QCD factorization and the jet cross section at hadron colliders

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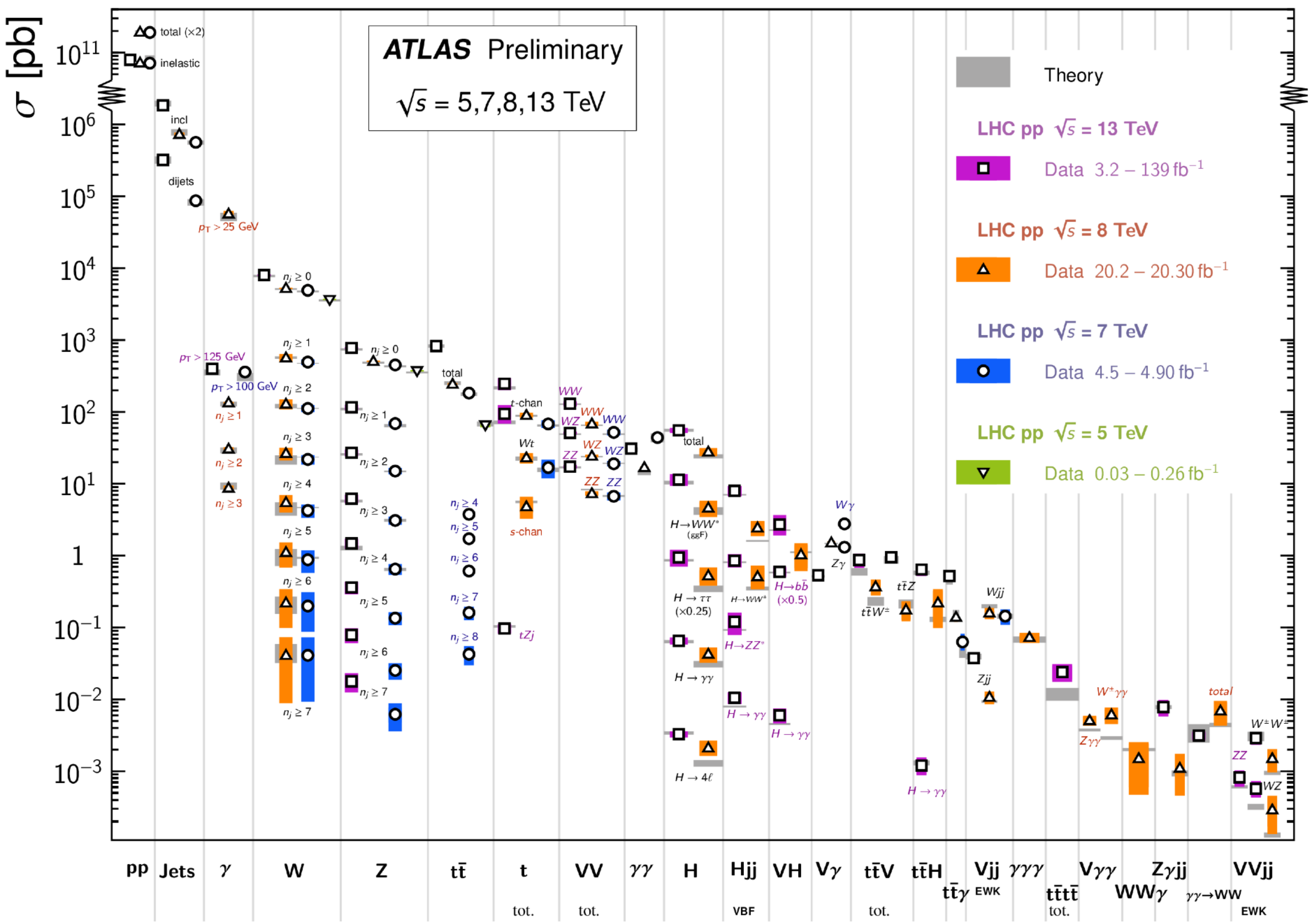
Standard Model Production Cross Section Measurements

Status: March 2021



Collinear factorization for inclusive observables

For inclusive observables, sensitive only to a single high-energy scale *Q*, we have

$$\sigma = \sum_{a,b} \int_0^1 dx_1 dx_2\, \hat{\sigma}_{ab}(Q, x_1, x_2, \mu_f)\, f_a(x_1, \mu_f)\, f_b(x_2, \mu_f) + \mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)$$

partonic cross sections: perturbation theory

parton distribution functions (PDFs): nonperturbative

power corrections nonperturbative

The right way to look at this formula is effective theory

$$\sigma = \sum_{a,b} \int_0^1 dx_1 dx_2\, C_{ab}(Q, x_1, x_2, \mu) \langle P(p_1)|O_a(x_1)|P(p_1)\rangle \, \langle P(p_2)|O_b(x_2)|P(p_2)\rangle + \mathcal{O}(\Lambda_{\rm QCD}/Q)$$

Wilson coefficient:
matching at $\mu \approx Q$
perturbation theory

RG-evolution

low-energy matrix elements
nonperturbative

power suppressed operators

The matching coefficient C_{ab} is independent of external states and insensitive to physics below the matching scale *μ*.

Can use quark and gluon states to perform the matching.

- **Trivial matrix elements**

$$\langle q_{a'}(x'p)|O_a(x)|q_{a'}(x'\,p)\rangle = \delta_{aa'}\,\delta(x'-x)$$

- **Wilson coefficients are partonic cross section**

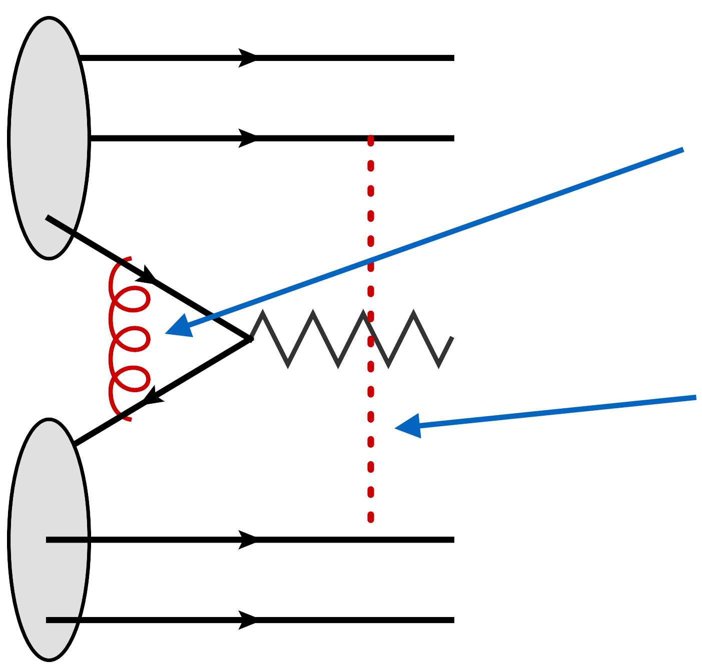
$$C_{ab}(Q,x_1,x_2) = \hat{\sigma}_{ab}(Q,x_1,x_2)$$

- **Bare Wilson coefficients have divergencies. Renormalization induces dependence on *μ*.**

Quite nontrivial that the low-energy matrix element factorizes into a product

$$\langle P(p_1)|O_a(x_1)|P(p_1)\rangle\,\langle P(p_2)|O_b(x_2)|P(p_2)\rangle$$

One should be worried about long-distance interactions mediated by soft gluons

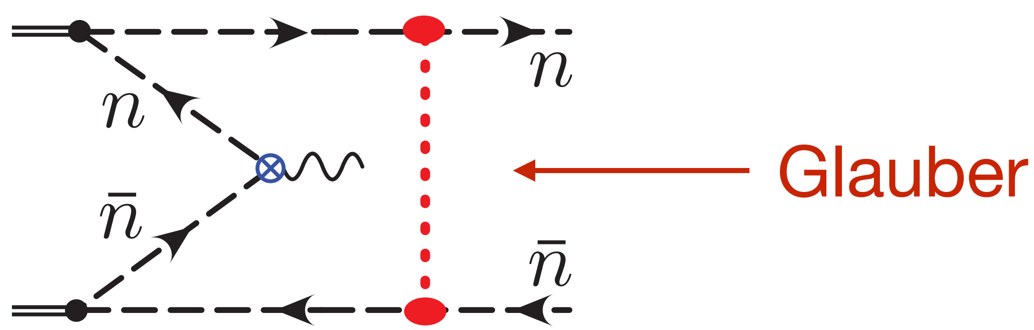


standard soft gluon

Glauber gluon

$$p^\mu \approx p_\perp^\mu$$

All proton collisions include forward component (proton remnants)



Absence of factorization-violation due to Glauber gluons is important element of factorization proof for Drell-Yan process. Bodwin '85; Collins, Soper, Sterman '85 '88 …

e.g. TMD factorization is violated in di-jet/di-hadron production

Collins, Qiu \`07; Collins \`07, Vogelsang, Yuan \`07; Rogers, Mulders \`10, …

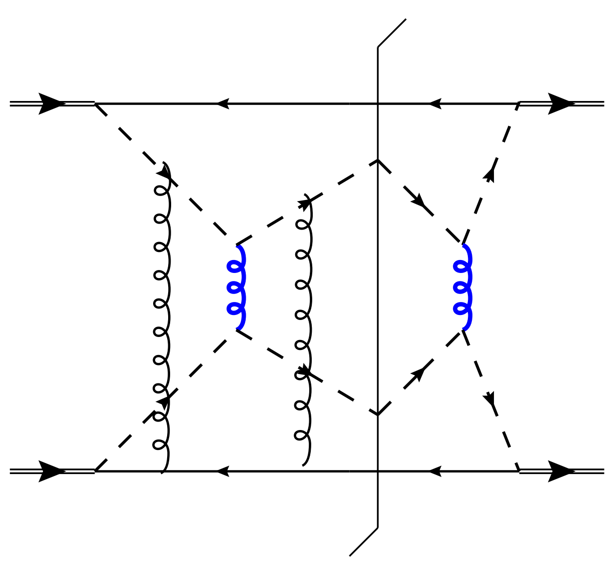
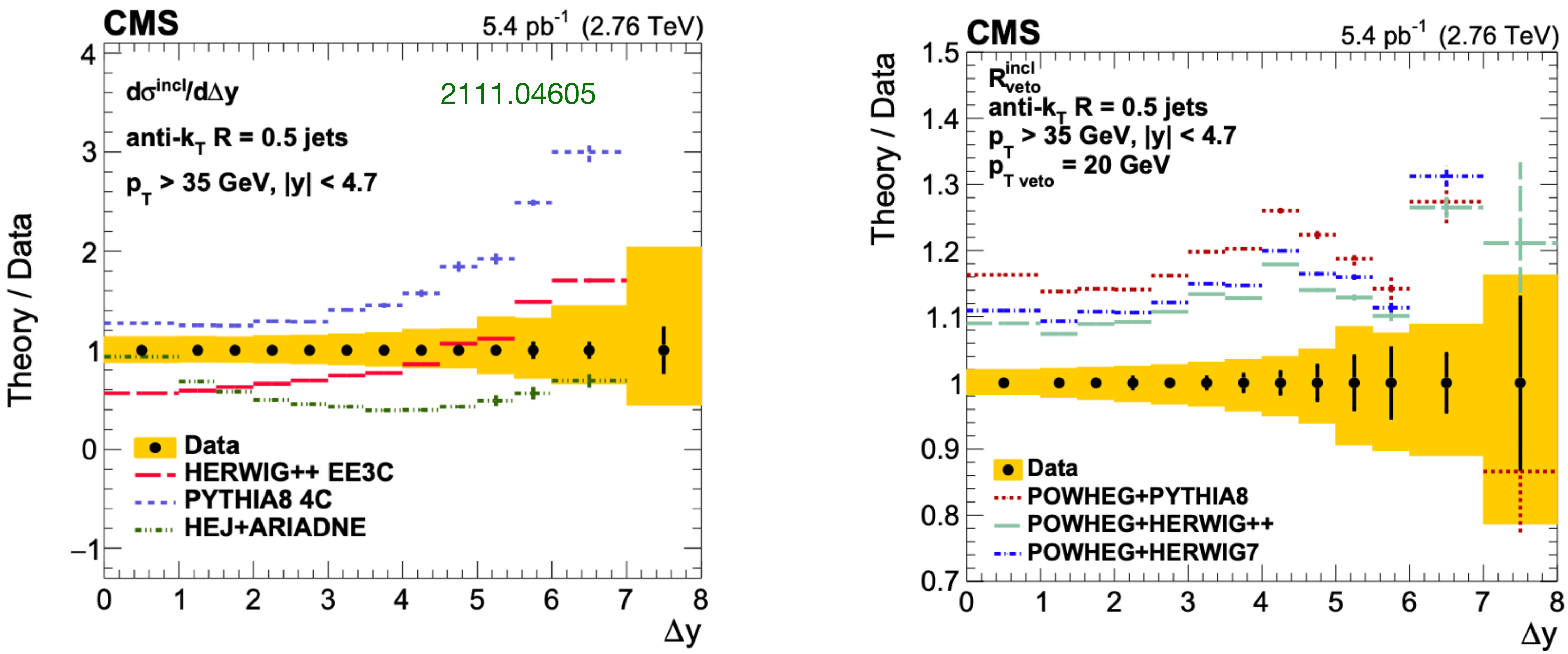


FIG. 8 (color online). The exchange of two extra gluons, as in this graph, will tend to give nonfactorization in unpolarized cross sections.

We remark that, because the TMD factorization breaking effects are due to the Glauber region where all components of gluon momentum are small, the interactions responsible for breaking TMD factorization are associated with large distance scales.

Rogers, Mulders \`10

Dijet events with large rapidity gap at the LHC



None of the DGLAP-based Monte Carlo generators using LO or NLO calculations can provide a complete description of all measured cross sections and their ratios.

Tools: Soft-Collinear Effective Theory

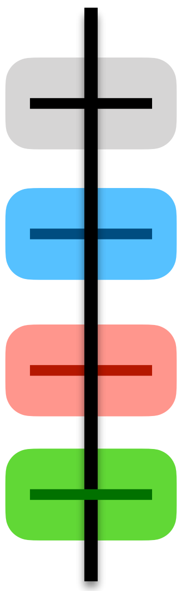
- **Technical challenges**
  - **Glauber gluons are offshell**
  - **Must be included as potential, not dynamical field in the effective Lagrangian**
  - **Glauber region is not well defined without additional rapidity regulator (on top of dim.reg.)** (Rothsten & Stewart ‘20)

- **study QCD factorization without Glauber region**
  - **Assign scaling behavior to fields**
  - **Expand Lagrangian to leading power**
  - **Resummation with Renormalization Group**

Jet radius and q_T joint resummation for boson-jet correlation

(Chien, DYS & Wu '19 JHEP)

$$N_1(P_1) + N_2(P_2) \to \underbrace{\text{boson}(p_V) + \text{jet}(p_J)}_{q_T} + X$$



$p_h \sim Q(1,1,1)$

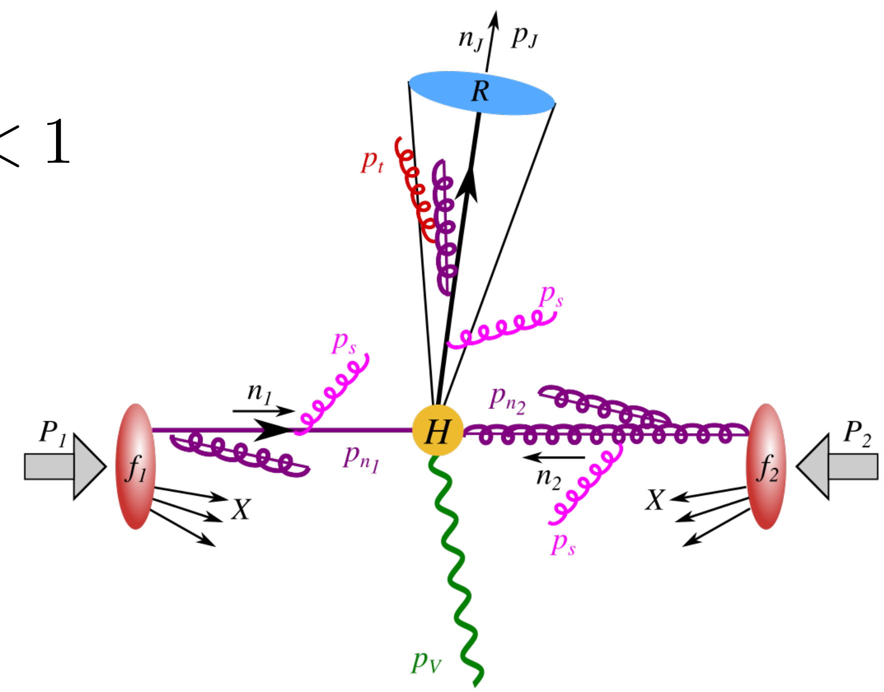
$p_{n_J} \sim p_T^J\,(R^2, 1, R)_{n_J \bar{n}_J}$

$p_{n_1} \sim (q_T^2/Q, Q, q_T)_{n_1 \bar{n}_1}$

$p_s \sim (q_T, q_T, q_T)$

$p_t \sim q_T\,(R^2, 1, R)_{n_J \bar{n}_J}$

$$q_T \ll Q, R \ll 1$$



Construction of the theory formalism

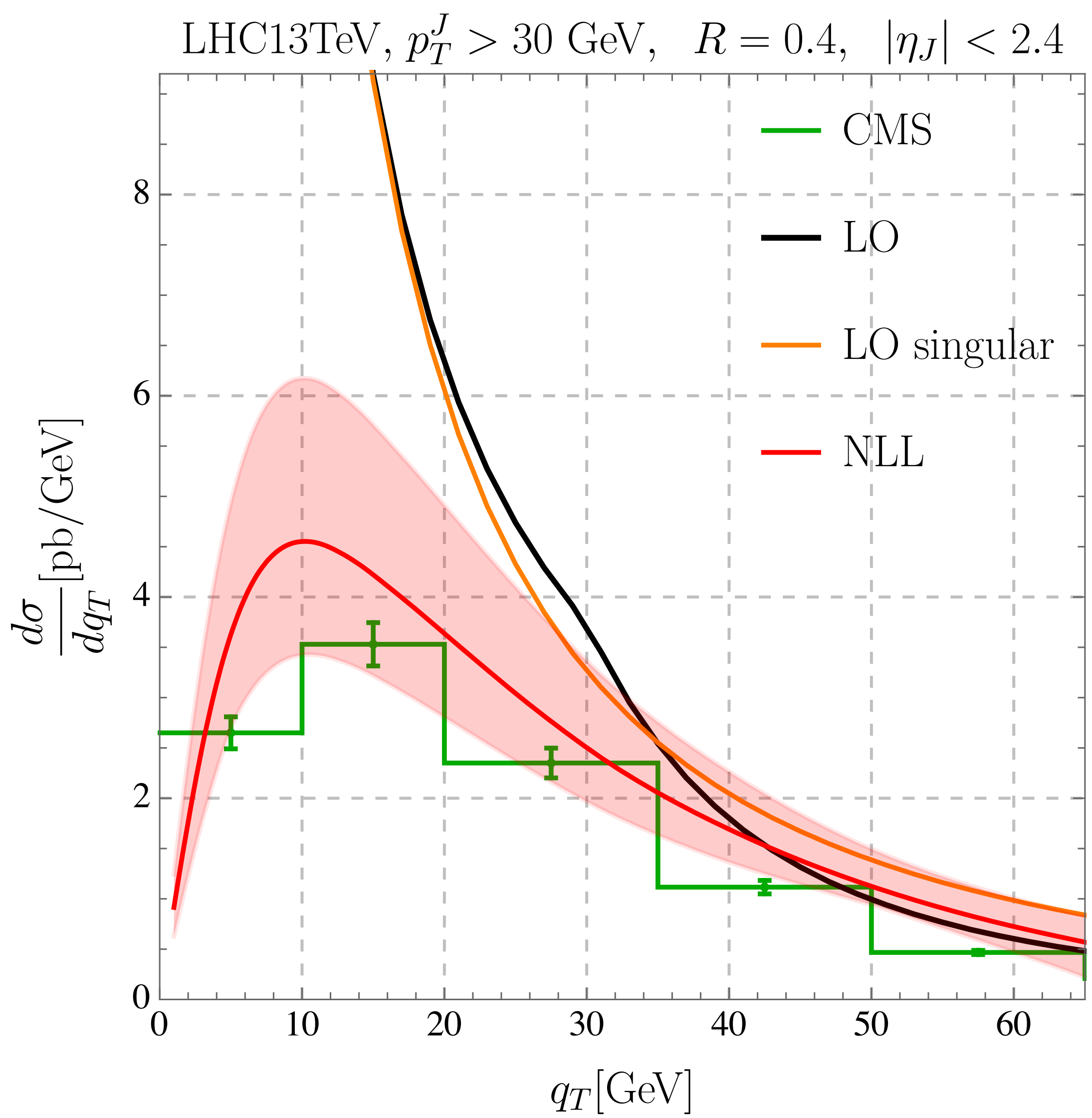
- **Multiple scales in the problem**
- **Rely on effective field theory: SCET + Jet Effective Theory** (Becher, Neubert, Rothen, DYS '16 PRL)

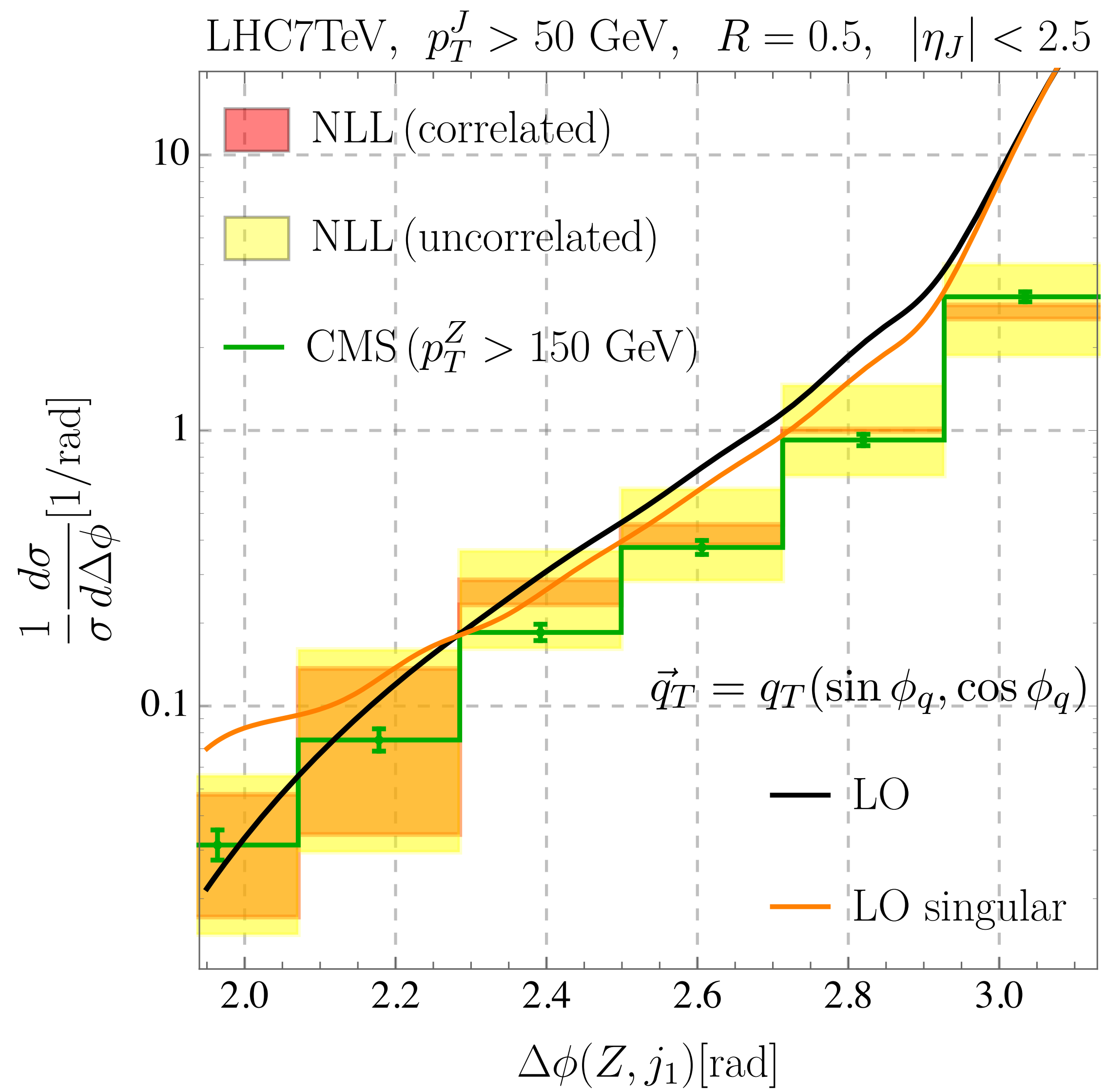
$$\frac{d\sigma}{d^2q_T d^2p_T d\eta_J dy_V} = \sum_{ijk} \int \frac{d^2x_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{x}_T} \mathcal{S}_{ij\to Vk}(\vec{x}_T,\epsilon)\mathcal{B}_{i/N_1}(\xi_1, x_T,\epsilon)\mathcal{B}_{j/N_2}(\xi_2, x_T,\epsilon)$$

$$\times\, \mathcal{H}_{ij\to Vk}(\hat{s},\hat{t},m_V,\epsilon) \sum_{m=1}^{\infty} \langle \boldsymbol{\mathcal{J}}_m^k(\{\underline{n_J}\}, R\,p_J,\epsilon) \otimes \boldsymbol{\mathcal{U}}_m^k(\{\underline{n_J}\}, R\,\vec{x}_T,\epsilon)\rangle$$

(also see Sun,Yuan,Yuan '14; Buffing,Kang,Lee,Liu '18,…)

Numerical results





- **NLL resummation is consistent with the LHC data (q_T & ΔΦ)**
- **ΔΦ distribution for dijet production can be a clean probe of *factorization violation***
  (Collins & Qiu '07, Rogers & Mulders '10, ……)
- **NLL result has 20-30% scale uncertainties. Higher-order resummation is necessary**

Jet definition

Which particles get put together?

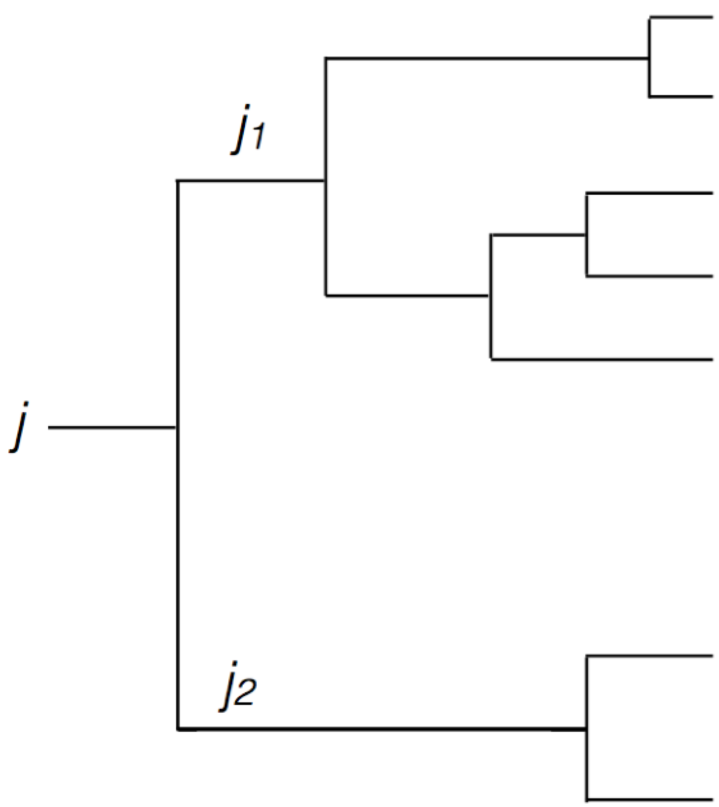
Jet algorithm

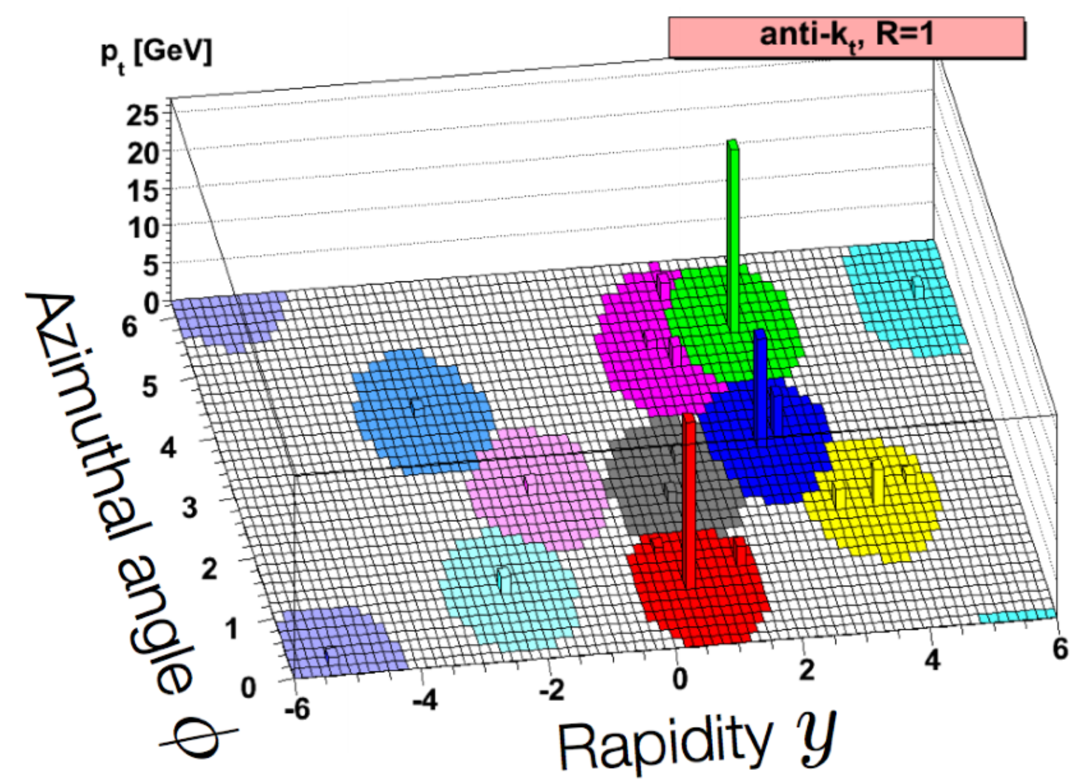
How to combine their momenta?

Recombination scheme

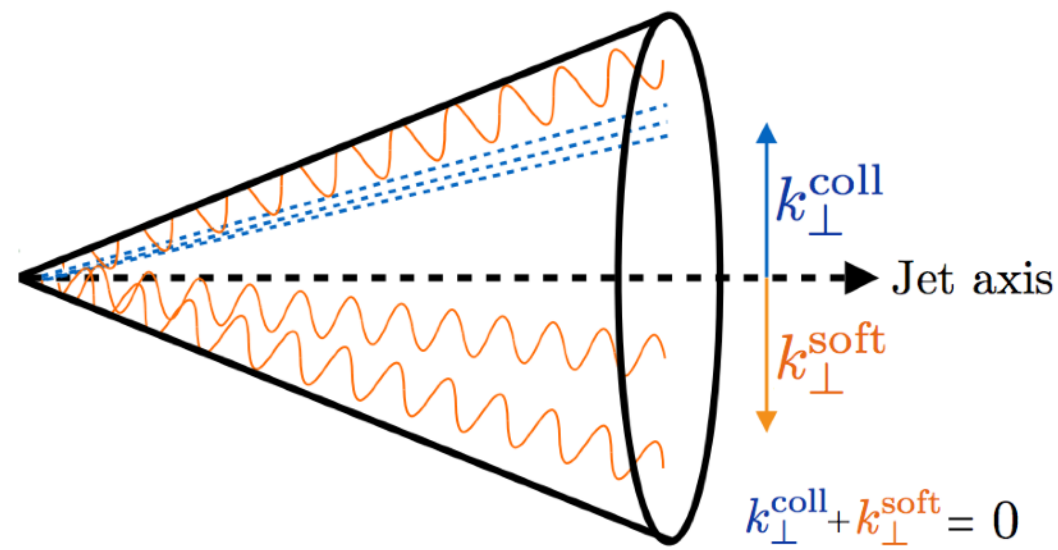
Jet definition with clustering algorithms

- **Determine distances between "particles"**
- **Recombine nearest "particles":** $p_i^\mu, p_j^\mu \to p_i^\mu + p_j^\mu$
- **Repeat until distances larger than jet radius R**





Recoil and the jet axis



Jet axis is along jet momentum: recoiled by soft radiation in jet

- **TH challenge: Non-linear evolution (Non-global logs)**
- **EX challenge: Contamination**

Recoil absent for the p_T-weighted recombination (Ellis, Soper '93)

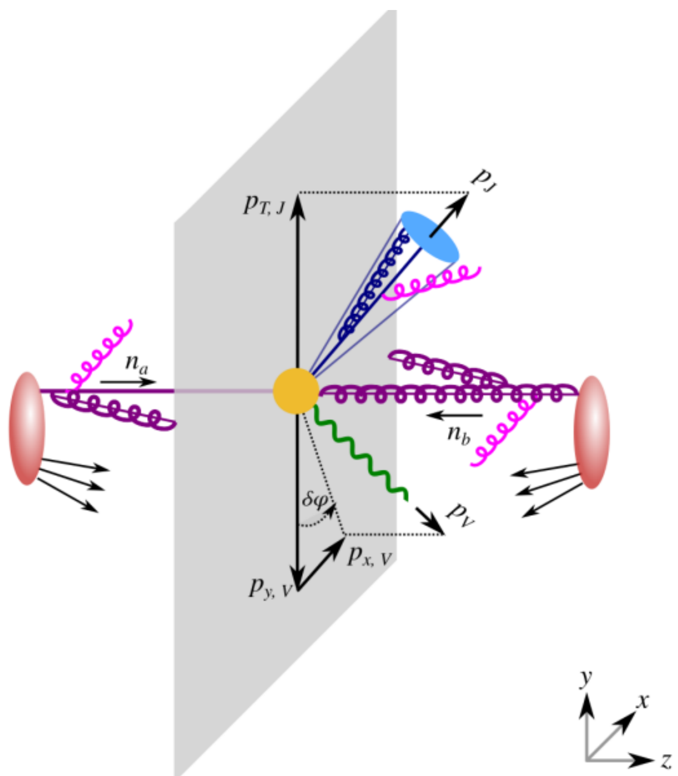
$$p_{t,r} = p_{t,i} + p_{t,j} \,,$$
$$\phi_r = (w_i\phi_i + w_j\phi_j)/(w_i + w_j)$$
$$y_r = (w_i y_i + w_j y_j)/(w_i + w_j)$$

$$w_i = p_t^n$$

$n \to \infty$ **(Winner-take-all scheme)** (Bertolini, Chan, Thaler '13)

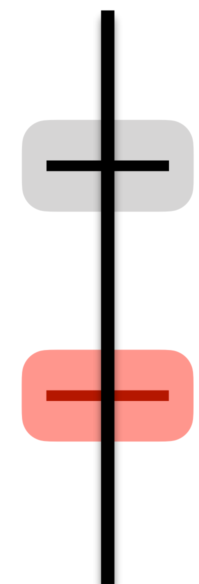
Recoil-free azimuthal angle for boson-jet correlation

(Chien, Rahn, Schrignder, DYS, Waalewijn & Wu ’21 PLB)



$$\pi - \Delta\phi \equiv \delta\phi \approx \sin(\delta\phi) = |p_{x,V}|/p_{T,V}$$

Standard SCET$_2$ (CSS, Ji-Ma-Yuan …)



$p_h \sim Q(1,1,1)$

$p_n \sim (p_x^2/Q, Q, p_x)_{n\bar{n}}$

$p_s \sim (p_x, p_x, p_x)$

(also see Gao,Li,Moult,Zhu ’19 PRL,…)

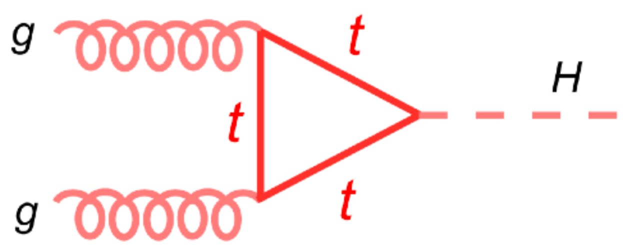
Following the standard steps in SCET$_2$ we obtain the following factorization formula

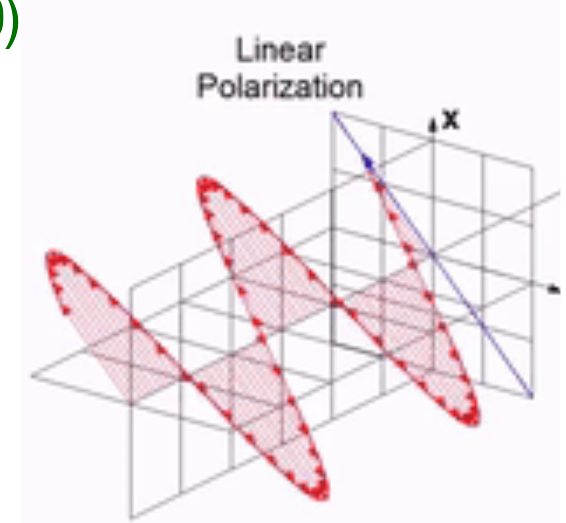
$$\frac{d\sigma}{dp_{x,V}\, dp_{T,J}\, dy_V\, d\eta_J} = \int \frac{db_x}{2\pi}\, e^{ip_{x,V} b_x} \sum_{i,j,k} B_i(x_a, b_x) B_j(x_b, b_x) S_{ijk}(b_x, \eta_J) H_{ij\to Vk}(p_{T,V}, y_V - \eta_J) J_k(b_x)$$

Fourier transformation in 1-D

Linearly-polarized gluon TMDs

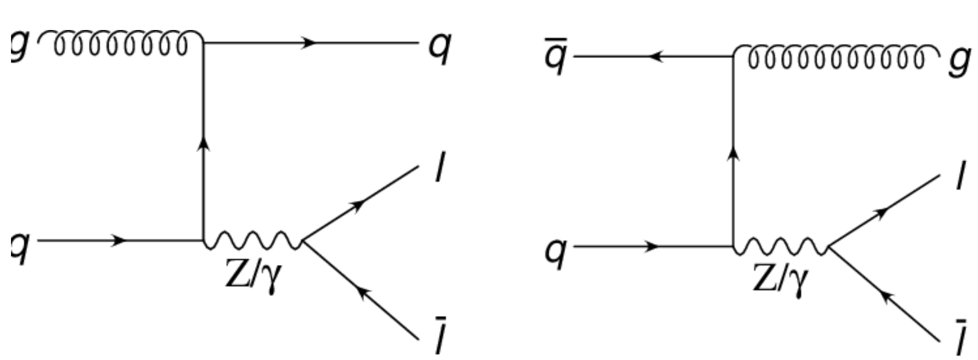
For Higgs production linearly-polarized gluon TMDs arises from spin interference between multiple initial-state gluons (Catani, Grazzini '10)

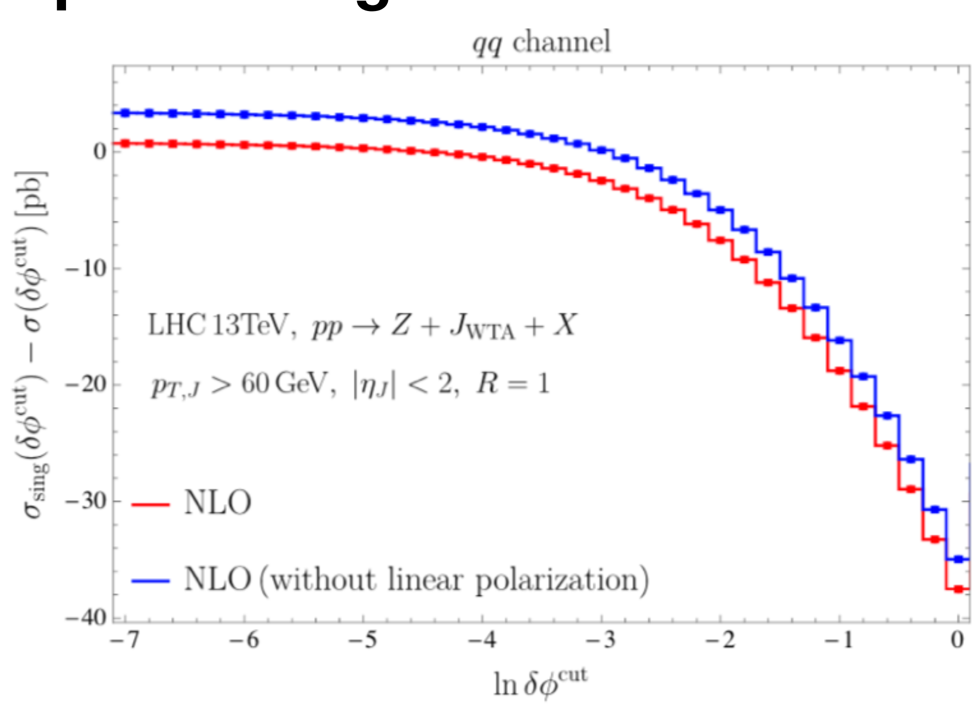




$$\Phi_g^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{n_\rho n_\sigma}{(p\cdot n)^2} \int \frac{d(\xi\cdot P) d^2\xi_T}{(2\pi)^3} e^{ip\cdot\xi} \langle P | \mathrm{Tr}\left[F^{\mu\rho}(0) F^{\nu\sigma}(\xi)\right] | P\rangle \Big|_{\mathrm{LF}}$$
$$= \frac{1}{2x}\left\{-g_T^{\mu\nu} f_1^g(x, \boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M^2}\right) h_1^{\perp g}(x, \boldsymbol{p}_T^2)\right\}$$

Boson-jet correlation can be used to probe linear-polarized gluon TMDs inside the proton (Boer, Mulders, Pisano, Zhou '16)





Linearly-polarized gluon jets

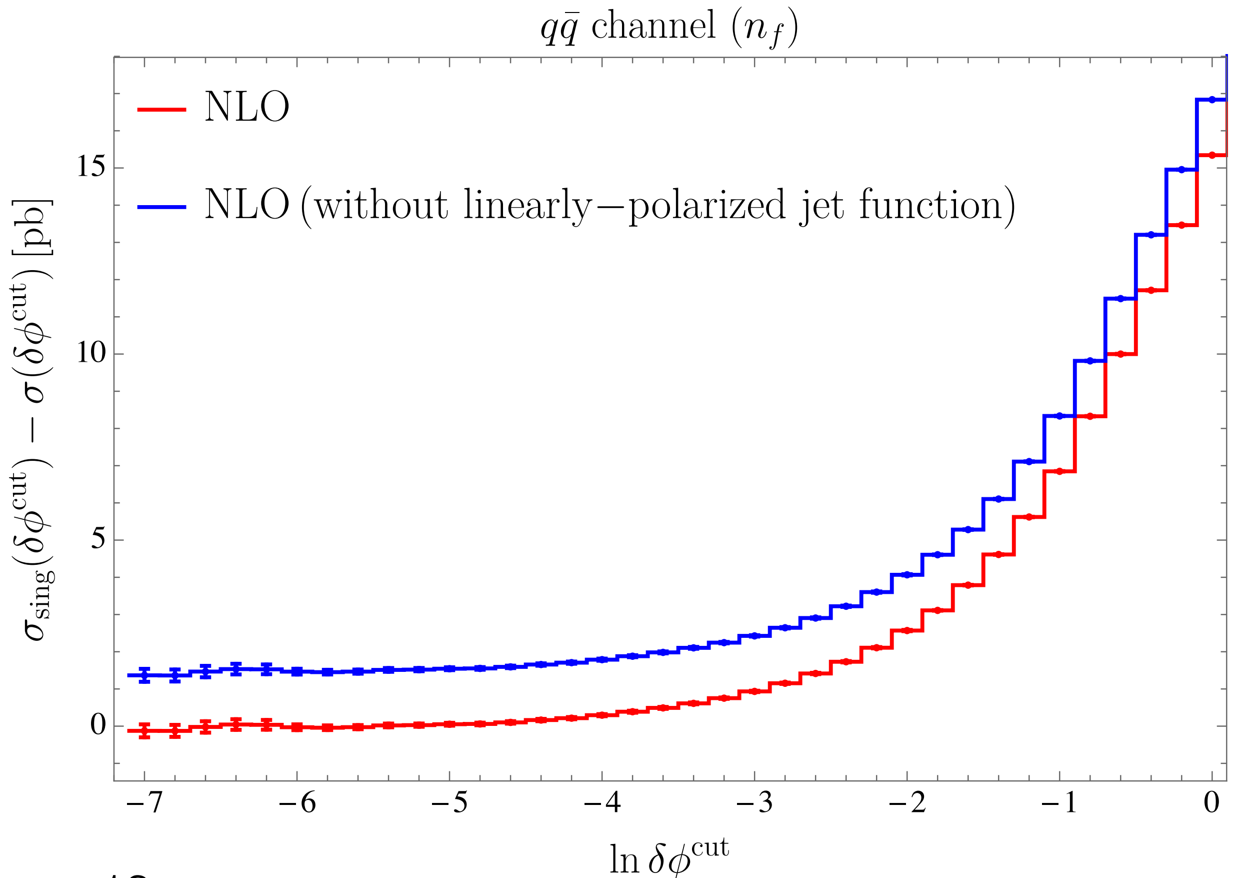
The linearly-polarized jet function describes the effect of a spin-superposition of the gluon initiating the jet

$$J_g^L(\vec{b}_\perp,\mu,\nu) = \left[\frac{1}{d-3}\left(\frac{g_\perp^{\mu\nu}}{d-2}+\frac{b_\perp^\mu b_\perp^\nu}{\vec{b}_\perp^2}\right)\right]\frac{2(2\pi)^{d-1}\omega}{N_c^2-1}\langle 0|\delta(\omega-\bar{n}\cdot\mathcal{P})\delta^{d-2}(\mathcal{P}_\perp)\mathcal{B}^a_{n\perp\mu}(0)e^{i\vec{b}_\perp\cdot\hat{\vec{k}}_\perp}\mathcal{B}^a_{n\perp\nu}(0)|0\rangle$$

The first non-vanishing order is one loop

$$J_g^{L(1)}(\vec{b}_\perp,\mu,\nu) = -\frac{1}{3}C_A+\frac{2}{3}T_F n_f$$

We provide evidence for contributions from linearly-polarized gluon jet functions using MCFM

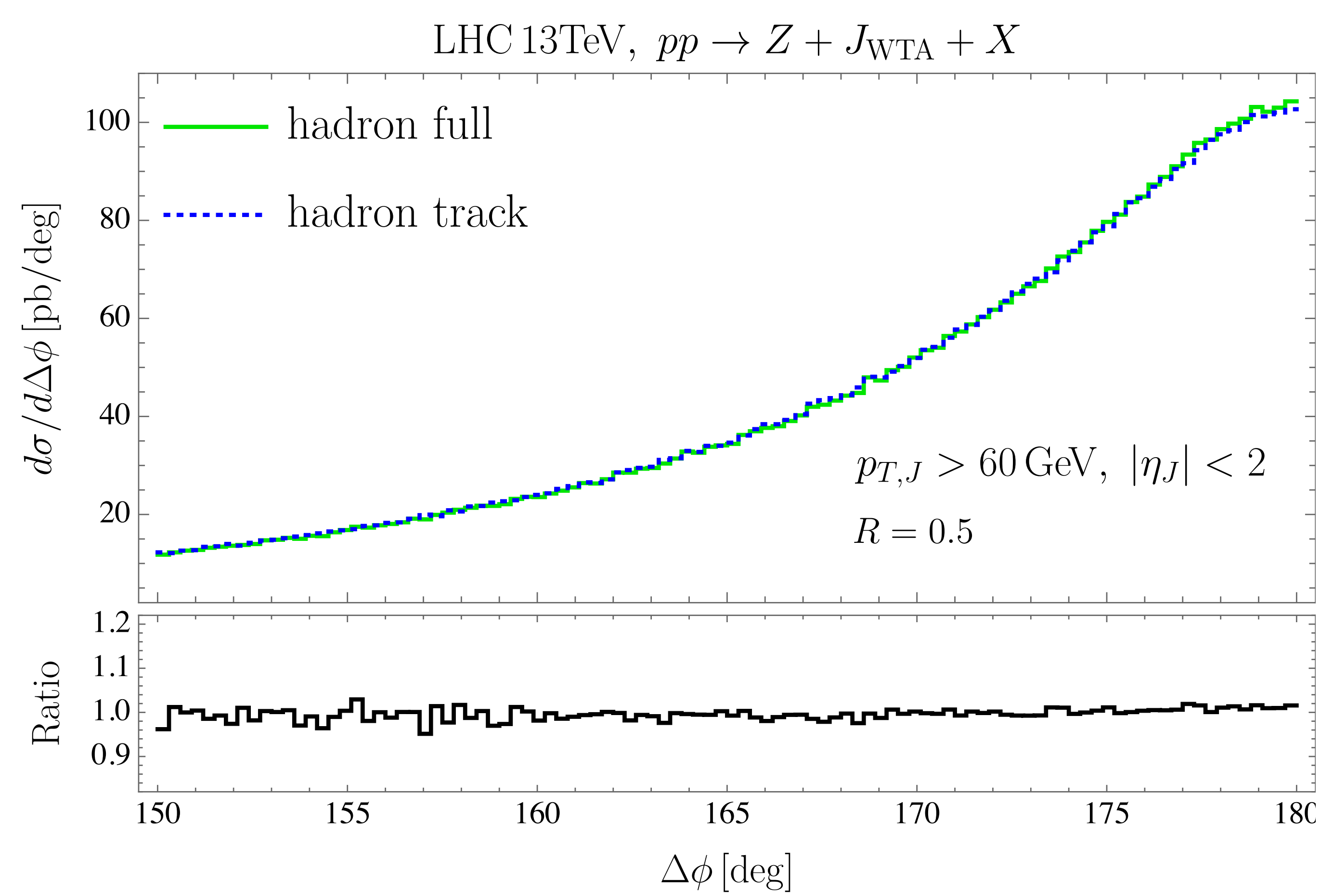


Better angular resolution

- **The angular resolution of jet measurements is about 0.1 radians, limiting access to the back-to-back region**
- **This can be overcome by measuring the jet using only charged particles, exploiting the superior angular resolution of the tracking systems at the LHC.**

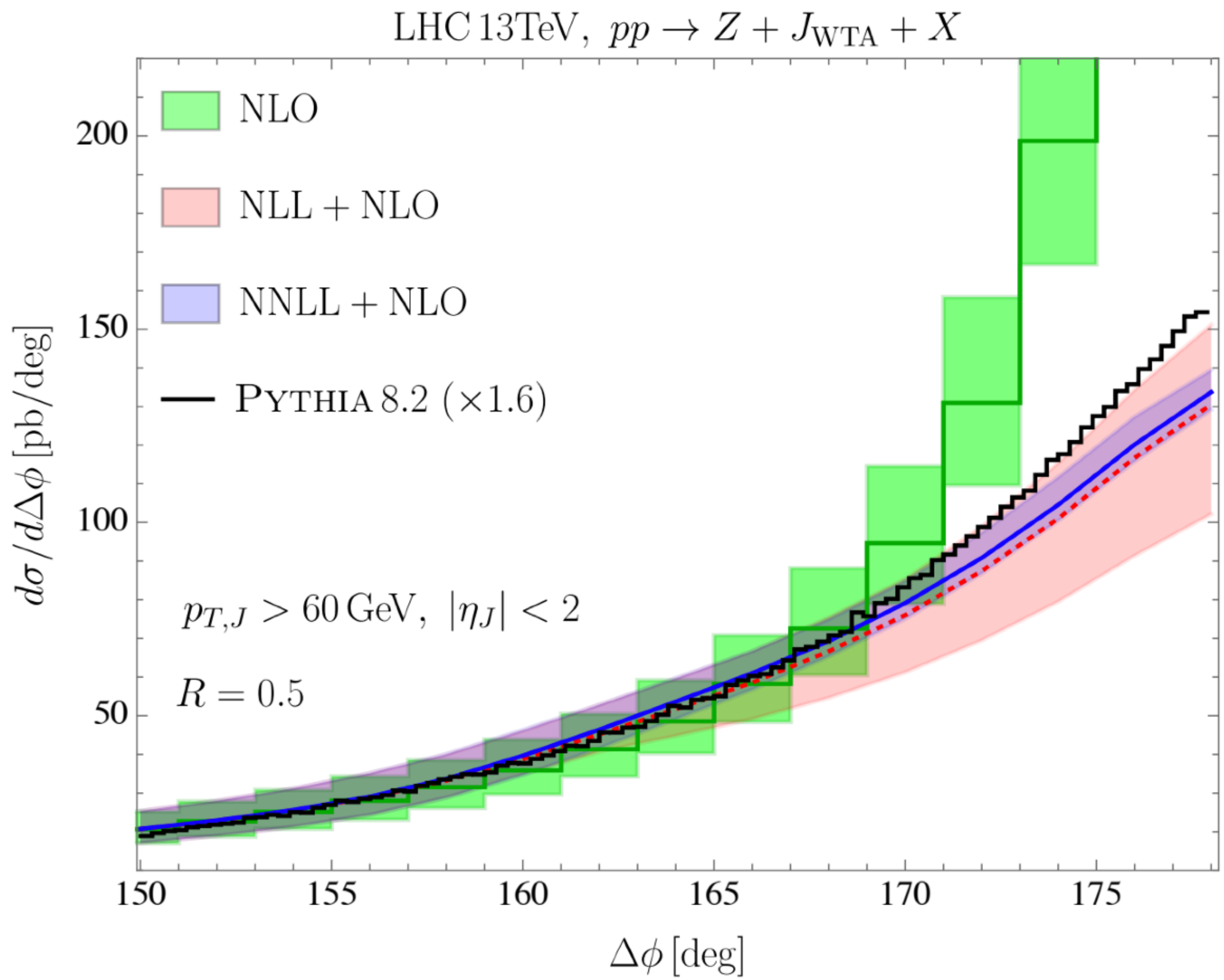
Tracking jet function:

$$\bar{\mathscr{J}}_q^{(1)} = \mathscr{J}_q^{(1)} + 4C_F \int_0^1 \mathrm{d}x\, \frac{1+x^2}{1-x} \ln \frac{x}{1-x} \int_0^1 \mathrm{d}z_1\, T_q(z_1,\mu) \times \int_0^1 \mathrm{d}z_2\, T_g(z_2,\mu) [\theta(z_1 x - z_2(1-x)) - \theta(x - \tfrac{1}{2})]$$



We have verified that using tracks only has a minimal effect on this measurement

Numerical results

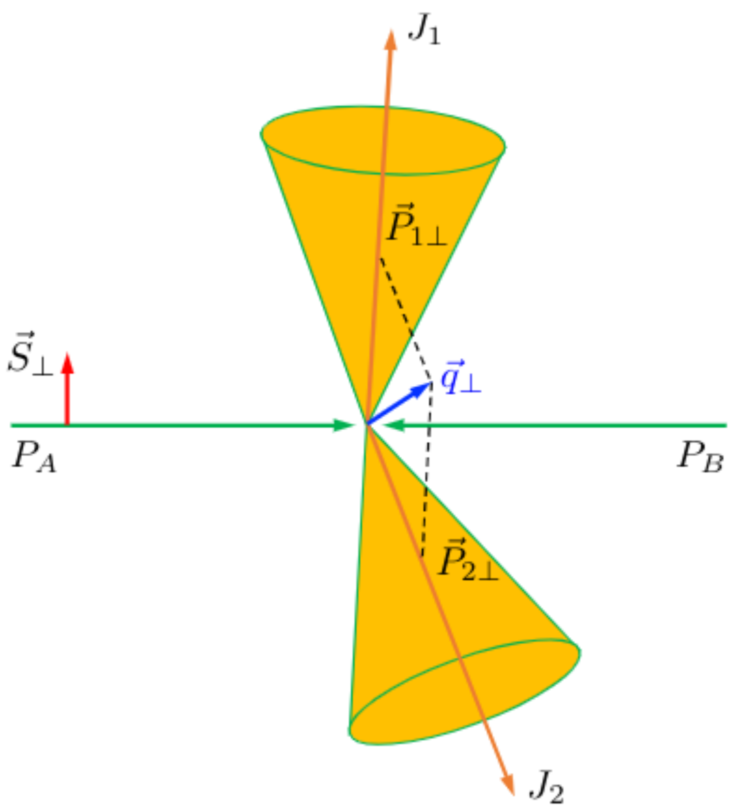


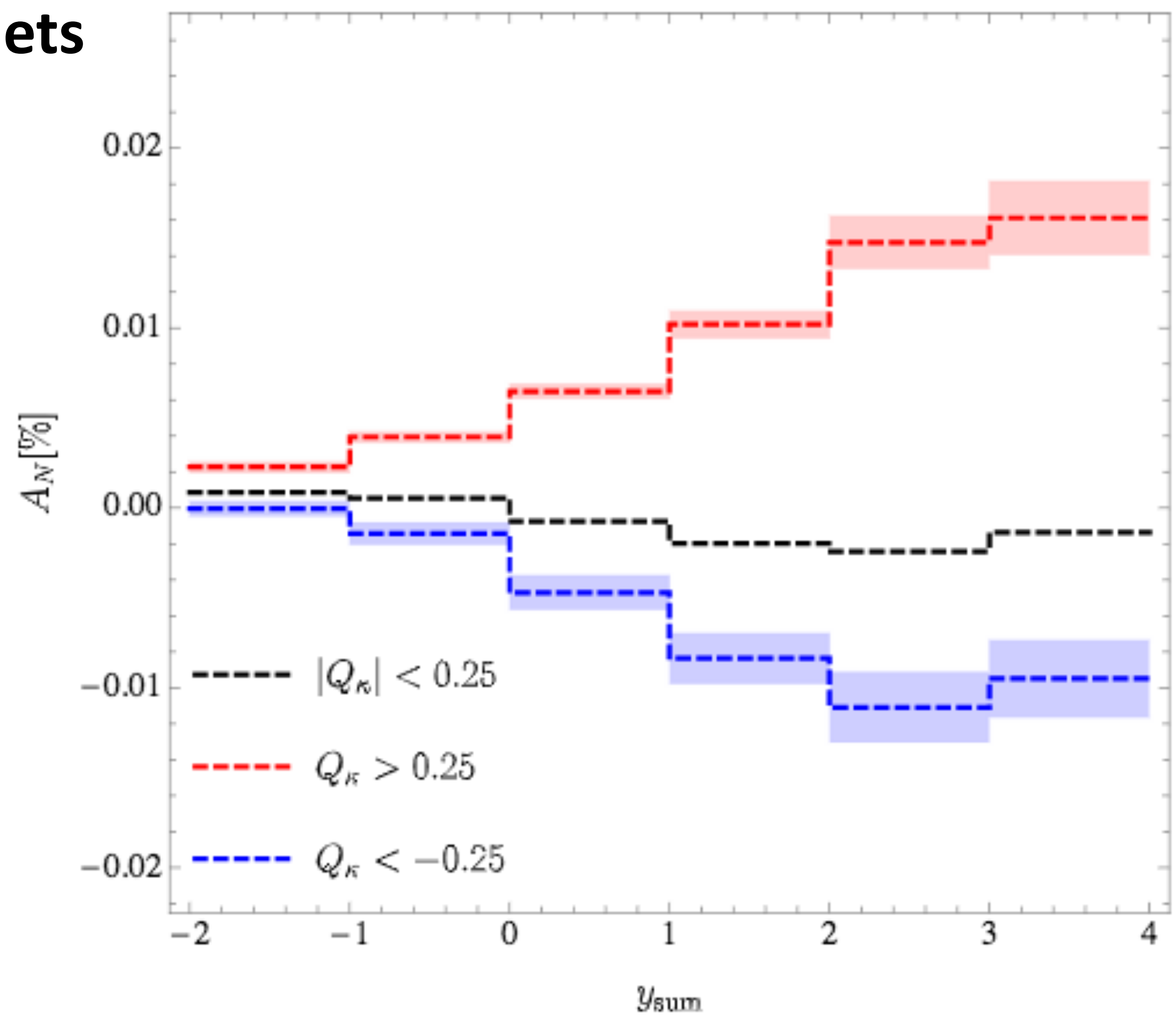
- first N²LL resummation including full jet dynamics
- good perturbative convergence
- Pythia agrees well
- Our work serves as a baseline for pinning down the factorization violation effects

Jet charge and Spin asymmetries at the RHIC

(Kang, Lee, DYS, Terry, '21 JHEP)

Transverse momentum imbalance between dijets



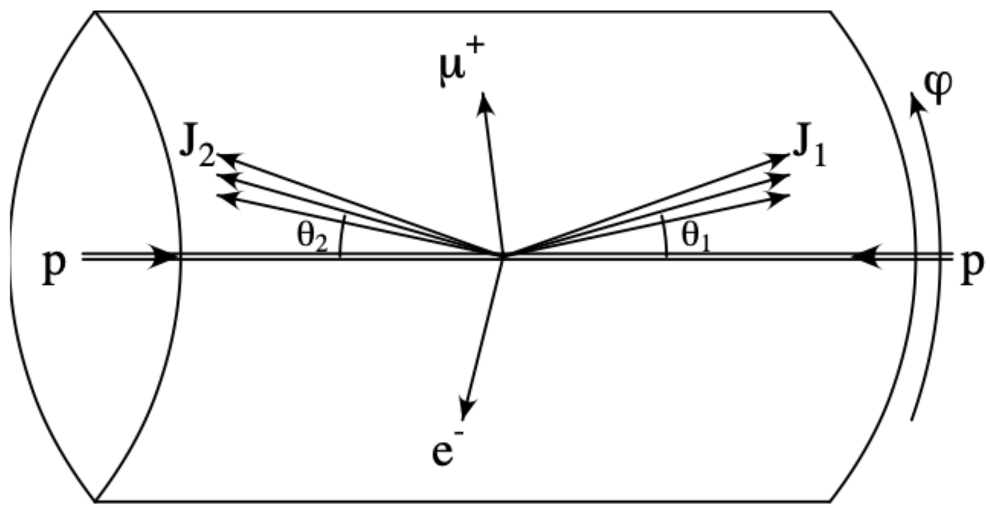


One-loop soft functions in the polarized case are different from the unpolarized counterpart beyond LL Liu, Ringer, Vogelsang, Yuan '20

We apply jet charge tagging to enhance the asymmetry Kang, Liu, Mantry, DYS '20 PRL

Gluon Sivers function: heavy flavor dijets (Kang, Reiten, DYS, Terry '21 JHEP)

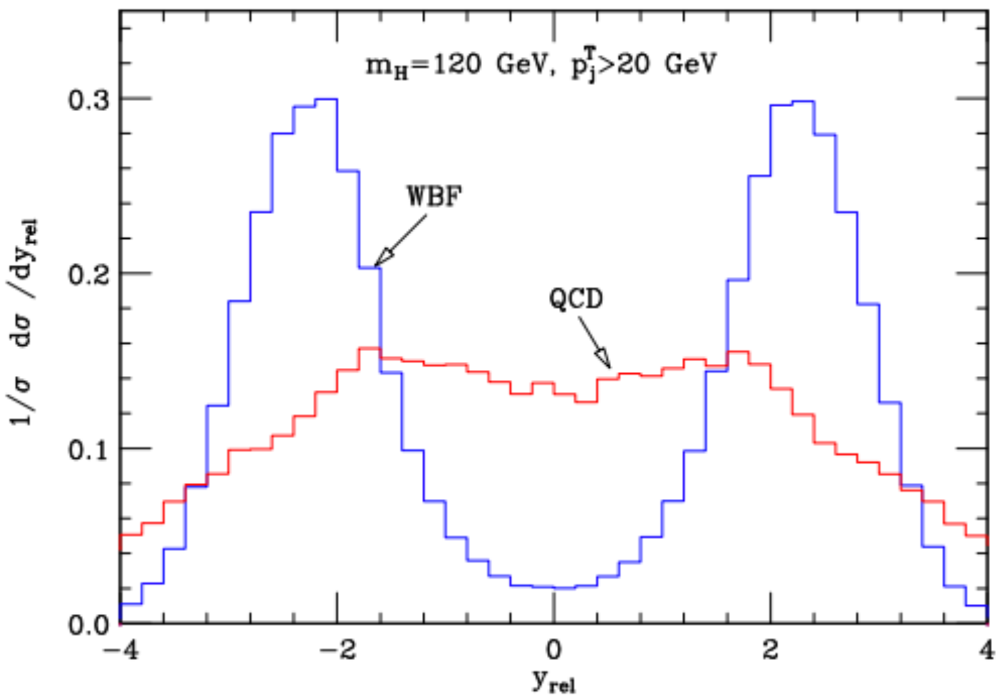
Central jet veto in Higgs production via VBF



VBF signature:

- **Energetic jets in the forward and backward directions**
- **Large rapidity separation and large invariant mass of two tagged jets**
- **Little radiation in the central-rapidity region**

- **Major QCD backgrounds: t-channel color octet exchange**
- **Central jet veto can suppresses QCD background**
- **Central jet veto: no extra jets between tagging jets**



Del Duca, Frizzo, Maltoni '05

Jet veto & QCD resummation

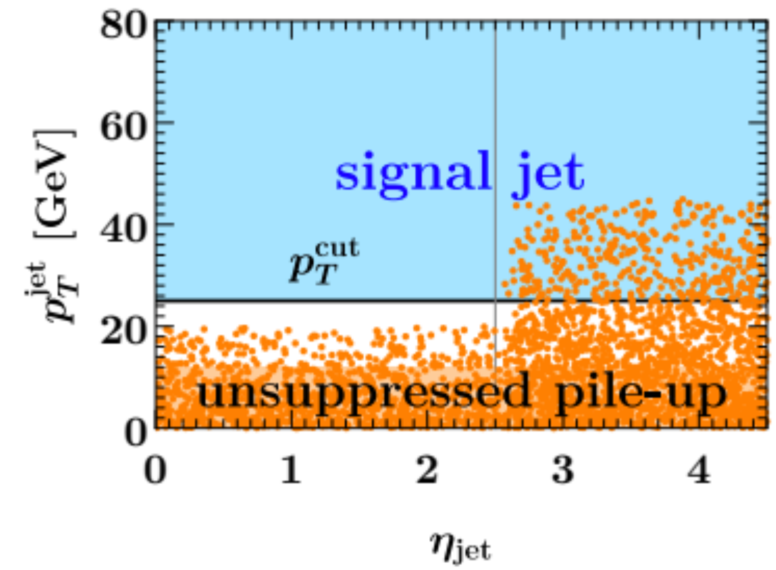
- **Due to existence of a small scale p_T^{veto}, the fixed order calculations are unreliable**
- **QCD resummation is necessary, the large log should be resumed to all order**
- **Standard jet veto resummation for gg->H processes**
  - **Rapidity cut independent**

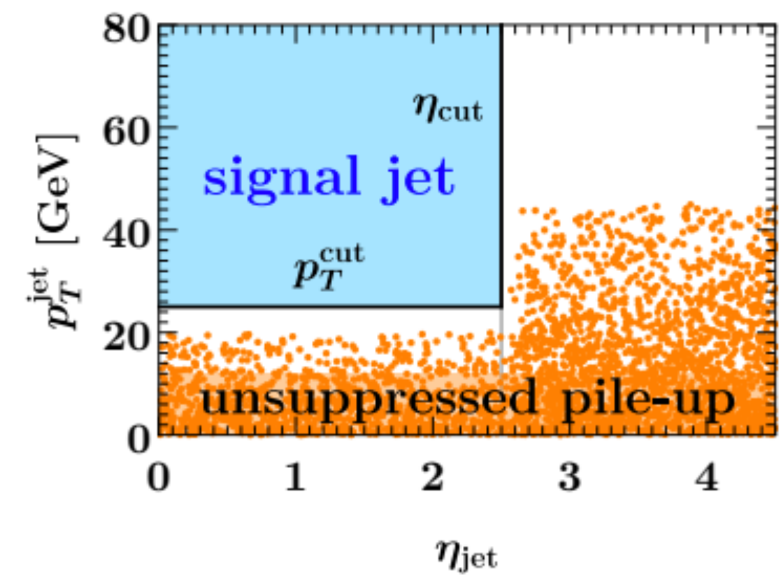
    Banfi, Monni, Salam, Zanderighi '12;

    Becher, Neubert, Rothen '12, '13;

    Stewart, Tackmann, Walsh, Zuberi '12, '13
  - **Rapidity cut dependent**

    Michel, Pietrulewicz, Tackmann '18
  - **Nonfactorizable jet veto in VBF: Superleading Logs**
    - **Four-loop** Forshaw, Kyrieleis, Seymour '06
    - **Five-loop** Keates, Seymour '09
    - **All-order** Becher, Neubert, DYS '21





Courtesy of Johannes Michel

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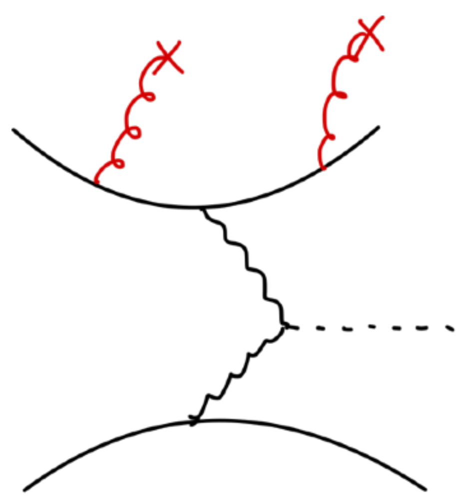

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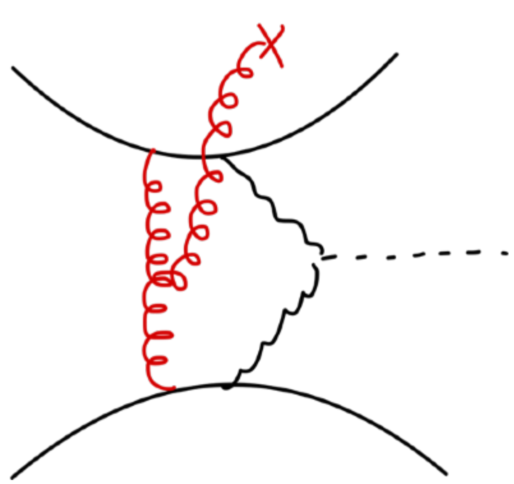
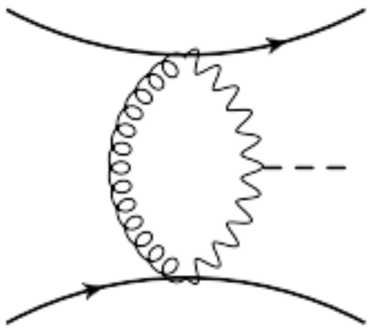
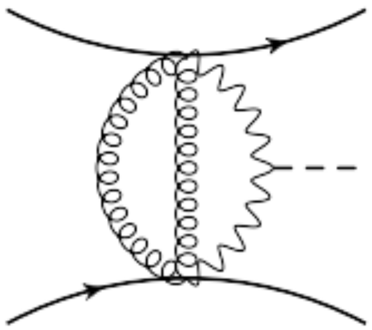


Nonfactorizable QCD effects in Higgs production via VBF

Liu, Melnikov, Penin '19





$$\mathcal{M}^{(2)} = -\frac{\tilde{\alpha}_s^2}{2!}\chi^{(2)}(\boldsymbol{q}_3, \boldsymbol{q}_4)\mathcal{M}^{(0)}$$

with $\chi^{(2)}(\boldsymbol{q}_3, \boldsymbol{q}_4) = \frac{1}{\pi^2}\int\left(\prod_{i=1}^{2}\frac{\mathrm{d}^2\boldsymbol{k}_i}{\boldsymbol{k}_i^2+\lambda^2}\right)$

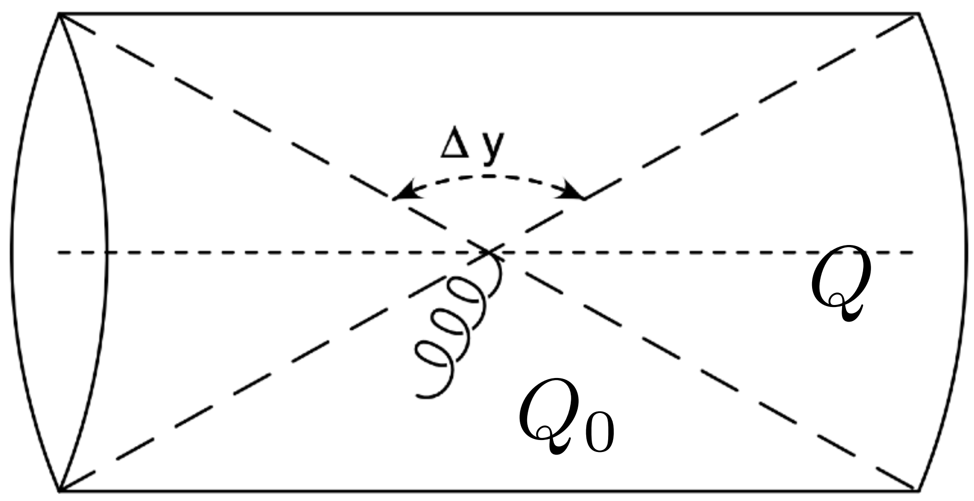
$$\times \frac{\boldsymbol{q}_3^2 + M_V^2}{(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{q}_3)^2 + M_V^2}\frac{\boldsymbol{q}_4^2 + M_V^2}{(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{q}_4)^2 + M_V^2}$$

nonfactorizable correction: $\Delta_{\mathrm{NF}} = \frac{\sigma_{\mathrm{VBF}}^{\mathrm{NNLO,NF}}}{\sigma_{\mathrm{VBF}}^{\mathrm{LO}}} \times 100\% = -0.39\%$

- **the nonfactorizable correction is comparable to the NNNLO QCD factorizable corrections**
- **appear for the first time at NNLO, scale dependence is large**

See also Gaunt '14, Schwartz, Yan & Zhu `17 `18 …

Central jet veto at the LHC

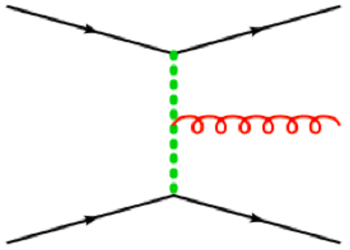


leading logs:

$$e^+e^-, ep: \quad \alpha_s^n \ln^n\left(\frac{Q}{Q_0}\right)$$

$$pp: \quad \cdots \quad + \alpha_s^3 (i\pi)^2 \ln^3\left(\frac{Q}{Q_0}\right) \times \alpha_s^n \ln^{2n}\left(\frac{Q}{Q_0}\right)$$

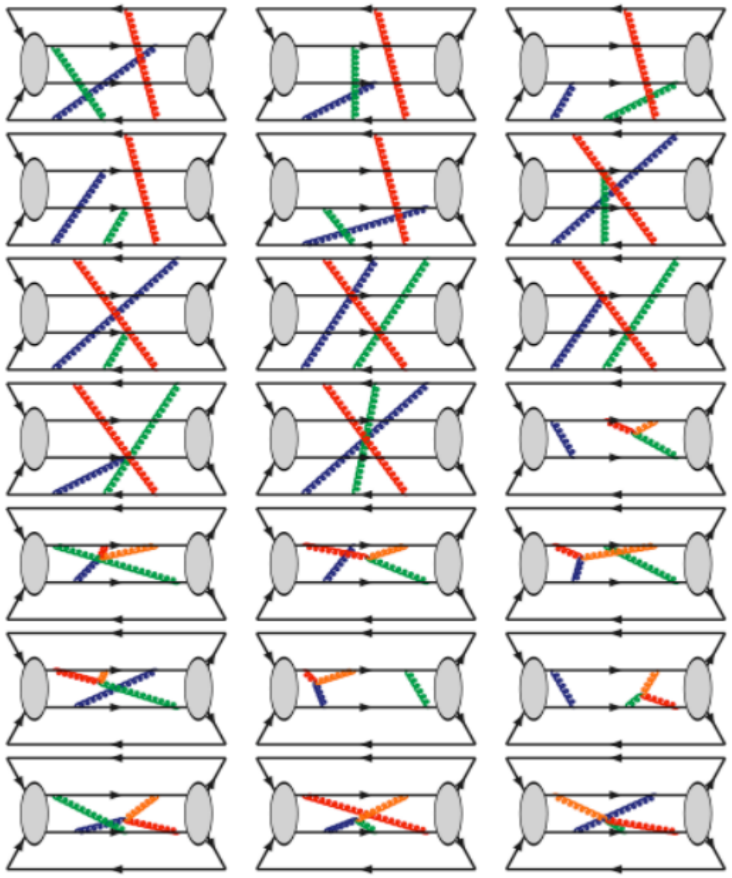
- Such events was originally suggested on the basis of color flow considerations in QCD Bjorken ’93
- Global Logs resummation is first done by Oderda & Sterman ’98
- Forshaw, Kyrieleis, Seymour ’06 have analyzed the effect of Glauber phases in non-global observables directly in QCD
    - Non-zero contributions starting at 3 loops
    - Collinear logarithms starting at 4 loops: Super-leading logs



wide angle soft gluon emission developing a sensitivity to emission at small angles

Fixed order calculation

- Gluons are added in all possible ways to trace diagrams and colour factors calculated using COLOUR
- Diagrams are then cut in all ways consistent with strong ordering
- At fourth order there are 10,529 diagrams and 1,746,272 after cutting.
- SLL terms are confirmed at fourth order and computed for the first time at 5th order



Keates and Seymour
arXiv:0902.0477 [hep-ph]

Simone Marzani's slide

All-order QCD resummation of super-leading logs

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Resummation of Super-Leading Logarithms

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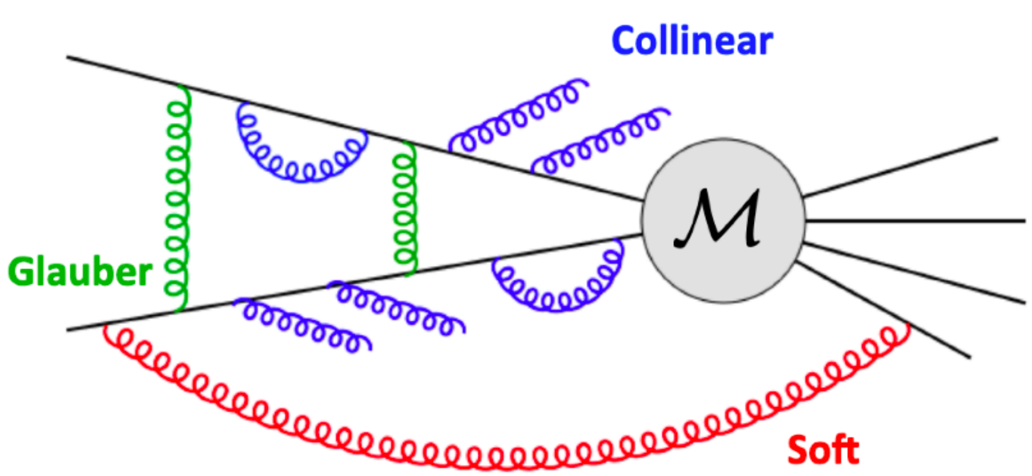


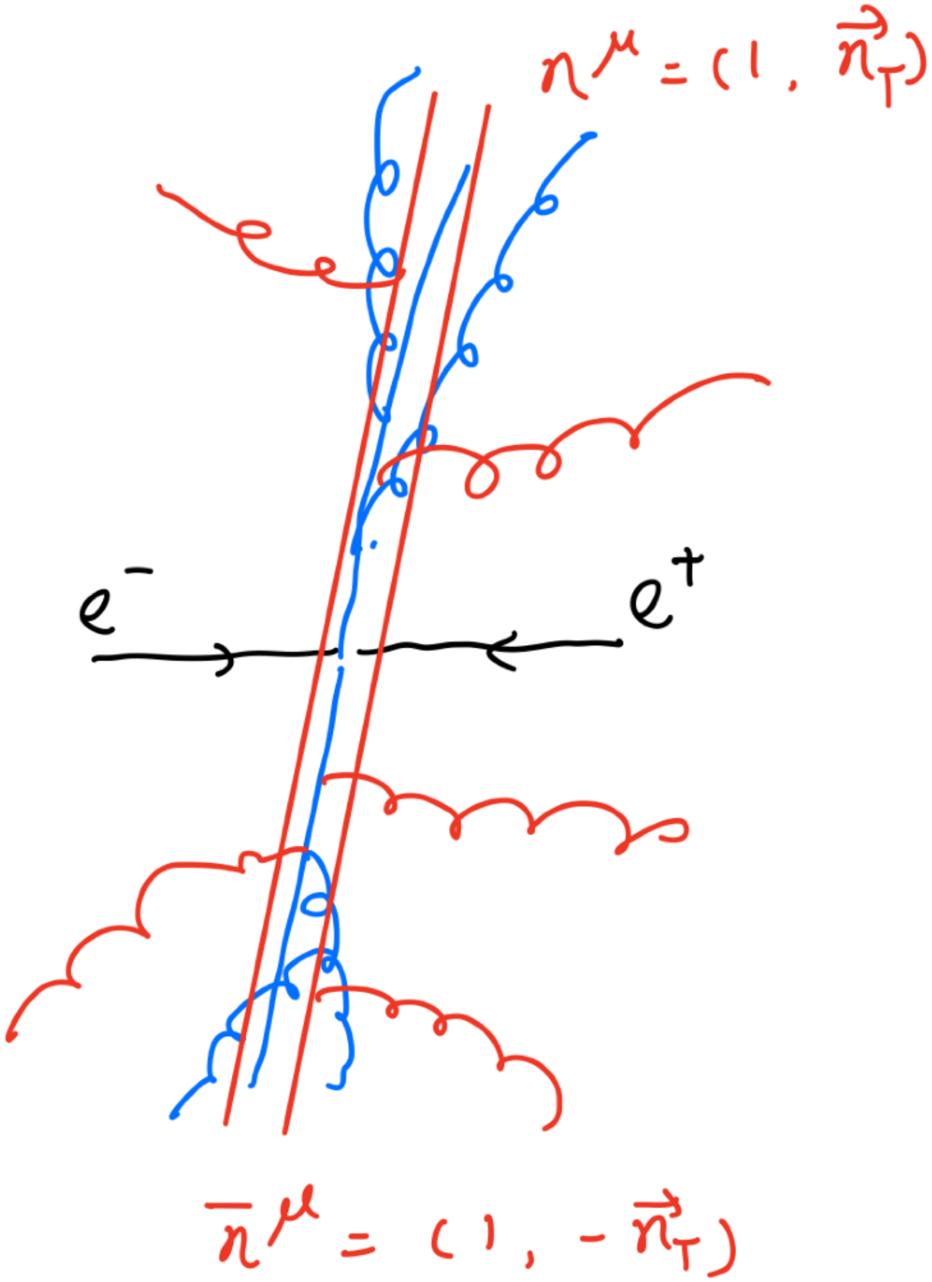
Super-leading logs from renormalization group evolution:

$$\boldsymbol{\mathcal{H}}_4\, \boldsymbol{U}(\mu_s,\mu_h) = \boldsymbol{\mathcal{H}}_4\, \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\,\boldsymbol{\Gamma}^H(Q,\mu)\right]$$
$$= \boldsymbol{\mathcal{H}}_4 + \int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\,\boldsymbol{\mathcal{H}}_4\,\boldsymbol{\Gamma}^H(Q,\mu) + \int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\int_{\mu}^{\mu_h}\frac{d\mu'}{\mu'}\boldsymbol{\mathcal{H}}_4\,\boldsymbol{\Gamma}^H(Q,\mu')\,\boldsymbol{\Gamma}^H(Q,\mu)$$

Becher, Neubert, Rothen, DYS ’16 PRL

Factorization in global event shapes

E.g. Thrust $T \sim 1$

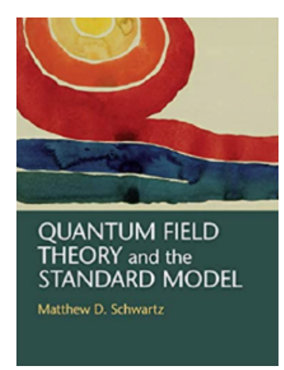


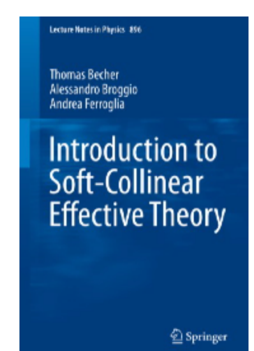
$$\frac{d\sigma}{dT} = H \cdot J \otimes S$$

Soft radiation does not resolve individual energetic patrons. Sensitive only to direction and total charge of the jets

$$S \sim \sum_{X_s} \left| \langle X_s | \boldsymbol{S}(n) \boldsymbol{S}(\bar{n}) | 0 \rangle \right|^2$$

Simple structure -> N^{3}LL resummation





...

Factorization for gap between jets in e+e-

(Becher, Neubert, Rothen, DYS, ’16 PRL, ’16 JHEP; Caron-Huot ’15 JHEP)

Hard function
m hard partons along
fixed directions {n_1, …, n_m}
$\boldsymbol{\mathcal{H}}_m \propto |\mathcal{M}_m\rangle\langle\mathcal{M}_m|$

Soft function
squared amplitude
with *m* Wilson lines

Color Trace

Hard scale

Soft scale

$$\sigma(Q, Q_\Omega) \sim \sum_{m=2}^{\infty} \prod_{i=1}^{m} \int \frac{d\Omega(\vec{n}_i)}{4\pi} \mathrm{Tr}_c \left[\mathcal{H}_m(\{\vec{n}_1, \cdots, \vec{n}_m\}, Q, \mu) \mathcal{S}_m(\{\vec{n}_1, \cdots, \vec{n}_m\}, Q_\Omega, \mu)\right]$$

# of jet not fixed

Integrate the angles for hard partons

Collinear singularities and SLLs at hadron colliders

One-loop anomalous dimension:

$$\boldsymbol{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

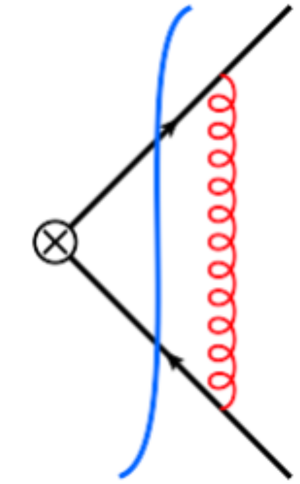
$$W_{ij}^k = \frac{n_i \cdot n_j}{n_i \cdot n_k \, n_j \cdot n_k}$$

$$\boldsymbol{V}_m = -2 \sum_{(ij)} \int \frac{d\Omega(n_k)}{4\pi} \left(\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R}\right) W_{ij}^k \left[\Theta_{\text{in}}^{n\bar{n}}(k) + \Theta_{\text{out}}^{n\bar{n}}(k)\right]$$

$$+ 2\, i\pi \sum_{(ij)} \left(\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R}\right) \Pi_{ij},$$

$$\boldsymbol{R}_m = 4 \sum_{(ij)} \boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,R} \, W_{ij}^{m+1} \, \Theta_{\text{in}}(n_{m+1}) \, .$$

$\Pi_{ij} = 1$ **if both incoming or outgoing**



Individually R_m and V_m contain singularities when emitted gluon k gets collinear to parsons i or j.

- **Expect cancellation in inclusive soft observables such as gaps between jets at lepton colliders**
- **Glauber phases spoil this cancellation: soft+collinear double logs! "Super-leading logs"**

Simplification of the imaginary part

Imaginary part of the anomalous dimension:

For e+e-:

$$\sum_{i\neq j} \mathbf{T}_i \cdot \mathbf{T}_j = -\sum_i \mathbf{T}_i^2 = -\sum_i C_i$$

For pp:

$$\begin{aligned} \sum_{(ij)} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \, \Pi_{ij} &= 2\,\boldsymbol{T}_1 \cdot \boldsymbol{T}_2 + \sum_{i=3}^{m} \boldsymbol{T}_i \cdot (-\boldsymbol{T}_1 - \boldsymbol{T}_2 - \boldsymbol{T}_i) \\ &= 2\,\boldsymbol{T}_1 \cdot \boldsymbol{T}_2 + (\boldsymbol{T}_1 + \boldsymbol{T}_2) \cdot (\boldsymbol{T}_1 + \boldsymbol{T}_2) - \sum_{i=3}^{m} C_i \\ &= 4\,\boldsymbol{T}_1 \cdot \boldsymbol{T}_2 + C_1 + C_2 - \sum_{i=3}^{m} C_i \end{aligned}$$

$$\sum_{(ij)} (\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R})\, \Pi_{ij} = 4\,(\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R})$$

Extracting the collinear singularities: $\overline{W}_{ij}^k = \frac{n_i \cdot n_j}{n_i \cdot n_k \, n_j \cdot n_k} - \frac{\delta(n_k - n_i)}{n_i \cdot n_k} - \frac{\delta(n_k - n_j)}{n_j \cdot n_k}$

The one-loop anomalous dimension is

$$\boldsymbol{V}_m = \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \ln \frac{\mu^2}{\hat{s}}$$

$$\boldsymbol{R}_m = \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \ln \frac{\mu^2}{\hat{s}} ,$$

with

$$\overline{\boldsymbol{V}}_m = 2 \sum_{(ij)} \left(\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R} \right) \int \frac{d\Omega\,(n_k)}{4\pi} \bar{W}_{ij}^k$$

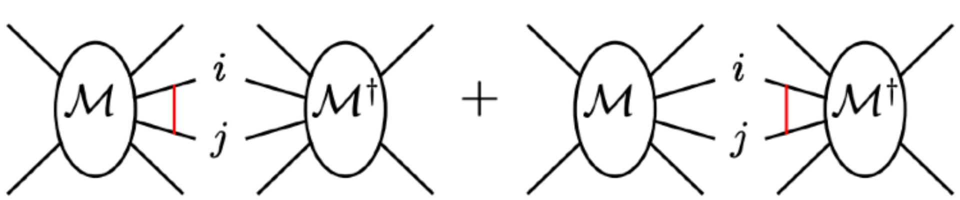
$$\boldsymbol{V}_i^c = 4 C_i \mathbf{1}$$

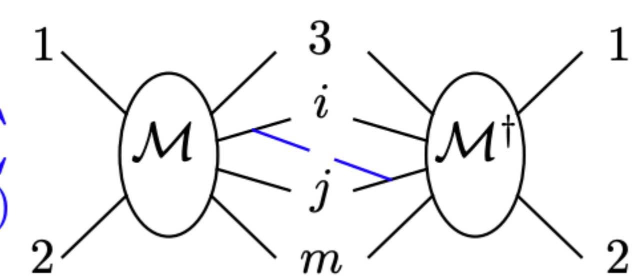
$$\boldsymbol{V}^G = -8 i \pi \left(\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R} \right)$$

$$\overline{\boldsymbol{R}}_m = -4 \sum_{(ij)} \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{j,R} \bar{W}_{ij}^{m+1} \Theta_{\text{hard}}\left(n_{m+1}\right)$$

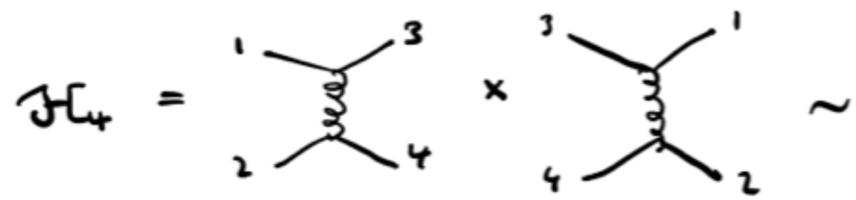
$$\boldsymbol{R}_i^c = -4 \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{i,R} \delta\left(n_k - n_i\right)$$

$$\boxed{\boldsymbol{\mathcal{H}}_m \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{j,R} = \boldsymbol{T}_i^a \boldsymbol{\mathcal{H}}_m \boldsymbol{T}_j^{\tilde{a}}}$$

$\boldsymbol{\mathcal{H}}_m \overline{\boldsymbol{V}}_m = \sum_{(ij)}$ 

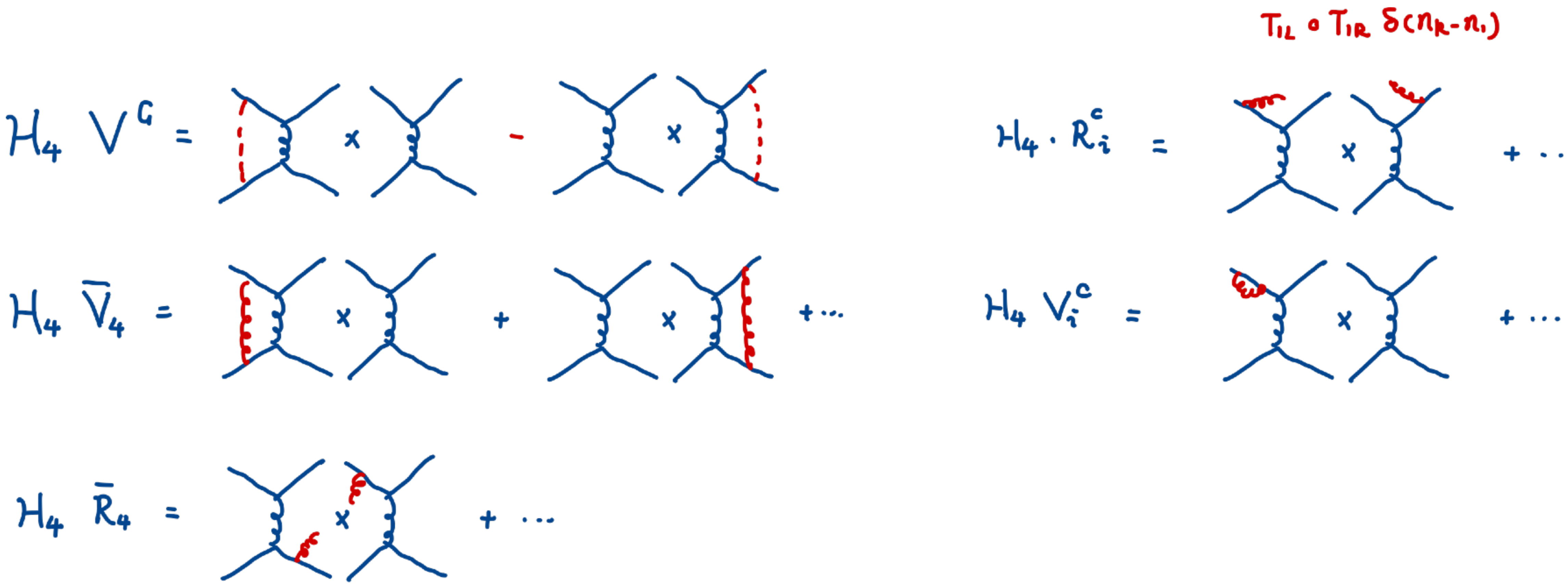
$\boldsymbol{\mathcal{H}}_m \overline{\boldsymbol{R}}_m = \sum_{(ij)}$ 

Hard function for octet exchange:



$$\sim\; t^a_{\alpha_3\alpha_1} t^a_{\alpha_4\alpha_2} t^b_{\beta_1\beta_3} t^b_{\beta_2\beta_4} \sigma_0$$

Action of the anomalous dimension



Compute $\mathcal{H}_4 \, \boldsymbol{U}(\mu_s, \mu_h) = \mathcal{H}_4 \, \mathbf{P} \exp\left[\int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \boldsymbol{\Gamma}^H(Q, \mu) \right]$

$$= \mathcal{H}_4 + \int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \mathcal{H}_4 \, \boldsymbol{\Gamma}^H(Q, \mu) + \int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \int_{\mu}^{\mu_h} \frac{d\mu'}{\mu'} \mathcal{H}_4 \, \boldsymbol{\Gamma}^H(Q, \mu') \, \boldsymbol{\Gamma}^H(Q, \mu)$$

Leading super-leading logs

1. **Want the maximum numbers of logs, i.e. the maximum power of Γ^c**
2. **Need two imaginary parts V^G to spoil cancellation of collinear singularities**
3. **Need at least one real emission $\bar{\Gamma}$ to resolve the gap region**

Three properties of the anomalous dimension greatly simplify the calculations

- **Color coherence**

$$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{\Gamma}^c \, \overline{\boldsymbol{\Gamma}} = \boldsymbol{\mathcal{H}}_m \, \overline{\boldsymbol{\Gamma}} \, \boldsymbol{\Gamma}^c$$

- **Cyclicity of the trace**

$$\langle \boldsymbol{\mathcal{H}}_m \, \boldsymbol{\Gamma}^c \otimes \mathbf{1} \rangle = 0$$
$$\big\langle \boldsymbol{\mathcal{H}}_m \, \boldsymbol{V}^G \otimes \mathbf{1} \big\rangle = 0$$

Leading super-leading logs

The super-leading logs at (3+n) order are associated with color traces of the form

$$C_{rn} = \left\langle \boldsymbol{\mathcal{H}}_4 \left(\boldsymbol{\Gamma}^c\right)^r \boldsymbol{V}^G \left(\boldsymbol{\Gamma}^c\right)^{n-r} \boldsymbol{V}^G \,\overline{\boldsymbol{\Gamma}} \otimes \boldsymbol{1} \right\rangle \qquad\qquad 0 \leq r \leq n$$

The SLLs first appear at four loop (n=1)

The three loop terms (n=0) can be numerically significant

We consider the case where particles 1 and 2 transform in the fundamental representation of SU(N_c)

$$C_{rn} = 2^{8-r} \pi^2 \left(4N_c\right)^n \Big\{ \sum_{j>2} J_j \langle \mathcal{H}_4 [(\boldsymbol{T}_2 - \boldsymbol{T}_1) \cdot \boldsymbol{T}_j + 2^{r-1} N_c \left(\sigma_1 - \sigma_2\right) d_{abc} \boldsymbol{T}_1^a \boldsymbol{T}_2^b \boldsymbol{T}_j^c] \rangle$$
$$+ 2 \left(1 - \delta_{r0}\right) J_2 \left\langle \mathcal{H}_4 \left[C_F + \left(2^r - 1\right) \boldsymbol{T}_1 \cdot \boldsymbol{T}_2 \right] \right\rangle \Big\}$$

with the angular integrals: $J_j = \int \frac{d\Omega(n_k)}{4\pi} \left(W_{1j}^k - W_{2j}^k\right) \Theta_{\rm veto}(n_k)$

All-order results of leading SLLs

(Becher, Neubert, DYS '21 PRL)

Owen's T function

$$f_1(w) = \frac{\sqrt{\pi}}{2w} \int_0^{\sqrt{\frac{w}{2}}} \frac{dz}{z^2} \left[\mathrm{erf}(z) - \frac{e^{-2z^2}}{i} \mathrm{erf}(iz) \right]$$

hypergeometric function

$$f_\delta(w) = \frac{1}{3}\, {}_2F_2\left(1, 1; 2, \frac{5}{2}; -w\right)$$

error function

$$f_2(w) = \frac{1}{w} - \frac{\sqrt{\pi}}{2w^{3/2}} \mathrm{erf}(\sqrt{w})$$

$$\omega \sim \alpha_s L^2$$

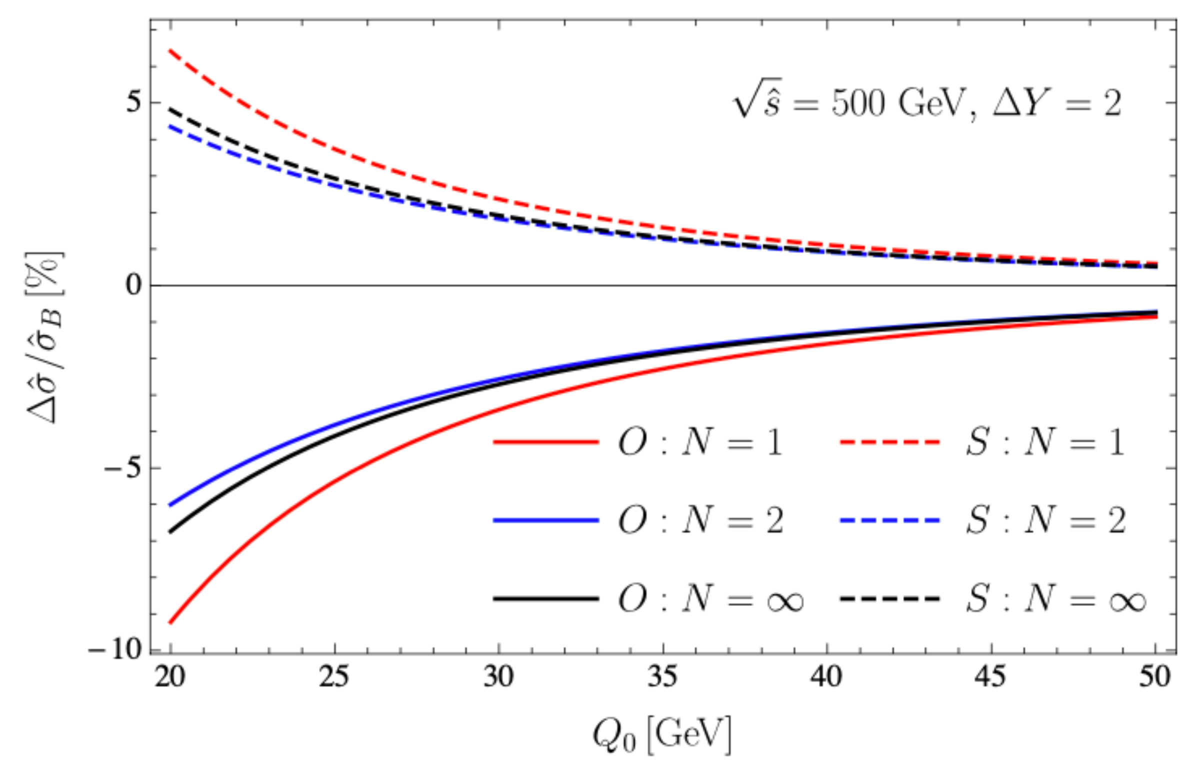
$$S_O = \left(\frac{\alpha_s}{\pi}\right)^3 \pi^2 \ln^3 \frac{Q}{\mu_s} \frac{1}{N_c} \left[N_c^2 \left(4 f_1(w) - 2 f_\delta(w)\right] - 4 f_2(\omega) + 2 f_\delta(w)\right) \Delta Y \, \sigma_0$$

Sudakov suppression of the superleading logarithms is weaker than the one present for global observables

Global logs ⟶ $e^{-\omega}$

Superleading logs $\xrightarrow{\omega \to \infty}$ $\frac{1}{\omega}$

Numerical results



Red: Four loop **Blue: Five loop** **Black: all order**

Summary and outlook

- Factorization is at the heart of any quantitative prediction using pQCD at hadron colliders
- We investigate naive factorization violation effects using jet processes
  - Azimuthal decorrelation: tracking jets
  - Spin Asymmetry: charge tagged jets
  - Gap fraction: all-order results of superseding logs
- Our EFT and the renormalization group-based approach provide a transparent understanding of the underlying dynamics
- Our findings indicate that SLLs could have an appreciable effect on precision observables, e.g. Higgs production via VBF
- Understand the low energy theory from Glauber gluons ?
- High order super-leading logs ?

Thank you

Welcome to Fudan!!!

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