Study of Z line-shape scan at CEPC

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IHEP

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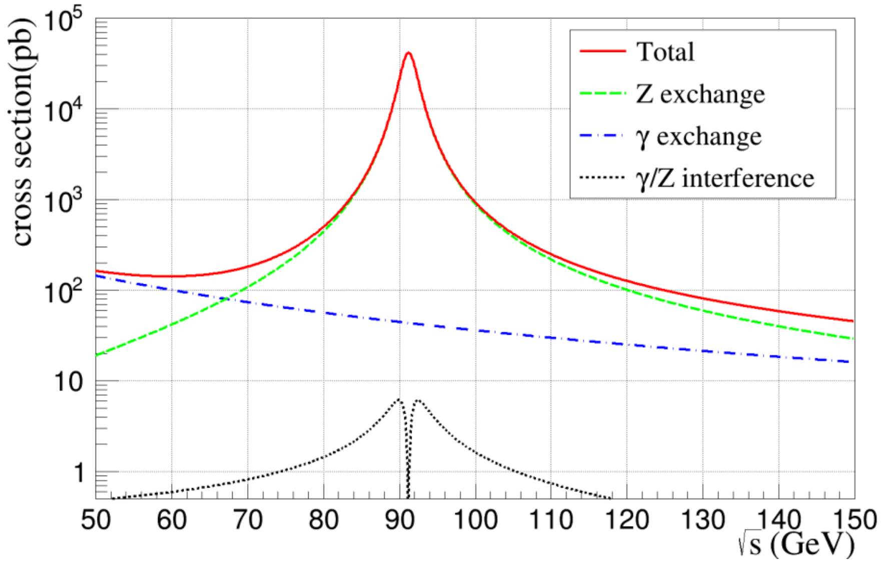
Outline

- Motivation
- Theoretical basis & Methodology
- Statistical and systematic uncertainties
- Data-taking strategy
- Updates using `ZFITTER`
- Summary & Outlook

Motivation

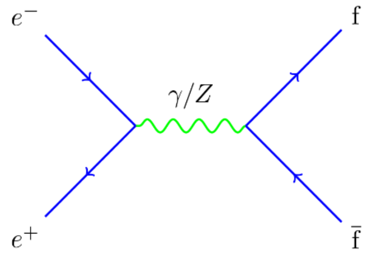
- Properties of Z boson are basic parameters of nature and could provide stringent tests of the Standard Model.
- Possible new physics beyond the SM might be revealed through the subtle changes of measured results.
- Develop data-taking strategies to make good use of the performance of future colliders & provide reference for CEPC design.



Theoretical basis & Methodology

Theoretical basis & Methodology

- Main process: $e^+e^- \to f\bar{f}$ at ~91 GeV



- In lowest order & neglecting fermion masses :

$$\frac{2s}{\pi N_c}\frac{d\sigma_{f\bar{f}}}{d\cos\theta} = \alpha^2 Q_f^2(1+\cos^2\theta)$$
$$+ 8\,\mathrm{Re}\left\{\alpha Q_f \chi^*(s)\left[\, C^s_{\gamma Z}(1+\cos^2\theta) + 2C^a_{\gamma Z}\cos\theta\,\right]\right\}$$
$$+ 16|\chi(s)|^2\left[\, C^s_{ZZ}(1+\cos^2\theta) + 8C^a_{ZZ}\cos\theta\,\right] \;,$$

with

$$\chi(s) = \frac{G_F M_Z^2}{8\pi\sqrt{2}}\frac{s}{s - M_Z^2 + iM_Z\Gamma_Z} \;,$$

$$C^s_{\gamma Z} = g_{Ve}g_{Vf}\;, \qquad C^a_{\gamma Z} = g_{Ae}g_{Af}\;,$$
$$C^s_{ZZ} = (g^2_{Ve} + g^2_{Ae})(g^2_{Vf} + g^2_{Af})\;, \qquad C^a_{ZZ} = g_{Ve}g_{Ae}g_{Vf}g_{Af}\;.$$

Eur.Phys.J.C 19 (2001) 587-651

Theoretical basis & Methodology

- Integrated over the full angular space：

  Z-exchange term：
$$\sigma_{\mathrm{f\bar{f}}}^{Z} = \sigma_{\mathrm{f}}^{0} \frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2},$$

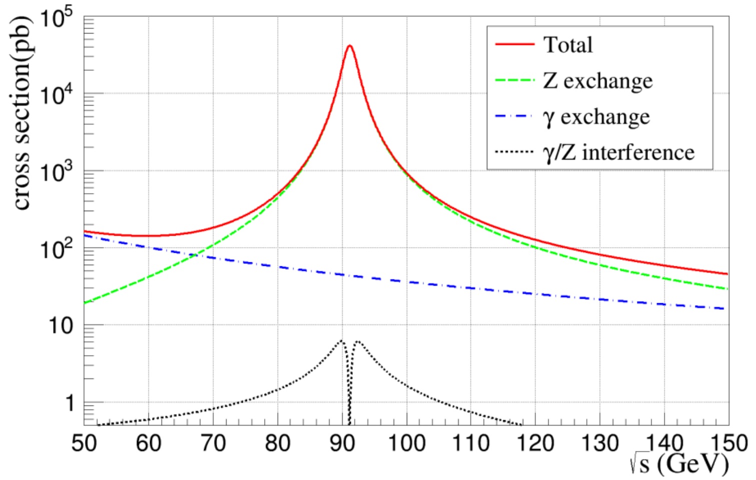
$$\sigma_{\mathrm{f}}^{0} = \frac{12\pi}{M_Z^2}\frac{\Gamma_{\mathrm{ee}}\Gamma_{\mathrm{ff}}}{\Gamma_Z^2} = \frac{C_{ZZ}^{\mathrm{s}} N_{\mathrm{c}}}{6\pi}\left(\frac{M_Z^2 G_{\mathrm{F}}}{\Gamma_Z}\right)^2$$

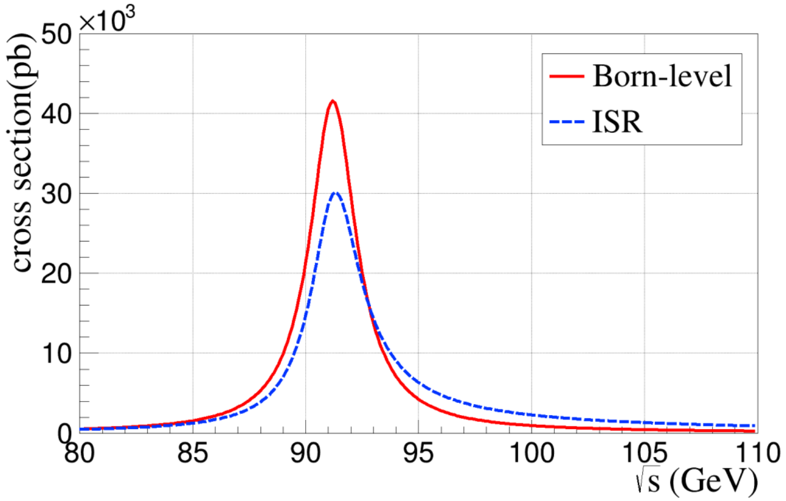
- Forward-backward asymmetry A_{FB}:

$$A_{\mathrm{FB}} = \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}}$$

- ISR

$$\sigma_{\mathrm{f\bar{f}}}^{\mathrm{obs}}(s) = \int_0^{1-s_m/s} dx\,\sigma(s(1-x))F(x,s)$$





Theoretical basis & Methodology

- $\sigma_{\mathrm{f\bar{f}}} = \sigma_{\mathrm{f\bar{f}}}\,(M_Z, \Gamma_Z, \sigma^0_{\mathrm{had}}, C^{\mathrm{s}}_{\mathrm{ZZ}}, C^{\mathrm{s}}_{\mathrm{\gamma Z}})$, $A^{ll}_{\mathrm{FB}} = A_{\mathrm{FB}}(M_Z, \Gamma_Z, C^{\mathrm{s}}_{\mathrm{ZZ}}, C^{\mathrm{a}}_{\mathrm{ZZ}}, C^{\mathrm{s}}_{\mathrm{\gamma Z}}, C^{\mathrm{a}}_{\mathrm{\gamma Z}})$

- Parameter set: M_{Z} Γ_{Z} σ^0_{had} $C^{\mathrm{s}}_{\mathrm{ZZ}}$ $C^{\mathrm{a}}_{\mathrm{ZZ}}$ $C^{\mathrm{s}}_{\mathrm{\gamma Z}}$ $C^{\mathrm{a}}_{\mathrm{\gamma Z}}$ Eur.Phys.J.C 19 (2001) 587-651
- Parameters can be obtained by fitting the $N_{obs}(\sigma_{\mathrm{f\bar{f}}}, A^{ll}_{\mathrm{FB}})$ with the theoretical result.

- Focusing on uncertainties, we use toy Monte Carlo method to generate N_{obs} and perform χ^2 fits to get the uncertainties of measured parameters.

Statistical and systematic uncertainties

Statistical and systematic uncertainties

- **Statistical uncertainties**
- For single data point:
- Cross-section :

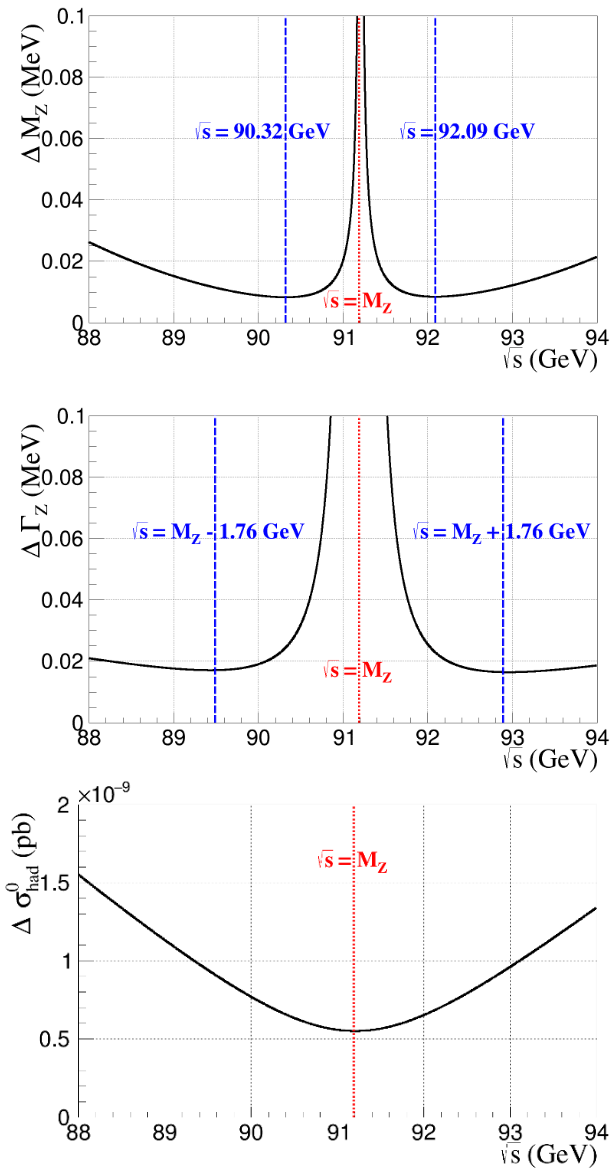
$$\sigma_{\text{meas}} = \frac{N_{\text{obs}} - N_{\text{bkg}}}{\epsilon \mathcal{L}} = \frac{N_{\text{meas}}}{\epsilon \mathcal{L}}$$

- N_{meas} subject to Poisson distribution

$$\Delta\sigma_{\text{meas}}(\text{stat}) = \frac{\sqrt{N_{\text{meas}}}}{\epsilon \mathcal{L}}$$

- Statistical uncertainty of a parameter P:

$$\Delta P(\text{stat}) = \left|\frac{\partial P}{\partial \sigma_{\text{meas}}}\right| \Delta\sigma_{\text{meas}}(\text{stat}) = \left|\frac{\partial \sigma_{\text{meas}}}{\partial P}\right|^{-1} \Delta\sigma_{\text{meas}}(\text{stat})$$



Statistical and systematic uncertainties

- **Statistical uncertainties**
- For two data points or more:
- The χ^2 defined as ($\xi = N$ or A_{FB}):

$$\chi^2 = \sum_i \frac{(\xi^i_{\mathrm{meas}} - \xi^i_{\mathrm{th}})^2}{\delta_i^2}$$

- Covariance matrix calculated by MINUIT:

$$C = \begin{bmatrix}
\frac{\partial^2\chi^2}{\partial M_Z^2} & \frac{\partial^2\chi^2}{\partial M_Z \partial \Gamma_Z} & \frac{\partial^2\chi^2}{\partial M_Z \partial \sigma^0_{\mathrm{had}}} & \frac{\partial^2\chi^2}{\partial M_Z \partial C^{\mathrm{s}}_{\mathrm{ZZ}}} & \frac{\partial^2\chi^2}{\partial M_Z \partial C^{\mathrm{a}}_{\mathrm{ZZ}}} & \frac{\partial^2\chi^2}{\partial M_Z \partial C^{\mathrm{a}}_{\gamma\mathrm{Z}}} & \frac{\partial^2\chi^2}{\partial M_Z \partial C^{\mathrm{s}}_{\gamma\mathrm{Z}}} \\
\frac{\partial^2\chi^2}{\partial M_Z \partial \Gamma_Z} & \frac{\partial^2\chi^2}{\partial \Gamma_Z^2} & \frac{\partial^2\chi^2}{\partial \Gamma_Z \partial \sigma^0_{\mathrm{had}}} & \frac{\partial^2\chi^2}{\partial \Gamma_Z \partial C^{\mathrm{s}}_{\mathrm{ZZ}}} & \frac{\partial^2\chi^2}{\partial \Gamma_Z \partial C^{\mathrm{a}}_{\mathrm{ZZ}}} & \frac{\partial^2\chi^2}{\partial \Gamma_Z \partial C^{\mathrm{a}}_{\gamma\mathrm{Z}}} & \frac{\partial^2\chi^2}{\partial \Gamma_Z \partial C^{\mathrm{s}}_{\gamma\mathrm{Z}}} \\
\frac{\partial^2\chi^2}{\partial M_Z \partial \sigma^0_{\mathrm{had}}} & \frac{\partial^2\chi^2}{\partial \Gamma_Z \partial \sigma^0_{\mathrm{had}}} & \frac{\partial^2\chi^2}{\partial \sigma^{0\;2}_{\mathrm{had}}} & \frac{\partial^2\chi^2}{\partial \sigma^0_{\mathrm{had}} \partial C^{\mathrm{s}}_{\mathrm{ZZ}}} & \frac{\partial^2\chi^2}{\partial \sigma^0_{\mathrm{had}} \partial C^{\mathrm{a}}_{\mathrm{ZZ}}} & \frac{\partial^2\chi^2}{\partial \sigma^0_{\mathrm{had}} \partial C^{\mathrm{a}}_{\gamma\mathrm{Z}}} & \frac{\partial^2\chi^2}{\partial \sigma^0_{\mathrm{had}} \partial C^{\mathrm{s}}_{\gamma\mathrm{Z}}} \\
\frac{\partial^2\chi^2}{\partial M_Z \partial C^{\mathrm{s}}_{\mathrm{ZZ}}} & \frac{\partial^2\chi^2}{\partial \Gamma_Z \partial C^{\mathrm{s}}_{\mathrm{ZZ}}} & \frac{\partial^2\chi^2}{\partial \sigma^0_{\mathrm{had}} \partial C^{\mathrm{s}}_{\mathrm{ZZ}}} & \frac{\partial^2\chi^2}{\partial C^{\mathrm{s}\;2}_{\mathrm{ZZ}}} & \frac{\partial^2\chi^2}{\partial C^{\mathrm{s}}_{\mathrm{ZZ}} \partial C^{\mathrm{a}}_{\mathrm{ZZ}}} & \frac{\partial^2\chi^2}{\partial C^{\mathrm{s}}_{\mathrm{ZZ}} \partial C^{\mathrm{a}}_{\gamma\mathrm{Z}}} & \frac{\partial^2\chi^2}{\partial C^{\mathrm{s}}_{\mathrm{ZZ}} \partial C^{\mathrm{s}}_{\gamma\mathrm{Z}}} \\
\frac{\partial^2\chi^2}{\partial M_Z \partial C^{\mathrm{a}}_{\mathrm{ZZ}}} & \frac{\partial^2\chi^2}{\partial \Gamma_Z \partial C^{\mathrm{a}}_{\mathrm{ZZ}}} & \frac{\partial^2\chi^2}{\partial \sigma^0_{\mathrm{had}} \partial C^{\mathrm{a}}_{\mathrm{ZZ}}} & \frac{\partial^2\chi^2}{\partial C^{\mathrm{s}}_{\mathrm{ZZ}} \partial C^{\mathrm{a}}_{\mathrm{ZZ}}} & \frac{\partial^2\chi^2}{\partial C^{\mathrm{a}\;2}_{\mathrm{ZZ}}} & \frac{\partial^2\chi^2}{\partial C^{\mathrm{a}}_{\mathrm{ZZ}} \partial C^{\mathrm{a}}_{\gamma\mathrm{Z}}} & \frac{\partial^2\chi^2}{\partial C^{\mathrm{a}}_{\mathrm{ZZ}} \partial C^{\mathrm{s}}_{\gamma\mathrm{Z}}} \\
\frac{\partial^2\chi^2}{\partial M_Z \partial C^{\mathrm{a}}_{\gamma\mathrm{Z}}} & \frac{\partial^2\chi^2}{\partial \Gamma_Z \partial C^{\mathrm{a}}_{\gamma\mathrm{Z}}} & \frac{\partial^2\chi^2}{\partial \sigma^0_{\mathrm{had}} \partial C^{\mathrm{a}}_{\gamma\mathrm{Z}}} & \frac{\partial^2\chi^2}{\partial C^{\mathrm{s}}_{\mathrm{ZZ}} \partial C^{\mathrm{a}}_{\gamma\mathrm{Z}}} & \frac{\partial^2\chi^2}{\partial C^{\mathrm{a}}_{\mathrm{ZZ}} \partial C^{\mathrm{a}}_{\gamma\mathrm{Z}}} & \frac{\partial^2\chi^2}{\partial C^{\mathrm{a}\;2}_{\gamma\mathrm{Z}}} & \frac{\partial^2\chi^2}{\partial C^{\mathrm{a}}_{\gamma\mathrm{Z}} \partial C^{\mathrm{s}}_{\gamma\mathrm{Z}}} \\
\frac{\partial^2\chi^2}{\partial M_Z \partial C^{\mathrm{s}}_{\gamma\mathrm{Z}}} & \frac{\partial^2\chi^2}{\partial \Gamma_Z \partial C^{\mathrm{s}}_{\gamma\mathrm{Z}}} & \frac{\partial^2\chi^2}{\partial \sigma^0_{\mathrm{had}} \partial C^{\mathrm{s}}_{\gamma\mathrm{Z}}} & \frac{\partial^2\chi^2}{\partial C^{\mathrm{s}}_{\mathrm{ZZ}} \partial C^{\mathrm{a}}_{\gamma\mathrm{Z}}} & \frac{\partial^2\chi^2}{\partial C^{\mathrm{a}}_{\mathrm{ZZ}} \partial C^{\mathrm{s}}_{\gamma\mathrm{Z}}} & \frac{\partial^2\chi^2}{\partial C^{\mathrm{a}}_{\gamma\mathrm{Z}} \partial C^{\mathrm{s}}_{\gamma\mathrm{Z}}} & \frac{\partial^2\chi^2}{\partial C^{\mathrm{s}\;2}_{\gamma\mathrm{Z}}}
\end{bmatrix}^{-1}$$

Statistical and systematic uncertainties

- **Systematic uncertainties**
- Sources of systematic uncertainties that we have studied:

$$\sigma_E, \quad \Delta E, \quad \Delta\sigma_E, \quad \Delta L$$

- σ_E beam energy spread :

$$\sigma_{f\bar{f}}(E_0, \sigma_E^0) = \int \sigma_{f\bar{f}}(E')G(E_0, \sigma_E^0)dE'$$
$$= \int \sigma_{f\bar{f}}(E') \cdot \frac{1}{\sqrt{2\pi}\sigma_E^0} e^{\frac{-(E_0-E')^2}{2\sigma_E^{0\,2}}} dE'$$

- Assume that E, σ_E subject to normal distribution:

$\mathrm{E} = G(\mathrm{E}_0, \Delta\mathrm{E})$、$\sigma_\mathrm{E} = G(\sigma_\mathrm{E}^0, \Delta\sigma_\mathrm{E})$

- Effect of ΔE, $\Delta\sigma_E$ can be considered by

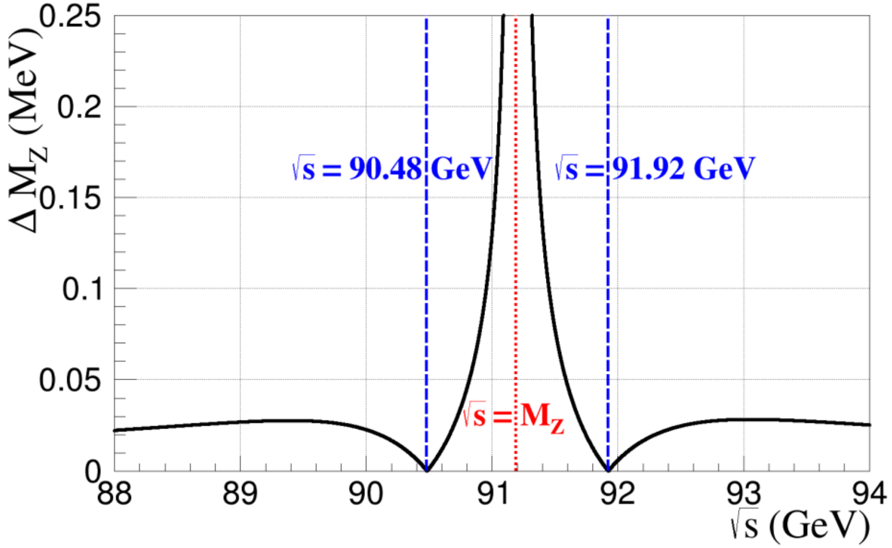
$$\sigma_{f\bar{f}}(E_0, \sigma_E^0) = \int \sigma_{f\bar{f}}(E')G(E_0, \sigma_E^0)dE'$$
$$= \int \sigma_{f\bar{f}}(E') \cdot \frac{1}{\sqrt{2\pi}\sigma_E^0} e^{\frac{-(E_0-E')^2}{2\sigma_E^{0\,2}}} dE'$$

$$\downarrow$$

$$\sigma_{f\bar{f}}(E, \sigma_E) = \int \sigma_{f\bar{f}}(E') \cdot \frac{1}{\sqrt{2\pi}\sigma_E} e^{\frac{-(E-E')^2}{2\sigma_E^2}} dE'$$

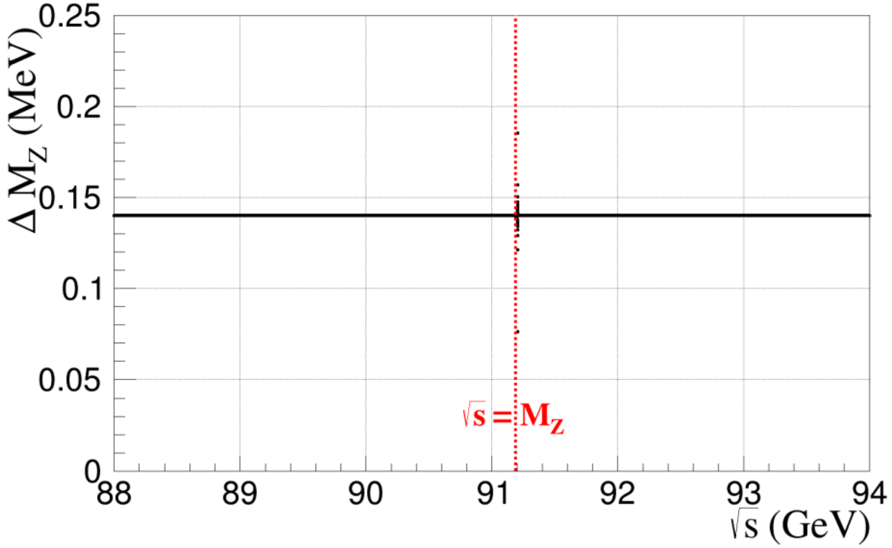
Statistical and systematic uncertainties

- **Systematic uncertainties**
- Effect of ΔE, $\Delta\sigma_E$ on ΔM_Z



$\Delta E = 0.14$ MeV

- At these two interesting energy points, ΔE have no effect on ΔM_Z



$\Delta\sigma_E = 0.57$ MeV

- For $\Delta\sigma_E$, ΔM_Z doesn't vary with $\sqrt{s}$ except for an energy point slightly above $\sqrt{s} = M_Z$

Statistical and systematic uncertainties

- **Systematic uncertainties**
- ΔL Uncertainty of integrated luminosity
- $\Delta\sigma$ caused by ΔL

$$\sigma_{\text{meas}} = \frac{N_{\text{obs}} - N_{\text{bkg}}}{\epsilon \mathcal{L}} = \frac{N_{\text{meas}}}{\epsilon \mathcal{L}}$$

$$\Delta\sigma(\Delta\mathcal{L}) = \sigma_{\text{meas}} \Delta\mathcal{L}$$

- The uncertainty of a parameter P :

$$\Delta P(\Delta\mathcal{L}) = \left|\frac{\partial P}{\partial \sigma_{\text{f}\bar{\text{f}}}}\right| \Delta\sigma(\Delta\mathcal{L}) = \left|\frac{\partial P}{\partial \sigma_{\text{f}\bar{\text{f}}}}\right| \cdot \sigma_{\text{meas}} \Delta\mathcal{L}$$

- In the simulation

$$\sigma_{\text{meas}} = G(\sigma^0_{\text{meas}}, \sigma^0_{\text{meas}} \cdot \Delta\mathcal{L})$$

Data-taking strategy

Data-taking strategy

- Global Determinant Parameter (GDP)
- For n parameters, it is defined as the determinant of the covariance matrix raised to the $\frac{1}{2n}$ power,

$$\mathrm{GDP} \equiv \sqrt[2n]{\det Cov}$$

- Here, GDP serve as an object parameter for optimization

[J. High Energ. Phys. 2017, 14 (2017).](#)

- To focus on uncertainties of M_Z Γ_Z σ^0_{had} , a transformation is performed:

$$C = V C_0 V^T \qquad V = diag(5,\ 5,\ 3,\ 1,\ 1,\ 1,\ 1)$$

Data-taking strategy

- Energy points selection :

$\sqrt{s}$ (GeV)	$\mathcal{L}$	$\sqrt{s}$ (GeV)	$\mathcal{L}$	$\sqrt{s}$ (GeV)	$\mathcal{L}$
$E_1 = 84.6$	$\mathcal{L}_1$	$E_6 = 90.4$	$\mathcal{L}_6$	$E_{10} = 93.2$	$\mathcal{L}_{10}$
$E_2 = 85.6$	$\mathcal{L}_2$	$E_7 = 91.2$	$\mathcal{L}_7$	$E_{11} = 94.3$	$\mathcal{L}_{11}$
$E_3 = 87.9$	$\mathcal{L}_3$	$E_8 = 92.0$	$\mathcal{L}_8$	$E_{12} = 95.3$	$\mathcal{L}_{12}$
$E_4 = 88.7$	$\mathcal{L}_4$	$E_9 = 92.5$	$\mathcal{L}_9$	$E_{13} = 96.2$	$\mathcal{L}_{13}$
$E_5 = 89.9$	$\mathcal{L}_5$				

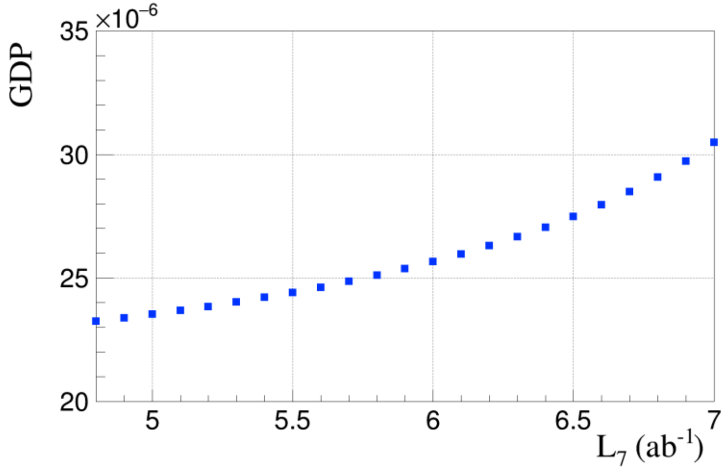
- Limitations for luminosity distribution :

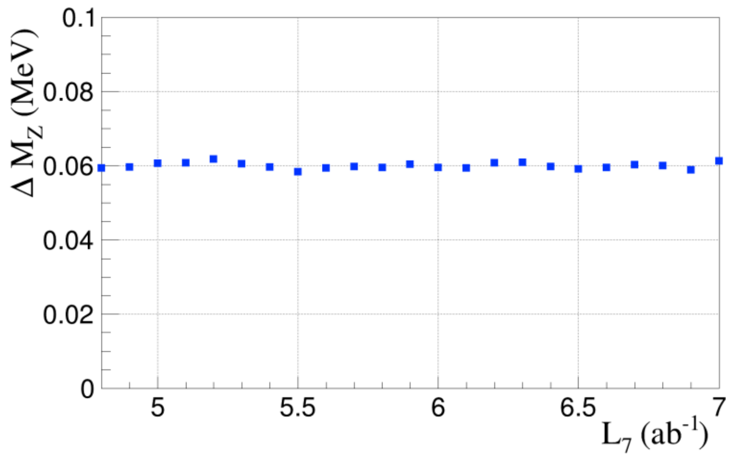
➡ Total integrated luminosity $\mathcal{L} = 8\ \mathrm{ab}^{-1}$

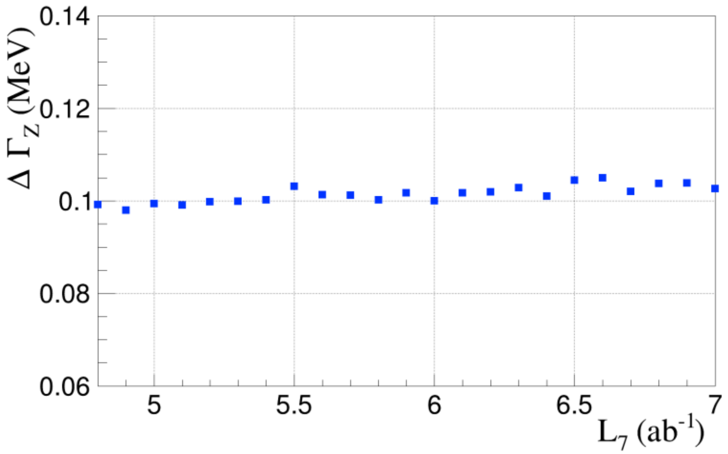
➡ At $E_7 = 91.2\ \mathrm{GeV}$ (Z pole) $\mathcal{L}_7 \geq 60\% \cdot \mathcal{L}$

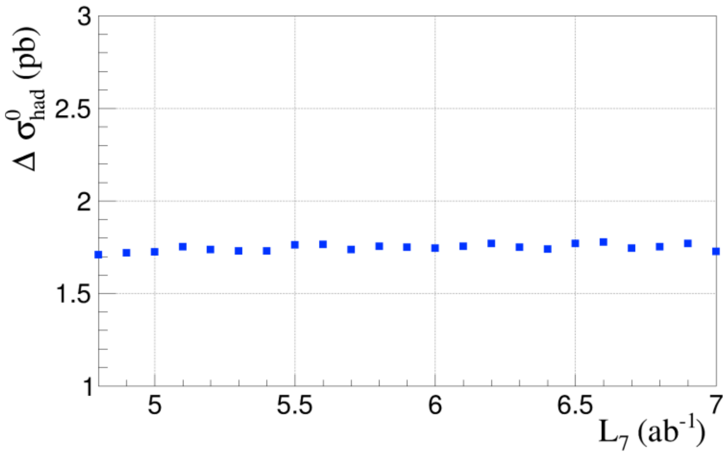
Data-taking strategy

- **Luminosity scan on E_7 (Z pole)**





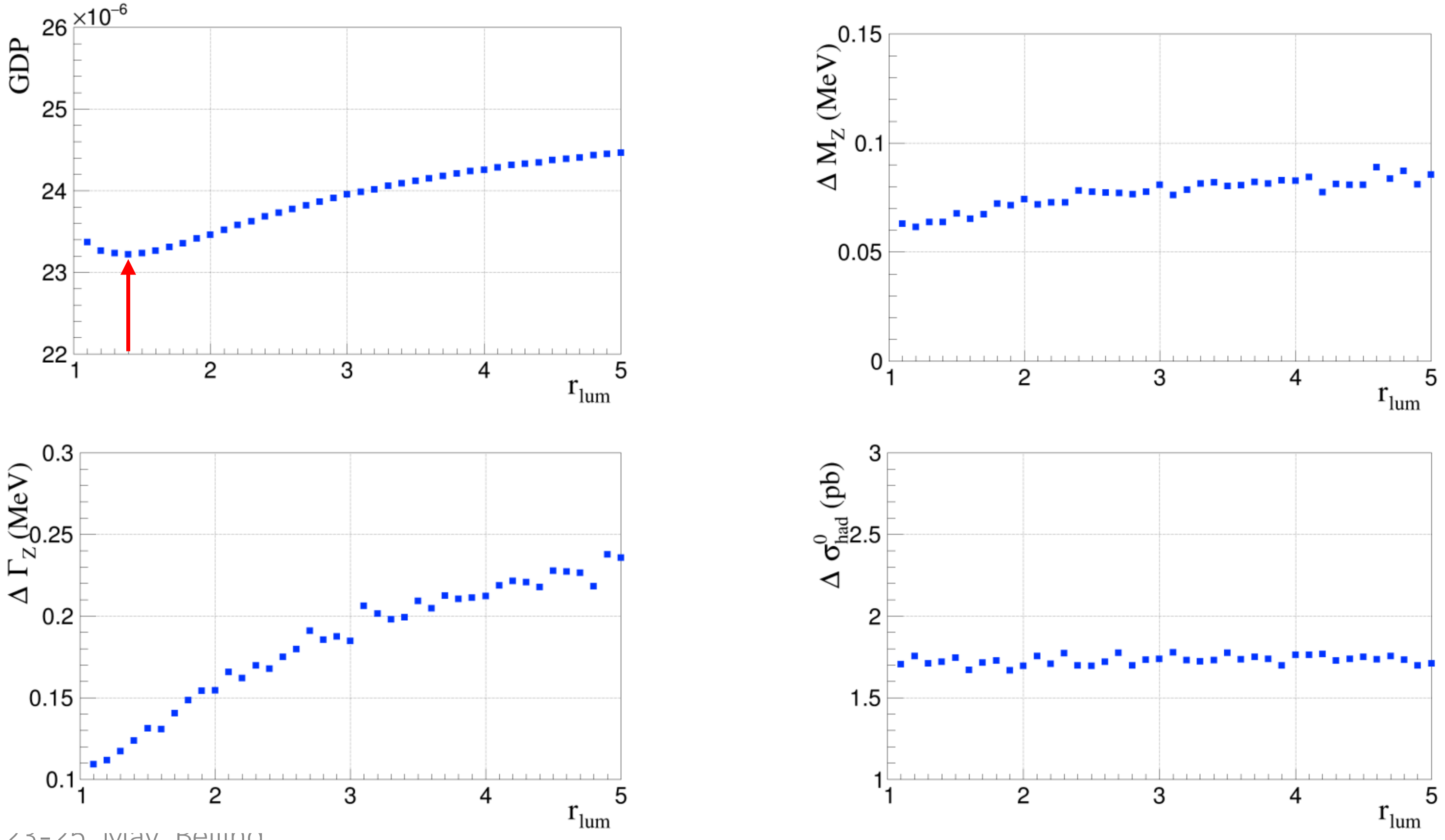




Data-taking strategy

- **Luminosity scan on off-peak energy points**
- Luminosity distribution is in the form of geometric series with common ratio $r_{\rm lum}$

$$\mathcal{L}_1 : \mathcal{L}_2 : \mathcal{L}_3 : \mathcal{L}_4 : \mathcal{L}_5 : \mathcal{L}_6 = r_{\rm lum}^0 : r_{\rm lum}^1 : r_{\rm lum}^2 : r_{\rm lum}^3 : r_{\rm lum}^4 : r_{\rm lum}^5$$



Data-taking strategy

- A preliminary data-taking scheme：

$\sqrt{s}$ (GeV)	$\mathcal{L}$ (ab^{-1})	$\sqrt{s}$ (GeV)	$\mathcal{L}$ (ab^{-1})	$\sqrt{s}$ (GeV)	$\mathcal{L}$(ab^{-1})
$E_1 = 84.6$	$\mathcal{L}_1 = 0.09$	$E_6 = 90.4$	$\mathcal{L}_6 = 0.50$	$E_{10} = 93.2$	$\mathcal{L}_{10} = 0.25$
$E_2 = 85.6$	$\mathcal{L}_2 = 0.13$	$E_7 = 91.2$	$\mathcal{L}_7 = 5.00$	$E_{11} = 94.3$	$\mathcal{L}_{11} = 0.18$
$E_3 = 87.9$	$\mathcal{L}_3 = 0.18$	$E_8 = 92.0$	$\mathcal{L}_8 = 0.50$	$E_{12} = 95.3$	$\mathcal{L}_{12} = 0.13$
$E_4 = 88.7$	$\mathcal{L}_4 = 0.25$	$E_9 = 92.5$	$\mathcal{L}_9 = 0.35$	$E_{13} = 96.2$	$\mathcal{L}_{13} = 0.09$
$E_5 = 89.9$	$\mathcal{L}_5 = 0.35$				

- Uncertainties

Parameter	δ_{stat}	δ_{total}
M_Z (KeV)	7	66
Γ_Z (KeV)	13	126
σ^0_{had} (pb)	0.09	1.73

Systematic dominant

(ISR effect not considered due to technical problems)

Updates with ZFITTER

Updates with ZFITTER

- ZFITTER is a famous Fortran program for the calculation of fermion pair production and radiative corrections at high energy e^+e^- colliders, and was intensively tested at LEP.

  Comput. Phys. Commun. **133** (2001), 229-395
  Comput. Phys. Commun. **174** (2006), 728-758

- Use ZFITTER v6.42 to calculate the cross section and perform the fit.
- All flags are same as those in the demo (subroutine ZFTEST) included in ZFITTER v6.42
- Preliminary fit result with only 2 parameters

```
COVARIANCE MATRIX CALCULATED SUCCESSFULLY
FCN=332.03 FROM MIGRAD    STATUS=CONVERGED      36 CALLS          37 TOTAL
                     EDM=6.20195e-09    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                                   STEP         FIRST
 NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
  1  mass         9.11877e+01   2.48340e-06   4.34817e-05   4.49254e+01
  2  width        2.49519e+00   1.03090e-06   1.18980e-06   2.47933e+00
EXTERNAL ERROR MATRIX.    NDIM=  25    NPAR=  2    ERR DEF=1
 6.167e-12 -2.245e-13
-2.245e-13  1.063e-12
PARAMETER  CORRELATION COEFFICIENTS
      NO.  GLOBAL      1      2
       1  0.08770   1.000 -0.088
       2  0.08770  -0.088  1.000
val[ 0] = 91187.6894,   0.002483 0.000003%
val[ 1] =  2495.1857,   0.001031 0.000041%
```

- statistical uncertainty: $\Delta M_Z = 2.5\ \text{keV}$, $\Delta\Gamma_Z = 1.1\ \text{keV}$

Summary & Outlook

Summary & Outlook

- **Summary:**
- Z line-shape scan is studied
- Statistical and some important systematic uncertainties are investigated
- A preliminary data-taking scheme is provided

- **Outlook:**
- Take ISR effect & other corrections into account (in progress using `ZFITTER`)
- A more comprehensive study on various uncertainties
- More efficient way to perform optimization (e.g. Genetic Algorithm)

STILL IN PROGRESS

Backup

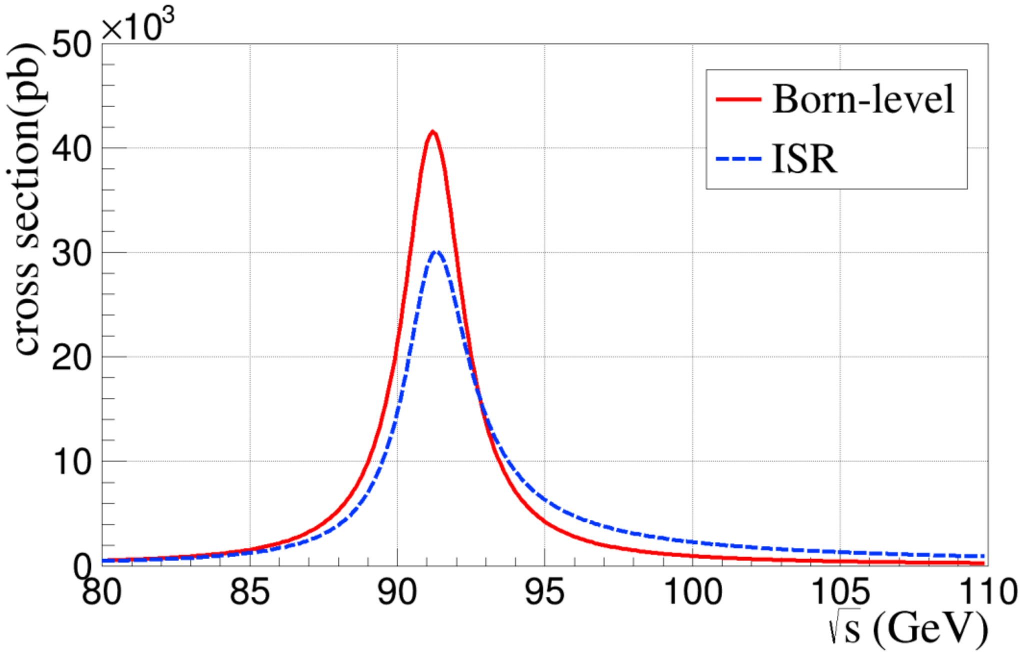
Backup

- Configurations (inputs) :

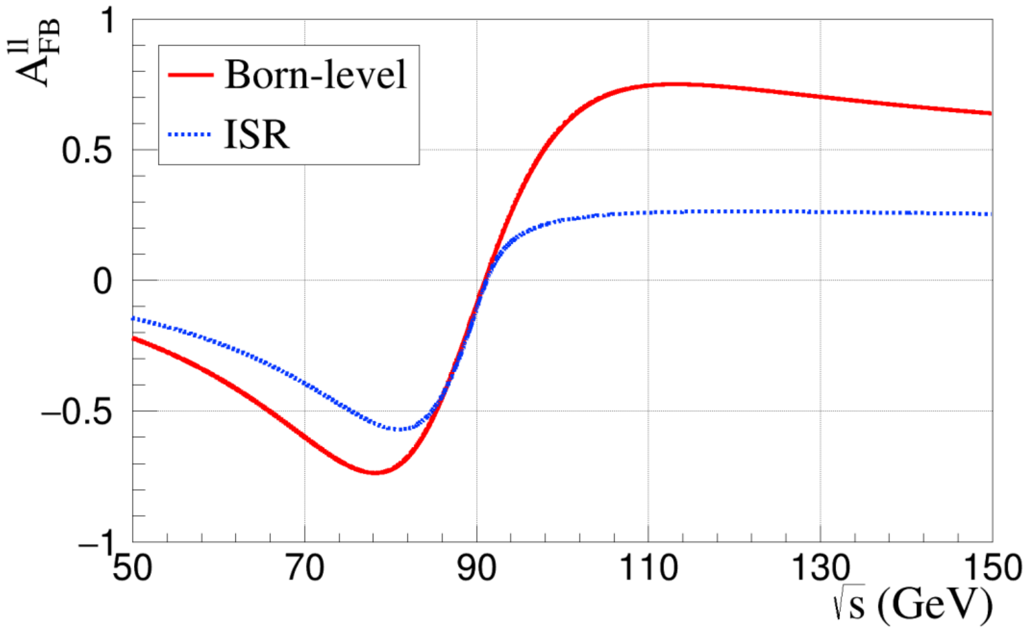
Parameter	Value
M_Z (GeV)	91.1876 ± 0.0021
Γ_Z (GeV)	2.4952 ± 0.0023
BR_{ee} (%)	(3.3632 ± 0.0042)
BR_{had} (%)	(69.911 ± 0.056)
$\sin^2 \theta_W$	0.23122(4)
$\mathcal{L}$ (ab^{-1})	8
ϵ	0.9
σ_E (%)	0.08
ΔE (MeV)	0.1
$\Delta \sigma_E$ (MeV)	0.4
$\Delta \mathcal{L}$(%)	0.05

Backup

- ISR:

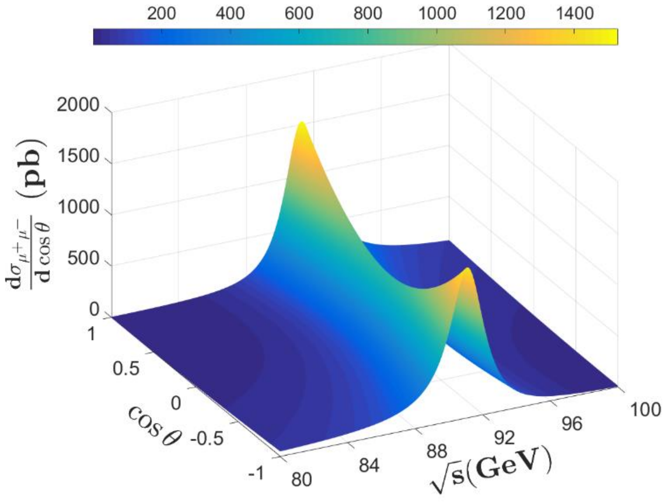


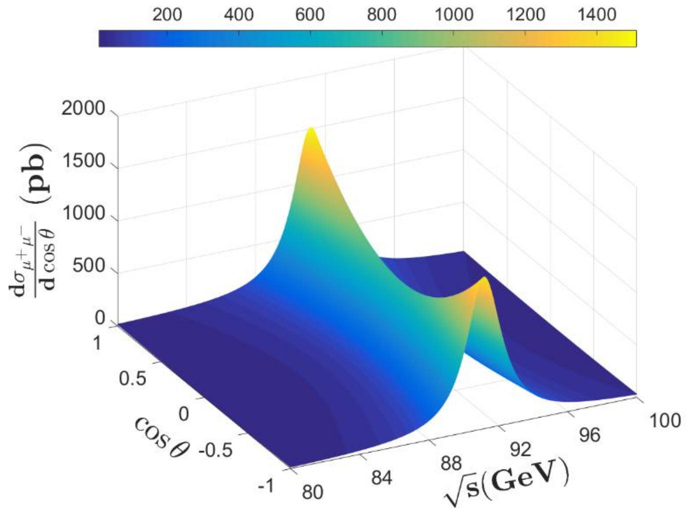
×10^3
cross section(pb)
50
40
30
20
10
0
80
85
90
95
100
105
110
√s (GeV)
Born-level
ISR

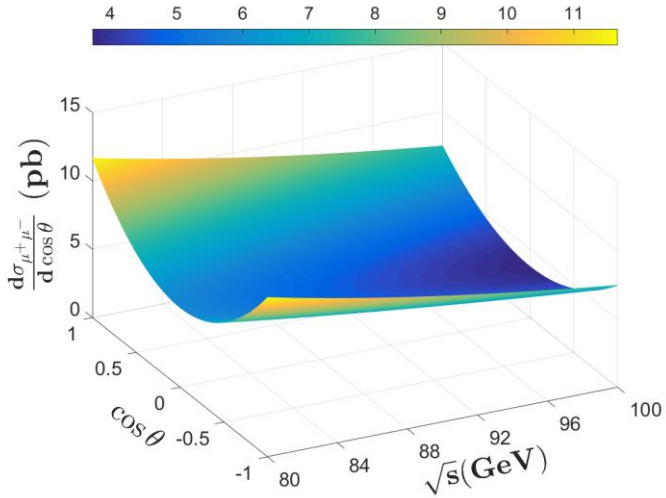


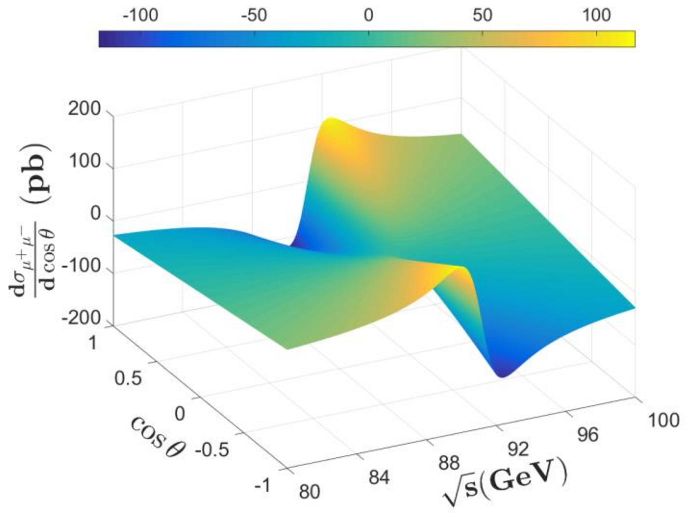
A_{FB}^{ll}
1
0.5
0
−0.5
−1
50
70
90
110
130
150
√s (GeV)
Born-level
ISR

Backup


200 400 600 800 1000 1200 1400
2000
1500
1000
500
0
$\frac{d\sigma_{\mu^+\mu^-}}{d\cos\theta}$ (pb)
1
0.5
0
-0.5
-1
$\cos\theta$
80
84
88
92
96
100
$\sqrt{s}$(GeV)

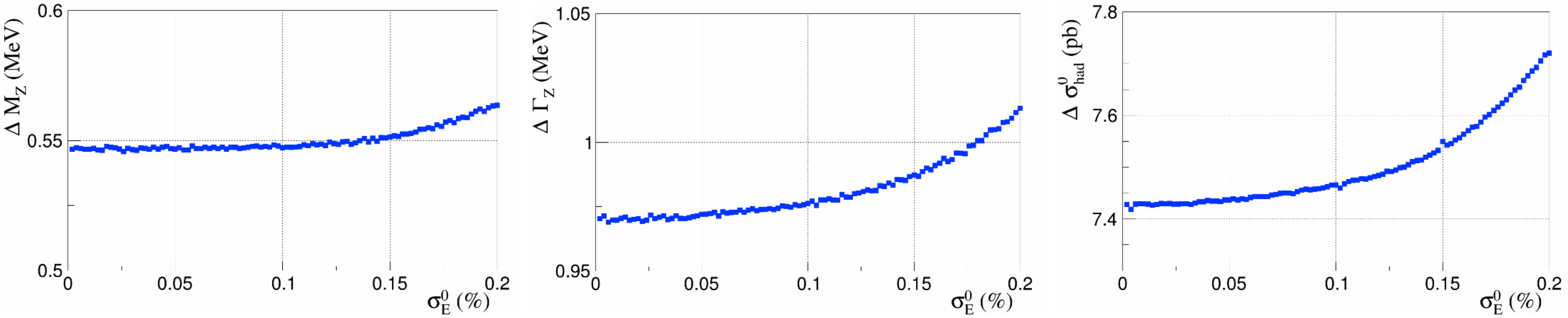

200 400 600 800 1000 1200 1400
2000
1500
1000
500
0
$\frac{d\sigma_{\mu^+\mu^-}}{d\cos\theta}$ (pb)
1
0.5
0
-0.5
-1
$\cos\theta$
80
84
88
92
96
100
$\sqrt{s}$(GeV)


4 5 6 7 8 9 10 11
15
10
5
0
$\frac{d\sigma_{\mu^+\mu^-}}{d\cos\theta}$ (pb)
1
0.5
0
-0.5
-1
$\cos\theta$
80
84
88
92
96
100
$\sqrt{s}$(GeV)


-100 -50 0 50 100
200
100
0
-100
-200
$\frac{d\sigma_{\mu^+\mu^-}}{d\cos\theta}$ (pb)
1
0.5
0
-0.5
-1
$\cos\theta$
80
84
88
92
96
100
$\sqrt{s}$(GeV)

Statistical and systematic uncertainties

- **Systematic uncertainties**
- Effect of σ_E :



Backup

- Analytical way to consider the effect of σ_E, ΔE, $\Delta\sigma_E$
- Taylor expansion of cross-section

$$\sigma_{\mathrm{f\bar{f}}}(E) = \sigma_{\mathrm{f\bar{f}}}(E_0) + \frac{d\sigma_{\mathrm{f\bar{f}}}}{dE}(E-E_0) + \frac{1}{2}\frac{d^2\sigma_{\mathrm{f\bar{f}}}}{dE^2}(E-E_0)^2 + \ldots + \frac{1}{n}\frac{d^n\sigma_{\mathrm{f\bar{f}}}}{dE^n}(E-E_0)^n + \ldots$$

- Perform the convolution

$$\sigma_{\mathrm{f\bar{f}}}(E_0,\sigma_E^0) = \int \sigma_{\mathrm{f\bar{f}}}(E')G(E_0,\sigma_E^0)dE'$$

$$\sigma_{\mathrm{f\bar{f}}}(E_0,\sigma_E^0) = \sigma_{\mathrm{f\bar{f}}}(E_0) + \frac{1}{2}\frac{d^2\sigma_{\mathrm{f\bar{f}}}}{dE^2}{\sigma_E^0}^2 + \frac{1}{8}\frac{d^4\sigma_{\mathrm{f\bar{f}}}}{dE^4}{\sigma_E^0}^4 + \ldots .$$

- Uncertainty caused by σ_{E}^{0}

$$\Delta\sigma_{\mathrm{f\bar{f}}}(E_0,\sigma_E^0) = \frac{1}{2}\frac{d^2\sigma_{\mathrm{f\bar{f}}}}{dE^2}{\sigma_E^0}^2$$

$$\Delta P(\sigma_E^0) = \left|\frac{\partial P}{\partial\sigma_{\mathrm{f\bar{f}}}}\Delta\sigma_{\mathrm{f\bar{f}}}(E_0,\sigma_E^0)\right| = \left|\frac{\partial P}{\partial\sigma_{\mathrm{f\bar{f}}}}\cdot\frac{1}{2}\frac{d^2\sigma_{\mathrm{f\bar{f}}}}{dE^2}{\sigma_E^0}^2\right|$$

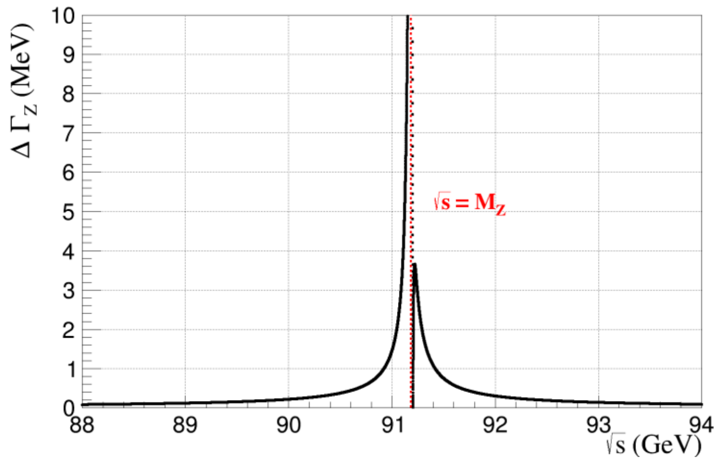
- Uncertainty caused by ΔE

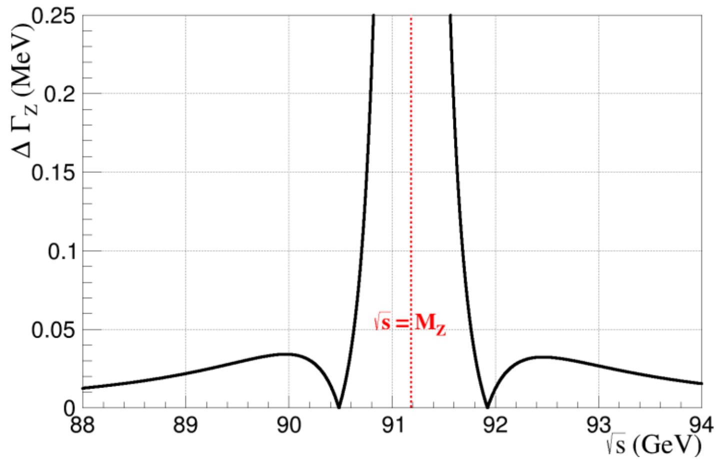
$$\Delta P(\Delta E) = \left|\frac{\partial P}{\partial\sigma_{\mathrm{f\bar{f}}}}\cdot\frac{\partial\sigma_{\mathrm{f\bar{f}}}}{\partial E}\right|\Delta E$$

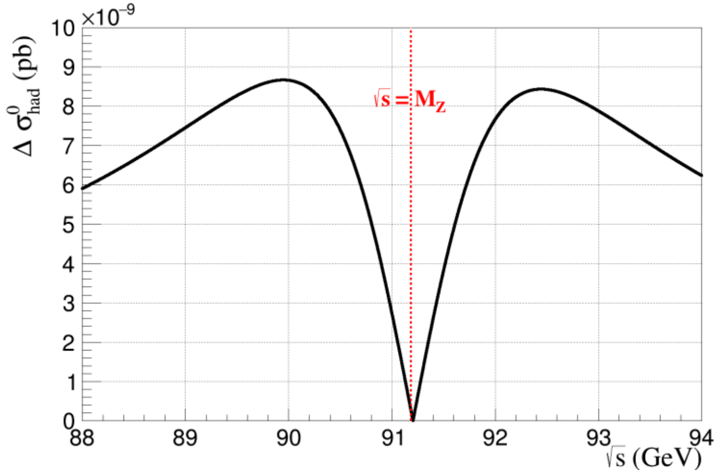
- Uncertainty caused by $\Delta\sigma_{\mathrm{E}}$

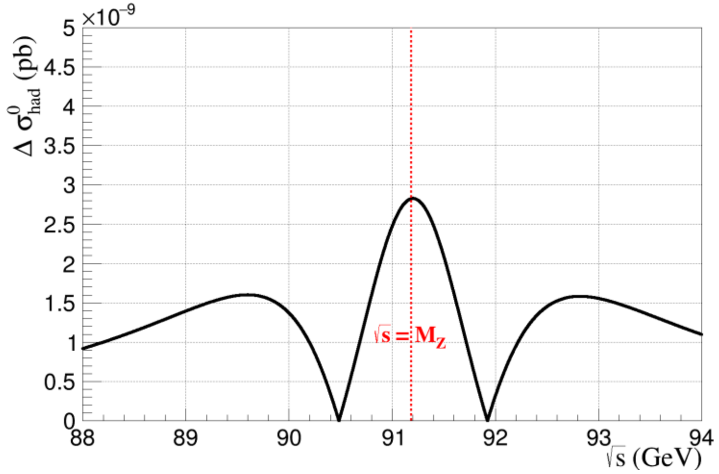
$$\Delta P(\Delta\sigma_E) = \left|\frac{\partial P}{\partial\sigma_{\mathrm{f\bar{f}}}}\cdot\frac{\partial\sigma_{\mathrm{f\bar{f}}}}{\partial\sigma_E}\right|\Delta\sigma_E = \left|\frac{\partial P}{\partial\sigma_{\mathrm{f\bar{f}}}}\cdot\frac{d^2\sigma_{\mathrm{f\bar{f}}}}{dE^2}\sigma_E^0\right|\Delta\sigma_E$$

Backup









$\Delta E = 0.14$ MeV

$\Delta \sigma_E = 0.57$ MeV

Backup

M_Z Γ_Z σ^0_{had} C^s_{ZZ} C^a_{ZZ} $C^s_{\gamma Z}$ $C^a_{\gamma Z}$

PARAMETER NO.	CORRELATION COEFFICIENTS GLOBAL	1	2	3	4	5	6	7
1	0.29059	1.000	-0.080	0.028	-0.094	0.141	0.001	-0.226
2	0.85195	-0.080	1.000	-0.349	0.846	0.025	0.151	0.005
3	0.34935	0.028	-0.349	1.000	-0.295	-0.009	-0.053	-0.002
4	0.84771	-0.094	0.846	-0.295	1.000	0.028	0.179	-0.003
5	0.17363	0.141	0.025	-0.009	0.028	1.000	-0.024	0.054
6	0.19692	0.001	0.151	-0.053	0.179	-0.024	1.000	-0.079
7	0.25487	-0.226	0.005	-0.002	-0.003	0.054	-0.079	1.000

Backup

- **Genetic Algorithm:**
- A genetic algorithm (GA) is a metaheuristic inspired by the process of natural selection that belongs to the larger class of evolutionary algorithms (EA).
- Genetic algorithms are commonly used to generate high-quality solutions to optimization and search problems by relying on biologically inspired operators such as mutation, crossover and selection.
- Debugging……

Flow Chart