rangle$$

- **Light-front Hamiltonian**

$$H = \int \mathrm{d}x^- \, \mathrm{d}^2 x_\perp \Big(i\psi_+^\dagger(-\not{D}_\perp + im)\frac{1}{\partial^+}(\not{D}_\perp + im)\psi_+ - g\psi_+^\dagger A^{-a}T^a\psi_+ + \frac{1}{4}F_\perp^{ija}F_{\perp ij}^a - \frac{1}{8}(\partial^+ A^{-a})^2 + \frac{1}{2}(\partial^+ A_\perp^{ia})(-\partial_i A^{-a} + gf^{abc}A^{-b}A_{\perp i}^c)\Big)$$

A^- field not dynamical

$$A^{-a} = \frac{2}{\partial^+}\partial^i A_\perp^{ia} - \frac{2g}{\partial^{+2}}\Big(f^{abc}(\partial^+ A_\perp^{ib})A_\perp^{ic} - 2\psi_+^\dagger T^a\psi_+\Big)$$

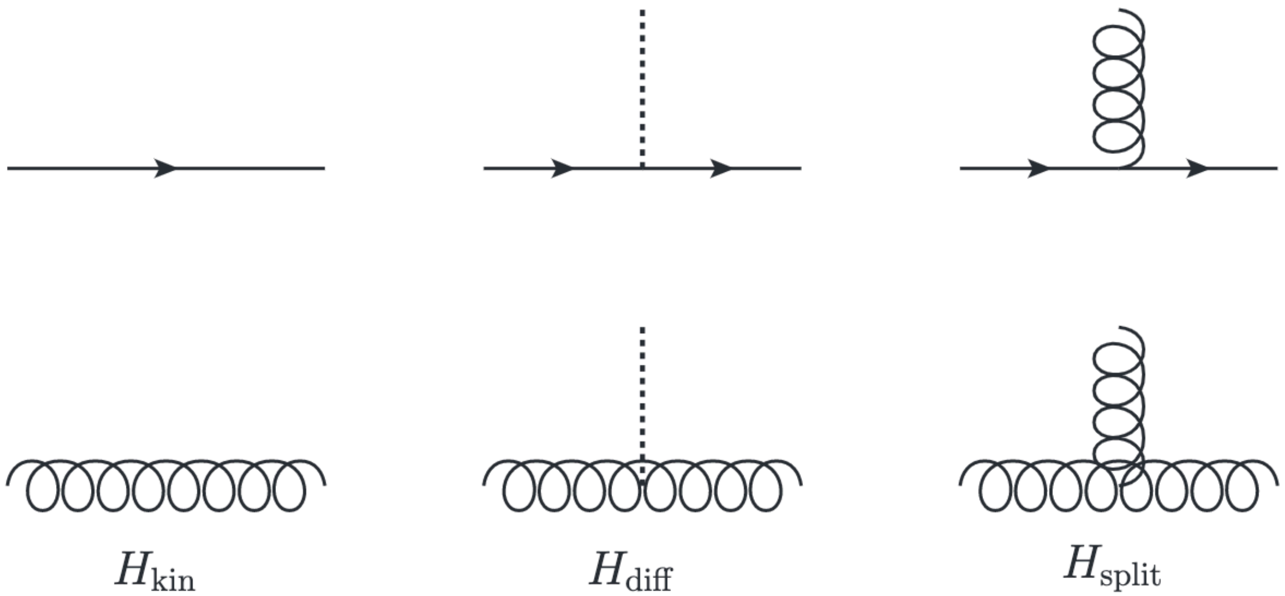
- **Background field $\bar{A}^-$ to describe medium effects**

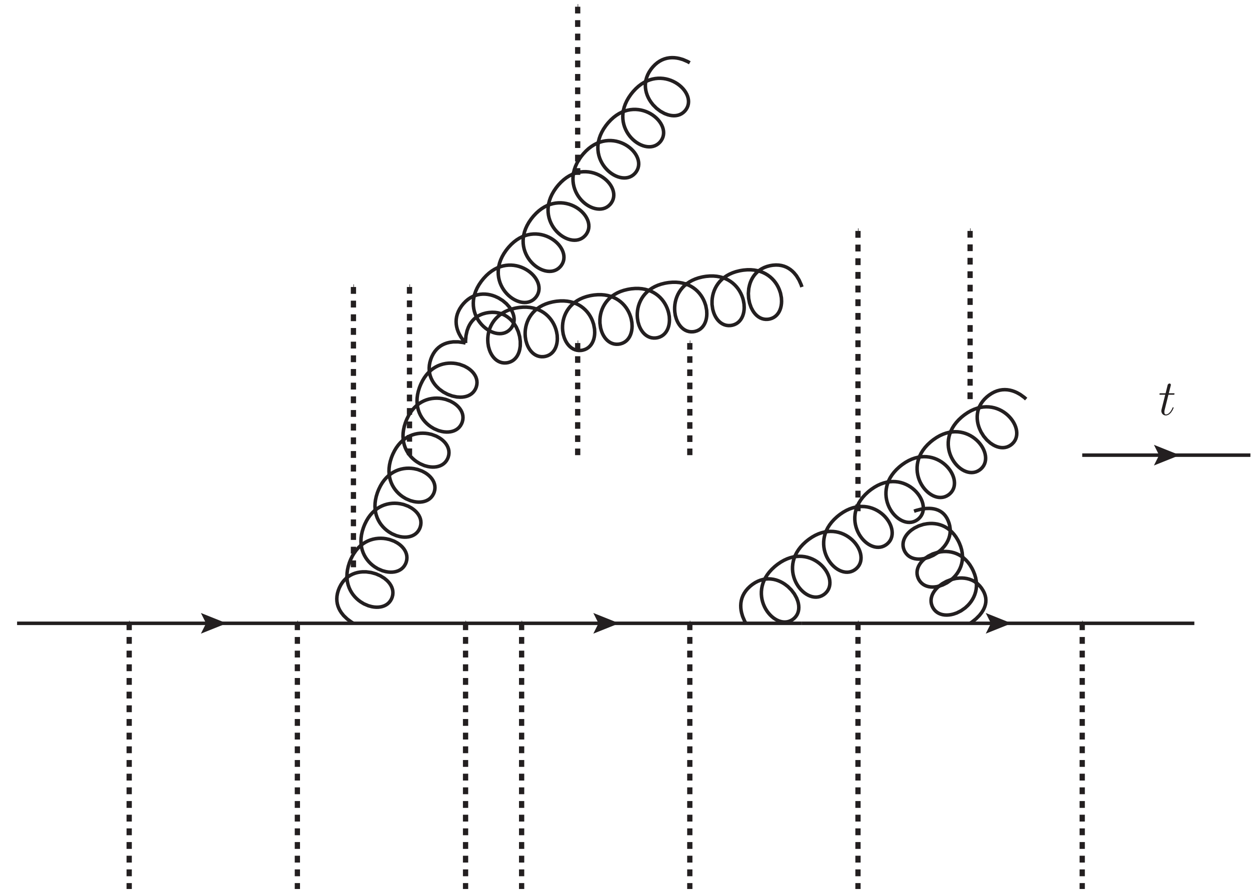
$$A^{-a} \to A^{-a} + \bar{A}^{-a}(x^+)$$

$$\big\langle \bar{A}^{-a}(x^+, x_\perp)\bar{A}^{-b}(y^+, y_\perp)\big\rangle = \delta^{ab}\delta(x^+ - y^+)\gamma(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)$$

Trotterization

$$H(x^+) = H_{\text{kin}} + H_{\text{diff}}(x^+) + H_{\text{split}}$$





Interference automatically included !

$$\left(e^{-i(H_{\text{kin}}+H_{\text{diff}}+H_{\text{split}})\Delta t}\right)^{N_t}|\Psi\rangle = \left(\prod_j e^{-iH_j\Delta t}e^{\mathcal{O}((\Delta t)^2)}\right)^{N_t}|\Psi\rangle$$

$$\sum_j H_j = H_{\text{kin}} + H_{\text{diff}} + H_{\text{split}} \qquad [H_j, H_k] \neq 0, \ j \neq k$$

See XY 2205.07902

Hilbert Space

- **Use n-particle state in momentum space**

$$\bigotimes_{i=1}^{n} \left| q/g,\ k^+ > 0,\ k_x,\ k_y,\ \text{color},\ \text{spin} \right\rangle_i$$

- **Size of Hilbert space with discrete momentum**

$$k^+ \in (0, K^+_{\text{max}}], \quad k_x \in [-K^\perp_{\text{max}}, K^\perp_{\text{max}}], \quad k_y \in [-K^\perp_{\text{max}}, K^\perp_{\text{max}}]$$

Qubit # for 1-particle Hilbert space: $\log_2(2^5 N^+ N_\perp^2)$

$$N^+ = \frac{K^+_{\text{max}}}{\Delta k^+}, \quad N_\perp = \frac{2K^\perp_{\text{max}}}{\Delta k^\perp} + 1$$

- **To study LPM effect with N particles, need 1-, 2-, … N-particle states**

$$\log_2 \left(\sum_{n=1}^{N} (2^5 N^+ N_\perp^2)^n \right)$$

For $N^+ = N_\perp = 100$

Need 100 qubits for N=4

See XY 2205.07902

Gluon Radiation on Small Momentum Lattice

- **Momentum discretization**

$$k^+ \in K^+_{\max}\{0.5, 1\}\,,\;\; k_x \in K^\perp_{\max}\{0, 1\}\,,\;\; k_y \in K^\perp_{\max}\{0, 1\}$$

For single particle:

$$|\underbrace{q_1 q_2 q_3}_{\text{describe momentum}} \overbrace{q_4 q_5 q_6}^{\text{describe color}} \underbrace{q_7}_{\text{describe polarization}}\rangle$$

- **LPM effect in cases with one initial particle, one splitting**

$$|\underbrace{q_1}_{\text{separate 1-gluon and 2-gluon states}} \overbrace{q_2 q_3 \cdots q_8}^{\text{describe 2nd gluon}} \underbrace{q_9 q_{10} \cdots q_{15}}_{\text{describe 1st gluon}}\rangle$$

1-gluon states: $|0\ 0000000\ q_9 q_{10} \ldots q_{15}\rangle$

2-gluon states: $|1\ q_2 q_3 \ldots q_8\ q_9 q_{10} \ldots q_{15}\rangle$

See XY 2205.07902

Discretized Light-Front Hamiltonians

- **Kinetic**

$$\left\langle g,\, k_1^+,\, k_{1\perp},\, a_1,\, \lambda_1 \middle| H_{g,\,\mathrm{kin}} \middle| g,\, k_2^+,\, k_{2\perp},\, a_2,\, \lambda_2 \right\rangle = \frac{\boldsymbol{k}_{1\perp}^2}{k_1^+} \delta_{k_1^+ k_2^+} \delta_{k_{1x} k_{2x}} \delta_{k_{1y} k_{2y}} \delta_{a_1 a_2} \delta_{\lambda_1 \lambda_2}$$

- **Diffusion**

$$\left\langle g,\, k_1^+,\, k_{1\perp},\, a_1,\, \lambda_1 \middle| H_{g,\,\mathrm{diff}}(x^+) \middle| g,\, k_2^+,\, k_{2\perp},\, a_2,\, \lambda_2 \right\rangle$$
$$= \frac{ig}{2(2\pi)^2} \Delta k_x \Delta k_y \delta_{k_1^+ k_2^+} \delta_{\lambda_1 \lambda_2} \left(f^{a_2 b a_1} - f^{a_1 b a_2} \right) \bar{A}^{-b}(x^+, \boldsymbol{k}_{1\perp} - \boldsymbol{k}_{2\perp})$$

$$\left\langle \bar{A}^{-a}(x^+, k_\perp) \bar{A}^{-a}(x^+, -k_\perp) \right\rangle = \frac{(2\pi)^2 \gamma(\boldsymbol{k}_\perp)}{\Delta x^+ \Delta k_x \Delta k_y}$$

$$\gamma(\boldsymbol{k}_\perp) = g^2 \frac{\pi T m_D^2}{(\boldsymbol{k}_\perp^2 + m_D^2)^2}$$

Discretized Light-Front Hamiltonians

- **Splitting**

$$\langle g, k_2^+, k_{2\perp}, a_2, \lambda_2; g, k_3^+, k_{3\perp}, a_3, \lambda_3 \big| H_{g,\,\mathrm{split}} \big| g, k_1^+, k_{1\perp}, a_1, \lambda_1 \rangle$$

$$= -ig\sqrt{\frac{\Delta k^+ \Delta k_x \Delta k_y}{2(2\pi)^3 k_1^+ k_2^+ k_3^+}} f^{a_1 a_2 a_3} \delta_{k_1^+,\, k_2^+ + q^+} \delta_{k_{1x},\, k_{2x}+q_x} \delta_{k_{1y},\, k_{2y}+q_y}$$

$$\Bigg(k_1^+ \epsilon_\perp^i(\lambda_1) \Big[\frac{k_{2\perp}^j}{k_2^+} \epsilon_\perp^j(\lambda_2) \epsilon_{\perp i}(\lambda_3) - \frac{k_{3\perp}^j}{k_3^+} \epsilon_\perp^j(\lambda_3) \epsilon_{\perp i}(\lambda_2) \Big] - k_2^+ \epsilon_\perp^i(\lambda_2) \Big[\frac{k_{3\perp}^j}{k_3^+} \epsilon_\perp^j(\lambda_3) \epsilon_{\perp i}(\lambda_1)$$

$$- \frac{k_{1\perp}^j}{k_1^+} \epsilon_\perp^j(\lambda_1) \epsilon_{\perp i}(\lambda_3) \Big] - k_3^+ \epsilon_\perp^i(\lambda_3) \Big[\frac{k_{1\perp}^j}{k_1^+} \epsilon_\perp^j(\lambda_1) \epsilon_{\perp i}(\lambda_2) - \frac{k_{2\perp}^j}{k_2^+} \epsilon_\perp^j(\lambda_2) \epsilon_{\perp i}(\lambda_1) \Big]$$

$$- k_{1\perp}^i \epsilon_\perp^j(\lambda_1) \Big[\epsilon_{\perp i}(\lambda_2) \epsilon_{\perp j}(\lambda_3) - \epsilon_{\perp i}(\lambda_3) \epsilon_{\perp j}(\lambda_2) \Big] + k_{2\perp}^i \epsilon_\perp^j(\lambda_2) \Big[\epsilon_{\perp i}(\lambda_3) \epsilon_{\perp j}(\lambda_1)$$

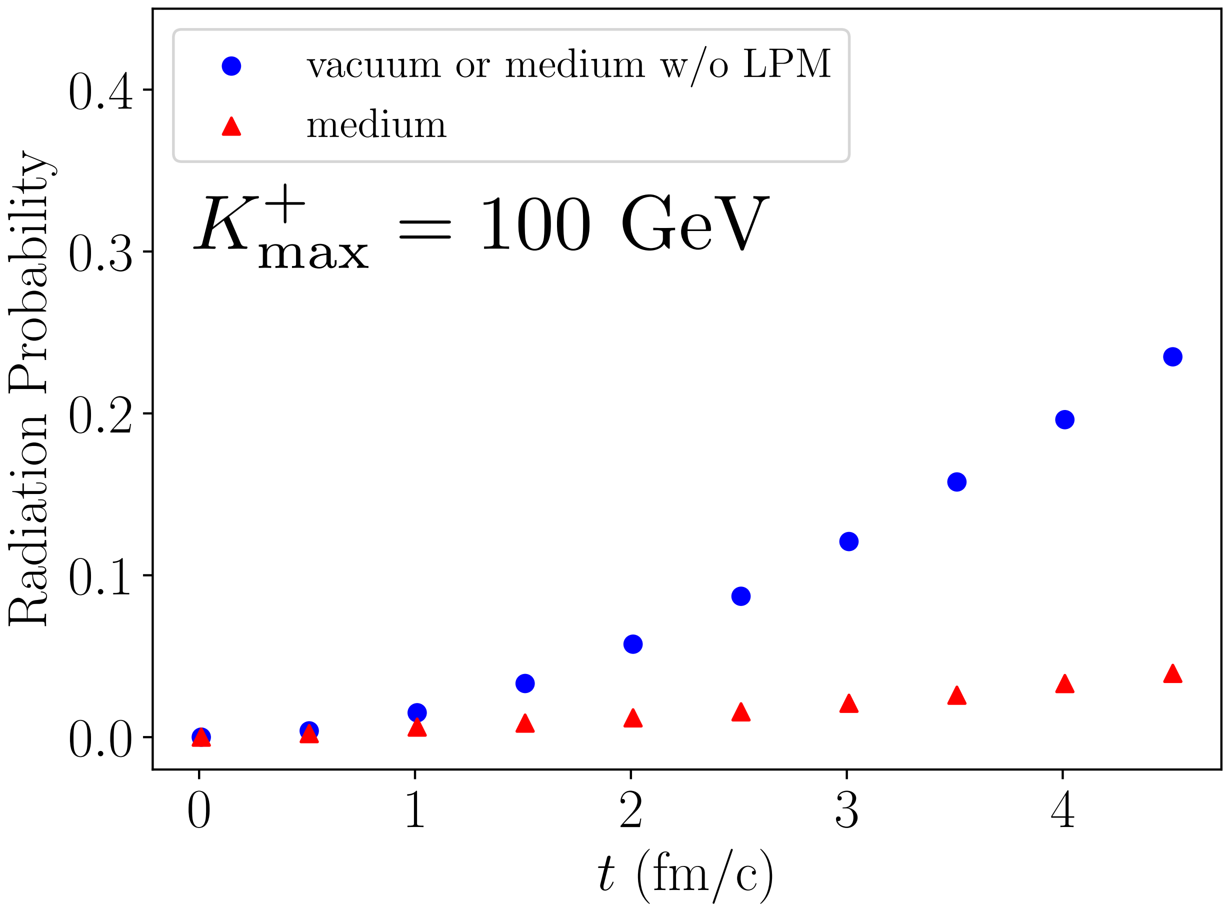
$$- \epsilon_{\perp i}(\lambda_1) \epsilon_{\perp j}(\lambda_3) \Big] + k_{3\perp}^i \epsilon_\perp^j(\lambda_3) \Big[\epsilon_{\perp i}(\lambda_1) \epsilon_{\perp j}(\lambda_2) - \epsilon_{\perp i}(\lambda_2) \epsilon_{\perp j}(\lambda_1) \Big] \Bigg) ,$$

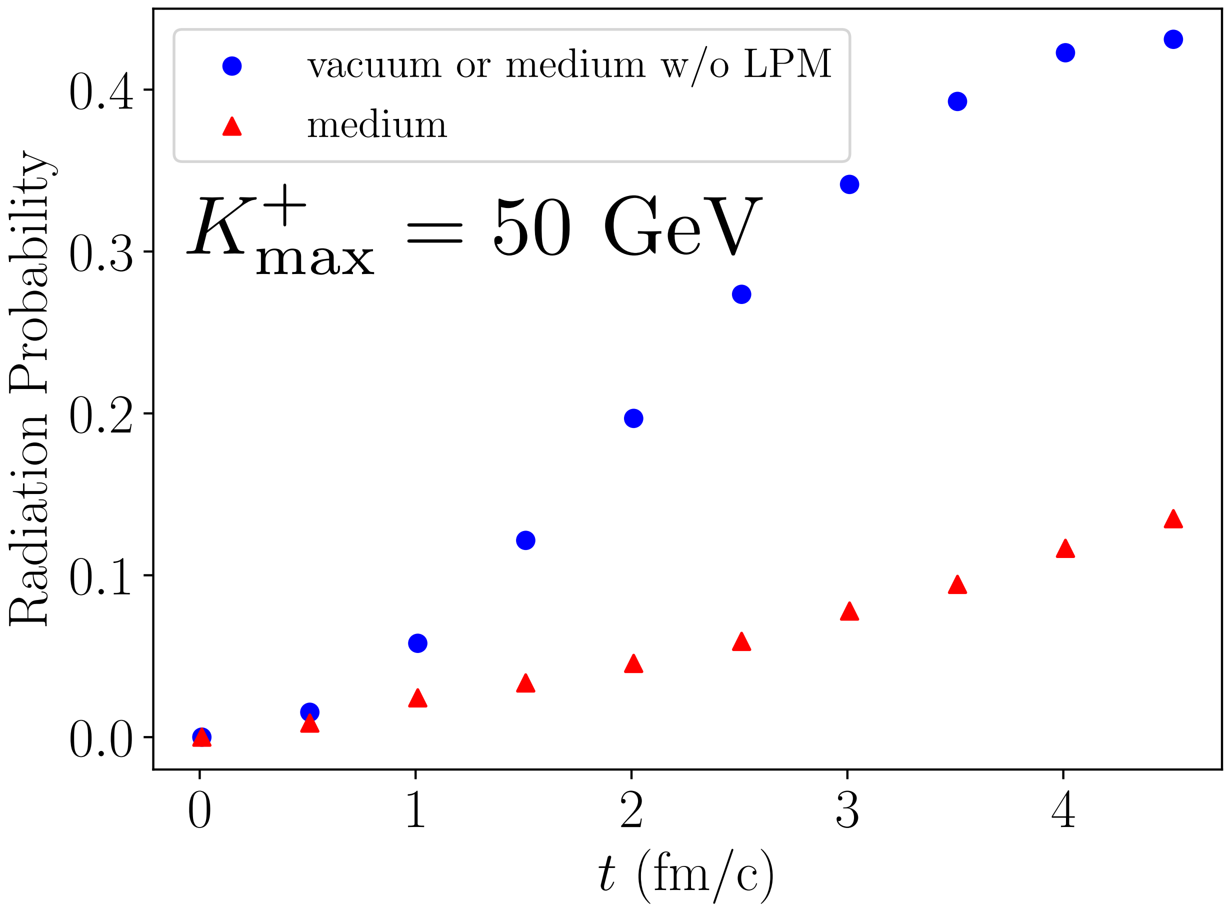
LPM Effect in Gluon Radiation

$$K^+_{\rm max} = 100\ {\rm GeV\ or\ } 50\ {\rm GeV},\ K^\perp_{\rm max} = 1\ {\rm GeV},\ g = 2,\ T = 300\ {\rm MeV}$$

$$|\psi(t=0)\rangle = |0\ 0000000\ 1110000\rangle$$

- **Radiation probabilities in vacuum v.s. medium**





Vacuum: averaged from 2^{20} shots (number of simulations)

Medium: 500 quantum trajectories are averaged, each trajectory has a different set of classical background fields sampled

See XY 2205.07902

Summary

- Quantum simulation for high energy nuclear physics
  - Open quantum systems
    - Schwinger model, thermalization, string breaking and reconnection
  - Jet quenching in nuclear environments
    - Light-front Hamiltonian approach, quantum interference automatically included
    - LPM effect seen in gluon radiation on a small lattice; With a few hundred fault-tolerant qubits, we can simulate LPM effect for more than two splittings and go beyond scope of state-of-the-art analyses