Partonic Structure by Quantum Computing

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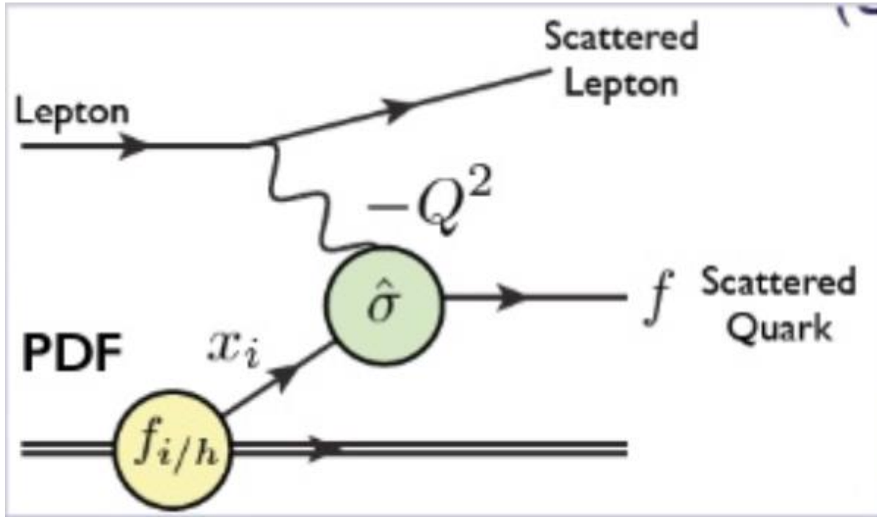
Contents

Parton Distribution Function(PDF)

- Properties:
  - Non-perturbative.
  - “Universal”.

- “Conventional” study method:
  - Global fitting of different experimental data.
  - Lattice QCD calculation: quasi-PDF, pseudo-PDF



PDF: Extensions

- More dimensions:
  - TMD PDF
  - Spin distribution: spin crisis
- More Particles
  - Mesons
  - Nuclears: cold nuclear effect, EMC, …

Quantum Computing

- "Hardware": Quantum gates →Quantum circuits.
- "Software": Quantum algorithms.
  - Shor's algorithm
  - Quantum Fourier transform
  - …
  - Hybrid algorithm
    - QAOA
    - VQE
    - Quantum Machine Learning
    - …

Quantum Computing

- Advantage:
  - Superposition and entanglement: speed up algorithms
  - Simulation of quantum systems

“… and if you want to make a simulation of nature, you’d better make it quantum mechanical, …” --Richard Feynman

Simulating Physics with Computers

Richard P. Feynman

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Quantum Computing

- Challenges in the algorithm field:
  - Limited qubits: contemporary quantum devices vs. classical simulation.
  - Noise.
  - Gauge fields: fermion effective theory, …
  - …

Quantum Computing of PDFs

- Hadronic tensor [Phys. Rev. Res. 2, 013272(2020)].
- Wilson loops [arXiv:1908.07051].
- Hybrid approach [Phys. Rev. D 102, 016007(2020)].
- Global analysis with quantum machine learning [Phys. Rev. D 103, 034027(2020)]
- …

Contents

Calculation of PDF

- Definition:

$$f_{q/h}(x) = \int \frac{dz}{4\pi} e^{-ixM_h z} \langle h | \bar{\psi}(z, -z) \gamma^+ \psi(0,0) | h \rangle$$

$$= \int \frac{dz}{4\pi} e^{-ixM_h z} \langle h | e^{iHz} \bar{\psi}(0, -z) e^{-iHz} \gamma^+ \psi(0,0) | h \rangle$$

$$\gamma^+ = \gamma^0 + \gamma^1$$

- Relevant dimension: 1+1D
- Preparation of the hadronic state.
- Calculation of the (dynamical) correlation function.

1+1D NJL model

$$\mathcal{L} = \bar{\psi}_\alpha\left(i\gamma^\mu\partial_\mu - m_\alpha\right)\psi_\alpha + g(\bar{\psi}_\alpha\psi_\alpha)^2$$

$$H = \int dx\,[-i\bar{\psi}_\alpha(i\gamma^1\partial_1 - m_\alpha)\psi_\alpha - g(\bar{\psi}_\alpha\psi_\alpha)^2]$$

- Discretization: staggered fermion approach

$$\psi_\alpha(x) = \begin{pmatrix}\psi_{\alpha,1}\\ \psi_{\alpha,2}\end{pmatrix} = \begin{pmatrix}\psi_{\alpha,2n}\\ \psi_{\alpha,2n+1}\end{pmatrix}$$

$$H = \sum_{\alpha,n}[-\frac{i}{2}\left(\psi^\dagger_{\alpha,n}\psi_{\alpha,n+1} - \psi^\dagger_{\alpha,n+1}\psi_{\alpha,n}\right) + (-1)^n m_\alpha\psi^\dagger_{\alpha,n}\psi_{\alpha,n}]$$

$$-g\sum_{\alpha,n=even}[\psi^\dagger_{\alpha,n}\psi_{\alpha,n} + \psi^\dagger_{\alpha,n+1}\psi_{\alpha,n+1} - 2\psi^\dagger_{\alpha,n}\psi_{\alpha,n}\psi^\dagger_{\alpha,n+1}\psi_{\alpha,n+1}]$$

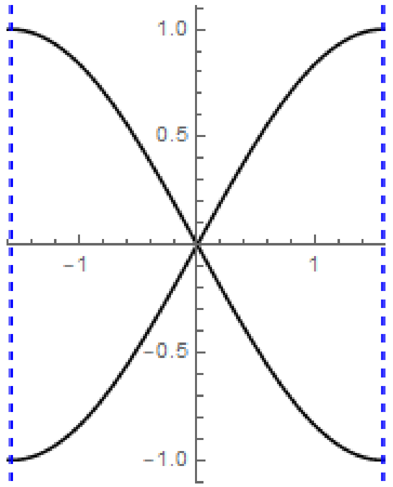
Staggered Fermion Approach

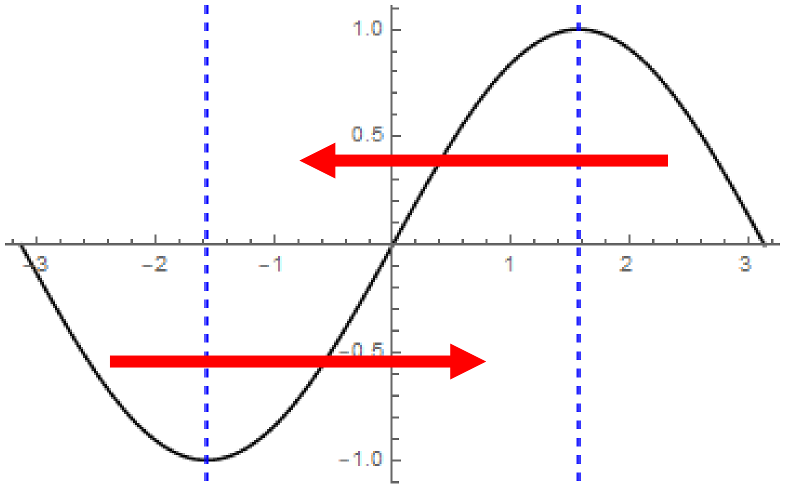
- Remaining redundant mode: $\frac{2^d}{k}$
- d: number of discretized dimensions
- k: number of fermion components

- 1+1D: $d = 1, k = 2$, good!

Staggered Fermion Approach

- Multi-component field ψ: $\gamma^0 E = \gamma^1 \sin p \rightarrow E^2 = \sin^2 p$
- One-component field χ: $E = \sin p$, reduced modes!





Staggered Fermion Approach

$$H_\chi = i\sum_n \chi_n^\dagger \frac{\chi_{n+1} - \chi_{n-1}}{a} + V \leftrightarrow$$

$$H_\psi = i\sum_n [\psi_{n,1}^\dagger \frac{\psi_{n,2} - \psi_{n-1,2}}{a} + \psi_{n,2}^\dagger \frac{\psi_{n+1,1} - \psi_{n,1}}{a}] + V$$

- Effectively modifies the momentum term
- Eigenvalues of $\begin{pmatrix} 0 & i\frac{1-e^{-iap}}{a} \\ i\frac{e^{iap}-1}{a} & 0 \end{pmatrix}$ is $\pm\frac{2\sin\frac{ap}{2}}{a}$ $(-\frac{\pi}{a} \leq p \leq \frac{\pi}{a})$

Jordan-Wigner transformation

- Express field operator as Pauli matrices that can be handled by a quantum computer.
- Keeps the anticommuting relation.

$$\psi_{\alpha,n} = \prod_{\beta=1}^{\alpha-1} \tilde{\sigma}^3_{\beta,\frac{N}{2}} \tilde{\sigma}^3_{\alpha,n} \sigma^+_{\alpha,n}$$

$$\tilde{\sigma}^3_{\alpha,n} = \prod_{i<n} \sigma^3_{\alpha,i}\,, \sigma^{\pm}_{\alpha,n} = \frac{1}{2}(\sigma^1_{\alpha,n} \pm i\sigma^2_{\alpha,n})$$

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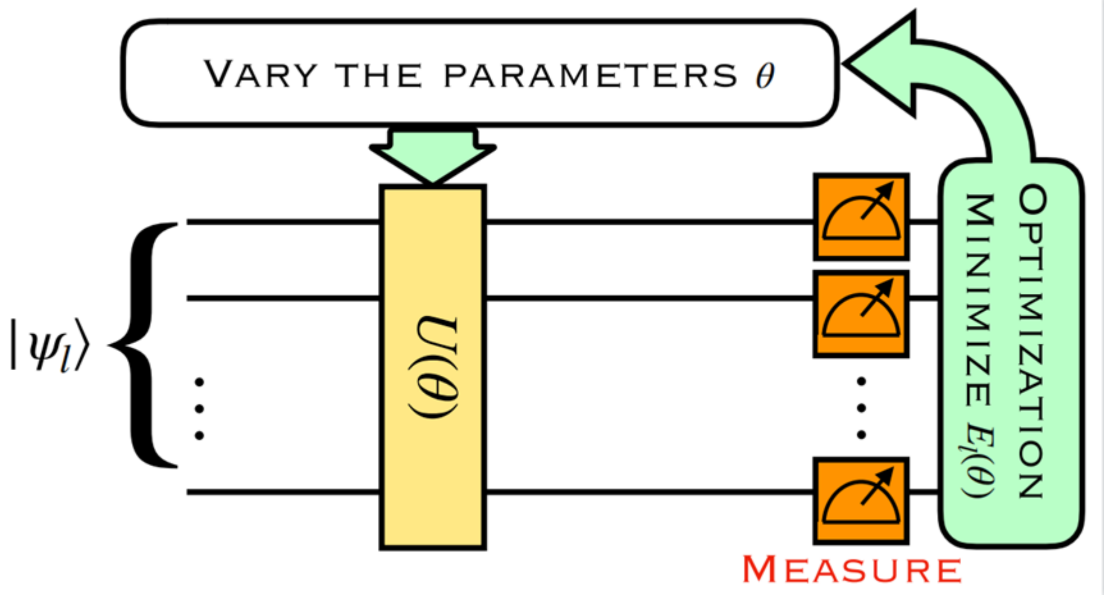
Hadronic State Preparation

- Hadron states are the (lowest few) eigenstates with fixed given quantum numbers.
- Constructing trial states: Quantum alternating operator ansatz(QAOA).
- Find eigenstates: minimizing the energy expectations.



QAOA[arXiv:1411.4028]

- Divide the Hamiltonian H into n pieces $H_1, H_2, \ldots, H_n$:
  - Each H_i has the same symmetry of H,
  - $[H_i, H_{i+1}] \neq 0$,
  - One of H_i is diagonal.
- Parameterized symmetry-preserving operator of p layers:

$$U(\theta) = \prod_{i=1}^{p} \prod_{j=1}^{n} e^{i\theta_{ij}H_j}$$

- For the k-th state with given quantum number l

$$|\psi_{lk}(\theta)\rangle = U(\theta)|\psi_{lk}\rangle$$

- The translational symmetry is also kept (for rest frame, $p = 0$).

Optimizing

- Cost function: weighted combination of the energy expectations

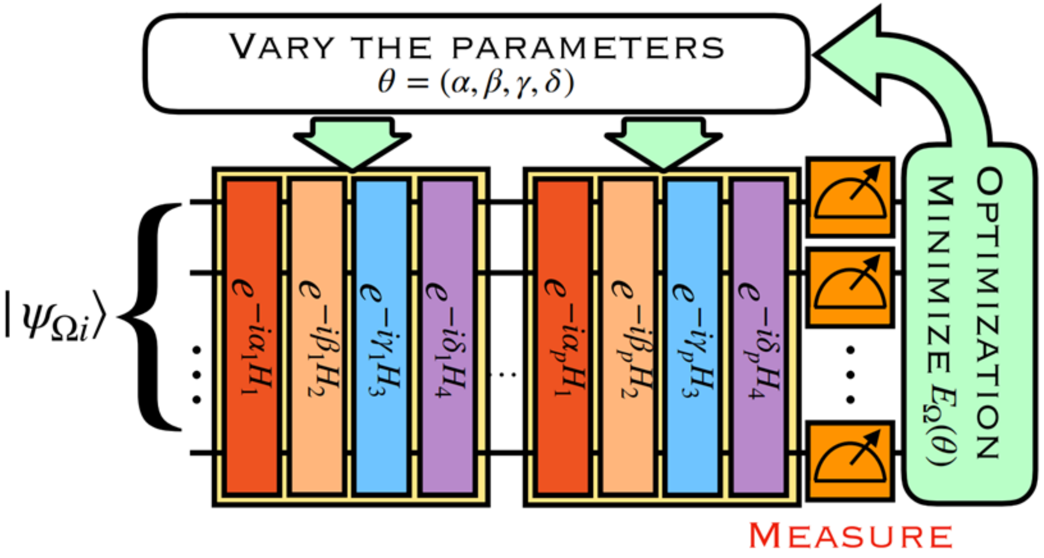
[Phys. Rev. Lett. 113, 020505]

$$E_l(\theta) = \sum_i^k \omega_{li} \langle \psi_{li}(\theta)|H|\psi_{li}(\theta)\rangle$$

$$\omega_{l1} > \omega_{l2} > \cdots > \omega_{lk}$$

$$\theta^* = argmin_\theta E(\theta)$$

- $U(\theta^*)|\psi_{lk}\rangle$ is the k-th excited hadron state with quantum number l.



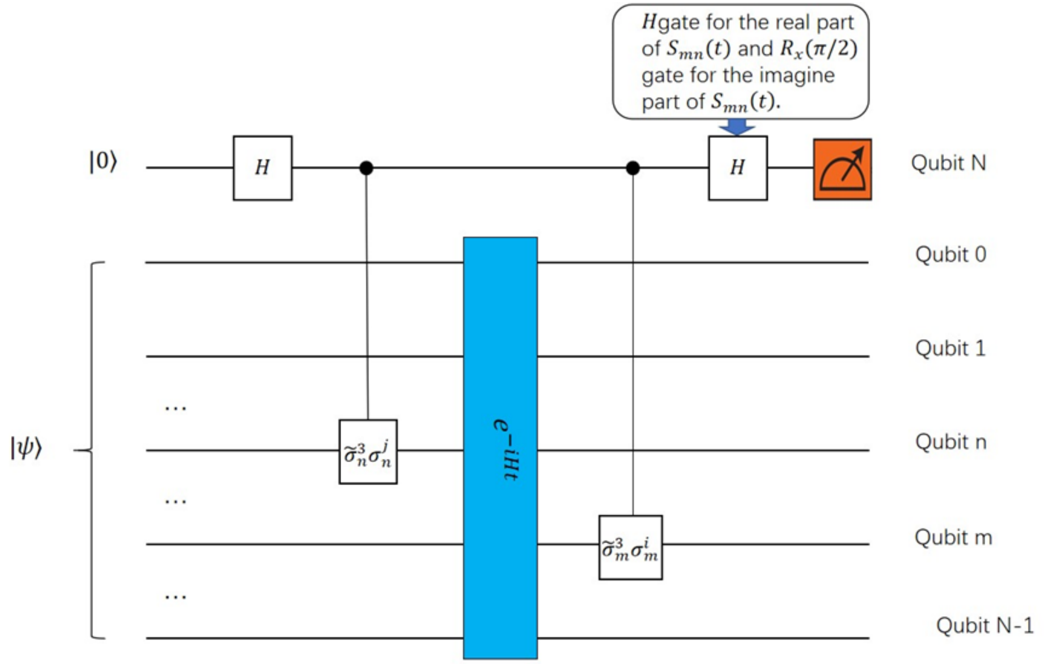
1-flavor meson state

- The 1-flavor meson state has the same quantum number with vacuum Ω.
- Two states needed: ground state—vacuum, 1st excited state—meson.
- $|\psi_{\Omega,1}\rangle = |0101 \ldots 01\rangle$.
- $|\psi_{\Omega,2}\rangle = \frac{1}{\sqrt{N/2}}(|1001 \ldots 01\rangle + |0110 \ldots 01\rangle + \cdots + |0101 \ldots 10\rangle)$.
- $\omega_{\Omega,1} = 1, \omega_{\Omega,2} = 0.5$

Dynamical correlation function

- The measurement on the quantum computer is achieved by an auxiliary qubit and the controlled gate. [Phys. Rev. B 101, 014411]
- What we measure:

$$S_{mn}(t) = \left\langle h \middle| e^{iHt} \tilde{\sigma}_m^3 \sigma_m^i e^{-iHt} \tilde{\sigma}_n^3 \sigma_n^j \middle| h \right\rangle$$



Measuring the correlation function

- Measurement in Quantum Computing
  - $\langle\alpha|\hat{O}|\alpha\rangle$
  - $\langle\alpha|\beta\rangle$

- Controlled gate: $|\alpha\rangle_a|0\rangle_b \rightarrow |\alpha\rangle_a|0\rangle_b, |\alpha\rangle_a|1\rangle_b \rightarrow |U\alpha\rangle_a|0\rangle_b$
- With this, we can create state: $|\psi\rangle = |\alpha\rangle_a|0\rangle_b + |\beta\rangle_a|1\rangle_b$
- $\langle\psi|I_a \otimes \sigma_b^x|\psi\rangle = 1 + 2\mathrm{Re}(\langle\alpha|\beta\rangle)$
- $\langle\psi|I_a \otimes \sigma_b^y|\psi\rangle = 1 - 2\mathrm{Im}(\langle\alpha|\beta\rangle)$

Measuring the correlation function

1.Prepare the intinal state as the derict product state $|0\rangle_A |0\rangle_S$. Where $|0\rangle_S$ is the vacuum of the Thirring model and $|0\rangle_A$ is the auxiliary qubit.

2. Act H gate on the auxiliary qubit, we have

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|0\rangle_S \otimes |0\rangle_A + |0\rangle_S \otimes |1\rangle]_A. \tag{42}$$

3. Act controled $\psi_\alpha^\dagger(0)$ gate on the vacuum. We have

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|0\rangle_S \otimes |0\rangle_A + \psi_\alpha^\dagger(0)|0\rangle_S \otimes |1\rangle]_A. \tag{43}$$

4. Act the unitary gate $e^{-iP(x-y)}$ on the vacuum

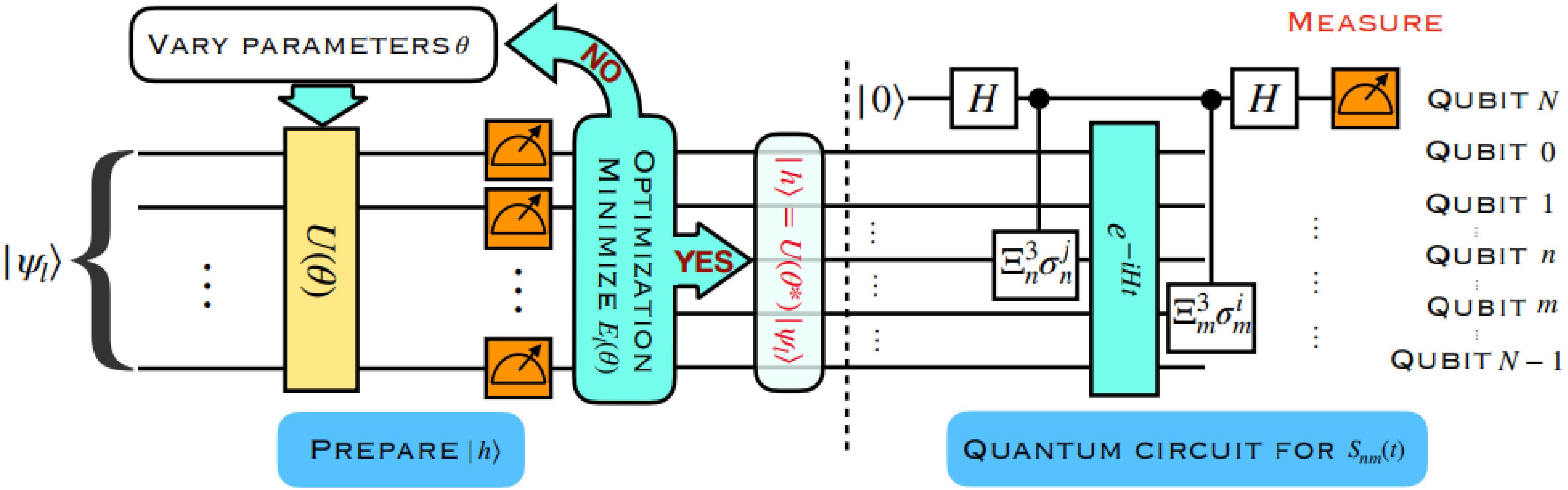
$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[e^{-iP(x-y)}|0\rangle_S \otimes |0\rangle_A + e^{-iP(x-y)}\psi_\alpha^\dagger(0)|0\rangle_S \otimes |1\rangle\right]_A. \tag{44}$$

5. Act the controled $\psi_\beta(0)$ gate on the vacuum

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[e^{-iP(x-y)}\,|0\rangle_S \otimes |0\rangle_A + \psi_\beta(0)e^{-iP(x-y)}\psi_\alpha^\dagger(0)\,|0\rangle_S \otimes |1\rangle_A\right]. \tag{45}$$

6. Mesaure the auxiliary qubit and obtain the two point function.

Full Quantum circuit



Meson Mass

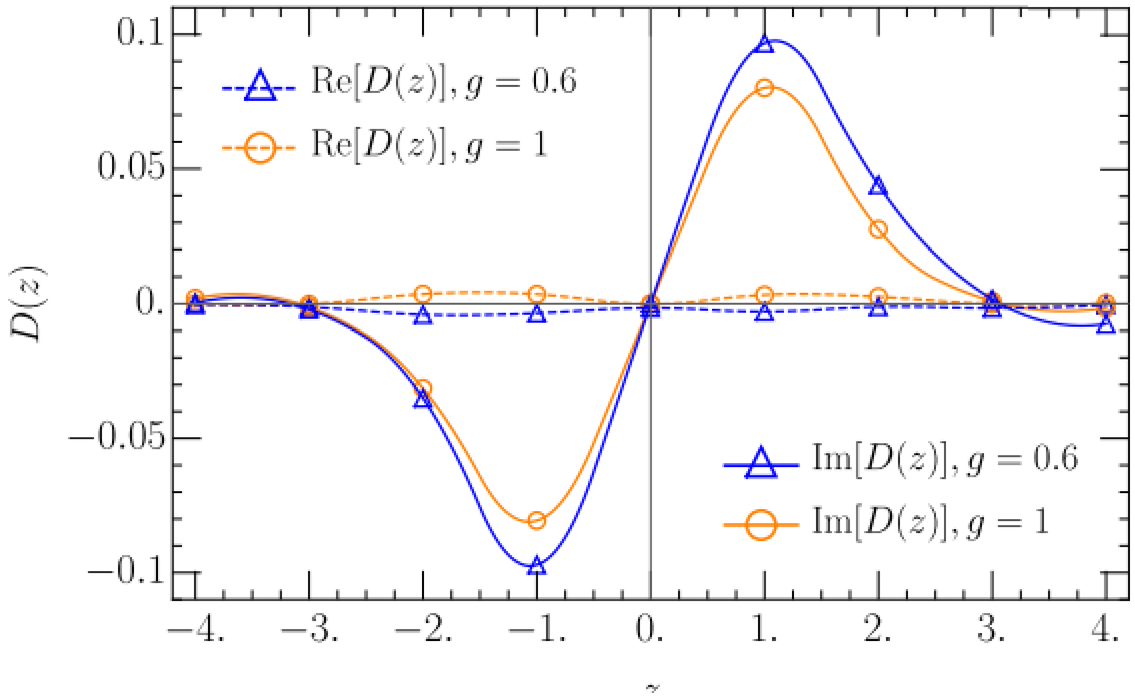
- $ma = 0.2$, 12 qubits

g	0.2	0.4	0.6	0.8	1.0
$M_{h,\mathrm{QC}}a$	1.002	1.810	2.674	3.534	4.352
$M_{h,\mathrm{NUM}}a$	1.001	1.801	2.659	3.509	4.342

- The majority of mass comes from interaction rather than quark masses.
- For $ma = 0.8$ the quark masses are dominant.

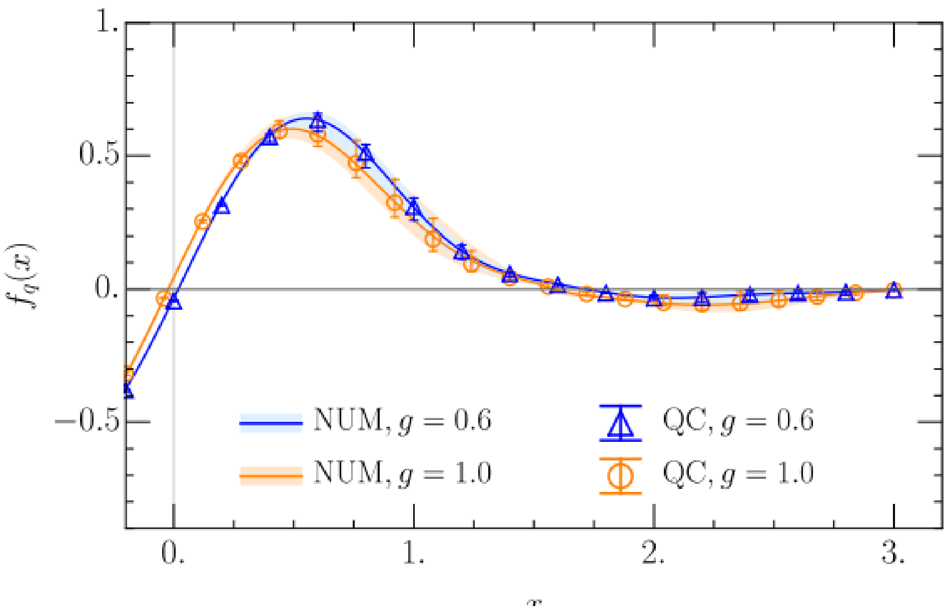
Correlation function

- Real part consistent with 0.
- Bounded state behavior.



Meson PDF

- Error bars/bands are from different interpolation methods.
- Quantum computation result matches well with conventional numerical solutions.
- Qualitative agreement with pion PDFs in 2d QCD.[Phys. Rev. D 98, 054011]



Contents

- Introduction
- 1+1D NJL model
- Parton Collinear Structure
- Light-Cone Distribution Amplitudes (arXiv:2207.13258)
- Summary and outlook

LCDA

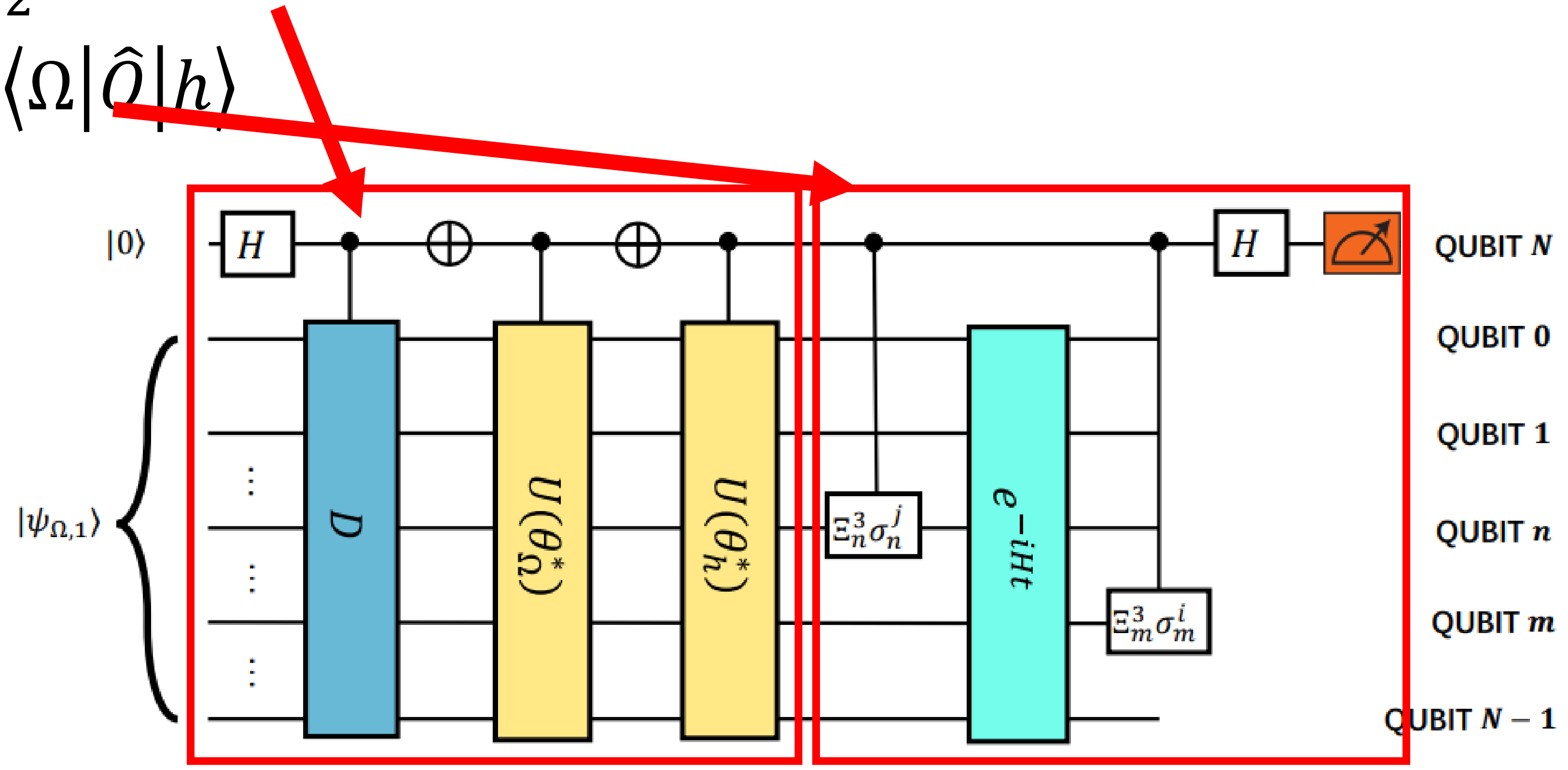
- The projection amplitude of a hadron onto multiple-parton states:
- Meson:

$$\phi_h(x) = \frac{1}{f}\int dz\, e^{-i(x-1)m_h z}\langle\Omega\left|e^{iHz}\bar{\psi}(0,-z)e^{-iHz}\gamma^+\psi(0,0)\right|h\rangle$$

- Describes the formation/decay of a hadron.
- In quantum computing language: $\left\langle\Omega\left|e^{iHt}\tilde{\sigma}_m^3\sigma_m^i e^{-iHt}\tilde{\sigma}_n^3\sigma_n^j\right|h\right\rangle$

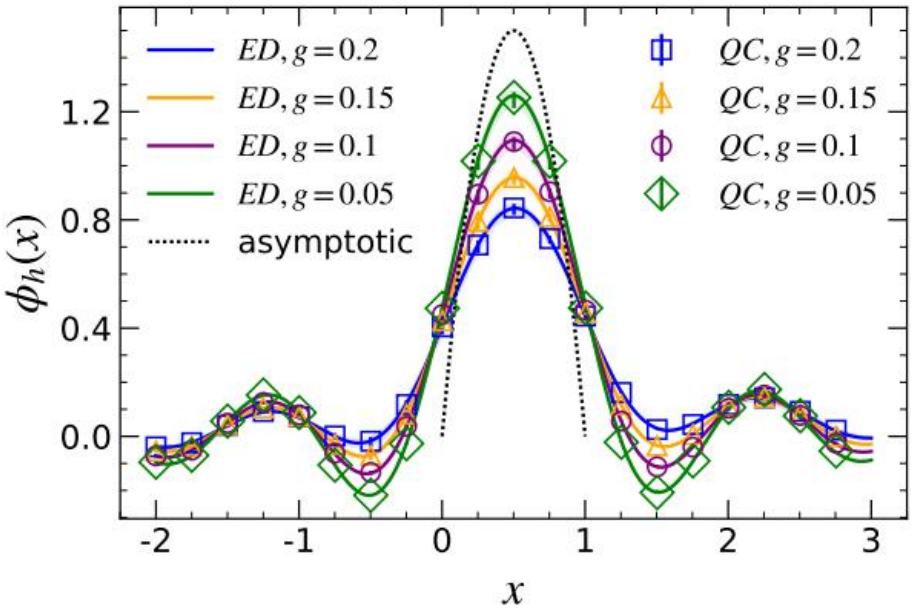
Measuring correlation function between different states

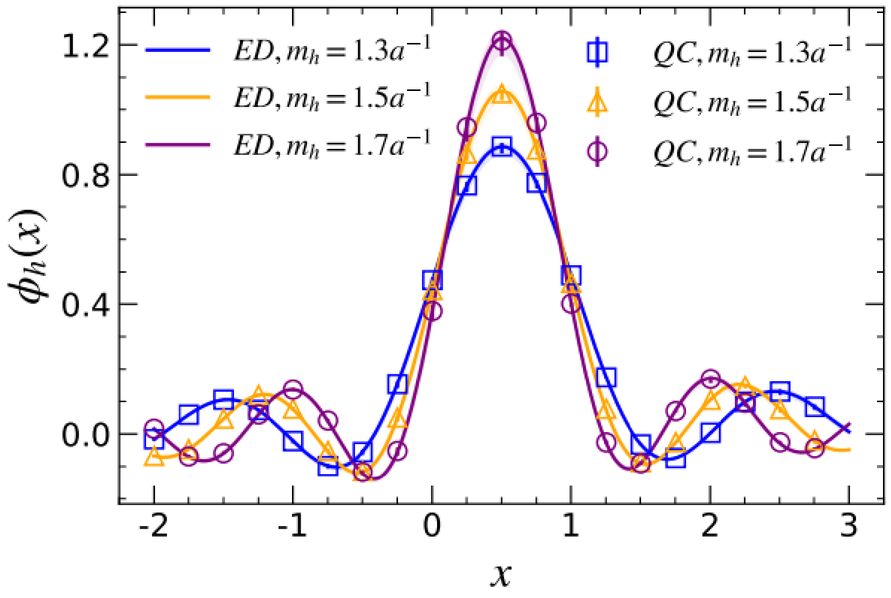
- Prepare $\frac{1}{\sqrt{2}}(|0\rangle|\Omega\rangle + |1\rangle|h\rangle)$
- Evaluate $\langle\Omega|\hat{O}|h\rangle$



Results

- Peak gets narrower when decreasing coupling constant or increasing hadron mass.
- Converges to asymptotic result.





Summary and outlook

- First direct simulation of the hadron partonic structure on a quantum computer
  - Prepare the hadron with quantum-number-resolving VQE.
  - Real-time evolution of the light-like correlation function.
- Proof-of-concept: 1 flavor 1+1D PDF and LCDA.
- Generally applicable to many questions:
  - Proton spin configuration.
  - Nucleon 3D structure.
  - Jet transport.
  - …

Thank you!