# Dynamics of electroweak phase transitions in the singlet-extended model

**Haibin Chen**

Higgs potential 2022, July 27th

Based on H. Chen, Y.Jiang, arXiv:2208.XXXXX appear soon

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# Contents:

# The cosmological significance of EWPTs

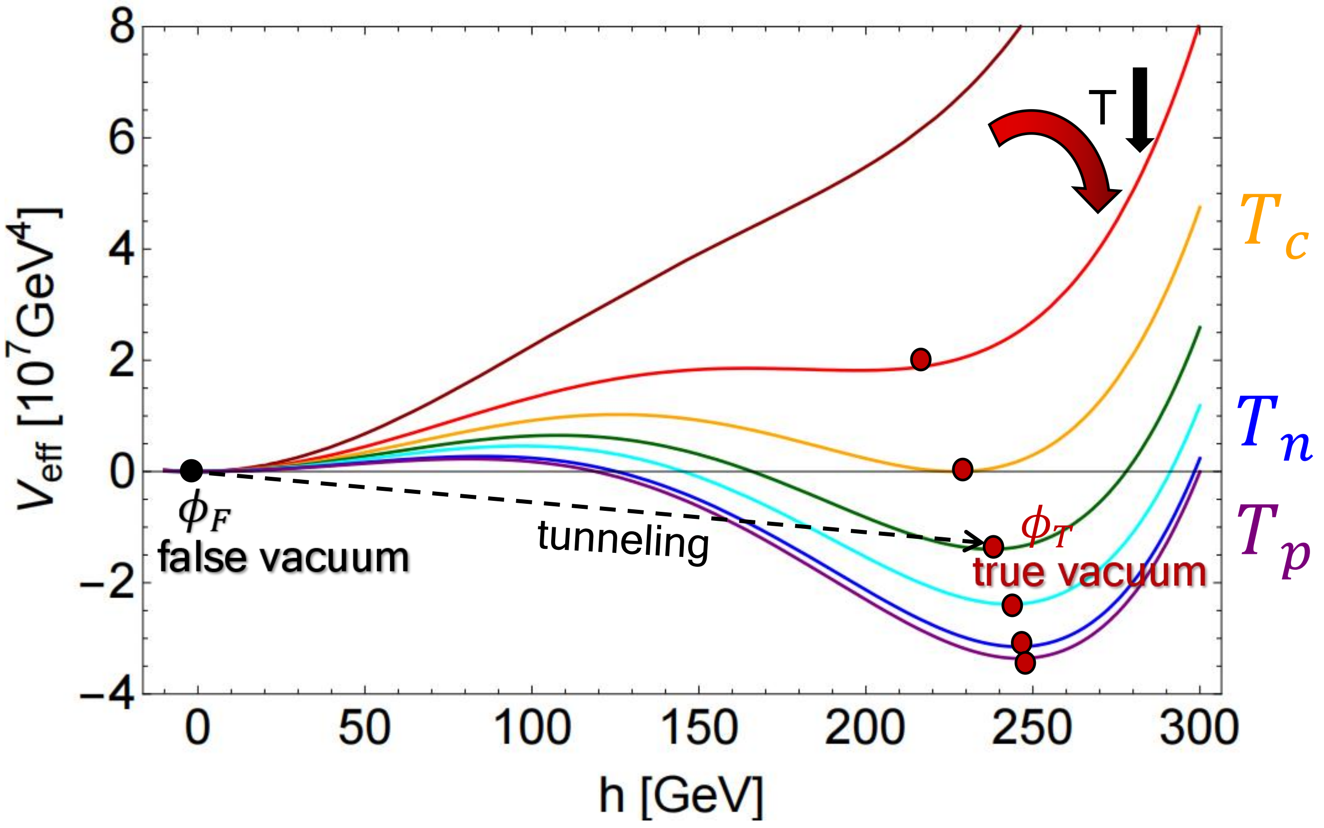
- Electroweak baryogenesis, producing the observed baryon asymmetry through this mechanism[1,2]

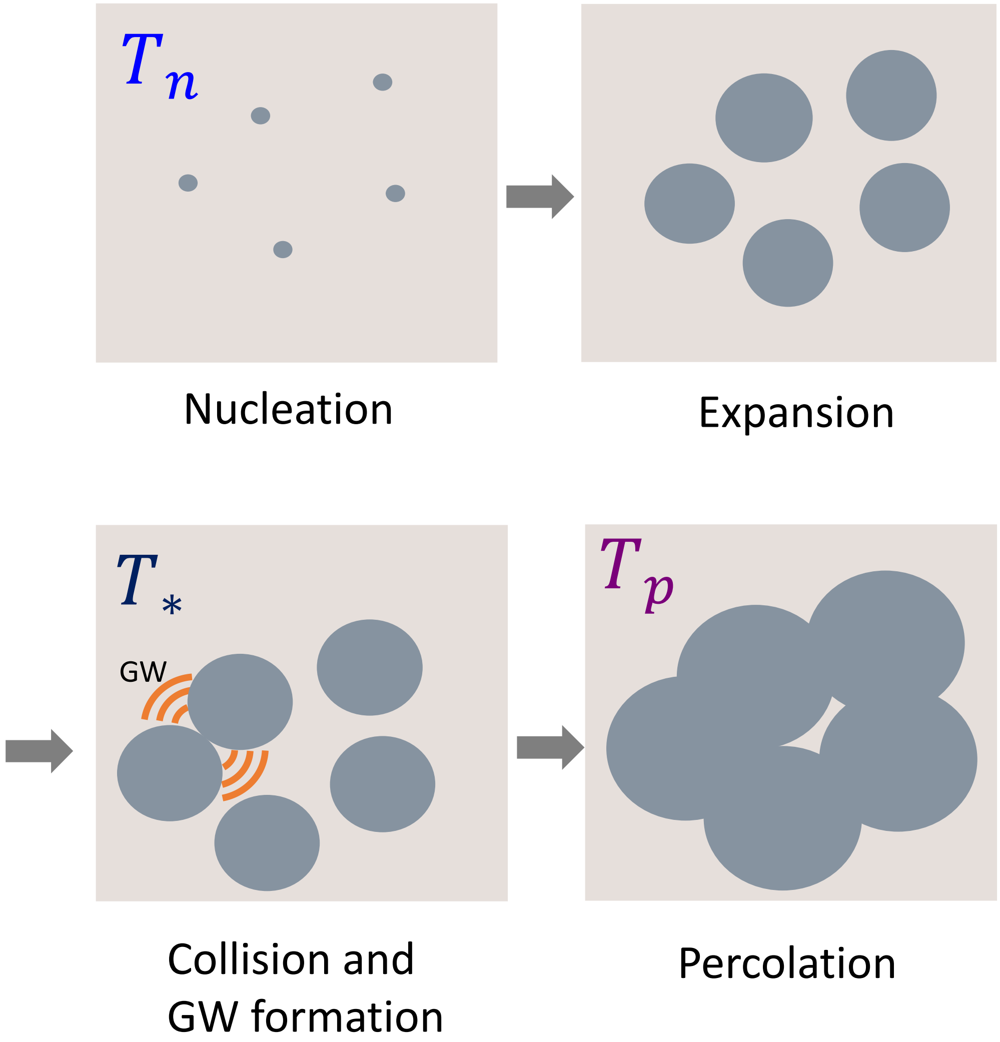
  Sakharov conditions: [3]
  - Baryon number violation
  - C and CP violation
  - Departure from thermal equilibrium   fulfilled by a **first-order phase transition** involved with EW symmetry breaking

- First-order EWPTs would produce gravitational waves that are potentially detectable by LISA, Tianqin, Taiji … [4,5,6]

[1]Trodden M. Electroweak baryogenesis[J]. Reviews of Modern Physics, 1999, 71(5): 1463.
[2]Morrissey D E, Ramsey-Musolf M J. Electroweak baryogenesis[J]. New Journal of Physics, 2012, 14(12): 125003.
[3]Sakharov A D. Violation of CPin variance, Casymmetry, and baryon asymmetry of the universe[J]. Sov. Phys. Usp, 1991, 34: 392.
[4]Weir D J. Gravitational waves from a first-order electroweak phase transition: a brief review[J]. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 2018, 376(2114): 20170126.
[5] Liang Z C, Hu Y M, Jiang Y, et al. Science with the TianQin Observatory: Preliminary results on stochastic gravitational-wave background[J]. Physical Review D, 2022, 105(2): 022001.
[6]Ruan W H, Guo Z K, Cai R G, et al. Taiji program: Gravitational-wave sources[J]. International Journal of Modern Physics A, 2020, 35(17): 2050075.]Liang Z C, Hu Y M, Jiang Y, et al. Science with the TianQin Observatory: Preliminary results on stochastic gravitational-wave background[J]. Physical Review D, 2022, 105(2): 022001.

# First-order phase transition



# Model

- SM has no first-order PTs   D'Onofrio M, Rummukainen K. Standard model cross-over on the lattice[J]. Physical Review D, 2016, 93(2): 025003.

Beyond standard model:

Higgs + scalar singlet   Damgaard P H et al. 2016, 2016(2): 1-28.  Vaskonen V. 2017, 95(12): 123515.
Two higgs doublet (2HDM)   Branco G C et al. 2012, 516(1-2): 1-102.  Bernon J, Bian L, Jiang Y. 2018, 2018(5): 1-43.
Three higgs doublet (3HDM)   Keus V, King S F, Moretti S. 2014, 2014(1): 1-55.
Next-to-minimal supersymmetric standard model (NMSSM)[7,8]   Ellwanger U, Hugonie C, Teixeira A M. 2010, 496(1-2): 1-77.
......

## Model: Higgs + real scalar singlet

$Z_2$ symmetry: $S \to -S$ , dark matter, domain walls...

$$V_0(H,S) = -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + \lambda_{HS} H^\dagger H S^2 - \frac{1}{2}\mu_S^2 S^2 + \frac{1}{4}\lambda_S S^4$$

# Vacuum structure at zero temperature

$$V_0(H,S) = -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + \lambda_{HS} H^\dagger H S^2 - \frac{1}{2}\mu_S^2 S^2 + \frac{1}{4}\lambda_S S^4$$

- The minima of the tree-level potential are determined by the minimization conditions

$$\begin{cases} \dfrac{\partial V_0}{\partial h} = h(\lambda_H h^2 + \lambda_{HS} s^2 - \mu_H^2) = 0, \\ \dfrac{\partial V_0}{\partial s} = s(\lambda_{HS} h^2 + \lambda_S s^2 - \mu_S^2) = 0. \end{cases}$$

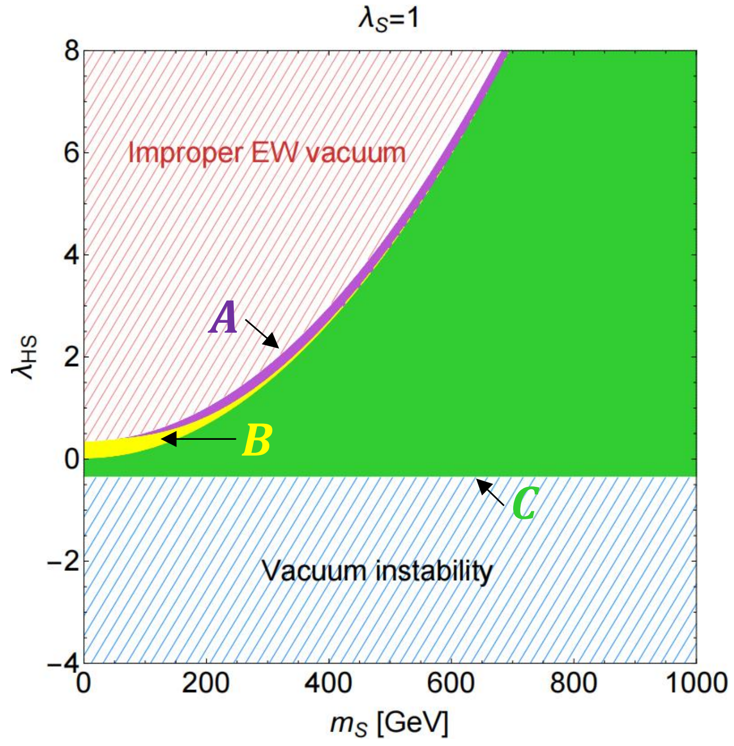
| Case | $(0,0)$ | $(v,0)$ | $(0,w)$ | $(v_h,w_s)$ | Existence |
|---|---|---|---|---|---|
| A | ✓LMax | ✓GMin | ✓LMin | ✓Sad | Yes (Purple) |
| B | ✓LMax | ✓GMin | ✓Sad | — | Yes (Yellow) |
| C | ✓Sad | ✓GMin | — | — | Yes (Green) |
| D | ✓ | ✓ | — | ✓ | No |
| E | ✓ | — | ✓ | ✓ | Excluded |
| F | ✓ | — | ✓ | — | Excluded |
| G | ✓ | — | — | ✓ | Excluded |
| H | ✓ | — | — | — | Excluded |

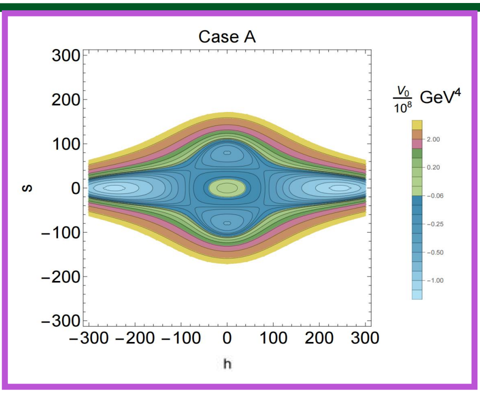
Cases A, B: $\mu_S^2 > 0$; Case C: $\mu_S^2 < 0$

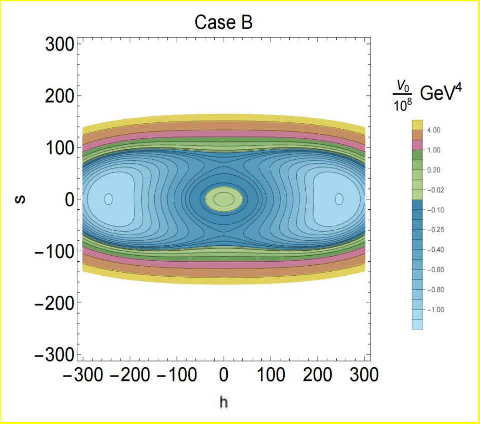
$v = \pm \frac{\mu_H}{\sqrt{\lambda_H}}$

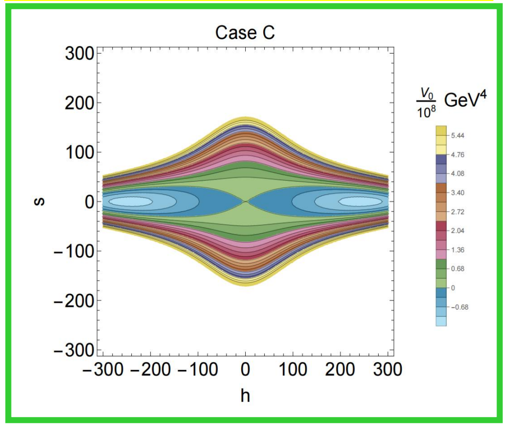
$w = \pm \frac{\mu_S}{\sqrt{\lambda_S}}$

$v_h$ and $w_s$ are two non-trivial solutions determined by the minimization conditions of the tree-level potential



# Analysis of phase transitions

## Finite-T effective potential

$$V_{\rm eff}(h,s,T) \approx V_0(h,s) + V_{\rm CW}(h,s) + V_{\rm CT}(h,s) + V_{1\rm T}(h,s,T) + V_{\rm daisy}(h,s,T)$$

$$V_0(h,s) = -\frac{1}{2}\mu_H^2 h^2 + \frac{1}{4}\lambda_H h^4 - \frac{1}{2}\mu_S^2 s^2 + \frac{1}{4}\lambda_S s^4 + \frac{1}{2}\lambda_{HS} h^2 s^2$$

$$V_{\rm CW}(h,s) = \frac{1}{64\pi^2}\sum_i (-1)^{2s_i} n_i m_i^4(h,s)\left[\ln\frac{m_i^2(h,s)}{Q^2} - C_i\right]$$

$$V_{\rm CT} = \frac{\delta\mu_H^2}{2}h^2 + \frac{\delta\lambda_H}{4}h^4 + \frac{\delta\mu_S^2}{2}s^2 + \frac{\delta\lambda_S}{4}s^4 + \frac{\delta\lambda_{HS}}{2}h^2 s^2$$

$$V_{1\rm T}(h,s,T) = \frac{T^4}{2\pi^2}\sum_i n_i J_{B,F}\left(\frac{m_i^2(h,s)}{T^2}\right)$$

$$V_{\rm daisy}(h,s,T) = -\frac{T}{12\pi}\sum_i n_i \left[\left(M_i^2(h,s,T)\right)^{\frac{3}{2}} - \left(m_i^2(h,s)\right)^{\frac{3}{2}}\right]$$

$$m_{h,s}^2(h,s) = \text{eigenvalues}\widehat{M}_{h,s}^2$$

$$m_{G^0,G^\pm}^2(h,s) = -\mu_H^2 + \lambda_H h^2 + \lambda_{HS} s^2$$

$$m_W^2(h,s) = \frac{1}{4}g^2 h^2$$

$$m_Z^2(h,s) = \frac{g^2+g'^2}{4}h^2$$

$$m_\gamma^2(h,s) = 0$$

$$m_t^2(h,s) = \frac{1}{2}y_t^2 h^2$$

field-dependent mass

$$M_i^2(h,s,T) = \text{eigenvalues}(\widehat{M}_i^2(h,s) + \Pi_i(T))$$

thermal mass corrections

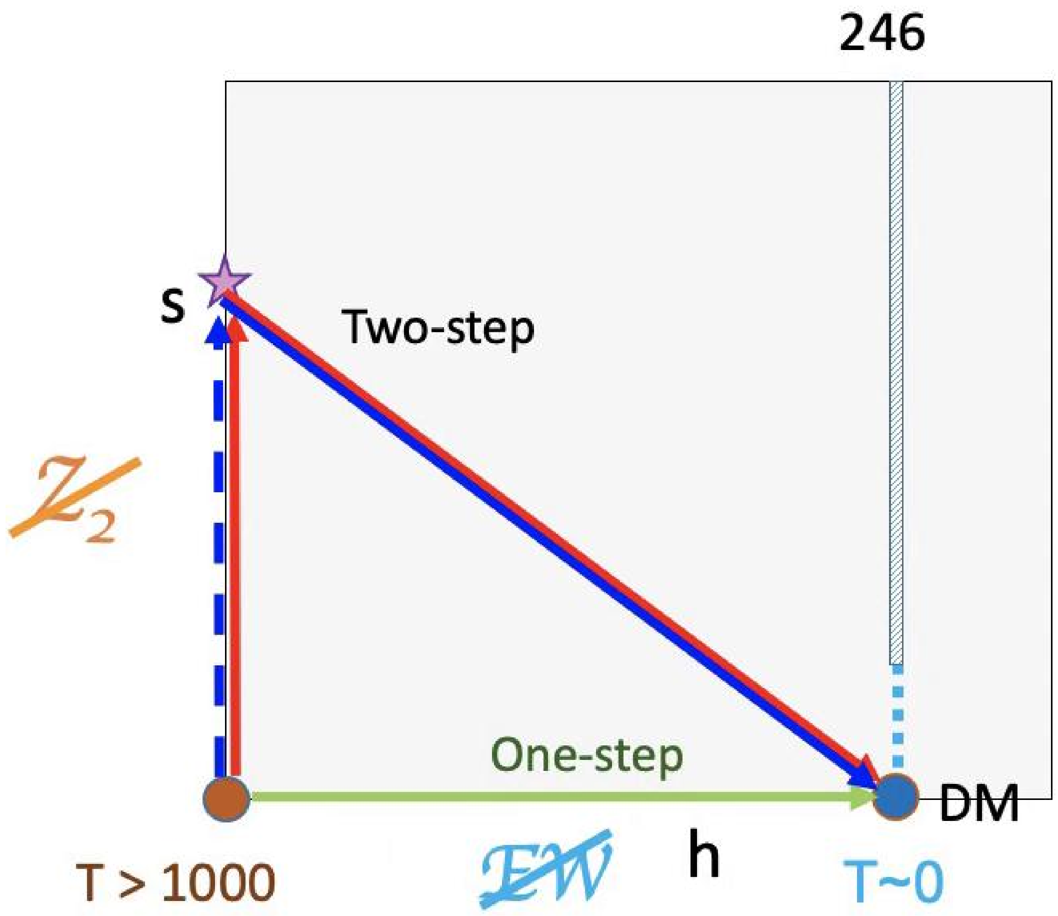
# Public package

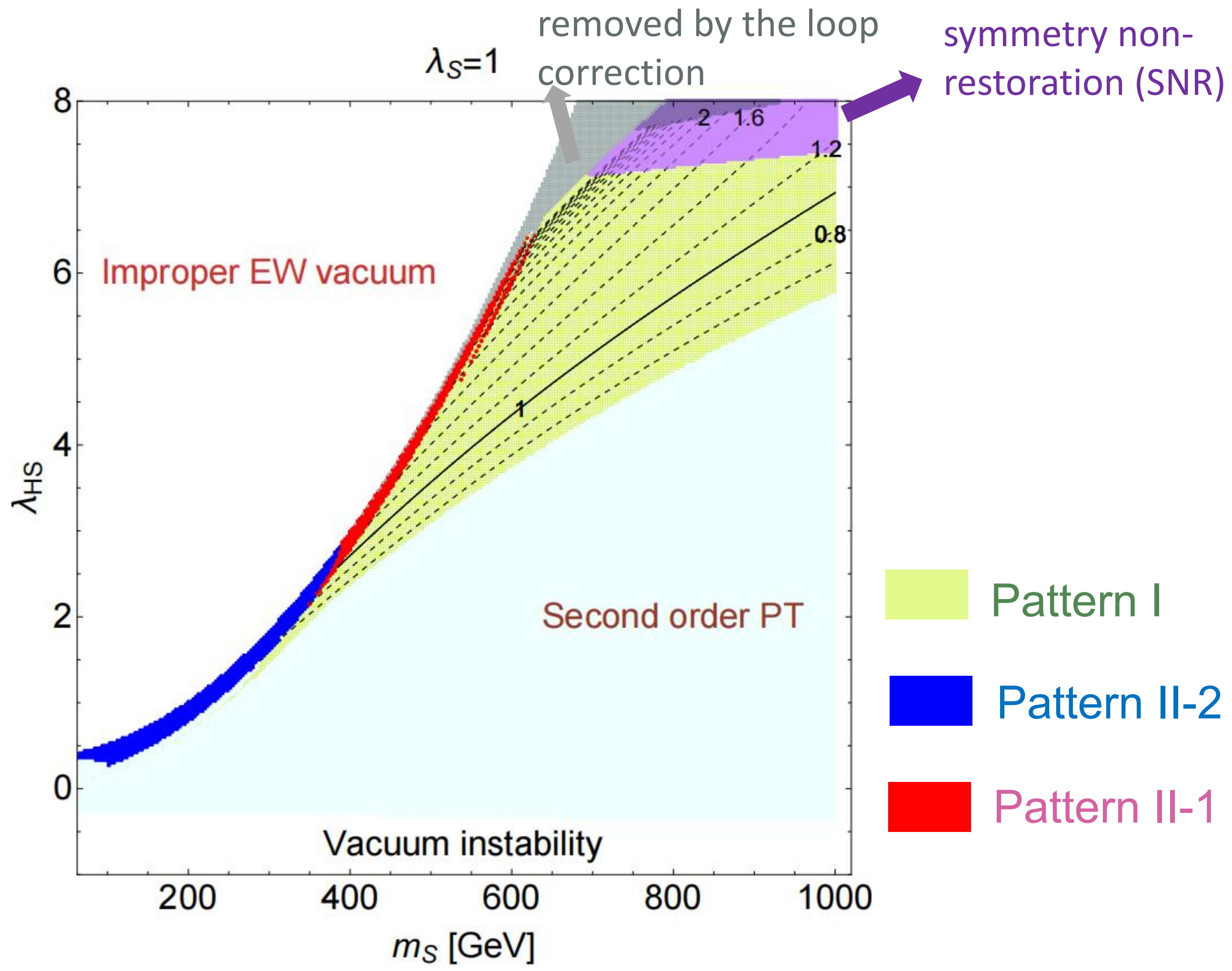
- numerical packages need to be used to find the lowest point of the effective potential and the patterns of the PTs and calculate $T_c$ :

- **CosmoTransition [1109.4189]**
- **BSMPT [2007.01725v2 ]**
- **PhaseTracer [2003.02859]**

# Possible PT patterns

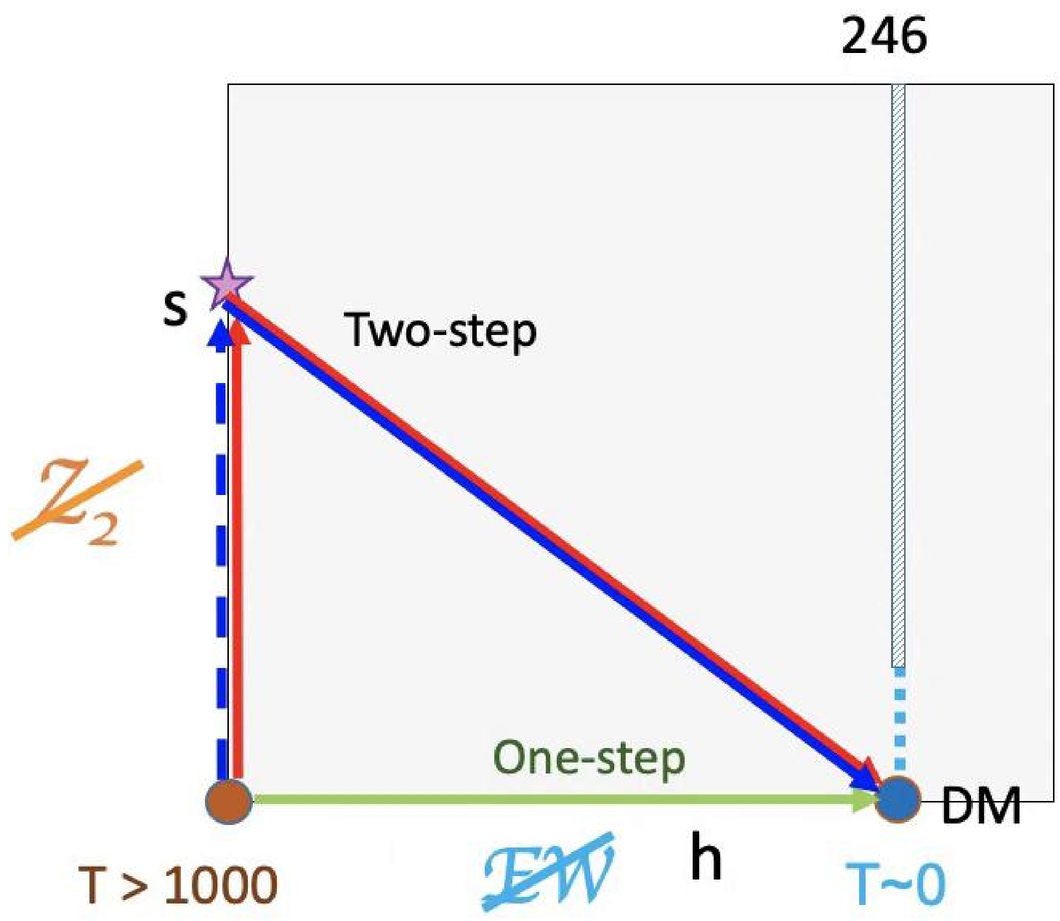
$$V_{\rm eff}(h,s,T) = V_0(h,s) + V_{\rm CW}(h,s) + V_{\rm CT}(h,s) + V_{\rm 1T}(h,s,T) + V_{\rm daisy}(h,s,T)$$

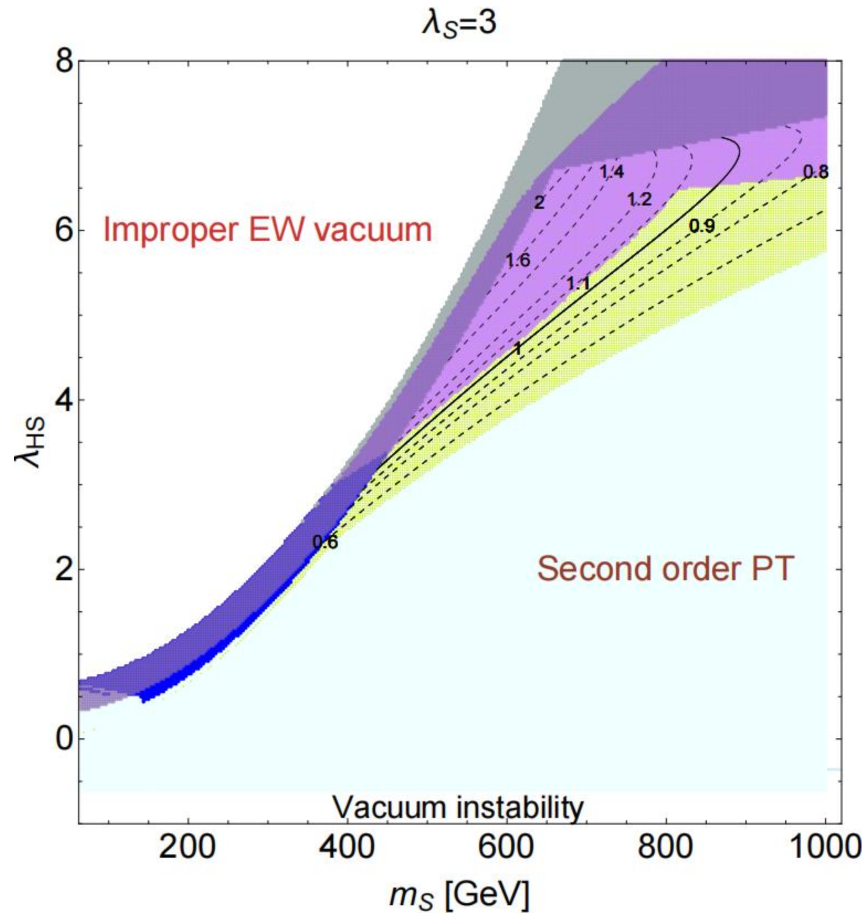


# Possible PT patterns

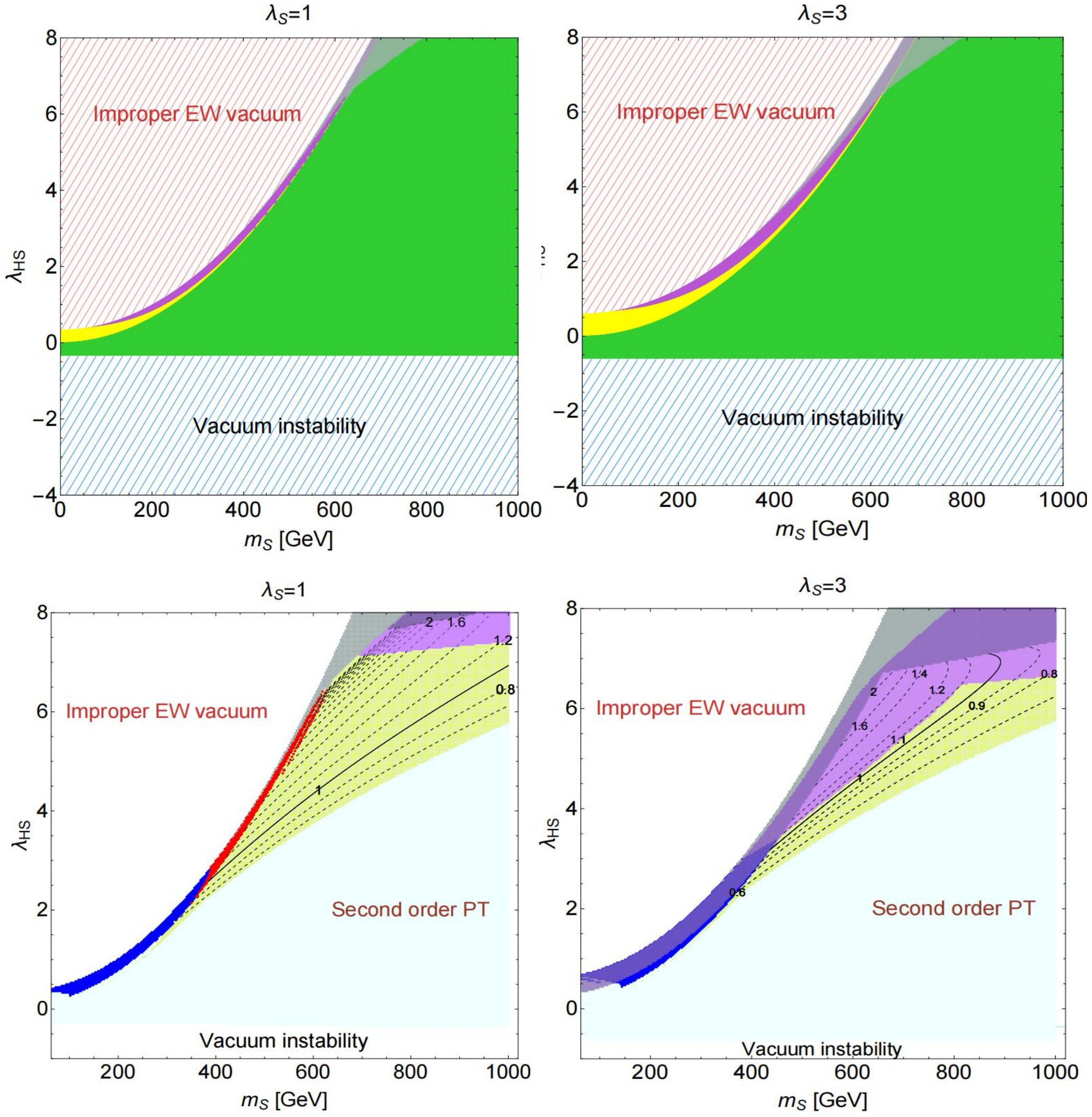
$$V_{\text{eff}}(h,s,T) = V_0(h,s) + V_{\text{CW}}(h,s) + V_{\text{CT}}(h,s) + V_{1\text{T}}(h,s,T) + V_{\text{daisy}}(h,s,T)$$




For a higher value of $\lambda_S = 3$, SNR constraint will eliminate the realization of two-step PTs (Pattern II-1 in the case) and also place an upper bound on the strength of the Pattern I EWPT, roughly $\xi \lesssim 1.1$

# The link between zero-temperature vacuum structures and PT patterns




vacuum structure at 0T

thermal transition history at finite T

| | | |
|---|---|---|
| Case C | favors | Pattern I EWPT |
| Case A,B | favor | two-step PTs |

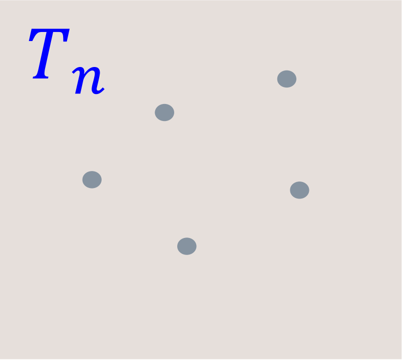
# Vacuum bubbles generated from the PTs

The rate of bubble nucleation is determined by tunneling action S

The bubble nucleation rate $\Gamma(T) \approx A(T)e^{-S(T)}$ → (high T) → $\Gamma(T) = T^4\left(\frac{S_3(T)}{2\pi T}\right)^{3/2} e^{-S_3(T)/T}$

$$N(T_n) = \int_{tc}^{tn} dt \frac{\Gamma(t)}{H(t)^3} = \int_{T_n}^{T_c} \frac{dT}{T}\frac{\Gamma(T)}{H(T)^4} = 1$$

For sufficiently fast EWPTs → $\frac{S_3(T_n)}{T_n} \approx 140$



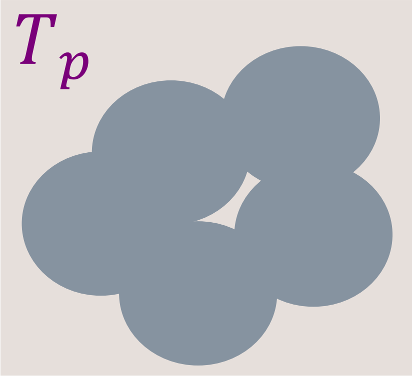
Nucleation

the volume fraction converted to the true vacuum $I(T)$

$$I(T) = \frac{4\pi}{3}\int_{T}^{Tc} dT' \frac{\Gamma(T')}{H(T')T'^4}\left(\int_{T}^{T'} dT'' \frac{v_w(T'')}{H(T'')}\right)^3$$

34% false vacuum turns into true vacuum at $T_p$

$T_p$ is defined such that $I(T_p) \approx 0.34$



Percolation

Quiros M. Summer school in high-energy physics and cosmology: Trieste1998, 1999: 187-259.

Leitao L, Megevand A, Sanchez A D. Journal of Cosmology and Astroparticle Physics, 2012, 2012(10): 024.

Kobakhidze A, Lagger C, Manning A, et al. The European Physical Journal C, 2017, 77(8): 1-13.

# Bubble nucleation and dynamics

$$\Gamma(T) = T^4 \left(\frac{S_3(T)}{2\pi T}\right)^{3/2} e^{-S_3(T)/T}$$

$$S_3(T) = \int d^3x \left[\frac{1}{2}(\nabla\phi)^2 + V_{\text{eff}}(\phi,T)\right] = 4\pi\int_0^\infty dr\, r^2 \left[\frac{1}{2}(\frac{d\phi}{dr})^2 + V_{\text{eff}}(\phi,T)\right]$$

## 1 Bounce solution

$$\frac{d^2\phi(r)}{dr^2} + \frac{2}{r}\frac{d\phi(r)}{dr} = \frac{\partial V_{\text{eff}}}{\partial\phi} \qquad \phi(r\to\infty) = 0\,,\ \frac{d\phi}{dr}|(r=0) = 0 \Rightarrow \phi_{\text{B}}(\text{r}) \Rightarrow S_3(\text{T}) \text{ and } \Gamma(\text{T})$$

- **CosmoTransition [1109.4189]**
- **AnyBubble [1610.06594 ]**
- **FindBounce [2002.00881]**
- **BubbleProfiler [1901.03714]**
- **SimpleBounce [1908.10868]**

more information about bubble dynamics:

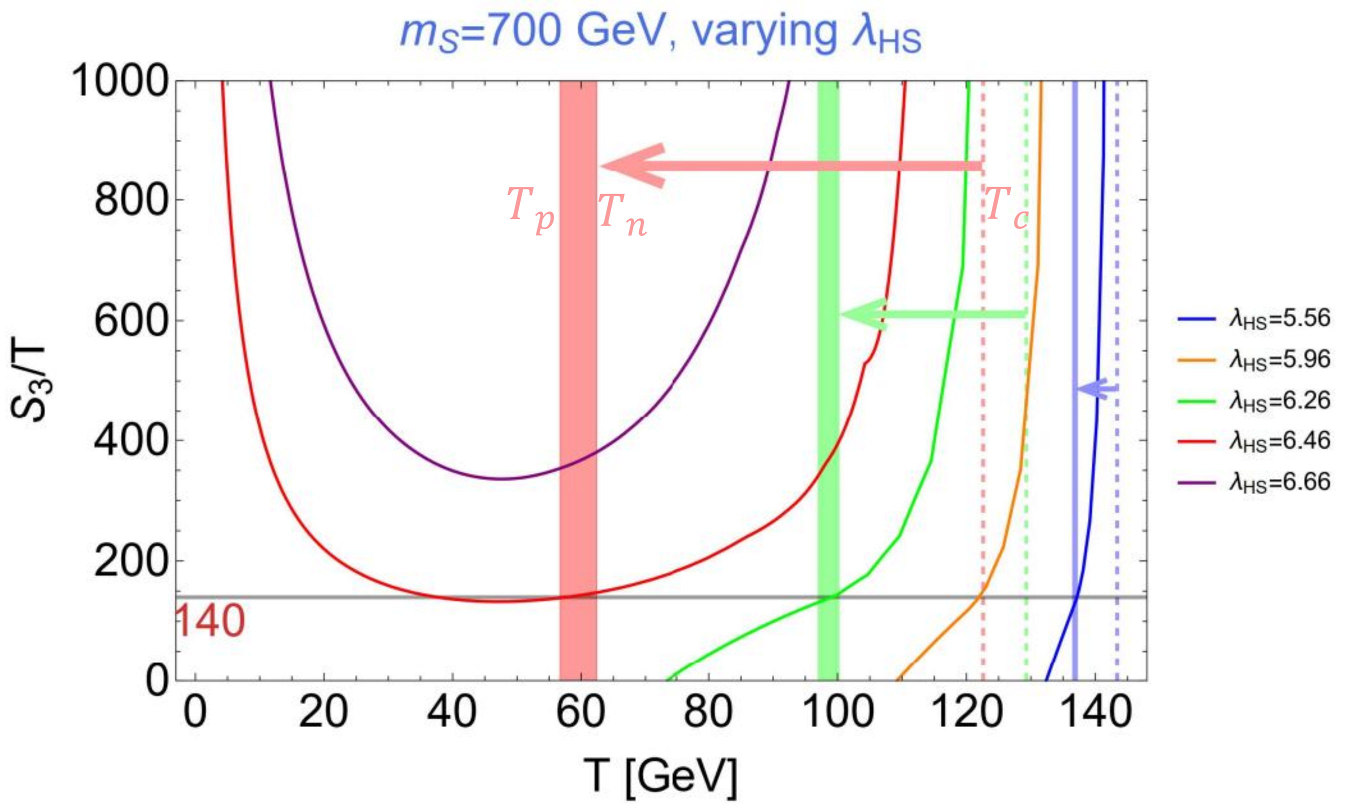
- Bubble size $R_c$
- Bubble thickness $L_w$

## 2 New method to calculate the action

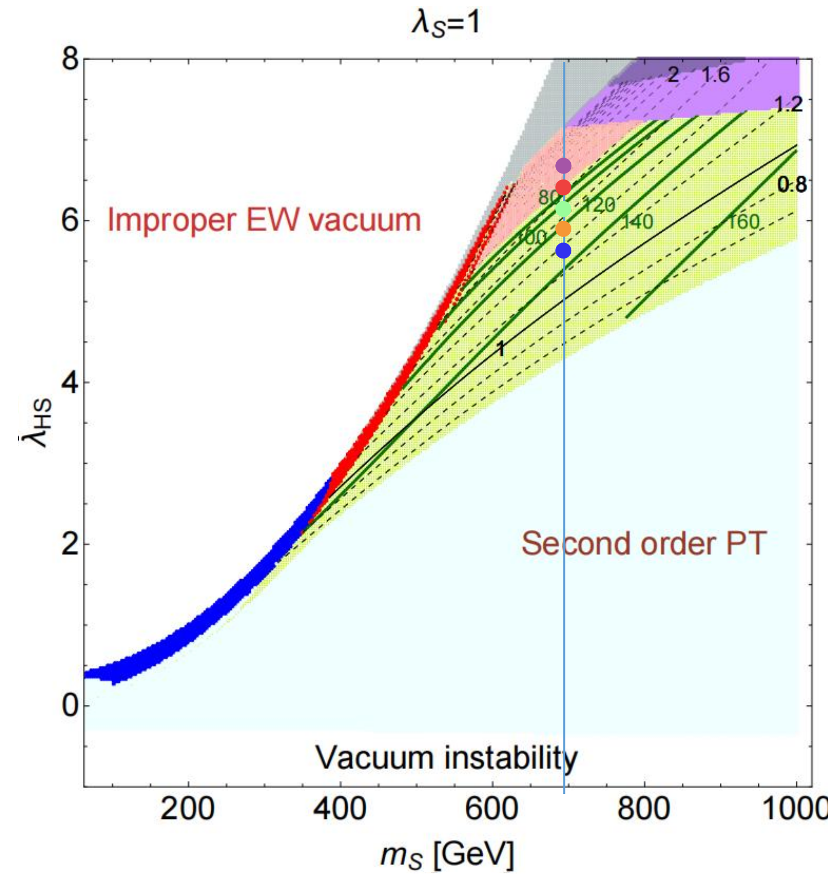
Using tunneling potential to get the tunneling action (see Espinosa J R. A fresh look at the calculation of tunneling actions[J]. JCAP, 2018, 2018(07): 036.)

# Bubble nucleation and dynamics： result analysis

- Focus on one-step PTs




| BP | $\lambda_{HS}$ | $T_c$[GeV] | $T_n$(def)/(app)[GeV] | $T_p$[GeV] |
|---|---|---|---|---|
| BP1 | 5.56 | 143.4 | 137.202 / 137.179 | 136.5 |
| BP2 | 5.96 | 136.5 | 122.1 / 121.8 | 120.6 |
| BP3 | 6.26 | 129.3 | 100.0 / 99.8 | 97.2 |
| BP4 | 6.46 | 122.6 | 62.4 / 55.2 | 56.7 |




Black contours: $\frac{v_c}{T_c}$

Green contours: $T_n$

To nucleate the bubble, there are bounds on the PT parameters, in this model

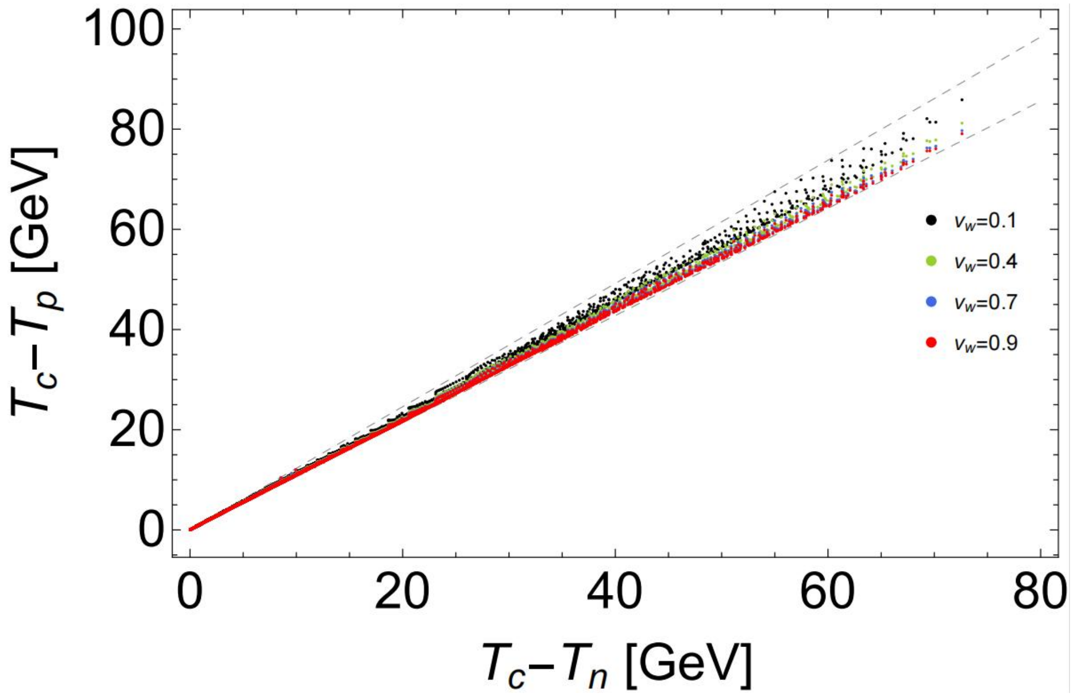
$$\frac{v_c}{T_c} \lesssim 2,\ T_n \gtrsim 60\ \text{GeV}$$

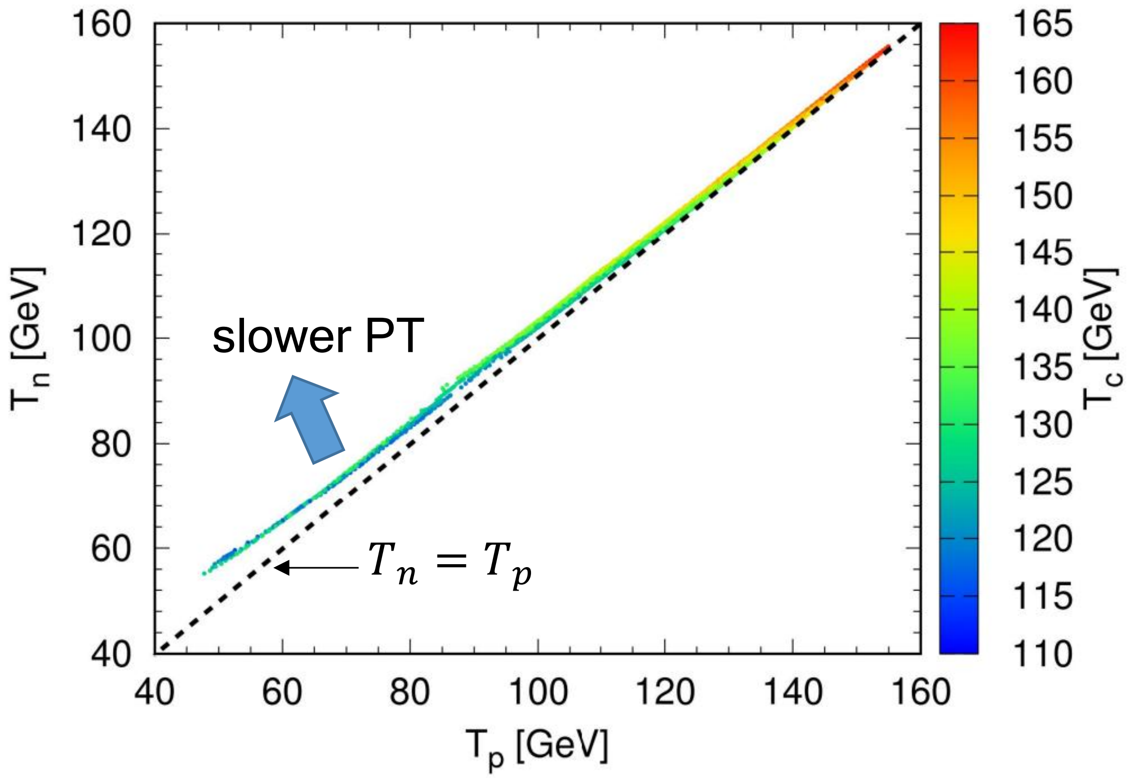
# Bubble nucleation and dynamics：result analysis

- Percolation temperature

[2009.14295], [2011.12903]

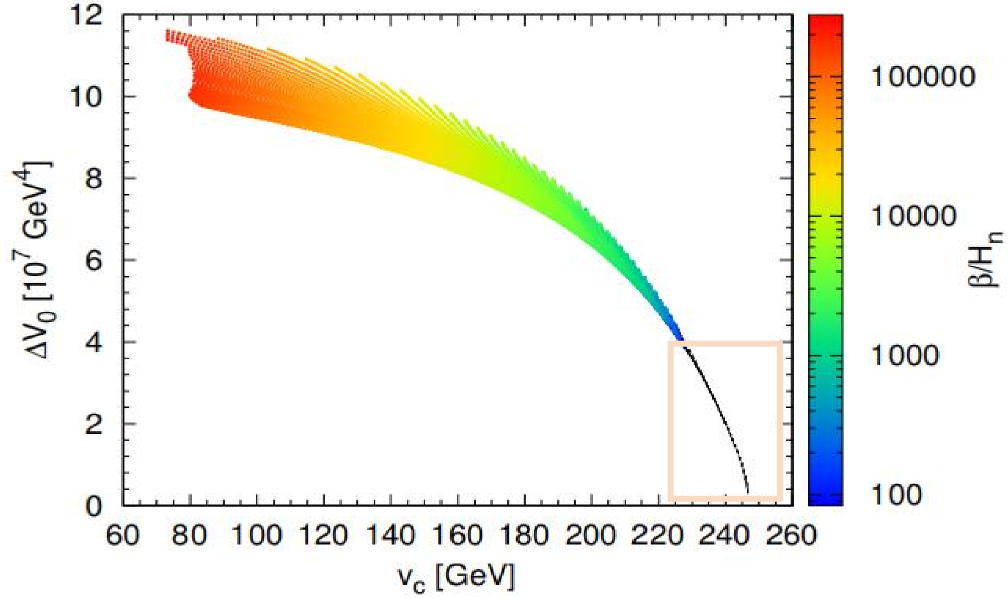
$$I(T_p) = 0.34 \qquad I(T) = \frac{4\pi}{3}\int_T^{T_c} dT' \frac{\Gamma(T')}{H(T')T'^4}\left(\int_T^{T'} dT'' \frac{v_w(T'')}{H(T'')}\right)^3$$




$$T_c - T_n \propto T_c - T_p$$

# Successful nucleation ?




**Conclusion:**

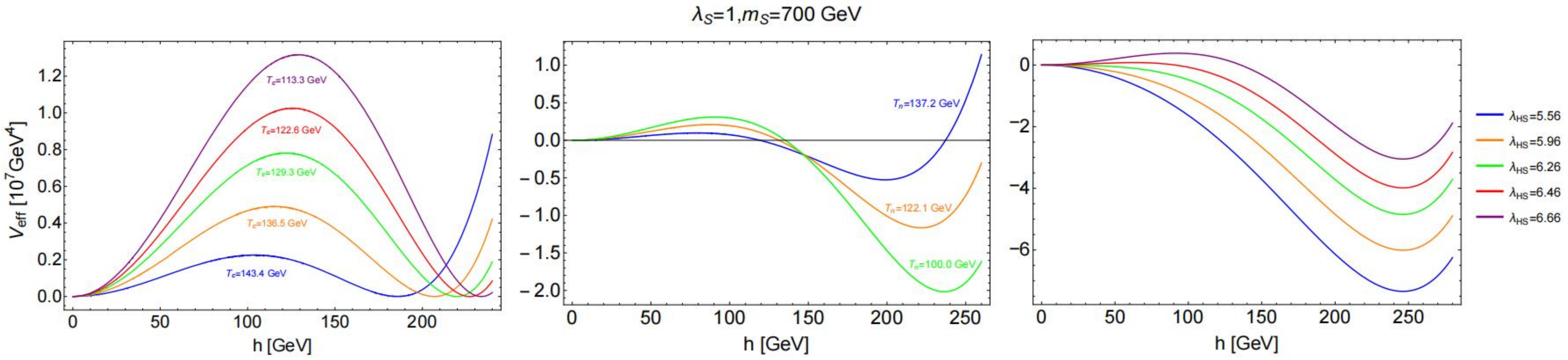
The potential difference at 0T $\Delta V_0$ and minima distance at $T_c$： $v_c$ can reflect the success of nucleation

$\Delta V_0$ ⬇

$v_c$ ⬆

~~nucleate~~

Similar findings: see Kurup PRD 2017, 96(1): 015036, Baum S, Carena M, Shah N R, et al. 2021, 2021(3): 1-57.




$V_{\rm eff}\ at\ T_c$ $V_{\rm eff}\ at\ T_n$ $V_{\rm eff}\ at\ 0\ T$

Coupling $\lambda_{HS}$ ↑

$\Delta V_0$ ↓

~~nucleate~~

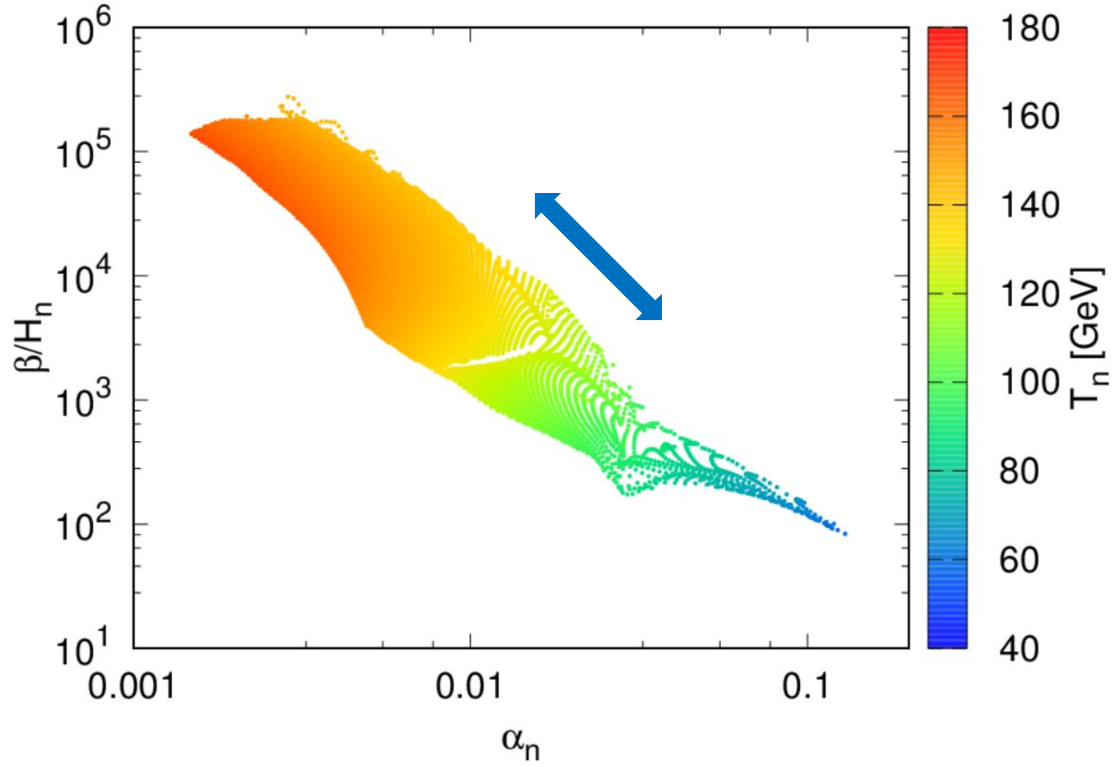
# Key PT parameters related to GWs

## Parameters on which PT gravitational wave calculations depend：

(1) PT characteristic temperature $T_*$ (2) Energy released by PT $\alpha_*$ (3) Tunneling rate $\beta/H_*$ (inverse of PT duration)

- Taking $T_*=T_n$ or $T_p$ may lead to a large theoretical uncertainty in predicting the GW spectrum

$$\alpha_n = \frac{\epsilon}{\rho_{\rm rad}}\Big|_{T=T_n} \quad \text{with } \epsilon = \rho_{\rm vac} - T\frac{\partial}{\partial T}\left[V_{\rm eff}(\phi_F,T) - V_{\rm eff}(\phi_T,T)\right] \qquad \beta = H(T)T\frac{dS}{dT}\Big|_{T=T_n}$$



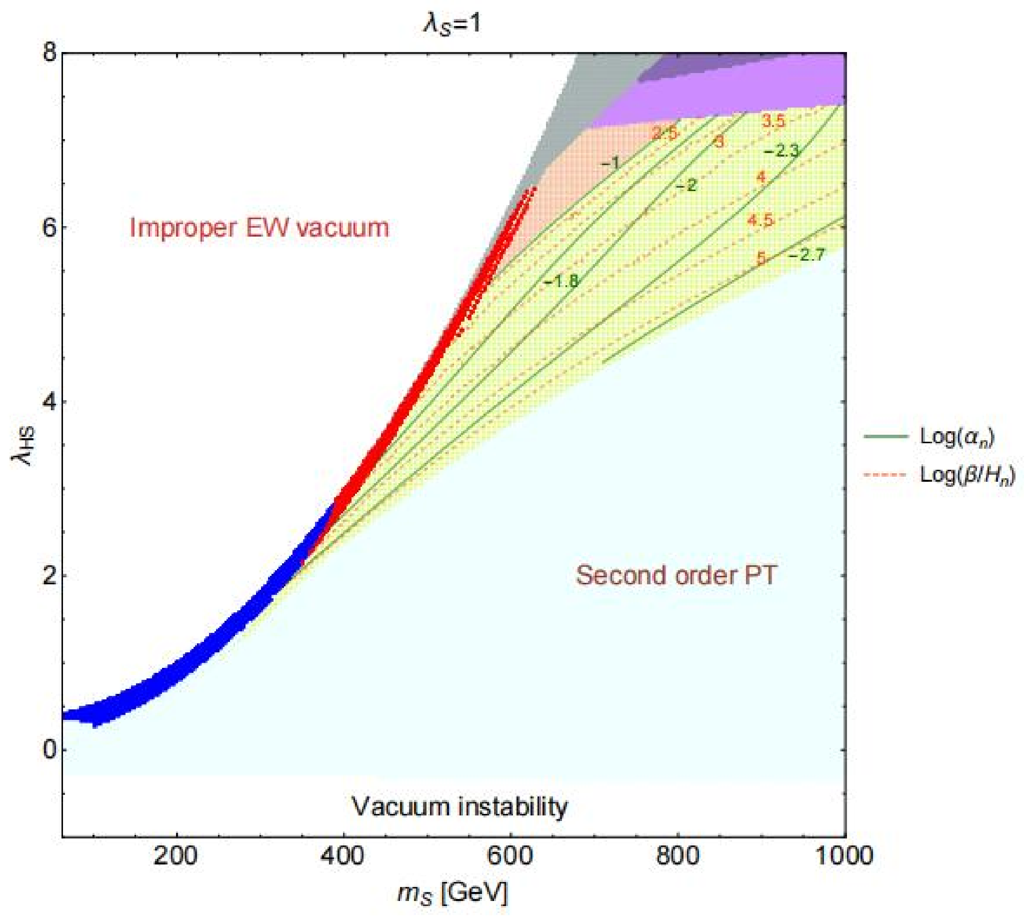

Key PT parameters determining GW spectrum are **NOT** independent

- Low $T_n$ Strong Slow EWPTs
- High $T_n$ Weak Fast EWPTs

| | |
|---|---|
| $\alpha \leq 0.1$ | Slight supercooling |
| $0.1 \leq \alpha \leq 0.5$ | Mild supercooling |
| $0.5 \leq \alpha \leq 1$ | Strong supercooling |
| $\alpha \geq 1$ | Ultra supercooling |

Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045

# Key PT parameters related to GWs




Sucessful nucleation: $\alpha_n \lesssim 0.15$, $\beta/H_n \gtrsim 80$

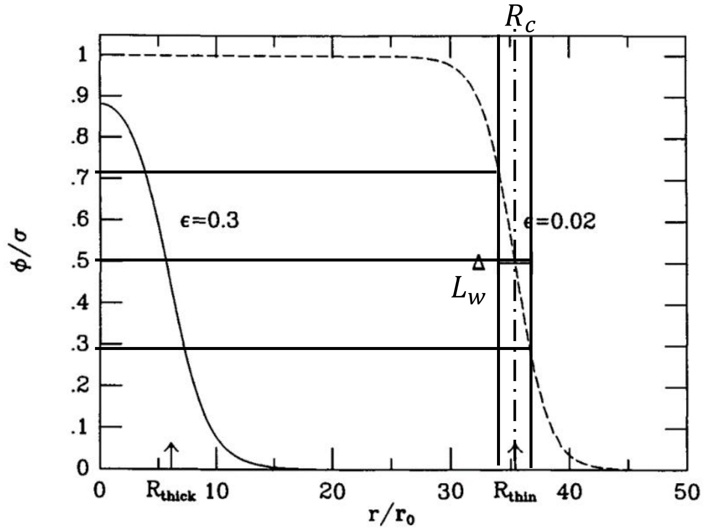
# Thin-wall approximation

## ◆ Thin-wall approximation

1 the difference in energy between the metastable and true vacua are small compared to the height of the barrier, it is possible to find a simple, approximate analytic expression for action $S_E$

2 Bubble size $R_c$>>Bubble thickness $L_w$

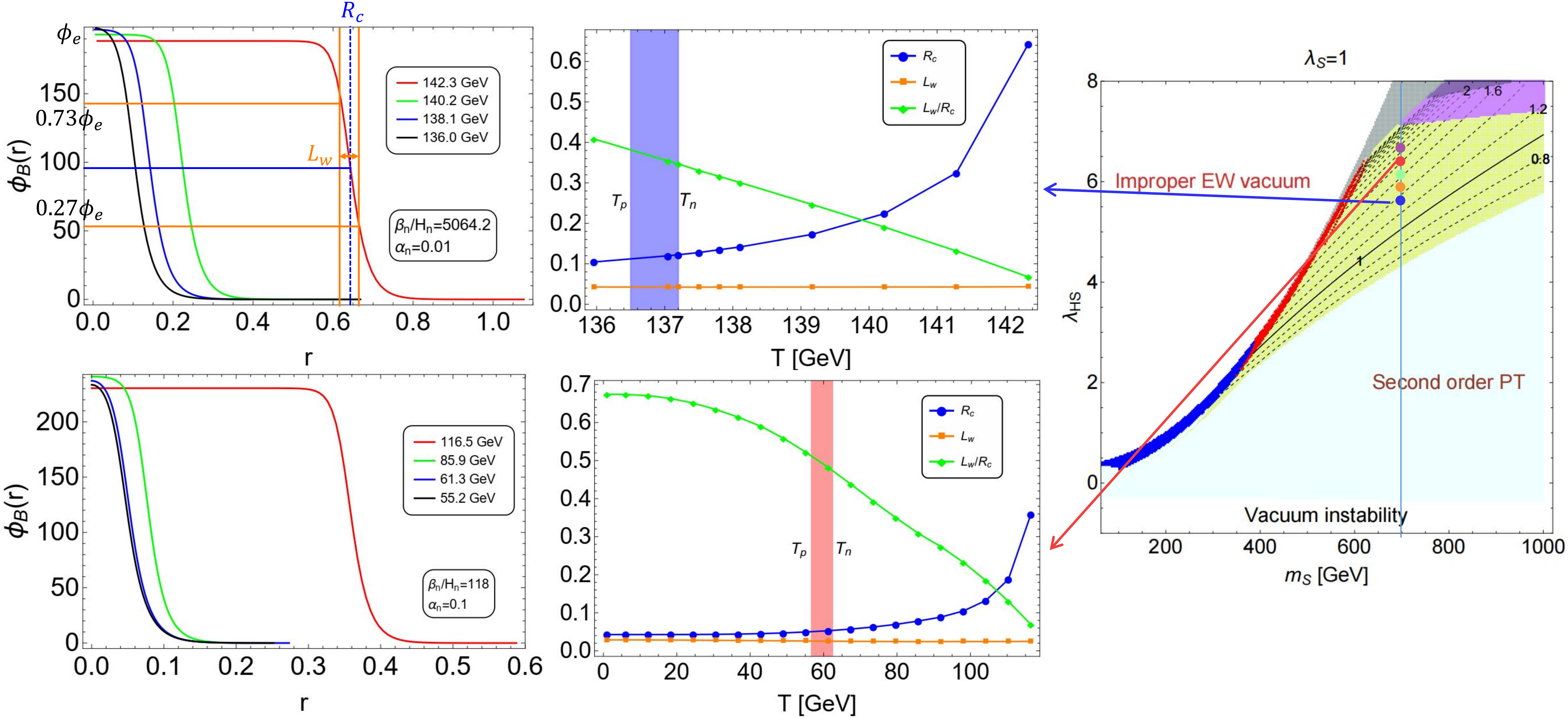
Kolb and M.S. Turner, The early universe, CRC press (2018)




Action: $\frac{S_3^{\rm tw}}{T} = \frac{16\pi[\int_0^{\phi_0} d\phi\sqrt{2V_0(\phi)}]^3}{3T(\Delta V)^2}$

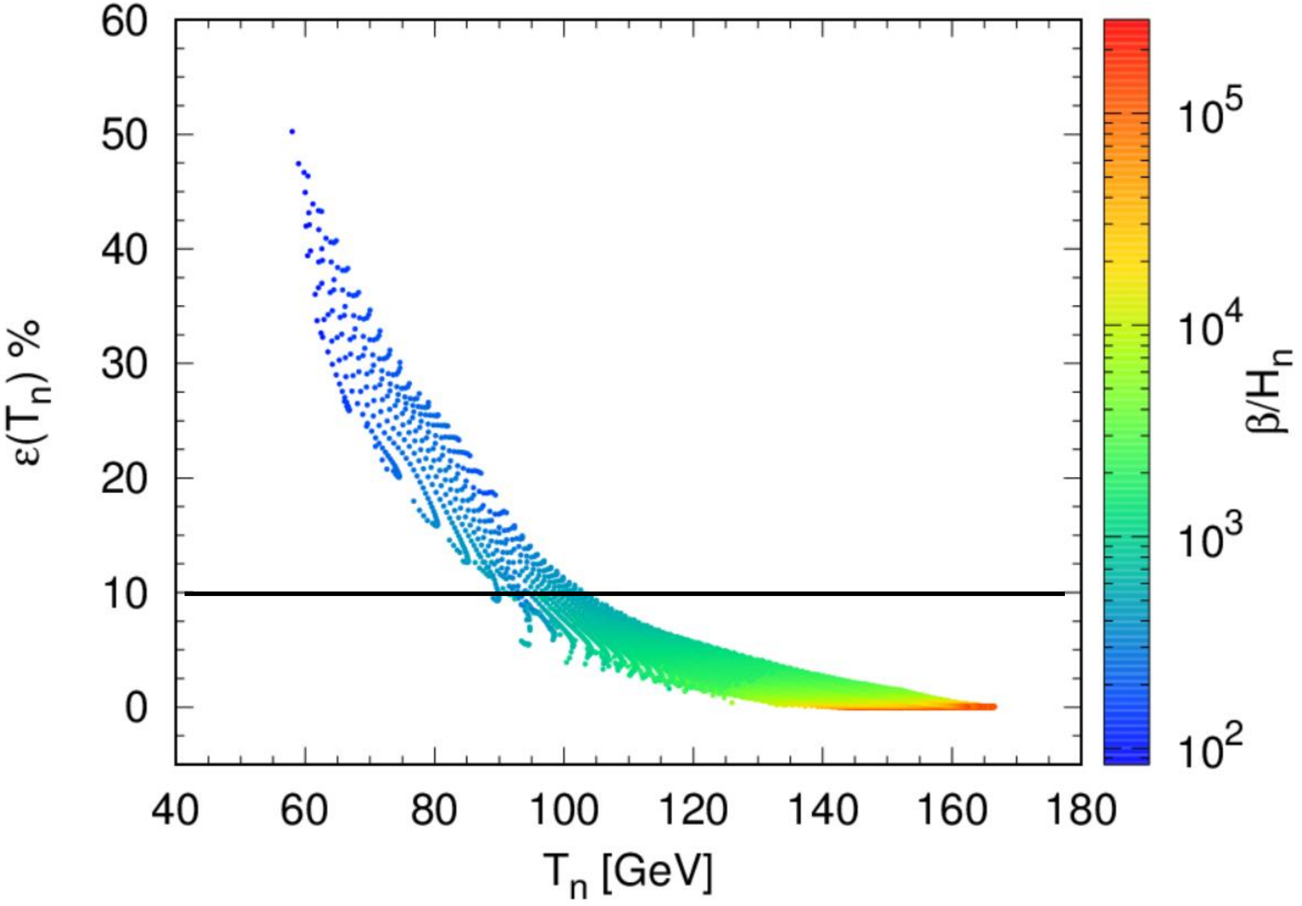
We define $\epsilon(X) = \frac{X^{\rm tw} - X}{X}$ Where $X = T_n$

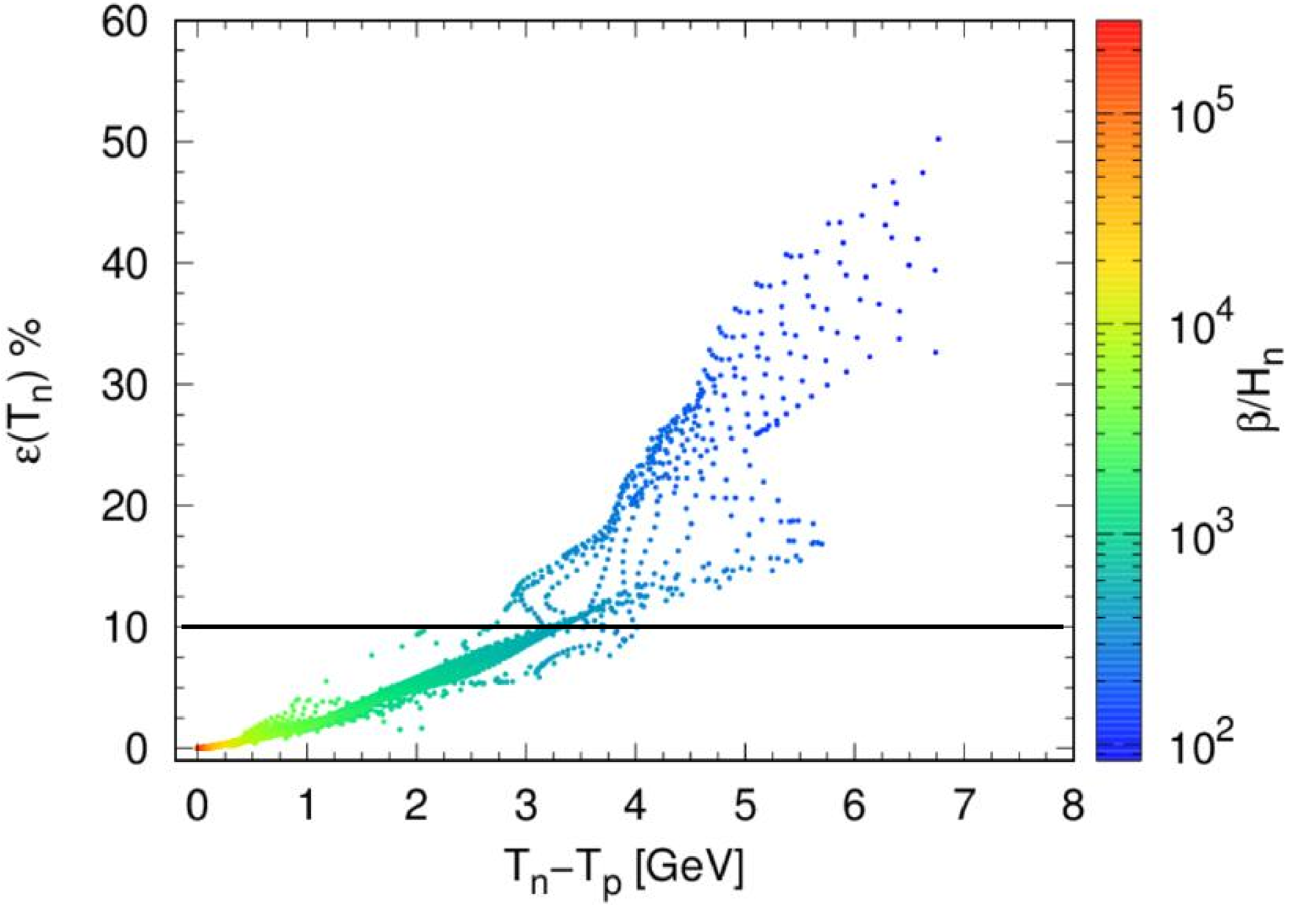
# Bubble nucleation and dynamics：result analysis



# Thin-wall approximation

Perhaps more attention is paid to the impact on $T_n$ calculations:




- If 10% is chosen as an acceptable error of $T_n$, $\beta/H_\mathrm{n}$ should be $\gtrsim$ 1000, $T_n - T_p \lesssim$ 4 GeV, $T_n \gtrsim 100$ GeV
- Thin wall limit well applies to fast PTs

# Validity of mean field approach

There is a simple approach to study PT: including the leading thermal correction in the scalar potential[1, 2], corresponding to terms as $\phi^2 T^2$ $(\phi = h, s)$ in the high temperature expansion of the 1-loop thermal potential

$$\Delta V_T(h,s,T) = c_h h^2 T^2 + c_s s^2 T^2$$

$$c_h = \frac{1}{48}\left(9g^2 + 3g'^2 + 24\lambda_H + 4\lambda_{HS} + 12y_t^2\right)$$
$$c_s = \frac{1}{12}\left(4\lambda_{HS} + 3\lambda_S\right)$$

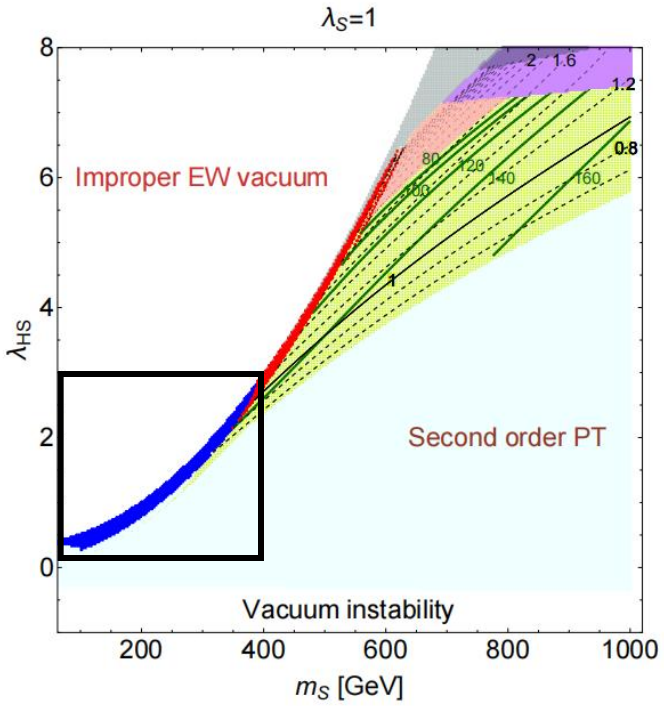
The resulting scalar potential with temperature is simply given by

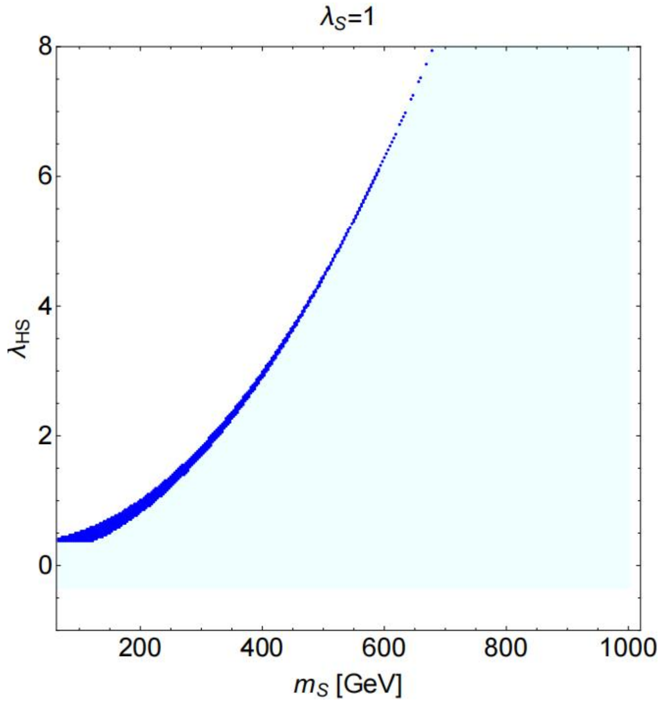
$$\begin{aligned} V(h,s,T) &= V_0(h,s) + \Delta V_T(h,s,T) \\ &= (-\frac{1}{2}\mu_H^2 + c_h T^2)h^2 + \frac{1}{4}\lambda_H h^4 + (-\frac{1}{2}\mu_S^2 + c_s T^2)s^2 + \frac{1}{4}\lambda_S s^4 + \frac{1}{2}\lambda_{HS} h^2 s^2 \end{aligned}$$

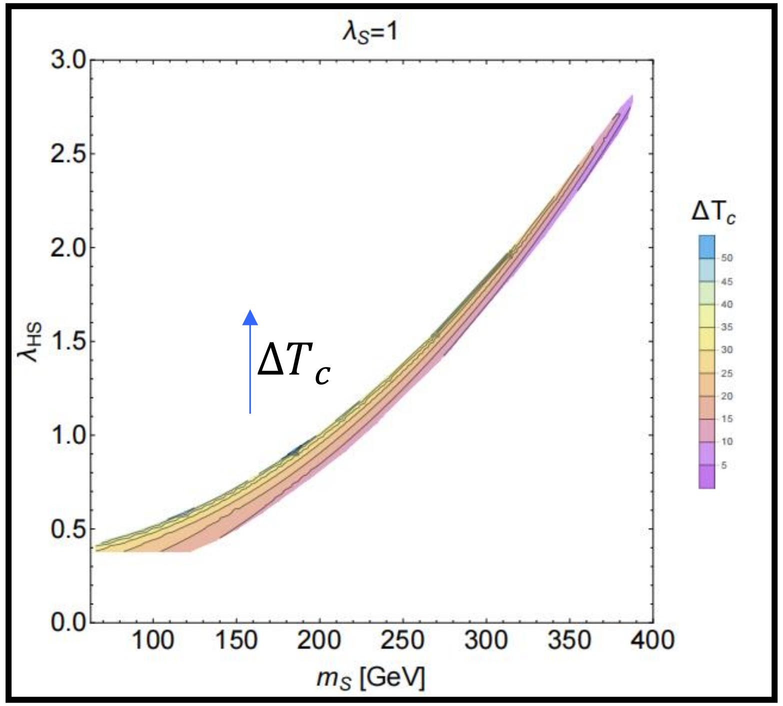
[1]Vaskonen V. PRD, 2017, 95(12): 123515.

[2]Ellis J, Lewicki M, No J M. JCAP, 2019, 2019(04): 003.

# Validity of mean field approach




| $m_S$[GeV] | $\lambda_{HS}$ | PT pattern | PT pattern(MFA) | $T_c$[GeV] | $T_c$(MFA)[GeV] |
|---|---|---|---|---|---|
| 350 | 2.36 | Pattern II-1 | Pattern II-1 | 66.2 | 44.1 |
| 500 | 4.34 | Pattern II-2 | SOFT | 181.2(first-step) 100.6(second-step) | — |
| 475 | 3.05 | Pattern I | SOFT | 146.7 | — |

Mean field approach improperly identifies most 1st PT parameter spaces as 2nd order, and for larger coupling $\lambda_{HS}$, there will be large differences in calculated $T_c$ .

# Summary and outlook

- Key PT parameters determining GW spectrum are not independent:
  Low $T_n$ favors strong and slow EWPTs

- For successful nucleation of bubbles, restrictions are necessary to place on the PT parameters

- The newly nucleated bubble is thin-walled at the start of the PT and getting thick-walled later, and thin-wall limit well applies to sufficiently fast, hot, weak PTs

Outlook

- Perform a similar analysis for another BSM model
- Focus on the dynamics of two-step phase transitions and their cosmological signatures

# Thank you !

Email: chenhb66@mail2.sysu.edu.cn

微信公众号

天琴中心大楼

激光测距台站

山洞实验室