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Fragmentation functions of heavy hadrons at CEPC

Ruilin Zhu
（朱瑞林）

Nanjing Normal University

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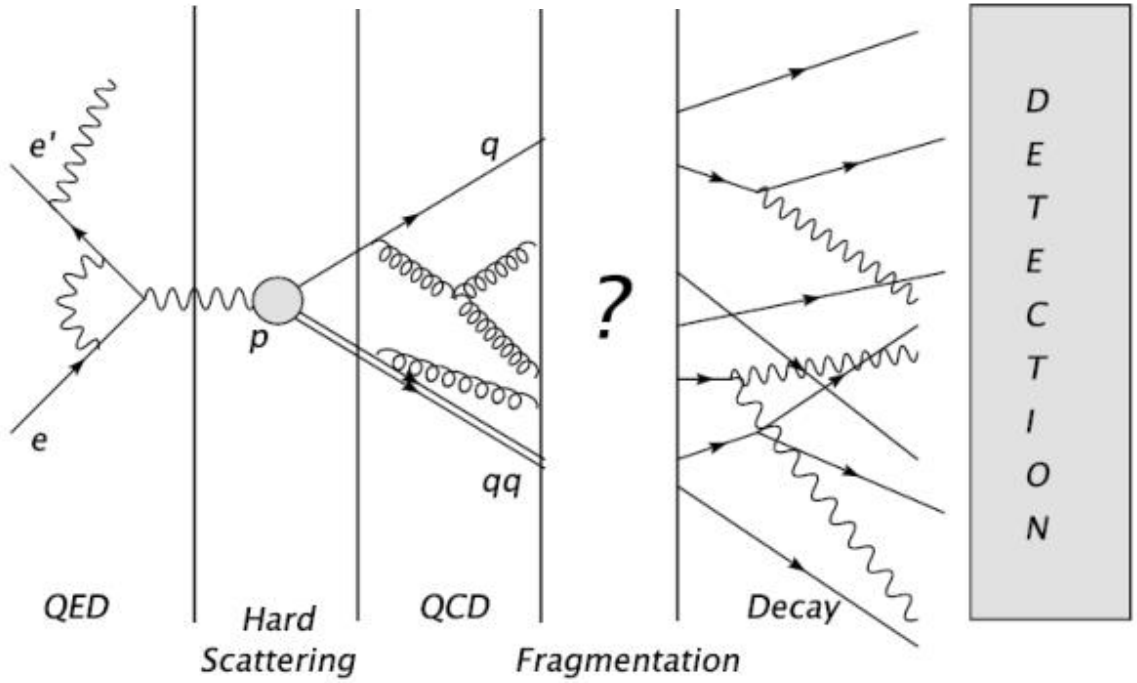
Content

- Introduction of Fragmentation functions(FFs)
- Opportunities of fragmentation functions of heavy hadrons at CEPC
- Summary

Welcome theorists and experimentalists join the studies of FFs of heavy hadrons at CEPC

Introduction

$$e^+ + e^- \rightarrow H + X$$

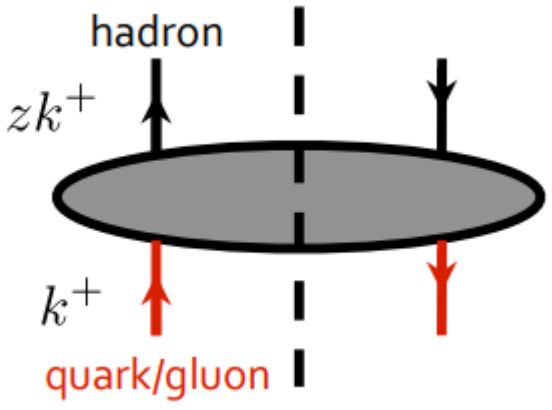


Fragmentation Functions (FFs) describe the parton hadronization process (nonperturbative but universal)

Fundamental to understand the QCD Color-Confinement

Fragmentation functions(FFs)

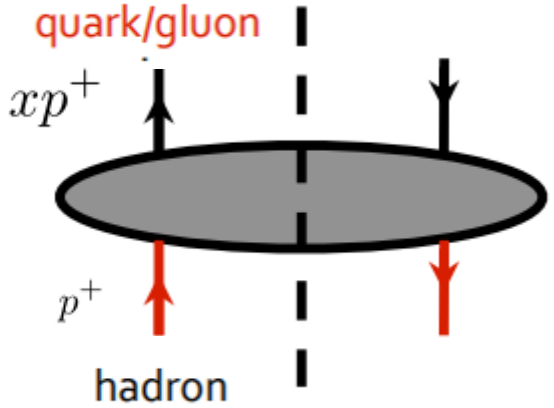
FFs $D_{H/i}(z,\mu)$



$$k=(k^+,\frac{k^2}{2k^+},\mathbf{0}_\perp)$$

$$p=(zk^+,\frac{m_H^2+\mathbf{p}_\perp^2}{2k^+},\mathbf{p}_\perp)$$

PDFs $f_{i/H}(x,\mu)$



$$p=(p^+,\frac{m_H^2}{2p^+},\mathbf{0}_\perp)$$

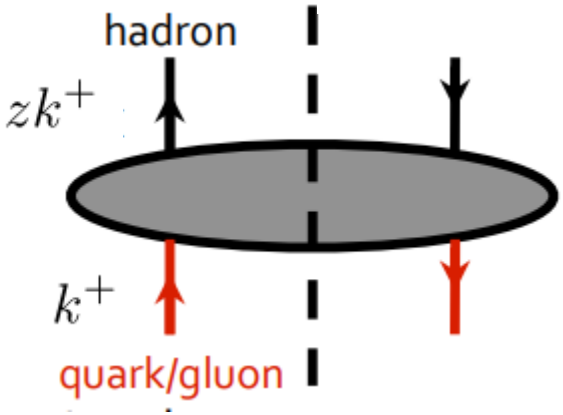
$$k=(xp^+,\frac{m_i^2+\mathbf{k}_\perp^2}{2k^+},\mathbf{k}_\perp)$$

FFs: probability densities for finding a hadron with certain momentum in the time-like parton

PDFs: probability densities for finding a parton with certain momentum in the nearly light-like hadron

Fragmentation functions(FFs)

FFs $D_{H/i}(z,\mu)$



Other partons and 3-D FFs can be defined similarly

$$D_1^{H/q}(z,\mu) = \frac{z}{4} Tr[\Delta^{H/q}(z;p_H,S_H)\,\gamma^+],$$

$$G_1^{H/q}(z,\mu) = \frac{z}{4S_L} Tr[\Delta^{H/q}(z;p_H,S_H)\,\gamma^+\gamma_5],$$

$$H_1^{H/q}(z,\mu) = \frac{z}{4S_T^j} Tr[\Delta^{H/q}(z;p_H,S_H)\,i\sigma^{j+}\gamma_5],$$

$$\Delta^{H/q}(z;p_H,S_H) = \sum_X \int \frac{d\xi^-}{2\pi} e^{ik^+\xi^-} \langle 0 | \mathcal{W}(\infty^-,\xi^-)\,\psi_q(\xi^-) | p_H,S_H;X\rangle$$

$$\times \langle p_H,S_h;X | \bar{\psi}_q(0)\,\mathcal{W}(0,\infty^-) | 0\rangle \ ,$$

$$\mathcal{W}(x^-,y^-) = \mathcal{P}\exp\left[-ig\int_{x^-}^{y^-} ds_\mu A^\mu(s^-)\right].$$

$$\sum_H \int_0^1 dz z D_i^H(z,\mu) = 1 \ .$$

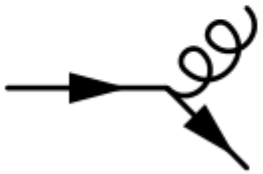
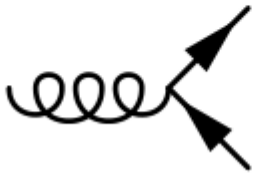
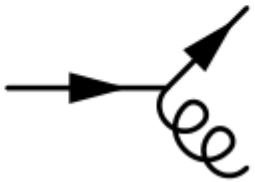
Berman et. al, PRD4,11(1971);
Field-Feynman,PRD15,9(1977);
Collins-Soper,NPB193,381(1981).

Evolution Equation (DGLAP)

$$\frac{\partial}{\partial \ln Q} D_{H/i}\left(z, Q^2\right) = \sum_j \int_z^1 \frac{dx}{x} \frac{\alpha_S}{2\pi} P_{ji}^t\left(x, \alpha_S\right) D_{H/j}\left(\frac{z}{x}, Q^2\right)$$

$$\frac{\partial}{\partial \ln Q} f_{i/H}\left(x, Q^2\right) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_S}{2\pi} P_{ji}^s\left(z, \alpha_S\right) f_{j/H}\left(\frac{x}{z}, Q^2\right)$$

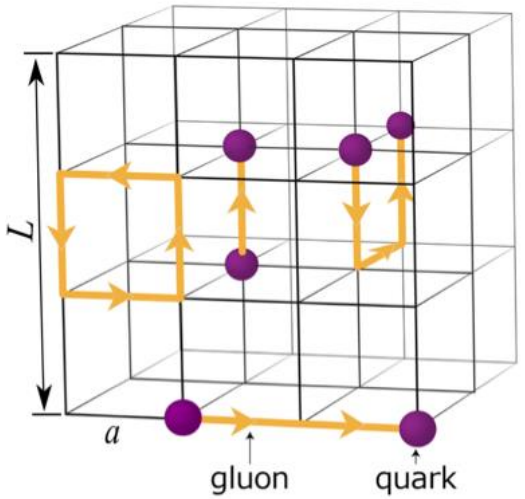
$$P_{ji}(z, \alpha_s) = \sum_{k=0} a_s^{k+1} P_{ji}^{(k)}(z), \qquad a_s = \alpha_s/(4\pi)$$

$P_{qq}^{(0)}$  $P_{gq}^{(0)}$  $P_{qg}^{(0)}$  $P_{gg}^{(0)}$ 

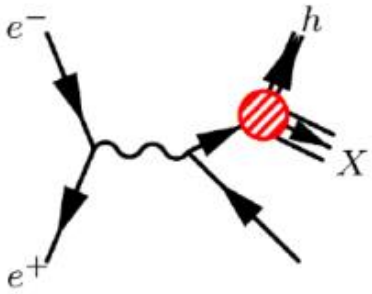
At NLO, the splitting functions are different in DGLAP equation for FFs and PDFs

Difficulties for FFs

- Nonperturbative
- Time-like correlators
- Need to sum unobserved states X
- Can not be calculated in discretized Euclidean space-time
- Attempts: Model-dependent(mostly) or rely on experimental data

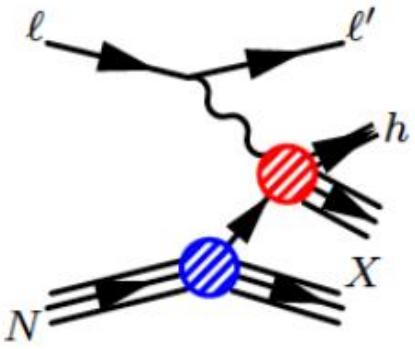


QCD Factorization



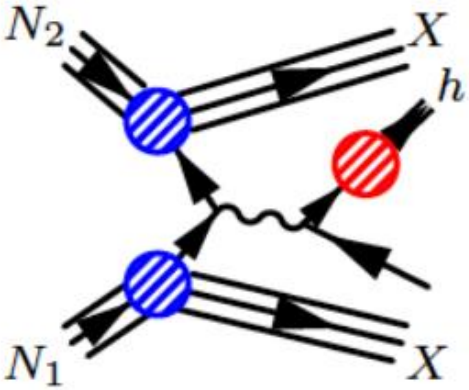
$e^+ + e^- \to h + X$

single-inclusive annihilation (SIA)



$\ell + N \to \ell' + h + X$

semi-inclusive deep-inelastic scattering (SIDIS)



$N_1 + N_2 \to h + X$

high-p_T hadron production in proton-proton collisions (PP)

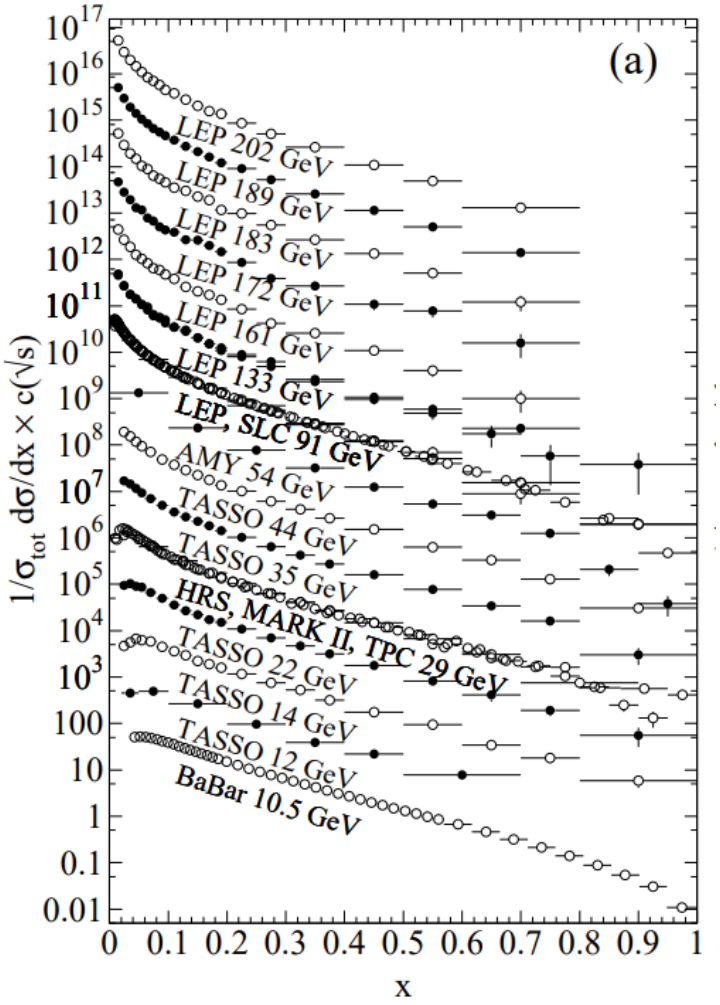
$$\sigma^{e^+e^-\to hX} = \hat{\sigma} \otimes FF\,,$$

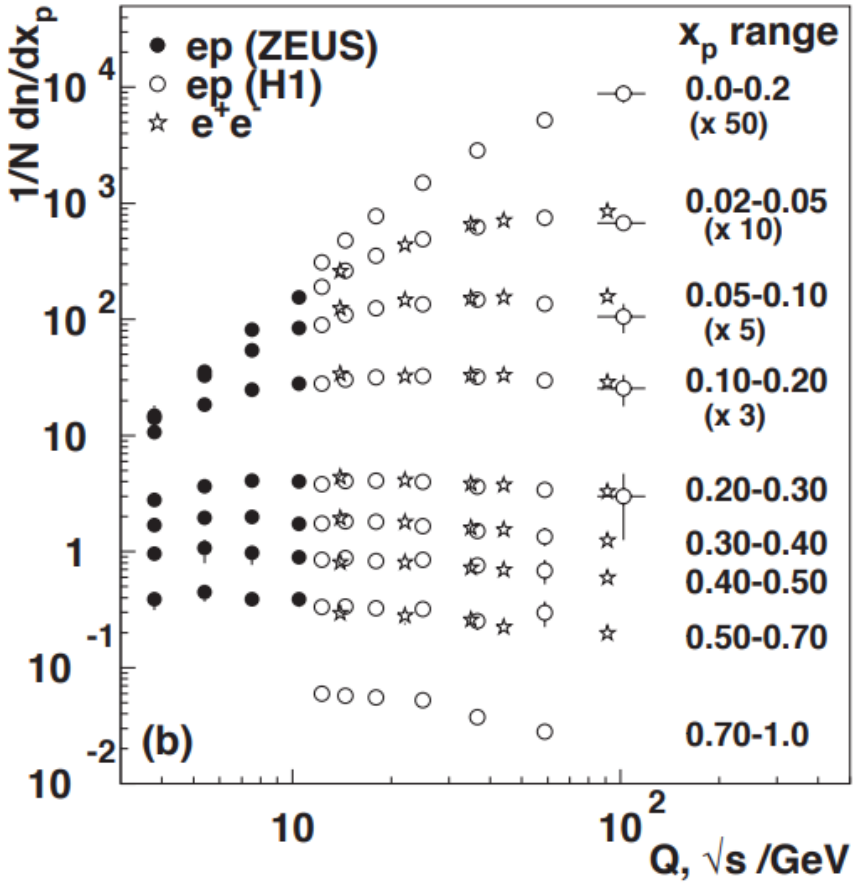
$$\sigma^{\ell N\to \ell hX} = \hat{\sigma} \otimes PDF \otimes FF\,,$$

$$\sigma^{pp\to hX} = \hat{\sigma} \otimes PDF \otimes PDF \otimes FF$$

Take from H. Khanpour

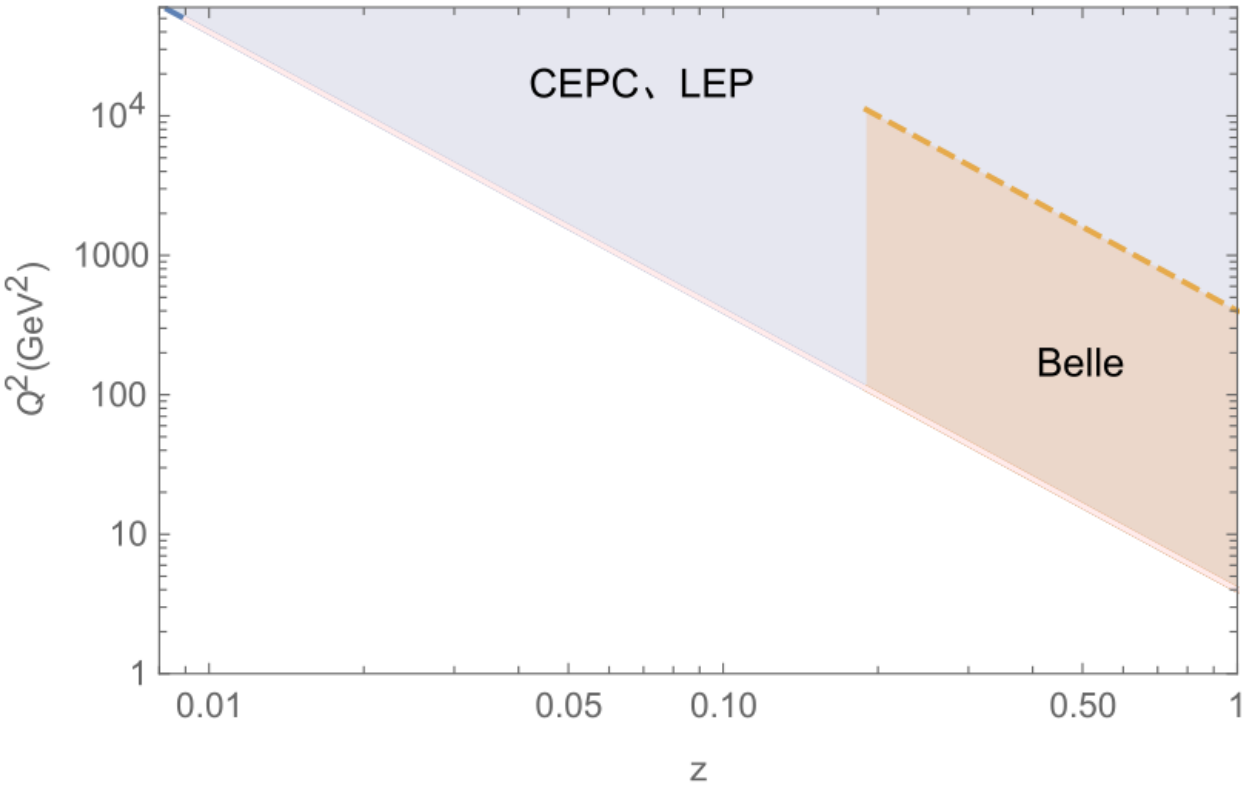
FFs for light hadrons





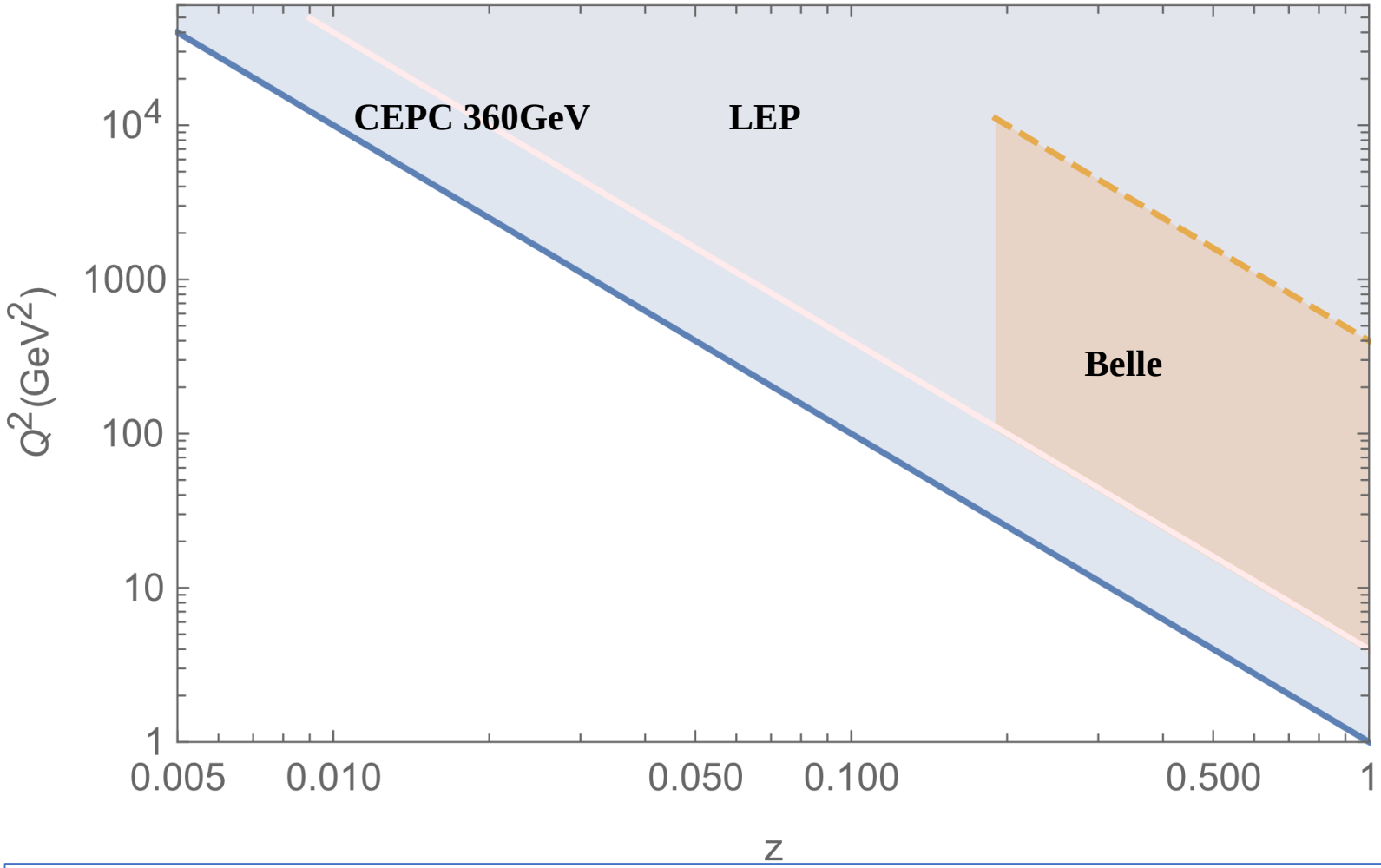
For charged hadrons, PDG 2020

FFs for light hadrons



CEPC has less advantage for studies on FFs of light hadrons, but can double-check the previous measurements at LEP,Belle, and eP colliders

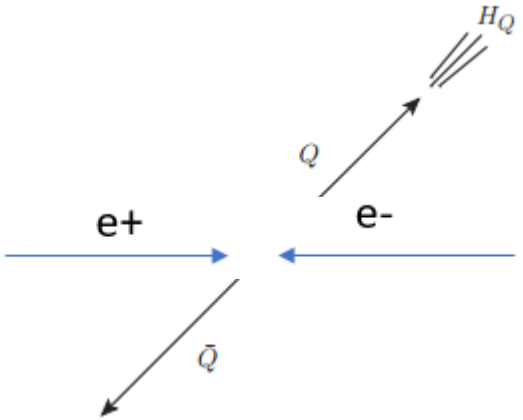
FFs for light hadrons



Run at 360GeV, CEPC covers more region than other ee collisions

Opportunities of FFs of Heavy Hadrons at CEPC

$$e^+ + e^- \to H + X$$



$$d\sigma(e^+ + e^- \to H + X) = \sum_i \int_0^1 dz D_i^H(z,\mu) d\hat{\sigma}_{e^+ + e^- \to i + X}(z,s,\mu) \ ,$$

$$\frac{d\sigma(e^+ + e^- \to H(p) + X)}{dz} = \sum_i D_i^H(z,\mu) d\hat{\sigma}_{e^+ + e^- \to i(k_i) + X}(z,s,\mu) \ ,$$

$$z = \frac{p_H^+}{k_i^+} = \frac{E_H + P_H^Z}{E_i + k_i^Z}$$

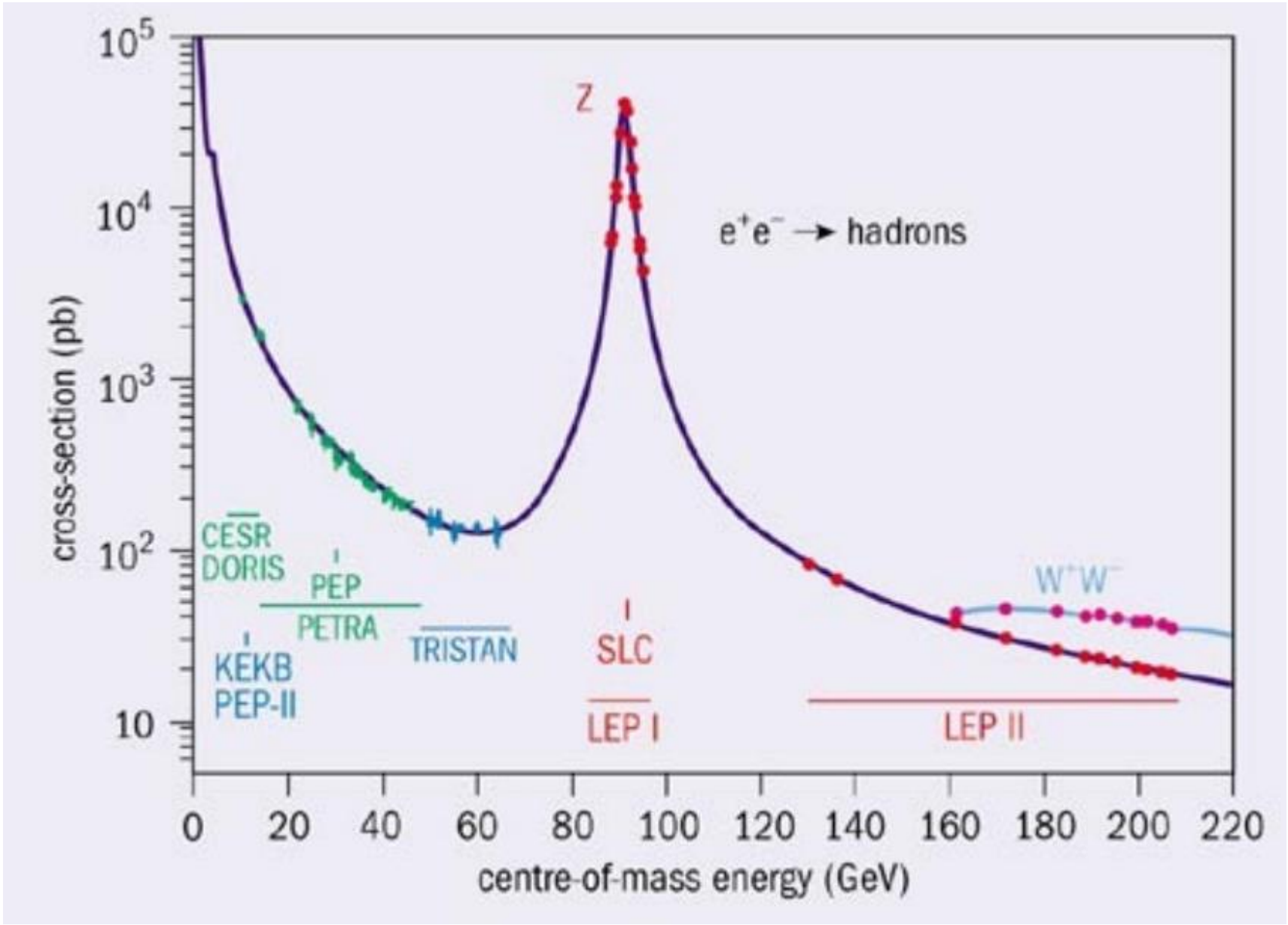
$$k = (k^+, \frac{k^2}{2k^+}, \mathbf{0}_\perp)$$

$$p = (zk^+, \frac{m_H^2 + \mathbf{p}_\perp^2}{2k^+}, \mathbf{p}_\perp)$$

Opportunities of FFs of Heavy Hadrons at CEPC

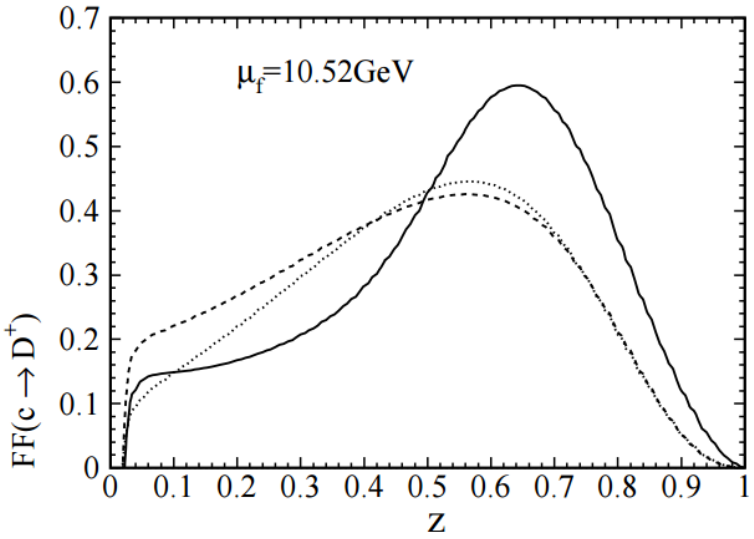
Advantage for the studies on FFs of Heavy hadrons at CEPC:

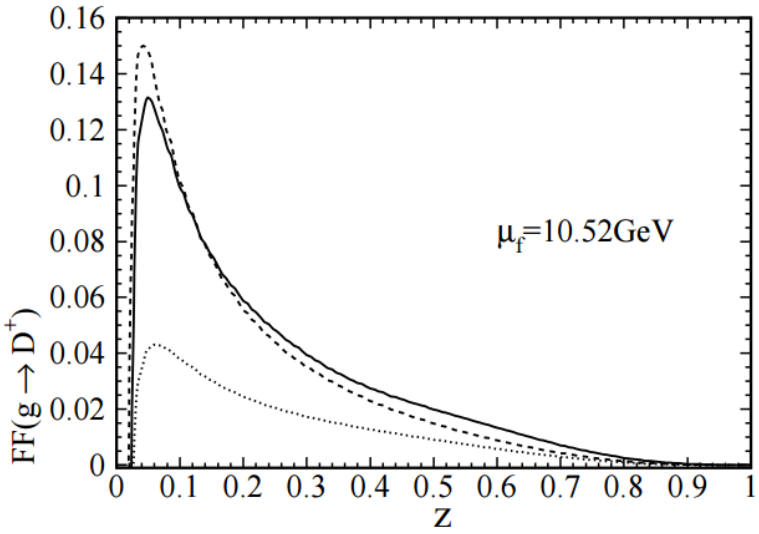
- Direct measurements of FFs of heavy-light, double-heavy hadrons with clear background
- Test the Fragmentation mechanisms、Color-Confining
- Test the QCD factorization formulae (Refactorization in doubly heavy hadrons)
- Disvovery potentials for doubly/triply heavy hadrons (or exotic hadrons)



Take from F. Sefkow

FFs for Heavy-Light hadrons





T. Kneesch et. al, NPB799,34(2008)

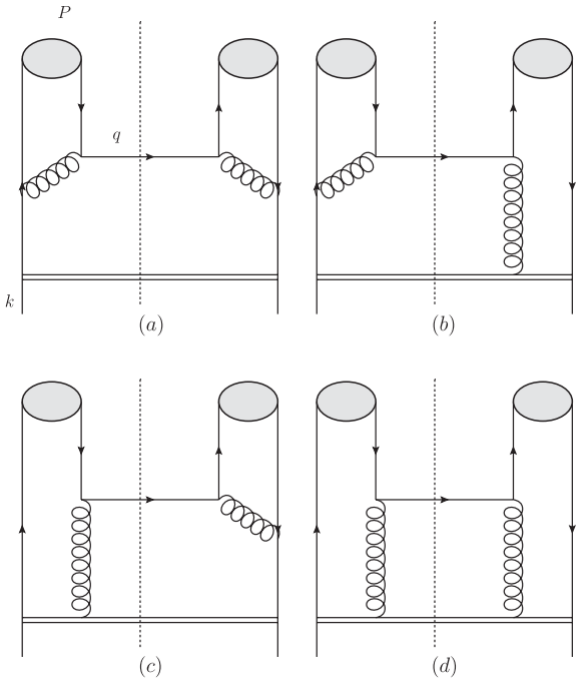
Fragmentation model:

$$D_{i\to D}\left(z,\,\mu_0\right) = N_i\, z^{-\left(1+\gamma_i^2\right)}\,(1-z)^a \exp\left(-\gamma_i^2/z\right),$$

~1 nb for D meson production at CEPC （91 GeV）

~10^9 D meson events for 1ab^(-1) data

FFs for Doubly heavy mesons

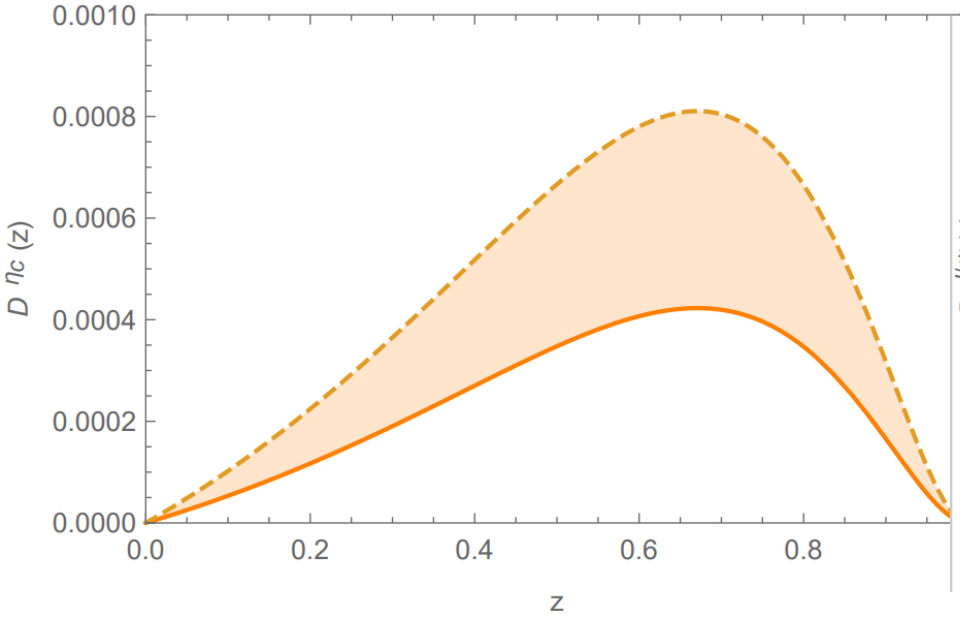


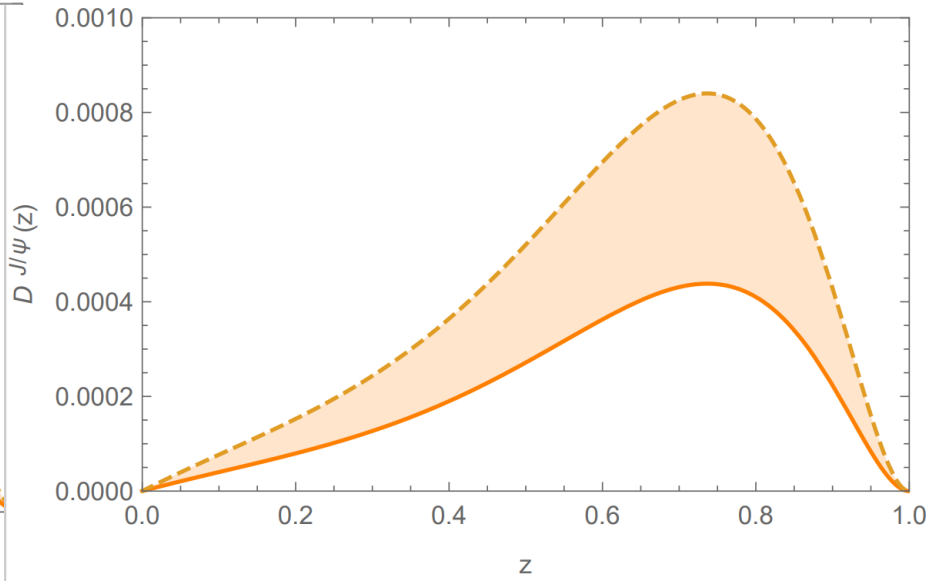
$$D_{b\to B_c^{(*)}}(z;\mu) = \frac{z^{d-3}}{4N_c}\sum_X \int_{-\infty}^{\infty} \frac{\mathrm{d}x^-}{2\pi} \mathrm{e}^{-\mathrm{i}p^+x^-/z}$$
$$\times \mathrm{Tr}\Big[\not{n}\langle 0|W(0)b(0)|B_c^{(*)}(p),X\rangle$$
$$\times\langle B_c^{(*)}(p),X|\bar{b}(x^-n)W^\dagger(x^-)|0\rangle\Big]$$
$$= \frac{z^{d-3}}{4N_c}\sum_X \delta(p^+/z - p^+ - p_X^+)$$
$$\times \mathrm{Tr}\Big[\not{n}\langle 0|W(0)b(0)|B_c^{(*)}(p),X\rangle$$
$$\times\langle B_c^{(*)}(p),X|\bar{b}(0)W^\dagger(0)|0\rangle\Big],$$

FFs can be perturbatively calculated in NRQCD

Previous calculations: Falk-Luke-Savage-Wise, PLB 312,486(1993) ;Braaten-Cheung - Yuan, PRD48,4230(1993);Chen, RPD48,11(1993);Ma,PLB332,398(1994)
For double parton FFs: Ma-Qiu-Zhang,PRD89,094029(2014);
For relativistic corrections: Sang-Chen, PRD80,014013(2009);
For gluon fragmentation: Zheng-Chang-Wu,2112.10520; Feng-Jia-Yang, 2112.15569;
For P wave states and heavy diquarks: Tao-Cao-Xiao-Zhu, to be prepared.

FFs for Doubly heavy mesons



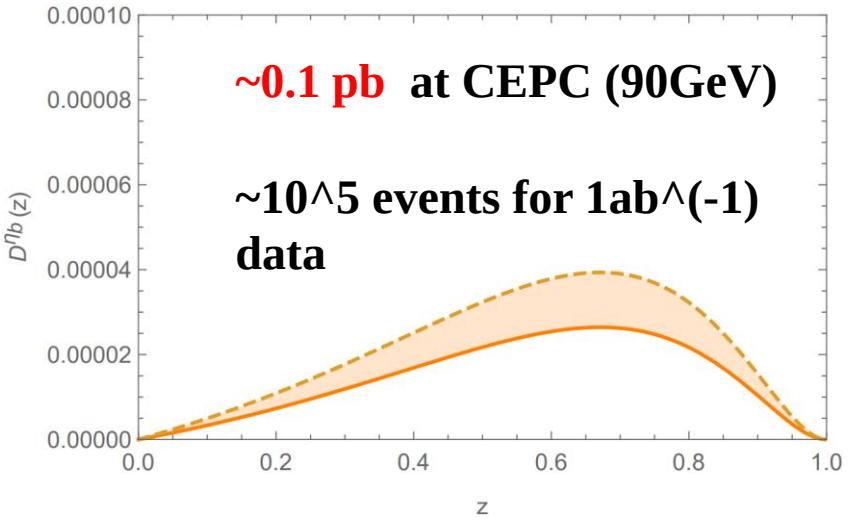


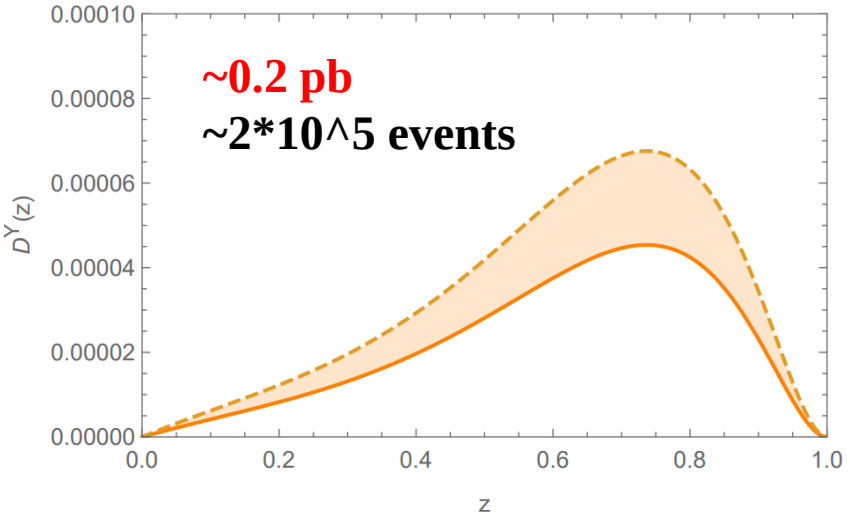
LO Results, mu~ from mQ to 2mQ;
Using them, we can calculate the cross section

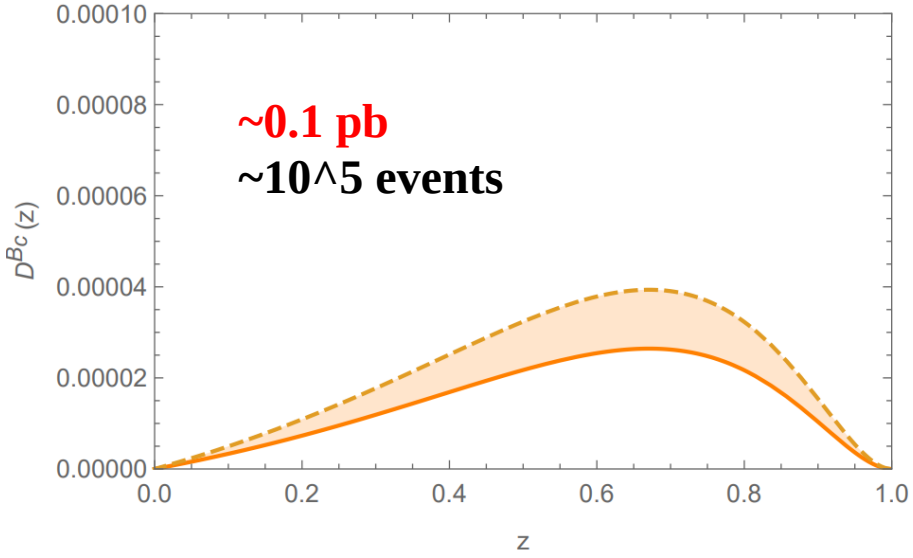
~5 pb for eta_c or J/psi production at CEPC (90GeV)

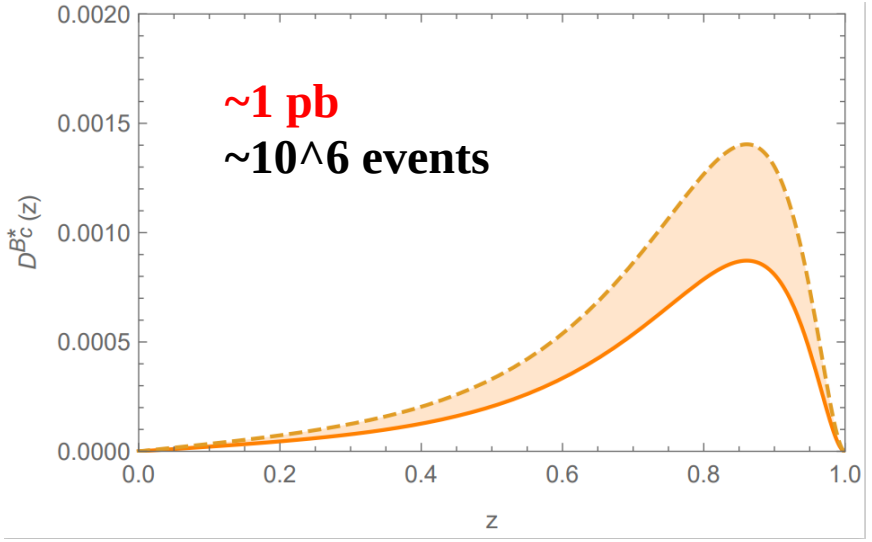
~5*10^6 eta_c or J/psi events for 1ab^(-1) data

FFs for Doubly heavy mesons

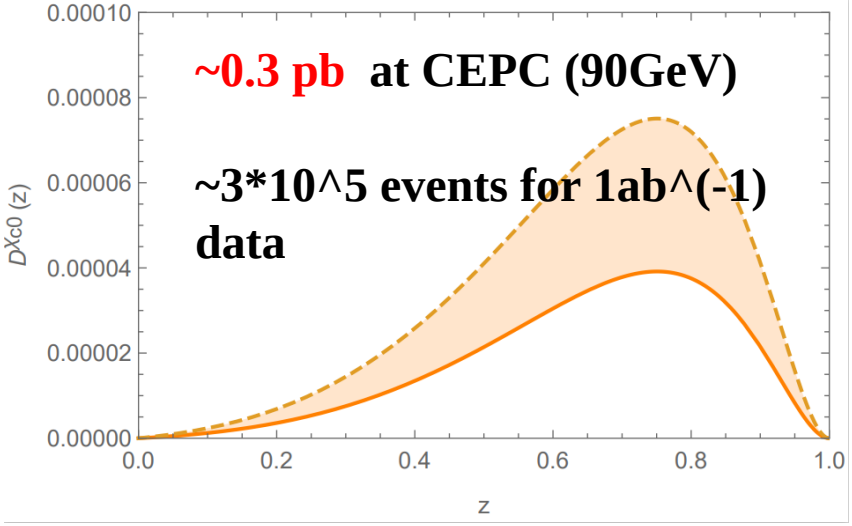


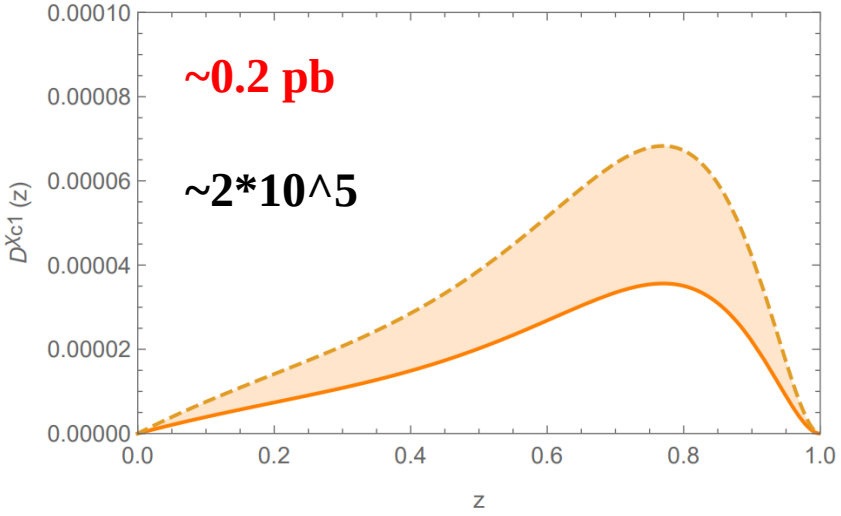


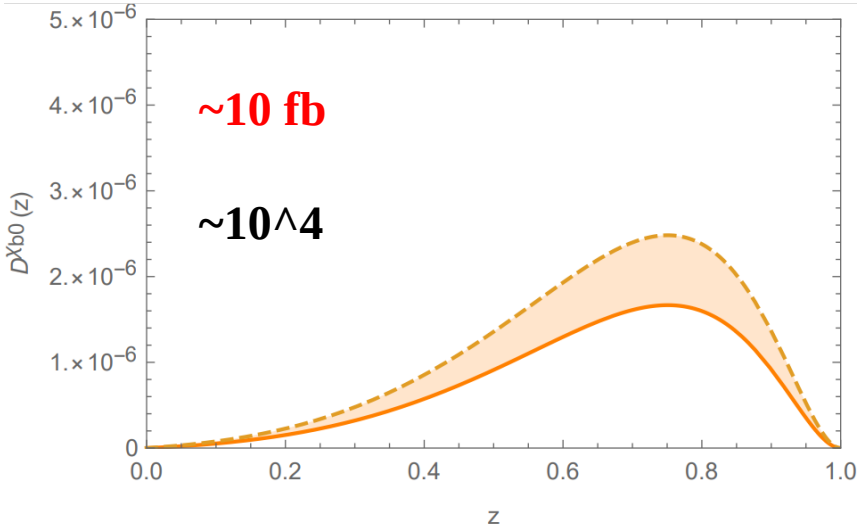


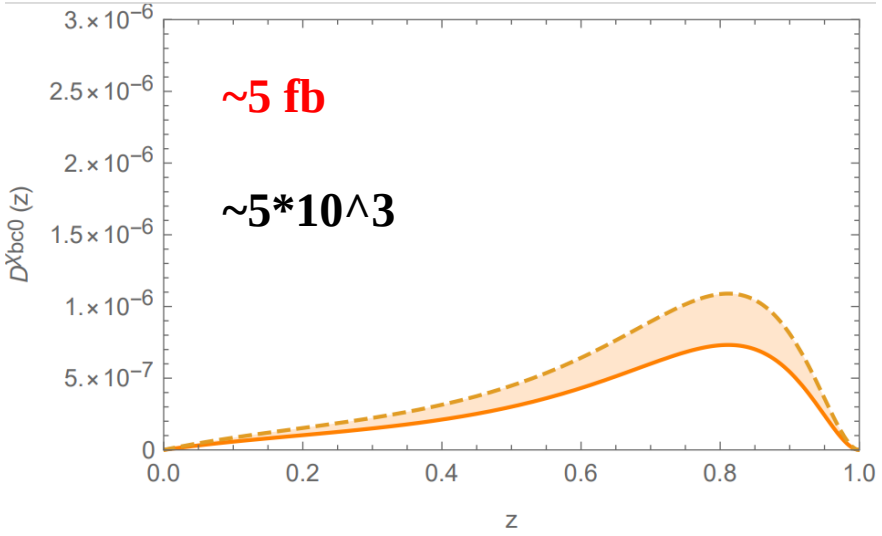


FFs for Doubly heavy mesons





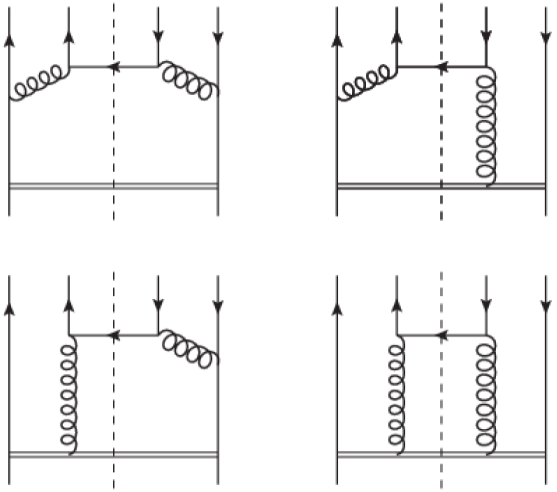




FFs for Doubly heavy baryons

$$d\sigma(e^+ + e^- \to \Xi_{cc} + X) \propto \int_0^1 dz D_{cc}^{\Xi_{cc}}(z,\mu) d\hat{\sigma}_{e^+ + e^- \to [cc] + X}(z,s,\mu) \ ,$$

$$d\sigma(e^+ + e^- \to [cc] + X) = \sum_i \int_0^1 dz D_i^{[cc]}(z,\mu) d\hat{\sigma}_{e^+ + e^- \to i + X}(z,s,\mu) \ ,$$



Fragmentation processes:

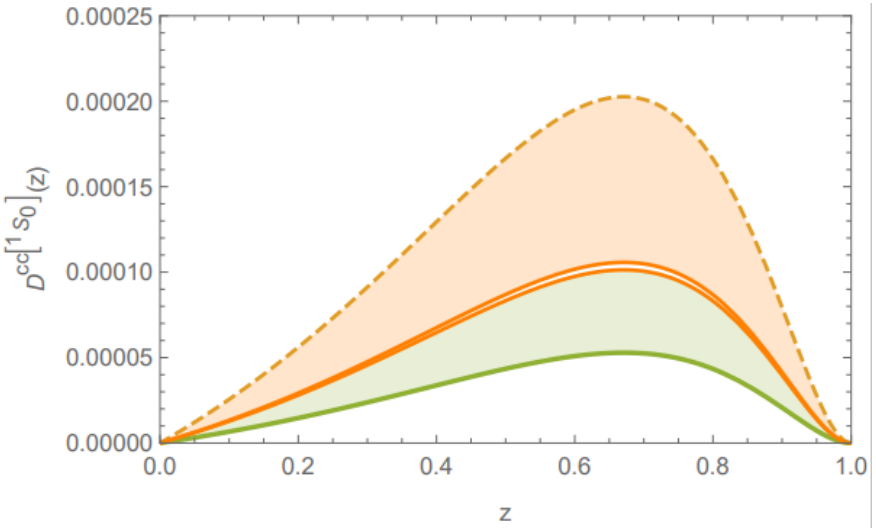
(1)a heavy quark fragmented into a diquark with certain probability;

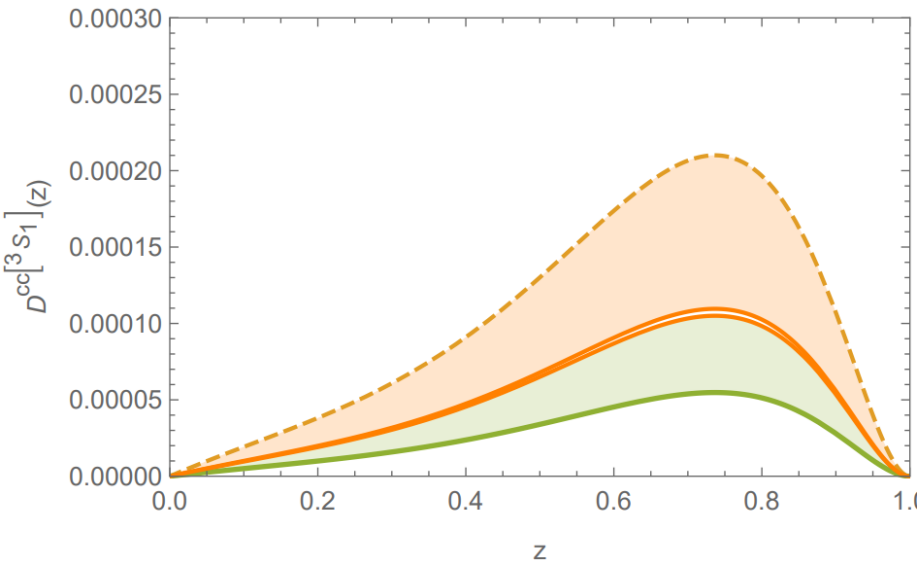
$$D_c^{[cc][\bar{3}]}(z,\mu)|_{LO} = \frac{64\psi_{[cc]}^2(0)(z-1)^2 z\left(5z^4 - 32z^3 + 72z^2 - 32z + 16\right)\alpha_s^2(\mu)}{27{m_{[cc]}}^3(z-2)^6}$$

(2)a heavy diquark fragmented into doubly heavy baryon with certain probability; (Similar to b->B)

$$D_{[cc]\to\Xi_{cc}}(z,\mu_0) = N_i z^{-\left(1+\gamma_i^2\right)}(1-z)^a \exp\left(-\gamma_i^2/z\right)$$

Tao-Cao-Xiao-Zhu, to be prepared.





~1 pb for diquark [cc] production at CEPC （90GeV）

~10^6 diquark [cc] events for 1ab^(-1) data

then

~0.05-0.3 pb for Xi_cc and T_cc production at CEPC （90GeV）

~10^4-10^5 Xi_cc and T_cc events for 1ab^(-1) data

Summary

Fragmentation Functions of heavy hadrons:
study them at CEPC (91GeV);
10^8-10^9 events for heavy-light hadron;
10^5-10^7 events for doubly heavy hadrons.

At CEPC (91GeV), we expect:

- ✓ Measure the FFs for heavy hadrons directly
- ✓ Test the fragmentation mechanism
- ✓ Discovery potentials for unobserved doubly/triply hadron

Thank you very much!