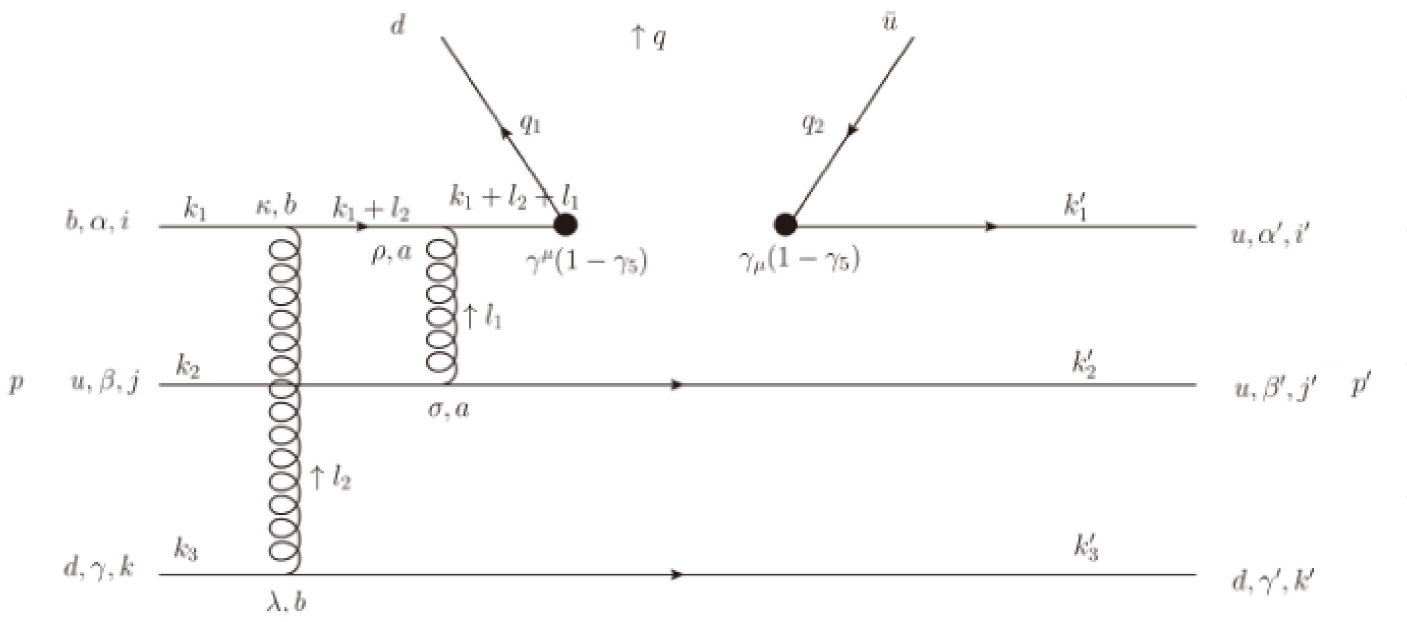

在光锥坐标系中:

$$P = \frac{M_{\Lambda b}}{\sqrt{2}}(1, 1, \vec{0}) \qquad p' = \frac{M_{\Lambda b}}{\sqrt{2}}(0, 1, \vec{0})$$

$$k_1 = (x_1 p^+, p^-, \vec{k}_{1T}) \qquad k_1' = (0, x_1' p'^-, \vec{k}_{1T}')$$

$$k_2 = (x_2 p^+, 0, \vec{k}_{2T}) \qquad k_2' = (0, x_2' p'^-, \vec{k}_{2T}')$$

$$k_3 = (x_3 p^+, 0, \vec{k}_{3T}) \qquad k_3' = (0, x_3' p'^-, \vec{k}_{3T}')$$

$$q = \frac{M_{\Lambda b}}{\sqrt{2}}(1, 0, \vec{0})$$

$$q_1 = (y q^+, 0, \vec{q}_T) \qquad q_2 = ((1-y) q^+, 0, -\vec{q}_T)$$

$$l_1 = k_2 - k_2' \qquad l_2 = k_3 - k_3'$$

其中 i, j, k 是颜色指标，α, β, γ 是旋量指标.

Λ_b 重子的波函数为:

$$(Y_{\Lambda b})_{\alpha\beta\gamma}(k_i, \mu) = \frac{f_{\Lambda b}}{8\sqrt{2} N_c}[(\not{p} + M_{\Lambda b})\gamma_5 C]_{\beta\gamma} \cdot [\Lambda_b]_\alpha \Psi_{\Lambda b}(k_i, \mu)$$

质子的波函数为:

$$\bar{Y}_{\alpha'\beta'\gamma'}(k_i', \mu) = -\frac{f_N}{8\sqrt{2} N_c}\left\{[\bar{N}'\gamma_5]_{\gamma'}[C^{-1}\not{p}']_{\alpha'\beta'}\phi^V + [\bar{N}']_{\gamma'}[C^{-1}\gamma_5\not{p}']_{\alpha'\beta'}\phi^A - [\bar{N}'\gamma_5\gamma^\mu]_{\gamma'}[C^{-1}\sigma_{\nu\mu}p'^\nu]_{\alpha'\beta'}\phi^T\right\}$$

赝标介子的波函数为:

$$\Phi_M = -\frac{i}{\sqrt{6}}\left[\gamma_5 \not{q}\, \phi_M^A + m_0 \gamma_5 \phi_M^P + m_0 \gamma_5 (\not{n}\not{v}-1)\phi_M^T\right]$$

上面的图是可因子化的，$(V-A)\otimes(V-A)$ 的贡献为:

$$\overline{Y}_{\alpha'\beta'\gamma'} (i g_s \gamma_\lambda T^b)_{\gamma'\gamma k'k} (i g_s \gamma_\sigma T^a)_{\beta'\beta \zeta'\zeta} \Big[\gamma_\mu (1-\gamma_5)\, \Phi_M \gamma^\mu (1-\gamma_5) \frac{i(\not{k}_1+\not{l}_1+\not{l}_2+m_b)}{(k_1+l_1+l_2)^2-m_b^2} i g_s \gamma_\rho T^a$$

$$\frac{i(\not{k}_1+\not{l}_2+m_b)}{(k_1+l_2)^2-m_b^2} i g_s \gamma_\kappa T^b\Big]_{\alpha'\alpha \xi'\xi} \frac{-i g^{\kappa\lambda}}{l_2^2} \frac{-i g^{\rho\sigma}}{l_1^2} Y_{\Lambda_b \alpha\beta\gamma} \cdot \underline{\Sigma_{\xi\zeta k}\, \Sigma_{\xi'\zeta' k'}}$$

全反对称张量，保证组成重子的三个夸克颜色不同

其中 $(T^b)_{k'k} (T^a)_{\zeta'\zeta} (T^a T^b)_{\xi'\xi} \Sigma_{\xi\zeta k} \Sigma_{\xi'\zeta' k'} = f_c = \frac{8}{3}$ 色因子

相同的指标表示求和

上式等于:

$$\frac{i\pi^2 \alpha_s^2 f_N f_{\Lambda_b} f_c}{8\sqrt{6} N_c^2} \Big\{[\bar{N}'\gamma_5]_{\gamma'} [C^{-1}\not{p}']_{\alpha'\beta'} \phi^V + [\bar{N}']_{\gamma'} [C^{-1}\gamma_5 \not{p}']_{\alpha'\beta'} \phi^A - [\bar{N}'\gamma_5 \gamma^\mu]_{\gamma'}$$

$$\cdot [C^{-1}\sigma_{\mu\nu} p'^\nu]_{\alpha'\beta'} \phi^T\Big\} \cdot (\gamma_\lambda)_{\gamma'\gamma} (\gamma^\rho)_{\beta'\beta} \Big[\gamma_\mu (1-\gamma_5) \cdot \Phi_M \gamma^\mu (1-\gamma_5) \frac{\not{k}_1+\not{l}_1+\not{l}_2+m_b}{(k_1+l_1+l_2)^2-m_b^2} \cdot \gamma_\rho \cdot$$

$$\frac{k_1+k_2+m_b}{(k_1+k_2)^2-m_b^2}\gamma^\lambda]_{\alpha'\alpha}\ \frac{1}{l_1^2 l_2^2}\ [(\not p+M_{\Lambda_b})\gamma_5 C]_{\beta\gamma}\ [\Lambda_b]_\alpha\ \psi_{\Lambda_b}$$

$$=\frac{\pi^2\alpha_s^2 f_N f_{\Lambda_b}\psi_{\Lambda_b}}{3\sqrt{6}N_c^2}\ \frac{N}{[(k_1+l_1+l_2)^2-m_b^2][(k_1+l_2)^2-m_b^2]l_1^2 l_2^2}$$

其中 $\bar N'$ 和 Λ_b 是旋量

N 中的 ϕ_N^V 项:

$[\bar N'\gamma_5]_{\gamma'}[C^{-1}\not p']_{\alpha\beta'}\ \phi_N^V\ [\gamma_\lambda]_{\gamma'\gamma}[\gamma^\rho]_{\beta'\beta}[\gamma_\delta(1-\gamma_5)(\gamma_5\not q\phi_M^A+\gamma_5 m_0\phi_M^P+m_0\gamma_5$

$(n\!\!\!/\, v\!\!\!/-1)\phi_M^T)\cdot\gamma^\delta(1-\gamma_5)(k_1+l_1+l_2+m_b)\gamma_\rho(k_1+l_2+m_b)\gamma^\lambda]_{\alpha'\alpha}[(\not p+M_{\Lambda_b})\gamma_5 C]_{\beta\gamma}[\Lambda_b]_\alpha$

$=\bar N'\gamma_5\gamma_\lambda C^{-1}\gamma_5^T(\not p+M_{\Lambda_b})^T\gamma^{\rho T}\not p'^T C\gamma_\delta(1-\gamma_5)[\gamma_5\not q\phi_M^A+\gamma_5 m_0\phi_M^P+m_0\gamma_5(n\!\!\!/\, v\!\!\!/-1)\phi_M^T]$

$\cdot\gamma^\delta(1-\gamma_5)(k_1+l_1+l_2+m_b)\gamma_\rho(k_1+l_2+m_b)\gamma^\lambda\Lambda_b\phi_N^V$, 利用 $\begin{cases} C\gamma_\mu^T C^{-1}=-\gamma_\mu \\ C\gamma_5^T C^{-1}=\gamma_5\end{cases}$

$=\bar N'\gamma_5\gamma_\lambda\gamma_5(-\not p+M_{\Lambda_b})\gamma^\rho\not p'\gamma_\delta(1-\gamma_5)[\gamma_5\not q\phi_M^A+\gamma_5 m_0\phi_M^P+m_0\gamma^5(n\!\!\!/\, v\!\!\!/-1)\phi_M^T]\cdot\gamma^\delta$

$(1-\gamma_5)(\not q+k_1'+m_b)\gamma_\rho(k_1+l_2+m_b)\gamma^\lambda\Lambda_b\phi_N^V$

针对旋量中间的矩阵：

$$-\gamma_\lambda(-\not p+M_{\Lambda b})\gamma^\rho\not p'\gamma_\delta\underline{(1-\gamma_5)}[\gamma_5\not q\phi_M^A+\gamma_5 m_0\phi_M^P+m_0\gamma_5(\not n\not v-1)\phi_M^T]\gamma^\delta\underline{(1-\gamma_5)}$$

$$(\not q+\not k_1'+m_b)\gamma_\rho(\not k_1+\not k_3-\not k_3'+m_b)\gamma^\lambda$$，m_b近似$M_{\Lambda b}$，为了简便都换为$M_{\Lambda b}$

利用 $P_L=\frac{1-\gamma_5}{2}$，$P_L\cdot P_R=0$，$P_L^2=P_L$

$$\text{上式}=4\gamma_\lambda(-\not p+M_{\Lambda b})P_R\gamma^\rho\not p'\gamma_\delta\not q\gamma^\delta(\not q+\not k_1'+M_{\Lambda b})\gamma_\rho\overbrace{(\not k_1+\not k_3-\not k_3'+m_{\Lambda b})}^{=\not p-\not k_2-\not k_3'+M_{\Lambda b}}\gamma^\lambda$$

$$=-8\gamma_\lambda(-\not p+M_{\Lambda b})P_R\gamma^\rho\not p'\not q(\not q+\not k_1'+M_{\Lambda b})\gamma_\rho(\not p-\not k_2-\not k_3'+M_{\Lambda b})\gamma^\lambda$$ $\quad q^2=0$

$$=-8\gamma_\lambda(-\not p+M_{\Lambda b})P_R(-2\not k_1'\not q\not p'+4M_{\Lambda b}p'\cdot q)(\not p-\not k_2-\not k_3'+M_{\Lambda b})\gamma^\lambda$$

$$=4(1+\gamma_5)\gamma_\lambda\not p(-2\not k_1'\not q\not p'+2M_{\Lambda b}^3)(\not p-\not k_2-\not k_3'+M_{\Lambda b})\gamma^\lambda \quad (1)$$

$$-4(1-\gamma_5)M_{\Lambda b}\gamma_\lambda(-2\not k_1'\not q\not p'+2M_{\Lambda b}^3)\cdot(\not p-\not k_2-\not k_3'+m_{\Lambda b})\gamma^\lambda \quad (2)$$

对于自旋$\frac{1}{2}$的旋量场，旋量的Dirac方程：

$$\begin{cases}(\not p-m)u^{(s)}(p)=0\\ \bar u^{(s)}(p')(\not p'-m')=0\end{cases}$$

对于旋量场，为了方便处理，我们将夸克的动量和介子的动量写成 p 和 p' 的组合形式：

$$
\begin{cases}
\not{q} = \not{p} - \not{p}' \\
\not{k}_1 = x_1\not{p} + (1-x_1)\not{p}' \\
\not{k}_2 = x_2\not{p} - x_2\not{p}' \\
\not{k}_3 = x_3\not{p} - x_3\not{p}'
\end{cases}
\qquad
\not{k}_1' = x_1'\not{p}' \qquad \not{k}_2' = x_2'\not{p}' \qquad \not{k}_3' = x_3'\not{p}'
$$

$$\not{p}'^2 = 0 \qquad p\cdot p' = \frac{m_{\Lambda b}^2}{2} \qquad p^2 = m_{\Lambda b}^2$$

先处理(1)式：

$$\not{k}_1'\not{q} = \gamma_\mu\gamma_\nu k_1'^\mu q^\nu = (2g_{\mu\nu} - \gamma_\nu\gamma_\mu)k_1'^\mu q^\nu$$
$$= k'\cdot q - \not{q}\not{k}_1'$$

① $-2\gamma_\lambda\not{p}\not{k}_1'\not{q}\overset{p-q}{\underline{\not{p}'}}\not{p}\gamma^\lambda = -2m_{\Lambda b}^2\gamma_\lambda\not{p}\not{k}_1'\not{q}\gamma^\lambda$

$= 4m_{\Lambda b}^2\not{q}\not{k}_1'\not{p} = 4m_{\Lambda b}^2(\not{p}-\not{p}')x_1'\not{p}'\not{p} = 4m_{\Lambda b}^2 x_1'[(2p\cdot p')\not{p} - p^2\not{p}']$

$= 4m_{\Lambda b}^4 x_1'(\not{p}-\not{p}')$

② $2\gamma_\lambda\not{p}\not{k}_1'\underset{p-p'}{\overline{\not{q}}}\not{p}'\not{k}_2\gamma^\lambda = 2\gamma_\lambda\not{p}(2p\cdot k_1' - \not{p}\not{k}_1')\not{p}'\not{k}_2\gamma^\lambda = -2m_{\Lambda b}^2 x_1'\gamma_\lambda\not{p}\not{p}'(x_2\not{p} - x_2\not{p}')\gamma^\lambda$

$= -4m_{\Lambda b}^2 x_1' x_2\not{p}\not{p}'\not{p} = -4m_{\Lambda b}^2 x_1' x_2(2p\cdot p'\not{p} - p^2\not{p}')$

$= -4m_{\Lambda b}^4 x_1' x_2(\not{p}-\not{p}')$

③ $2\gamma_\lambda \not p \not k_1' \not q \underline{\not p' \not k_3'} \gamma^\lambda = 0$

④ $-2\gamma_\lambda \not p \not k_1 \underbrace{\not q \not p'}_{= p - p'} \gamma^\lambda M_{\Lambda b} = -2M_{\Lambda b}\gamma_\lambda \not p (2p\cdot k_1' - \not p \not k_1') \not p' \gamma^\lambda$

$$= -2x_1' M_{\Lambda b}^3 \, 4p\cdot p' = -4M_{\Lambda b}^5 x_1'$$

⑤

$$2M_{\Lambda b}^3 \begin{cases} \gamma_\lambda \not p \not p \gamma^\lambda = 8M_{\Lambda b}^5 \\ -4p\cdot k_2 = -4M_{\Lambda b}^5 x_2 \\ -4p\cdot k_3' = -4M_{\Lambda b}^5 x_3' \\ -2M_{\Lambda b}\not p = -4M_{\Lambda b}^4 \not p \end{cases}$$

整理：第一项

(1) $4(1+\gamma_5) M_{\Lambda b}^4 [4x_1'(\not p - \not p') - 4x_1' x_2(\not p - \not p') - 4M_{\Lambda b} x_1' + 8M_{\Lambda b} - 4M_{\Lambda b} x_2' - 4M_{\Lambda b} x_3'$

$- 4\not p]$

下面计算第2项：

① $-2m_{\Lambda b}\gamma_\lambda \not{k}_1' \not{q} \not{p}' \not{p} \gamma^\lambda = -2m_{\Lambda b}\gamma_\lambda x_1' \not{p}' (\cancel{\not{p}} - \not{p}') \not{p}' \not{p} \gamma^\lambda$

$= -2m_{\Lambda b} x_1' \gamma_\lambda \not{p}' \not{p} \not{p}' \not{p} \gamma^\lambda = -2m_{\Lambda b} x_1' (2p\cdot p')(4p\cdot p') = -4m_{\Lambda b}^5 x_1'$

② $-2m_{\Lambda b}\gamma_\lambda \not{k}_1' \not{q} \not{p}' \not{k}_2 \gamma^\lambda = 2m_{\Lambda b}\gamma_\lambda x_1' \not{p}' (\cancel{\not{p}} - \not{p}') \not{p}' x_2 \not{p} \gamma^\lambda$

$= 2m_{\Lambda b} x_1' x_2 \not{p}' \not{p} \not{p}' \not{p} \gamma^\lambda = 4m_{\Lambda b}^5 x_1' x_2$

③ $2m_b \gamma_\lambda \not{k}_1' \not{q} \underline{\not{p}' \not{k}_3'} \gamma^\lambda = 0$

④ $-2m_{\Lambda b}^2 \gamma_\lambda \not{k}_1' \not{q} \not{p}' \gamma^\lambda = -2m_{\Lambda b}^2 \gamma_\lambda x_1' \not{p}' (\not{p} - \not{p}') \not{p}' \gamma^\lambda$

$= -2m_{\Lambda b}^2 \gamma_\lambda x_1' (2p\cdot p' - \not{p}\not{p}') \not{p}' \gamma^\lambda = 4m_{\Lambda b}^4 x_1' \not{p}'$

⑤

$$2m_{\Lambda b}^4 \left\{ \begin{array}{l} \gamma_\lambda \not{p} \gamma^\lambda = -4m_{\Lambda b}^4 \not{p} \\ -\gamma_\lambda \not{k}_2 \gamma^\lambda = -\gamma_\lambda (x_2 \not{p} - x_2 \not{p}') \gamma^\lambda = 4m_{\Lambda b}^4 (x_2 \not{p} - x_2 \not{p}') \\ -\gamma_\lambda \not{k}_3' \gamma^\lambda = -\gamma_\lambda x_3' \not{p}' \gamma^\lambda = 4m_{\Lambda b}^4 x_3' \not{p}' \\ 8m_{\Lambda b}^5 \end{array} \right.$$

$\mathcal{L} \supset -4(1-x_5)\left[-4m_{\Lambda_b}^5 x_1' + 4m_{\Lambda_b}^5 x_1' x_2 + 4m_{\Lambda_b}^4 x_1' \not{p}' - 4m_{\Lambda_b}^4 \not{p} + 4m_{\Lambda_b}^4 (x_2\not{p} - x_2\not{p}') + 4m_{\Lambda_b}^4 x_3' \not{p}' + 8m_{\Lambda_b}^5\right]$

由旋量的Dirac方程 $\Rightarrow \begin{cases} \not{p}\Lambda_b = m_{\Lambda_b}\Lambda_b \\ \bar{\nu}'\not{p}' = m_p\bar{\nu}' \approx 0 \end{cases}$ (与 m_{Λ_b} 相比是小量)

约化条件 $\begin{cases} \not{p}\gamma_5\Lambda_b = -\gamma_5\not{p}\Lambda_b = -m_{\Lambda_b}\gamma_5\Lambda_b \\ \bar{\nu}'\not{p}'\gamma_5 = 0 \end{cases}$

代入约化条件可得：

ϕ^ν 项为：

$\bar{\nu}'[16m_{\Lambda_b}^5(x_1' - x_1'x_2 - x_1' + 2 - x_2 - x_3' - 1 + x_1' - x_1'x_2 + 1 - x_2 - 2) + 16\gamma_5 m_{\Lambda_b}^5$

$(x_1' - x_1'x_2 - x_1' + 2 - x_2 - x_3' - 1 - x_1' + x_1'x_2 - 1 + x_2 + 2)]\Lambda_b \phi_M^A \phi_\nu^V$

$= \bar{\nu}'\left\{-16m_{\Lambda_b}^5\left[2x_1'x_2 - x_1' + 2x_2 + x_3' + \gamma_5(x_1' + x_3' - 2)\right]\right\}\Lambda_b \phi_M^A \phi_\nu^V$