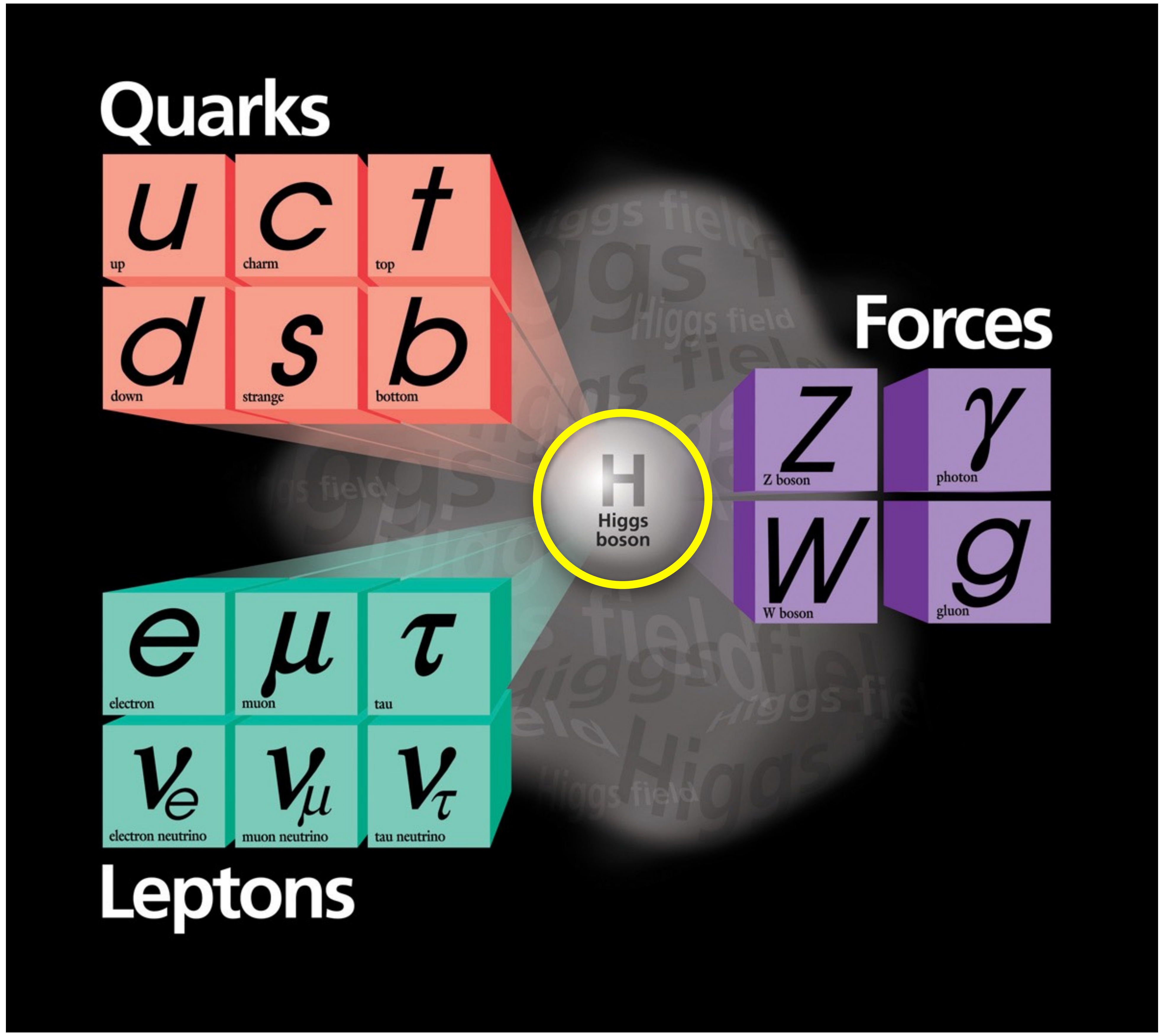
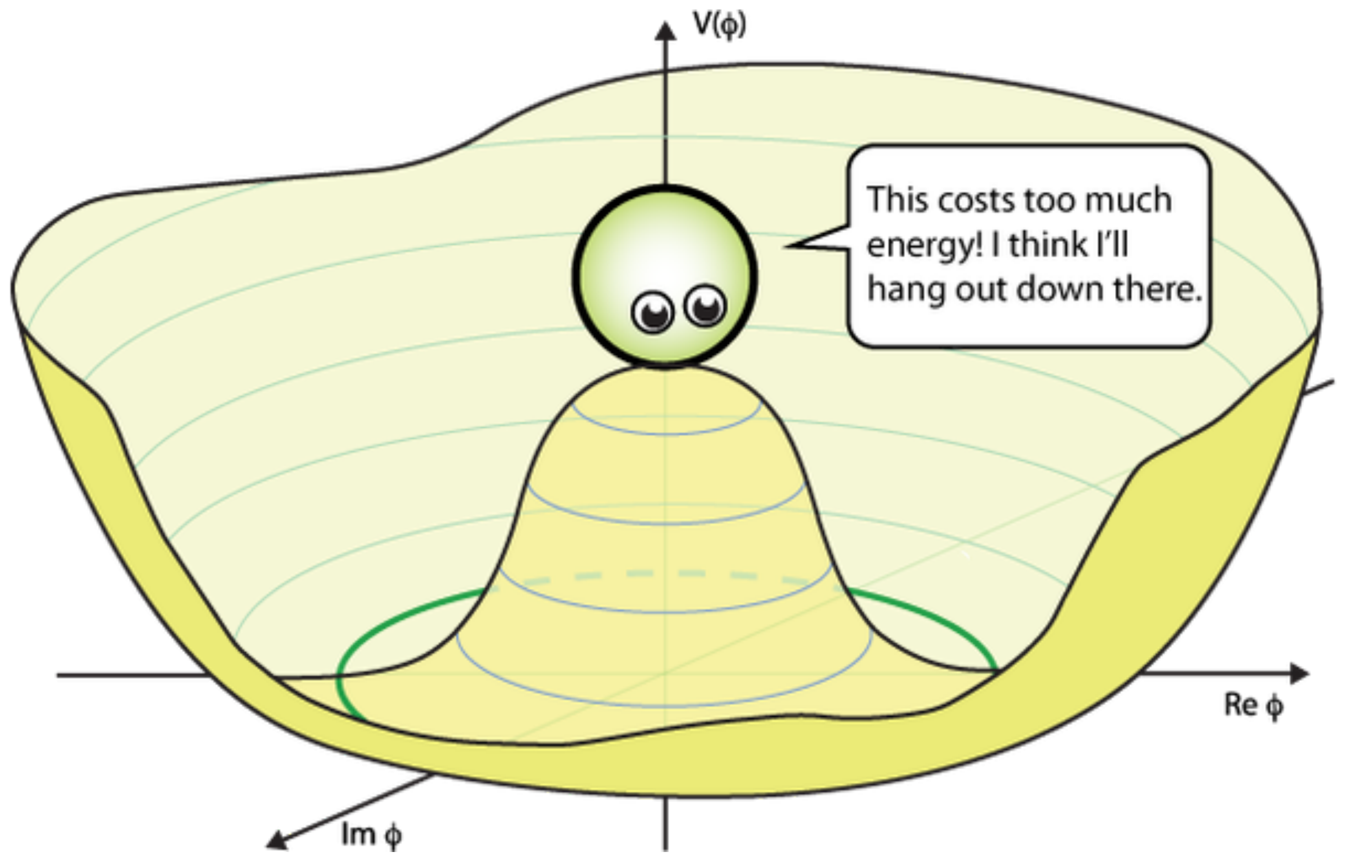
Higgs production and decay: theory

Li Lin Yang
Zhejiang University

The Higgs boson

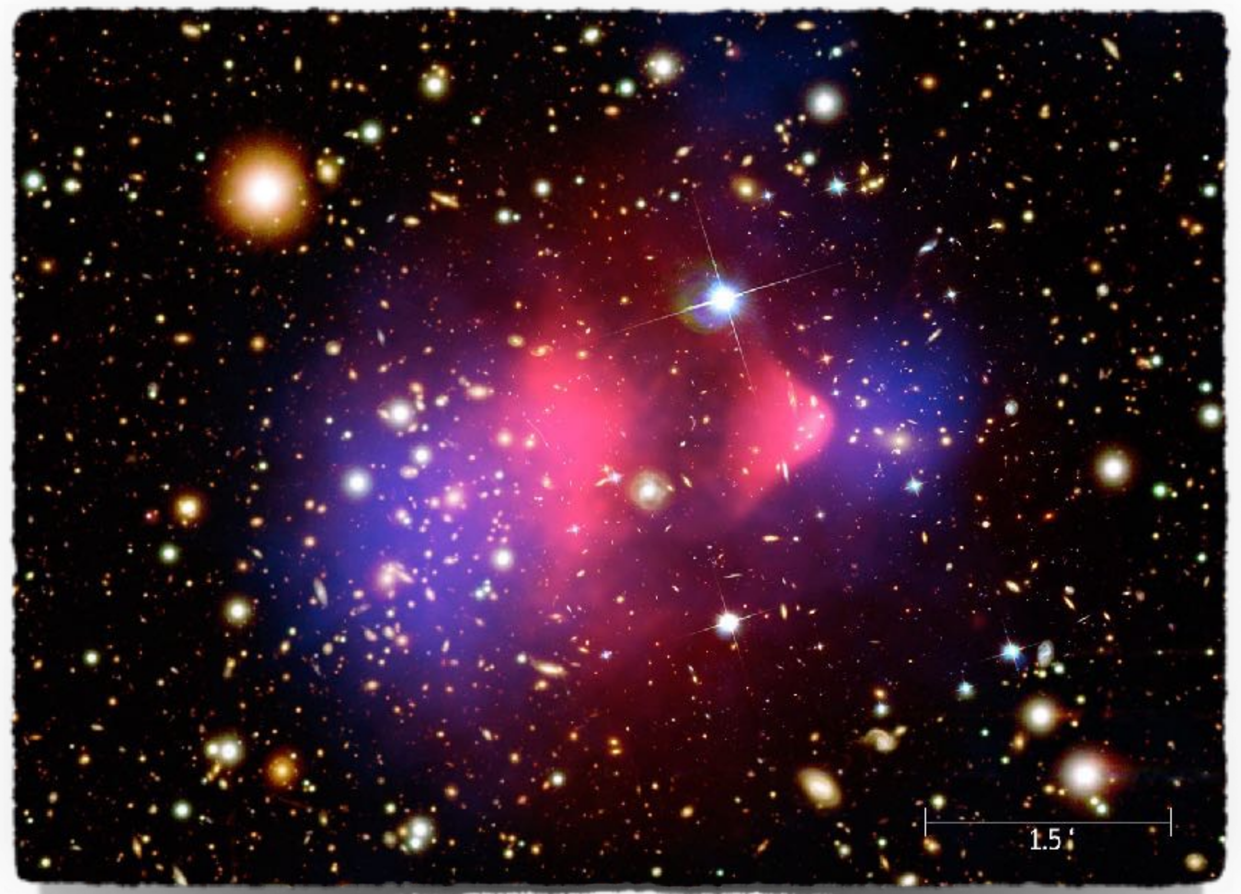


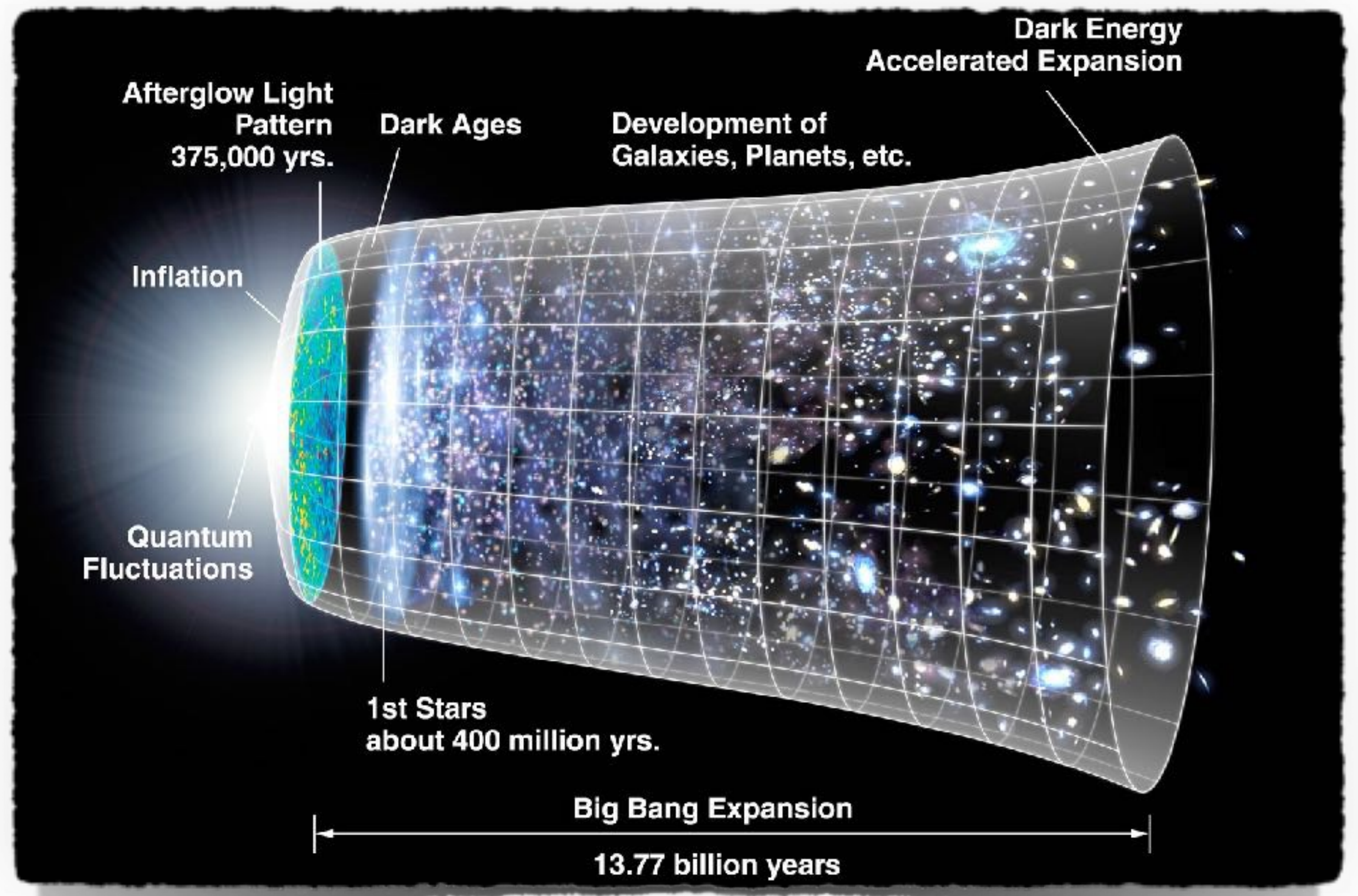
- Completes the standard model
- "Ghost" or "God" filling the vacuum of our universe
- Electroweak symmetry breaking → masses of weak gauge bosons
- Yukawa couplings → masses of fundamental matter particles

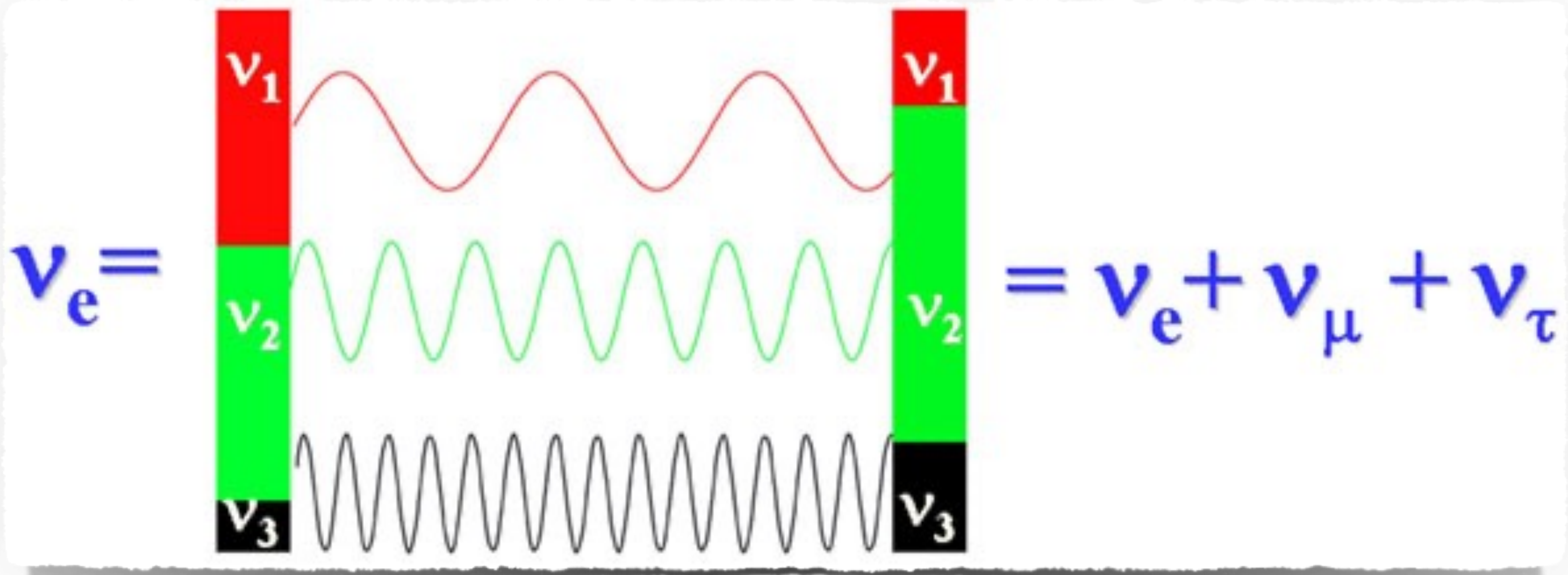


What's beyond the SM?

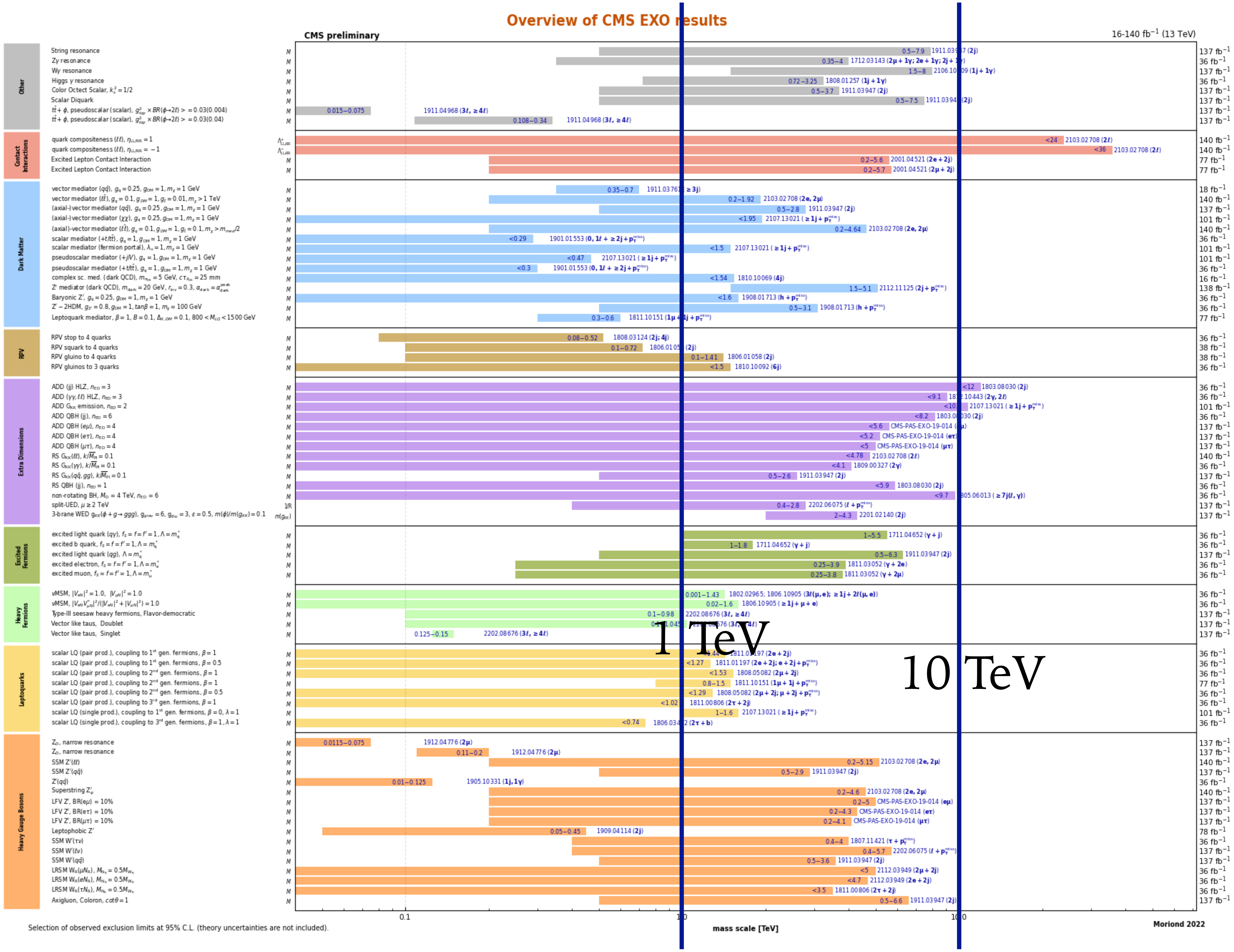
We know that there has to be something new at higher energies beyond the SM







What's beyond the SM?



Current LHC direct searches have pushed the scale of new physics very high

Supplementary information from precision measurements important!

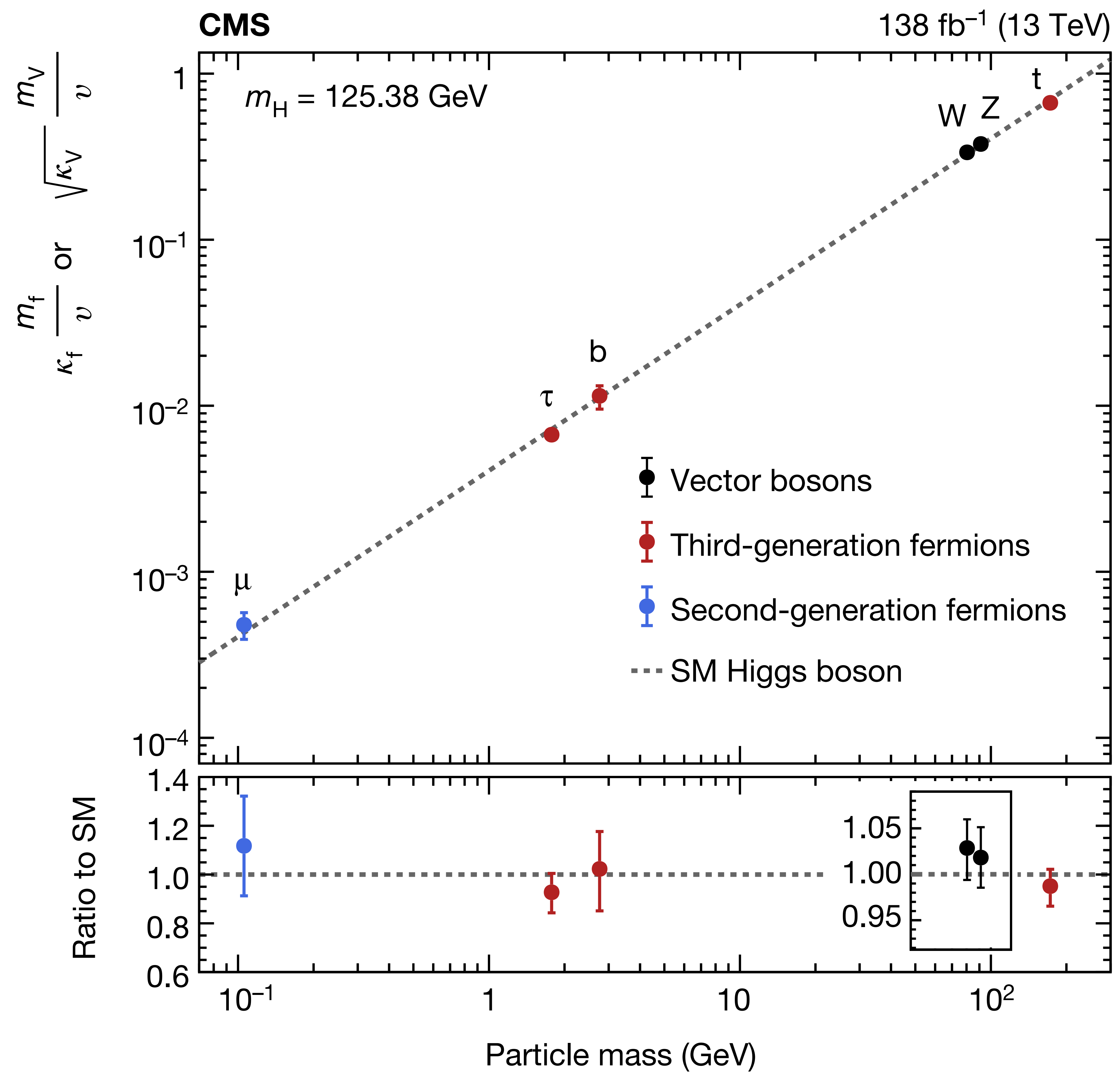
The need for precision!

Physics beyond the SM may reveal itself in various couplings of the Higgs boson

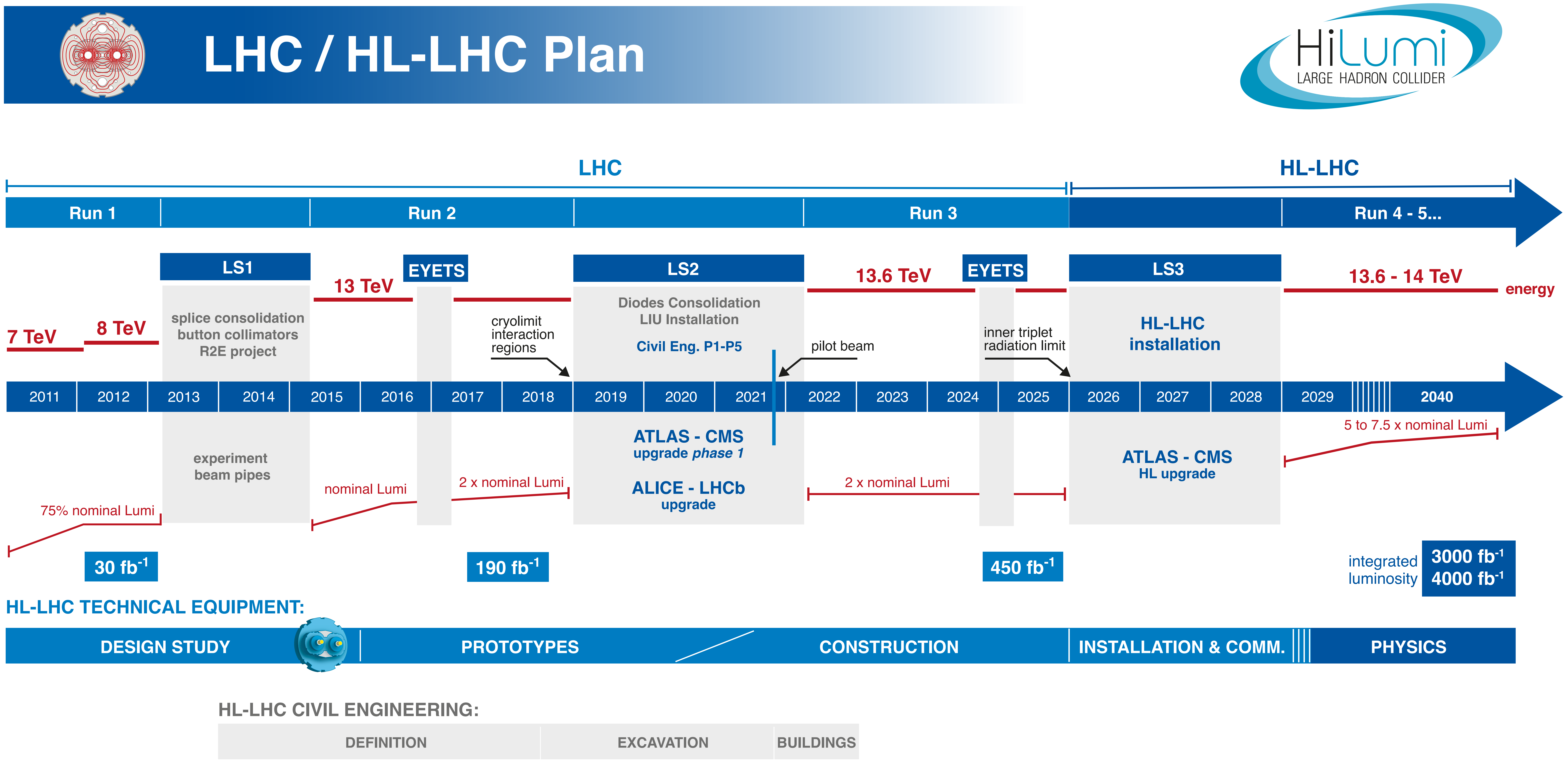
Typical deviations from the SM $\sim \left(\frac{v}{\Lambda}\right)^p$

Requires high-precision measurements!

Latest from CMS (July 4th)



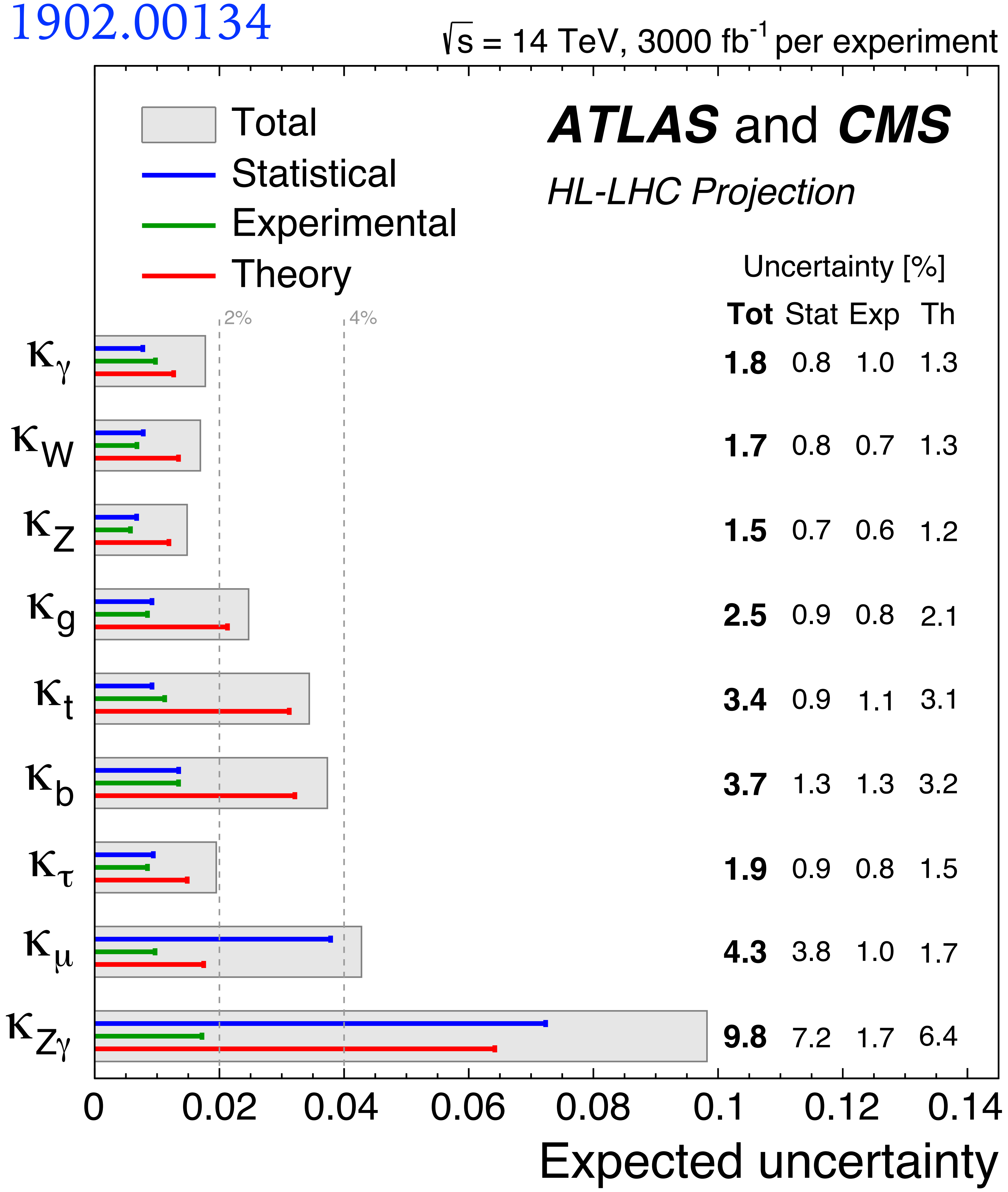
The future

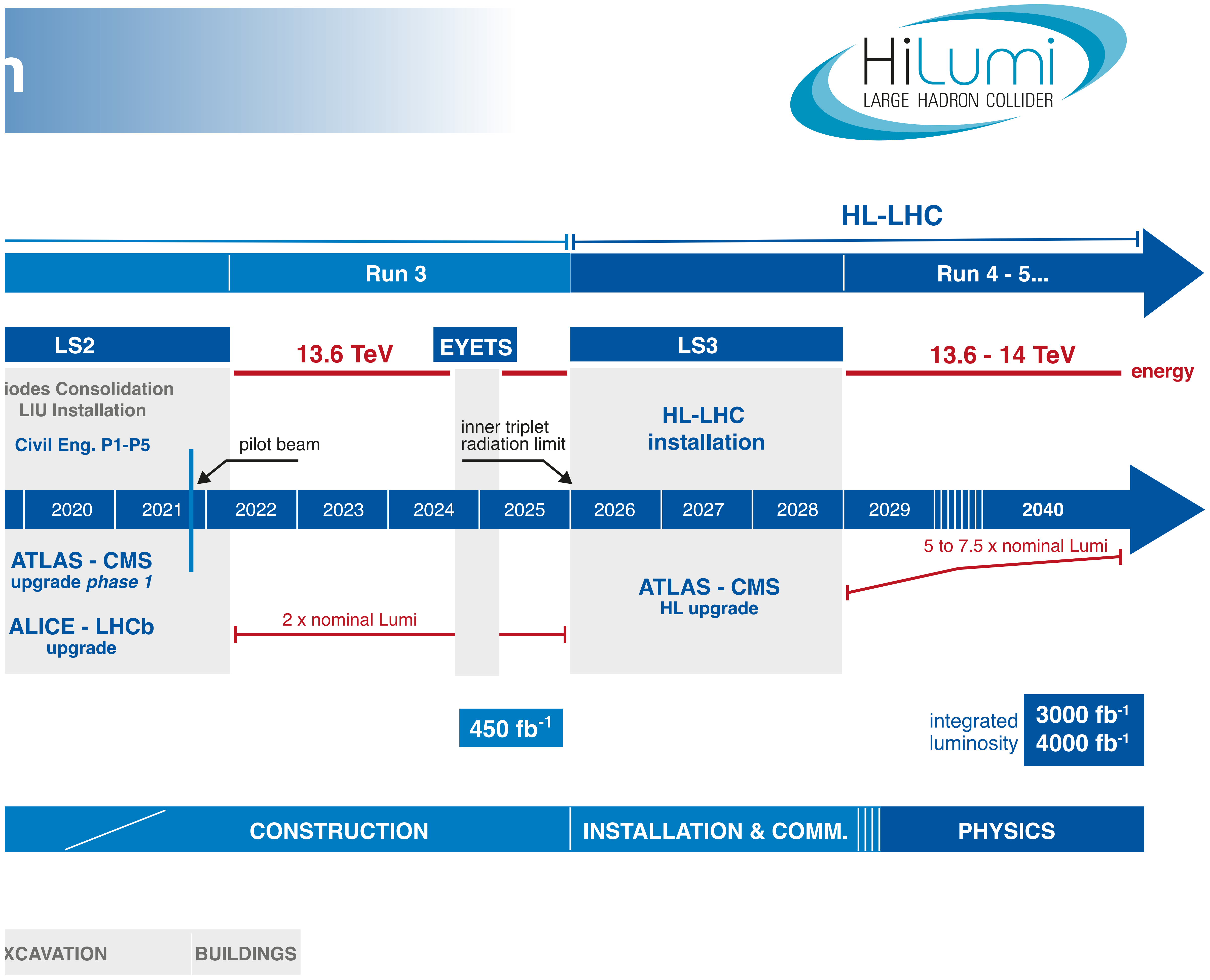


LHC / HL-LHC Plan
HiLumi
LARGE HADRON COLLIDER
LHC
HL-LHC
Run 1
Run 2
Run 3
Run 4 - 5...
LS1
EYETS
LS2
EYETS
LS3
7 TeV
8 TeV
13 TeV
13.6 TeV
13.6 - 14 TeV
energy
splice consolidation
button collimators
R2E project
cryolimit
interaction
regions
Diodes Consolidation
LIU Installation
Civil Eng. P1-P5
pilot beam
inner triplet
radiation limit
HL-LHC
installation
2011
2012
2013
2014
2015
2016
2017
2018
2019
2020
2021
2022
2023
2024
2025
2026
2027
2028
2029
2040
experiment
beam pipes
ATLAS - CMS
upgrade phase 1
ALICE - LHCb
upgrade
ATLAS - CMS
HL upgrade
5 to 7.5 x nominal Lumi
75% nominal Lumi
nominal Lumi
2 x nominal Lumi
2 x nominal Lumi
30 fb-1
190 fb-1
450 fb-1
integrated
luminosity
3000 fb-1
4000 fb-1
HL-LHC TECHNICAL EQUIPMENT:
DESIGN STUDY
PROTOTYPES
CONSTRUCTION
INSTALLATION & COMM.
PHYSICS
HL-LHC CIVIL ENGINEERING:
DEFINITION
EXCAVATION
BUILDINGS

The future

1902.00134

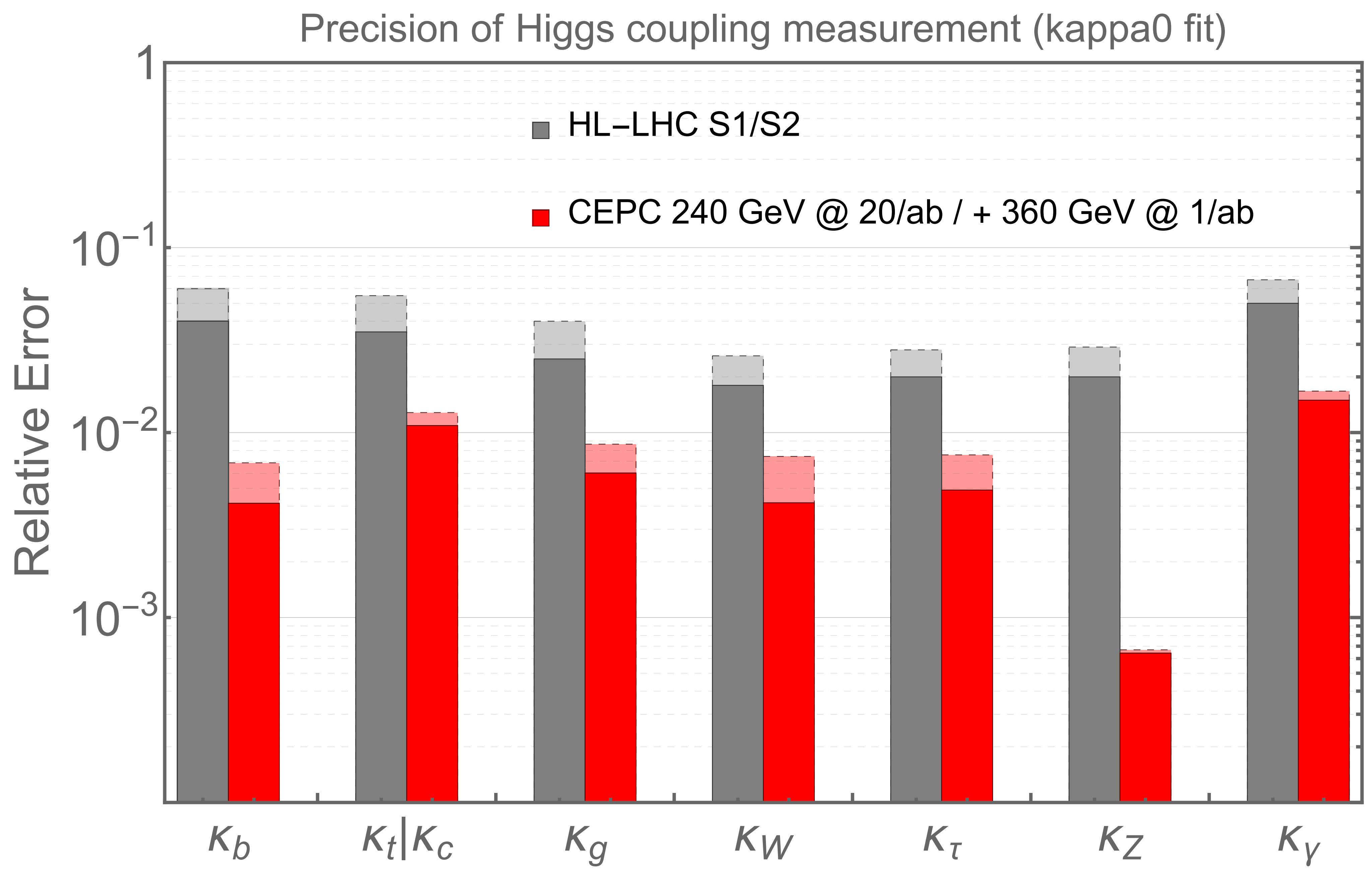




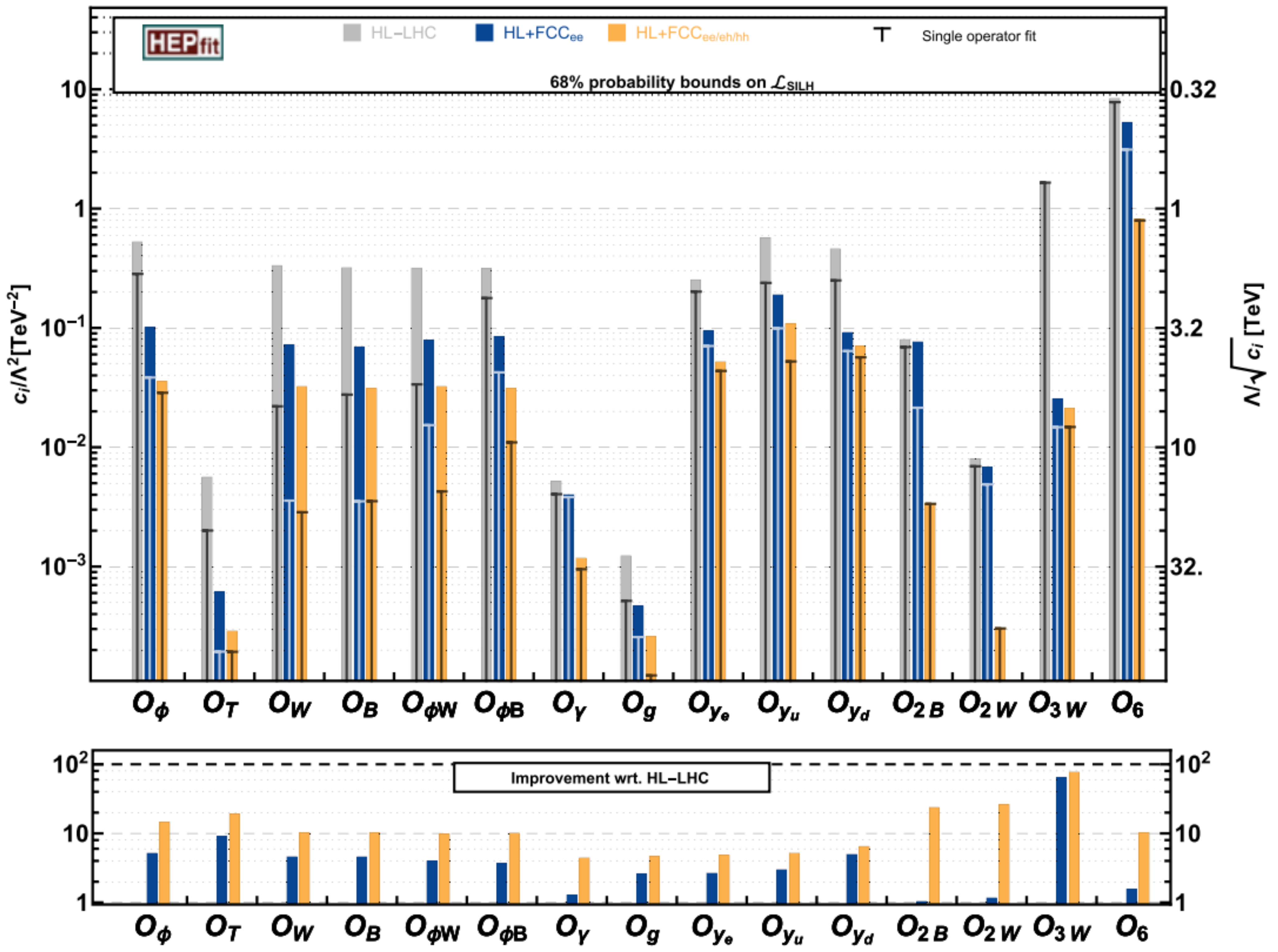
Much higher precision can be achieved at the HL-LHC

The future

Future Higgs factories can provide even better accuracies



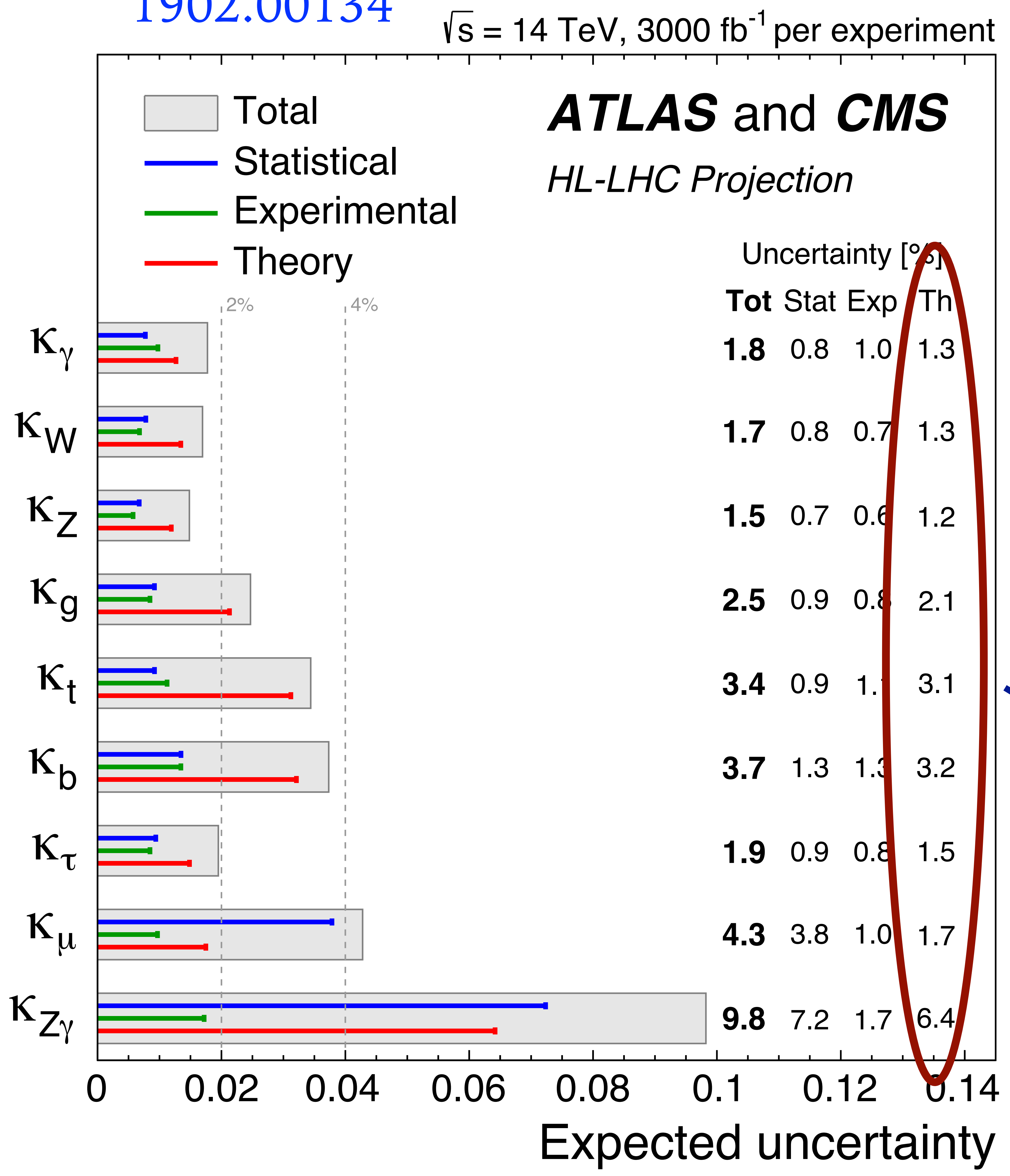
2205.08553



2203.06520

The need for theoretical precision!

1902.00134



The upcoming experimental accuracies are demanding **much better** theoretical precision for various scattering processes

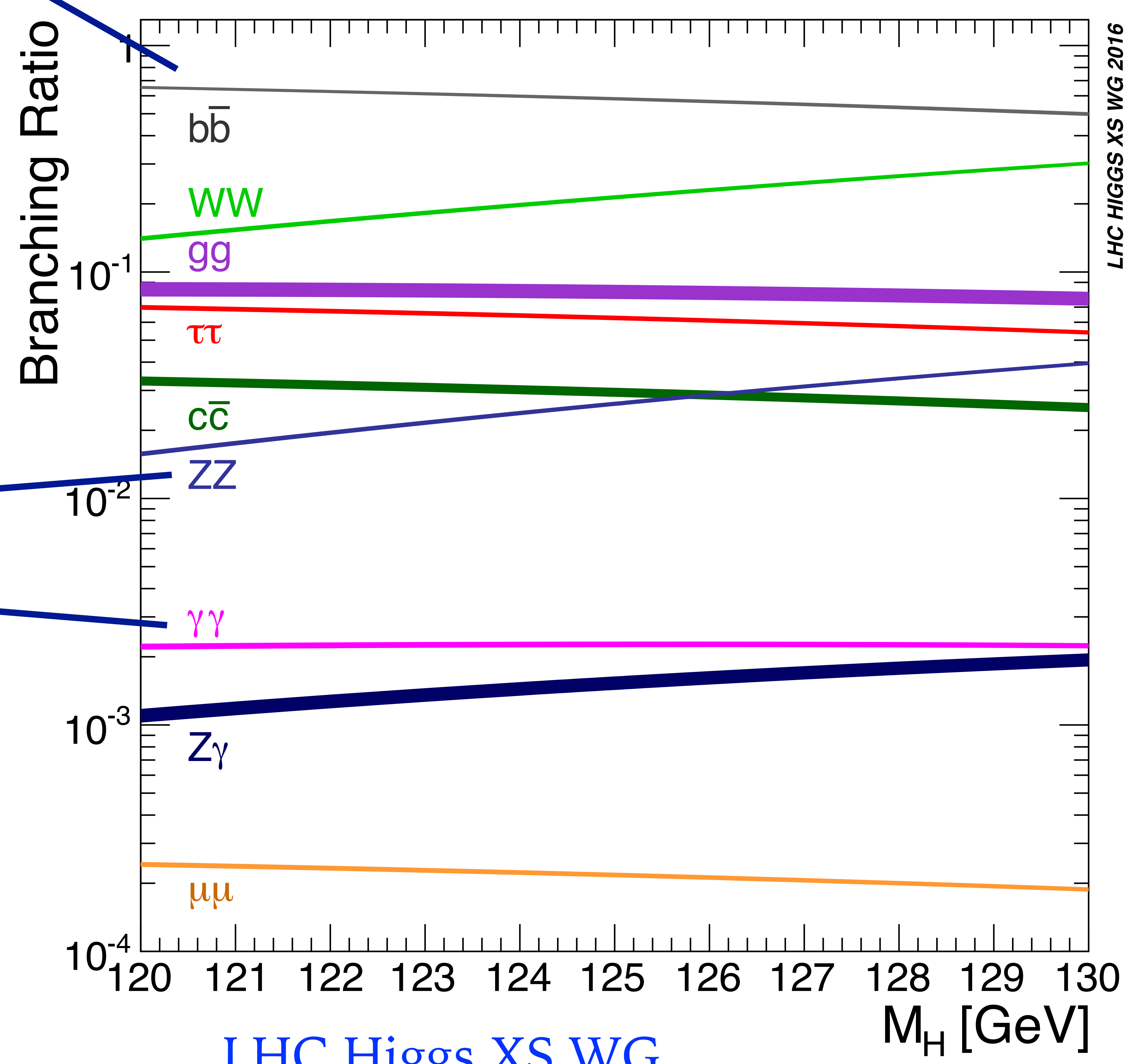
Estimated theoretical uncertainties that can be achieved during the HL-LHC run

(reduced by a factor of 2~3 w.r.t. current values)

Decay of the Higgs boson

Most frequent; relevant for $Hb\bar{b}$; but not easy to detect

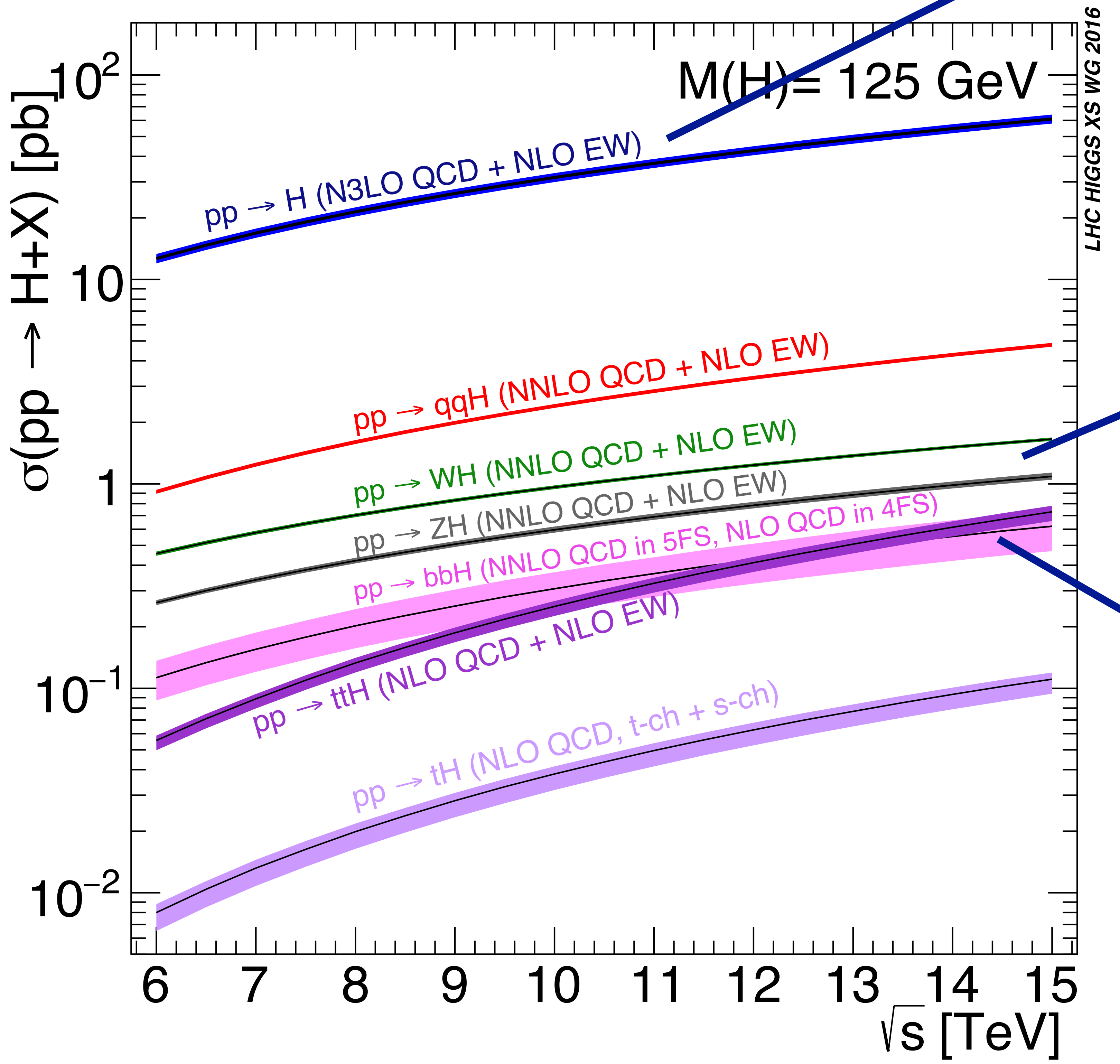
Rare but clean



LHC Higgs XS WG

Single Higgs production at the LHC

LHC Higgs XS WG

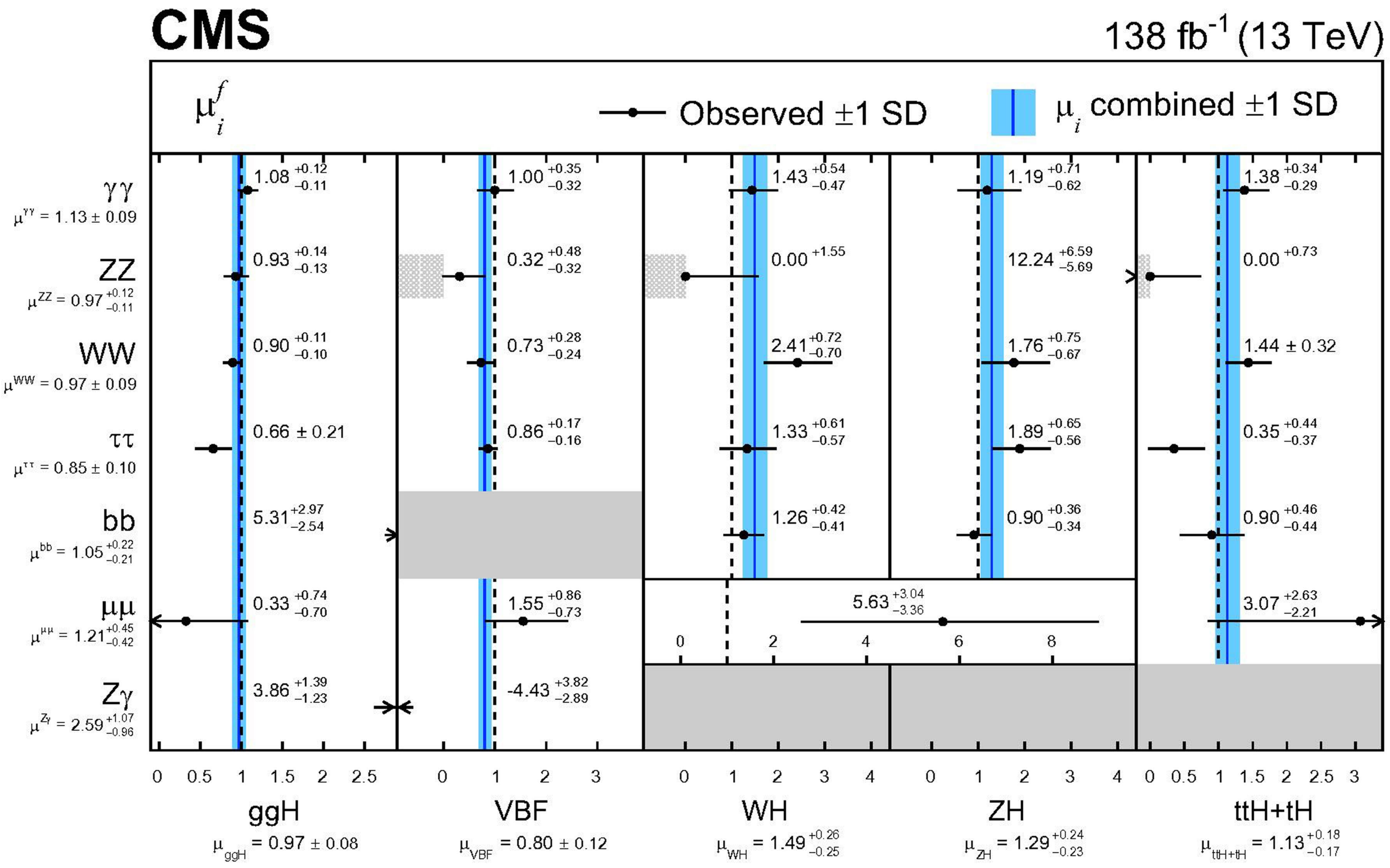


Gluon-fusion; largest rate; but only clean decay channels

VH-associated production, probing $b\bar{b}$ decay mode

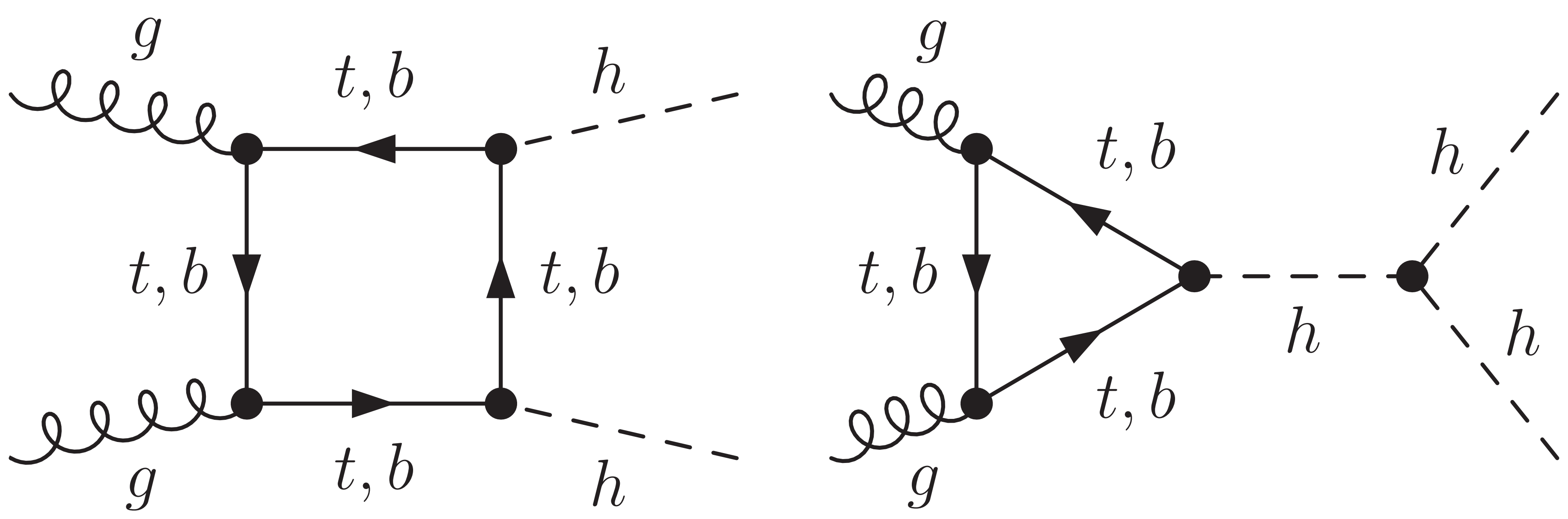
$t\bar{t}H$ associated production, probing top Yukawa

Single Higgs production at the LHC



Double Higgs production at the LHC

Higgs bosons can also be produced in pairs



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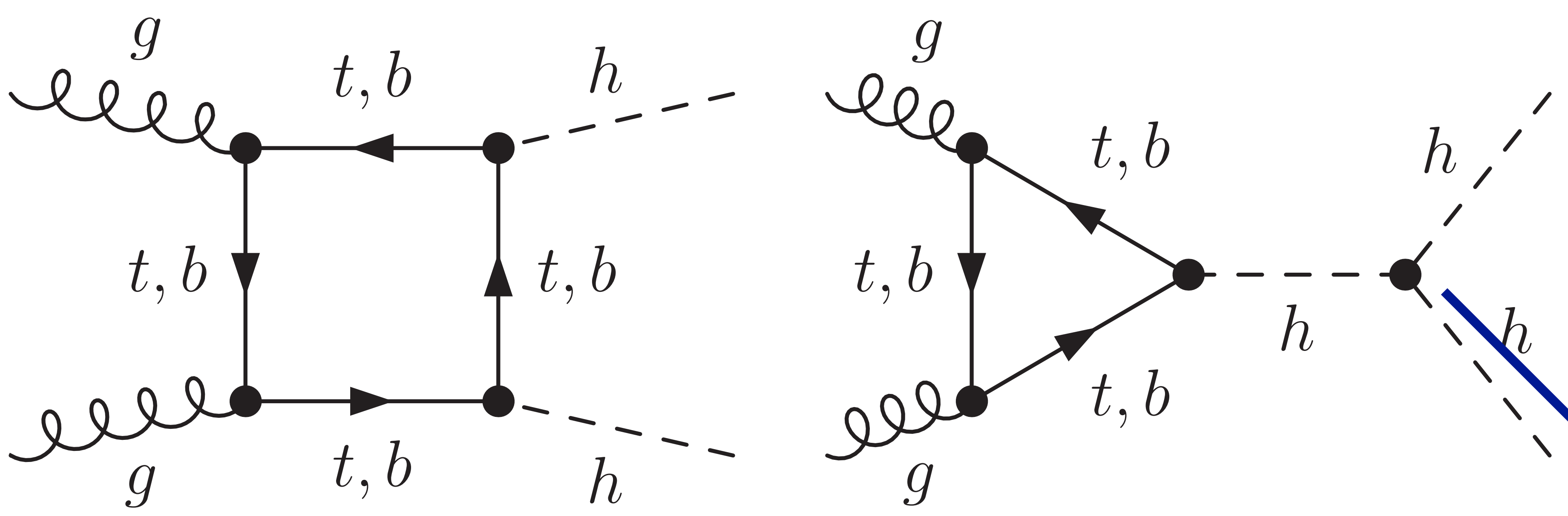
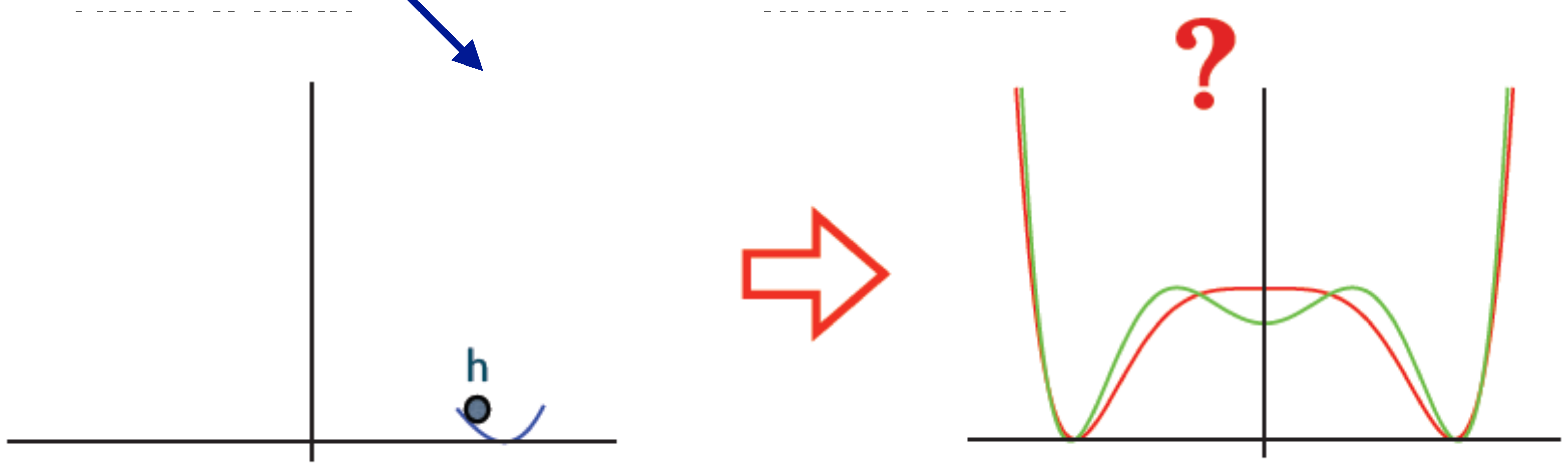
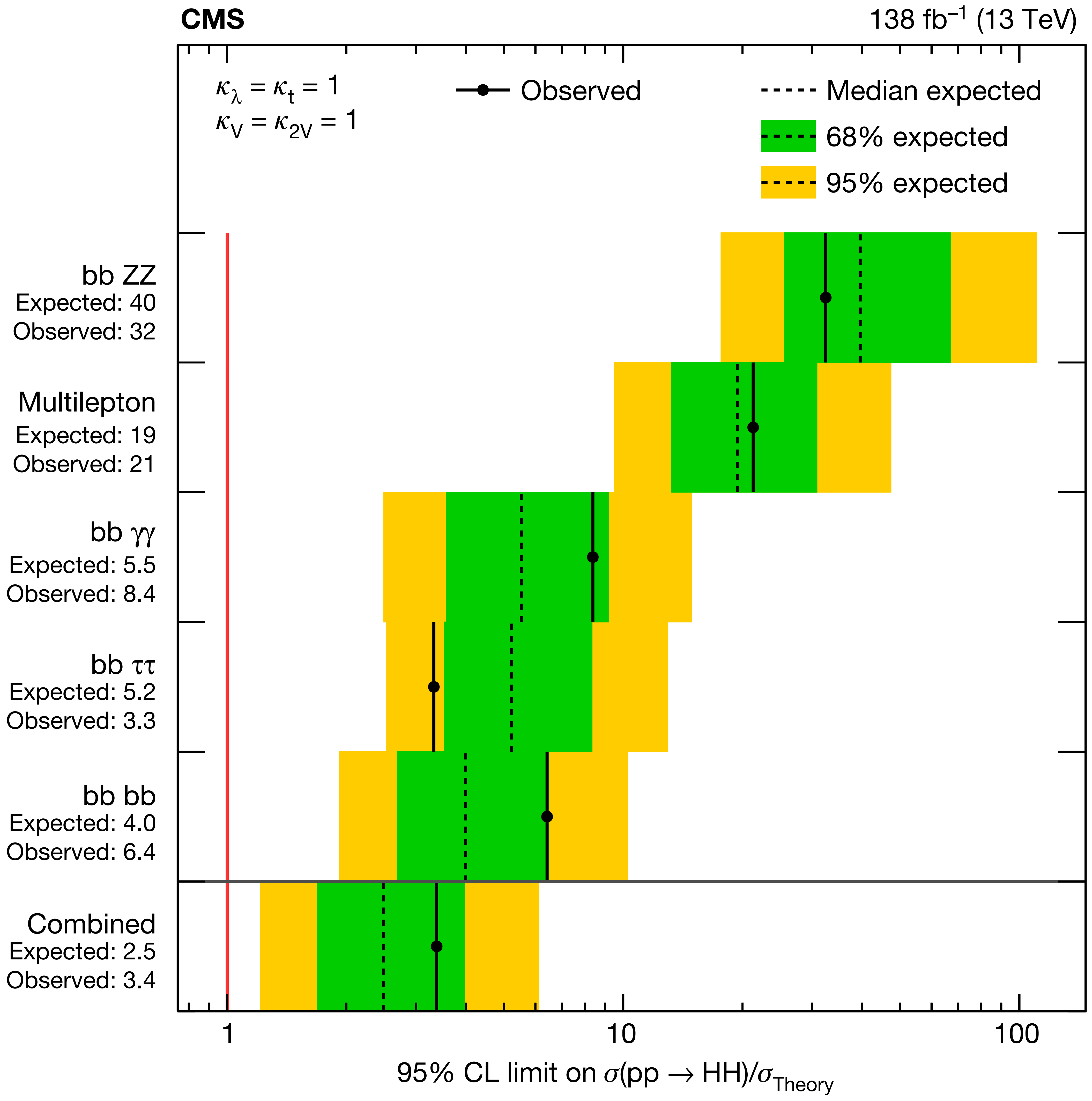


Probing the Higgs potential

$$V(h) = \frac{m_h^2}{2}h^2 + \lambda_3 h^3 + \cdots$$



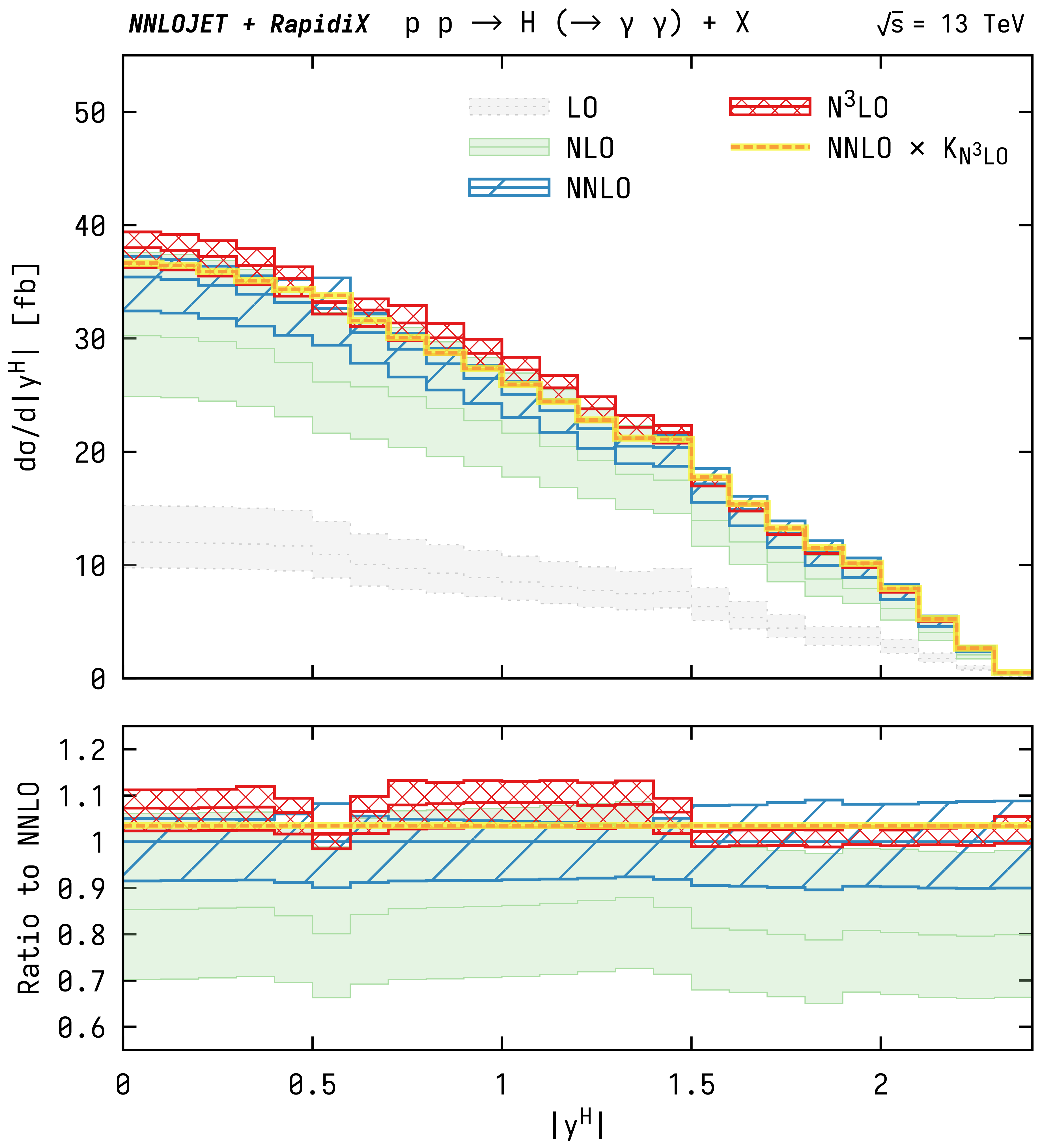
We are not there yet…



Higgs boson pair production is extremely difficult to detect

We'll need HL-LHC…

Single Higgs from gluon-fusion

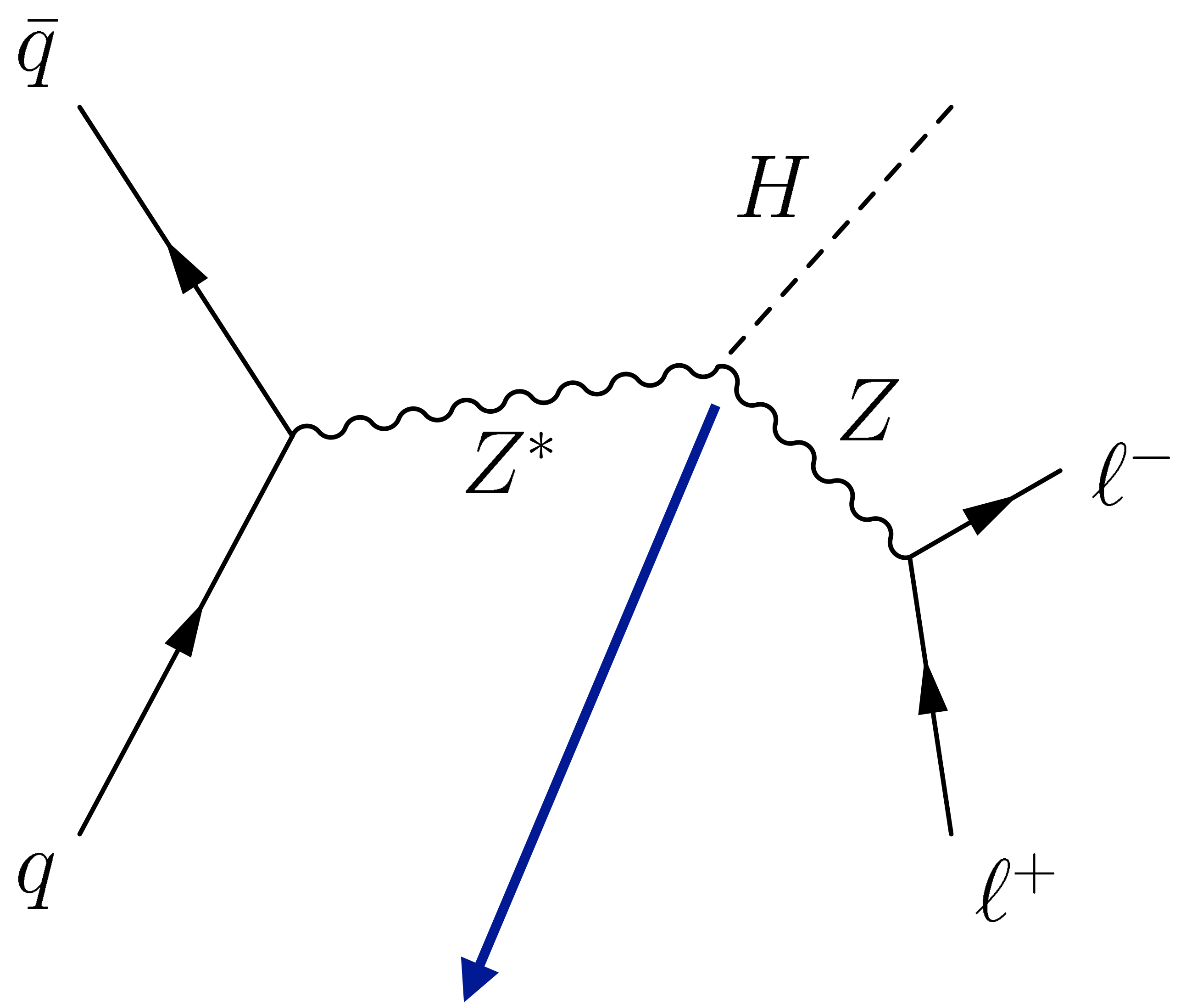


Fully differential cross sections at N^3LO

Chen et al.: 2102.07607

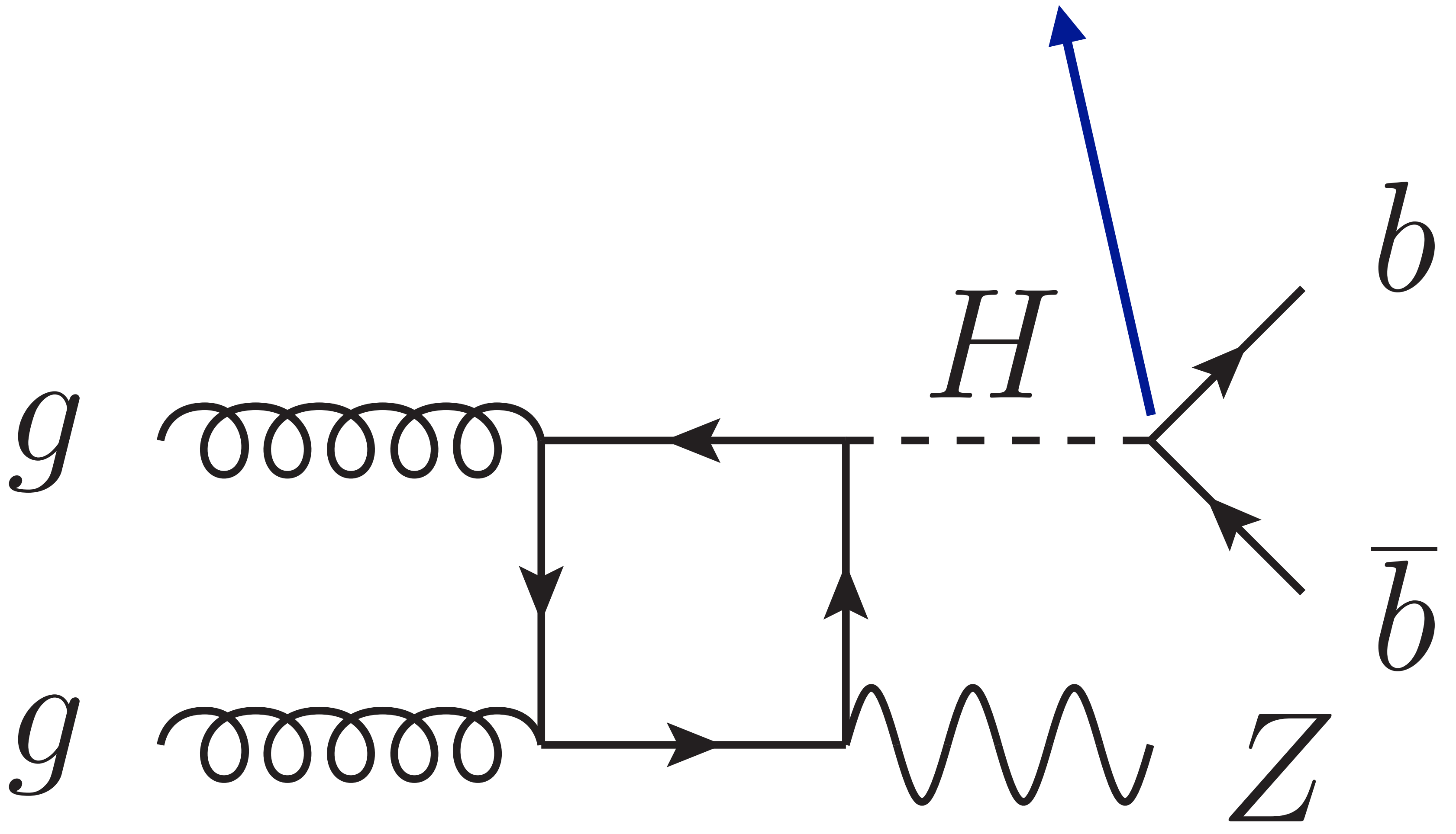
VH associated production (Higgs-strahlung)

Important for measuring gauge and Yukawa couplings



Electroweak symmetry breaking

Fermion mass



Gluon-fusion channel unique for ZH-production

Formally higher order, but enhanced
by gluon luminosity at the LHC

Theoretical uncertainties for ZH-production

Theoretical uncertainties dominated by missing higher order corrections

Table 10: Cross-section for the process $pp \to ZH$. The predictions for the $gg \to ZH$ channel are computed at LO, rescaled by the NLO K-factor in the $m_t \to \infty$ limit, and supplemented by the $\mathrm{NLL_{soft}}$ resummation. The photon contribution is omitted. Results are given for a Higgs boson mass $m_H = 125.09$ GeV.

1902.00134

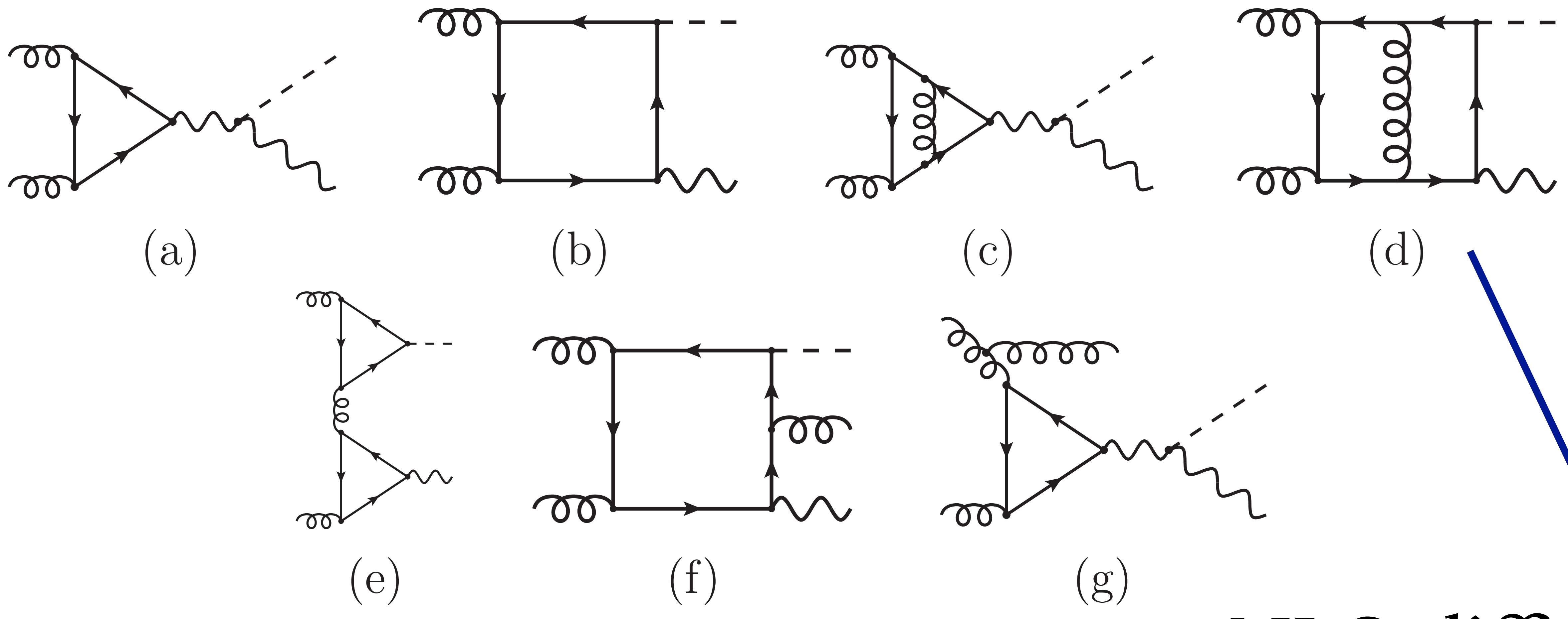
$\sqrt{s}$ [TeV]	$\sigma_{\mathrm{NNLO\ QCD \otimes NLO\ EW}}$ [pb]	Δ_{scale} [%]	$\Delta_{\mathrm{PDF}\oplus\alpha_s}$ [%]
13	0.880	$^{+3.50}_{-2.68}$	1.65
14	0.981	$^{+3.61}_{-2.94}$	1.90
27	2.463	$^{+5.42}_{-4.00}$	2.24

Mainly come from gg→ZH

gg→ZH

Loop induced

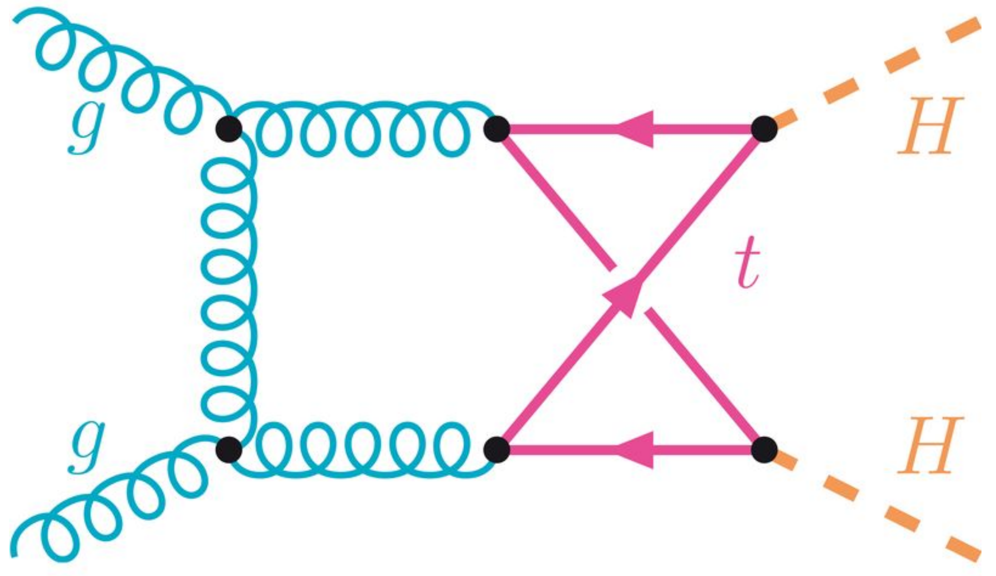
LO → formally start at α_s^2



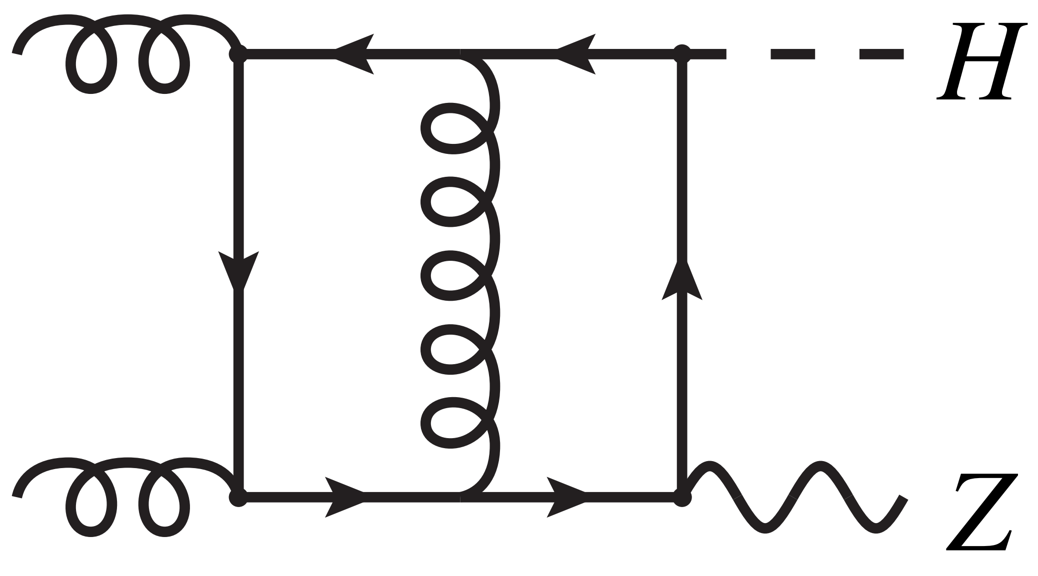
NLO difficult: two-loop four-point amplitude with 5 physical scales

Heavy top EFT not good for distributions…

Approximations with small-mass expansion



4 scales: s, t, m_t, m_H



5 scales: s, t, m_t, m_H, m_Z

Difficult to solve: integral reduction? master integrals?

An approximation: $m_H^2, m_Z^2 \ll |s|, |t|, m_t^2$

Valid for rather generic physical kinematics

Small-mass expansion

Xu, **LLY**: 1810.12002
Wang, Wang, Xu, Xu, **LLY**: 2010.15649

For HH: $$F_{HH}(s,t_1,m_t^2,m_H^2) = \sum_{n=0}^{\infty} \left(m_H^2\right)^n F_{HH}^{(n)}(s,t_1,m_t^2)$$

Same master integrals!

We know how to solve…

For ZH: $$F_{ZH}(s,t_1,m_t^2,m_H^2,m_Z^2) = \sum_n \sum_i \left(m_H^2\right)^i \left(m_Z^2\right)^{n-i} F_{ZH}^{(n,i)}(s,t_1,m_t^2)$$

Wang, Xu, Xu, **LLY**: 2107.08206

Small-mass expansion

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For ZH: $F_{ZH}(s, t_1, m_t^2, m_H^2, m_Z^2) = \sum_n \sum_i \left(m_H^2\right)^i \left(m_Z^2\right)^{n-i} F_{ZH}^{(n,i)}(s, t_1, m_t^2)$ Wang, Xu, Xu, **LLY**: 2107.08206

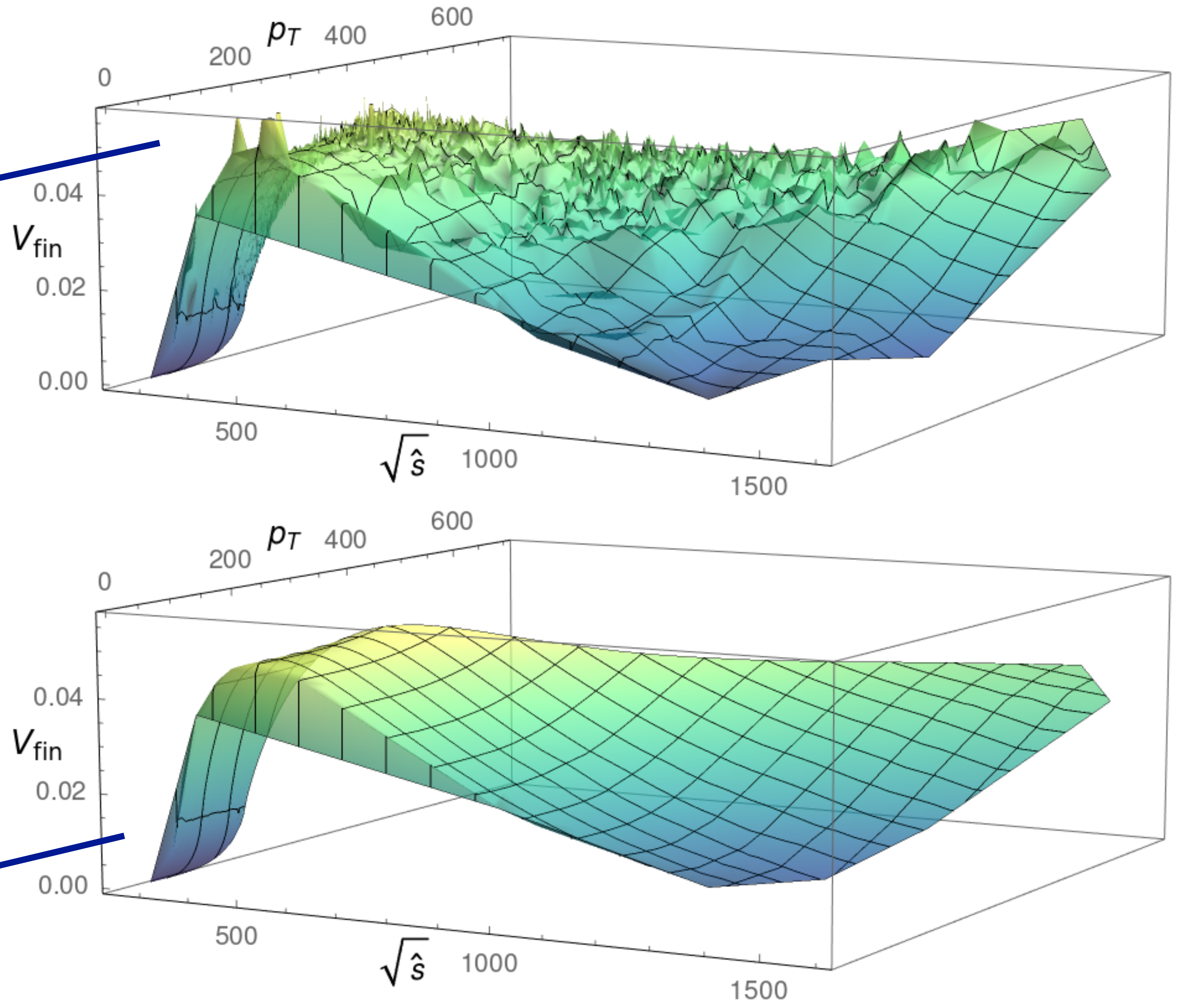
A slight complication: polarization sum of the Z boson $-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_Z^2}$

Consistent power-counting required

Numeric results for gg→HH

Wang, Wang, Xu, Xu, **LLY**: 2010.15649

UV and IR finite part of the two-loop amplitude



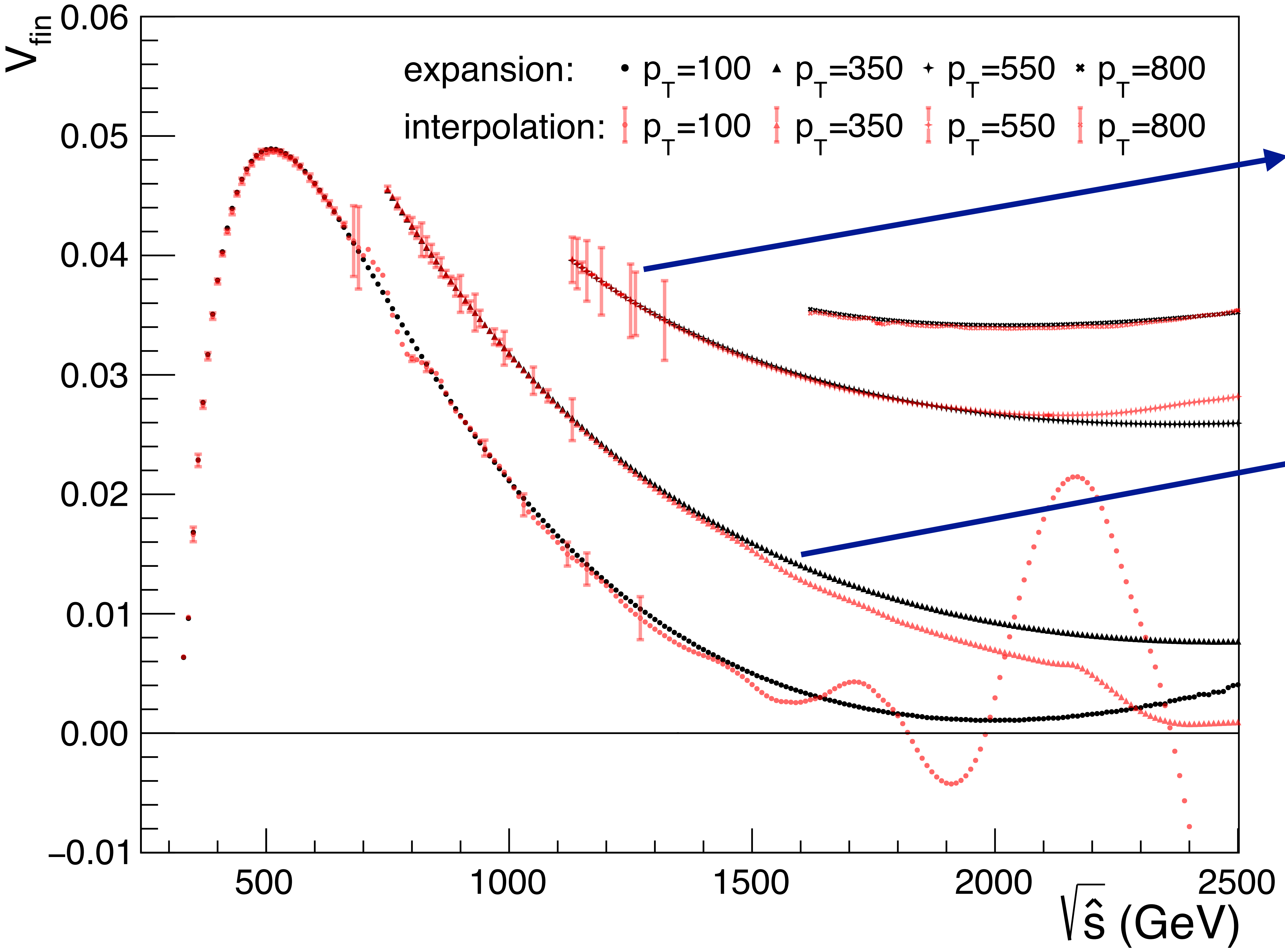
Interpolated from hhgrid (sector decomposition)

Heinrich et al. 7 GPGPU hours per phase space point

Small-mass expansion

10 CPU seconds per phase space point

Numeric results for gg→HH



Interpolated from hhgrid (sector decomposition + high energy expansion)

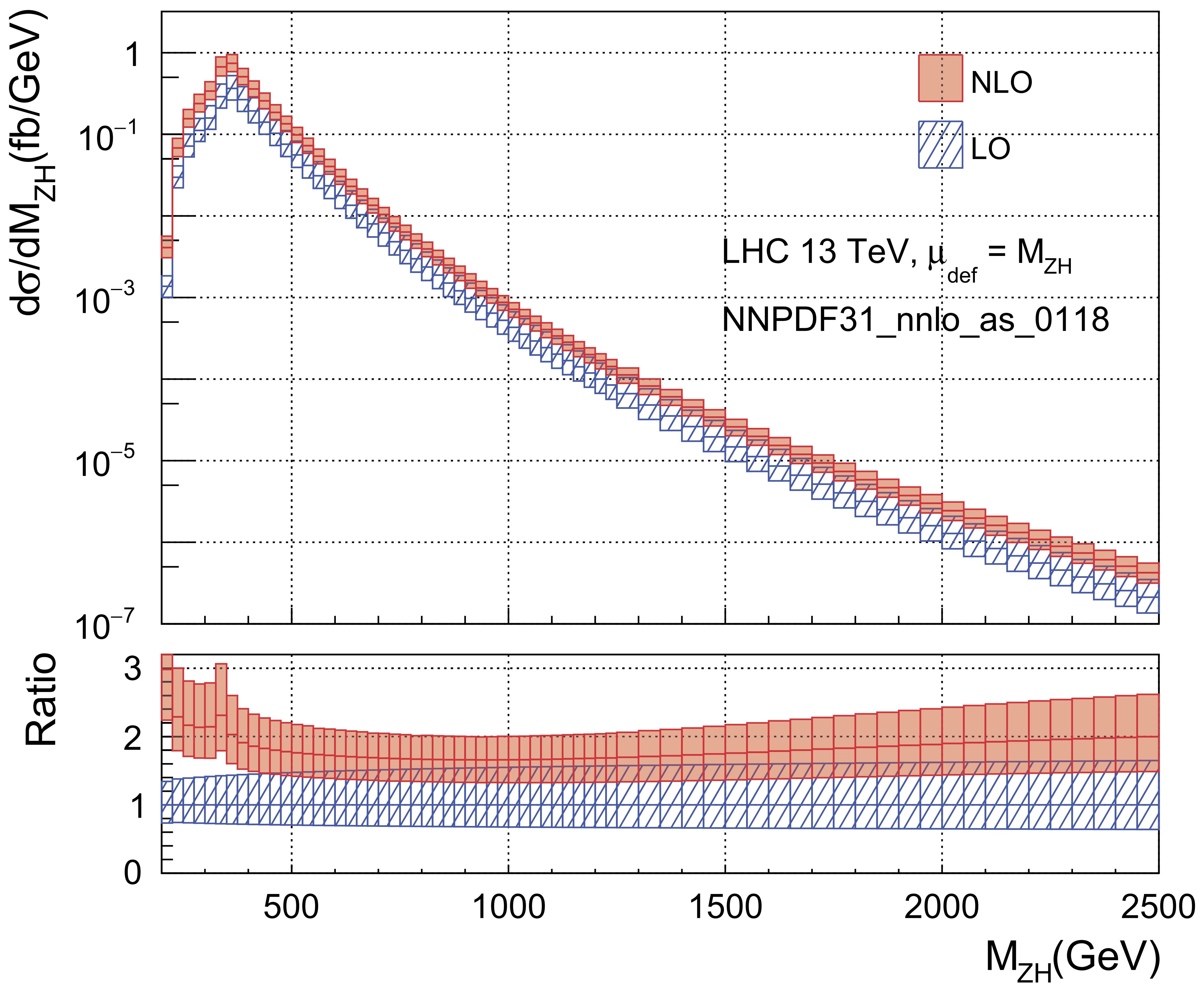
Small-mass expansion

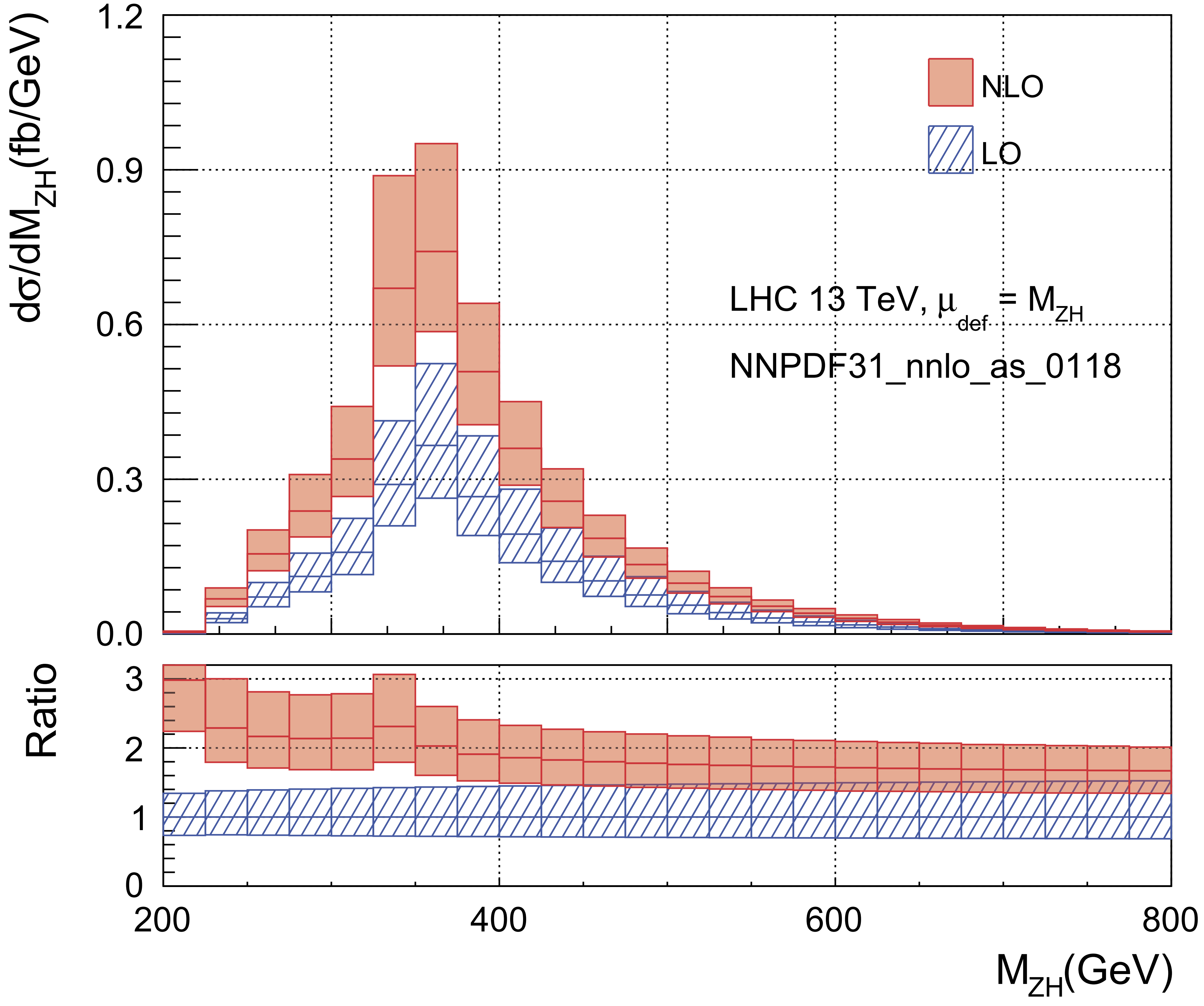
Numeric results for $gg \to ZH$

Wang, Xu, Xu, **LLY**: 2107.08206

NLO predictions for both total and differential cross sections including top quark mass dependence (**first time ever**)

$$\sigma_{pp\to ZH} = 882.9^{+3.5\%}_{-2.5\%}\ \text{fb}$$

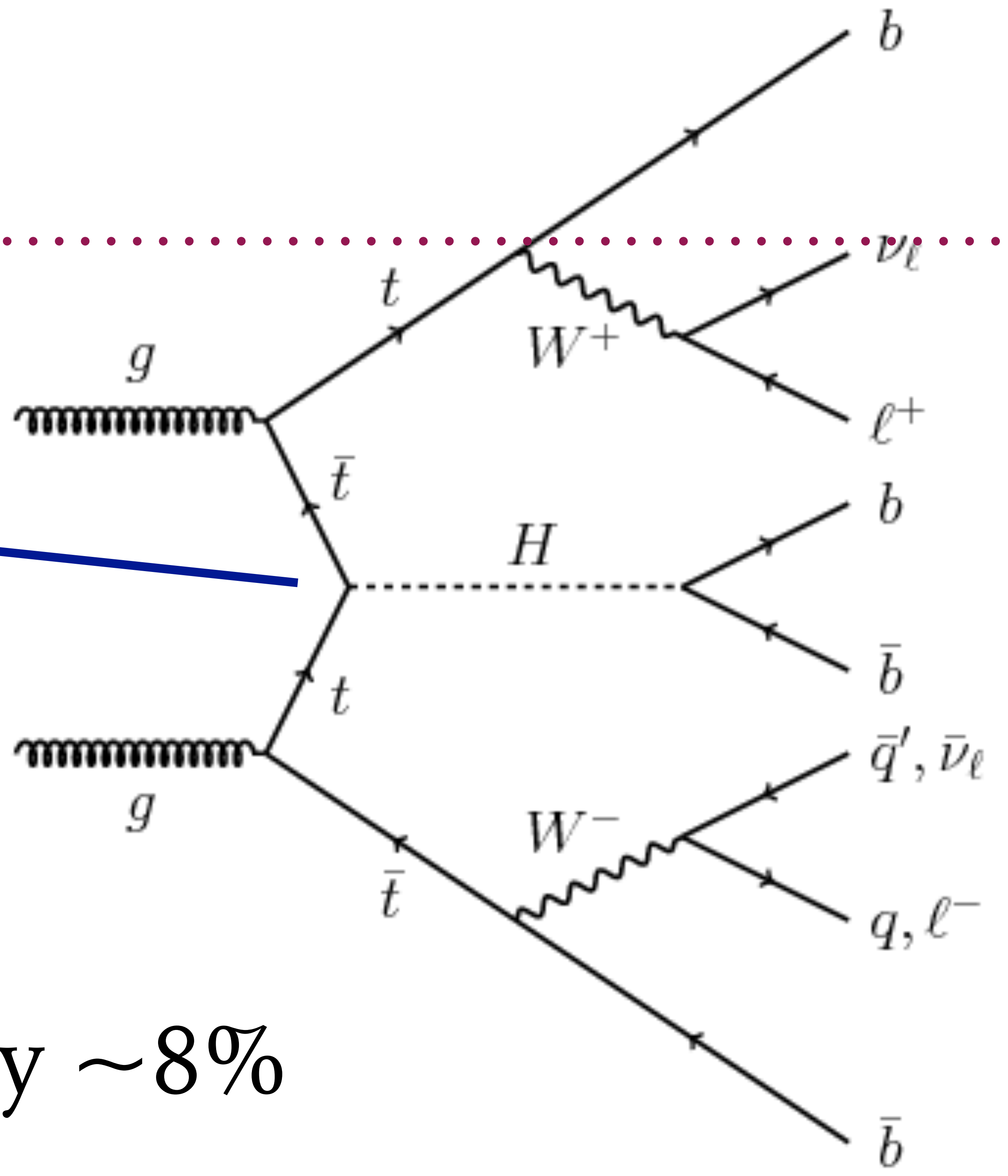




Non-trivial kinematic dependence: not an overall K-factor

What about NNLO?

Top-quark pair associated production



Probing the top-quark Yukawa coupling

Current state-of-the-art: NLO+NNLL in QCD, NLO in EW

Broggio, Ferroglia, Pecjak, **LLY**: 1601.00049
Ju, **LLY**: 1904.08744

See also 1610.07922 and references therein

Residue scale uncertainty ~8%

Top-quark pair associated production

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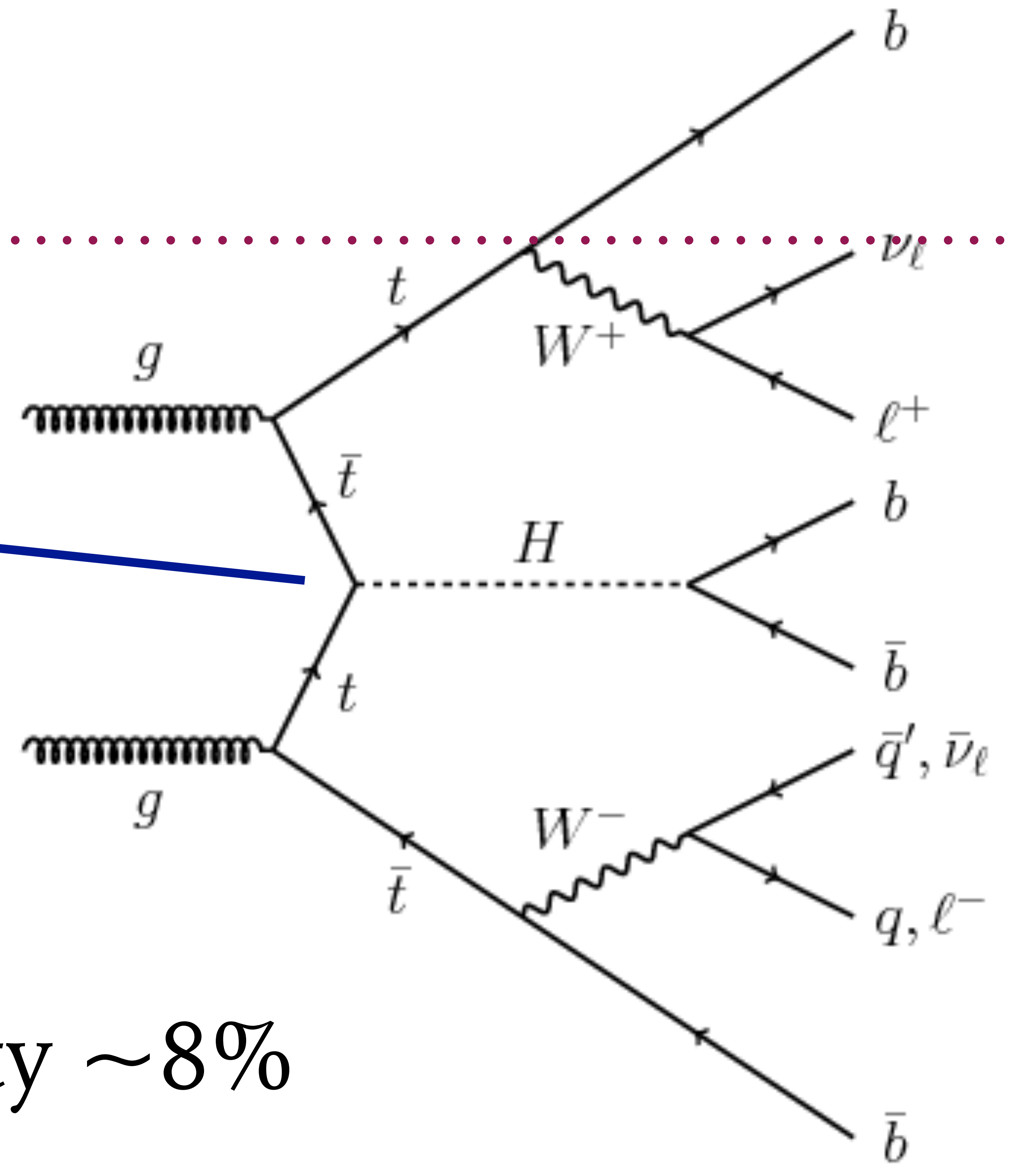

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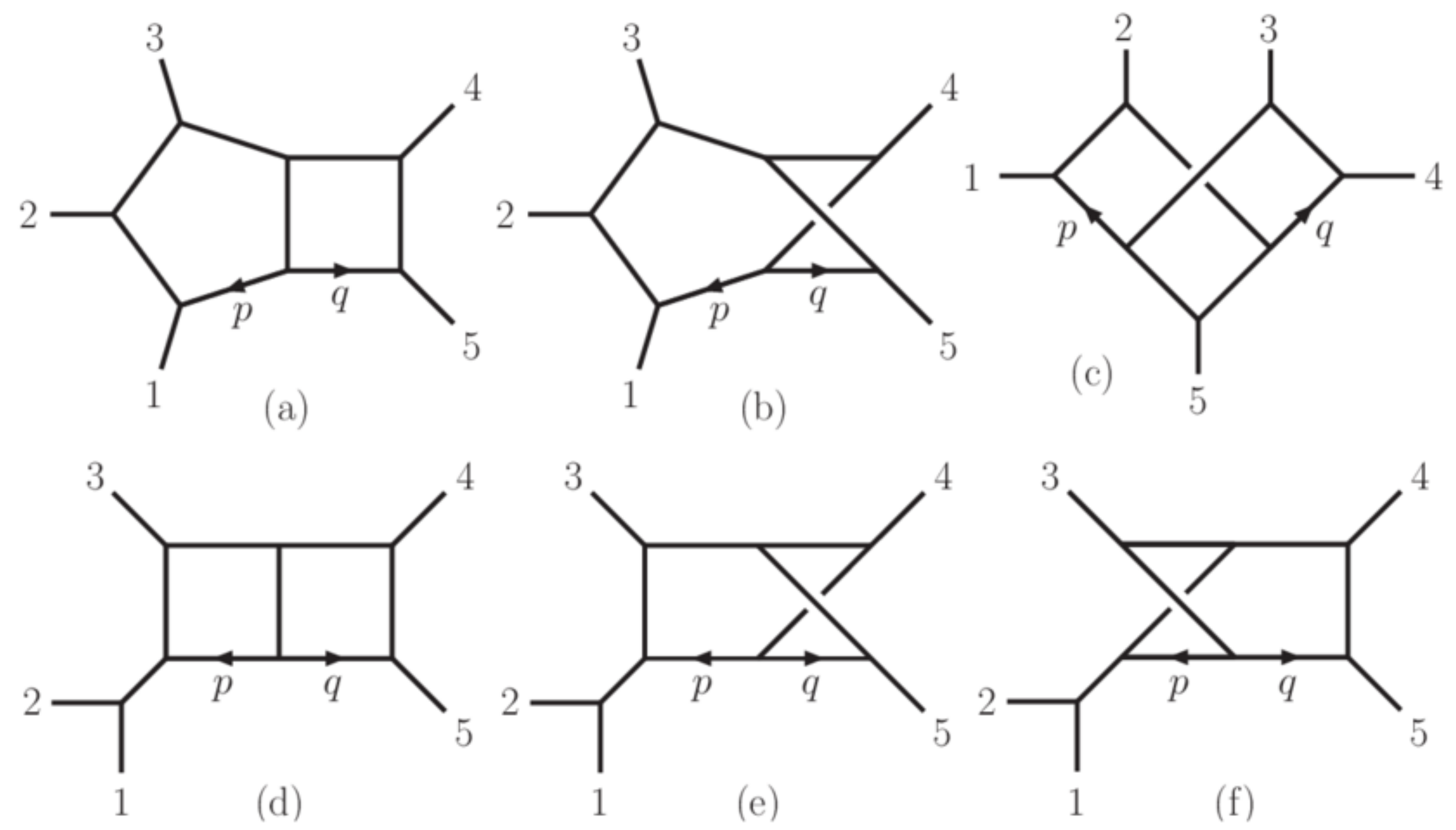
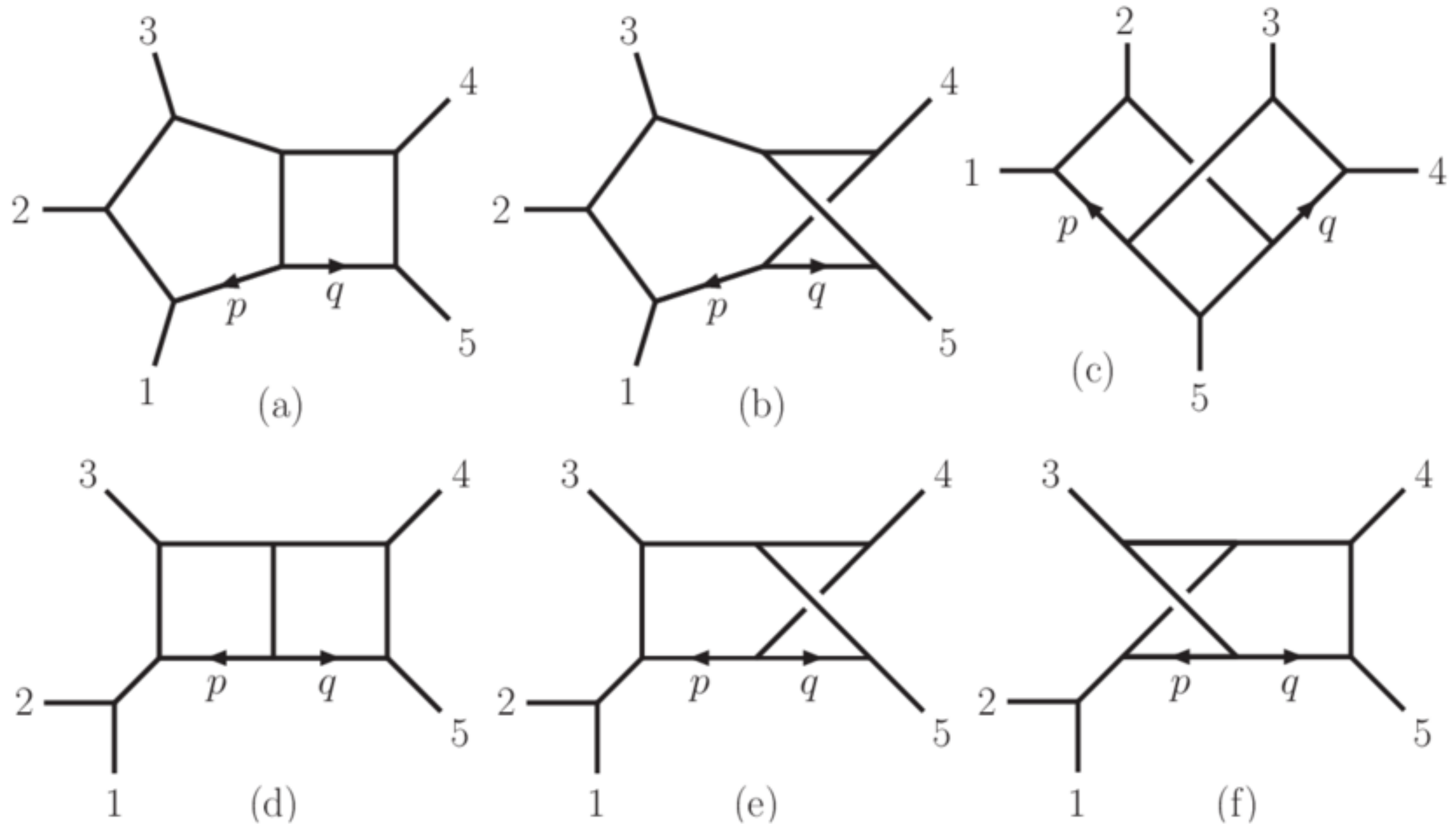


NNLO QCD extremely difficult

(two-loop integrals with 7 physical scales)

Two-loop IR divergences for ttH



The full two-loop amplitude is too difficult…

But it is possible to compute the IR divergent part of the amplitude!

- Needs to be exactly cancelled against real corrections
- Provide an independent check for future (most likely numeric) calculations of the full amplitude

The universal structure of two-loop IR divergences

The IR divergences of any two-loop amplitude in gauge theories can be determined given the corresponding one-loop amplitudes (up to order ϵ^1 in DREG)
and a universal anomalous dimension matrix

$$\begin{aligned}\boldsymbol{\Gamma}(\{\underline{p}\},\{\underline{m}\},\mu) = &\sum_{(i,j)} \frac{\boldsymbol{T}_i\cdot\boldsymbol{T}_j}{2}\,\gamma_{\rm cusp}(\alpha_s)\,\ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)\\ &- \sum_{(I,J)} \frac{\boldsymbol{T}_I\cdot\boldsymbol{T}_J}{2}\,\gamma_{\rm cusp}(\beta_{IJ},\alpha_s) + \sum_I \gamma^I(\alpha_s) + \sum_{I,j} \boldsymbol{T}_I\cdot\boldsymbol{T}_j\,\gamma_{\rm cusp}(\alpha_s)\,\ln\frac{m_I\mu}{-s_{Ij}}\\ &+ \sum_{(I,J,K)} if^{abc}\,\boldsymbol{T}_I^a\,\boldsymbol{T}_J^b\,\boldsymbol{T}_K^c\,F_1(\beta_{IJ},\beta_{JK},\beta_{KI}) \qquad (5)\\ &+ \sum_{(I,J)}\sum_k if^{abc}\,\boldsymbol{T}_I^a\,\boldsymbol{T}_J^b\,\boldsymbol{T}_k^c\,f_2\Big(\beta_{IJ},\ln\frac{-\sigma_{Jk}\,v_J\cdot p_k}{-\sigma_{Ik}\,v_I\cdot p_k}\Big) + \mathcal{O}(\alpha_s^3)\,.\end{aligned}$$

Ferroglia, Neubert, Pecjak, **LLY**: 0907.4791, 0908.3676

A fact from soft-collinear factorization

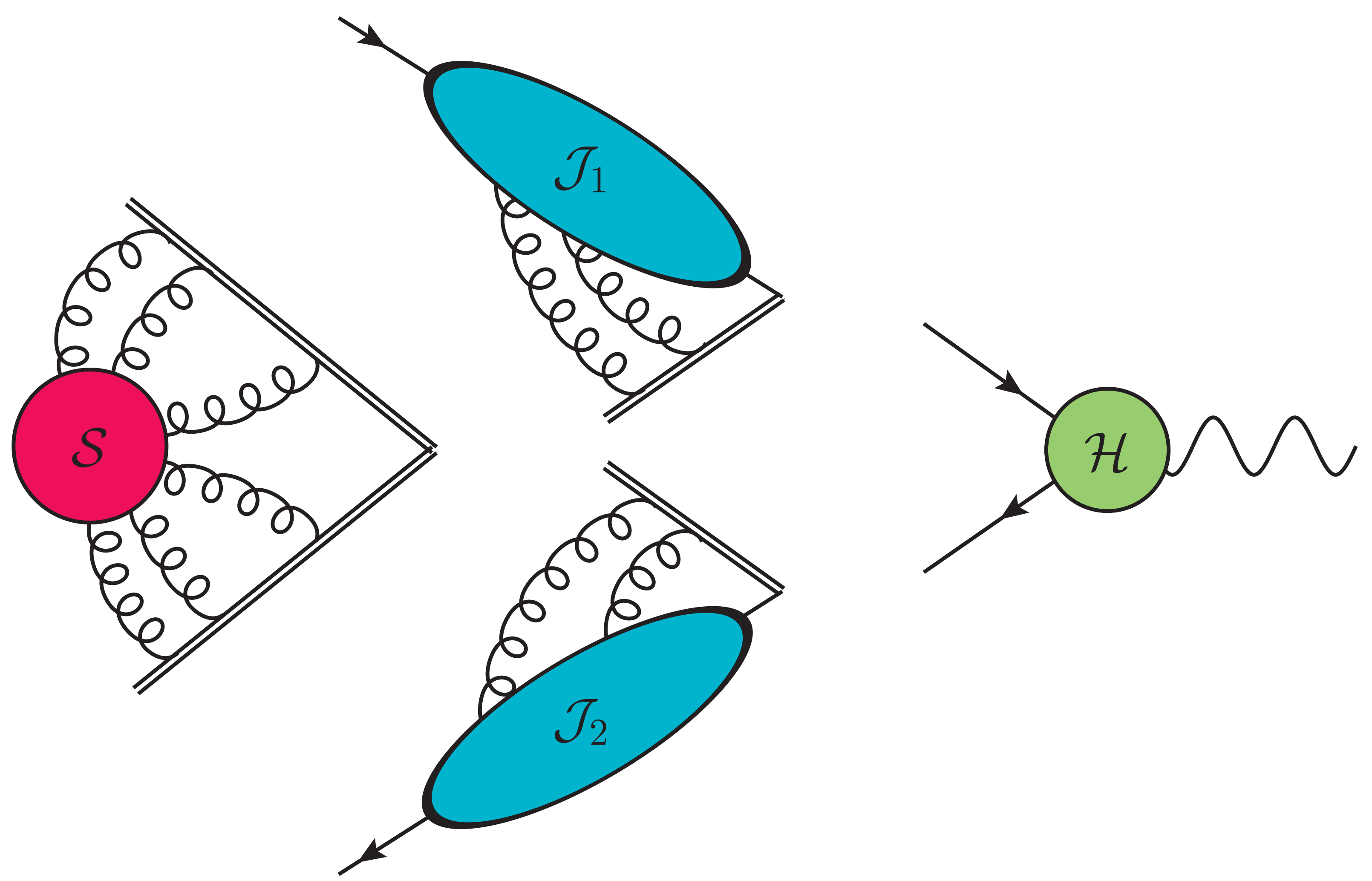


Figure from 2112.07099

One-loop integrals to higher orders in ϵ

The one-loop integrals up to the finite term have been obtained long long ago

't Hooft, Veltman (1979)

However, the problem of higher order terms in ϵ has not been generically solved!

One-loop integrals to higher orders in ϵ

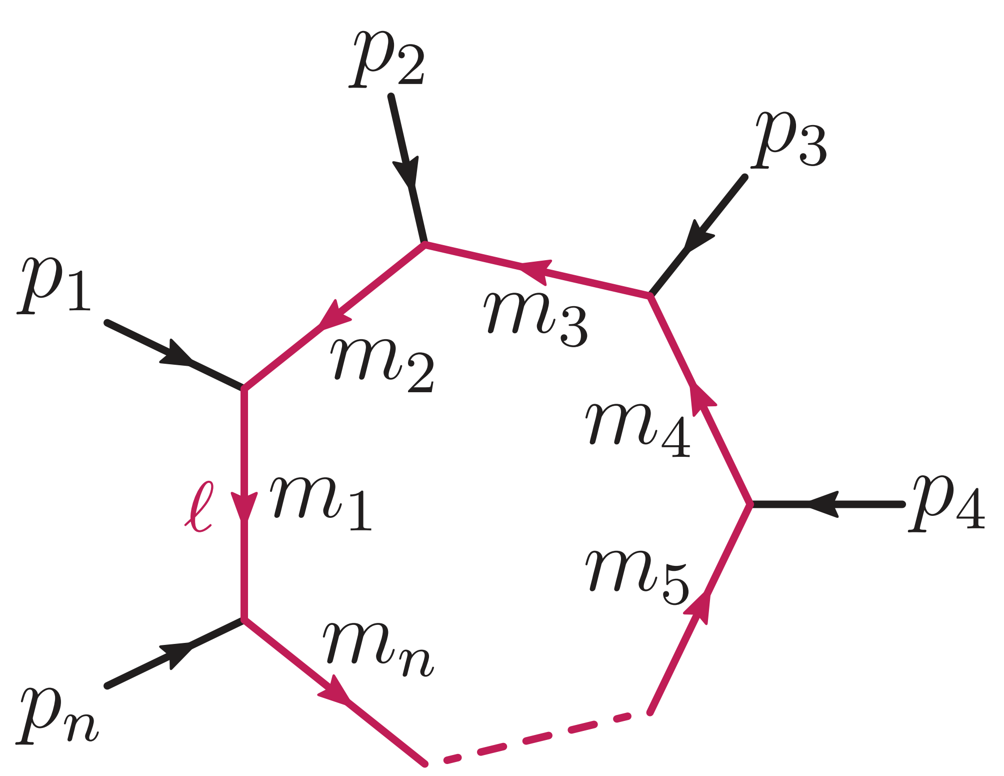
The one-loop integrals up to the finite term have been obtained long long ago

’t Hooft, Veltman (1979)

However, the problem of higher order terms in ϵ has not been generically solved!

Fortunately, it is known that all one-loop integral families admit canonical bases

Bourjaily, Gardi, McLeod, Vergu: 1912.11067
Chen, Jiang, Xu, **LLY**: 2008.03045
Chen, Jiang, Ma, Xu, **LLY**: 2202.08127



ϵ-form differential equations:

$$d\vec{f}(z,\epsilon) = \epsilon\, dA(z)\, \vec{f}(z,\epsilon)$$

$$\vec{f}(z,\epsilon) = \sum_n \epsilon^n \vec{f}^{(n)}(z)$$

Generic result of the one-loop alphabet

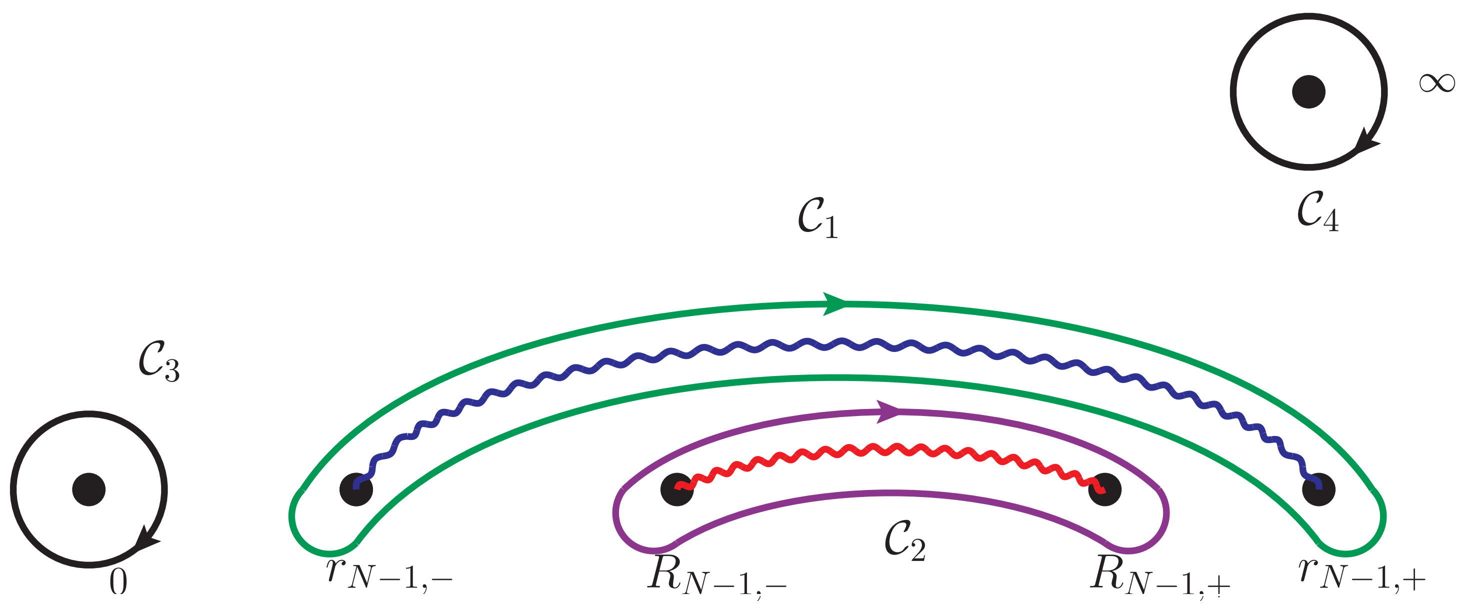
Chen, Ma, **LLY**: 2201.12998
Abreu et al.: 1704.07931

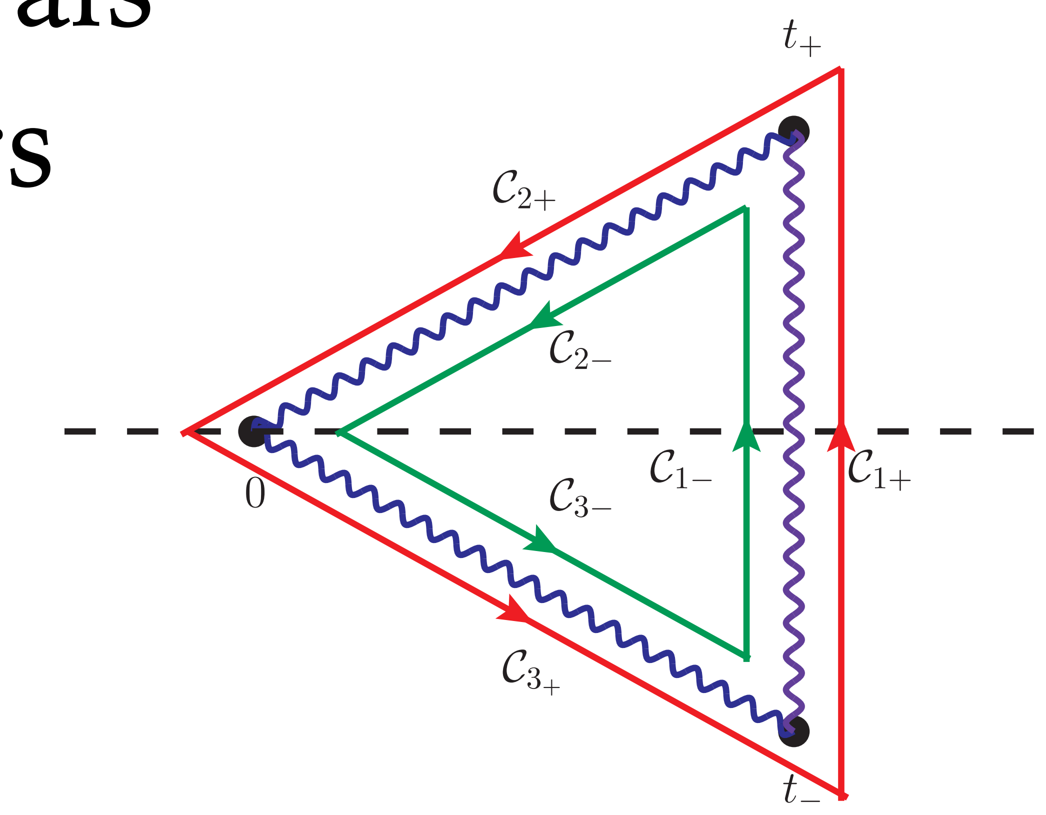
Solutions consist of iterated integrals

$$\vec{f}^{(n)}(z) \supset \int_{z_0}^{z} d\log(\alpha_n(z_n)) \cdots \int_{z_0}^{z_3} d\log(\alpha_2(z_2)) \int_{z_0}^{z_2} d\log(\alpha_1(z_1))$$

Lots of information about the solutions is contained in the "alphabet"

We managed to obtain generic result of the alphabet using cut integrals in the Baikov representation and cleverly-chosen integration contours

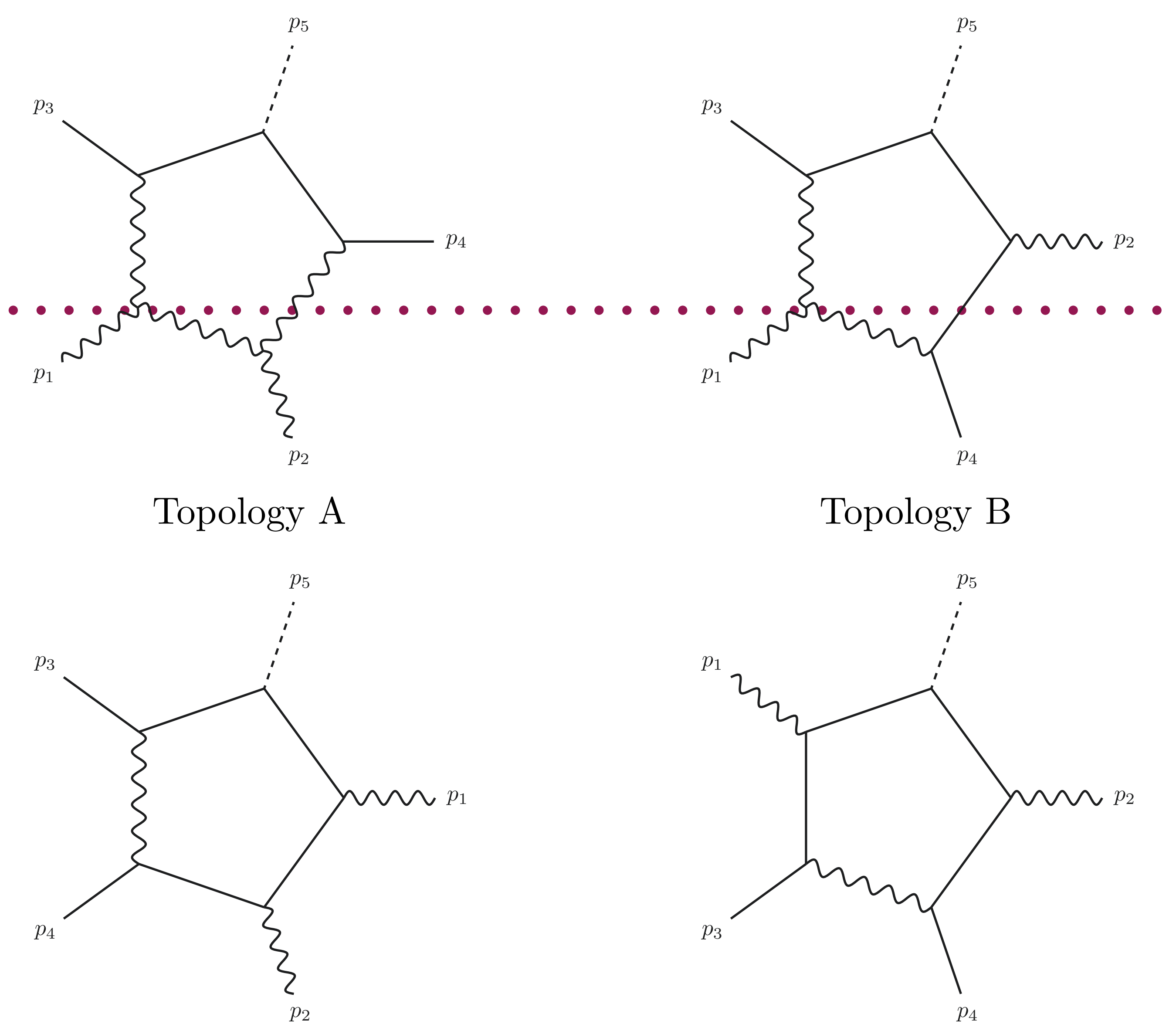




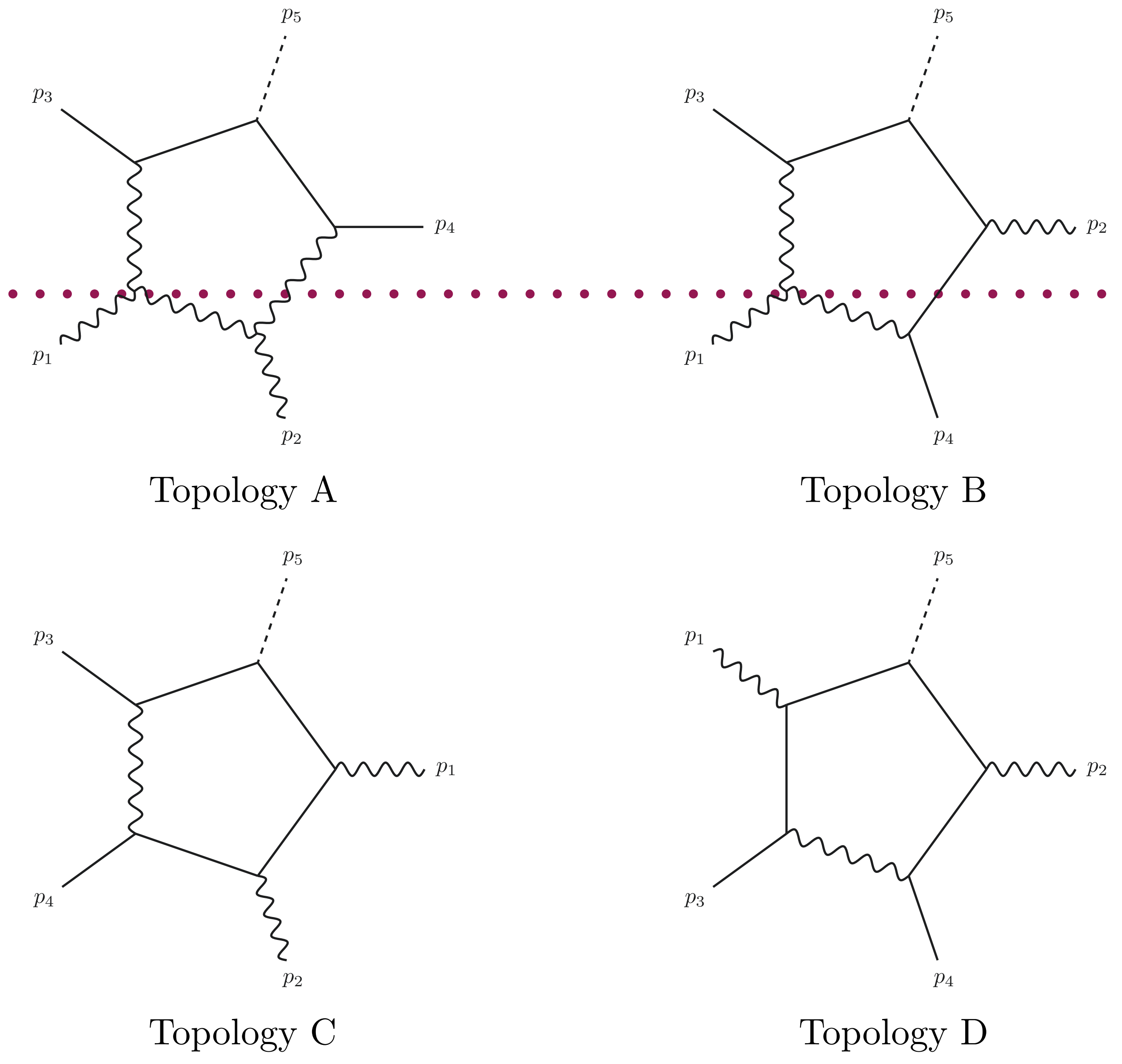
Applied to the ttH case

4 topologies, 6 independent dimensionless kinematic variables

$$x_{ij} = \frac{s_{ij}}{m_t^2}, \quad x_h = \frac{m_H^2}{m_t^2}$$


Topology A Topology B
Topology C Topology D

Applied to the ttH case



4 topologies, 6 independent dimensionless kinematic variables

$$x_{ij} = \frac{s_{ij}}{m_t^2}\,, \quad x_h = \frac{m_H^2}{m_t^2}$$

The letters are sometimes rather complicated!

e.g.: $$\frac{C_5 - \sqrt{-\mathcal{K}_5\mathcal{K}_3}}{C_5 + \sqrt{-\mathcal{K}_5\mathcal{K}_3}}$$

$$\begin{aligned} C_5 &= G(-p_3, p_1 + p_2, -p_4; -p_3, p_1 + p_2, -p_2) \\ &= \frac{1}{8}\Big(x_{45}x_{12}x_h - x_{12}^2 x_h - 2x_{13}x_{12}x_h + x_{12}x_h - x_{13}x_{12}^2 + x_{24}x_{12}^2 + 2x_{13}x_{12} \\ &\quad + x_{12} - 2x_{24}x_{12} + x_{13}x_{35}x_{12} + x_{13}x_{45}x_{12} - 2x_{24}x_{45}x_{12} + x_{35}x_{45}x_{12} \\ &\quad + x_{24}x_{45}^2 - x_{35}x_{45}^2 - x_{13}x_{45} - 2x_{24}x_{45} + x_{13}x_{35}x_{45} + x_{13} - x_{13}x_{35} \\ &\quad + x_{35}x_{45} + x_{45} + x_{24} - 2x_{45}x_{12} - 1\Big)\,, \\ \mathcal{K}_3 &= G(-p_3, p_1 + p_2) = -\frac{1}{4}\left(x_{12}^2 + x_{45}^2 - 2x_{45}x_{12} - 2x_{12} - 2x_{45} + 1\right), \end{aligned}$$

$$\begin{aligned} \mathcal{K}_5 &= G(-p_3, p_1, p_2, -p_4) \\ &= \frac{1}{16}\Big(x_{12}^2x_h^2 - 2x_{13}x_{12}^2x_h - 2x_{24}x_{12}^2x_h + 2x_{13}x_{12}x_h - 4x_{13}x_{24}x_{12}x_h + 2x_{24}x_{12}x_h \\ &\quad + 2x_{12}x_{35}x_{13}x_h - 2x_{12}x_h + 2x_{12}x_{24}x_{45}x_h - 2x_{12}x_{35}x_{45}x_h + x_{12}^2x_{13}^2 + x_{12}^2x_{24}^2 \\ &\quad - 2x_{13}x_{24}x_{12}^2 - 2x_{13}^2x_{12} - 2x_{24}^2x_{12} + 2x_{13}x_{12} + 4x_{13}x_{24}x_{12} - 2x_{13}^2x_{35}x_{12} \\ &\quad - 2x_{12}x_{45}x_{24}^2 + 2x_{12}x_{13}x_{35}x_{24} - 4x_{12}x_{45}x_{24} + 2x_{12}x_{13}x_{45}x_{24} - 4x_{12}x_{13}x_{35} \\ &\quad + x_{35}^2x_{13}^2 + x_{13}^2 + 2x_{12}x_{35}x_{45}x_{13} - 2x_{13} + x_{24}^2x_{45}^2 + x_{35}^2x_{45}^2 + 2x_{12}x_{24}x_{35}x_{45} \\ &\quad - 2x_{35}x_{13}^2 + 2x_{24}x_{13} - 2x_{24}x_{35}x_{13} + 2x_{35}x_{13} - 2x_{24}x_{35}x_{45}^2 - 2x_{24} - 2x_{24}^2x_{45} \\ &\quad - 2x_{13}x_{45}x_{35}^2 + 2x_{13}x_{45}x_{35} + 2x_{13}x_{24}x_{45}x_{35} - 2x_{13}x_{24}x_{45} + 2x_{24}x_{45} \\ &\quad + 2x_{24}x_{35}x_{45} - 2x_{35}x_{45} + 2x_{24}x_{12} + x_{24}^2 + 1\Big)\,, \end{aligned}$$

Easily obtained using our method

Results for the two-loop IR poles

Chen, Ma, Wang, **LLY**, Ye: 2202.02913

The square amplitudes can be decomposed into color coefficients

$$2\,\mathrm{Re}\left\langle \mathcal{M}_q^{(0)} \middle| \mathcal{M}_q^{(2)} \right\rangle = 2(N^2-1)\left(N^2 A^q + B^q + \frac{1}{N^2} C^q + N n_l D_l^q + N n_h D_h^q + \frac{n_l}{N} E_l^q + \frac{n_h}{N} E_h^q + n_l^2 F_l^q + n_l n_h F_{lh}^q + n_h^2 F_h^q \right),$$

$$2\,\mathrm{Re}\left\langle \mathcal{M}_g^{(0)} \middle| \mathcal{M}_g^{(2)} \right\rangle = (N^2-1)\left(N^3 A^g + N B^g + \frac{1}{N} C^g + \frac{1}{N^3} D^g + N^2 n_l E_l^g + N^2 n_h E_h^g + n_l F_l^g + n_h F_h^g + \frac{n_l}{N^2} G_l^g + \frac{n_h}{N^2} G_h^g + N n_l^2 H_l^g + N n_l n_h H_{lh}^g + N n_h^2 H_h^g + \frac{n_l^2}{N} I_l^g + \frac{n_l n_h}{N} I_{lh}^g + \frac{n_h^2}{N} I_h^g \right)$$

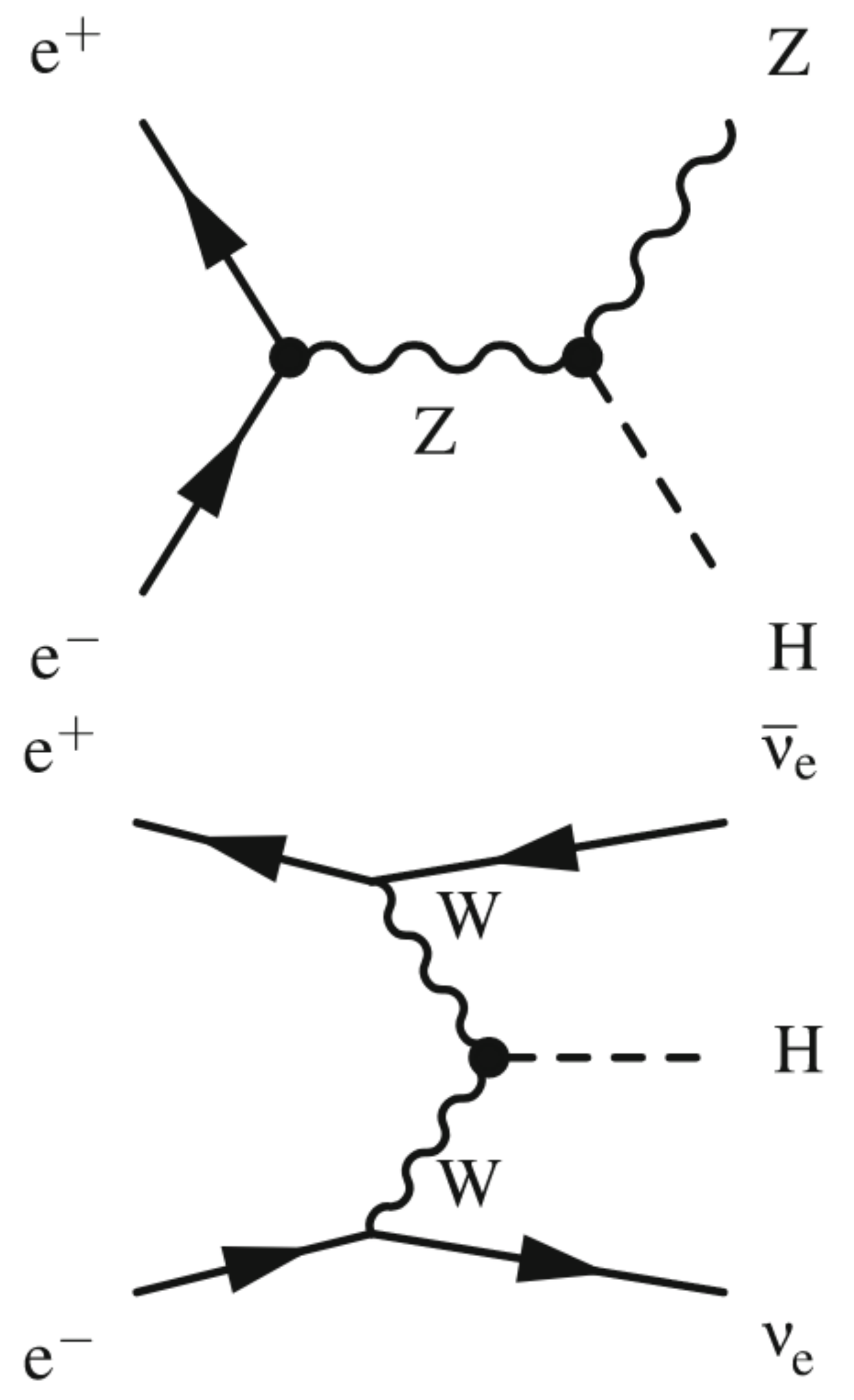
Results at a sample phase-space point

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}
A^g	17.37022326	6.277797530	−162.1830217	559.8062598
B^g	−32.49510001	−34.75486260	−624.1343773	3901.332369
C^g		−9.463444735	−54.41556200	−497.5350517
D^g			143.6321997	−578.4857199
E_l^g		−20.26526047	46.54471184	−10.69967085
E_h^g			−24.23013938	79.68650479
F_l^g		37.91095001	−74.94866603	71.66904977
F_h^g			43.70151160	−132.3384924
G_l^g			4.731722368	85.25318119
G_h^g				6.363526190
H_l^g			3.860049613	−10.52987601
H_{lh}^g				8.076713126
H_h^g				
I_l^g			−7.221133335	19.49234494
I_{lh}^g				−14.56717053
I_h^g				
A^q	2.390051823	15.03938540	0.597121534	−34.95784899
B^q	−4.780103646	−22.69017086	49.54607207	106.0851578
C^q	2.390051823	7.650785464	−186.5751188	−21.39439443
D_l^q		−2.390051823	0.308675876	−6.605875838
D_h^q			6.244349191	4.860387981
E_l^q		2.390051823	1.610219156	77.52356965
E_h^q			−6.244349191	19.76269918
F_l^q				
F_{lh}^q				
F_h^q				

Table 1. IR poles decomposed as color coefficients for the phase-space point $x_{12} = 10$, $x_{13} = -1339/920$, $x_{14} = -2269/465$, $x_{23} = -1951/620$, $x_{24} = -1803/1810$ and $x_{34} = 5$.

Higgs production at e+e- colliders

Higgs-strahlung (ZH)



W-fusion (WWH)

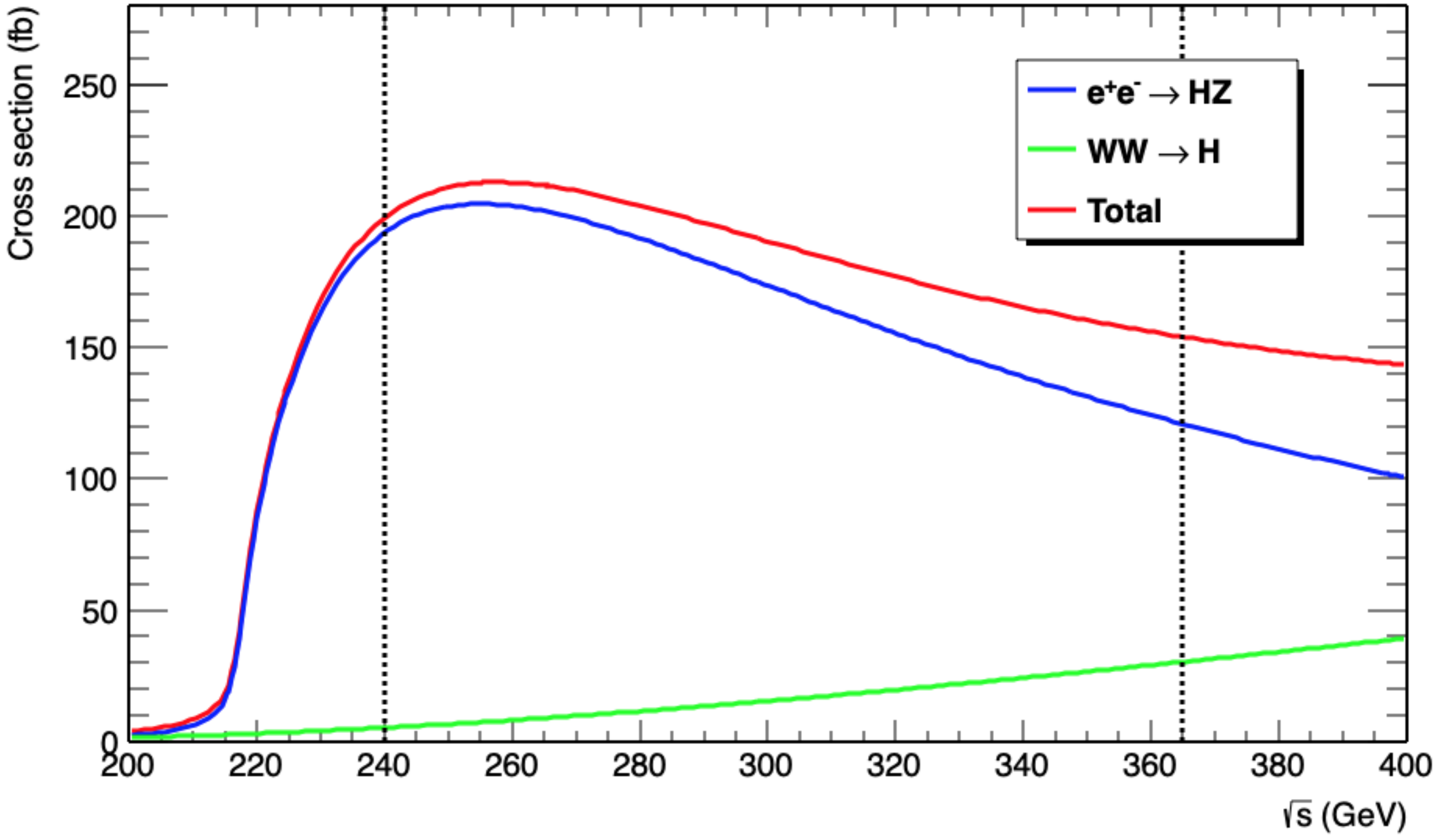


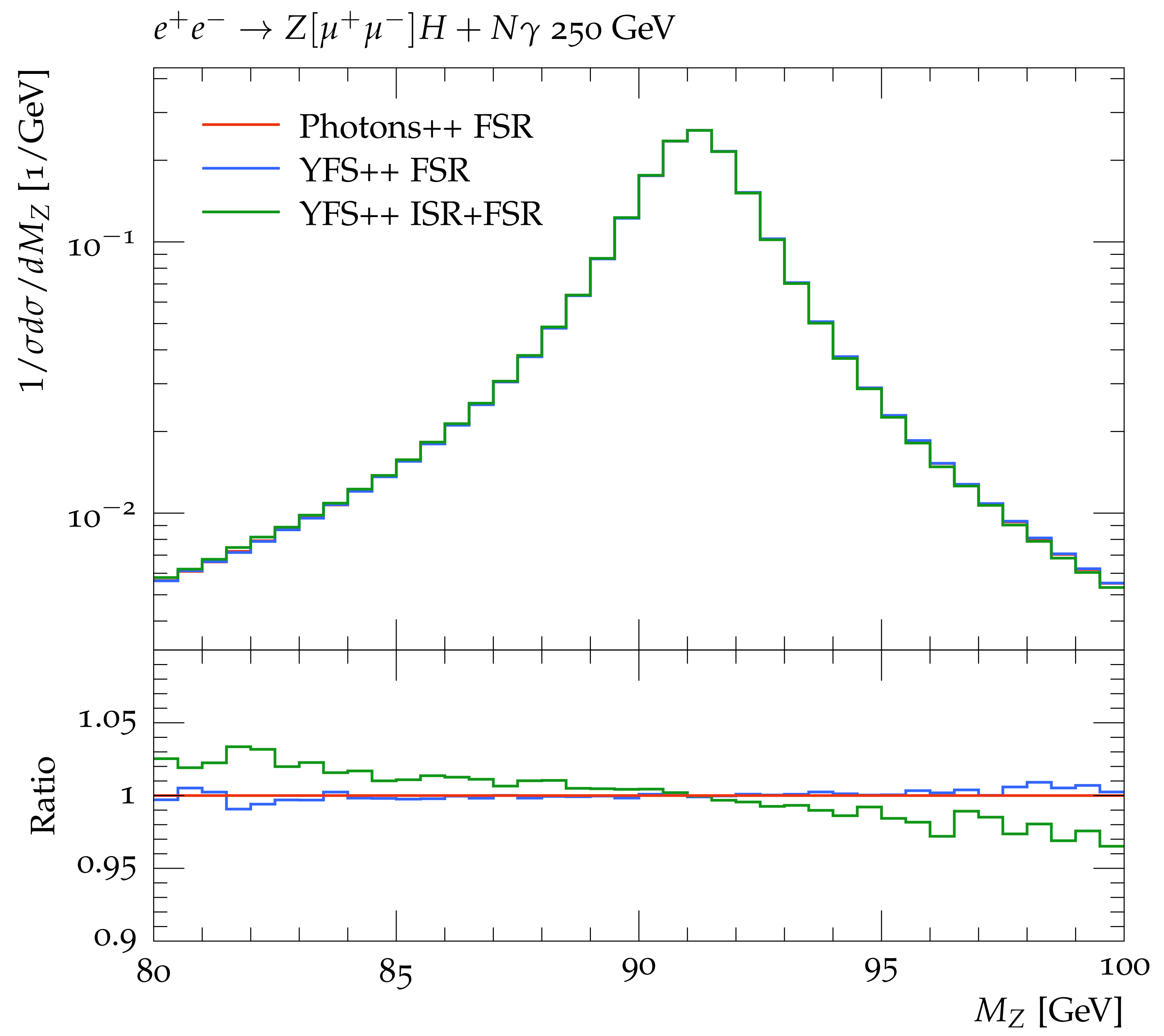
Figure from 2106.15438

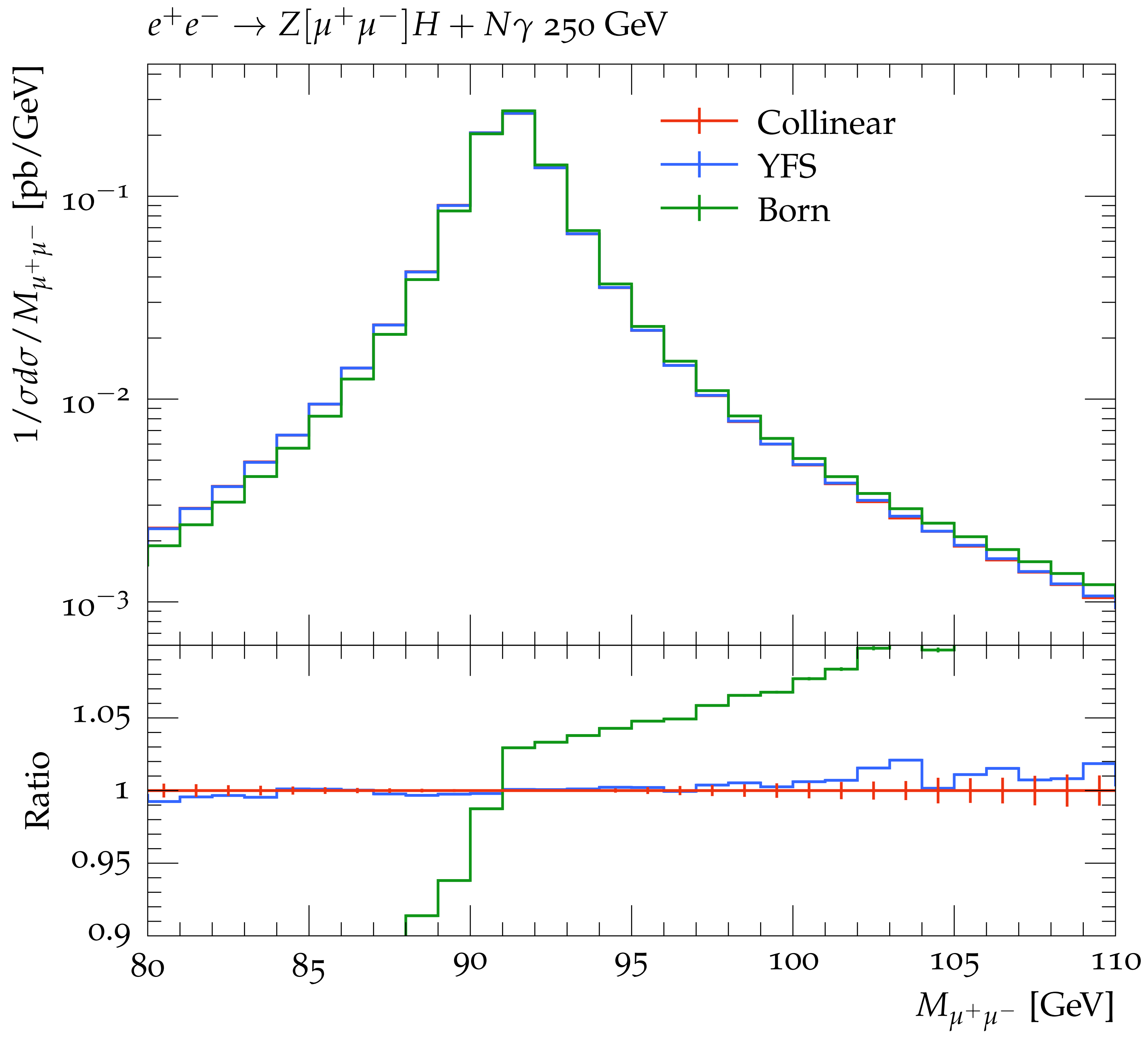
NLO EW + QED radiations built in Monte Carlo event generators

QED ISR and FSR effects

Critically re-examined very recently

Blümlein, Schönwald: 2202.08476
Krauss, Price, Schönherr: 2203.10948
Frixione et al.: 2203.12557
and many more references therein





Figures from 2203.10948

Mixed QCD-EW corrections to ZH

Gong, Li, Xu, **LLY**, Zhao: 1609.03955

the $\alpha(m_Z)$ scheme.

$\sqrt{s}$ (GeV)	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NNLO}$ (fb)
240	252.0	228.6	231.5
250	252.0	227.9	230.8
300	190.0	170.7	172.9
350	135.6	122.5	124.2
500	60.12	54.03	54.42

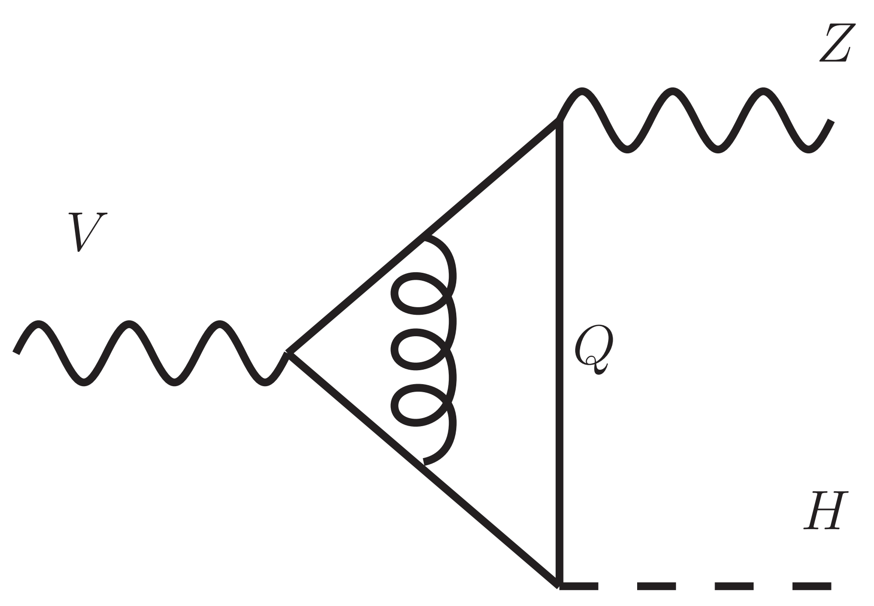
Corrections at the level of $\sim$1%: non-negligible compared to the $\sim$0.3% experimental accuracy

Sun, Feng, Jia, Sang: 1609.03995

$\sqrt{s}$	Schemes	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NNLO}$ (fb)
240	$\alpha(0)$	223.14 ± 0.47	229.78 ± 0.77	$232.21^{+0.75+0.10}_{-0.75-0.21}$
	$\alpha(M_Z)$	252.03 ± 0.60	$228.36^{+0.82}_{-0.81}$	$231.28^{+0.80+0.12}_{-0.79-0.25}$
	G_μ	239.64 ± 0.06	$232.46^{+0.07}_{-0.07}$	$233.29^{+0.07+0.03}_{-0.06-0.07}$

Residue dependence on renormalization schemes

Calculation methods back then



Bottleneck was the two-loop triangle integrals

→ Purely numeric evaluation with sector decomposition

Private code of 1508.02512 (employed by 1609.03955)

FIESTA/CubPack (employed by 1609.03995)

Slow; bad convergence around or above $2m_Q$ threshold

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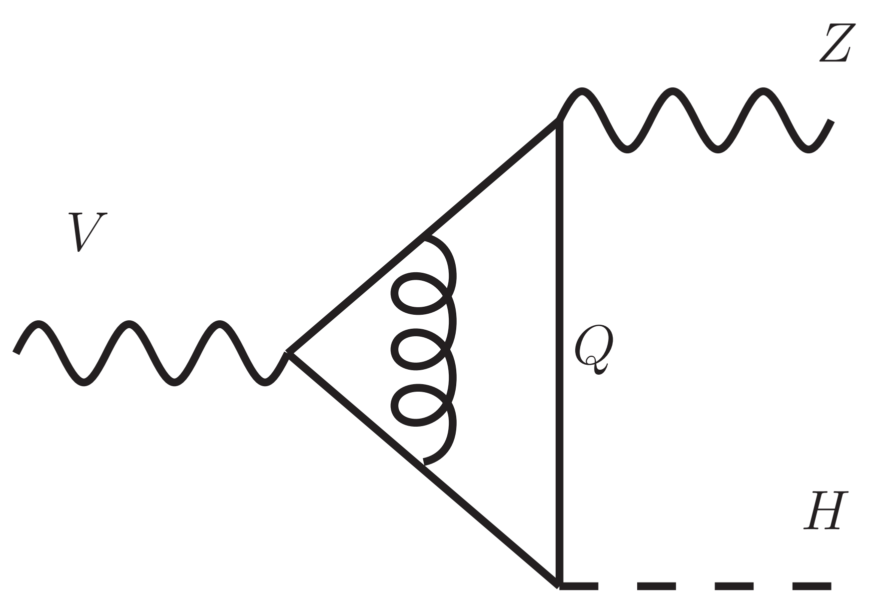

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Alternative method: $1/m_t$ expansion

Gong, Li, Xu, **LLY**, Zhao: 1609.03955

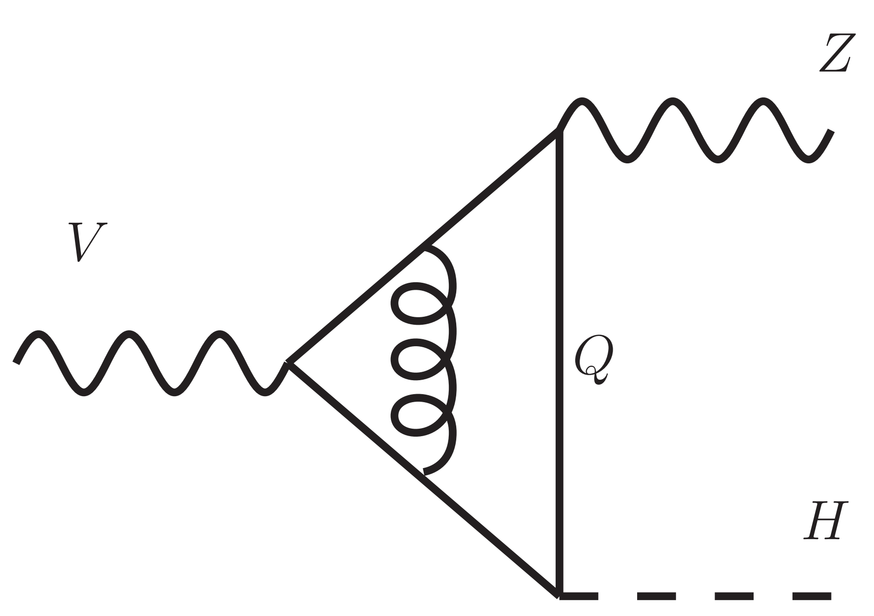
$\sqrt{s}$ (GeV)	$\mathcal{O}(m_t^2)$	$\mathcal{O}(m_t^0)$	$\mathcal{O}(m_t^{-2})$	$\mathcal{O}(m_t^{-4})$
240	81.8%	16.2%	1.4%	0.4%
250	81.7%	16.1%	1.5%	0.5%
300	80.0%	15.2%	2.1%	1.1%
350	69.7%	12.6%	2.7%	2.1%
500	137%	18.6%	17.3%	31.1%

Good approximation for low energies: analytic expressions easy to implement in Monte-Carlo

Not valid for high energies…

A new calculation for the HZV two-loop diagrams

Wang, Xu, **LLY**: 1905.11463



$$x = -\frac{q^2}{4m_Q^2}$$

$$y = -\frac{p_Z^2}{4m_Q^2}$$

$$z = -\frac{p_H^2}{4m_Q^2}$$

Constructed a canonical basis of master integrals

$$d\vec{f}(x, y, z; \epsilon) = \epsilon\, dA(x, y, z)\, \vec{f}(x, y, z; \epsilon)$$

$$= \epsilon \sum_i A_i\, d\log(\alpha_i)\, \vec{f}(x, y, z; \epsilon)$$

Alphabet contains 4 kinds of square roots

$$\sqrt{x(x+1)} \quad \sqrt{y(y+1)} \quad \sqrt{z(z+1)} \quad \sqrt{x^2 + y^2 + z^2 - 2xy - 2yz - 2zx}$$

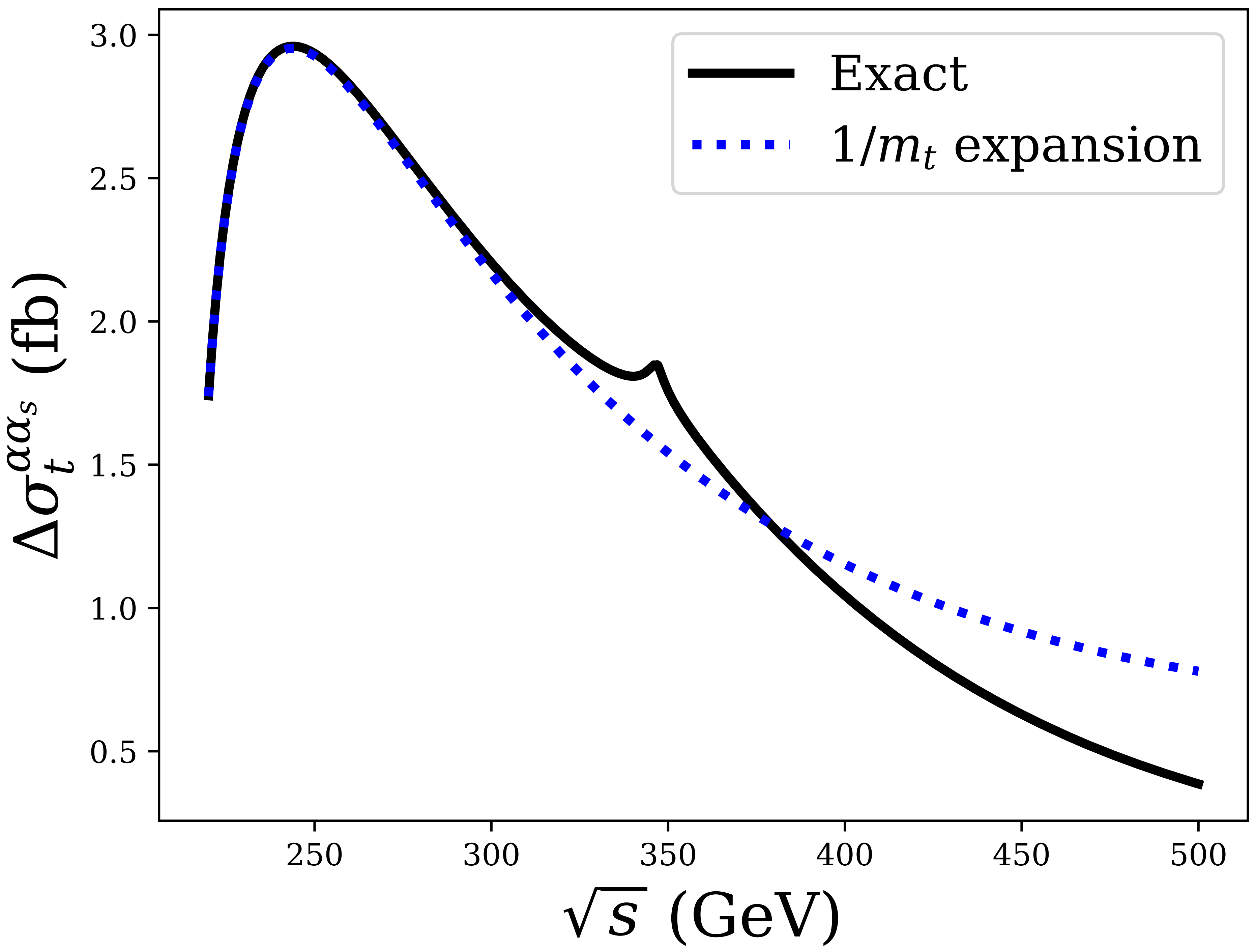
Solutions up to weight-3 written in terms of GPLs

Weight-4 parts expressed as one-fold integrals (not ideal, but usable)

A new calculation for the HZV two-loop diagrams

Wang, Xu, **LLY**: 1905.11463

The new result works well for all kinematic configurations



NNLO $\mathcal{O}(\alpha\alpha_s)$ corrections to ZH cross section

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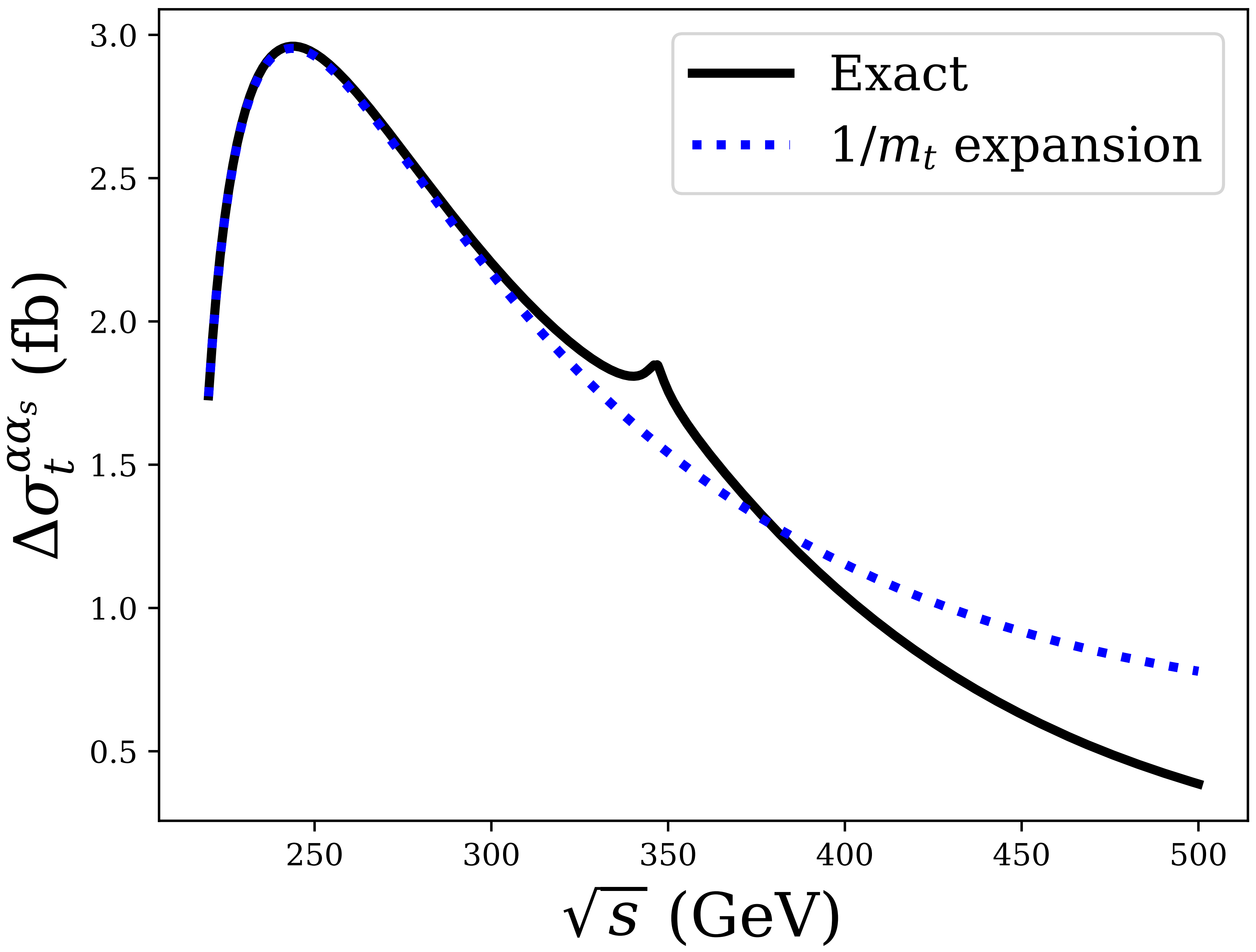
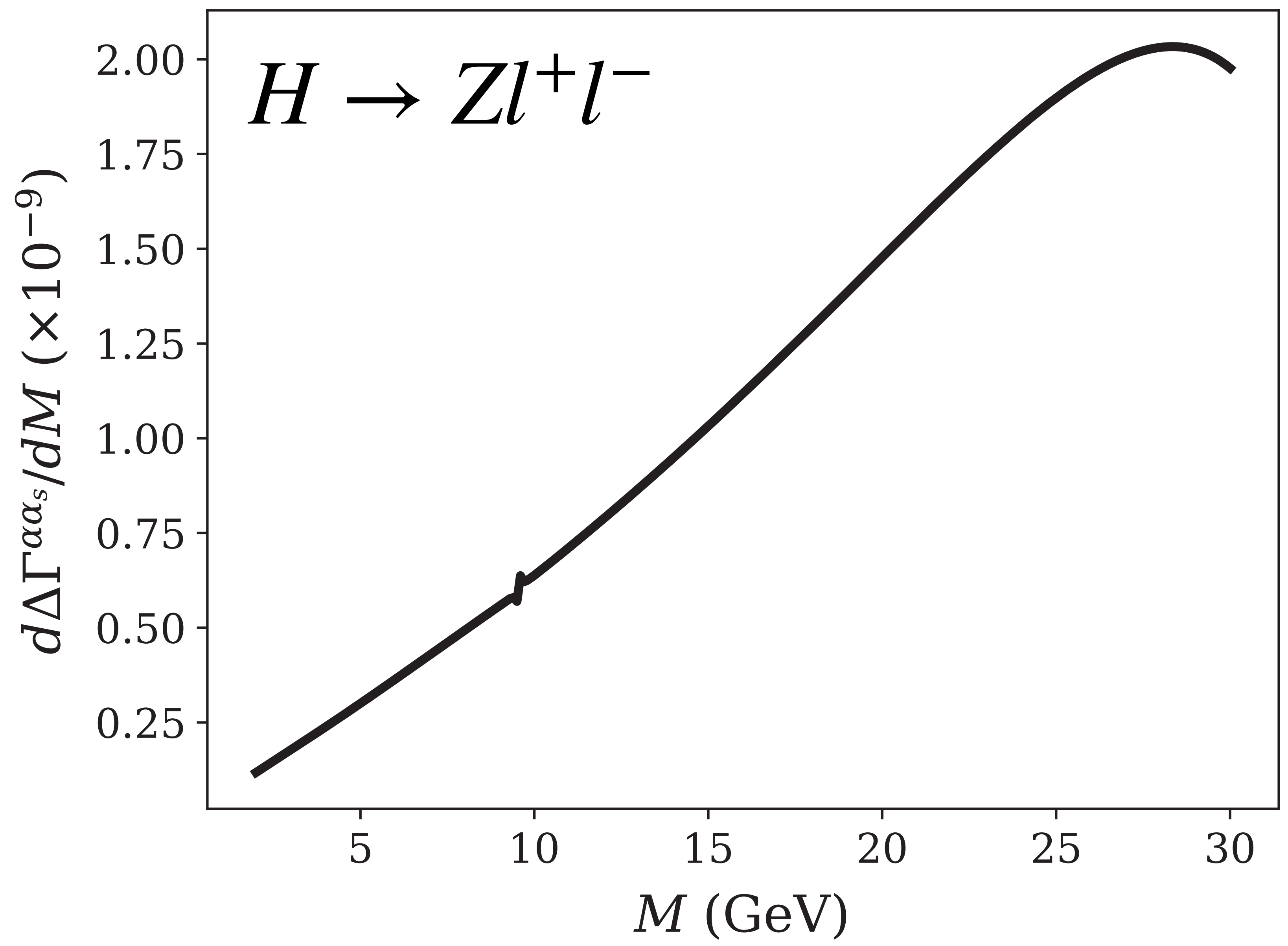


NNLO $\mathcal{O}(\alpha\alpha_s)$ corrections to ZH cross section

Also for bottom quark loops



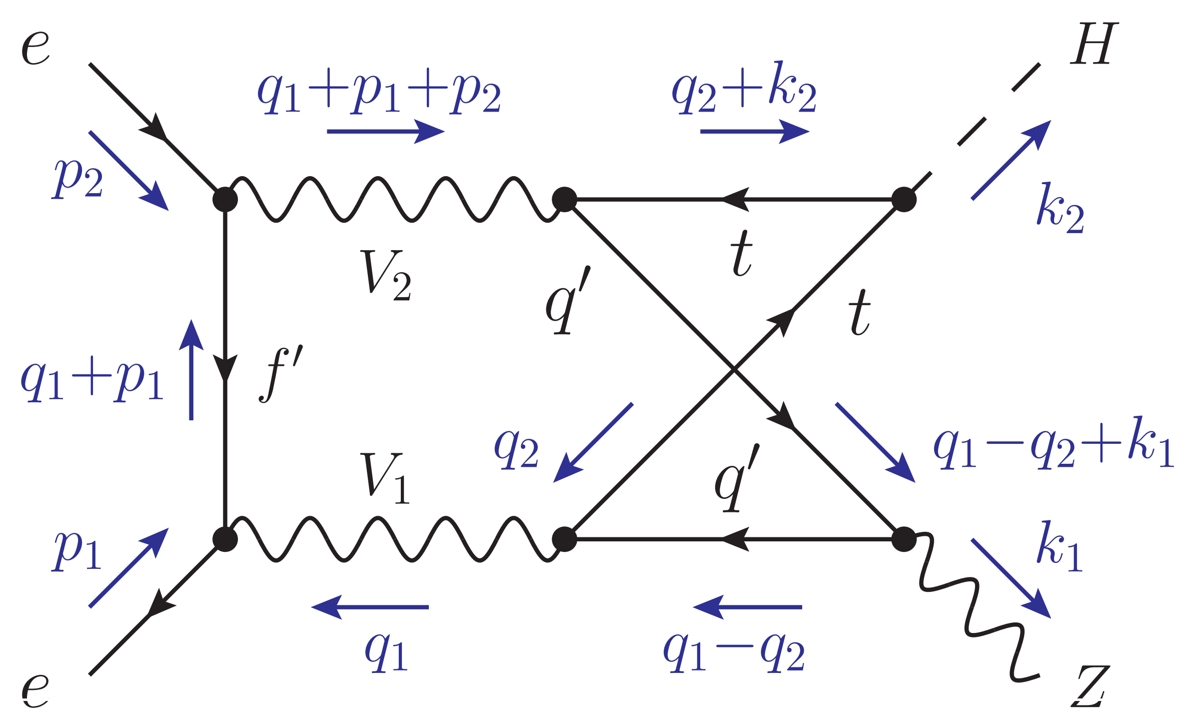
Bottom contribution to the M_{ll} distribution

Towards two-loop EW corrections to ZH

A must to match the ~0.3% experimental accuracy

A rather challenging task: ~20000 diagrams, a lot of physical scales Li, Wang, Wu: 2012.12513

Evaluation of a class of double boxes with a top quark loop Song, Freitas: 2101.00308



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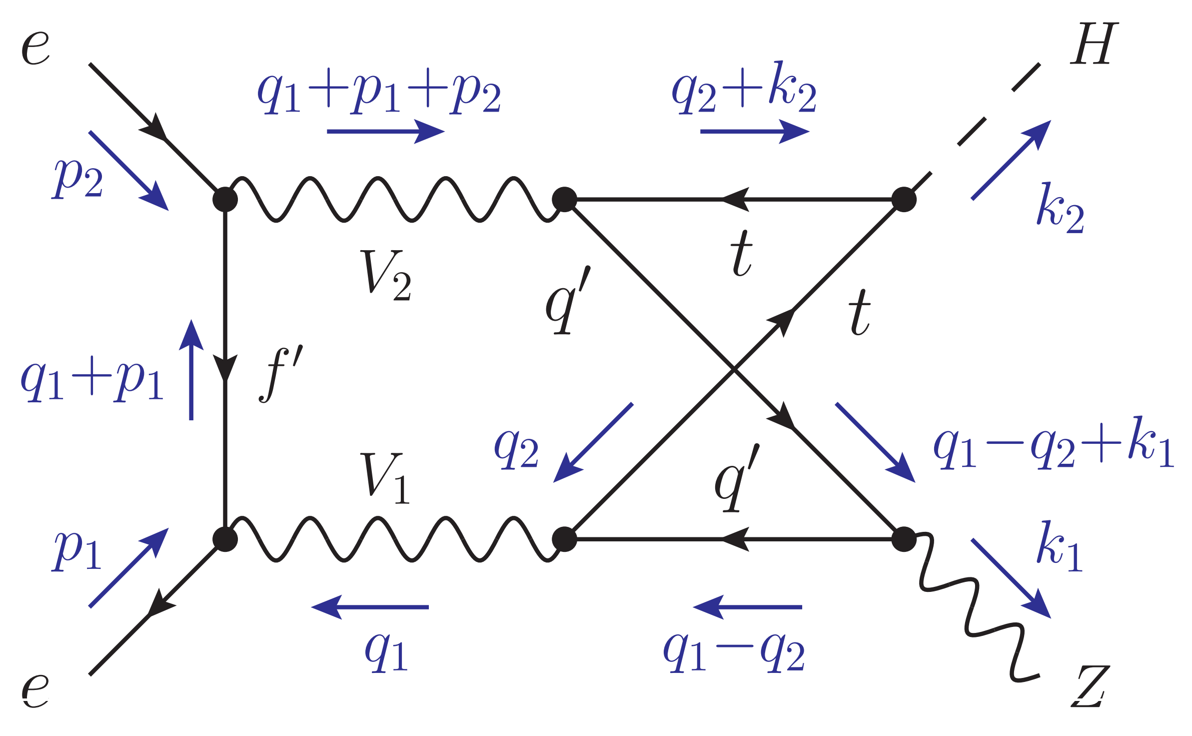

Further development of computational techniques required!

e.g.: Canonical differential equations in both GPL sectors and elliptic sectors

Numeric solutions (pySecDec, DiffExp, AMFlow, …)

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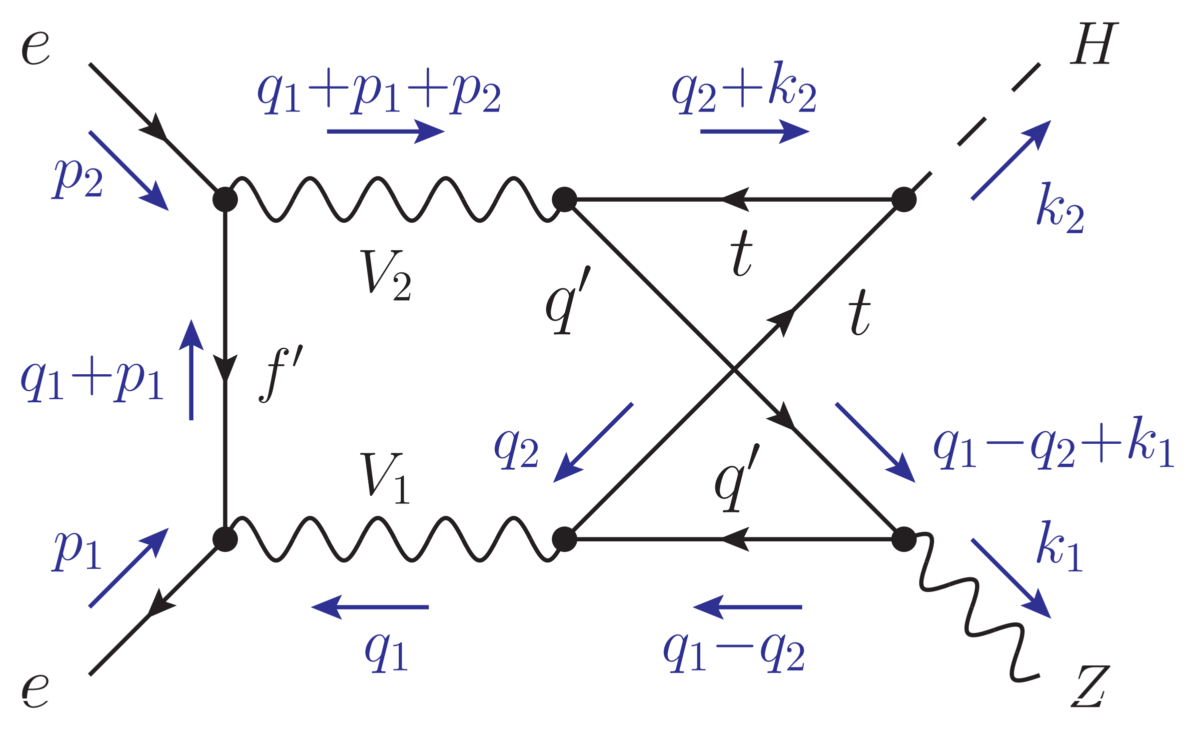

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e.g.: Canonical differential equations in both GPL sectors and elliptic sectors

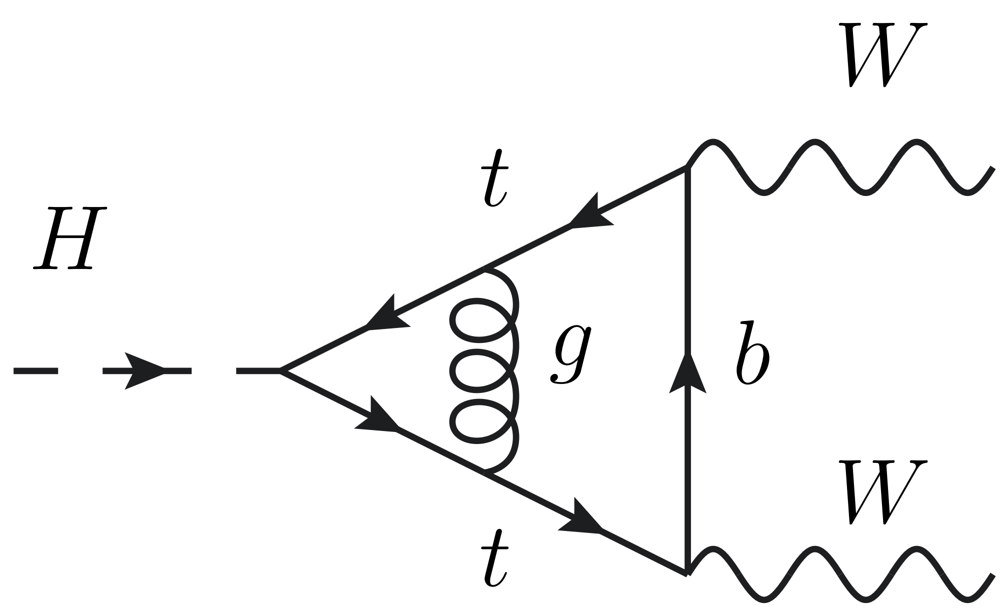
Numeric solutions (pySecDec, DiffExp, AMFlow, …)

Perhaps some kind of approximate result is good enough

→ Vague thought: asymptotic expansion in the limit $m^2_{\text{everything}} \ll s, m_t^2$?

Mixed QCD-EW corrections to WWH

Di Vita, Mastrolia, Primo, Schubert: 1702.07331
Ma, Wang, Xu, **LLY**, Zhou: 2105.06316
Wang, **LLY**, Zhou: 2112.04122



The two-loop amplitude can be written in a fully-analytic form (involving a lot of weight-4 GPLs)

$H \to Wl\nu$

$\alpha(m_Z)$	LO	NLO EW	NNLO QCD-EW
Γ (10^{-5} GeV)	4.597	4.474	4.518

G_μ	LO	NLO EW	NNLO QCD-EW
Γ (10^{-5} GeV)	4.374	4.524	4.531

$e^+e^- \to \nu\bar{\nu}H$

$\sqrt{s}$ (GeV)	$\sigma_{\rm LO}$ (fb)	$\delta\sigma_{\rm NNLO}$ (fb)
250	7.88	0.010
350	30.6	0.040
500	74.8	0.101

Rather small corrections

Note: did not consider mixing with $Z(\to \nu\bar{\nu}) + H$

In the future: two-loop EW? Perhaps only some approximations…

The numeric evaluation of GPLs

In all the above calculations one needs numeric evaluations of a large amount of GPLs

Gehrmann, Remiddi: hep-ph/0111255
Vollinga, Weinzierl: hep-ph/0410259
Ablinger, Blümlein, Schneider: 1302.0378

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handyG: newer implementation using double-precision or quad-precision numbers, aimed for usage in Monte-Carlo Naterop, Signer, Ulrich: 1909.01656

The numeric evaluation of GPLs

The algorithm is recursive: one transforms the target GPL to a sum of so-called "convergent" GPLs, which can be evaluated by series expansion

A problem of **numerically** recursive implementations: to evaluate a single GPL, sometimes a transformed GPL needs to be computed for many many times!

- Greatly slows down the computation speed
- May lose accuracy due to repeated floating-point cancellations

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- May lose accuracy due to repeated floating-point cancellations

We have encountered such situations in the calculation of $e^+e^- \to \nu\bar{\nu}H$: in general handyG can evaluate a weight-4 GPL in far less than a second, but sometimes it takes several seconds

$$\text{e.g.:}\quad \mathtt{G}(1.0025, 0.989, 0.45, 0.89 + 0.24\mathtt{i}; 1)$$

The problem becomes much worse at higher weights: at three-loops one needs weight-6

FastGPL

https://github.com/llyang/FastGPL

A re-implementation of the algorithm: hybrid analytic/numeric

The reduction to convergent GPLs are (mostly) done in a Mathematica package (to be released)

```
<< reduceGPL`
map[{1, 0, 1, 1}, 3]
{a, 0, b, b}
There is no any artificial divergence!
{a, 0, b, c}
There is artificial divergence when c=x!
We need to rescale indices and argument of GPLs!
complex<double> G4_a0bc_b(complex<double> a, complex<double> b, complex<double> c, int sa, int sb, int sc, double x) {
a=a/x;
b=b/x;
c=c/x;
x=1.;
if(b==c) {
const vector<complex<double>> sy = {G({a, b}, {sa, sb}, x), G({a}, {sa}, x), G({0, a/b, 1}, 1), G({b}, {sb}, x)};
complex<double> res= sy[1]*sy[2] - sy[2]*sy[3] + sy[0]*G({0, 1}, 1) - sy[1]*G({0, 0, 1}, 1) + sy[3]*G({0, a/b, x/b}, 1) + G({0, a/b,
  0, 1}, 1) - G({0, a/b, x/b, 1}, 1) + G({0, a/b}, 1)*(-sy[0] + G({0, b}, {1, sb}, x)) + G({a, 0, 0, b}, {sa, 1, 1, sb}, x);
return res;
 }
else {
const vector<complex<double>> sy = {Log(b, sb), G({a}, {sa}, x), G({c/b}, 1), G({a,
  c}, {sa, sc}, x), G({0, a}, {1, sa}, x), G({0, a/b, c/b}, 1), G({0, c/b, a/b}, 1), G({c/b, 0, a/b}, 1)};
complex<double> res= sy[0]*(sy[2]*sy[4] - sy[5] - sy[6] - sy[7]) + sy[3]*G({0, c/b}, 1) + 2.*G({0, 0, a/b, c/b}, 1) + 2.*G({0, 0, c/b, a/b},
  1) + G({0, a/b, 0, c/b}, 1) + G({0, a/b, c/b, x/b}, 1) + 2.*G({0, c/b, 0, a/b}, 1) + G({0, c/b, a/b, x/b}, 1) + 2.*G({c/b, 0, 0,
  a/b}, 1) + G({c/b, 0, a/b, x/b}, 1) + G({0, a/b}, 1)*(-sy[3] + G({0, c}, {1, sc}, x)) + sy[2]*(G({0, 0, a}, {1, 1, sa}, x) - G({a,
  0, c}, {sa, 1, sc}, x)) + G({a, 0, 0, c}, {sa, 1, 1, sc}, x) + (-(sy[0]*sy[1]*sy[2]) - sy[2]*sy[4] + sy[5] + sy[6] + sy[7])*Log(-x,
  sb) + (sy[1]*sy[2]*pow(sy[0], 2.))/2. + sy[1]*(-sy[6] - sy[7] - G({0, 0, c/b}, 1) + sy[2]*(-G({0, 0}, {1, 1}, x) - 2.*Zeta(2)));
if(c!=x) res += (-sy[5] + G({0, a/b, x/b}, 1))*G({c}, {sc}, x);
return res;
 }
}
```

Generate numeric codes automatically

↓

The FastGPL library (up to weight-4 well-tested, up to weight-6 implemented)

Aiming at fast evaluations using double-precision numbers

Comparison of speed

Wang, **LLY**, Zhou: 2112.04122

	t_f (s)	t_h (s)	t_h/t_f
`G(1.0025, 0.989, 0.45, 0.89 + 0.24i; 1)`	0.006	2.2	~ 400
`G(0.998, 1.0545 + 0.127i, 0.91 + 0.25i, −0.226; 1)`	0.004	1.5	~ 400
`G(−1.04, −0.97, 0.25, −0.84 + 0.45i; 1)`	0.004	1.1	~ 300

Table 2: Average evaluation times of several GPLs which require many iterations.

	$0aBC$	$0abC$	$0abc$	$00aB$	$00ab$
t_f (ms)	0.22	0.25	0.20	0.08	0.05
t_h (ms)	3.1	5.8	4.5	1.3	0.80
t_h/t_f	~ 14	~ 23	~ 23	~ 17	~ 16

	$ABCD$	$abCD$	$abcD$	$abcd$
t_f (ms)	0.22	0.47	0.50	0.42
t_h (ms)	1.7	7.4	11.0	9.1
t_h/t_f	~ 7.5	~ 16	~ 22	~ 22

Table 3: Average evaluation times of a few categories of weight-4 GPLs.

$e^+e^- \to \nu\bar{\nu}H$

$\sqrt{s}$ (GeV)	σ_{LO} (fb)	$\delta\sigma_{\mathrm{NNLO}}$ (fb)	t_f (h)	t_h (h)	t_h/t_f
250	7.88	0.010	0.45	8.60	~ 19
350	30.6	0.040	0.51	9.02	~ 18
500	74.8	0.101	0.52	9.24	~ 18

10000 sample phase-space points

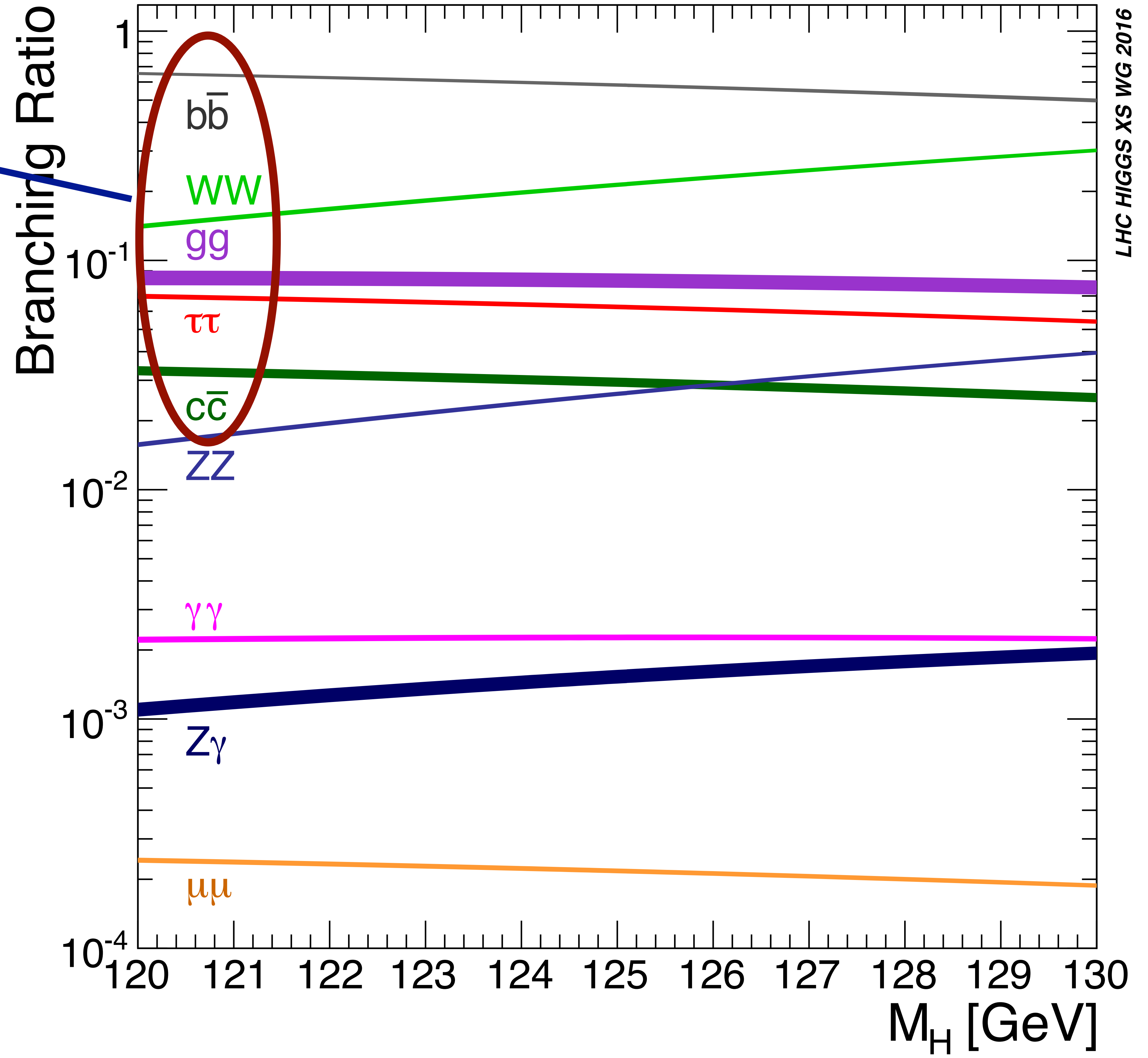
several thousand GPLs per point

FastGPL is faster in general, and is much faster for special cases

Preliminary tests show that the speed-boost is much larger at weight-6

Higgs decay

Weakness of the LHC



The hadronic channels

$H \to b\bar{b}$ Important for HZZ and $Hb\bar{b}$ couplings

$H \to gg$ Probes new particles running in the loop

$H \to c\bar{c}$ Unique window to charm Yukawa

Partial widths

Freitas (2021) and references therein

- $H \to q\bar{q}$
  - $\mathcal{O}(\alpha_s^4)$ in the limit of massless quarks
  - $\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha\alpha_s)$ and partial $\mathcal{O}(\alpha^2)$
- $H \to gg$

Herzog et al.: 1707.01044

  - $\mathcal{O}(\alpha_s^4)$ with infinite m_t

$$\Gamma_{\mathrm{N^4LO}}(H \to gg) = \Gamma_0 \left(1.844 \pm 0.011_{\,\mathrm{series}} \pm 0.045_{\,\alpha_s(M_Z),1\%}\right)$$

  - $\mathcal{O}(\alpha_s^2)$ with $1/m_t$ expansion
  - $\mathcal{O}(\alpha_s^2)$ three-loop form factor with full m_t dependence (hence also bottom loop)

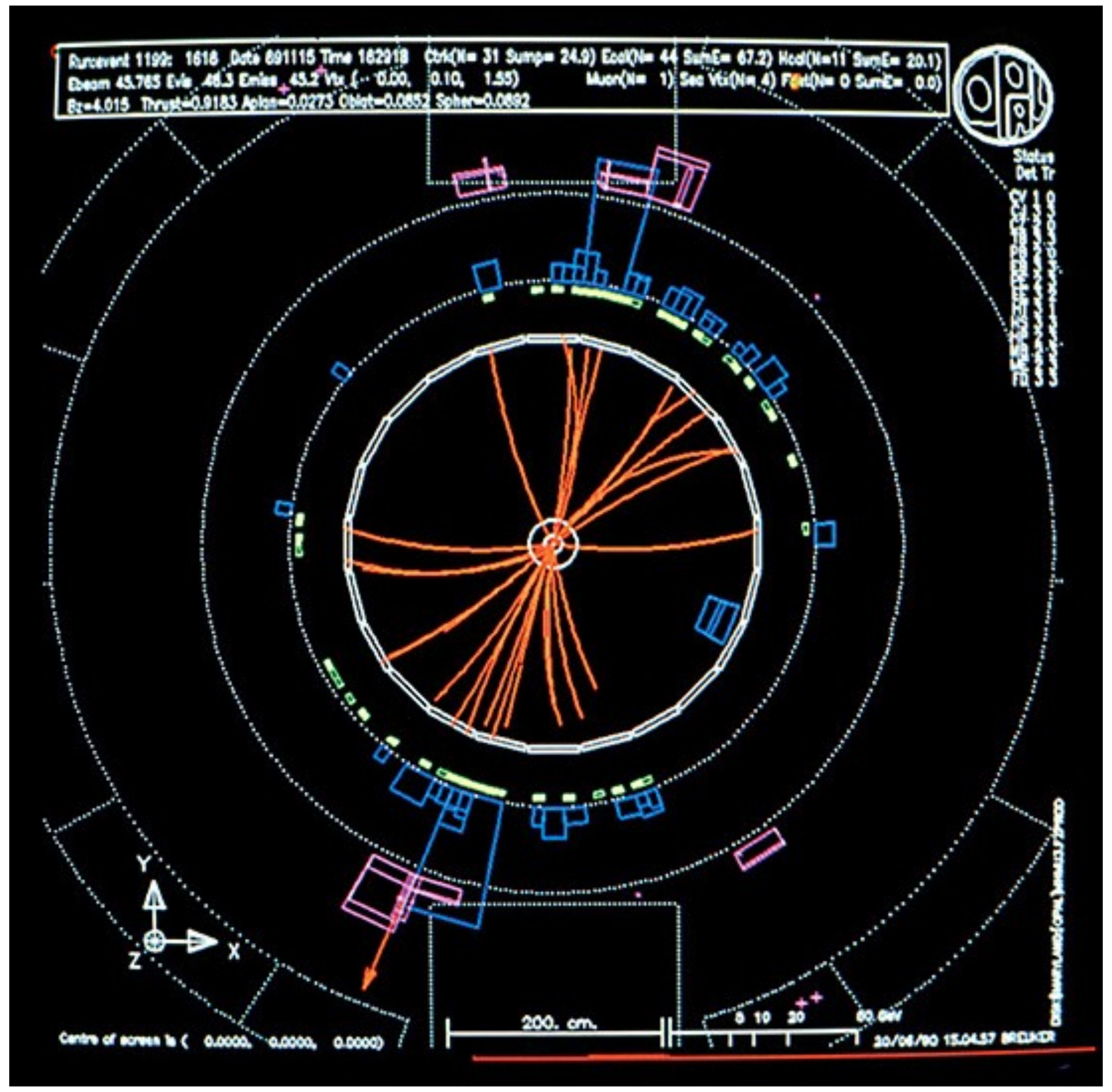
Czakon, Niggetiedt: 2001.03008

  - $\mathcal{O}(\alpha)$ EW corrections

Event shapes

Event shapes provide more information than the total rates

- Discrimination between quark and gluon final states
- Probing kinematic dependence of the Hgg vertex
- New-physics enhanced light-quark Yukawa couplings?



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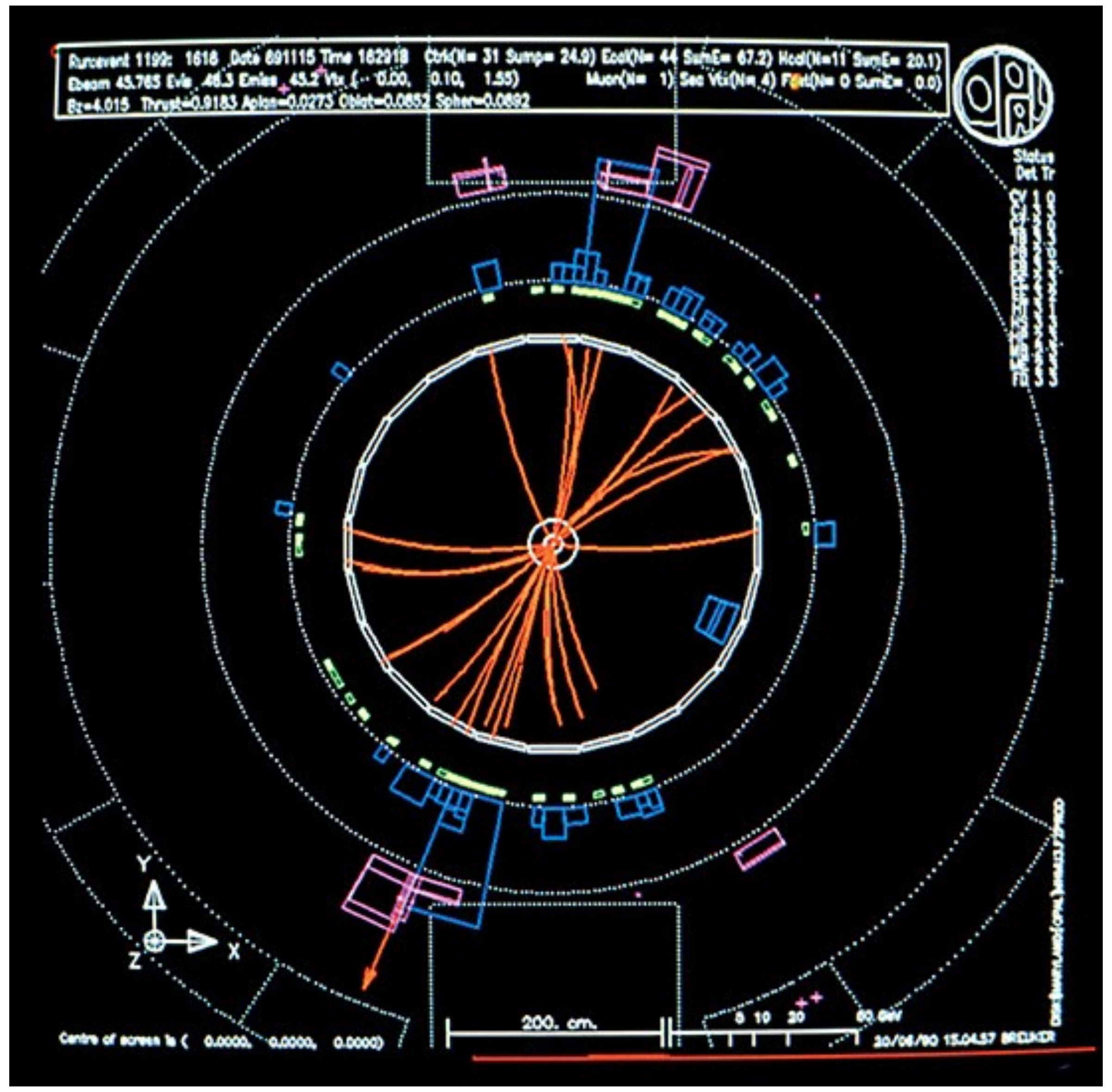


I'll focus on one particular variable: thrust

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|}$$

$$\tau = 1 - T$$

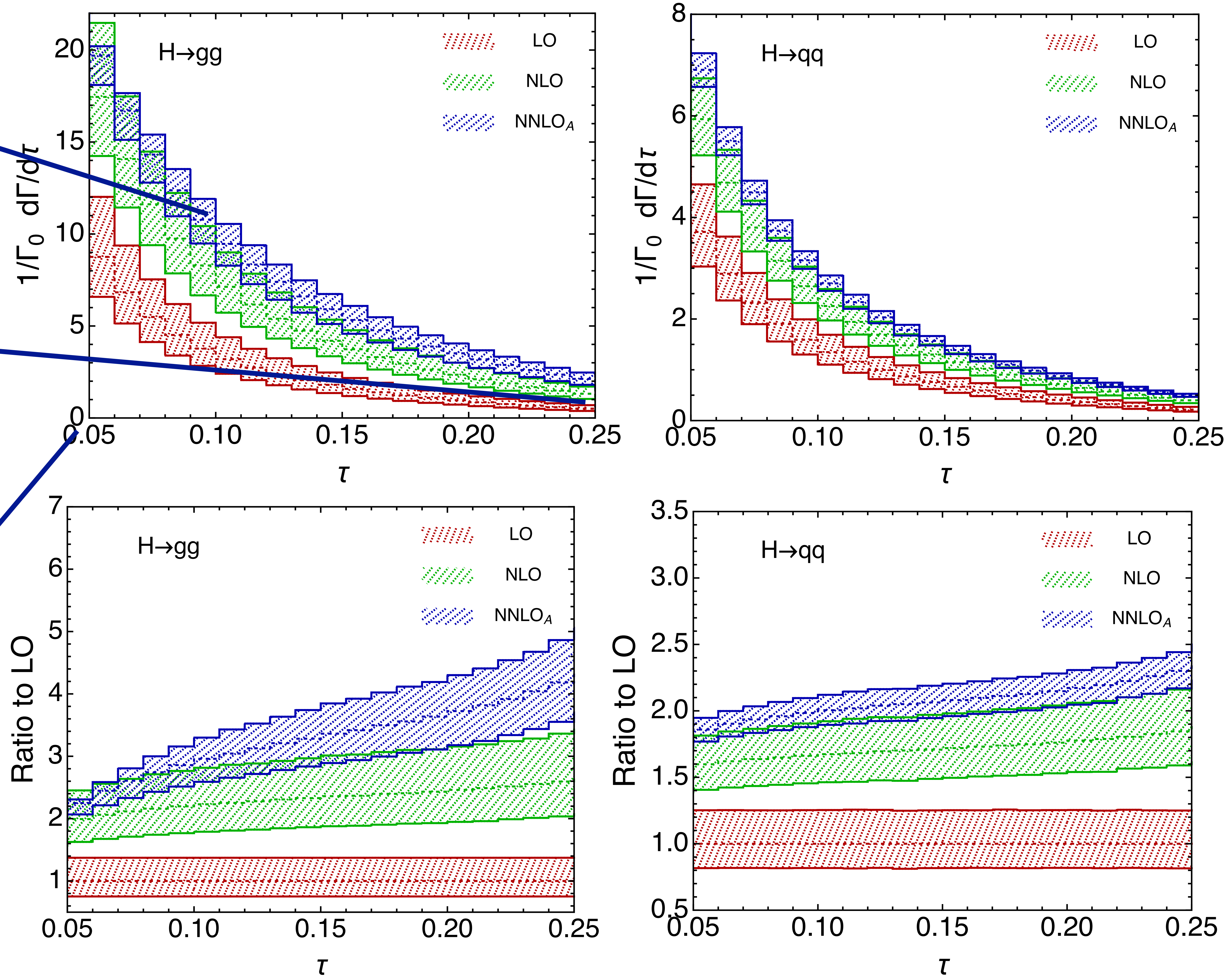
Fixed-order predictions for thrust distribution

Gao, Gong, Ju, **LLY**: 1901.02253

Large corrections, especially in the gluon channel; N^3LO needed?

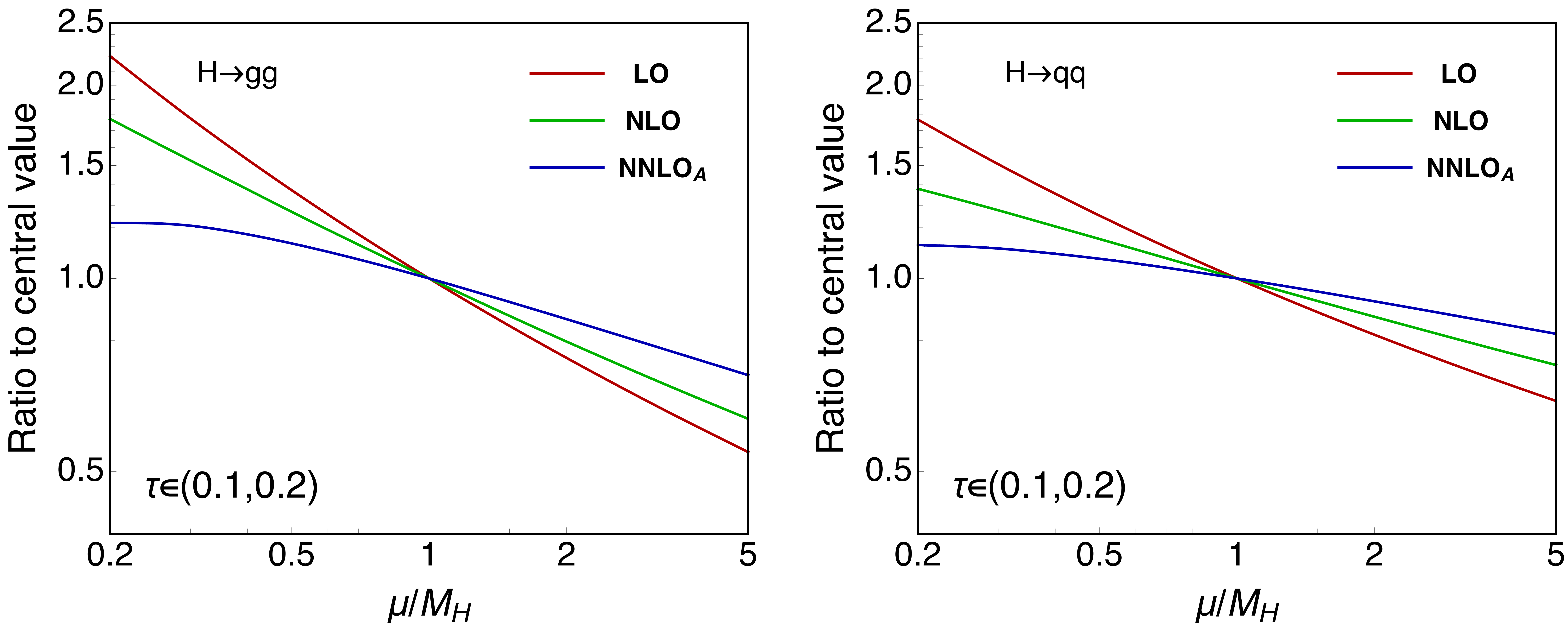
Soft-collinear approximation not valid for larger τ; a full NNLO calculation required!

Parton shower and/or resummation needed for smaller τ



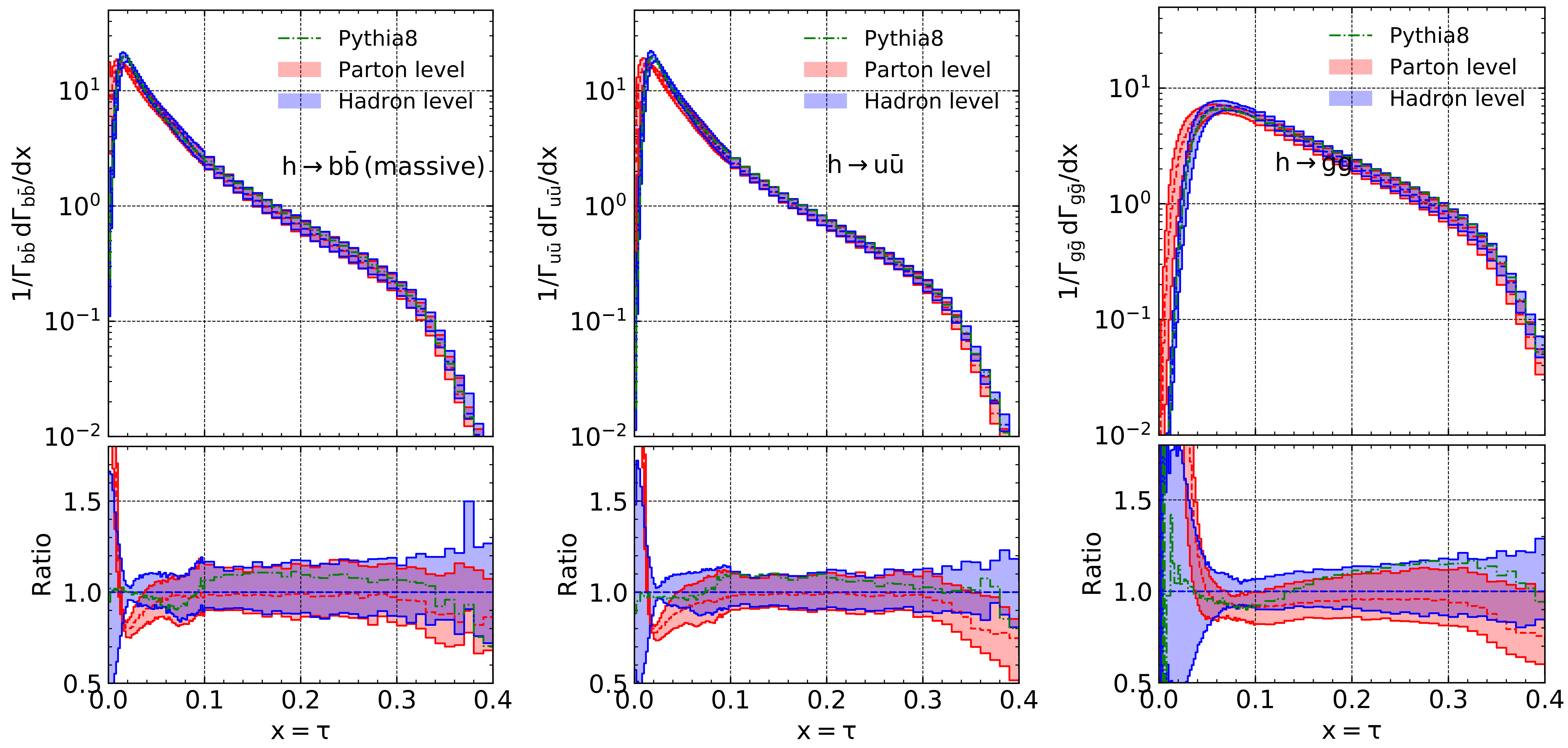
Scale dependence

Gao, Gong, Ju, **LLY**: 1901.02253



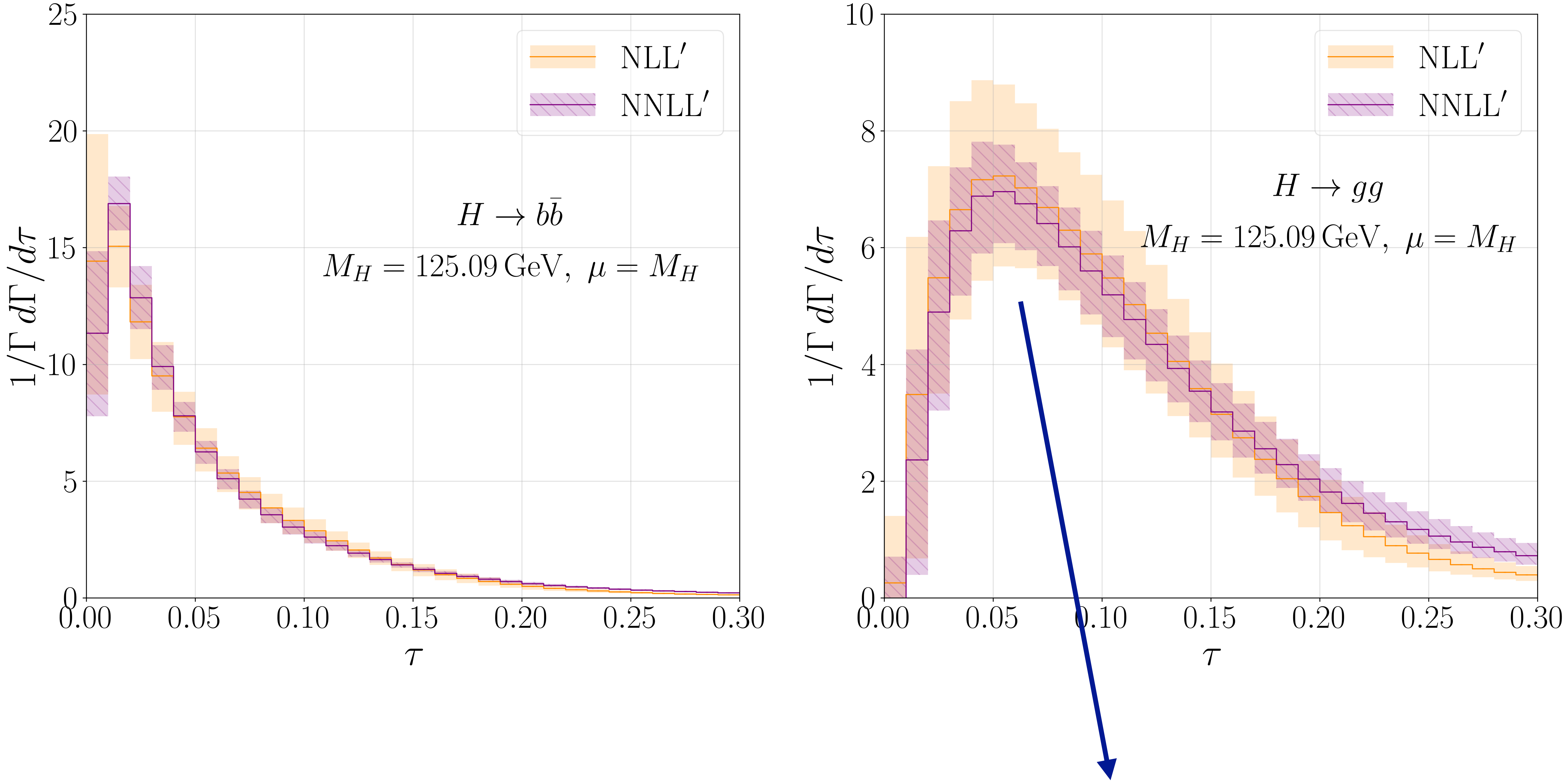
Matched with parton shower

Hu, Sun, Shen, Gao: 2101.08916



Resumed predictions

Alioli et al.: 2009.13533



Large uncertainties in the gluon channel; N3LL or N3LL' needed?

Towards N^{3}LL' thrust resummation

	hard, jet, soft functions	hard, jet, soft anomalous dimensions	cusp anomalous dimension, beta function
NNLL'	2-loop	2-loop	3-loop
N^{3}LL	2-loop	3-loop	4-loop
N^{3}LL'	3-loop	3-loop	4-loop
		available	available

available except the non-logarithmic term of the 3-loop soft function

The 3-loop soft function

The non-logarithmic term of the 3-loop soft function for quarks was extracted from the numeric result of EERAD3

$$c_3^S = 2s_3 + 691 = -19988 \pm 1440\,(\text{stat.}) \pm 4000\,(\text{syst.})$$

Brüser, Liu, Stahlhofen: 1804.09722

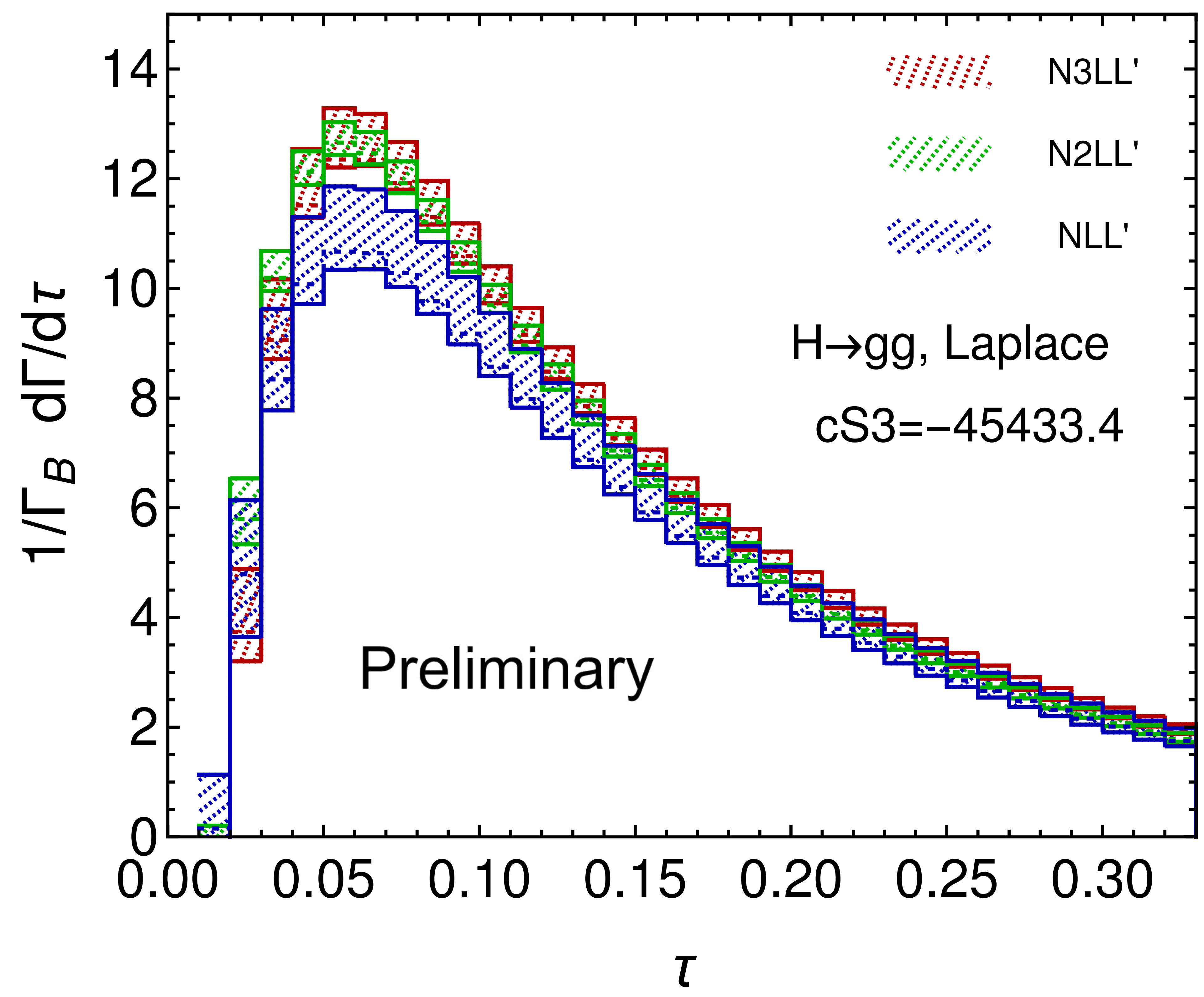
With a Casimir scaling , the corresponding term for gluons

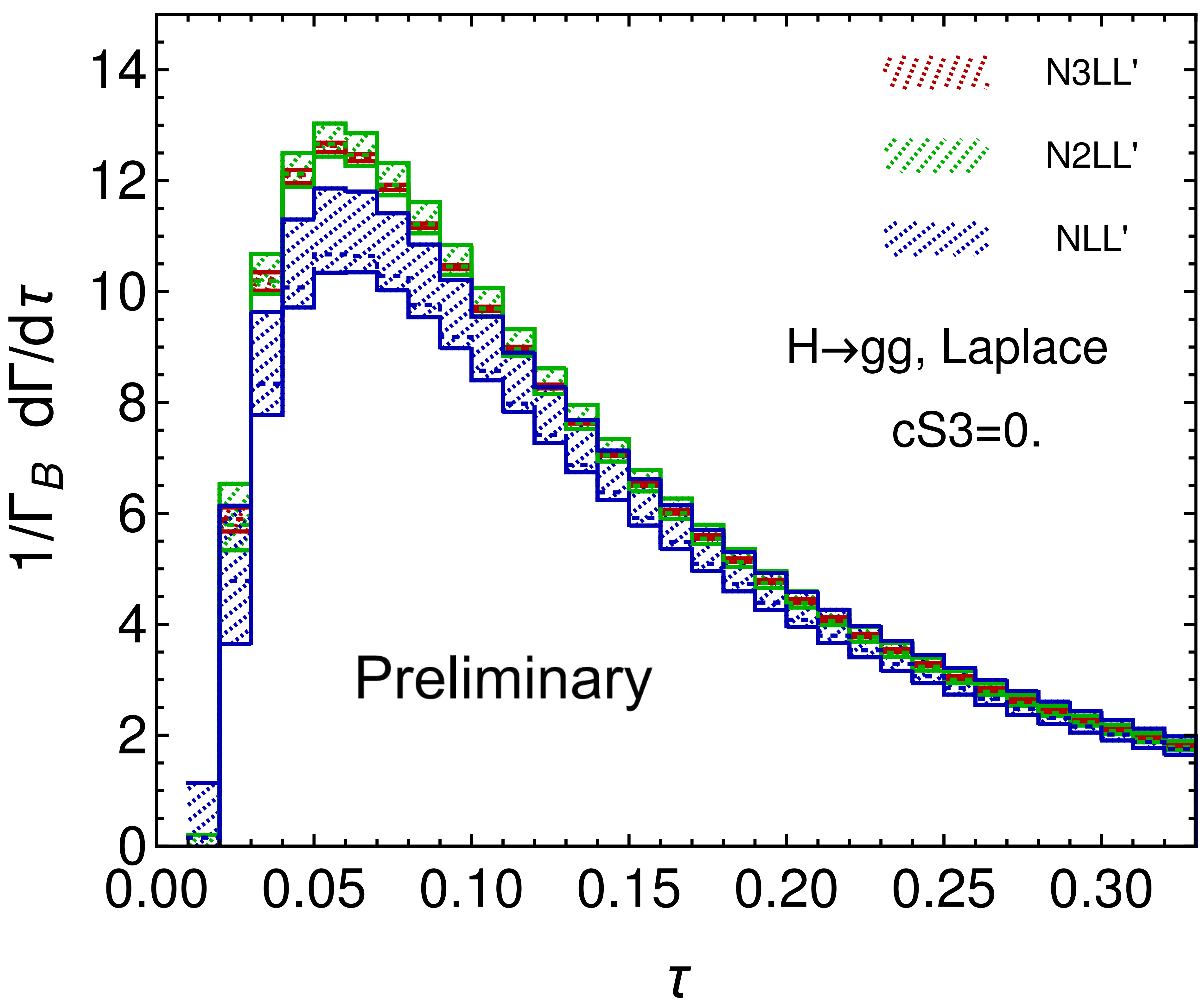
$$c_3^S \sim -45000 \pm 10000$$

A rather large constant term, one might worry about convergence!

Especially it multiplies $\alpha_s(\mu_s)$ at the low scale $\mu_s \sim \tau m_H$

Towards N³LL' thrust resummation





Preliminary result shows that the non-logarithmic term of the 3-loop soft function has a large impact!

We want to know its precise value! A part of the result: Chen, Feng, Jia, Liu: 2206.12323

Thank you!