

中山大學天琴中心

TIANQIN CENTER FOR GRAVITATIONAL PHYSICS, SYSU

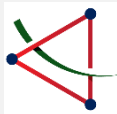


Phase transition and gravitational wave

—Higgs potential from 10^{-11} s of early universe to 2022

Fa Peng Huang (黄发朋)

Higgs potential 2022@Peking University, July 26, 2022
(Online talk)



Outline

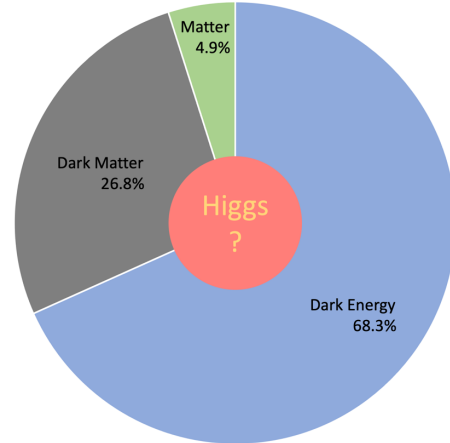
- 1. Motivation**
- 2. Electroweak Phase transition and phase transition gravitational wave in a nutshell**
- 3. Six important directions on Electroweak Phase transition**
- 4. Summary and outlook**



Motivation

Higgs@10:Higgs particle cosmology

After the discovery of 125 GeV scalar, it becomes a realistic portal to study the fundamental physics and its deep connections to cosmology



What is the role of Higgs in the early universe?

*Hi, I am Higgs boson!
I have many questions??*

*What is the my role in
Baryon asymmetry of
the universe??
Electroweak
baryogenesis??*

*What is the my role in
dark matter models??*

How to test?

*Hi, Higgs !
Let us help you to
explore your
confusion!*

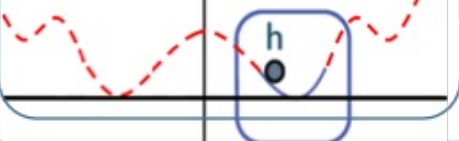
Future lepton colliders
(ILC, CEPC, FCC-ee)



Future Gravitational
wave experiments(LISA)

*Why my mass is so light
compared to Planck mass??
Cosmological relaxion??*

*What is my potential??
??*

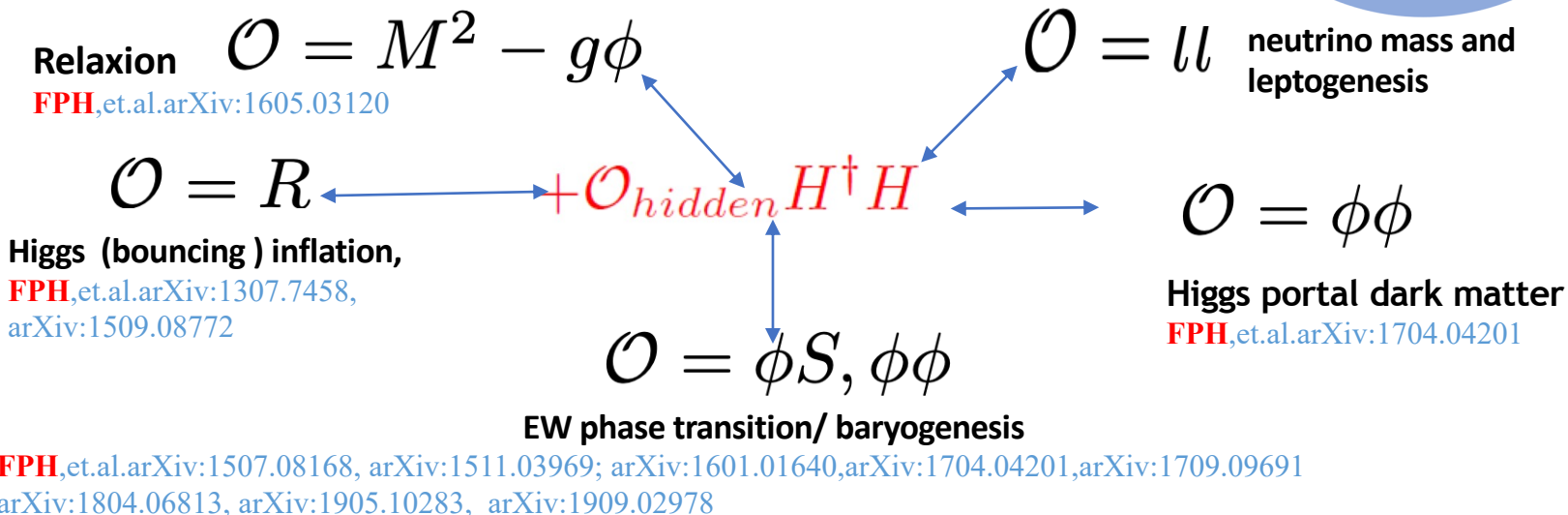
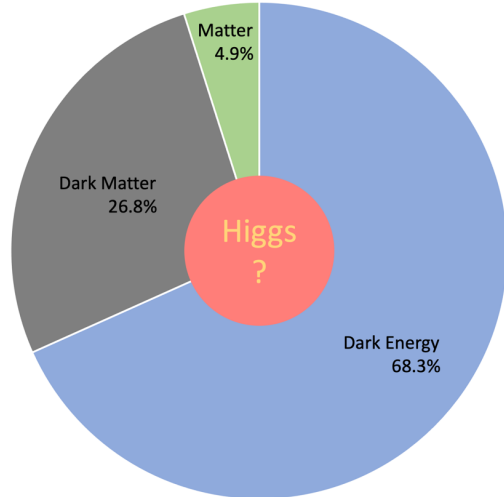




Motivation

Higgs@10: Higgs particle cosmology

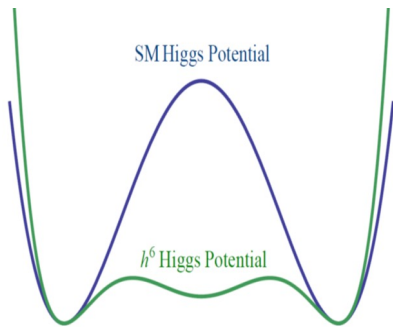
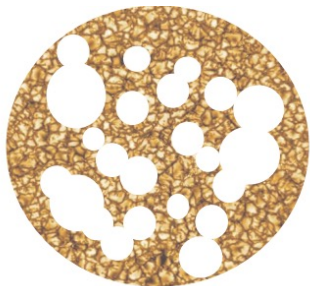
Higgs' deep connections to cosmology, such as Higgs (portal) inflation, Neutrino mass (leptogenesis), cosmological relaxation, **EW phase transition/baryogenesis, dark matter(DM)...**



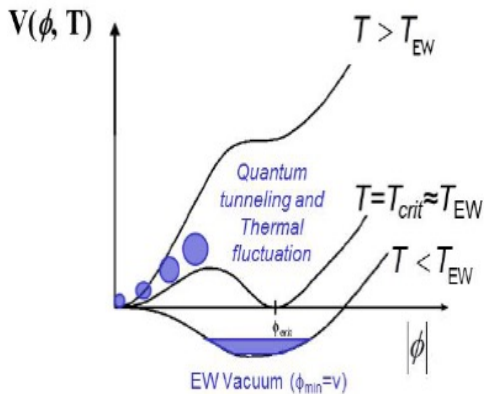


Motivation

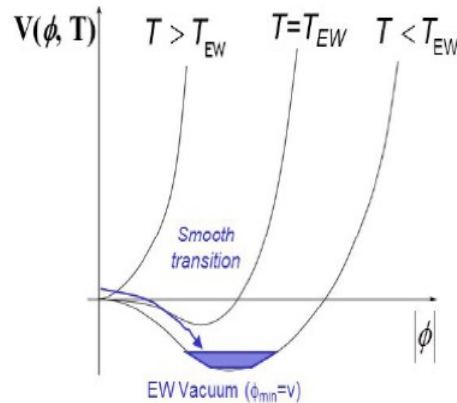
From
lattice
simulation



SFOPT for $m_H < 75$ GeV



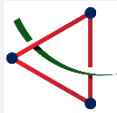
Cross over for $m_H > 75$ GeV



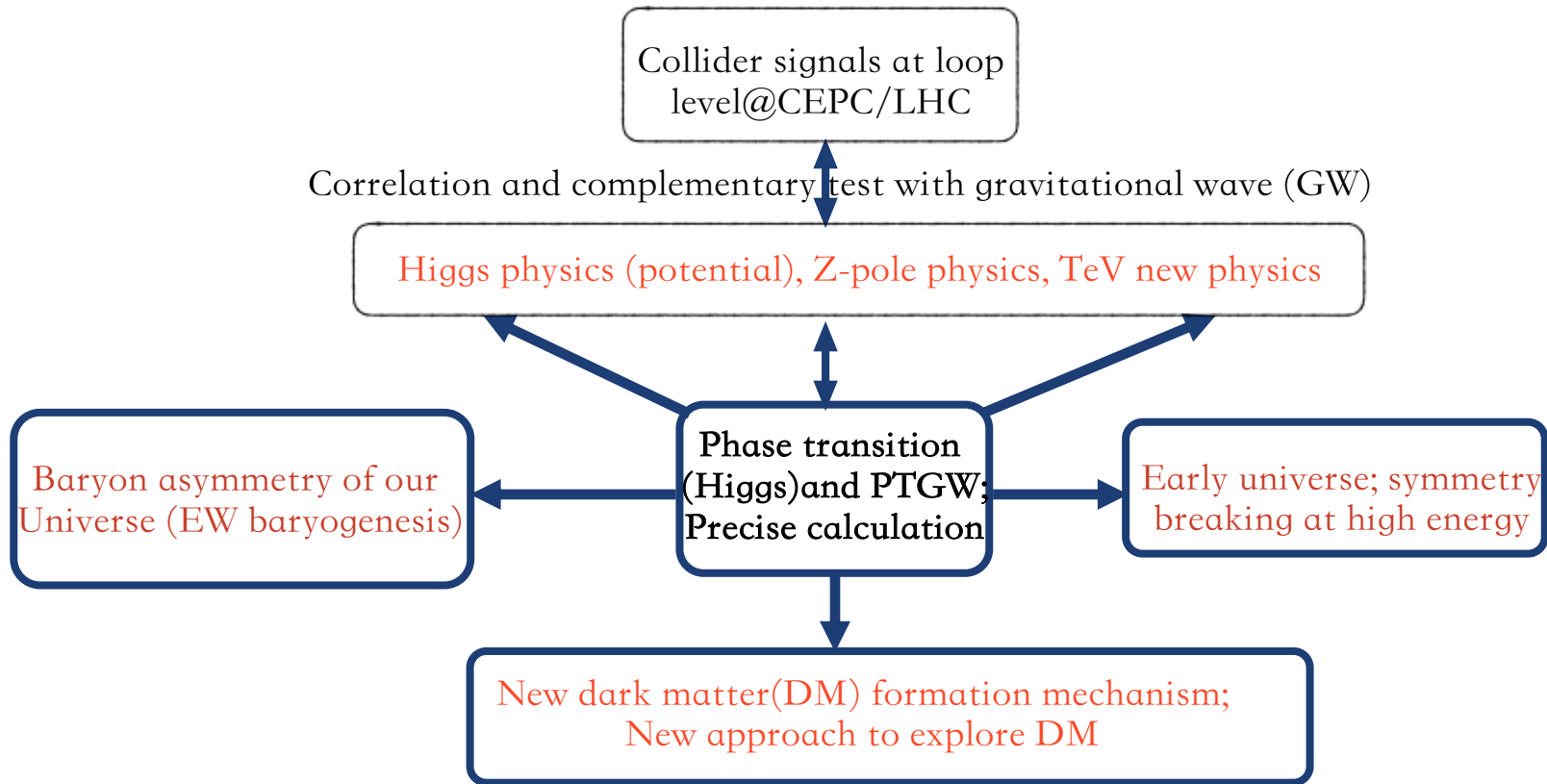
Extension of the Higgs sector is needed to SFOPT for 125 GeV Higgs boson.

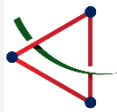
We discuss well-motivated extensions (baryogenesis, DM...) of Higgs section to realize strong first-order phase transition (SFOPT) with abundant cosmological effects.

EW phase transition and its GW signals becomes realistic after the discovery of Higgs by LHC and GW by LIGO.

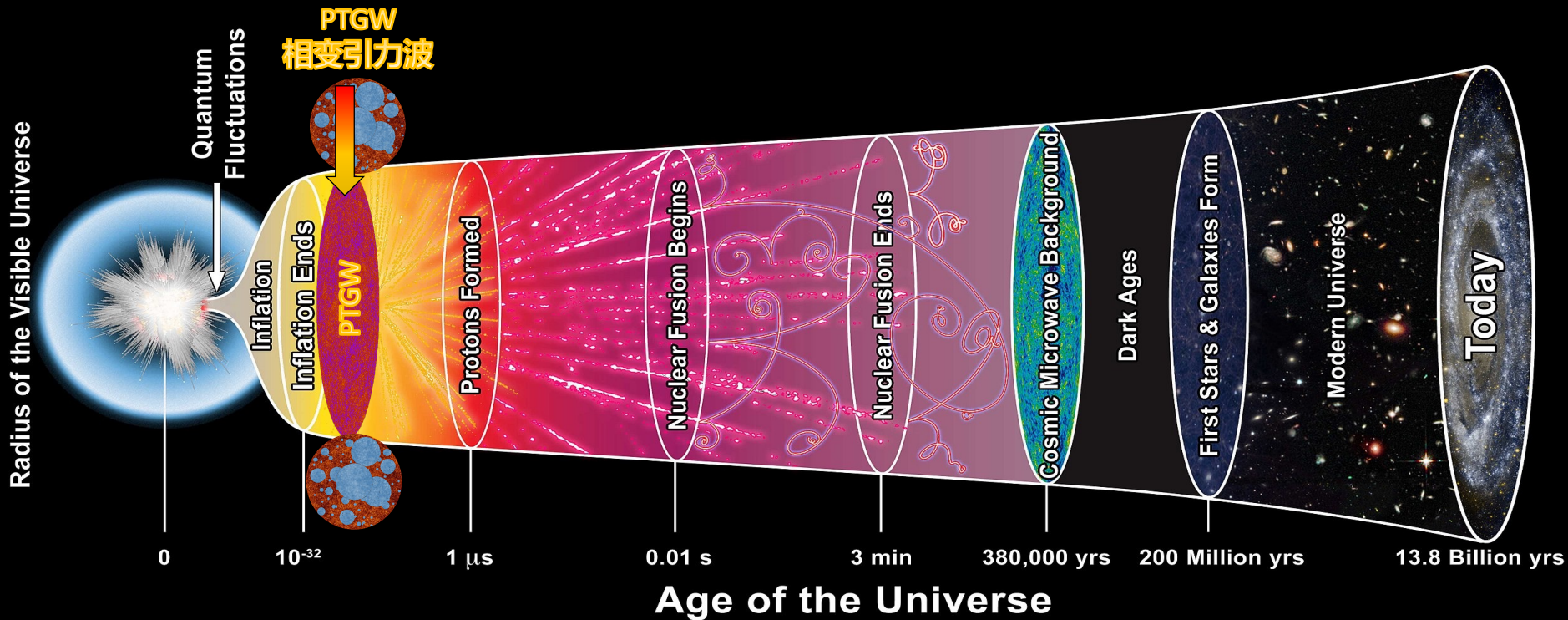


Motivation





Motivation

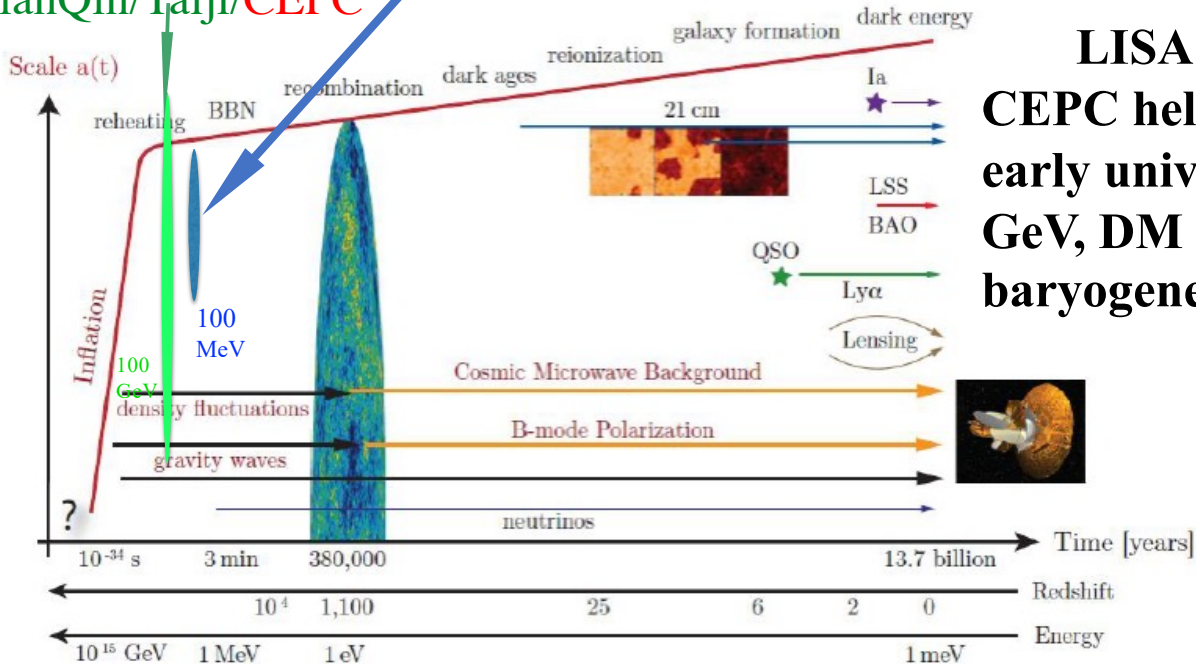




Motivation

EW phase transition/
baryogenesis:
LISA/TianQin/Taiji/CEPC

QCD phase transition and
axion cold DM:
SKA, FAST, GBT



LISA in synergy with CEPC helps to explore the early universe around 100 GeV, DM and baryogenesis.

credit:D.Baumann

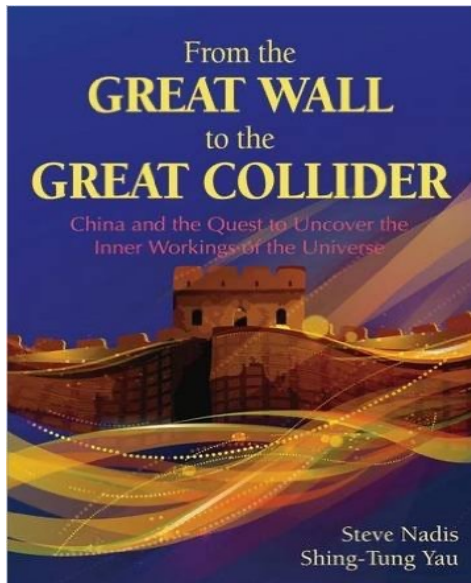


Motivation

Complementary

Particle approach
CEPC/SppC, FCC etc.

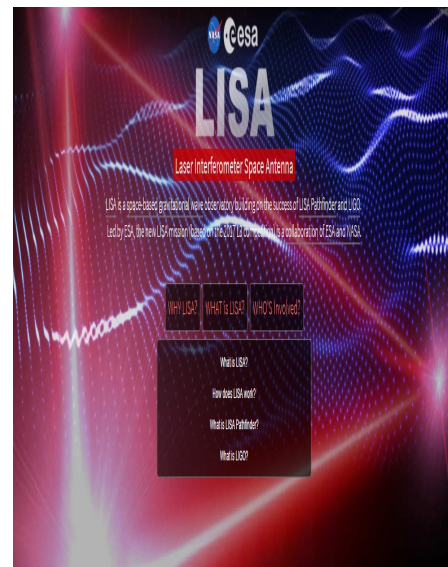
Wave approach
LISA/TianQin/Taiji ~2034



Relate by Higgs physics: EW phase transition/baryogenesis



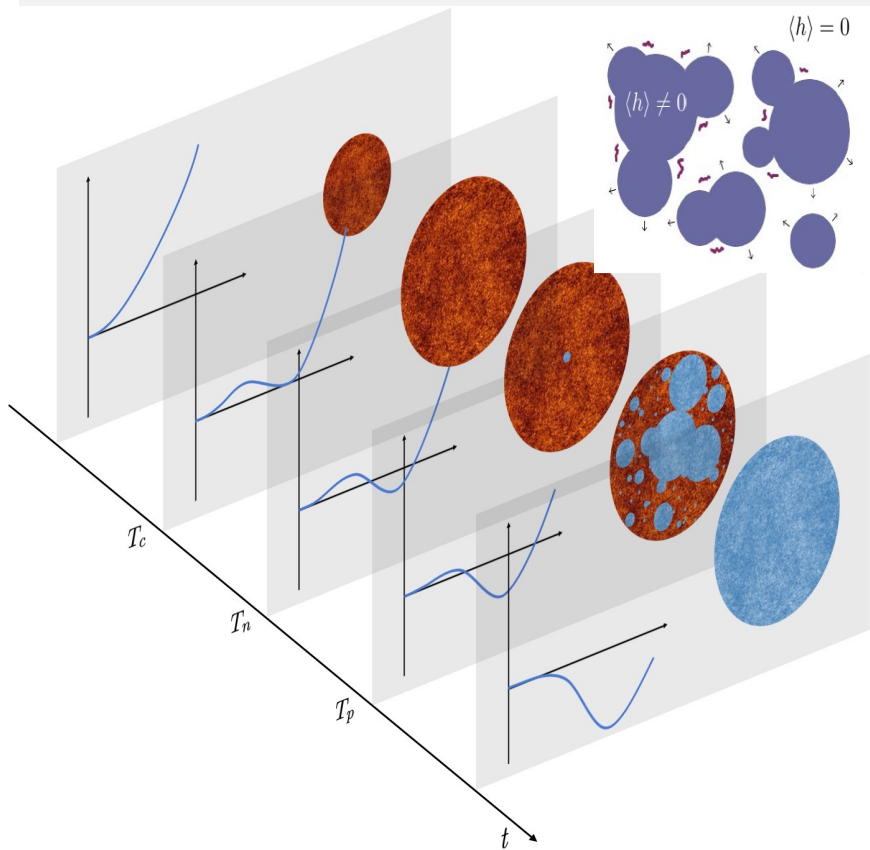
Double test on the Higgs potential and baryogenesis, DM



“天琴”
“Harpe in space”



Phase transition GW in a nutshell



calculate the finite-temperature effective potential using the thermal field theory: free energy density.

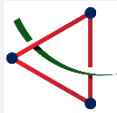
$$V_{\text{eff}}^{(1)}(\bar{\phi}) = \sum_i n_i \left[\int \frac{d^D p}{(2\pi)^D} \ln(p^2 + m_i^2(\bar{\phi})) + J_{\text{B,F}} \left(\frac{m_i^2(\bar{\phi})}{T^2} \right) \right]$$

$$S(T) = \int d^4 x \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 + V_{\text{eff}}(\phi, T) \right]$$

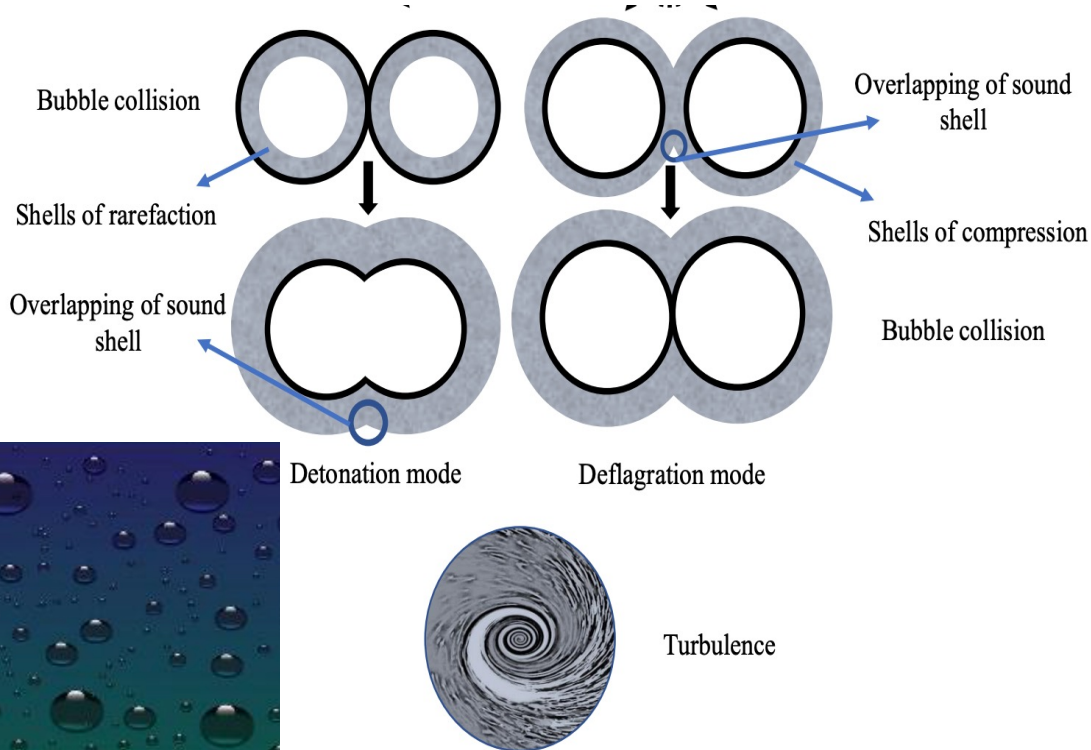
$$\Gamma = \Gamma_0 e^{-S(T)}$$

这世上的热闹，源自遂穿

Xiao Wang, **FPH**, Xinmin Zhang, JCAP05(2020)045

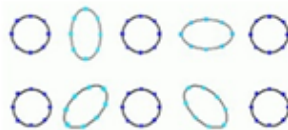


Phase transition GW in a nutshell



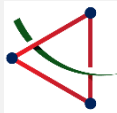
$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$h_{ij} \simeq \frac{2G}{c^4 r} \ddot{Q}_{ij}^{TT}(t - r/c)$$



E. Witten, Phys. Rev. D 30, 272 (1984)
C. J. Hogan, Phys. Lett. B 133, 172 (1983);
M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D 49, 2837 (1994))
EW phase transition GW becomes more interesting and realistic after the discovery of Higgs by LHC and GW by LIGO.

Xiao Wang, **FPH**, Xinmin Zhang, JCAP05(2020)045



Phase transition GW in a nutshell

$$\ddot{h}_{ij}(\mathbf{x}, t) + 3H \dot{h}_{ij}(\mathbf{x}, t) - \frac{\nabla^2}{a^2} h_{ij}(\mathbf{x}, t) = 16\pi G \overset{\cdot}{\Pi}_{ij}(\mathbf{x}, t)$$

Possible sources of **tensor anisotropic stress** in the early universe

- Scalar field gradients $\Pi_{ij} \sim [\partial_i \phi \partial_j \phi]^{TT}$ eg. Collisions of bubble walls
- Bulk fluid motion $\Pi_{ij} \sim [\gamma^2 (\rho + p) v_i v_j]^{TT}$ eg. Sound waves and turbulence in the fluid
- Gauge fields $\Pi_{ij} \sim [-E_i E_j - B_i B_j]^{TT}$ eg. Primordial magnetic fields (MHD turbulence)
- Second order scalar perturbations, Π_{ij} from a combination of $\partial_i \Psi, \partial_i \Phi$
- ... [arXiv:1801.04268](https://arxiv.org/abs/1801.04268)



Phase transition GW in a nutshell

Bubble collisions

$$h^2\Omega_{\text{co}}(f) \simeq 1.67 \times 10^{-5} \left(\frac{H_* R_*}{(8\pi)^{1/3}} \right)^2 \left(\frac{\kappa_\phi \alpha}{1 + \alpha} \right)^2 \left(\frac{100}{g_*} \right)^{1/3} \frac{0.11 v_b}{0.42 + v_b^2} \frac{3.8 (f/f_{\text{co}})^{2.8}}{1 + 2.8 (f/f_{\text{co}})^{3.8}}$$

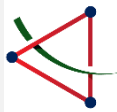
Turbulence

$$h^2\Omega_{\text{turb}}(f) \simeq 1.14 \times 10^{-4} H_* R_* \left(\frac{\kappa_{\text{turb}} \alpha}{1 + \alpha} \right)^{3/2} \left(\frac{100}{g_*} \right)^{1/3} \frac{(f/f_{\text{turb}})^3}{(1 + f/f_{\text{turb}})^{11/3} (1 + 8\pi f/H_*)}$$

Sound wave

$$h^2\Omega_{\text{sw}}(f) \simeq 1.64 \times 10^{-6} (H_* \tau_{\text{sw}}) (H_* R_*) \left(\frac{\kappa_v \alpha}{1 + \alpha} \right)^2 \left(\frac{100}{g_*} \right)^{1/3} (f/f_{\text{sw}})^3 \left(\frac{7}{4 + 3(f/f_{\text{sw}})^2} \right)^{7/2}$$

E. Witten, Phys. Rev. D 30, 272 (1984); C. J. Hogan, Phys. Lett. B 133, 172 (1983);
M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D 49, 2837 (1994); Mark
Hindmarsh, et al., PRL 112, 041301 (2014); Lots of unlisted papers.
See Huai-ke and Ligong's works for current constraints from GW data



Phase transition Dynamics

Precise predictions on the phase transition dynamics and its GW signals

1. Precisely calculate the finite-temperature effective potential $V_{eff}(\phi, T)$

(1). Daisy resummation problem: [Pawani scheme vs. Arnold scheme](#)

(2). Gauge dependence problem: [see Michael J. Ramsey-Musolf's works](#)

(3). No perturbative calculations: lattice calculations, ([see Michael J. Ramsey-Musolf's works](#), also [Ligong's recent work](#))

and dim-reduction method: by [D. Weir, Michael J. Ramsey-Musolf et.al](#)

2. Reliable calculations of bubble wall velocity v_b

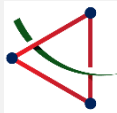
α T_p $R_* H_*$
S. Hoche, J. Kozaczuk, A. J. Long, J. Turner and Y. Wang, [arXiv:2007.10343](#),
[Avi Friedlander](#), [Ian Banta](#), [James M. Cline](#), [David Tucker-Smith](#), [arXiv:2009.14295v2](#)

Xiao Wang, **FPH**, Xinmin Zhang, [arXiv:2011.12903](#)

3. Energy budget during phase transition κ

F. Giese, T. Konstandin, K. Schmitz and J. van de Vis, [arXiv:2010.09744](#)

Xiao Wang, **FPH** and Xinmin Zhang, [arXiv:2010.13770](#)

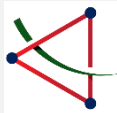


Phase transition Dynamics

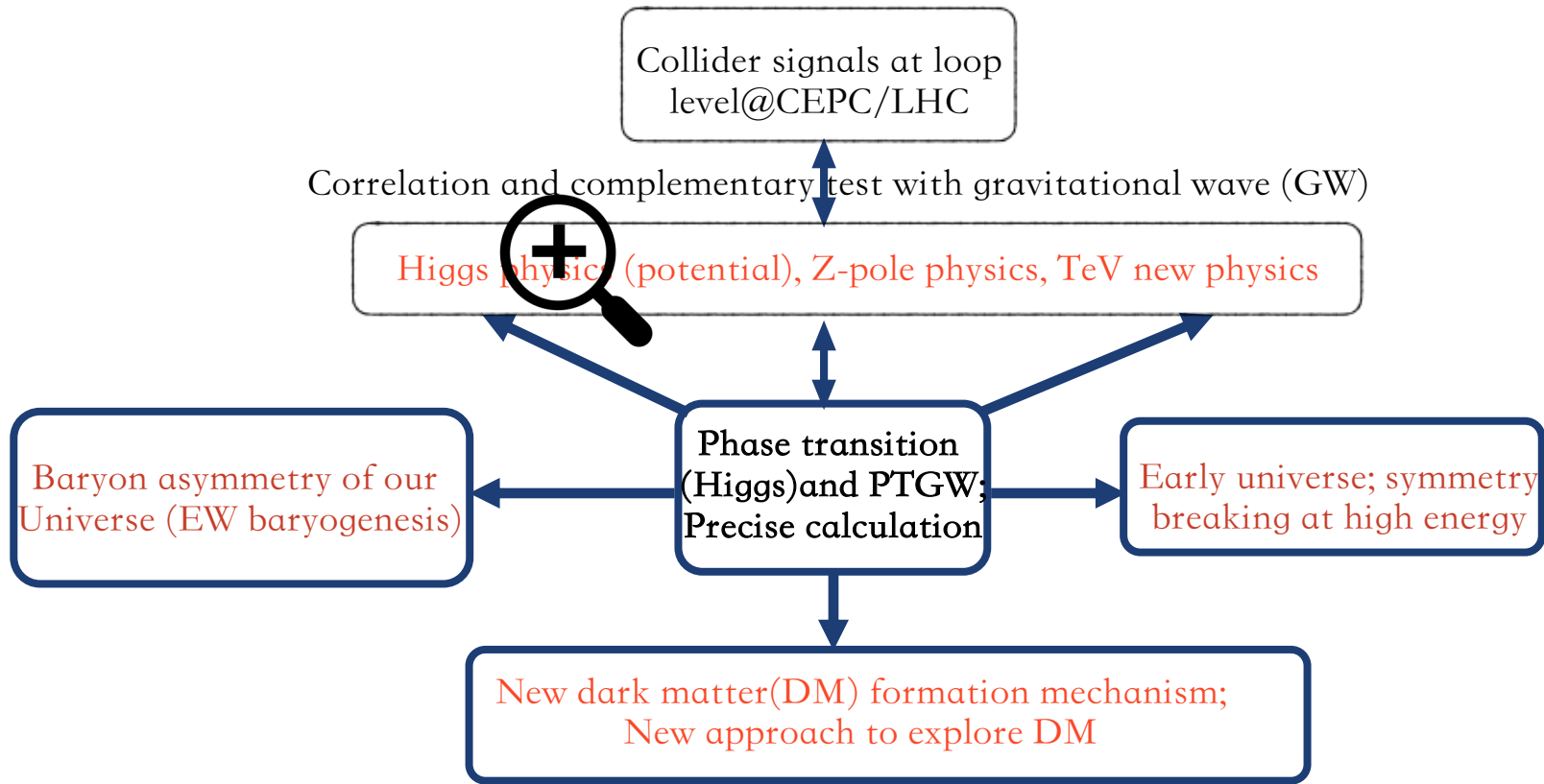
Classify the SFOPT into four cases:

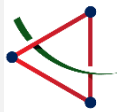
- **Slight supercooling:** $\alpha_p < 0.1$ the GW is too weak to be detected by LISA, might be within the sensitivity of BBO and ultimate-DECIGO.
- **Mild supercooling:** $0.1 < \alpha_p < 0.5$ the GW could be detected by LISA, TianQin, Taiji, BBO, DECIGO.
- **Strong supercooling:** $0.5 < \alpha_p < 1$
- **Ultra supercooling:** $\alpha_p > 1$

Xiao Wang, **FPH**, Xinmin Zhang, JCAP05(2020)045



Motivation

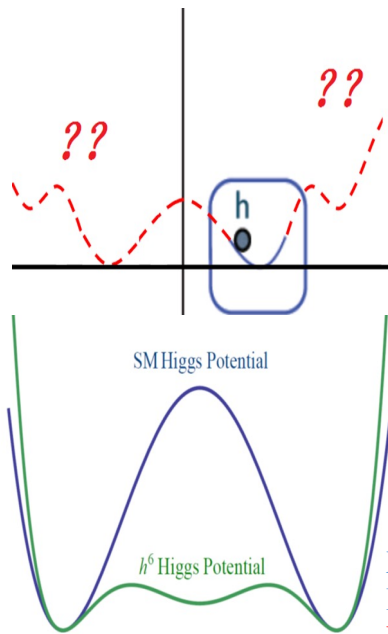




SFOPT and Higgs potential

What is the shape of Higgs potential?

Current data tells us nothing but the quadratic oscillation around the VEV 246 GeV with 125 GeV mass.



$$V(h) = \frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4}h^4$$

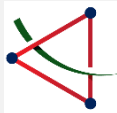
or
$$V(h) = \frac{1}{2}\mu^2 h^2 - \frac{\lambda}{4}h^4 + \frac{1}{\Lambda^2}h^6$$

Produce a SFOPT, large deviation of Higgs trilinear coupling and GW

Xinmin Zhang Phys.Rev. D47 (1993) 3065-3067; C. Grojean, G. Servant, J. Well PRD71(2005)036001
D.J.H. Chung, A. J. Long, Lian-tao Wang Phys.Rev. D87(2013) 023509

FPH, et.al, Phys.Rev.D94(2016)no.4,041702 ; FPH, et.al, Phys.Rev.D93 (2016) no.10,103515

arXiv:1511.06495, Nima Arkani-Hamed et. al.; PreCDR of CEPC; arXiv: [1811.10545](https://arxiv.org/abs/1811.10545), CDR of CEPC



SFOPT and Higgs potential

SFOPT leads to obvious deviation of the tri-linear Higgs coupling

See Prof. Lilin Yang's talk

$$\mathcal{L}_{hhh} = -\frac{1}{3!}(1 + \delta_h)A_h h^3$$

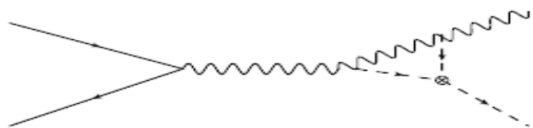
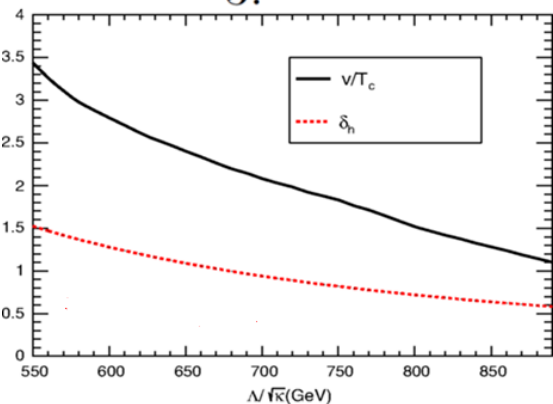
At one-loop level, deviation of the tri-linear Higgs coupling

$$\delta_h \in (0.6, 1.5)$$

The Circular Electron Positron Collider (CEPC), ILC, FCC-ee can precisely test this scenario by precise measurements of the hZ cross section ($e^-e^+ \rightarrow hZ$).

SM NNLO hZ cross section recently by Lilin Yang, et al 2016, Yu Jia et al 2016

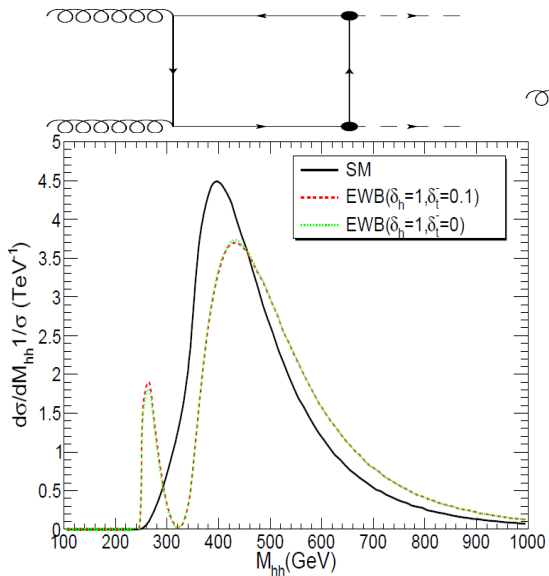
$$\delta_\sigma = \frac{\sigma_{hz, \delta_h \neq 0}}{\sigma_{hz, SM}} - 1$$





SFOPT and Higgs potential

Hints at hadron collider: Modify the invariant mass distribution of Higgs pair due to interference effects:



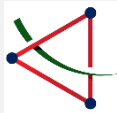
➤ Two peaks for the baryogenesis scenario, one peak for the SM.

➤ Due to the difficulties to suppress backgrounds at the LHC, it will be difficult to completely pin down these anomalous coupling at 14 TeV LHC, even with 3000 ab^{-1} integrated luminosity.

➤ Exploiting boosted tricks helps to increase ability to extract the anomalous couplings.

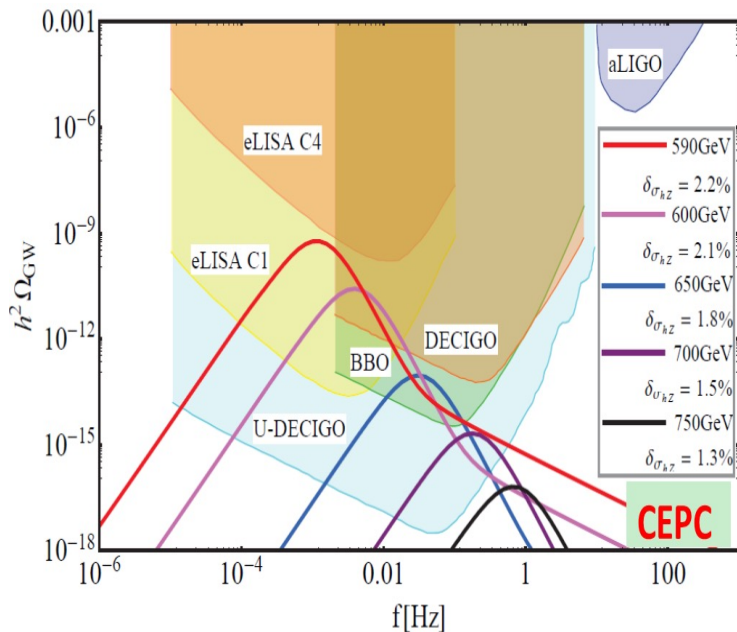
➤ More precise information may come from future 100 TeV hadron collider, such as SppC, or future lepton collider, such as CEPC.

See Prof. Shufang Su,
Lilin Yang,
Yaquan FANG,
Zhen Liu's talk



SFOPT and Higgs potential

Correlate particle collider and GW signals: double test on Higgs potential from particle to wave



For CEPC with 10 ab^{-1} at $\sqrt{s} = 240 \text{ GeV}$, precision of σ_{zh} may be about 0.4% and can test the scenario.

- LISA, BBO, U-DECIGO are capable of detection
- The study on EW phase transition naturally bridges the particle physics at collider with GW survey and baryogenesis

FPH, et.al, Phys.Rev.D94(2016)no.4,041702 ; FPH, et.al, Phys.Rev.D93 (2016) no.10,103515



SFOPT and Higgs potential

SM EFT

$$\mathcal{L} \supset -\mu^2 |H|^2 - \lambda |H|^4 + c_6 |H|^6$$

$$+ c_T \mathcal{O}_T + c_{WW} \mathcal{O}_{WW} + \text{other dimension-six operators}$$

$$\delta_{\sigma(hZ)} \approx (0.26c_{WW} + 0.01c_{BB} + 0.04c_{WB} - 0.06c_H - 0.04c_T + 0.74c_L^{(3)\ell}$$

$$+ 0.28c_{LL}^{(3)\ell} + 1.03c_L^\ell - 0.76c_R^e) \times 1 \text{ TeV}^2 + 0.016\delta_h,$$

SFOPT produces large modification of trilinear Higgs coupling

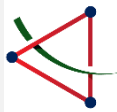
δ_h

c_6

dominates the hZ cross section deviation

Taking a general study of the scalar extended models and the composite Higgs model as examples, we find that the Higgs sextic scenario still works well after considering all the dim-6 operators and the precise measurements.

Qing-Hong Cao, FPH, Ke-Pan Xie, Xinmin Zhang, arXiv:1708.0473,
See Jianghao Yu's papers for more systematical study from SM EFT



SFOPT and Higgs potential

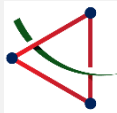
One examples

Qing-Hong Cao, **FPH**, Ke-Pan Xie, Xinmin Zhang, arXiv:1708.0473,
See Jianghao Yu's papers for more systematical study from SM EFT

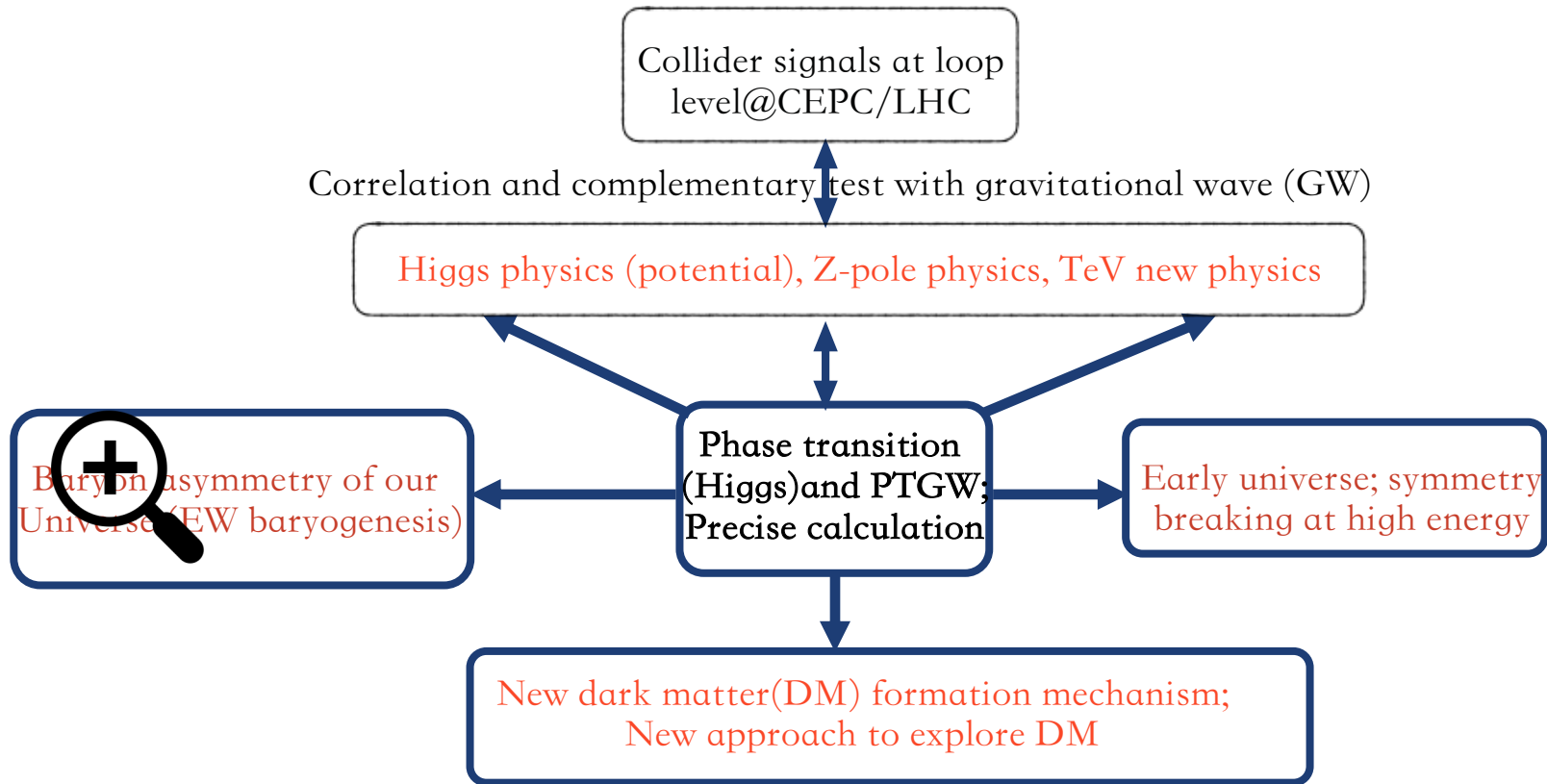
$$\delta\mathcal{L} = D_\mu\Phi^\dagger D^\mu\Phi - M_\Phi^2\Phi^\dagger\Phi - \frac{\lambda_\Phi}{4}(\Phi^\dagger\Phi)^2 - \lambda_1\Phi^\dagger\Phi H^\dagger H - \lambda_2|\Phi \cdot H|^2 \\ - \lambda_3[(\Phi \cdot H)^2 + h.c.] + (\eta_H|H|^2 + \eta_\Phi|\Phi|^2)(\Phi \cdot H + h.c.),$$

Using **Covariant Derivative Expansion method**, the matched dim-6 operators and their coefficients in the doublet scalar models are obtained:

Dimension-six operator	Wilson coefficient
$\mathcal{O}_{WW} = g^2 H ^2W_{\mu\nu}^aW^{a,\mu\nu}$	$c_{WW} = \frac{1}{(4\pi)^2} \frac{1}{48} (2\lambda_1 + \lambda_2) \frac{1}{M_\Phi^2}$
$\mathcal{O}_{2W} = -\frac{1}{2}(D^\mu W_{\mu\nu}^a)^2$	$c_{2W} = \frac{1}{(4\pi)^2} \frac{g^2}{60} \frac{1}{M_\Phi^2}$
$\mathcal{O}_{3W} = \frac{1}{3!} g\epsilon^{abc}W_\rho^{a\mu}W_\mu^{b\nu}W_\nu^{c\rho}$	$c_{3W} = \frac{1}{(4\pi)^2} \frac{g^2}{60} \frac{1}{M_\Phi^2}$
$\mathcal{O}_{BB} = g'^2 H ^2B_{\mu\nu}B^{\mu\nu}$	$c_{BB} = \frac{1}{(4\pi)^2} \frac{1}{48} (2\lambda_1 + \lambda_2) \frac{1}{M_\Phi^2}$
$\mathcal{O}_{WB} = gg'H^\dagger\sigma^aHW_{\mu\nu}^aB^{\mu\nu}$	$c_{WB} = \frac{1}{(4\pi)^2} \frac{\lambda_2}{24} \frac{1}{M_\Phi^2}$
$\mathcal{O}_{2B} = -\frac{1}{2}(\partial^\mu B^{\mu\nu})^2$	$c_{2B} = \frac{1}{(4\pi)^2} \frac{g'^2}{60} \frac{1}{M_\Phi^2}$
$\mathcal{O}_H = \frac{1}{2}(\partial_\mu H ^2)^2$	$c_H = \frac{1}{(4\pi)^2} [6\eta_\Phi\eta_H + \frac{1}{12}(4\lambda_1^2 + 4\lambda_1\lambda_2 + \lambda_2^2 + 4\lambda_3^2)] \frac{1}{M_\Phi^2}$
$\mathcal{O}_T = \frac{1}{2}(H^\dagger\overleftrightarrow{D}_\mu H)^2$	$c_T = \frac{1}{(4\pi)^2} \frac{1}{12} (\lambda_2^2 - 4\lambda_3^2) \frac{1}{M_\Phi^2}$
$\mathcal{O}_r = H ^2 D_\mu H ^2$	$c_r = \frac{1}{(4\pi)^2} (6\eta_\Phi\eta_H + \frac{1}{6}(\lambda_2^2 + 4\lambda_3^2)) \frac{1}{M_\Phi^2}$
$\mathcal{O}_6 = H ^6$	$c_6 = \eta_H^2 + \frac{1}{(4\pi)^2} [\frac{3}{2}\lambda_\Phi\eta_H^2 + 6\eta_\Phi(\lambda_1 + \lambda_2) - \frac{1}{6}(2\lambda_1^3 + 3\lambda_1^2\lambda_2 + 3\lambda_1\lambda_2^2 + \lambda_2^3) - 2(\lambda_1 + \lambda_2)\lambda_3^2] \frac{1}{M_\Phi^2}$



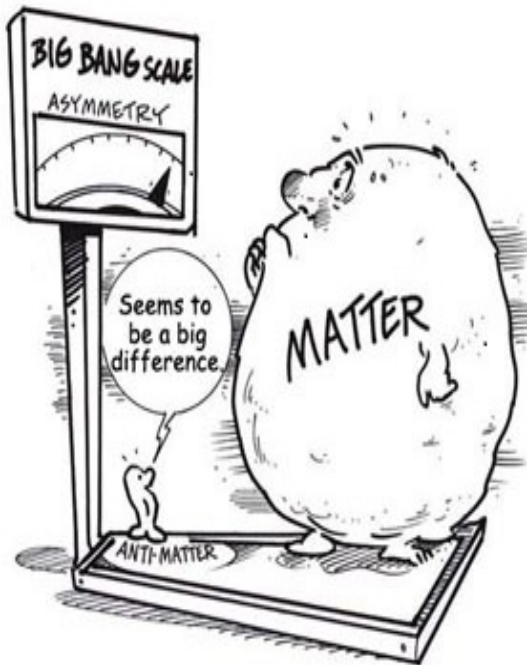
Motivation





SFOPT and EW baryogenesis

A long standing problem in particle cosmology is the origin of baryon asymmetry of the universe (BAU).



After the discovery of the Higgs boson by LHC and GW by aLIGO, electroweak (EW) baryogenesis becomes a timely and testable scenario for explaining the BAU.

$$\eta_B = n_B/n_\gamma = 5.8 - 6.6 \times 10^{-10}$$

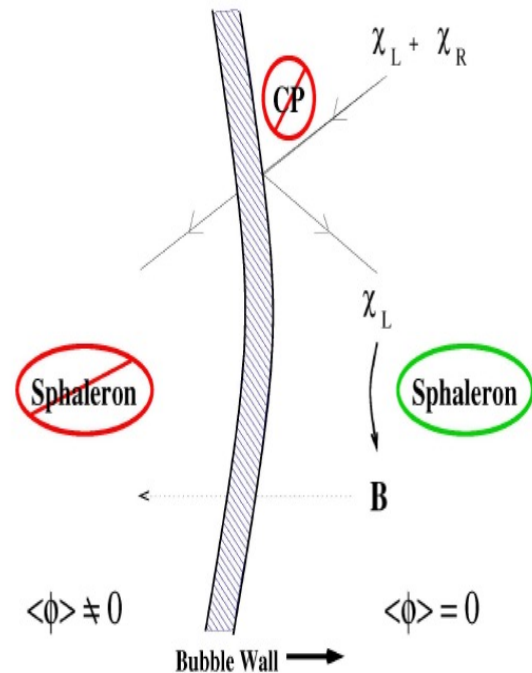
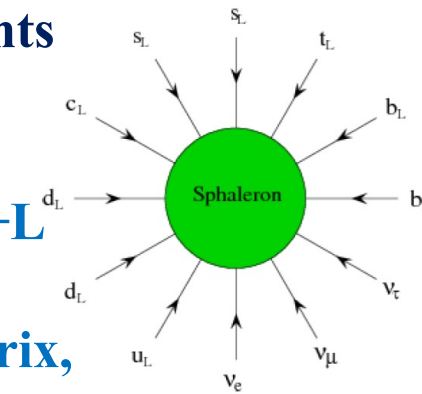
(CMB, BBN)



SFOPT and EW baryogenesis

SM technically has all the 3 elements for baryogenesis (Sakharov conditions)

- **B violation from anomaly in B+L current;**
- **C and CP-violation: CKM matrix, but too weak, need new CP-violating sources;**
- **Departure from thermal equilibrium: SFOPT with expanding Higgs bubble wall**



D. E. Morrissey and M. J. Ramsey-Musolf, *New J. Phys.* 14, 125003 (2012).

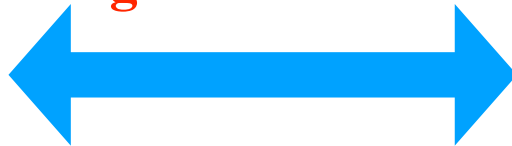


SFOPT and EW baryogenesis

Dynamical CP violation for baryogenesis

Large enough
CP-violating source
for successful
EW baryogenesis

Strong tension in most cases



pretty small
CP-violation
to avoid strong EDM
constraints

EDM, ACME Collaboration, i.e. $|d_e| < 1.1 \times 10^{-29} \text{ cm} \cdot e$ at 90% C.L. (Nature vol.562,357,2018)

How to alleviate this tension for successful baryogenesis?



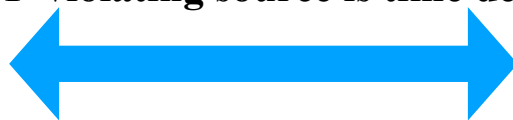
SFOPT and EW baryogenesis

Question: How to alleviate the tension for successful baryogenesis ?

Answer: Dynamical CP-violating source

Large enough
CP-violating source
for successful
EW baryogenesis

Alleviate by assuming the
CP-violating source is time dependent



Dynamical/cosmological evolve

Negligible
CP-violating source
at current time
to avoid strong EDM
constraints

- **Effective field theory:** **FPH**, Zhuoni Qian, Mengchao Zhang, Phys.Rev. D98 (2018) no.1, 015014; **FPH**, Chong Sheng Li, Phys. Rev. D 92, 075014 (2015);
- **Renormalizable model:** Complex 2HDM, Xiao Wang, FPH, Xinmin Zhang, arXiv: 1909.02978, work in progress with Eibun Senaha, Xiao Wang in an extended IDM model

Baltes, T. Konstandin and G. Servant, arXiv:1604.04526; I. Baltes, T. Konstandin and G. Servant, JHEP 1612, 073 (2016); S. Bruggisser, T. Konstandin and G. Servant, JCAP 1711, no. 11, 034 (2017)



SFOPT and EW baryogenesis

Taking the effective scenario as a representative example:

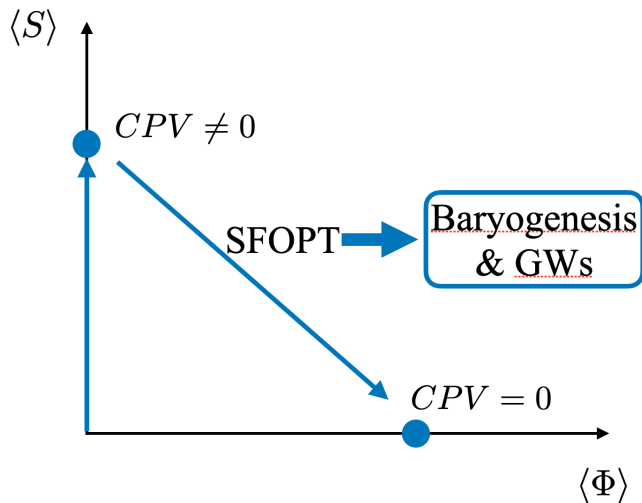
$$\mathcal{L}_{\text{SM}} - y_t \frac{\eta}{\Lambda} S \bar{Q}_L \tilde{\Phi} t_R + \text{H.c} + \frac{1}{2} \partial_\mu S \partial^\mu S + \frac{1}{2} \mu^2 S^2 - \frac{1}{4} \lambda S^4 - \frac{1}{2} \kappa S^2 (\Phi^\dagger \Phi)$$

Phys.Rev. D98 (2018) no.1, 015014 , **FPH**, Zhuoni Qian, Mengchao Zhang

The singlet and the dim-5 operator can come From many types composite Higgs model, arXiv:0902.1483 , arXiv:1703.10624 ,arXiv:1704.08911,

J. R. Espinosa, B. Gripaios, T. Konstandin and F. Riva, JCAP **1201**, 012 (2012)

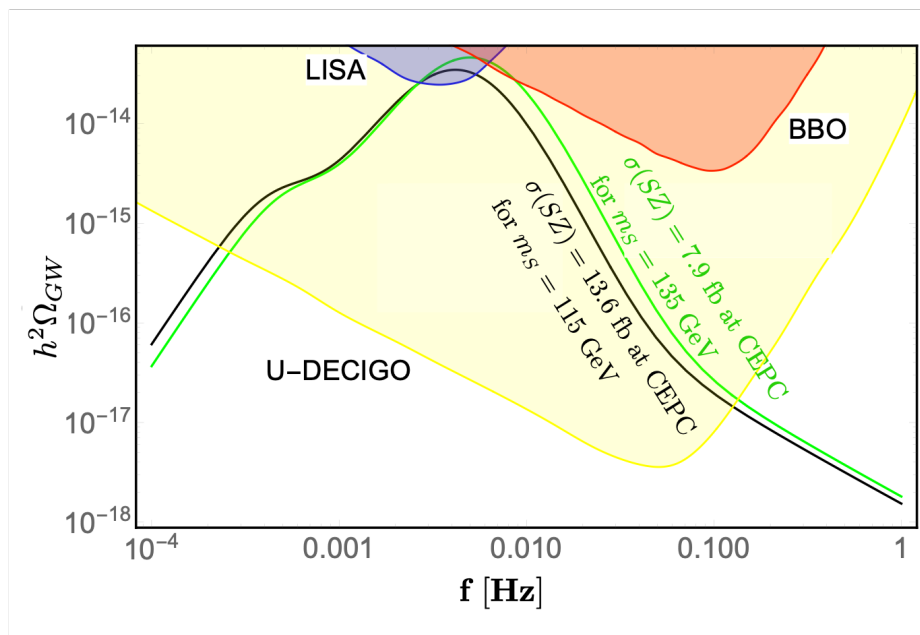
J. M. Cline and K. Kainulainen, JCAP **1301**, 012 (2013)



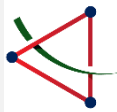


SFOPT and EW baryogenesis

The correlation between the future GW and collider signals



Phys.Rev. D98 (2018) no.1, 015014 , **FPH**, Zhuoni Qian, Mengchao Zhang



SFOPT and EW baryogenesis

We study a simple model for the successful DM and EW baryogenesis with dynamical CP-violating source.

[arXiv:1905.10283](https://arxiv.org/abs/1905.10283), FPH, Eibun Senaha and work in progress with Eibun Senaha

$$V_0(\Phi, \eta) = \mu_1^2 \Phi^\dagger \Phi + \mu_2^2 \eta^\dagger \eta + \frac{\lambda_1}{2} (\Phi^\dagger \Phi)^2 + \frac{\lambda_2}{2} (\eta^\dagger \eta)^2 + \lambda_3 (\Phi^\dagger \Phi) (\eta^\dagger \eta) \\ + \lambda_4 (\Phi^\dagger \eta) (\eta^\dagger \Phi) + \left[\frac{\lambda_5}{2} (\Phi^\dagger \eta)^2 + \text{h.c.} \right],$$

The new lepton Yukawa interaction is

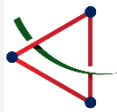
$$-\mathcal{L}_Y \ni y_{ij} \bar{\ell}_{iL} \eta E_{jR} + m_{E_i} \bar{E}_{iL} E_{iR} + \text{h.c.}$$

vector-like lepton (E_i)

D. Borah, S. Sadhukhan and S. Sahoo, Phys. Lett. B **771**, 624 (2017).

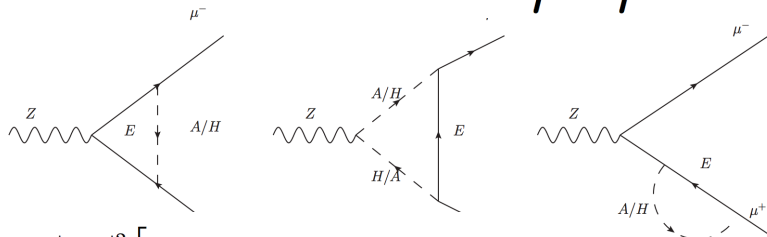
L. Calibbi, R. Ziegler and J. Zupan, JHEP **1807**, 046 (2018).

D. Borah, P. S. B. Dev and A. Kumar, Phys. Rev. D **99**, no. 5, 055012 (2019).



SFOPT and EW baryogenesis

Indirect search by $Z \rightarrow \mu^+ \mu^-$



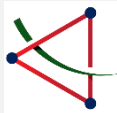
$$\Delta g_{Z\bar{\mu}\mu}^L = \frac{3|y_{\mu E}|^2}{32\pi^2} \left[\tilde{F}_3(m_E, m_H, m_A) + \sum_{\phi=H,A} \left\{ \left(-\frac{1}{2} + s_W^2 \right) F_2(m_E, m_\phi) + s_W^2 F_3(m_E, m_\phi) \right\} \right] R_{\mu/e} = \frac{\Gamma(Z \rightarrow \mu^+ \mu^-)}{\Gamma(Z \rightarrow e^+ e^-)}$$

An important missing observable in many previous study!

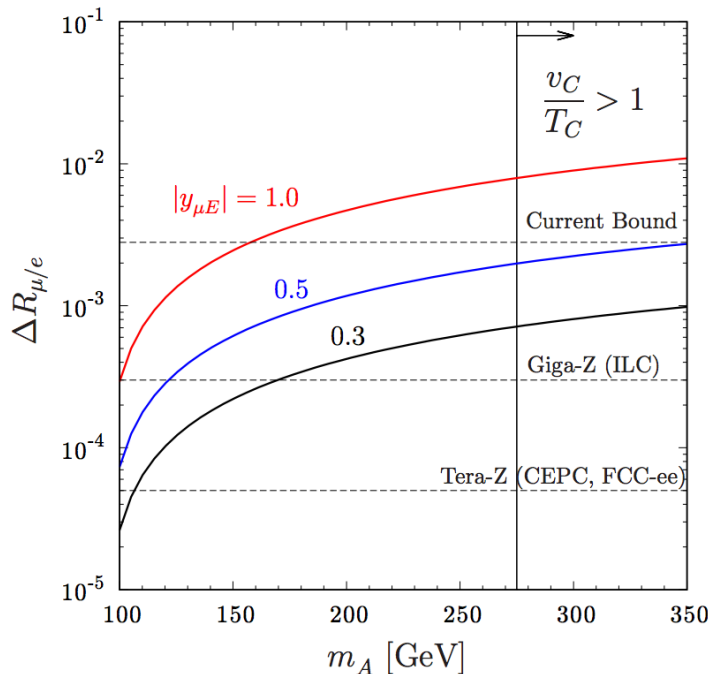
deviation of $R_{\mu/e}$ from the SM value as

Large enhancement of Z boson decay by the requirements of EW baryogenesis and DM. Further generalisation of this enhancement effects from the aspects of symmetry breaking is working in progress.

$$\Delta R_{\mu/e} \equiv \frac{R_{\mu/e} - R_{\mu/e}^{\text{SM}}}{R_{\mu/e}^{\text{SM}}} \simeq \frac{2g_{Z\bar{\mu}\mu}^{L,\text{SM}} \text{Re}(\Delta g_{Z\bar{\mu}\mu}^L) + |\Delta g_{Z\bar{\mu}\mu}^L|^2}{|g_{Z\bar{\mu}\mu}^{L,\text{SM}}|^2 + |g_{Z\bar{\mu}\mu}^{R,\text{SM}}|^2}$$

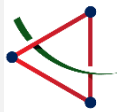


SFOPT and EW baryogenesis

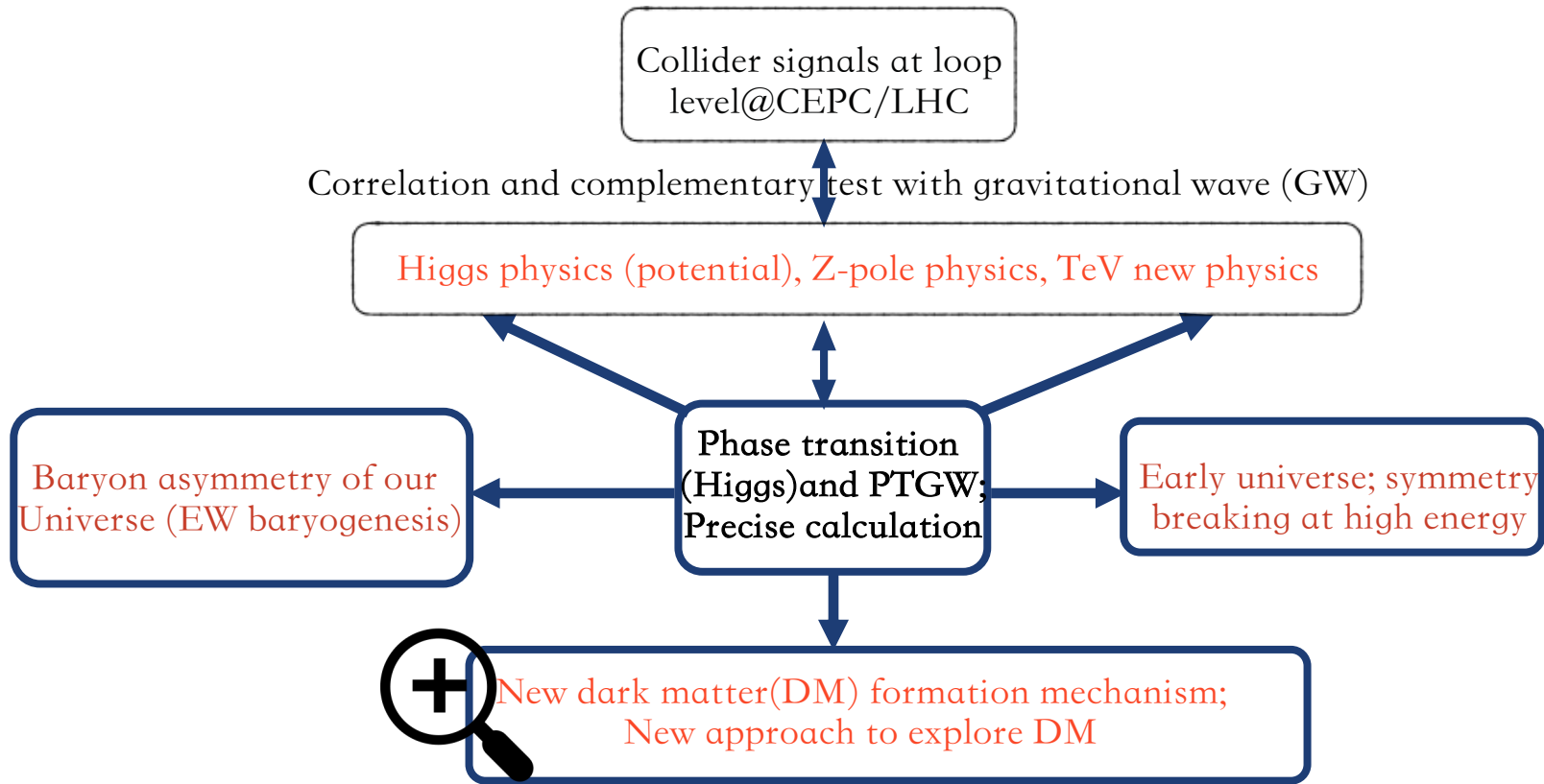


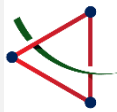
To satisfy the EW strong first-order phase transition (baryogenesis) and DM it requires the large mass splitting of the scalar mass spectrum in the same multiplet, which leads to significant enhancement of the Z boson decay.

Tera-Z can be a new indirect search to explore DM and baryogenesis.



Motivation





SFOPT and new DM mechanism/signal

New dark matter (DM) production scenario filtered by the bubbles of the SFOPT in the early universe.

The cosmic phase transition with Q-balls production can explain baryogenesis and DM simultaneously, where constraints on DM mass and reverse dilution are significantly relaxed. We study how to probe this scenario by GW signals and collider signals at QCD NLO.

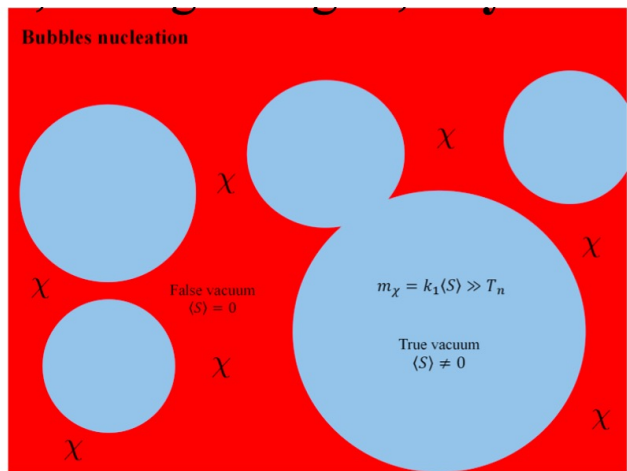
FPH, Chong Sheng Li, *Phys. Rev. D*96 (2017) no.9, 095028

$$\rho_{\text{DM}}^4 v_b^{3/4} = 73.5 (2\eta_B s_0)^3 \lambda_S \sigma^4 \Gamma^{3/4}$$

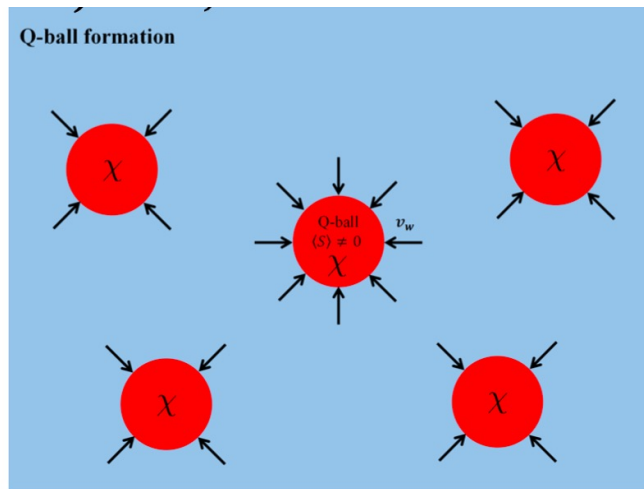


SFOPT and new DM mechanism/signal

SFOPT naturally correlates DM, baryogenesis, particle collider and GW signals.

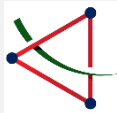


(a) Bubble nucleation: χ particles trapped in the false vacuum due to Boltzmann suppression

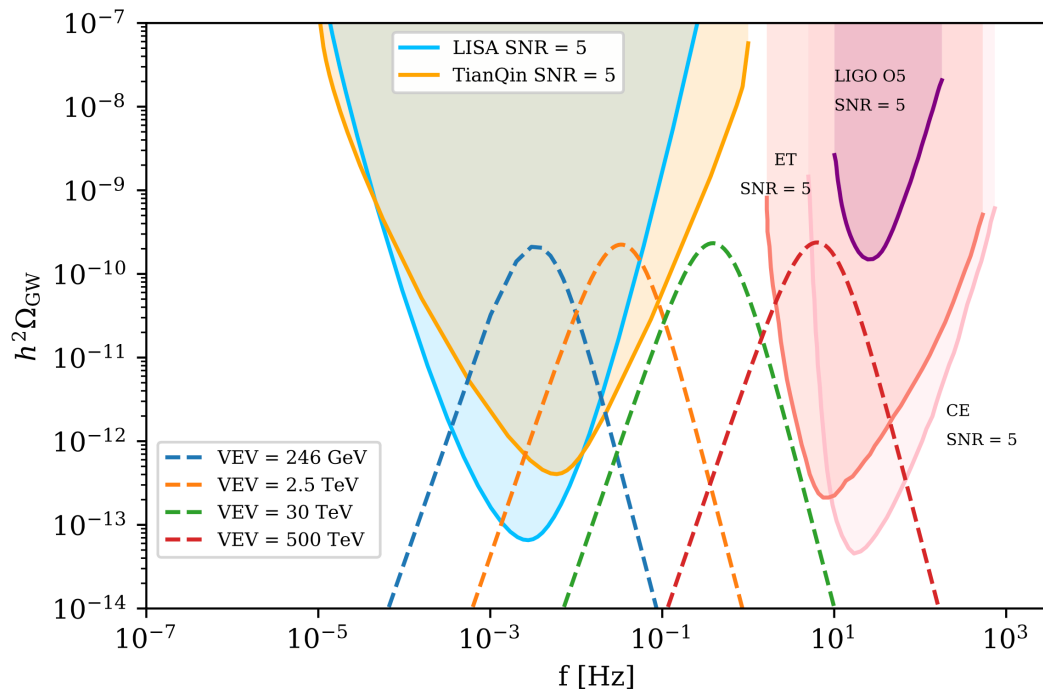


(b) Q-ball formation: After the formation of Q-balls, they should be squeezed by the true vacuum

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028; FPH, Xiao Wang, this work:arXiv:2208:xxxxx



SFOPT and new DM mechanism/signal



FPH, Xiao Wang, this work:arXiv:2208:xxxxx



SFOPT and new DM mechanism/signal

In the recent two years, this dynamical DM formed by phase transition has become a new idea and attracted more and more attentions.

Namely, bubbles in SFOPT can be the “filters” to packet your needed heavy DM.

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028

arXiv:1912.04238, Dongjin Chway, Tae Hyun Jung, Chang Sub Shin

arXiv:1912.02830, Michael J. Baker, Joachim Kopp, and Andrew J. Long

arXiv:2012.15113, Wei Chao, Xiu-Fei Li, Lei Wang

arXiv:2101.05721, Aleksandr Azatov, Miguel Vanvlasselaer, Wen Yin

arXiv:2103.09827, Pouya Asadi, Eric D. Kramer, Eric Kuflik, Gregory W. Ridgway, Tracy R. Slatyer, J. Smirnov

arXiv:2103.09822, Pouya Asadi, Eric D. Kramer, Eric Kuflik, Gregory W. Ridgway, Tracy R. Slatyer, J. Smirnov

arXiv:2008.04430 Jeong-Pyong Hong, Sunghoon Jung, Ke-pan Xie

See Ke-Pan's talk for the primordial black hole DM formed by EWPT





SFOPT and new DM mechanism/signal

- **New** The observation of GW by LIGO has initiated a new era of exploring DM by GW.
- **DM can trigger a SFOPT in the early universe, which can leads to detectable GW signals.**

Hearing the signal of dark sectors with gravitational wave detectors

J.Jaekel, V. V. Khoze, M. Spannowsky, Phys.Rev. D94 (2016) no.10, 103519

Zhaofeng Kang, et.al. arXiv:2101.03795

Zhaofeng Kang, et. al. arXiv:2003.02465

Yan Wang, Chong Sheng Li, and **FPH**, arXiv:2012.03920

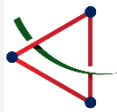
FPH, Eibun Senaha Phys.Rev. D100 (2019) no.3, 03501

FPH PoS ICHEP2018 (2019) 397

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028

FPH, Jiang-Hao Yu, Phys.Rev. D98 (2018) no.9, 095022

FPH, Xinmin Zhang, Phys.Lett. B788 (2019) 288-29



SFOPT and new DM mechanism/signal

Inert Doublet Models

$$V_0 = M_D^2 D^\dagger D + \lambda_D (D^\dagger D)^2 + \lambda_3 \Phi^\dagger \Phi D^\dagger D \\ + \lambda_4 |\Phi^\dagger D|^2 + (\lambda_5/2)[(\Phi^\dagger D)^2 + h.c.],$$

mixed singlet-doublet model

$$V_0 = \frac{1}{2} M_S^2 S^2 + M_D^2 H_2^\dagger H_2 + \frac{1}{2} \lambda_S S^2 |\Phi|^2 + \lambda_3 \Phi^\dagger \Phi H_2^\dagger H_2 \\ + \lambda_4 |\Phi^\dagger H_2|^2 + \frac{\lambda_5}{2} [(\Phi^\dagger H_2)^2 + H.c.] + A [S \Phi H_2^\dagger + H.c.].$$

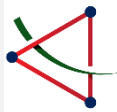
mixed singlet-triplet model

$$V_0 = \frac{1}{2} M_S^2 S^2 + M_\Sigma^2 \text{Tr}(H_3^2) + \kappa_\Sigma \Phi^\dagger \Phi \text{Tr}(H_3^2) \\ + \frac{\kappa}{2} |\Phi|^2 S^2 + \xi S \Phi^\dagger H_3 \Phi.$$

provide natural
DM candidate

produce SFOPT and phase transition
GW

Yan Wang, Chong Sheng Li, and **FPH**, arXiv:2012.03920; **FPH**, Jiang-Hao Yu, Phys.Rev. D98 (2018) no.9, 095022

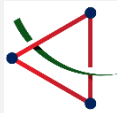


SFOPT and new DM mechanism/signal

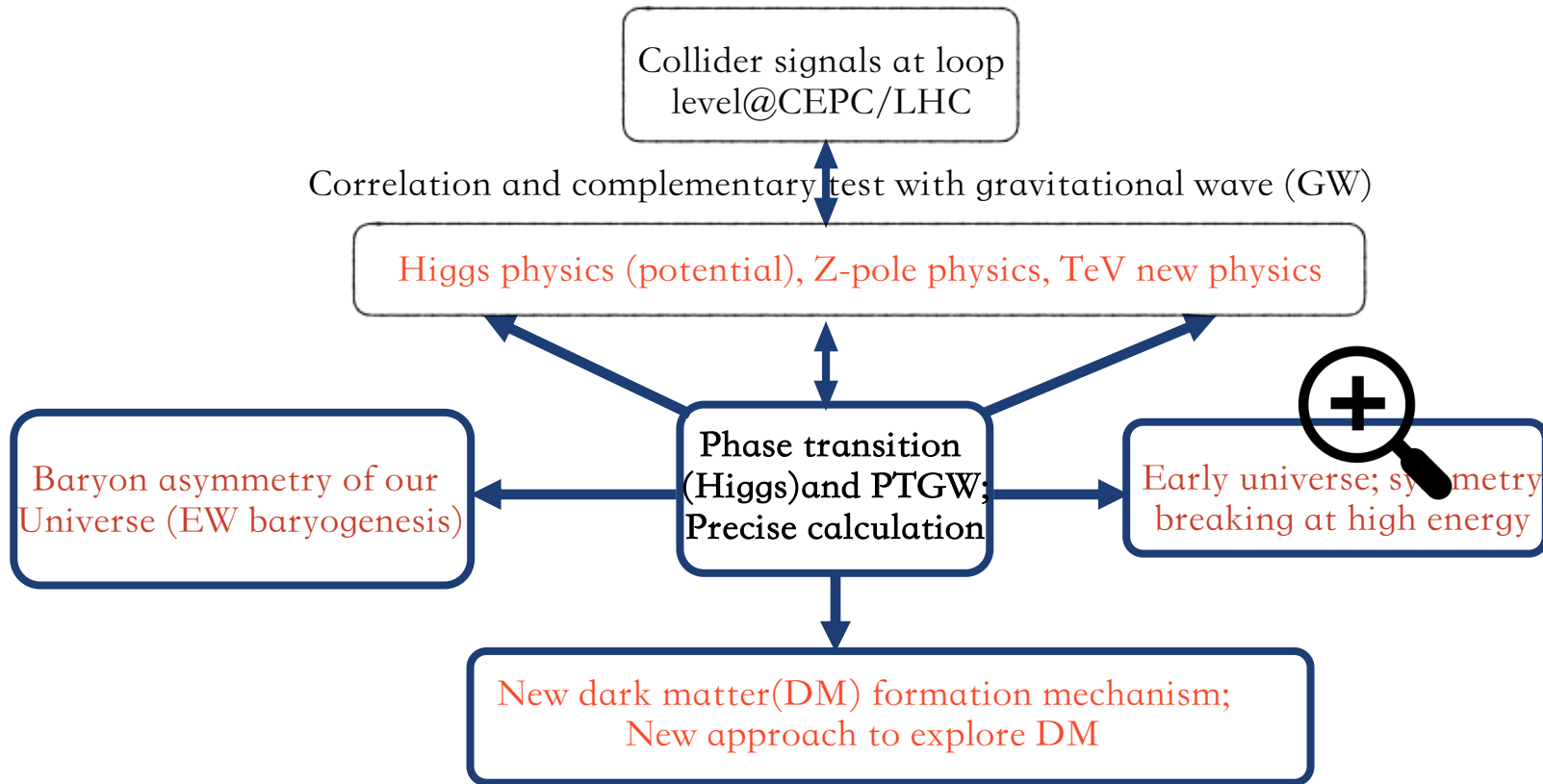
Phase transition GW can provide a unique way to probe many important physics processes: inflation, PQ-symmetry breaking, neutrino physics, axion physics, extra dimension, primordial magnetic field, cosmic defects...

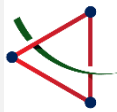
Wei-Chih Huang, et. al, arXiv: 2012.11614
Mark Hindmarsh, et. al. arXiv: 2011.12878
Bhupal Dev, et. al. arXiv: 1905.00891
Yiyang Zhang, et. al. arXiv:1902.02751
Yang Bai, et. al. arXiv:1810.04360
Andrew Long, et. al. arXiv:1703.04902
Graciela Gelmini, et. al. arXiv:2009.01903
Stephen King, et.al. arXiv:2005.13549
Bhupal Dev, et.al. arXiv:1602.04203
Astrid Eichhorn et.al. arXiv:2010.00017

Yuefeng Di, et. al., arXiv: 2012.15625
Haipeng An, et.al. arXiv:2009.12381
FPH, Xinmin Zhang, Phys.Lett. B788 (2019) 288-29,
Jia Liu, et.al. arXiv:2104.06421
Zhao Zhang, et. al. arXiv:2102.01588
Wei Liu, et.al. arXiv:2101.10469
Cheng-wei Chiang, et.al. arXiv:2012.14071
Ke-Pan Xie, et.al. arXiv:2011.04821
Ligong Bian, et.al. arxiv:1907.13589
Zhaofeng Kang, et.al. arXiv:2101.03795
Zhaofeng Kang, et. al. arXiv:2003.02465
a lot of new and nice works unmentioned here



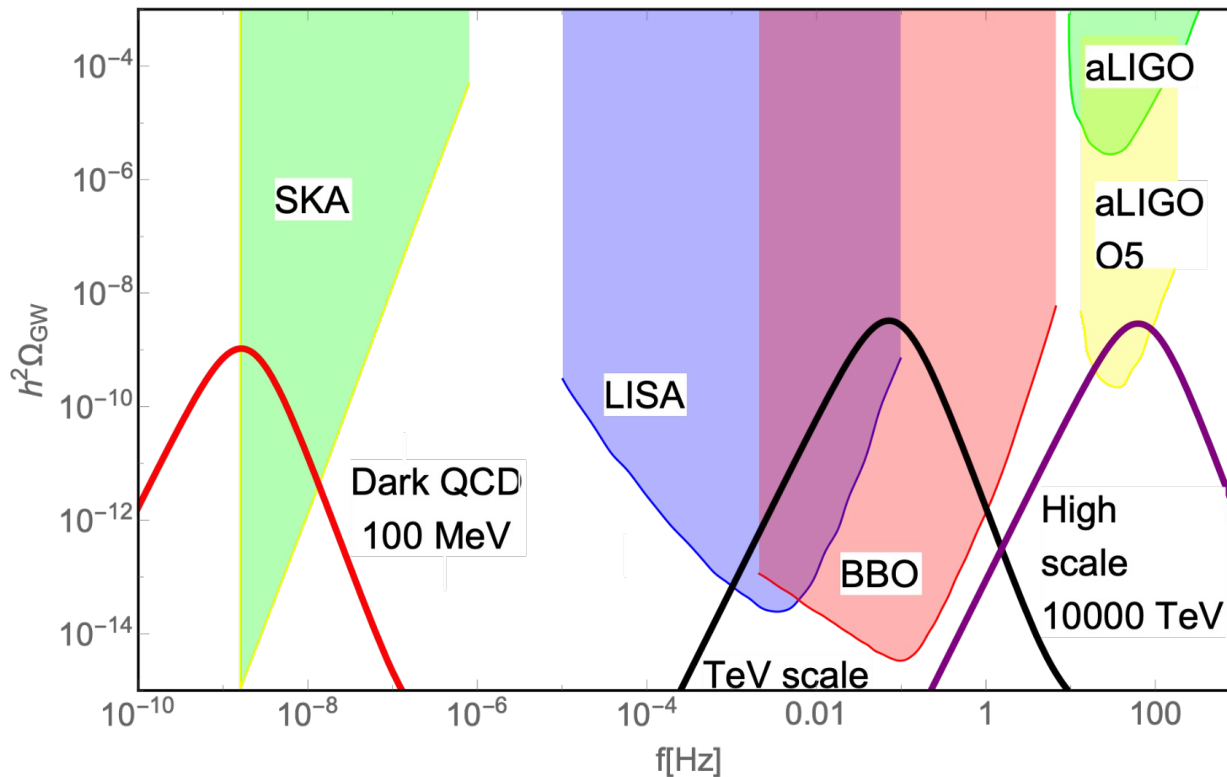
Motivation



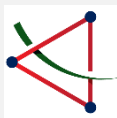


SFOPT and new physics/early universe

Generally



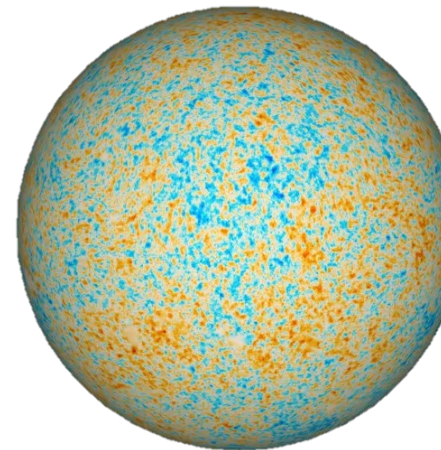
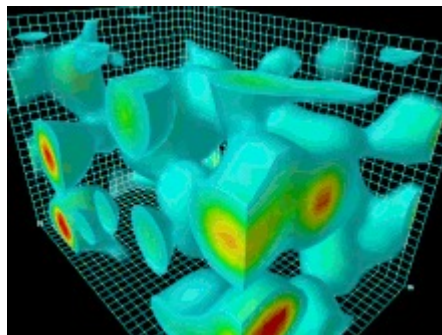
FPH, Xinmin Zhang, Phys.Lett. B788 (2019) 288-29,



PTGW anisotropy and its implication for primordial seeds of our universe

Primordial power spectra

$$P_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \left\langle |\mathcal{R}_k|^2 \right\rangle \Big|_{aH=k} = \frac{H^2}{\pi \epsilon_{\text{sr}} m_{\text{pl}}^2} \Big|_{aH=k} = A_s \left(\frac{k}{k_*} \right)^{n_s - 1}$$

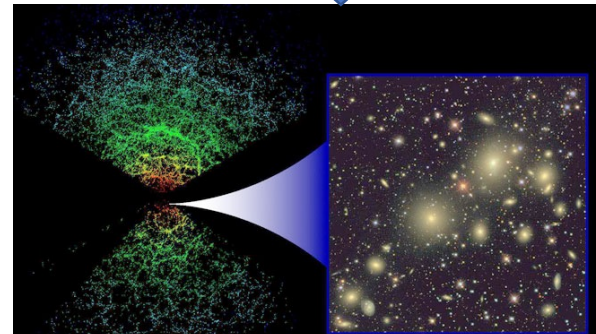


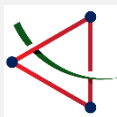
angular power spectra

$$C_l = \frac{2}{\pi} \int_0^\infty dk k^2 P_{\mathcal{R}}(k) |\mathcal{T}_l(k)|^2$$

$$C_l^{\text{obs}} = \frac{1}{2l+1} \sum_{\ell m} a_{\ell m} a_{\ell m}^*$$

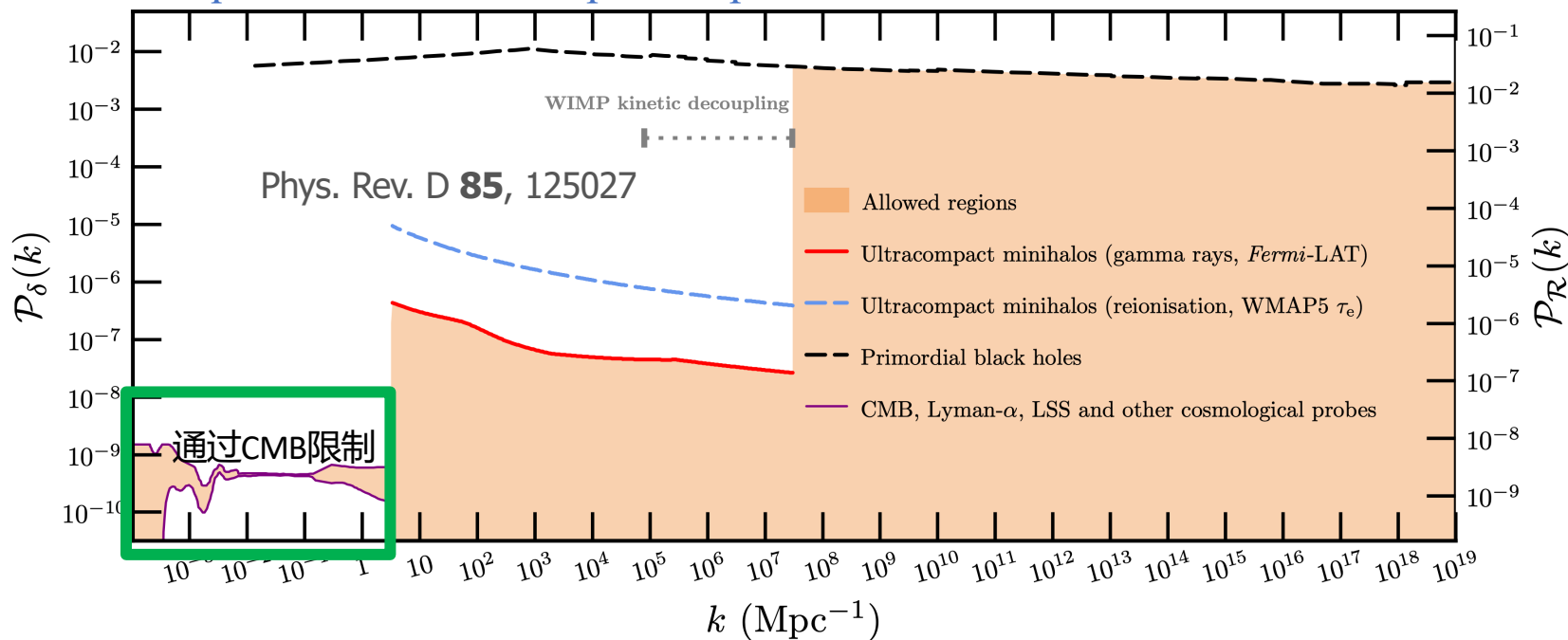
The Primordial density Perturbation (quantum fluctuation) from inflation or alternative as the origin of structure



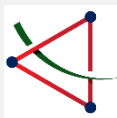


PTGW anisotropy and its implication for primordial seeds of our universe

The quest for small-scale power spectrum——The road less traveled



- ✓ A complete model of inflation requires a solid understanding of the small-scale primordial power spectrum;
- ✓ however, it is hard!



PTGW anisotropy and its implication for primordial seeds of our universe

$$H_*^2 = \rho / 3M_{\text{pl}}^2$$

$$h^2 \Omega_{\text{GW}}(f) \simeq 1.64 \times 10^{-6} \left(\frac{4}{3}\right)^{\frac{1}{2}} \left[(H_* R_*)^2 \left(\frac{\kappa_v \alpha}{1 + \alpha}\right)^{\frac{3}{2}} \times \left(\frac{100}{g_*}\right)^{\frac{1}{3}} (f/f_{\text{sw}})^3 \left(\frac{7}{4 + 3(f/f_{\text{sw}})^2}\right)^{\frac{7}{2}} \right]$$

$$f_{\text{sw}} \simeq 2.6 \times 10^{-5} \text{ Hz} \frac{1}{H_* R_*} \left(\frac{T_*}{100 \text{ GeV}}\right) \left(\frac{g_*}{100}\right)^{\frac{1}{6}} \quad \text{Peak frequency}$$

QCD-like DM model

phase transition strength

$$\alpha = 0.5$$

D.O.F

$$g_* = 10$$

efficiency factor

$$\kappa_v \approx 0.44$$

bubble wall velocity

$$v_b = 0.95$$

characteristic temperature

$$T_* = 1 \text{ MeV}$$

$$T_* = 5 \text{ MeV}$$

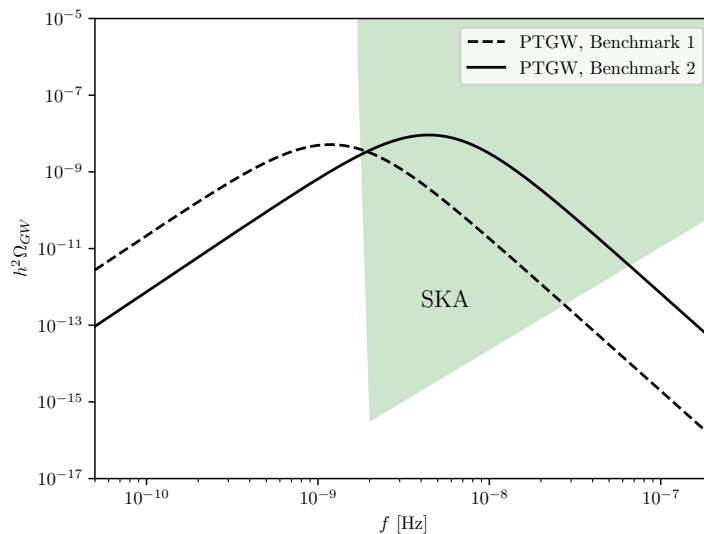
mean bubble separation

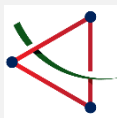
$$H_* R_* = 0.15$$

$$H_* R_* = 0.2$$

Benchmark 1

Benchmark 2





PTGW anisotropy and its implication for primordial seeds of our universe

Phys. Lett. B, 2017, 771: 9-12 ; Phys. Rev. D, 2019, 100(12): 121501 ; Phys. Rev. D, 2021, 103 (2): 023522 ; Phys. Rev. Lett., 2021, 127(27): 271301

distribution function $f(\eta, \mathbf{x}, \mathbf{p}) = \bar{f}(\eta, p) - p \frac{\partial \bar{f}(\eta, p)}{\partial p} \mathcal{G}(\eta, \mathbf{x}, \hat{\mathbf{p}})$

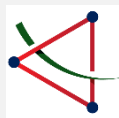
$$\mathcal{G}(\eta, \mathbf{x}, \hat{\mathbf{p}})$$

quantify the inhomogeneity of graviton

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 - 2\Phi)\delta_{ij}dx^i dx^j$$

Boltzmann equation $\frac{df}{dt} = C[f] \Rightarrow \mathcal{G}' + ik\mu\mathcal{G} = \Phi' - ik\mu\Psi$

$$\begin{aligned} \mathcal{G}(\eta_0, k, \mu) &= \mathcal{G}(\eta_{\text{pt}}, k, \mu) e^{ik\mu(\eta_{\text{pt}} - \eta_0)} + \int_{\eta_{\text{pt}}}^{\eta_0} d\eta [\Phi'(\eta, k) - ik\mu\Psi(\eta, k)] e^{ik\mu(\eta - \eta_0)} \\ &= \mathcal{G}(\eta_{\text{pt}}, k, \mu) e^{ik\mu(\eta_{\text{pt}} - \eta_0)} + \int_{\eta_{\text{pt}}}^{\eta_0} d\eta \left[\Phi'(\eta, k) e^{ik\mu(\eta - \eta_0)} - \frac{d}{d\eta} (\Psi(\eta, k) e^{ik\mu(\eta - \eta_0)}) + \Psi'(\eta, k) e^{ik\mu(\eta - \eta_0)} \right] \\ &= \underbrace{[\mathcal{G}(\eta_{\text{pt}}, k) + \Psi(\eta_{\text{pt}}, k)] e^{ik\mu(\eta_{\text{pt}} - \eta_0)}}_{\text{SW}} + \underbrace{\int_{\eta_{\text{pt}}}^{\eta_0} d\eta [\Phi'(\eta, k) + \Psi'(\eta, k)] e^{ik\mu(\eta - \eta_0)}}_{\text{ISW}} \end{aligned}$$



PTGW anisotropy and its implication for primordial seeds of our universe

$$(\mathcal{G} + \Psi)(\eta_{\text{pt}}, k) = -\frac{1}{3}\mathcal{R}(k) \quad \text{radiation dominate}$$

$$\mathcal{G}_\ell^{\text{SW}}(\eta_0, k) = (\mathcal{G} + \Psi)(\eta_{\text{pt}}, k) j_\ell[k(\eta_{\text{pt}} - \eta_0)]$$

$$C_\ell^{\mathcal{G}, \text{SW}} = \frac{4\pi}{9} \int_0^\infty \frac{dk}{k} P_{\mathcal{R}}(k) j_\ell^2[k(\eta_0 - \eta_{\text{pt}})]$$

$$\int_{\eta_{\text{pt}}}^{\eta_0} d\eta [\Phi'(\eta, k) + \Psi'(\eta, k)] e^{ik\mu(\eta - \eta_0)}$$

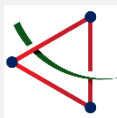
$$\mathcal{G}_\ell^{\text{ISW}}(\eta_0, k) = \int_{\eta_{\text{pt}}}^{\eta_0} d\eta (\Phi' + \Psi')(\eta, k) j_\ell[k(\eta - \eta_0)]$$

$$(\Phi' + \Psi')(\eta, k) \approx -(\Phi + \Psi)(\eta_{\text{pt}}, k) \delta(\eta - \eta_k)$$

$$(\Phi + \Psi)(\eta_{\text{pt}}, k) = -\frac{4}{3}\mathcal{R}(k)$$

small scale

$$C_\ell^{\mathcal{G}, \text{ISW}} = \frac{64\pi}{9} \int_0^\infty \frac{dk}{k} P_{\mathcal{R}}(k) j_\ell^2[k(\eta_0 - \eta_{\text{pt}})]$$



PTGW anisotropy and its implication for primordial seeds of our universe

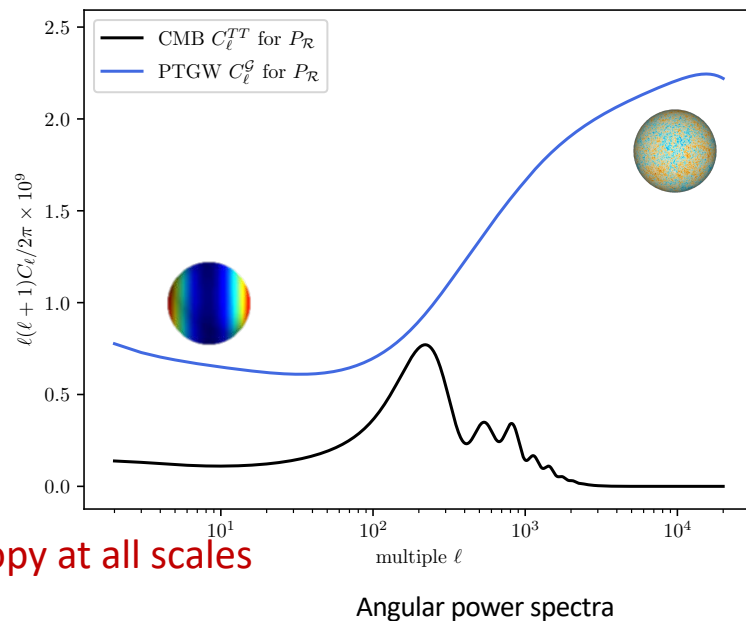
$$C_\ell^{\mathcal{G}} \approx C_\ell^{\mathcal{G},\text{SW}} = \frac{4\pi}{9} \int_0^\infty \frac{dk}{k} P_{\mathcal{R}}(k) j_\ell^2 [k(\eta_0 - \eta_{\text{pt}})]$$

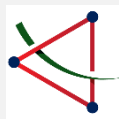
$$\begin{aligned} \mathcal{G}_\ell(\eta_0, k) &= (\mathcal{G}_\ell^{\text{SW}} + \mathcal{G}_\ell^{\text{ISW}})(\eta_0, k) \\ &\approx [(\mathcal{G} + \Psi) - (\Phi + \Psi)](\eta_{\text{pt}}, k) j_\ell [k(\eta_{\text{pt}} - \eta_0)] \end{aligned}$$

$$C_\ell^{\mathcal{G}} = 4\pi \int_0^\infty \frac{dk}{k} P_{\mathcal{R}}(k) j_\ell^2 [k(\eta_0 - \eta_{\text{pt}})]$$

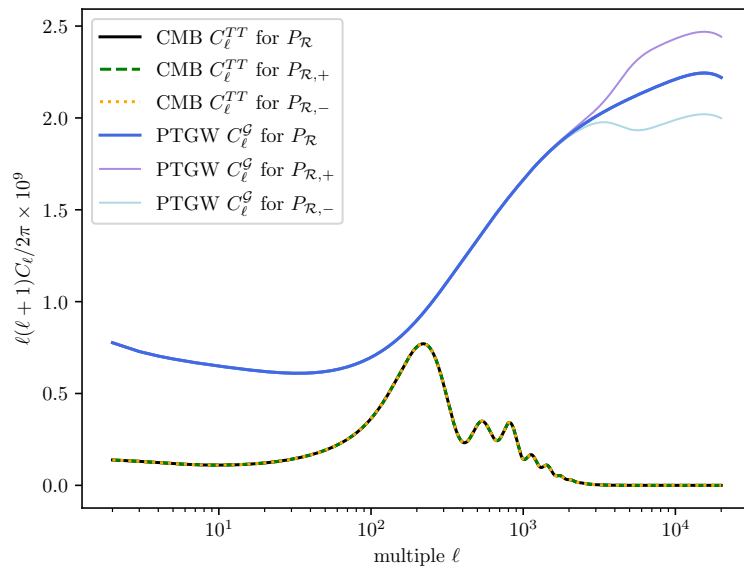
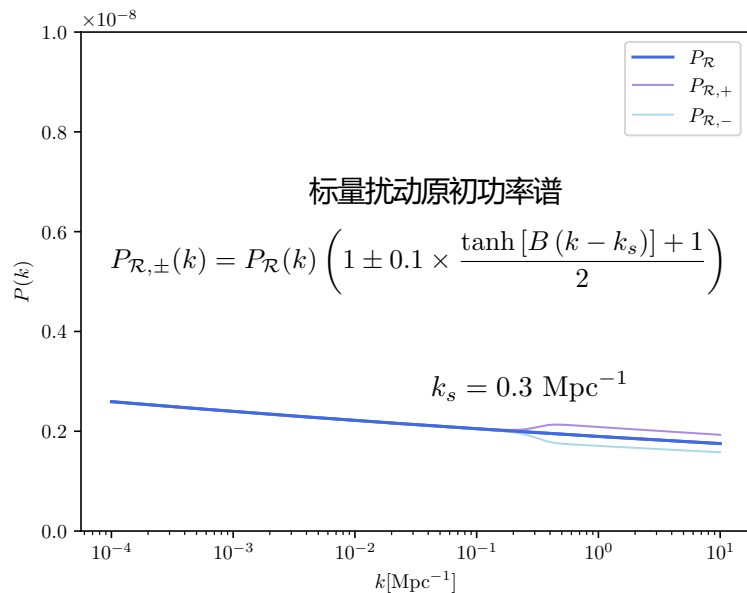
- ✓ The anisotropy of PTGW is stronger than CMB anisotropy at all scales
- ✓ The anisotropy of PTGW goes up at small scale

Planck: $\ln(10^{10} A_s) = 3.040 \pm 0.016$ $n_s = 0.9626 \pm 0.0057$

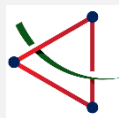




PTGW anisotropy and its implication for primordial seeds of our universe



✓ The anisotropy of PTGW keeps more small-scale information.

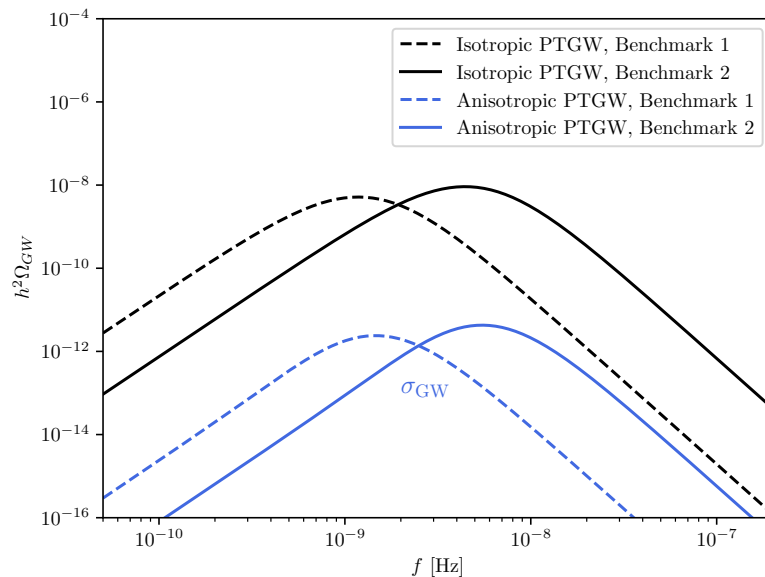


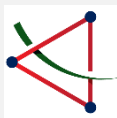
PTGW anisotropy and its implication for primordial seeds of our universe

$$\text{Var}^{\mathcal{G}} = \frac{1}{4\pi} \sum_{\ell} (2\ell + 1) C_{\ell}^{\mathcal{G}}$$

$$\sigma_{\text{GW}}(p) \equiv h^2 \Omega_{\text{GW}}(p) \sqrt{\text{Var}^{\delta_{\text{GW}}}(p)}$$

CMB TT anisotropy	4×10^{-5}
PTGW anisotropy	1×10^{-4}
PTGW energy spectra anisotropy	$8 \times 10^{-4} (> f_{\text{sw}})$





PTGW anisotropy and its implication for primordial seeds of our universe

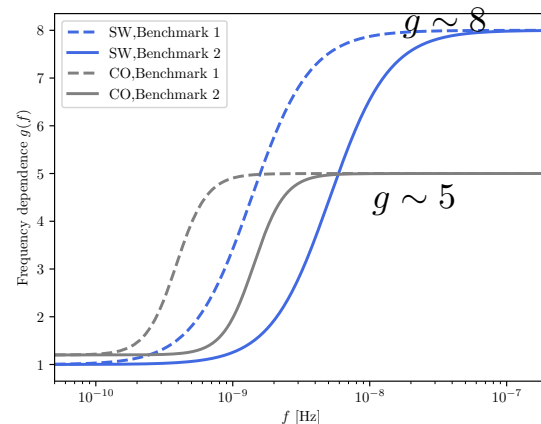
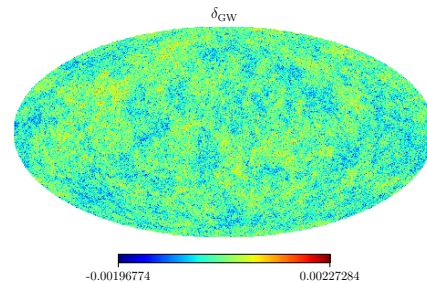
$$\rho_{\text{GW}}(\eta, \mathbf{x}) = \int d^3 p p f(\eta, \mathbf{x}, p) = \int dp d\hat{p} p^3 f(\eta, \mathbf{x}, p, \hat{p})$$

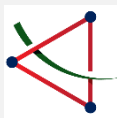
$$\begin{aligned} \Omega_{\text{GW}}(\eta, \mathbf{x}, p) &= \int \frac{d\hat{p}}{4\pi} \bar{\Omega}_{\text{GW}}(\eta, p) [1 + \delta_{\text{GW}}(\eta, \mathbf{x}, p, \hat{p})] \\ &= \int d\hat{p} \frac{p^4}{\rho_c} \left[\bar{f}(\eta, p) - p \frac{\partial \bar{f}(\eta, p)}{\partial p} \mathcal{G}(\eta, \mathbf{x}, \hat{p}) \right] \end{aligned}$$

anisotropy of GW energy spectra $\delta_{\text{GW}} = \frac{\delta \Omega_{\text{GW}}(\eta, \mathbf{x}, p, \hat{p})}{\Omega_{\text{GW}}(\eta, p)}$

$$\delta_{\text{GW}}(\eta, \mathbf{x}, p, \hat{p}) = \left[4 - \frac{\partial \ln \bar{\Omega}_{\text{GW}}(\eta, p)}{\partial \ln p} \right] \mathcal{G}(\eta, \mathbf{x}, \hat{p})$$

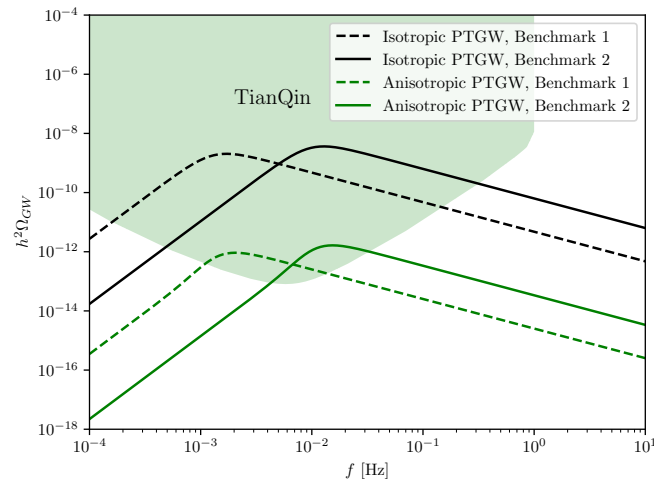
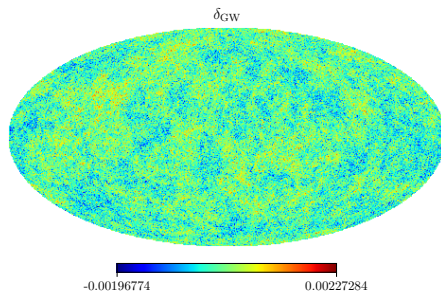
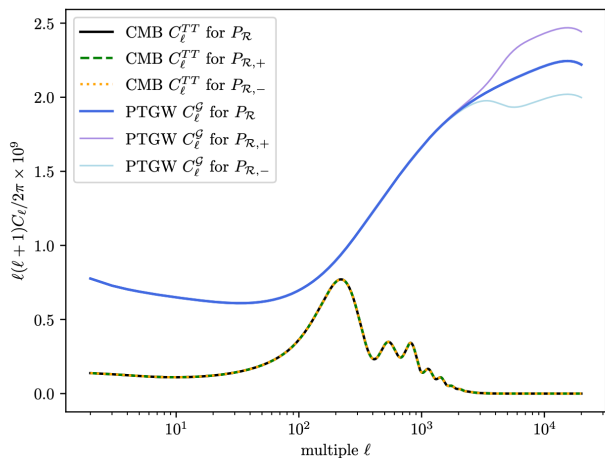
$$C_l^{\delta_{\text{GW}}}(p) = g^2(p) C_l^{\mathcal{G}}$$





PTGW anisotropy and its implication for primordial seeds of our universe

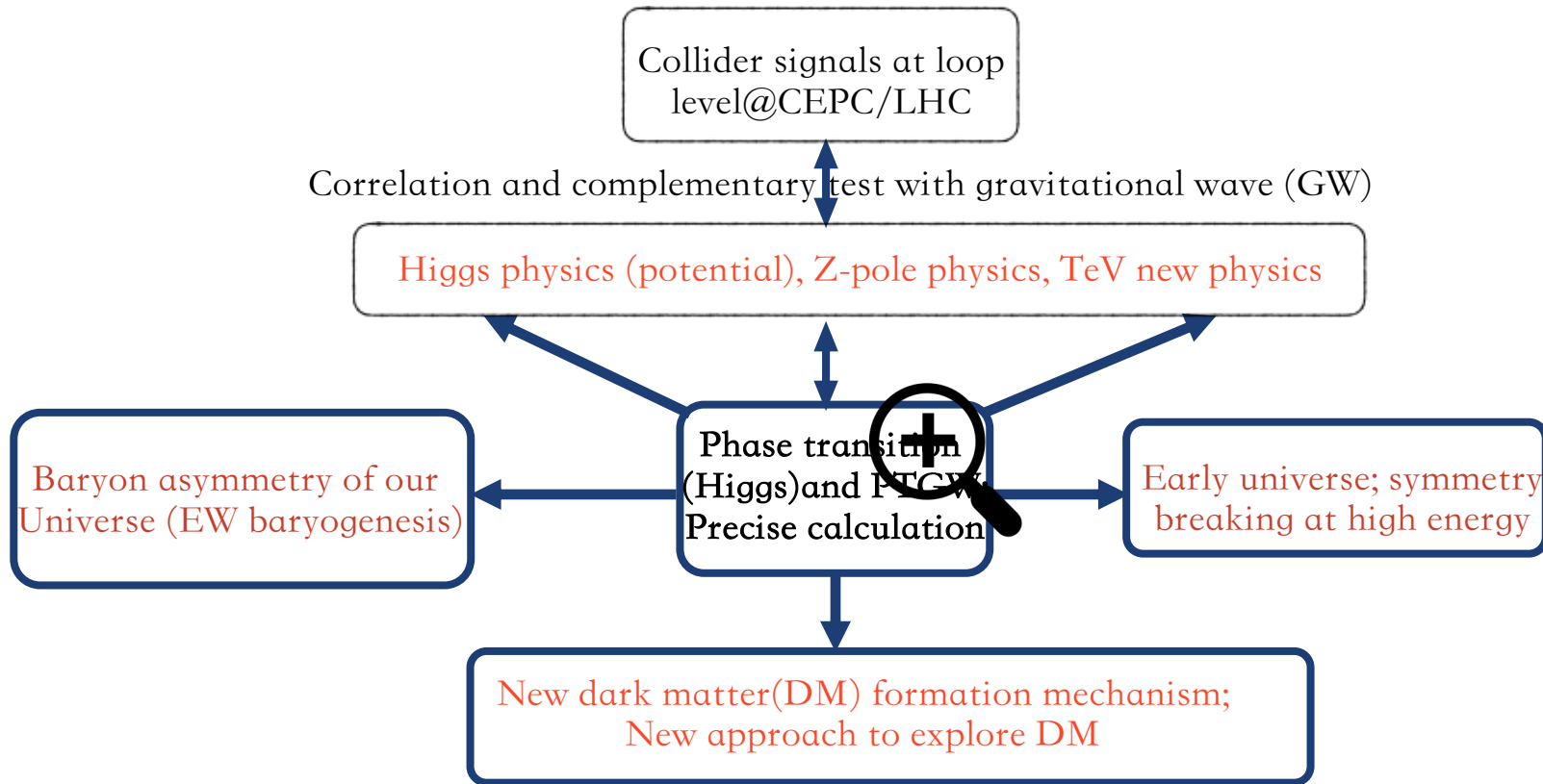
Application to EWPT

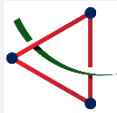


Yongping Li, **FPH**, Xiao Wang, Xinmin Zhang, Phys.Rev.D 105 (2022) 083527
 Yongping Li, **FPH**, Xiao Wang, work in progress



Motivation





Phase transition Dynamics

Precise predictions on the phase transition dynamics and its GW signals

1. precisely calculate the finite-temperature effective potential $V_{eff}(\phi, T)$

(1). Daisy resummation problem: [Pawani scheme vs. Arnold scheme](#)

(2). Gauge dependence problem: [see Michael J. Ramsey-Musolf's works](#)

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and dim-reduction method: by D. Weir, Michael J. Ramsey-Musolf et.al

α T_p $R_* H_*$

2. Reliable calculations of bubble wall velocity v_b

[S. Hoche, J. Kozaczuk, A. J. Long, J. Turner and Y. Wang, arXiv:2007.10343](#),
[Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith, arXiv:2009.14295v2](#)

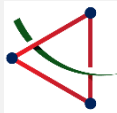
Xiao Wang, **FPH**, Xinmin Zhang, arXiv:2011.12903



3. energy budget during phase transition κ

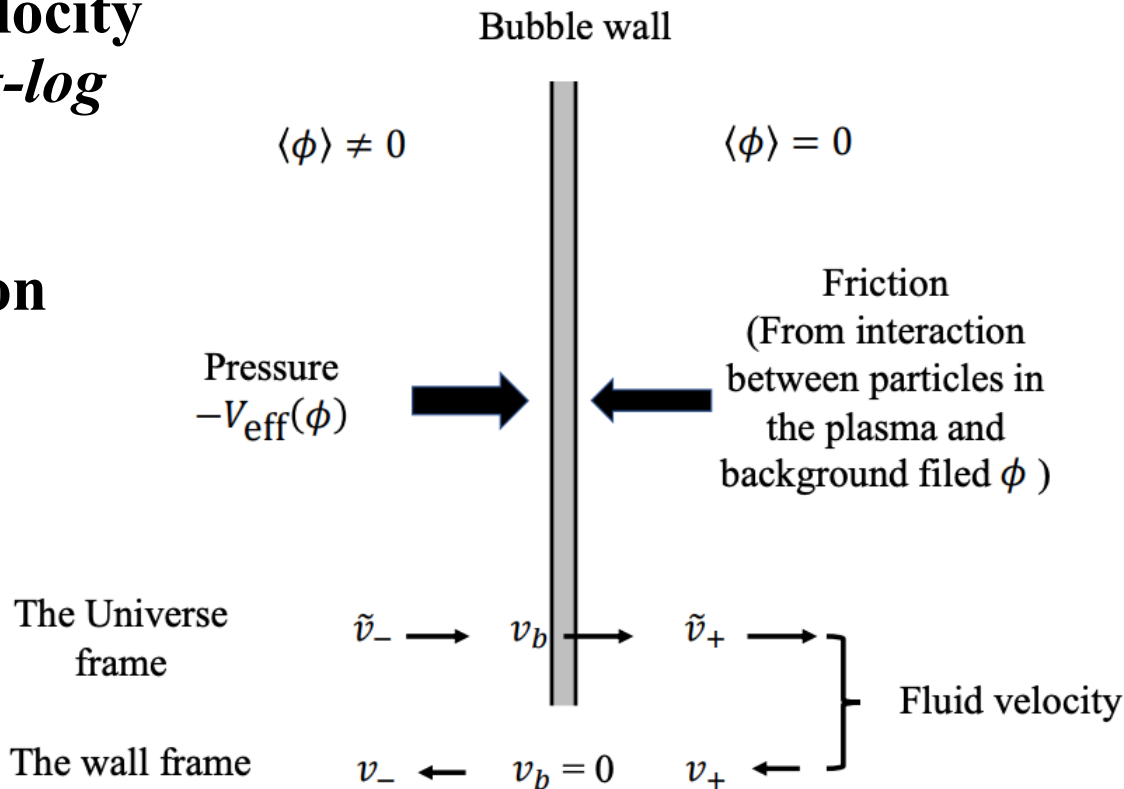
[F. Giese, T. Konstandin, K. Schmitz and J. van de Vis, arXiv:2010.09744](#)

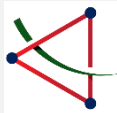
Xiao Wang, **FPH** and Xinmin Zhang, arXiv:2010.13770



Phase transition Dynamics

**Bubble wall velocity
beyond *leading-log*
approximation
in electroweak
phase transition**





Phase transition Dynamics

Bubble wall velocity beyond *leading-log* approximation in electroweak phase transition

The EOM of Higgs field (order parameter field)

- Energy momentum conservation of scalar-plasma system in WKB approximation

$$\square\phi + \frac{\partial V_0(\phi)}{\partial\phi} + \sum \frac{dm^2}{d\phi} \int \frac{d^3p}{(2\pi)^3 2E} f(p, x) = 0,$$

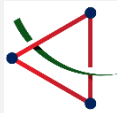
Zero-temperature part of effective potential

$$f \equiv f_0 + \delta f.$$
$$\square\phi + \frac{\partial V_{\text{eff}}(\phi, T)}{\partial\phi} + \sum \frac{dm^2}{d\phi} \int \frac{d^3p}{(2\pi)^3 2E} \delta f(p, x) = 0,$$

Full thermal effective potential

Friction term

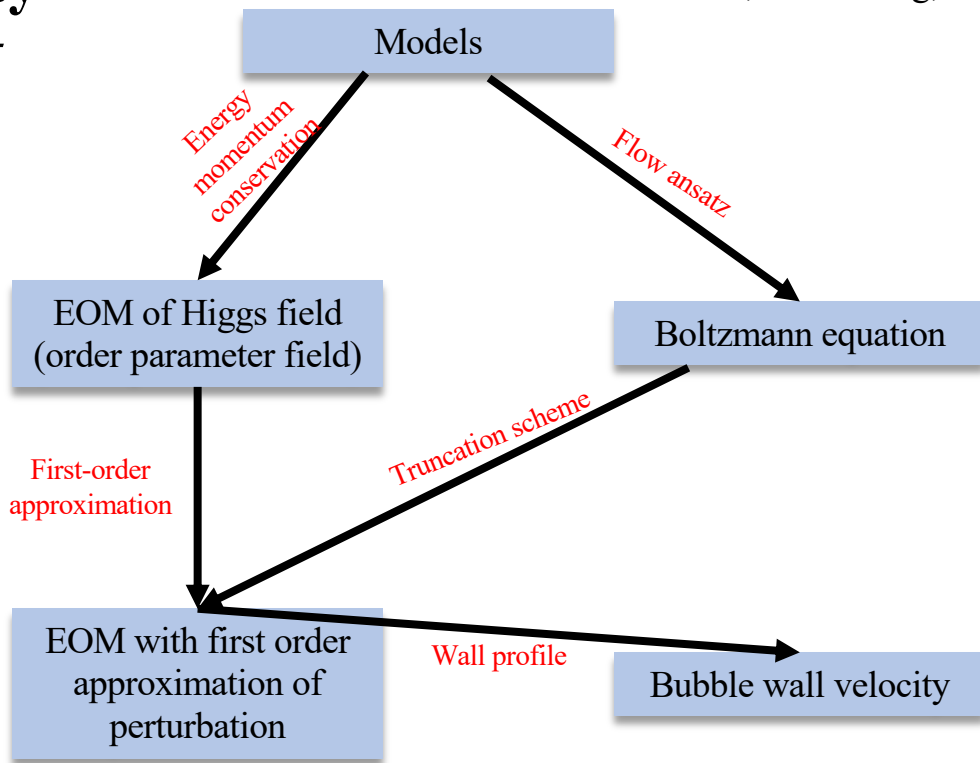
Deviation from equilibrium
Field dependent mass

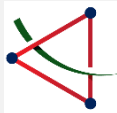


Phase transition Dynamics

**Bubble wall velocity
beyond *leading-log*
approximation
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arXiv:2011.12903, Xiao wang, **FPH**, Xinmin Zhang





Phase transition Dynamics

The deviation distribution part for each massive particle is crucial

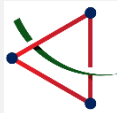
With WKB approximation $p \gg 1/L_w$, we can describe it with Boltzmann equation

$$\frac{d}{dt}f = \left(\frac{\partial}{\partial t} + \dot{z} \frac{\partial}{\partial z} + \dot{p}_z \frac{\partial}{\partial p_z} \right) f = -C[f],$$

- For thermal equilibrium situation, the collision term is zero, hence the non-equilibrium part can be derived from this equation

To solve the Boltzmann equation

- *Appropriate form for the distribution function (flow ansatz)*
- *Specific truncation scheme*
- *Proper treatment of collision term*



Phase transition Dynamics

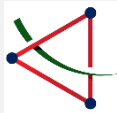
Key point: calculate the particle scattering amplitude in thermal plasma

The collision term in Boltzmann equation

$$C[f] = \sum_i \frac{1}{2E_p} \int \frac{d^3k d^3p' d^3k'}{(2\pi)^9 2E_k 2E_{p'} 2E_{k'}} \bar{\Sigma} |M|^2 (2\pi)^4 \delta^4(p + k - p' - k') \mathcal{P}[f_i],$$
$$\mathcal{P}[f_i] = f_1 f_2 (1 \pm f_3) (1 \pm f_4) - f_3 f_4 (1 \pm f_1) (1 \pm f_2).$$

To the first-order a

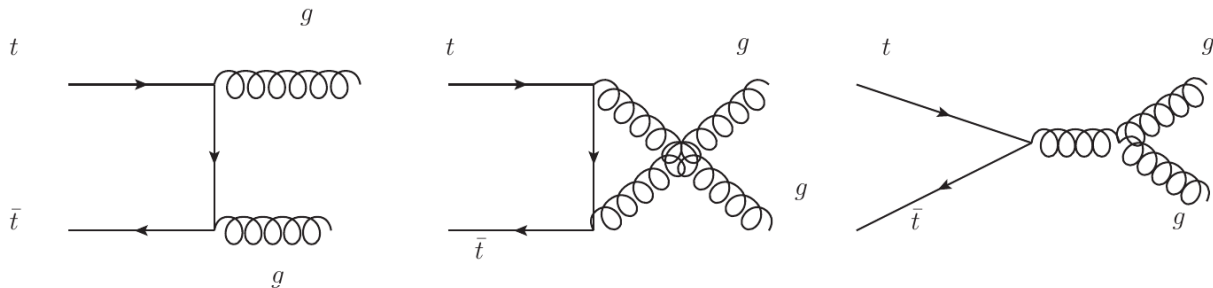
$$\mathcal{P}[f_i] \approx \frac{\delta_1 + \delta_2 - \delta_3 - \delta_4}{T} f_1 f_2 (1 \pm f_3) (1 \pm f_4).$$

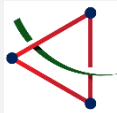


Phase transition Dynamics

Top quark: $t\bar{t} \rightarrow gg$

- Interactions of $O(g_s^4)$

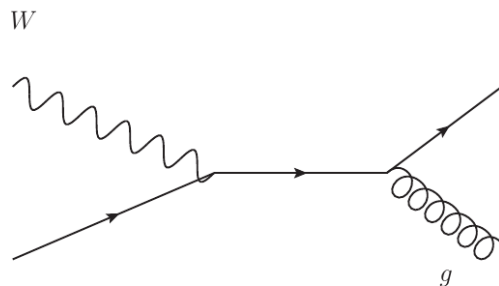
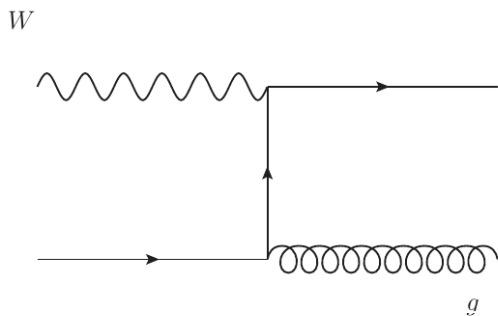


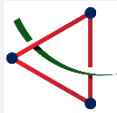


Phase transition Dynamics

W/Z boson: $Wu \rightarrow dg$

- Interactions of $O(g_s^2 g_w^2)$

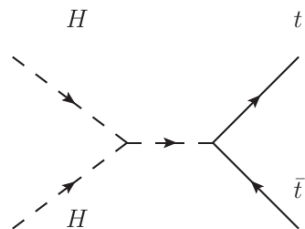
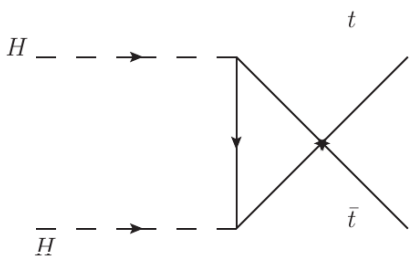
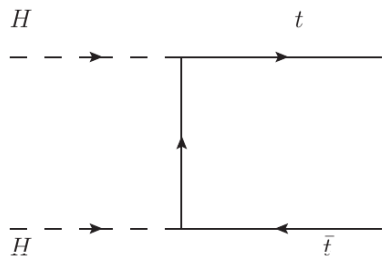


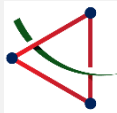


Phase transition Dynamics

Higgs boson: $HH \rightarrow t\bar{t}$

- Interactions of $O(y_t^4)$





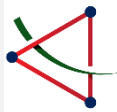
Phase transition Dynamics

To the first order of perturbations, we can derive the following **EOM of the Higgs field**.

$$\begin{aligned} & -(1 - v_w^2)\phi'' + \frac{\partial V_{\text{eff}}(\phi, T)}{\partial \phi} + \delta T_{bg} \frac{\partial^2 V_{\text{eff}}(\phi, T)}{\partial T \partial \phi} \\ & + \frac{N_t T}{2} \frac{dm_t^2}{d\phi} (c_1^f \mu_t + c_2^f (\delta T_t + \delta T_{bg})) \\ & + \frac{N_W T}{2} \frac{dm_W^2}{d\phi} (c_1^b \mu_W + c_2^b (\delta T_W + \delta T_{bg})) \\ & + \frac{N_H T}{2} \frac{dm_H^2}{d\phi} (c_1^b \mu_H + c_2^b (\delta T_H + \delta T_{bg})) = 0 \end{aligned}$$

$$T = T_+ + \delta T_{bg}(z) \longrightarrow T_n = T_+ = T$$

Here we take an approximation



Phase transition Dynamics

Bubble wall velocity beyond *leading-log* approximation in electroweak phase transition

- For a specific benchmark point, the results of beyond leading-log approximation with the contribution of Higgs boson

	$\Lambda/\sqrt{\kappa}$ [GeV]	v_w	$L_w T$	v_c/T_c	T_n [GeV]
Two-particle	780	0.3382	20.1863	1.1044	100.977
Three-particle	780	0.2499	18.1759	1.1044	100.977

arXiv:2011.12903, Xiao wang, **FPH**, Xinmin Zhang



Phase transition Dynamics

Precise predictions on the phase transition dynamics and its GW signals

1. precisely calculate the finite-temperature effective potential $V_{eff}(\phi, T)$

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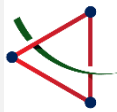
Xiao Wang, **FPH**, Xinmin Zhang, arXiv:2011.12903

3. energy budget during phase transition κ

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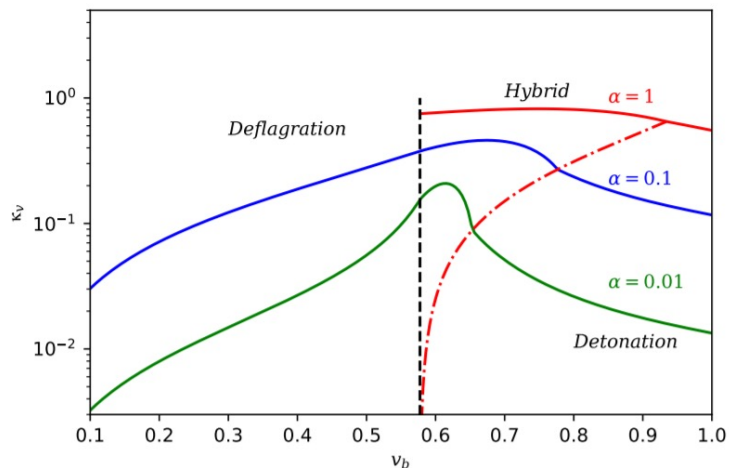
Phase transition Dynamics

Energy budget for phase transition GW

Energy budget (to measure the efficiency of the energy released by a SFOPT converting to the kinetic energy of sounding plasma)

$$h^2 \Omega_{\text{sw}}(f) \simeq 1.64 \times 10^{-6} (H_* \tau_{\text{sw}}) (H_* R_*) \left(\frac{\kappa_v \alpha}{1 + \alpha} \right)^2 \left(\frac{100}{g_*} \right)^{1/3} (f/f_{\text{sw}})^3 \left(\frac{7}{4 + 3(f/f_{\text{sw}})^2} \right)^{7/2}$$

LISA Cosmology working group *JCAP* 03 (2020) 024



The efficiency parameter (based on the bag model), the energy budget/efficiency parameter beyond the bag model are study in our recent work:

Xiao Wang, **FPH** and Xinmin Zhang, *PRD* 103 (2021) 10, 103520

Xiao Wang, **FPH** and Xinmin Zhang, *JCAP* 05, 045 (2020)

J. R. Espinosa, T. Konstandin, J. M. No, and G. Servant, *JCAP* 06 (2010) 028.



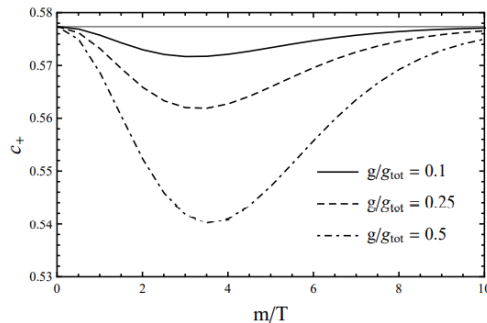
Phase transition Dynamics

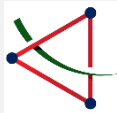
Energy budget for phase transition GW

- GW signals generated during a SFOPT are directly related to efficiency parameter:
 $h\Omega_{GW} \propto \kappa^2$ or $\kappa^{\bar{2}}$;
- Most of current studies of the efficiency parameter are based on bag EoS, which assume the sound velocity is $1/\sqrt{3}$ in both phases. But for a realistic SFOPT, particle can obtain the mass, hence, the sound velocity can deviate from pure radiation phase.

$$K \equiv \frac{\rho_{\text{fl}}}{e_+} = \frac{3}{ev_w^3} \int w(\xi) v^2 \gamma^2 \xi^2 d\xi, \quad \rho_{\text{fl}} = \frac{3}{v_w^3} \int \xi^2 v^2 \gamma^2 w d\xi$$

$$\Omega_{GW} \propto \left(\frac{\rho_{\text{fl}}}{e_+} \right)^2, \quad \kappa_{\bar{\theta}} = \frac{4\rho_{\text{fl}}}{\Delta\theta} \quad K = \left(\frac{\Delta\bar{\theta}}{4e_+} \kappa_{\bar{\theta}} \right)$$





Phase transition Dynamics

Energy budget for phase transition GW

Bag EoS

$$p_+ = \frac{1}{3}a_+T_+^4 - \epsilon_+, \quad e_+ = a_+T_+^4 + \epsilon_+,$$

$$p_- = \frac{1}{3}a_-T_-^4 - \epsilon_-, \quad e_- = a_-T_-^4 + \epsilon_-.$$

$$\alpha_\theta = \frac{4}{3} \frac{\Delta\epsilon}{w_+}, \quad \epsilon_\pm = \frac{1}{4}(e_\pm - 3p_\pm)$$

Strength parameter

$$\partial p / \partial e = c_s^2 = \text{constant}$$

EoS with different sound velocity (DSVM)

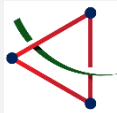
$$p_+ = c_+^2 a_+ T_+^4 - \epsilon_+, \quad e_+ = a_+ T_+^4 + \epsilon_+,$$

$$p_- = c_-^2 a_- T_-^4 - \epsilon_-, \quad e_- = a_- T_-^4 + \epsilon_- ,$$

$$\alpha_{\bar{\theta}} = \frac{\Delta\bar{\theta}}{3w_+}, \quad \bar{\theta} = e - p/c_-^2$$

Strength parameter

F. Giese, T. Konstandin, and J. van de Vis, JCAP. 07 (2020) 057.



Phase transition Dynamics

Energy budget for phase transition GW

- Matching condition

$$\begin{aligned} w_- v_-^2 \gamma_-^2 + p_- &= w_+ v_+^2 \gamma_+^2 + p_+, \\ w_- v_- \gamma_-^2 &= w_+ v_+ \gamma_+^2. \end{aligned} \quad \longrightarrow \quad v_+ v_- = \frac{p_+ - p_-}{e_+ - e_-}, \quad v_+ = \frac{e_- + p_+}{e_+ + p_-}.$$

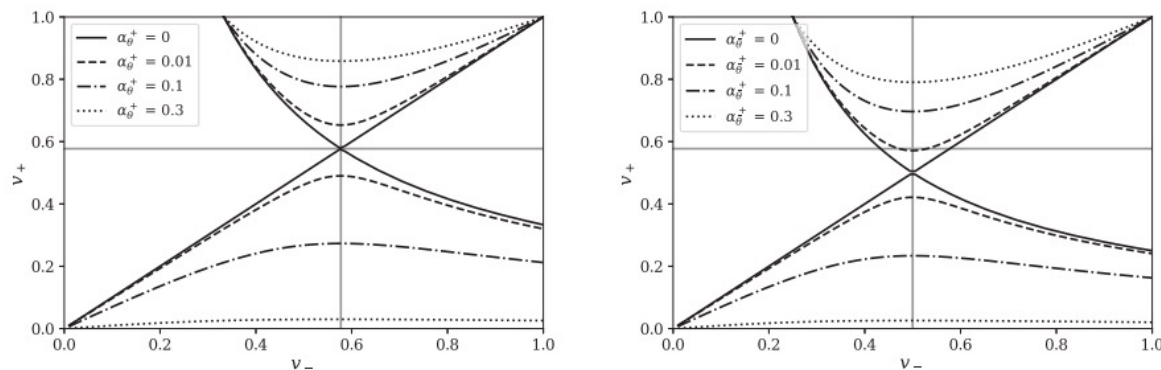
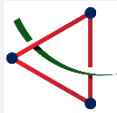


FIG. 1. The fluid velocities v_+ and v_- in the reference frame of bubble wall for different definitions and values of phase transition strength parameter. The horizontal and vertical gray lines indicate the sound velocities of symmetric and broken phase. Left panel: the bag model. Right panel: the DSVM with $c_+^2 = 1/3$ and $c_-^2 = 0.25$.

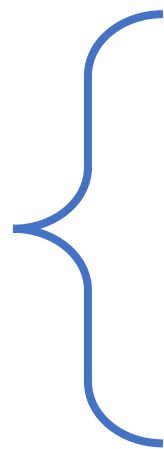


Phase transition Dynamics

Energy budget for phase transition GW

- Energy momentum conservation
derive fluid equation:

$$(\xi - v) \frac{\partial_\xi e}{w} = 2 \frac{v}{\xi} + \gamma^2 (1 - v\xi) \partial_\xi v ,$$
$$(1 - v\xi) \frac{\partial_\xi p}{w} = \gamma^2 (\xi - v) \partial_\xi v .$$



$$2 \frac{v}{\xi} = \gamma^2 (1 - v\xi) \left[\frac{\mu^2}{c_s^2} - 1 \right] \partial_\xi v$$

Velocity profile

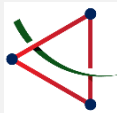
$$\frac{\partial_\xi w}{w} = \left(1 + \frac{1}{c_s^2} \right) \mu \gamma^2 \partial_\xi v$$

Enthalpy profile

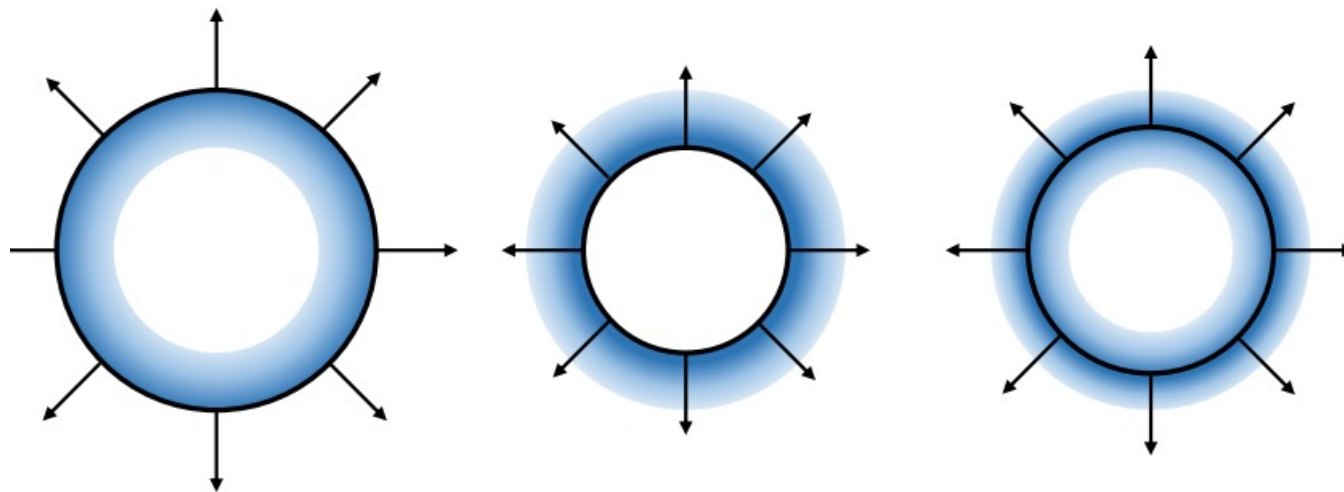
$$\frac{\partial_\xi T}{T} = \gamma^2 \mu \partial_\xi v$$

Temperature profile

Different boundary conditions give different hydrodynamical modes.



Phase transition Dynamics



Detonation

$$v_b > c_s$$

Deflagration

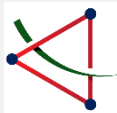
$$v_b < c_s$$

Hybrid

$$v_b > c_s$$

Stronger GW signal favored

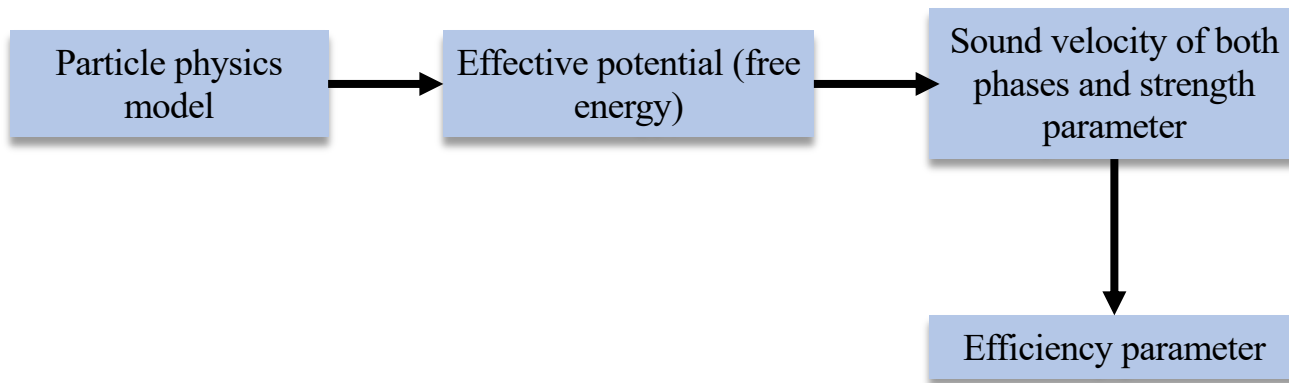
EW baryogenesis favored

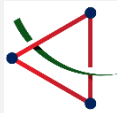


Phase transition Dynamics

Energy budget for phase transition GW

- The method to map a particle physics model on the DSVM to get efficiency parameter



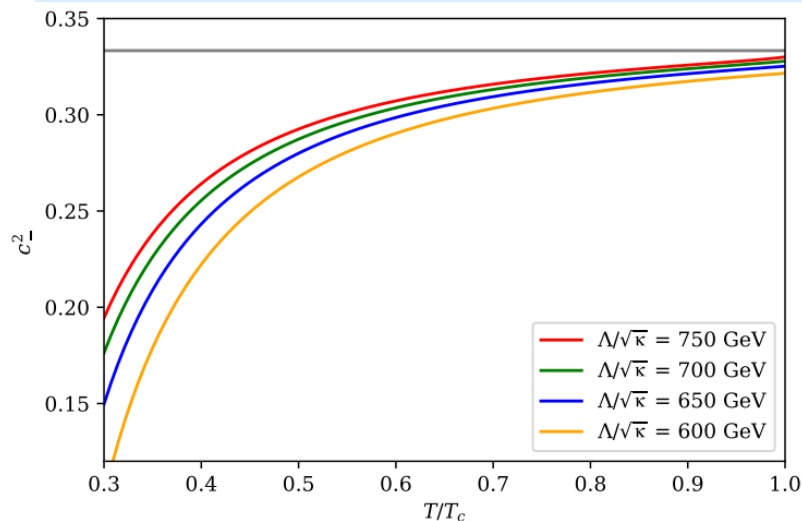


Phase transition Dynamics

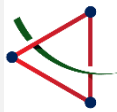
Energy budget for phase transition GW

The evolution of sound velocity of broken phase and symmetric phase in Dim-6 effective model:

$$\mathcal{F}(\phi, T) \approx -\frac{a_{\pm}}{3}T^4 + \frac{\mu^2 + cT^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{\kappa}{8\Lambda^2}\phi^6$$



X. Wang, F. P. Huang and X. Zhang, PRD 103 (2021) 10, 103520



Phase transition Dynamics

Energy budget for phase transition GW

GW spectrum and SNR for different EoS with different parameter combination:

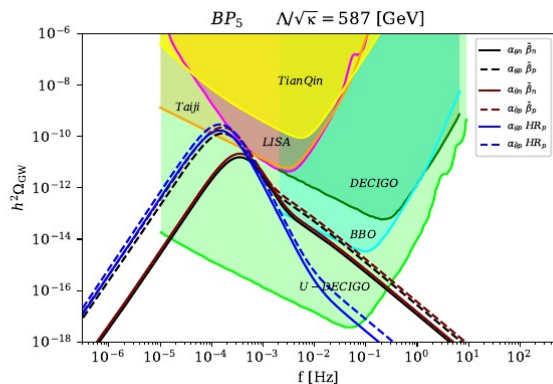


TABLE II. The SNR of BP_5 for different experiment configurations with different combinations of phase transition parameters and models of EOS.

	$\alpha_{0n} \tilde{\beta}_n$	$\alpha_{0p} \tilde{\beta}_p$	$\alpha_{0n} \tilde{\beta}_n$	$\alpha_{0p} \tilde{\beta}_p$	$\alpha_{0p} HR_p$	$\alpha_{0n} HR_p$
$SNR_{(LISA)}$	7.949	16.930	10.913	28.836	16.009	27.468
$SNR_{(Taiji)}$	14.760	58.607	20.271	100.343	66.216	113.609
$SNR_{(TianQin)}$	0.452	1.506	0.620	2.576	1.629	2.794

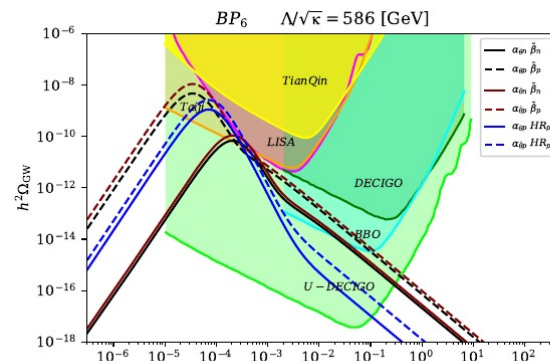
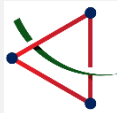


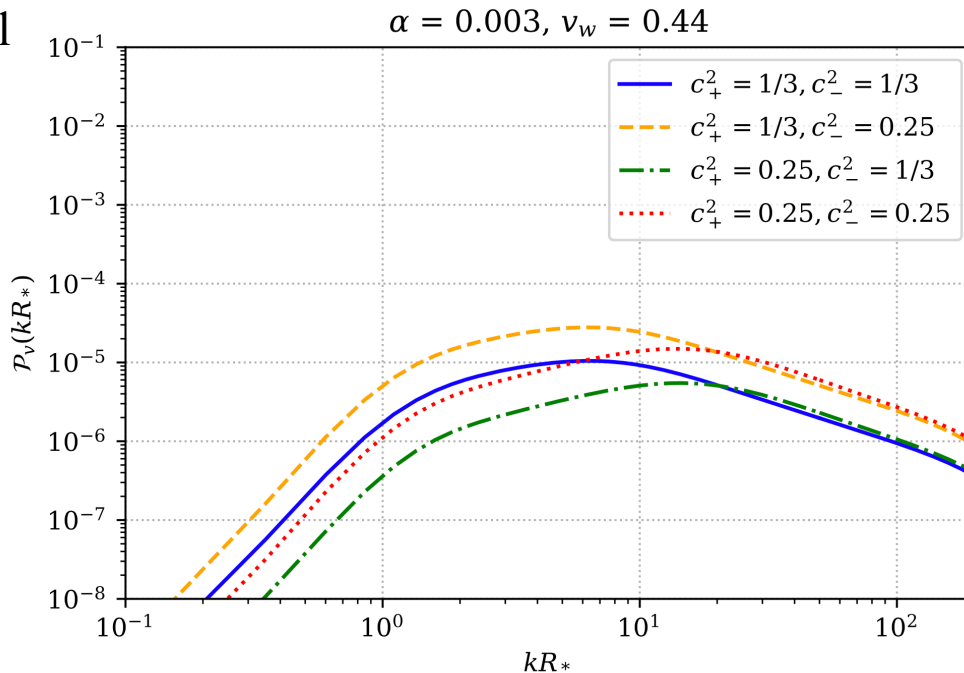
TABLE III. The SNR of BP_6 for different experiment configurations with different combinations of phase transition parameters and models of EOS.

	$\alpha_{0n} \tilde{\beta}_n$	$\alpha_{0p} \tilde{\beta}_p$	$\alpha_{0n} \tilde{\beta}_n$	$\alpha_{0p} \tilde{\beta}_p$	$\alpha_{0p} HR_p$	$\alpha_{0n} HR_p$
$SNR_{(LISA)}$	14.230	15.368	22.470	26.382	17.367	40.816
$SNR_{(Taiji)}$	38.666	427.813	61.208	1000.501	213.123	500.668
$SNR_{(TianQin)}$	1.060	5.569	1.678	12.934	3.973	9.333



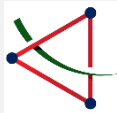
Phase transition Dynamics

Sound velocity effects on the phase transition gravitational wave spectrum in the Sound Shell Model

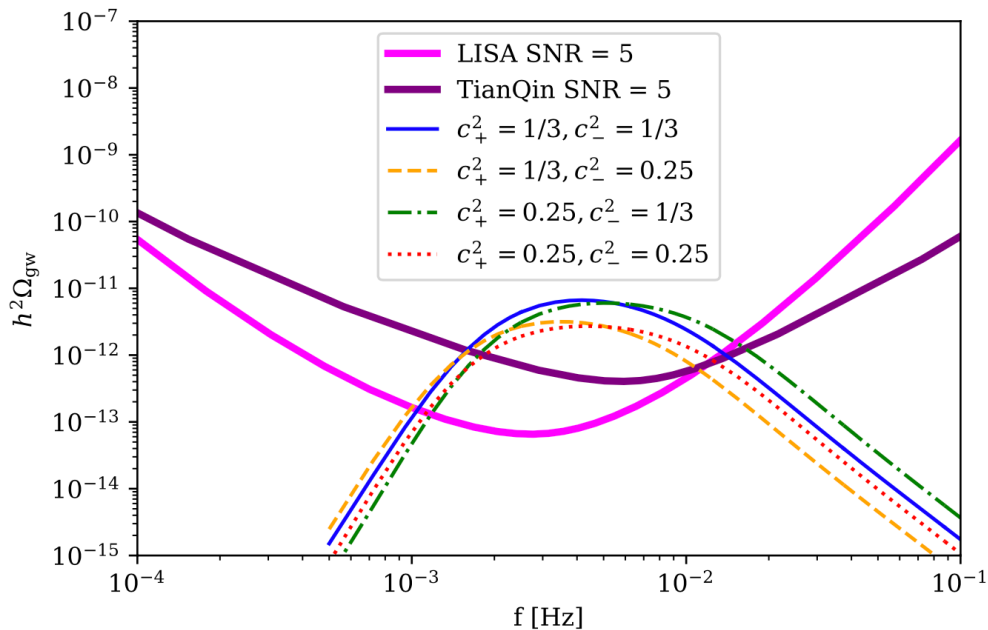


Xiao Wang, **FPH**, Yongping Li, Sound velocity effects on the phase transition gravitational wave spectrum in the Sound Shell Model, arXiv:2112.14650

Xiao Wang, **FPH**, Xinmin Zhang, Energy budget and the gravitational wave spectra beyond the bag model, Phys.Rev.D 103 (2021) 10, 103520

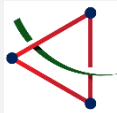


Phase transition Dynamics

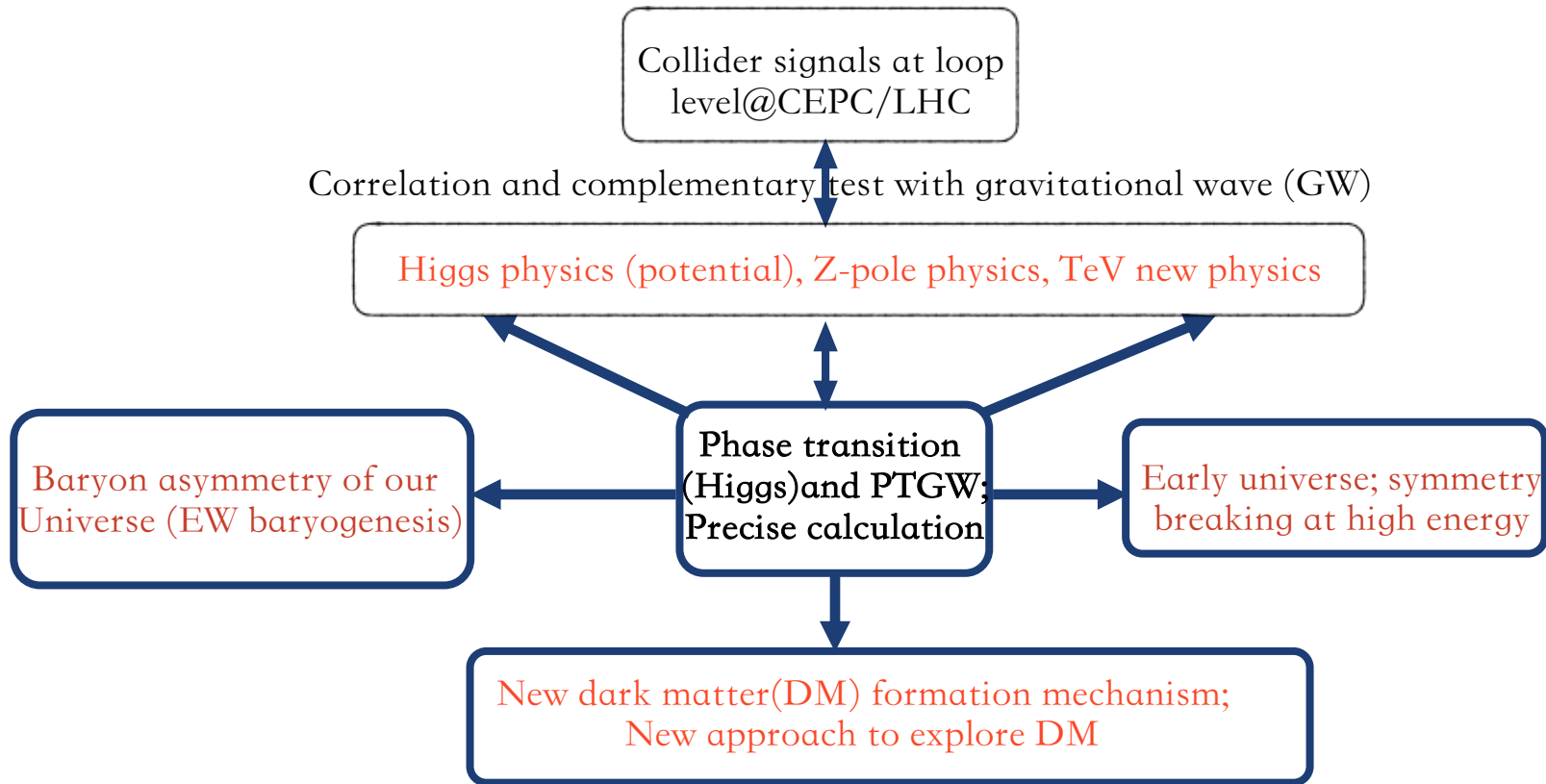


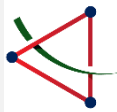
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Summary



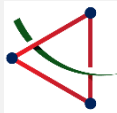


Summary and Outlook

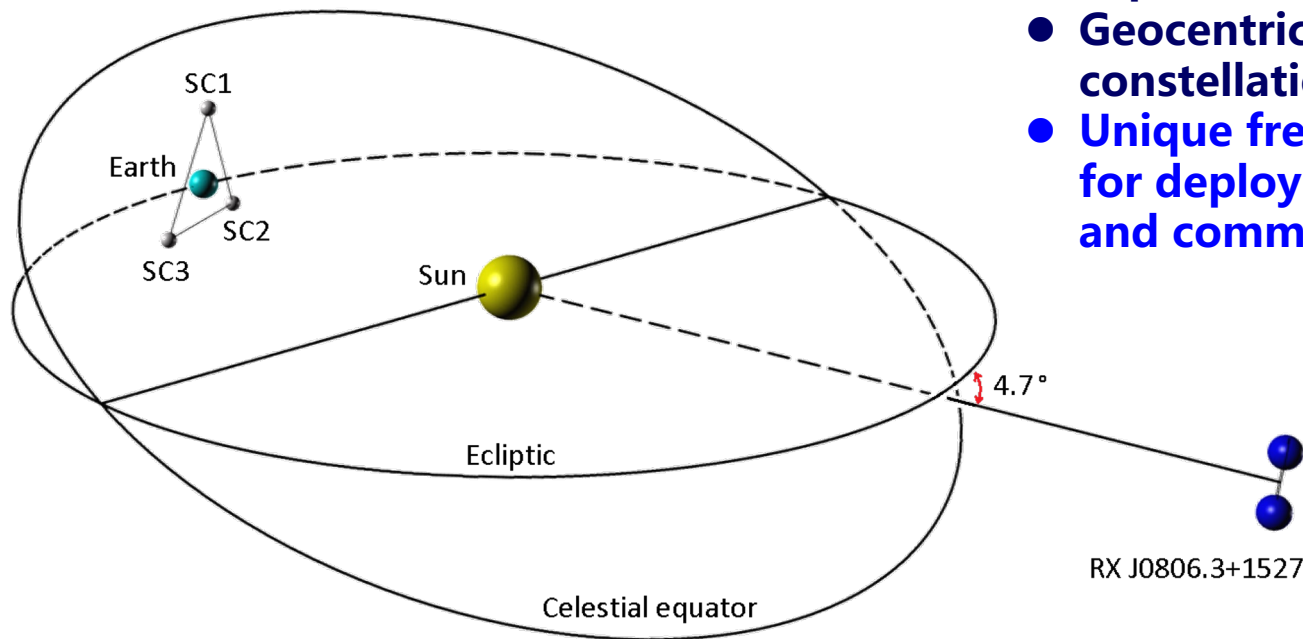
- **EW first-order phase transition has abundant collider and cosmological effects in baryogenesis, dark matter, GW...**
- **The correlation between GW and collider signals at CEPC can make complementary test on the Higgs nature, baryogenesis, dark matter and the cosmic evolution history at 100 GeV.**
- **More precise study are needed: reliable resummation, non-perturbative calculations, bubble dynamics (wall velocity, energy budget).**

Thanks! **Comments and collaborations are welcome!**

Email: huangfp8@sysu.edu.cn



TianQin



- Expected in 2035
- Geocentric orbit, normal triangle constellation, radius $\sim 10^5$ km
- Unique frequency band, easier for deployment, tracking, control, and communication



“天琴”

“Harpe in space”

J. Luo et al. TianQin: a space-borne gravitational wave detector, Class. Quant. Grav. 33 (2016) no.3, 035010.