Axion-like particle solution to muon g-2 and its test at Z-factory

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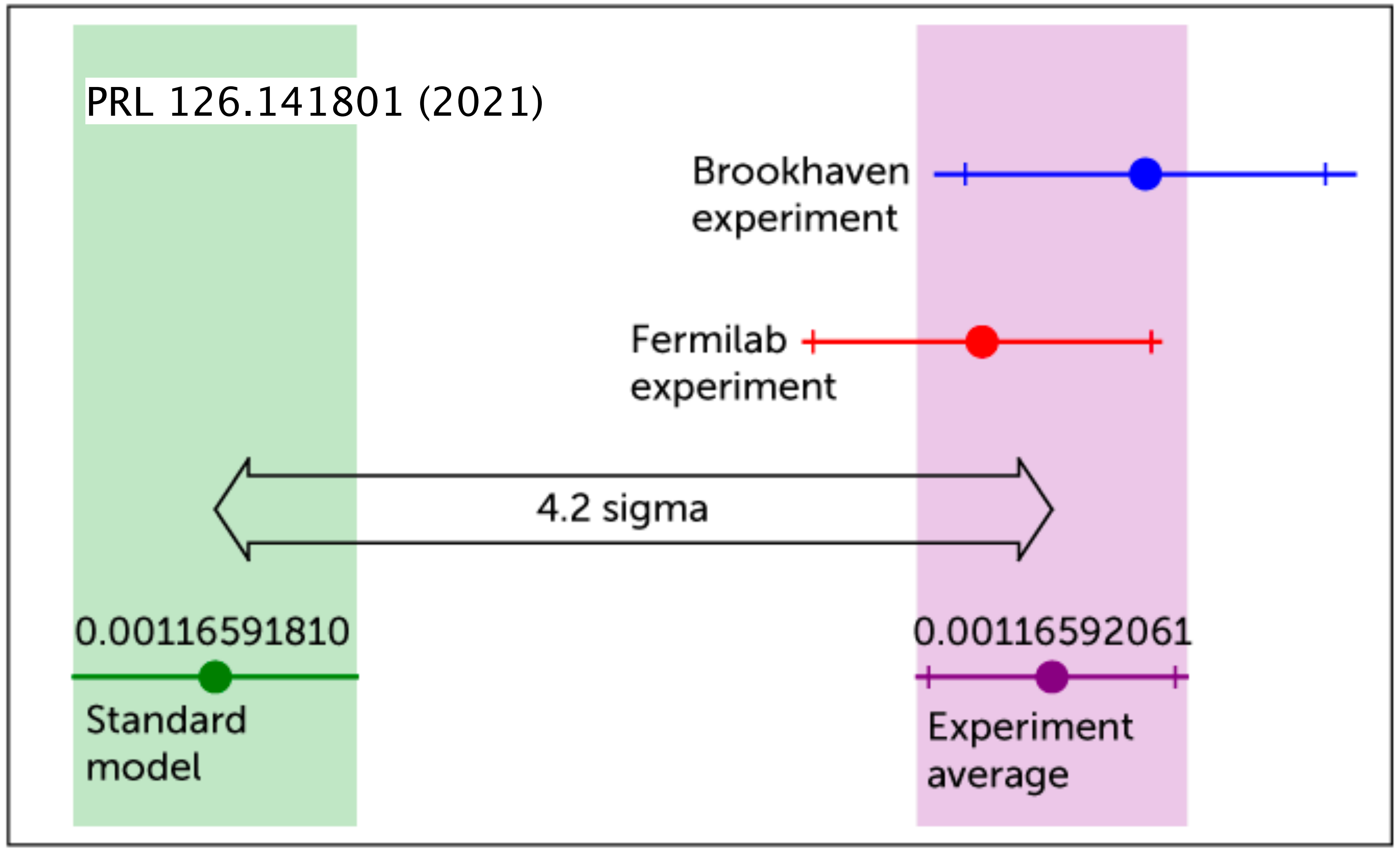
Preliminary results

Joint Workshop of the CEPC Physics, Software and New Detector Concept
2022-05-24

The Muon g-2 anomaly

- Positive value and a 4.2 σ (Fermilab + Brookhaven)

$$\Delta a_\mu = a_\mu^{\exp} - a_\mu^{\text{th}} = (25.1 \pm 5.9) \times 10^{-10}$$



Muon magnetic anomaly

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NEWS FEATURE

The Era of Anomalies

May 14, 2020 • *Physics* 13, 79

Particle physicists are faced with a growing list of “anomalies”—experimental results that conflict with the standard model but fail to overturn it for lack of sufficient evidence.

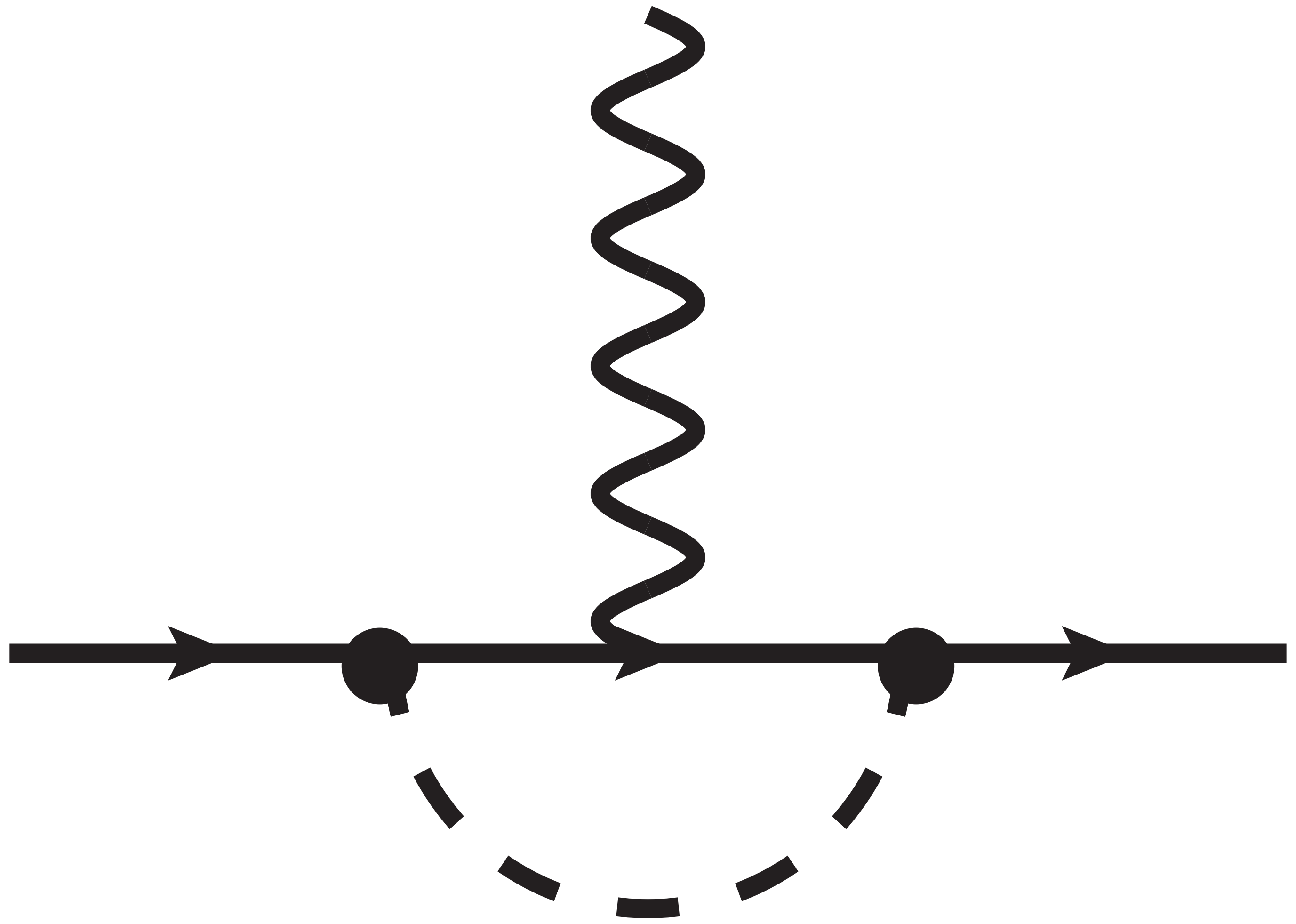
The (pseudo)scalar answer to g-2

- The (pseudo)scalar Yukawa coupling to lepton

$$\mathscr{L}_{\text{yuk}} = \phi \bar{\ell} \left(g_R + i g_I \gamma_5 \right) \ell$$

- The 1-loop contribution to g-2

$$\Delta a_\ell = \frac{1}{8\pi^2} \int_0^1 dx \frac{(1-x)^2 \left((1+x) g_R^2 - (1-x) g_I^2 \right)}{(1-x)^2 + x \left(m_\phi / m_\ell \right)^2}$$



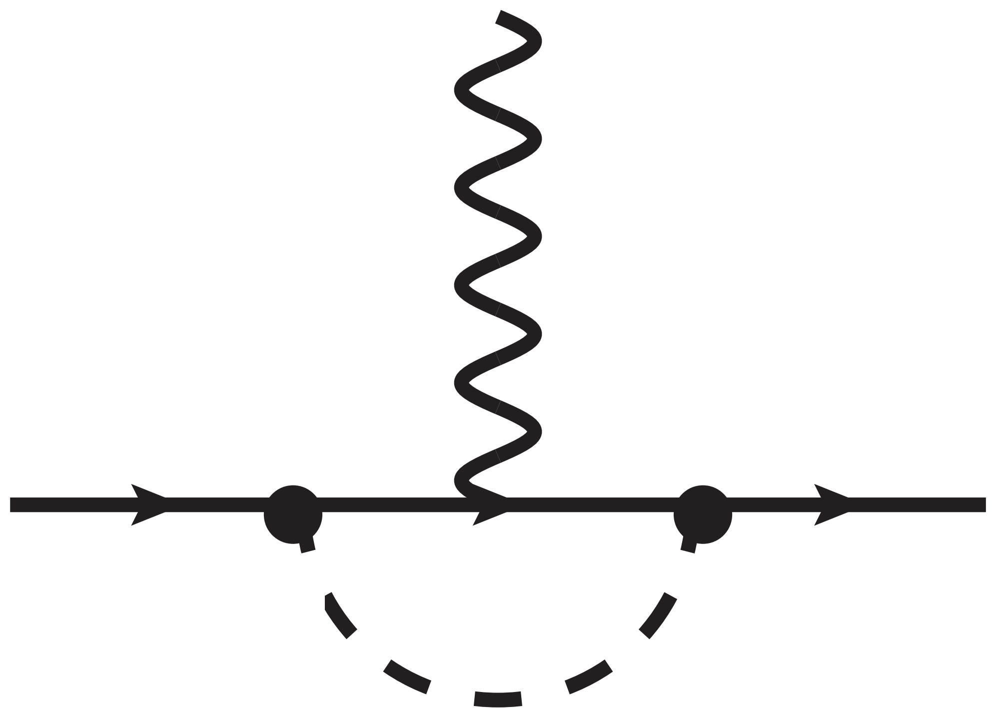
- For scalar, $\Delta a_\ell > 0$

- For (psudo)scalar, $\Delta a_\ell < 0$

The pseudo-scalar solution

- Further requiring photon coupling

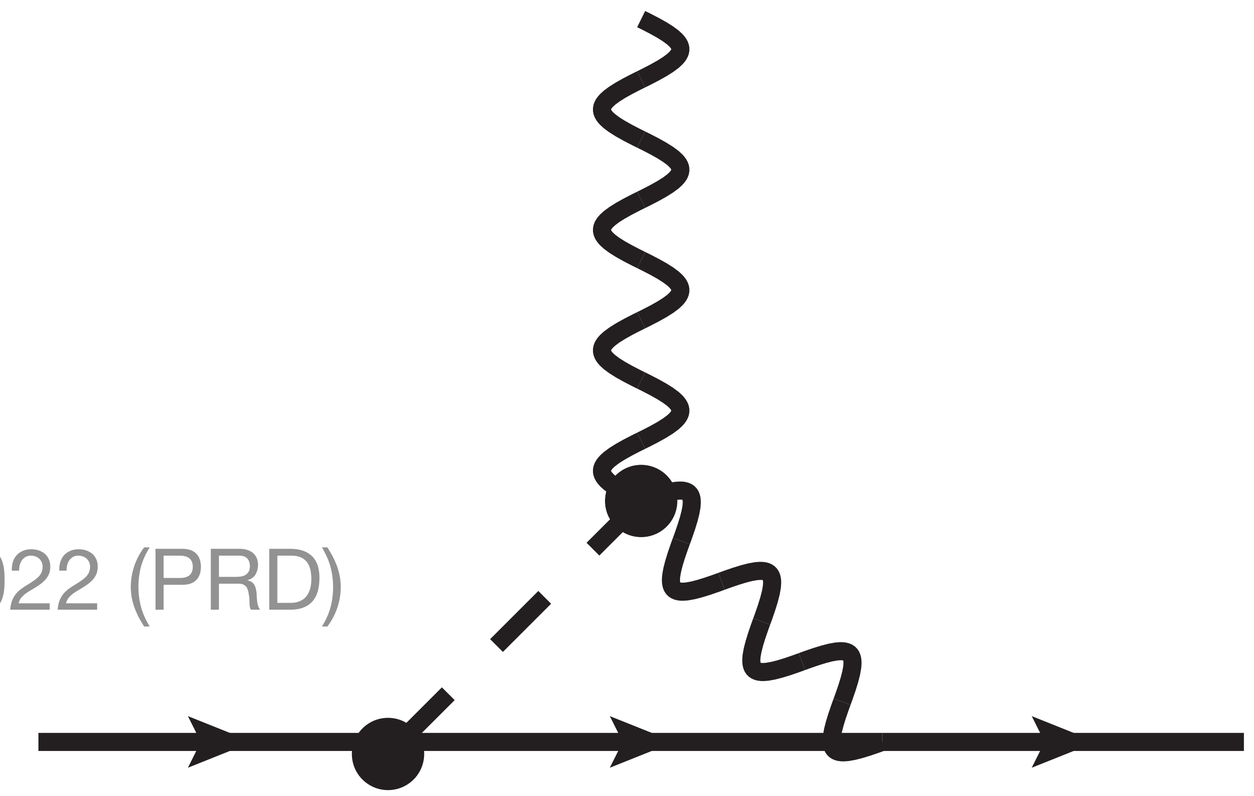
$$\mathscr{L} = \frac{1}{4} g_{a\gamma\gamma} \tilde{F} F + i y_{a\psi} a \bar{\psi} \gamma_5 \psi$$



- The 1-loop BZ contribution

$$\Delta a_\ell^{\mathrm{BZ}} \approx \frac{m_\ell}{4\pi^2} g_{a\gamma\gamma} y_{a\ell} \log \frac{\Lambda}{m_a}$$

Marciano et al, 1607.01022 (PRD)



- Assumes $g_{a\gamma\gamma}$ remains essentially constant throughout the integration over virtual photon-loop momentum

- $g_{a\gamma\gamma}$ and $y_{a\ell}$ can adjust its sign to give positive result

A complete calculation for axion-like particle

- The axion-like particle Lagrangian

$$\mathscr{L}_{\text{eff}} = \sum_f \frac{C_{ff}}{2}\frac{\partial^\mu a}{f_a}\bar{f}\gamma_\mu\gamma_5 f + \frac{g^2}{16\pi^2}C_{WW}\frac{a}{f_a}W^i_{\mu\nu}\tilde{W}^{\mu\nu,i} + \frac{g'^2}{16\pi^2}C_{BB}\frac{a}{f_a}B_{\mu\nu}\tilde{B}^{\mu\nu}$$

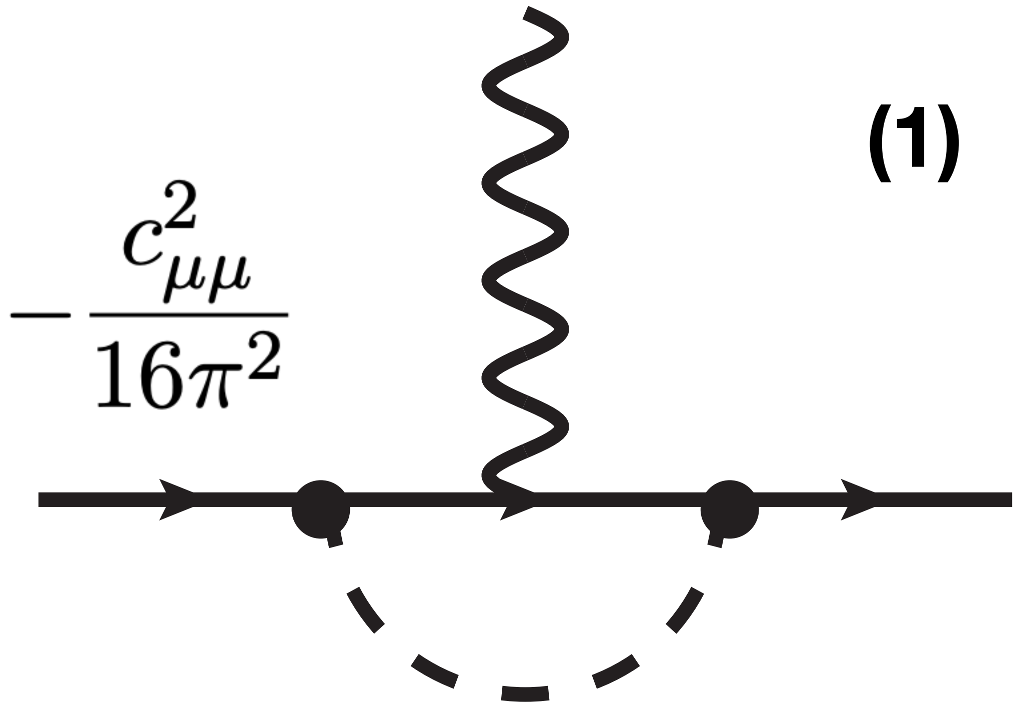
- The 3rd diagram subtlety:

Bauer et al, 1708.00443 (JHEP)

  - Previously, calculate Barr-Zee diagram for on-shell ALP and photon and use the result of 2nd diagram
  - Recently, calculate off-shell ALP and photon

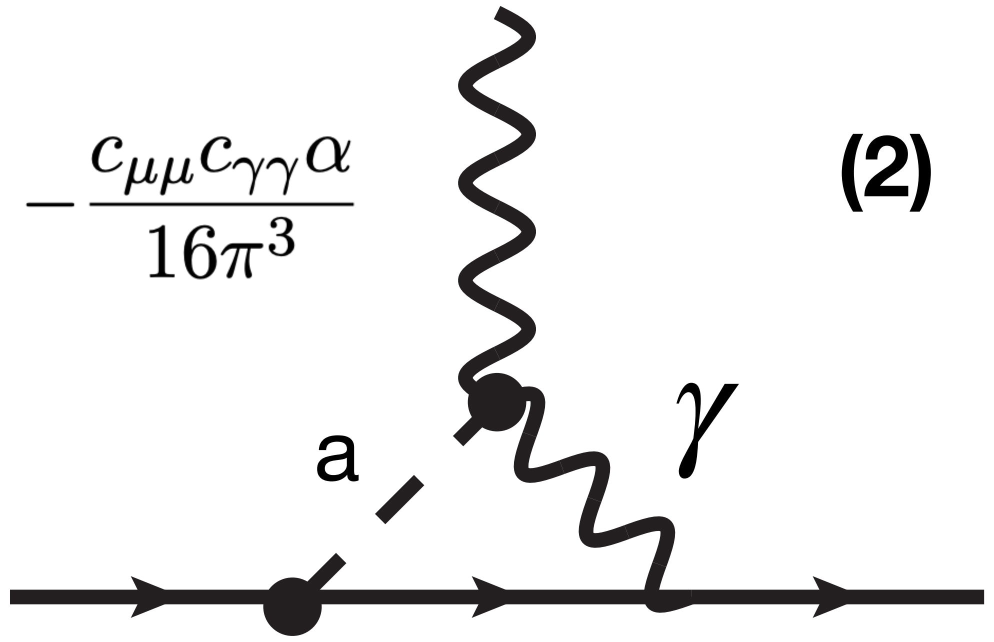
Buen-Abad et al, 2104.03267 (JHEP)

$$\Delta a_\mu^{(1)} \propto -\frac{c_{\mu\mu}^2}{16\pi^2}$$



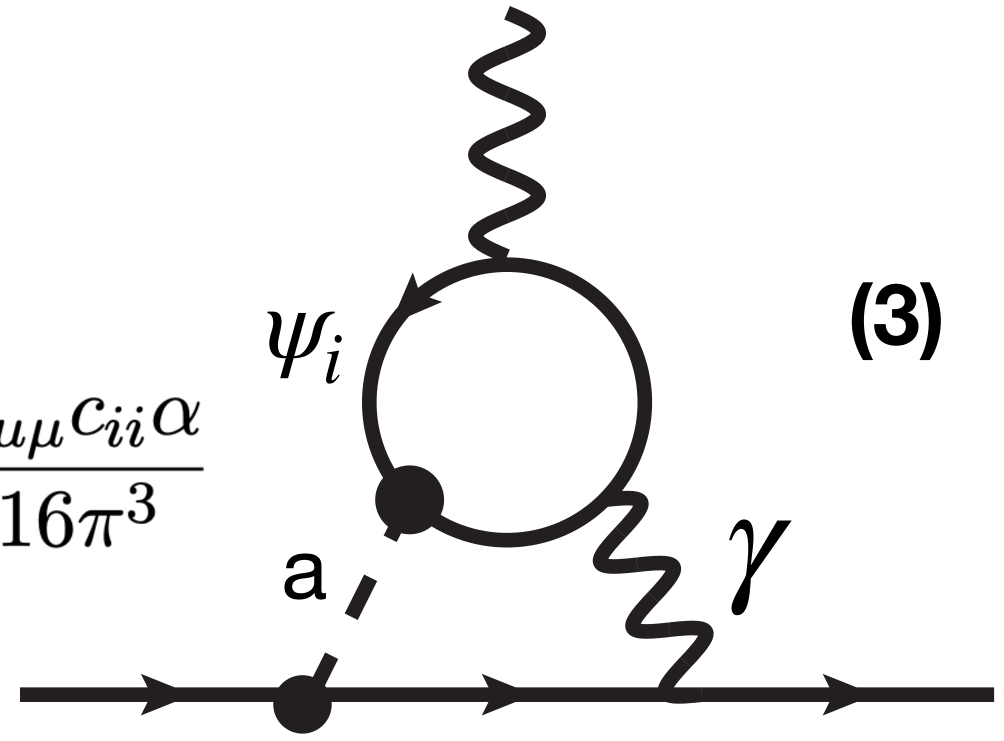
(1)

$$\Delta a_\mu^{(2)} \propto -\frac{c_{\mu\mu}c_{\gamma\gamma}\alpha}{16\pi^3}$$



(2)

$$\Delta a_\mu^{(3)} \propto -\frac{c_{\mu\mu}c_{ii}\alpha}{16\pi^3}$$



(3)

A complete calculation for axion-like particle

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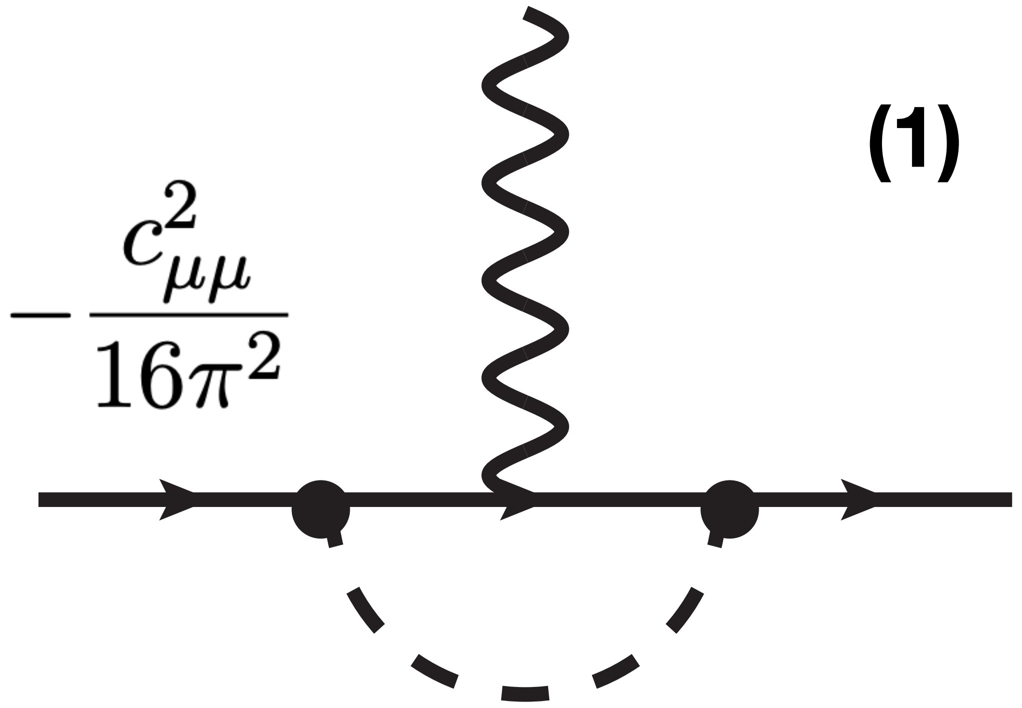

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  - Previously, calculate Barr-Zee diagram for on-shell ALP and photon and use the result of 2nd diagram

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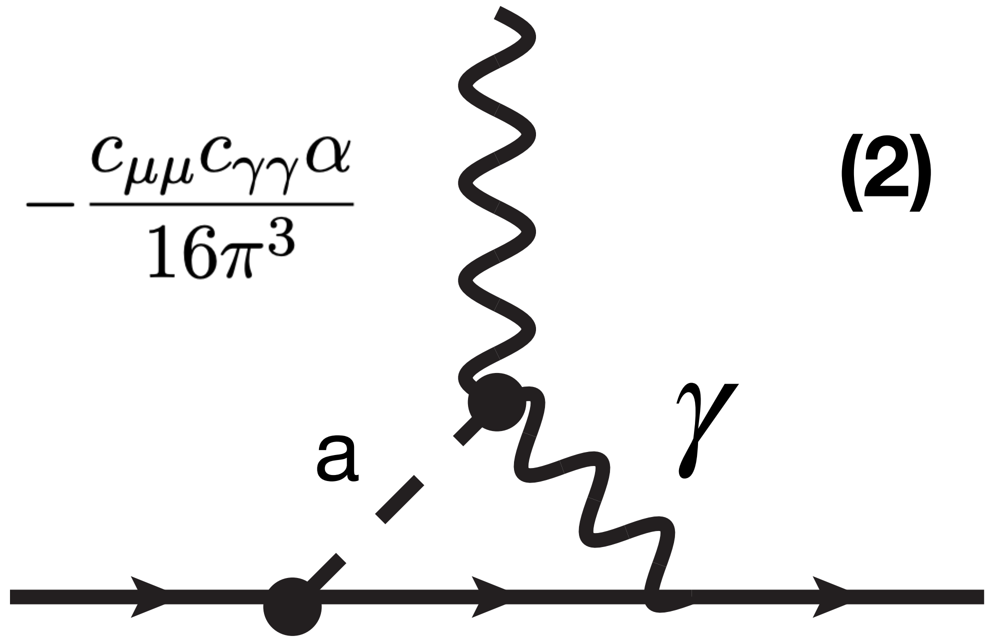


  - Recently, calculate off-shell ALP and photon

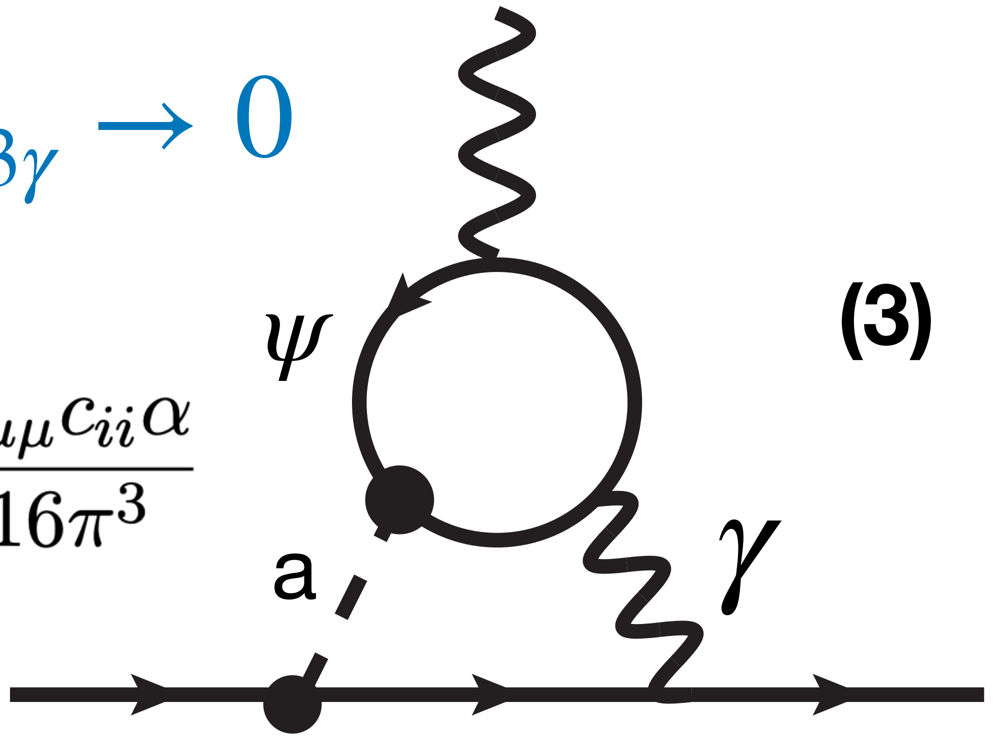
Buen-Abad et al, 2104.03267 (JHEP)

$$\Delta a_{2\gamma} = -\frac{m_\mu^2 c_{\mu\mu} c_{\gamma\gamma}\alpha}{8\pi^3 f_a^2}\cdot h_\gamma(x,\mu) \qquad x \equiv \frac{m_a^2}{m_\mu^2}$$

$$m_\psi \to \infty,\ \Delta a_{3\gamma} \to 0$$

$$\Delta a_{3\gamma} = -\frac{c_{\mu\mu}^2\alpha}{8\pi^3}\frac{m_\mu^2}{f_a^2}\left[H_\gamma(\textbf{two loop}) + h_\gamma(\textbf{counter term})\right]$$

$$\Delta a_\mu^{(3)} \propto -\frac{c_{\mu\mu}c_{ii}\alpha}{16\pi^3}$$



- Different sign for $c_{\mu\mu}$ and $c_{\gamma\gamma}$ is needed

A complete calculation for axion-like particle

- The axion-like particle Lagrangian

$$\mathscr{L}_{\text{eff}} = \sum_f \frac{C_{ff}}{2}\frac{\partial^\mu a}{f_a}\bar{f}\gamma_\mu\gamma_5 f + \frac{\alpha C_{\gamma\gamma}}{4\pi}\frac{a}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\alpha C_{\gamma Z}}{2\pi s_w c_w}\frac{a}{f_a}F_{\mu\nu}\tilde{Z}^{\mu\nu} + \frac{\alpha C_{ZZ}}{4\pi s_w^2 c_w^2}\frac{a}{f_a}Z_{\mu\nu}\tilde{Z}^{\mu\nu}$$

- Our calculation

$$\Delta a_{2Z} = \frac{\alpha c_{\gamma z} c_{\mu\mu} m_\mu^2\left(4s_w^2 - 1\right)}{32\pi^3 c_w^2 s_w^2 f_a^2} \cdot h_Z(x, y, \mu)$$

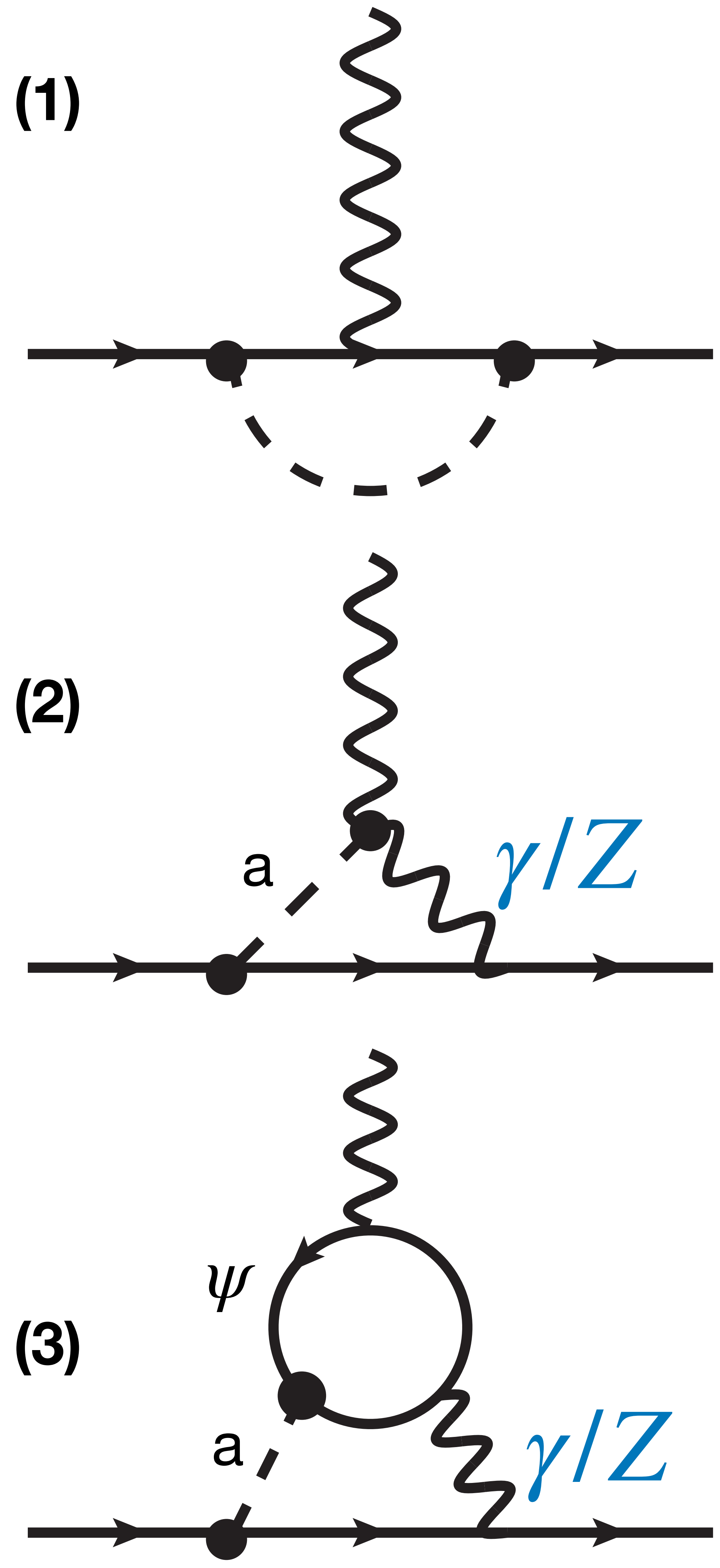
$$m_\psi \to \infty,\ \Delta a_3 \to 0$$

$$\Delta a_{3Z} = -\frac{\alpha c_{\mu\mu}^2 m_\mu^2\left(4s_w^2 - 1\right)^2}{128\pi^3 c_w^2 s_w^2 f_a^2}\left[H_Z(\textbf{two loop}) + h_Z(\textbf{counter term})\right]$$

$$x \equiv \frac{m_a^2}{m_\mu^2}\ ,\ y \equiv \frac{m_Z^2}{m_\mu^2}$$

- Calculation in another basis

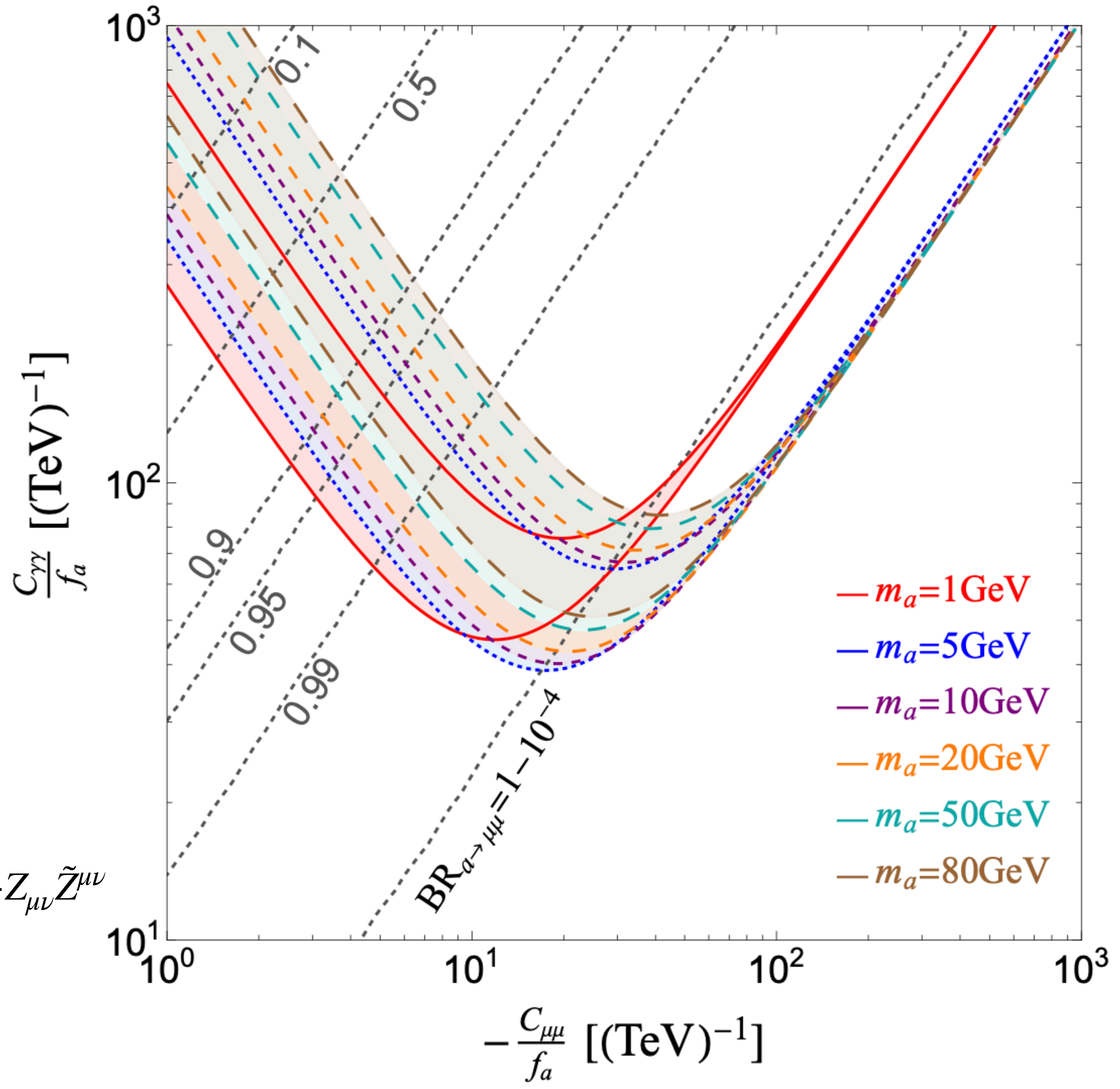
$$\frac{c_{\mu\mu}}{2}\frac{\partial^\mu a}{f_a}\bar{\mu}\gamma_\mu\gamma_5\mu \to -\frac{ic_{\mu\mu}m_\mu}{f_a}a\bar{\mu}\gamma_5\mu + \frac{\alpha c_{\mu\mu}}{4\pi f_a}aF\tilde{F} + \frac{\alpha c_{\mu\mu}(s_w^2 Q - \frac{1}{2}T_3)}{2\pi c_w s_w f_a}aF^{\mu\nu}\tilde{Z}_{\mu\nu}$$



Muon g-2 solution and ALP decay BR

- Assume only coupling to Muon and Photon
- In g-2 solution region, mostly decay to $a \to \mu^+\mu^-$
- The inclusion of Z diagram makes some difference for large m_a
- Exotic Z decay should happen

$$\mathcal{L}_{\rm eff} = \sum_f \frac{C_{ff}}{2}\frac{\partial^\mu a}{f_a}\bar{f}\gamma_\mu\gamma_5 f + \frac{\alpha C_{\gamma\gamma}}{4\pi}\frac{a}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\alpha C_{\gamma Z}}{2\pi s_w c_w}\frac{a}{f_a}F_{\mu\nu}\tilde{Z}^{\mu\nu} + \frac{\alpha C_{ZZ}}{4\pi s_w^2 c_w^2}\frac{a}{f_a}Z_{\mu\nu}\tilde{Z}^{\mu\nu}$$



Muon g-2 solution and ALP decay BR

- The BR($a \to \gamma\gamma, Z \to \gamma a$) needs to consider 1-loop contribution from fermion loop

$$\Gamma(Z \to \gamma a) = \frac{\alpha^2(m_Z) m_Z^3}{96\pi^3 s_w^2 c_w^2 f_a^2} \left|C_{\gamma Z}^{\text{eff}}\right|^2 \left(1 - \frac{m_a^2}{m_Z^2}\right)^3 ,$$

$$\Gamma(a \to \mu^+\mu^-) = \frac{m_a m_\mu^2}{8\pi f_a^2} |C_{\mu\mu}|^2 \sqrt{1 - \frac{4m_\mu^2}{m_a^2}},$$

$$\Gamma(a \to \gamma\gamma) = \frac{\alpha^2 m_a^3}{64\pi^3 f_a^2} \left|C_{\gamma\gamma}^{\text{eff}}\right|^2 ,$$

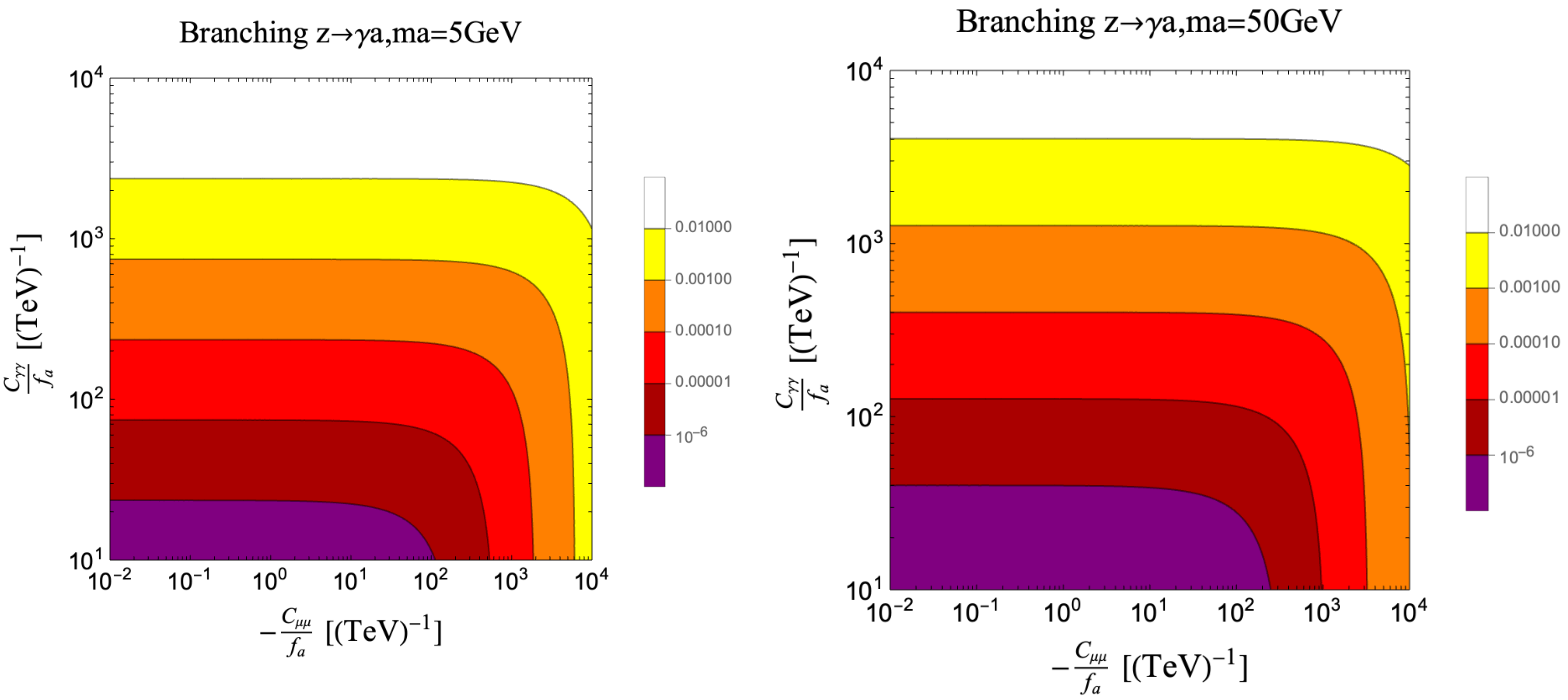
$$C_{\gamma\gamma}^{\text{eff}} = C_{\gamma\gamma} + C_{\mu\mu} \left[1 + \frac{m_\mu^2}{m_a^2} \cdot \mathcal{F}\left(\frac{m_a^2}{m_\mu^2}\right)\right] + \mathcal{O}(\alpha),$$

$$C_{\gamma Z}^{\text{eff}} = C_{\gamma Z} + C_{\mu\mu}\left(\frac{1}{4} - s_w^2\right) \left[1 + \frac{m_\mu^2}{m_a^2 - m_Z^2} \cdot \left(\mathcal{F}\left(\frac{m_a^2}{m_\mu^2}\right) - \mathcal{F}\left(\frac{m_Z^2}{m_\mu^2}\right)\right)\right] + \mathcal{O}(\alpha)$$

$$\mathcal{F}(x) \equiv \ln^2\left(\frac{\sqrt{x(x-4)} - x + 2}{2}\right)$$

The relevant search at Z-factory (CEPC/FCC-ee)

- The exotic Z decay: $Z \to a + \gamma$ and $Z \to a + \mu^+ + \mu^-$



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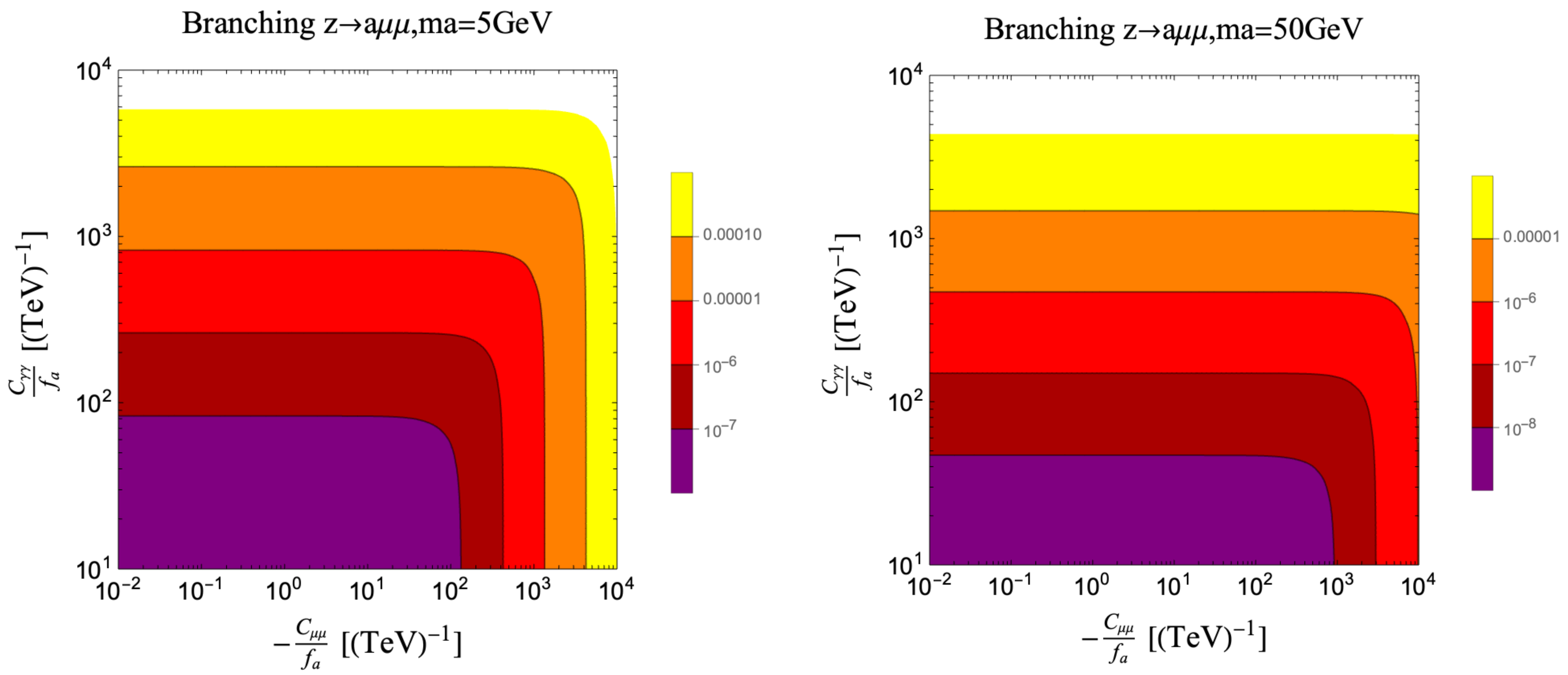
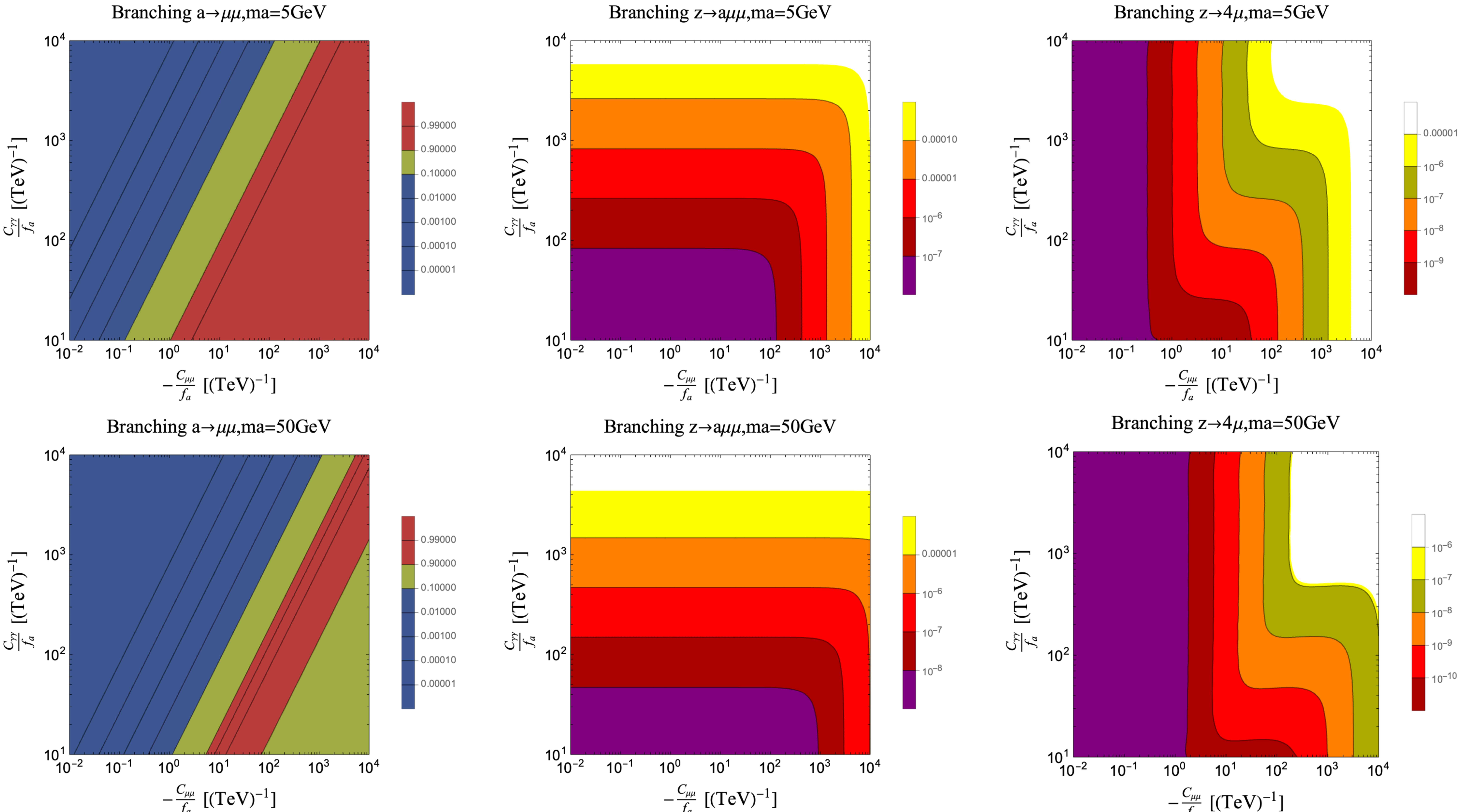
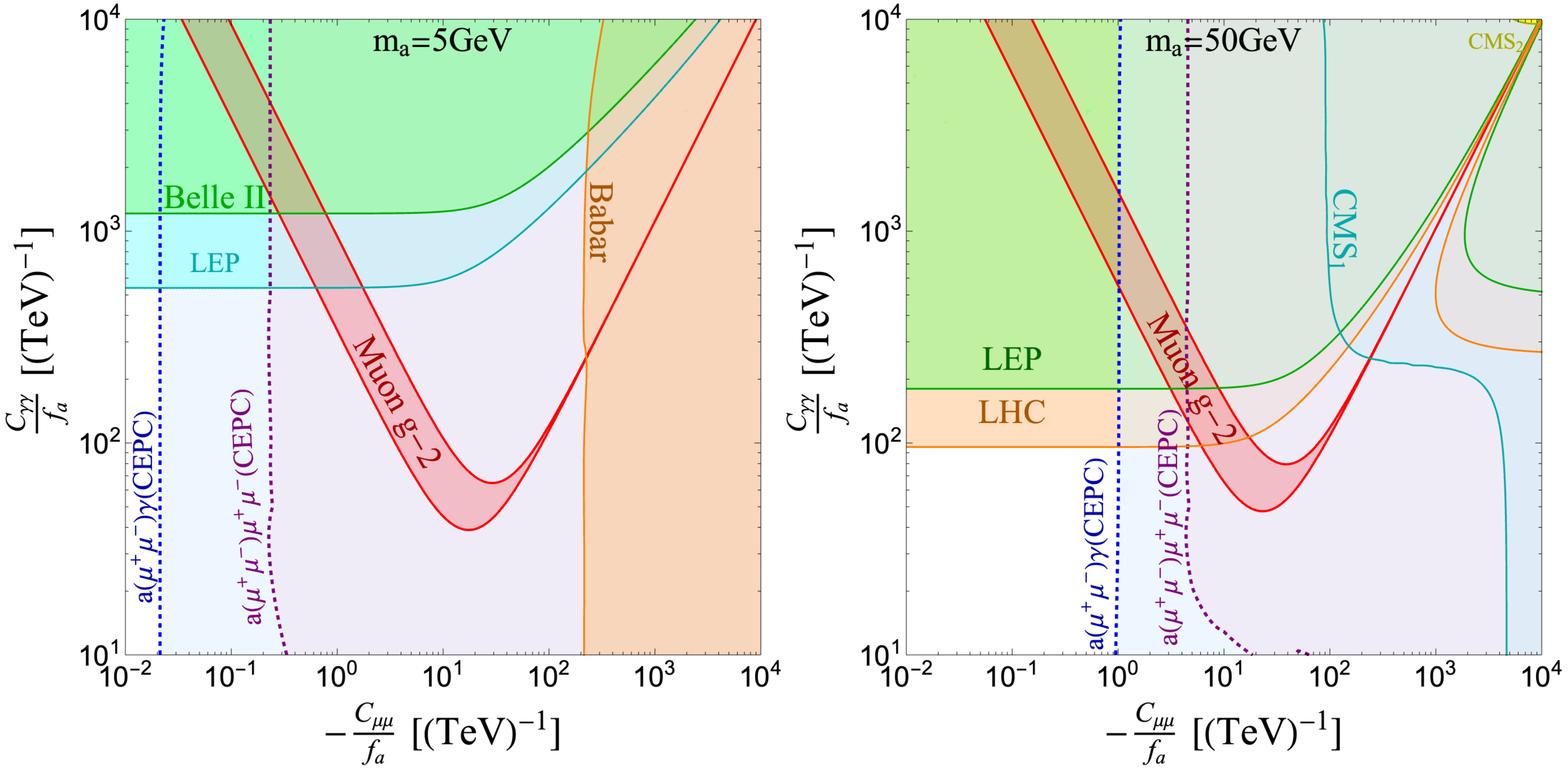


ALP $a \to \mu\mu$ branching ratio and effects



The existing constraints for ALP

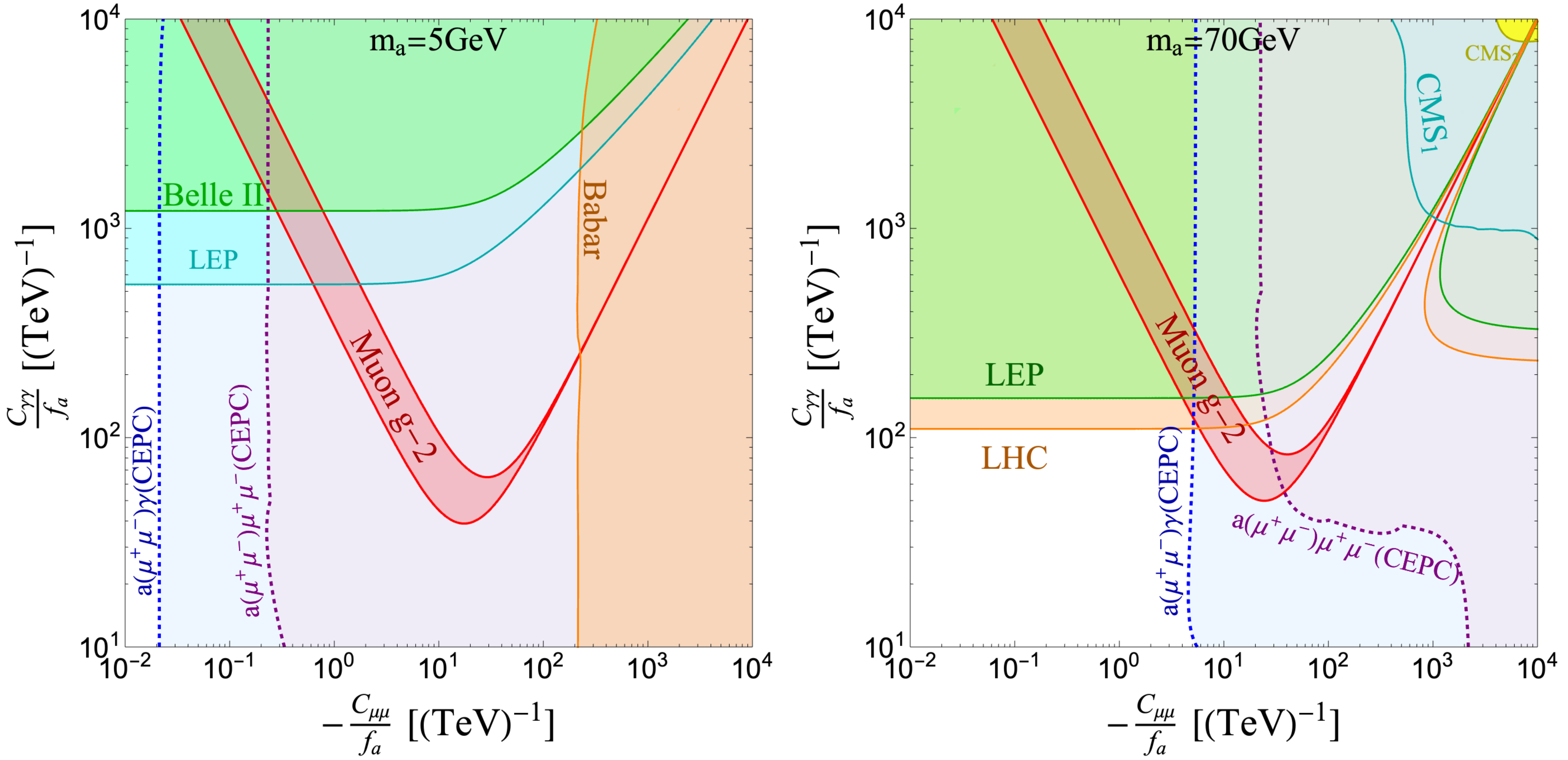
- Constraining a-γ coupling only:
  - Belle-II, LEP: $e^+e^- \to a\gamma \to (\gamma\gamma)\gamma$
  - LHC: $pp \to a\gamma \to (\gamma\gamma)\gamma$
- Constraining a-μ coupling only:
  - BaBar: recast $e^+e^- \to \mu^+\mu^- Z'$
  - CMS(4μ): $pp \to \mu^+\mu^-\phi$
- Constraining both coupling
  - CMS($\bar{t}t + 2\mu$): $pp \to \bar{t}t\phi \to \bar{t}t(\mu^+\mu^-)$



Preliminary results

The relevant search at Z-factory ($C_{WW} = 0$)

- The exotic Z decay: $Z \to a + \gamma$ and $Z \to a + \mu^+ + \mu^-$
- With ALP decay: $a \to \mu^+\mu^-$
- Relevant SM background: $\mu^+\mu^-\gamma$ and 4μ
    - Cuts: $E_\gamma > 2\text{GeV}$, $p_T^\mu > 5\text{GeV}$, $|\eta| < 3$, $\Delta R_{\mu\mu} > 0.1$, dimuon resolution: $\left|\frac{m_{\mu\mu}}{m_a} - 1\right| < 0.19\,\%$



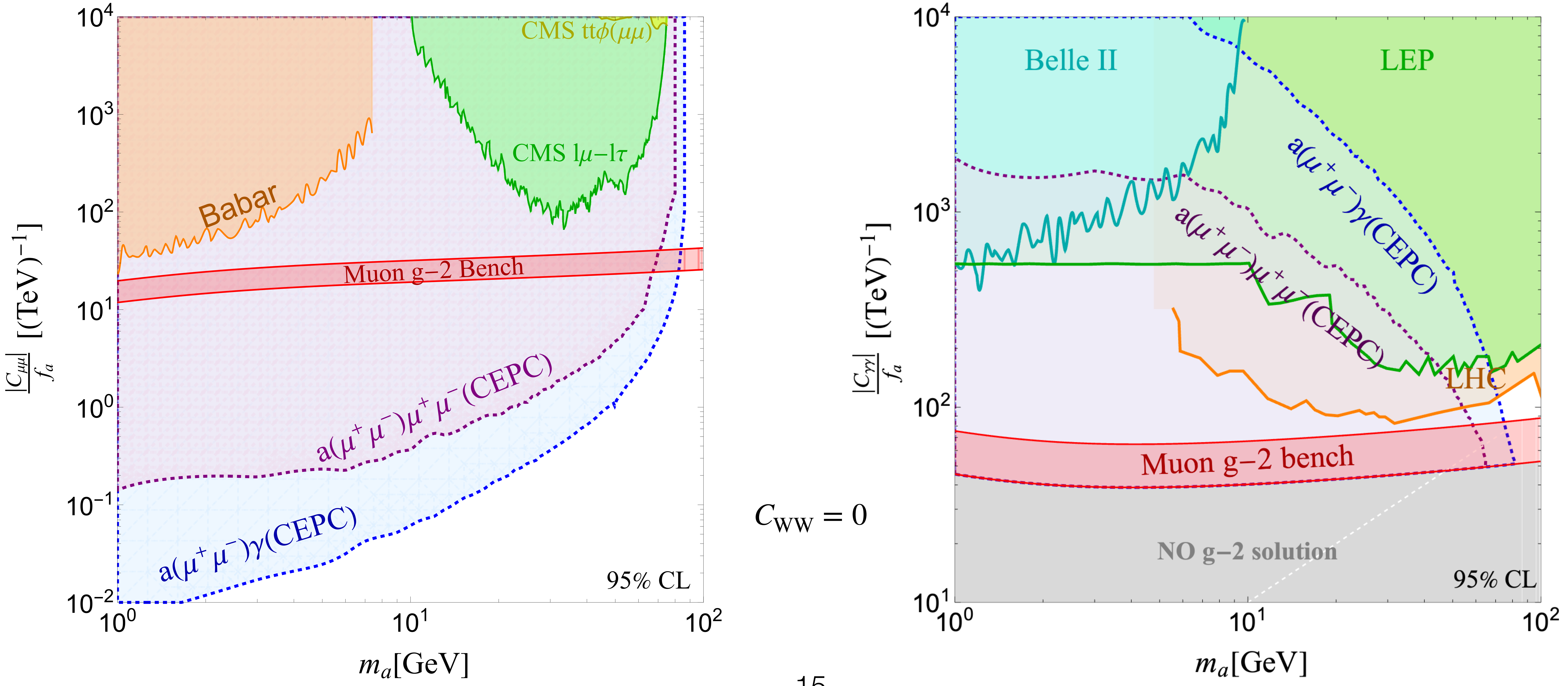
$C_{WW} = 0$

- Large m_a prefers decay $a \to \gamma\gamma$
- SM bkg: $\mu^+\mu^-\gamma$ dominated by soft γ

The relevant search at Z-factory ($C_{WW} = 0$)

- Marginalize one coupling $C_{\gamma\gamma}$ or $C_{\mu\mu}$

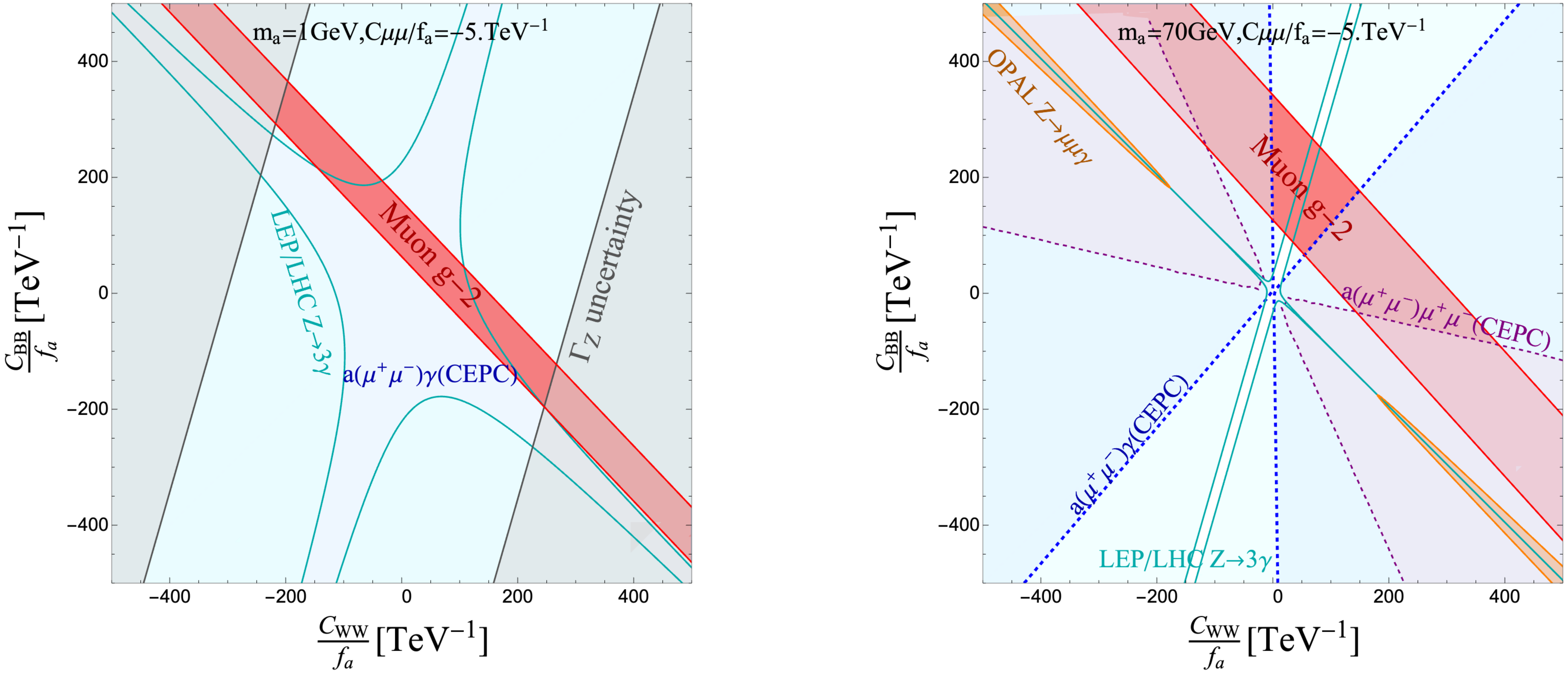
Preliminary results



The relevant search at Z-factory ($C_{WW} \neq 0$)

- 4 parameters: $m_a, C_{\mu\mu}, C_{BB}, C_{WW}$
- Z-factory $a \to \mu^+\mu^-$ search is sensitive to light ALP
- Z-factory $a \to \gamma\gamma$ search will be sensitive to heavy ALP (in progress)

Preliminary results



Summary

- Muon g-2 experiments show 4.2σ discrepancy with SM
- ALP can provide a solution with couplings $C_{\mu\mu}$ and $C_{\gamma\gamma}$
- In UV model, $C_{\gamma\gamma}$ comes from C_{WW} and C_{BB}, leads to $C_{\gamma Z}$
- At Z-factory, it leads to exotic Z decay:
  - $Z \to a\gamma, a\mu^+\mu^-$
  - Future Z factory can provide sensitivity covers most of the g-2 region for $m_a \lesssim m_Z$

Thank you!

Backup slides

- Special functions

$$x \equiv \frac{m_a^2}{m_\mu^2}\ ,\ y \equiv \frac{m_Z^2}{m_\mu^2}\ ,\ \Delta^2 \equiv \frac{m_\mu^2}{z(1-z)} \tag{15}$$

$$B(m_\mu^2, m_{a/Z}, m_\mu) = \sqrt{m_{a/Z}^2(m_{a/Z}^2 - 4m_\mu^2)}\ln\left(\frac{\sqrt{m_{a/Z}^2 - 4m_\mu^2} + m_{a/Z}}{2m_\mu}\right)/m_\mu^2 \tag{16}$$

$$H_\gamma = \int_0^1 dz \frac{\Delta^2}{2(\Delta^2 - m_a^2)}\left[-\frac{2}{3}\frac{m_a^2 + 2m_\mu^2}{m_\mu^2}B(m_\mu^2, m_a, m_\mu) + \frac{2}{3}\frac{\Delta^2 + 2m_\mu^2}{m_\mu^2}B(m_\mu^2, m_a, \Delta)\right.$$
$$\left. + \frac{\Delta^4}{3m_\mu^4}\ln\left(\frac{m_\mu^2}{\Delta^2}\right) + \frac{m_a^4}{3m_\mu^4}\ln\left(\frac{m_a^2}{m_\mu^2}\right) + \frac{2}{3}\frac{\Delta^2 - m_a^2}{m_\mu^2}\right] \tag{17}$$

$$H_Z = \int_0^1 dz\ \frac{\Delta^2}{\Delta^2 - m_a^2}\left[-\frac{ma^2(m_a^2 + 2m_\mu^2)B(m_\mu^2, m_a, m_\mu)}{3(m_a^2 - m_Z^2)m_\mu^2} + \frac{m_a^6}{6m_\mu^4(m_a^2 - m_Z^2)}\ln\left(\frac{m_a^2}{m_\mu^2}\right)\right]$$
$$+ \frac{\Delta^2}{\Delta^2 - m_Z^2}\left[\frac{m_Z^2(2m_\mu^2 + m_Z^2)B(m_\mu^2, m_Z, m_\mu)}{3m_\mu^2(m_a^2 - m_Z^2)} + \frac{m_Z^6}{6m_\mu^4(m_Z^2 - m_a^2)}\ln\left(\frac{m_Z^2}{m_\mu^2}\right)\right]$$
$$+ \frac{\Delta^2}{3m_\mu^2} + \frac{\Delta^4(\Delta^2 + 2m_\mu^2)B(m_\mu^2, m_\mu, \Delta)}{3m_\mu^2(\Delta^2 - m_a^2)(\Delta^2 - m_Z^2)} + \frac{\Delta^8}{6m_\mu^4(\Delta^2 - m_a^2)(\Delta^2 - m_Z^2)}\ln\left(\frac{m_\mu^2}{\Delta^2}\right) \tag{18}$$

$$h_\gamma = \ln\left(\frac{\mu^2}{m_\mu^2}\right) + 2 - \frac{x^2}{6}\ln(x) + \frac{x}{3} + \frac{x+2}{3}\sqrt{x(x-4)}\ln\left(\frac{\sqrt{x-4} + \sqrt{x}}{2}\right) \tag{19}$$

$$h_Z = \ln\left(\frac{\mu^2}{m_Z^2}\right) + \frac{x(x+2)B(m_\mu^2, m_a, m_\mu)}{3(x-y)} + \frac{y(2+y)B(m_\mu^2, m_Z, m_\mu)}{3(y-x)}$$
$$+ \frac{(x+3+y)}{3} - \frac{6x + y^3 - 6y}{6(x-y)}\ln\left(\frac{x}{y}\right) - \frac{x^2 + xy - 6 + y^2}{6}\ln(x) \tag{20}$$