Extraction of the CKM angle α

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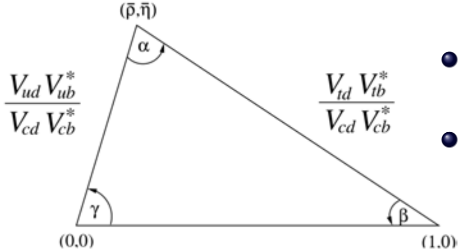
CEPC workshop, 24/5/22

The CKM matrix

In SM, flavour dynamics related to weak charged transitions which mix quarks of different generations

Encoded in unitary CKM matrix $V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$



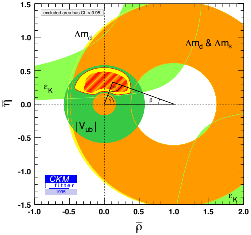
- 3 generations $\Longrightarrow$ 1 phase, only source of *CP*-violation in SM
- Wolfenstein parametrisation, defined to hold to all orders in λ and rephasing invariant

$$\lambda^2 = \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2} \qquad A^2\lambda^4 = \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2} \qquad \bar\rho + i\bar\eta = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

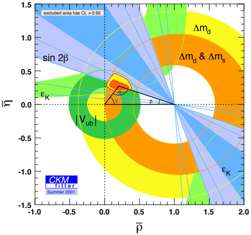
$\Longrightarrow$ 4 parameters describing the CKM matrix

Two and a half decades of CKM(fitter)

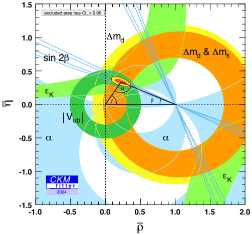
[LEP, KTeV, NA48, Babar, Belle, CDF, DØ, LHCb, ATLAS, CMS, Belle II. . .]



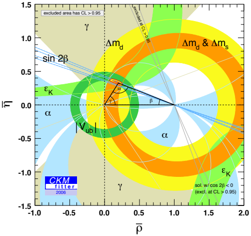
1995



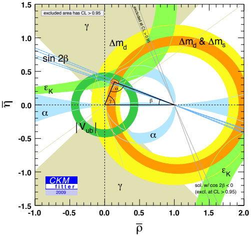
2001



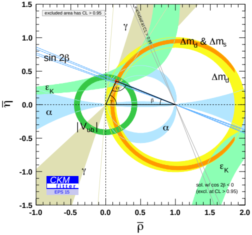
2004



2006

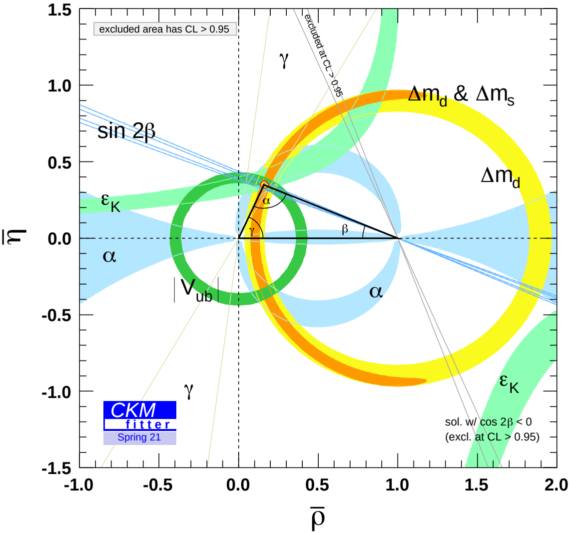


2009



2015

Spring 2021



$|V_{ud}|$, $|V_{us}|$

$|V_{cb}|$, $|V_{ub}|_{SL}$

$B \to \tau\nu$

$|V_{ub}/V_{cb}|_{\Lambda_b}$

Δm_d, Δm_s

ϵ_K

$\sin 2\beta$

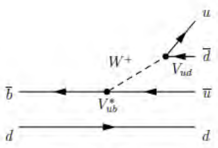
α

γ

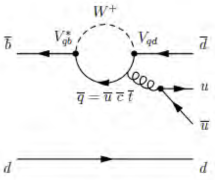
$$A = 0.813^{+0.012}_{-0.006}$$
$$\lambda = 0.225^{+0.0002}_{-0.0002}$$
$$\bar\rho = 0.157^{+0.000}_{-0.005}$$
$$\bar\eta = 0.347^{+0.012}_{-0.005}$$
(68% CL)

α compatible with rest of the fit, but less precise than other constraints

α from $\pi\pi$



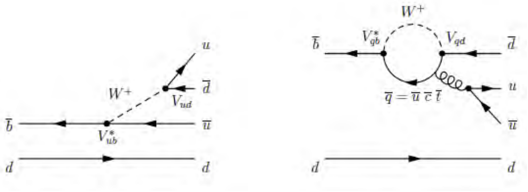
Tree



Penguin

$$A(B^0 \to \pi^+\pi^-) = V_{ud}V_{ub}^* t + \sum_{q=u,c,t} V_{qd}V_{qb}^* p_q = V_{ud}V_{ub}^* t^{+-} + V_{td}V_{tb}^* p^{+-}$$

α from $\pi\pi$



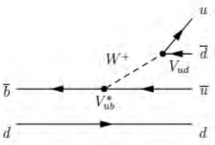
Tree　　　　Penguin

$$A(B^0 \to \pi^+\pi^-) = V_{ud}V^*_{ub}t + \sum_{q=u,c,t} V_{qd}V^*_{qb}p_q = V_{ud}V^*_{ub}t^{+-} + V_{td}V^*_{tb}p^{+-}$$

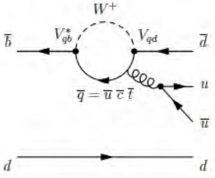
Time-dependent CP asymmetry : $A(t) = \frac{\Gamma(\bar{B}^0(t)\to\pi^+\pi^-)-\Gamma(B^0(t)\to\pi^+\pi^-)}{\Gamma(\bar{B}^0(t)\to\pi^+\pi^-)+\Gamma(B^0(t)\to\pi^+\pi^-)}$

$$\begin{aligned} A(t) &= S^{+-}\sin(\Delta m t) - C^{+-}\cos(\Delta m t) \\ &= \sqrt{1-(C^{+-})^2}\,\sin 2\alpha_{eff}\,\sin(\Delta m t) - C^{+-}\cos(\Delta m t) \end{aligned}$$

α from $\pi\pi$



Tree



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Combining CKM for t^{+-} and B-$\bar{B}$ mixing: $S^{+-} = \sin(2\alpha) + O(\frac{p^{+-}}{t^{+-}})$

$\Longrightarrow$Penguin pollution: handle on p^{+-} and t^{+-} to extract $\sin(2\alpha)$

Isospin analysis for $B \to \pi\pi$

In terms of isospin quantities $Q^{(I)}_{I_z}$

- Two operators for $\bar{b} \to \bar{u}u\bar{d}$: $O^{(3/2)}_{1/2}$ and $O^{(1/2)}_{1/2}$
- Two inital states: $|B^+\rangle = |B^{(1/2)}_{1/2}\rangle$ and $|B^0\rangle = |B^{(1/2)}_{-1/2}\rangle$
- Three final states: [$I = 1$ forbidden by Bose symmetry]

$$\langle \pi^+\pi^0| = \langle \pi\pi^{(2)}_1| \qquad \begin{aligned} \langle \pi^+\pi^-| &= \sqrt{\frac{1}{3}}\langle \pi\pi^{(2)}_0| + \sqrt{\frac{2}{3}}\langle \pi\pi^{(0)}_0| \\ \langle \pi^0\pi^0| &= \sqrt{\frac{2}{3}}\langle \pi\pi^{(2)}_0| - \sqrt{\frac{1}{3}}\langle \pi\pi^{(0)}_0| \end{aligned}$$

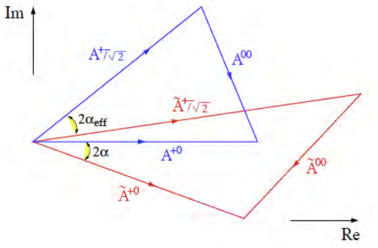
From $B^{(1/2)}$, $O^{(3/2)}$ can only yield $I = 2$ final states,
and $O^{(1/2)}$ only $I = 0$, so two reduced amplitudes

$$A_2 = \frac{1}{2\sqrt{3}}\langle \pi\pi^{(2)}||O^{(3/2)}||B^{(1/2)}\rangle \qquad A_0 = -\frac{1}{\sqrt{6}}\langle \pi\pi^{(0)}||O^{(1/2)}||B^{(1/2)}\rangle$$

Trapping the penguin in $B \to \pi\pi$

$$B^+, B^0 \;:\; A^{+0} = 3A_2 \qquad A^{+-} = \sqrt{2}(A_2 - A_0) \qquad A^{00} = 2A_2 + A_0$$
$$B^-, B^0 \;:\; \bar{A}^{+0} = 3\bar{A}_2 \qquad \bar{A}^{+-} = \sqrt{2}(\bar{A}_2 - \bar{A}_0) \qquad \bar{A}^{00} = 2\bar{A}_2 + \bar{A}_0$$

A^{+0} is $I = 2$ $\pi\pi$, only from tree and (negligible) $I = 3/2$ penguins



Two triangular relations

$$A^{+-} + \sqrt{2}A^{00} = \sqrt{2}A^{+0}$$
$$\bar{A}^{+-} + \sqrt{2}\bar{A}^{00} = \sqrt{2}\bar{A}^{+0}$$

from BR (B^{+0}, B^{+-}, B^{00}) and CP-asyms. (C^{+-}, S^{+-}, C^{00})

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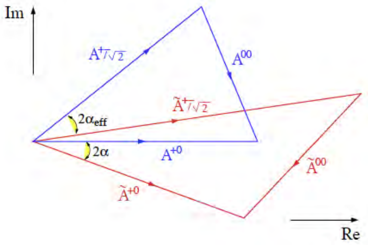

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from BR (B^{+0}, B^{+-}, B^{00}) and CP-asyms. (C^{+-}, S^{+-}, C^{00})

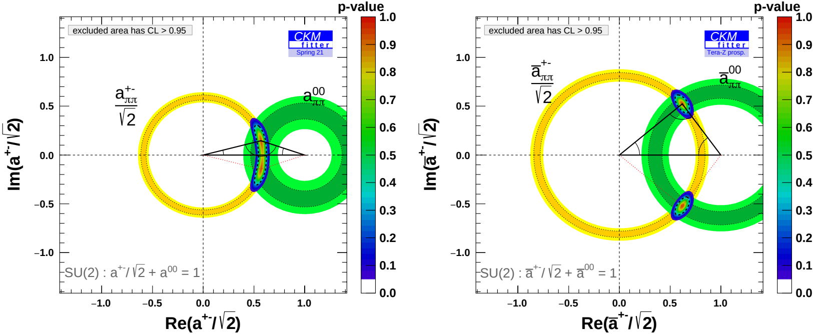
Introducing $\tilde{A}^{ij} = exp(-2i\beta)\bar{A}^{ij}$,

2α between A^{+0} and $\tilde{A}^{+0}$, $\qquad 2\alpha_{eff}$ between A^{+-} and $\tilde{A}^{+-}$

- Measure mixed CP-asymmetry in $\pi^+\pi^-$ as $\sin(2\alpha_{eff})$
- Reconstruct the triangles
- Up to discrete ambiguities, possible to determine $\sin(2\alpha)$

$\pi\pi$ triangles

Reconstructing the two triangles (normalised by $+0$) to get α from $\alpha_{\rm eff}$



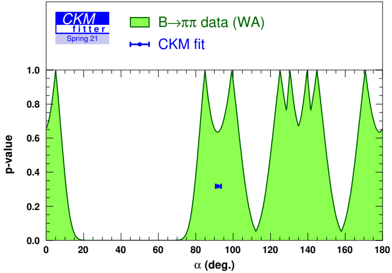
The A triangle is (almost) flat, whereas the $\bar{A}$ triangle is not (two-fold degeneracy of the eight solutions : only four distinct solutions)

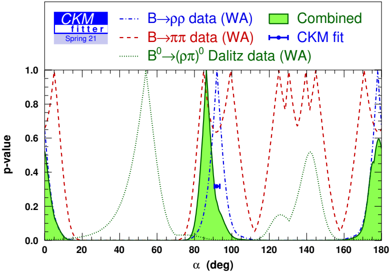
Possibility to extend the analysis to $\rho\rho$ and $\rho\pi$ systems

Determination of α

- Four-fold ambiguity for $\pi\pi$
- Two-fold ambiguity for $\rho\rho$
- No ambiguity for $\rho\pi$

(agreement only at 3σ with other determinations)





Including all modes: $\alpha[\text{combined}] = (86.4^{+4.3}_{-4.0} \cup 178.5^{+3.1}_{-5.2})^\circ$

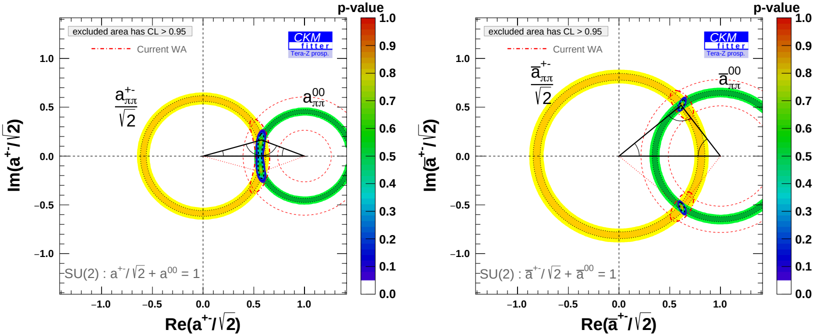
indirect fit determination (not including α measurements):
$\alpha[\text{indirect}] = (91.9^{+1.6}_{-1.2})^\circ$ (pull of 1.3 σ)

TeraZ measurements for $B \to \pi^0\pi^0$ (1)

Impact of TeraZ/CEPC measurements of $\pi\pi$ on precision for α ?

[Wang, Li, Chen, Zhu, Ruan, Deschamps, SDG; see Yuexin Wang's talk]

$B \to \pi^0\pi^0$	World average	TeraZ
$\Delta BR/BR$	16%	0.44%
ΔC_{00}	0.22	0.01

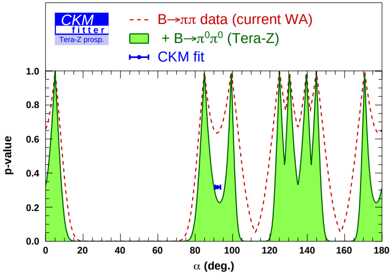


Assuming central values are unchanged

reduction of uncertainty on 00 side, but ambiguity not fully lifted

TeraZ measurements for $B \to \pi^0\pi^0$ (2)

$\alpha(\pi\pi)$ alone

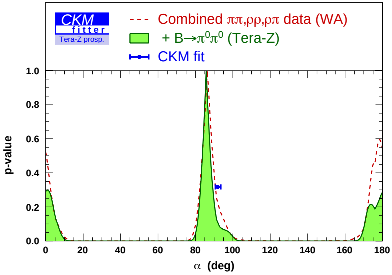


WA: $\alpha(\pi\pi) = (93.0 \pm 13.6)^\circ$

TeraZ : $\alpha(\pi\pi) = (84.9^{+5.2}_{-3.0} \cup 99.4^{+2.2}_{-3.6})^\circ$

Improved separation of mirror solutions

Combined with $\alpha(\rho\rho), \alpha(\rho\pi)$



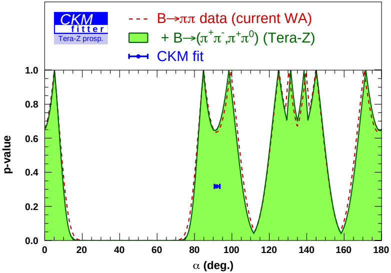
WA: $\alpha = (86.4^{+4.3}_{-4.0} \cup 178.5^{+3.1}_{-5.2})^\circ$

TeraZ: $\alpha = (85.8^{+3.4}_{-3.0})^\circ$

Overall resolution improved by 1°

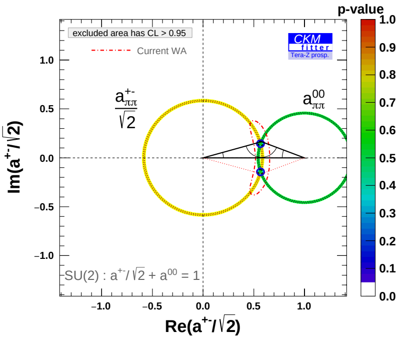
TeraZ measurements for $B \to \pi^+\pi^-, \pi^+\pi^0$

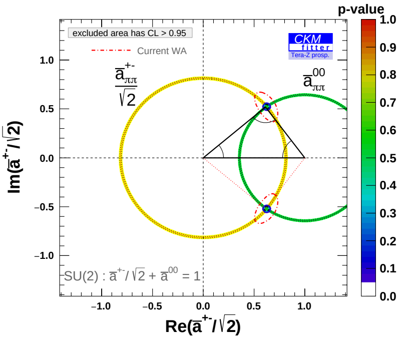
$B \to \pi\pi$ obs	World average	TeraZ
ΔBR($B \to \pi^+\pi^-$)/BR	3.7%	0.15%
ΔC_{+-}	0.030	0.0021
ΔS_{+-}	0.029	0.0018
ΔBR($B \to \pi^+\pi^0$)/BR	11.3%	0.16%



Only a slight improvement without $\pi^0\pi^0$ update
(the +- side is already quite well known)

Much better situation once all three modes improved
the separation of solutions for the (almost flat) *B* isospin triangle
requires improving both neutral and charged modes

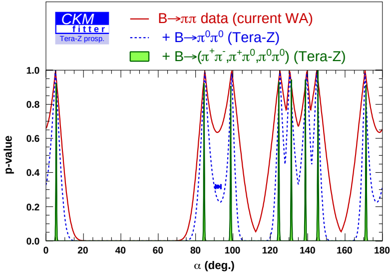




Caveat: Depends on the central values (kept unchanged here)

TeraZ measurements for $B \to \pi^+\pi^-, \pi^+\pi^0, \pi^0\pi^0$ (2)

TeraZ $\alpha(\pi\pi) = (84.5 \pm 0.3)^\circ$
+mirror solutions



At that level of accuracy, the statistical resolution becomes smaller than isospin breaking effects ($1^\circ - 2^\circ$)

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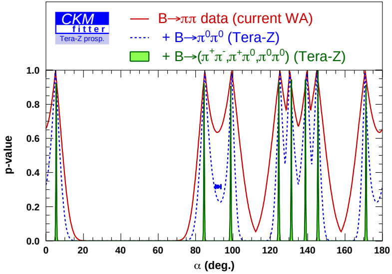

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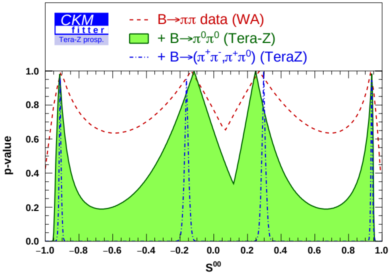
Several sources of isospin breaking

- $\Delta I = 3/2$ electroweak penguins: theoretical estimates available
- isospin breaking in $\pi\pi$: $\pi^0 - \eta - \eta'$ mixing could be better controlled with more data on $B^{0,+} \to \pi^{+,0}\eta(')$
- isospin breaking in $\rho\rho$: effect of the width of the ρ
- isospin breaking in $\rho\pi$: negligible according to theoretical estimates

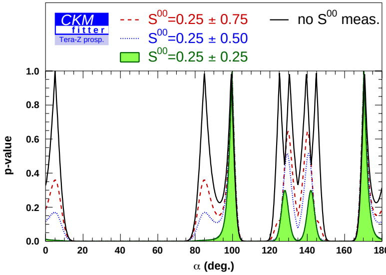
A missing observable

- Time-dependent asym for $B^0 \to \pi^0\pi^0$ (S^{00}) not yet measured
- Can be predicted from the outcome of the fit
- Would help to resolve ambiguities of mirror solutions
- As it yields information on relative phase of A_{00} and $\bar{A}_{00}$

S^{00} prediction
from rest of the fit



Impact of measuring S^{00}
on $\alpha(\pi\pi)$



Conclusions

α angle

- important ingredient to improve the determination of the CKM matrix
- based on $B \to \pi\pi, \rho\pi, \rho\rho$ measuremnts (BR, CP-asymmetries) to assess penguin contributions, through isospin analysis

TeraZ/CEPC can improve the situation through

- $B^0 \to \pi^0\pi^0$ measurements: better separation of mirror solutions, combined determination of α around 1°
- adding charged modes: around 0.3°, below systematics due to isospin breaking effects
- caveat: assuming central values unchanged
- $B^{0,+} \to \pi^{0,+}\eta(')$: improvement of isospin breaking systematics

Thanks for your attention. Hope to meet you soon in Beijing !

Bonus track

Extension to $\rho\rho$ and $\rho\pi$

$\rho\rho$

- Analysis for each helicity state, dominated by longit. polarisation
- B^{+-} and B^{+0} five times larger than $\pi\pi$, B^{00} similar
- indicating a smaller penguin contamination than $\pi\pi$
- same inputs + (loose) info from S^{00}

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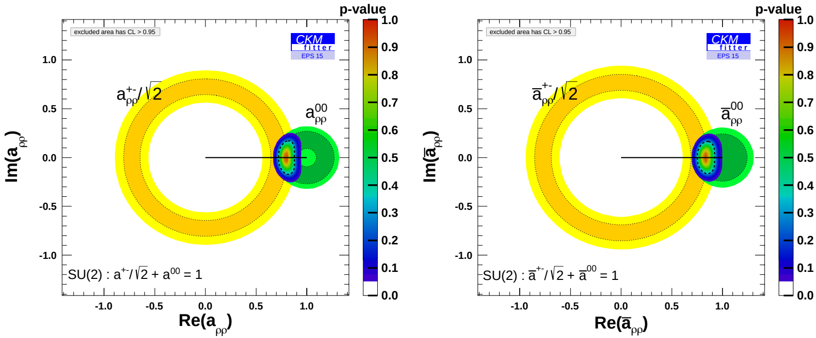
$\rho\pi$

- Dalitz plot for neutral B modes: $A_{3\pi} = f_+ A^{+-} + f_- A^{-+} + f_0 A^{00}$ with $A^{ij} = A(B^0 \to \rho^i \pi^j)$ and ρ^i line-shape f_i
- $\Gamma(t)$: interferences $f_i^* f_j$ and $A^{ij} A^{ij*}$ (coefficients $\mathcal{U}$ and $\mathcal{I}$)
- Coefficients $\mathcal{U}$ and $\mathcal{I}$ yield A^{ij} and $\bar{A}^{ij}$ for $+-$, $-+$, 00 providing α as relative phase between combinations of amplitudes
- Similar analysis for charged B modes, and since distinguishable particles, isospin yields pentagonal relations

$$A^{+-} + A^{-+} + 2A^{00} = \sqrt{2}(A^{+0} + A^{0+})$$

- Pentagonal rels. + Charged and neutral Dalitz $\Longrightarrow$ full α constraint

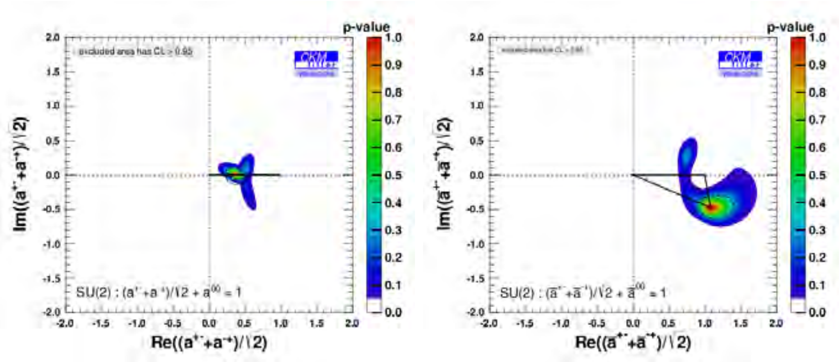
$\rho\rho$ triangle



Both triangles are flat

(four-fold degeneracy of the eight solutions : only two distinct solutions)

$\rho\pi$ analysis



Constraint on the reduced amplitude $(A^{+-}+A^{-+})/(A^{+0}+A^{0+})$

One of the two triangles is flat

Isospin breaking by penguins

Broken by electroweak $\Delta I = 1/2$ penguins in $B^+ \to h^+ h^0$

$$\begin{aligned} A^{+-} &= T^{+-} e^{-\imath\alpha} + P^{+-} \\ \sqrt{2} A^{00} &= T^{00} e^{-\imath\alpha} - P^{+-} + P_{EW}^{+0} \\ \sqrt{2} A^{+0} &= (T^{+-} + T^{00}) e^{-\imath\alpha} + P_{EW}^{+0} \end{aligned}$$

- Model-independent constraint from effective Hamiltonian analysis

$$\frac{P_{EW}^{+0}}{T^{+0} e^{-\imath\alpha}} \sim -\frac{3}{2} \frac{\mathcal{C}_9 + \mathcal{C}_{10}}{\mathcal{C}_1 + \mathcal{C}_2} \left| \frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right|$$

[Buras,Fleischer,Neubert,Rosner]

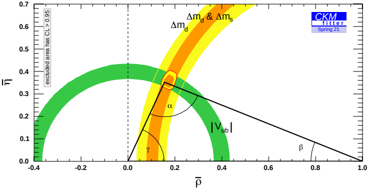
neglecting small ew ops $\mathcal{O}_7$ and $\mathcal{O}_8$, leading to theoretical estimate

$$\begin{aligned} r_{P_{EW}} &= \frac{P_{EW}^{+0}}{T^{+0}} = (3.23 \pm 0.30) \cdot 10^{-2} \\ \Delta\alpha &= \alpha - \alpha|_{r_{P_{EW}}=0} = \arcsin[r_{P_{EW}} \sin\alpha] < 1.9^\circ \end{aligned}$$

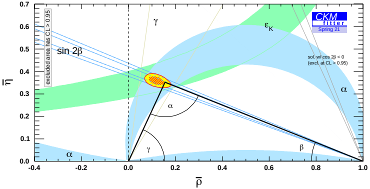
- Indirect constraint from the fit Charles, Deschamps, SDG, Niess

$$r_{P_{EW}}(\pi\pi) = (-8 \pm 16) \cdot 10^{-2} \qquad r_{P_{EW}}(\rho\rho) = (-2.3^{+10.5}_{-7.7}) \cdot 10^{-2}$$

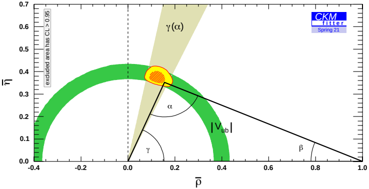
Consistency of the KM mechanism



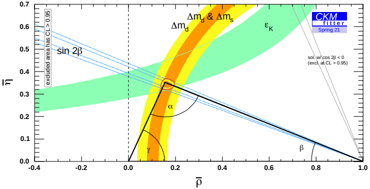
CP-allowed only



CP-violating only



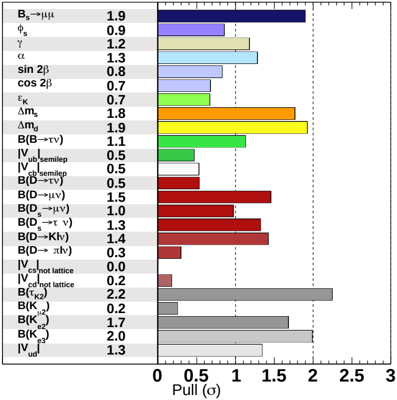
Tree only



Loop only

Validity of Kobayashi-Maskawa picture of *CP* violation

Pulls



- Pulls for various observables (included in the fit or not)
- For 1D, pull obs = $\sqrt{\chi^2_{\text{min; with obs}} - \chi^2_{\text{min; w/o obs}}}$
- If Gaussian errors, uncorrelated, random vars of mean 0 and variance 1
- Here correlations, and some pulls = 0 due to the Rfit model for syst

No indication of significant deviations from CKM picture