Higher-form symmetries of QFT from string theory

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Introduction to higher-form symmetry

Symmetries

- Symmetry is a central concept in physics

(1) Global symmetry: transformation parameter ϵ is independent of x^μ

(2) Local symmetry: $\epsilon(x^\mu)$ is space-time dependent

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- Examples of global symmetry:

1. Continuous space-time symmetry, e. g. Poincaré symmetry, conformal symmetry, global SUSY, ...
2. Discrete symmetry, e. g. C, P, T,...
3. Continuous internal symmetry, e. g. flavor symmetry, $U(1)_B$, $U(1)_L$
4. Higher-form symmetry, categorical symmetry

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- In differential form language,

$$Q(M^{(d-1)}) = \oint_{M^{(d-1)}} j\,. \tag{1}$$

- e. g. in EM, the $(d-1)$-form converved current j is given by the Maxwell's equation as

$$d * F = j\,. \tag{2}$$

Ordinary 0-form global symmetry

• Introduce the topological operator $U(g, M^{(d-1)})$ generating the 0-form symmetry, which corresponds to $g \in G$,

$$U(g, M^{(d-1)}) = g^{\oint_{M^{(d-1)}} j}\,, \qquad (3)$$

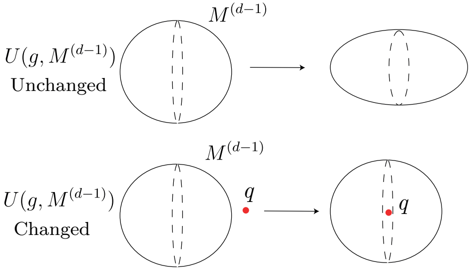
satisfying

$$U(g_1, M^{(d-1)})U(g_2, M^{(d-1)}) = U(g_1 g_2, M^{(d-1)})\,. \qquad (4)$$

• Acts on a 0-dim. local operator at a particular time slice in $\mathbb{R}^d$.

0-form symmetry

- The operator $U(g, M^{(d-1)})$ is unchanged by a small deformation of $M^{(d-1)}$, but changes when $M^{(d-1)}$ crossed through a charged particle (operator):



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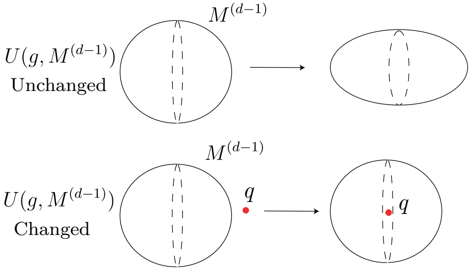

- For example, for an $M^{(d-1)}$ surrounding a point $\mathcal{P}$, and a local operator $V_i(\mathcal{P})$ charged under G, we have

$$U(g, M^{(d-1)})V_i(\mathcal{P}) = R_i^j(g)V_j(\mathcal{P}) . \quad (5)$$

Higher-form symmetry

- Extend the story to a p-form ($p > 0$) symmetry with group G (Gaiotto, Kapustin, Seiberg, Willett 14')
- A p-form symmetry is generated by a $(d-p-1)$-dimensional topological operator $U(g, M^{(d-p-1)})$:

$$U(g_1, M^{(d-p-1)})U(g_2, M^{(d-p-1)}) = U(g_1 g_2, M^{(d-p-1)})\,. \tag{6}$$

and acts on p-dimensional object(operator) $V_i(\mathcal{C}^{(p)})$.

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• $U(g, M^{(d-p-1)})$ has non-trivial action on $V_i(\mathcal{C}^{(p)})$ when $M^{(d-p-1)}$ and $\mathcal{C}^{(p)}$ are non-trivially linked.

U(1) gauge theory

- 4d pure $U(1)$ gauge theory, with action

$$S = \frac{1}{2g^2} \int F \wedge *F \ , \ F = dA \, , \tag{7}$$

there is a $U(1)_e \times U(1)_m$ 1-form symmetry.

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- $U(1)_e$ is generated by the 2-form current

$$j_e = \frac{2}{g^2} * F \,. \tag{9}$$

Closed by e.o.m. $d * F = 0$.

U(1) gauge theory

- The generator of the electric 1-form symmetry:

$$U_e(e^{i\alpha}, M^{(2)}) = \exp\left(i\frac{2\alpha}{g^2}\int_{M^{(2)}} *F\right) \tag{10}$$

here $\int_{M^{(2)}} *F$ is the electric flux through the surface $M^{(2)}$.

- The charged object under $U(1)_e$ is the Wilson loop operator

$$W_n(\mathcal{C}) = \exp(in\oint_C A) \tag{11}$$

U(1) gauge theory

(2) The magnetic 1-form symmetry $\Gamma_m^{(1)} = U(1)_m$ shifts the dual photon

$$\tilde{A} \to \tilde{A} + \tilde{\lambda}\ ,\ d\tilde{A} = *dA \tag{12}$$

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$$U_m(e^{i\eta}, M^{(2)}) = \exp\left(i \frac{\eta}{2\pi} \int_{M^{(2)}} F \right) \tag{14}$$

here $\int_{M^{(2)}} F$ is the magnetic flux through the surface $M^{(2)}$.

- The charged object under $U(1)_m$ is the 't Hooft loop operator

$$T_n(\mathcal{C}) = \exp(in \oint_C \tilde{A}) \tag{15}$$

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- The correct electric 1-form symmetry generators are

$$U_e(e^{i\alpha}, M^{(2)}) \;,\; \alpha = \frac{2\pi}{N}\,. \tag{18}$$

- Hence $U(1)_e$ is broken to $\mathbb{Z}_N$!

U(1) gauge theory with matter

- Equivalently, denote the generator of $\mathfrak{u}(1)$ gauge symmetry algebra by Q, which acts on each matter field with

$$Q \cdot C_i = q_i \,. \tag{19}$$

- In the language of Lie algebra, $Q \in$ root lattice, $C_i \in$ weight lattice.

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• In the language of Lie algebra, $Q \in$ root lattice, $C_i \in$ weight lattice.
• Now one try to find a multiple aQ, $a \in \mathbb{Q}$, where all the matter fields have integral charge under it:

$$aQ \cdot C_i \in \mathbb{Z} \tag{20}$$

When $\gcd(q_i) = N$, this $aQ = Q/N \pmod{Q}$, which generates the $\mathbb{Z}_N$ 1-form symmetry.

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• In other words, the generator of the 1-form symmetry is an element of the lattice $\Lambda_{\mathrm{coweight}}/\Lambda_{\mathrm{coroot}}$.

U(1) gauge theory with matter

- Extend the story to $G = U(1)^r$ gauge group with a number of matter fields ϕ_i with charge

$$q_{i,j} = C_i \cdot Q_j \quad (i = 1, \ldots, m)(j = 1, \ldots, r) \tag{21}$$

under the j-th $U(1)$.

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• To compute the electric 1-form symmetry, one need to write down linear combinations $Q^{(k)}=\sum_j b_{kj}Q_j$, such that all the matter fields have integral charges under $Q^{(k)}$:

$$C_i\cdot Q^{(k)}\in\mathbb{Z}\quad (\forall C_i)\,. \tag{22}$$

U(1) gauge theory with matter

• Solution in linear algebra - Smith Decomposition: take the $(m \times r)$-charge matrix q,

$$q = UDV\,, \tag{23}$$

D is a $(m \times r)$-matrix with the form

$$D = \begin{pmatrix} \alpha_1 & 0 & \dots & 0 \\ 0 & \alpha_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \alpha_r \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix} \tag{24}$$

$(\alpha_i \in \mathbb{Z})$, called the Smith normal form of q.
U is a $m \times m$ matrix and V is a $r \times r$ matrix.

U(1) gauge theory with matter

- Read off each column of V, the linear combination

$$Q^{(i)} = \frac{1}{\alpha_i} \cdot \sum_j V_{j,k} Q_j \in \Lambda_{\text{coweight}}/\Lambda_{\text{coroot}} \tag{25}$$

are the generators of the 1-form symmetry,

$$\Gamma_e^{(1)} = \bigoplus_i (\mathbb{Z}/\alpha_i \mathbb{Z}) \tag{26}$$

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- Examples: suppose we have $G = U(1)^2$ and matter fields with charges

$$q_1 = (2,-1)\ ,\ \ q_2 = (-1,2) \tag{27}$$

$$q = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \tag{28}$$

From the form of D, we can read off $\Gamma_e^{(1)} = \mathbb{Z}_3$, and the generator is

$$Q_{\mathbb{Z}_3} = \frac{1}{3}(-Q_1 + Q_2) \tag{29}$$

non-abelian gauge theory

(1) 4d pure $SU(N)$ gauge theory

- $SU(N)$ has a $\mathbb{Z}_N$ center
- Electric 1-form symmetry $\Gamma_e^{(1)} = \mathbb{Z}_N$ acting on the Wilson loop operator

$\rightarrow$ confining string

- Confined phase!

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(2) 4d pure $PSU(N)$ gauge theory

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- In a particular gauge theory with $\mathfrak{g} = \mathfrak{su}(N)$, the $\mathbb{Z}_N$ Wilson loop and 't Hooft loop cannot coexist $\rightarrow$ choice of polarization.
- This choice is not manifest in the action

$$S = \int \frac{1}{2g^2} tr(F \wedge *F) \,. \tag{30}$$

non-abelian gauge theory

• One can compute the 1-form symmetry using the $U(1)$ techniques after $SU(N) \to U(1)^{N-1}$ (Coulomb branch).

• Add matter fields in fundamental rep. of $SU(N) \to$ breaks $\mathbb{Z}_N$ 1-form symmetry

• Adj. matter does not break the $\mathbb{Z}_N$ 1-form symmetry.

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• Help us to characterize and understand non-Lagrangian QFTs (next parts)

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Idea of geometric engineering

- Putting high-dimensional superstring/M/F-theory on geometric spaces

$\rightarrow$ lower dimensional gravity/QFT.

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• String theory basics:

(1) Superstring in 10d; M-theory in 11d

(2) Extended objects: fundamental string, D-branes; M2-brane, M5-brane

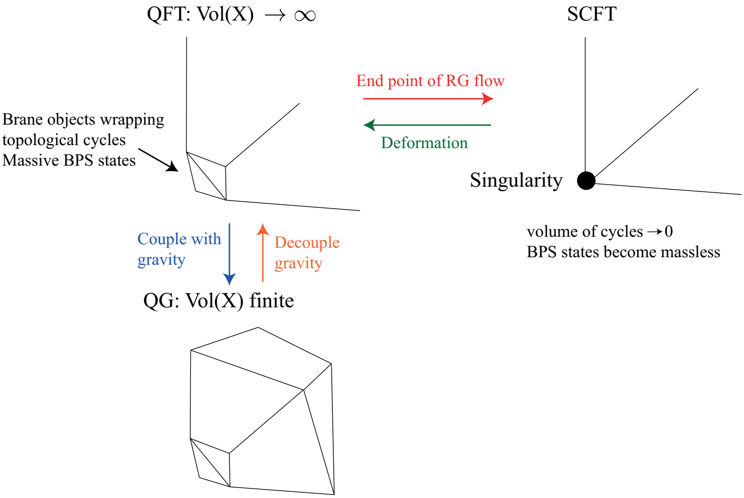
...

(3) Supersymmetry: extension of Poincaré algebra by fermionic operators (supercharges), into super-Poincaré algebra

(4) They are UV complete descriptions of quantum gravity

Idea of geometric engineering

- We consider a product space $\mathbb{R}^{d-1,1} \times X$



Idea of geometric engineering

- In this talk, X is a non-compact Calabi-Yau threefold with $SU(3)$ holonomy
- Number of supercharges: $32 \to 8$, 4d $\mathcal{N}=2$, 5d $\mathcal{N}=1$ or 6d (1,0)
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(1) 1-form symmetry of 5d $\mathcal{N}=1$ SCFTs

(Morrison, Schafer-Nameki, Willett 19')(Albertini, Del Zotto, Exterbarria, Hosseini 20')(Bhardwaj, Schafer-Nameki 20')(Tian, YNW 21')...

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(2) 1-form symmetry in 4d $\mathcal{N}=2$ SCFTs and 5d/4d correspondence

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(3) Higher-form symmetry of 6d (1,0) SCFTs

(Bhardwaj, Schafer-Nameki 20')(Apruzzi, Dierigl, Ling, 21')(Hubner, Morrison, Schafer-Nameki, YNW 22')...

Basics of 5d (SUSY) gauge theories

$$S=\int d^5x\left[\frac{1}{g_{\mathrm{YM}}^2}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu})+i\bar{\psi}\not{D}\psi+\ldots\right] \tag{31}$$

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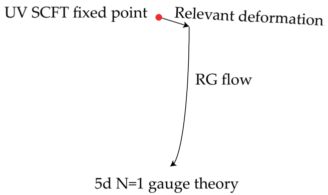
- For some 5d $\mathcal{N}=1$ supersymmetric gauge theories, it can be UV completed to a strongly coupled superconformal field theory when $g_{\rm YM}\to\infty$ (Seiberg 96')(Intriligator, Morrison, Seiberg 96').



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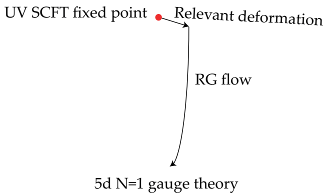
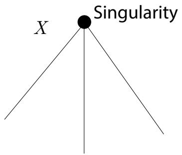


- UV completion of $\mathfrak{su}(2)+N\boldsymbol{F}$: Seiberg $E_{N+1}(N\leq 7)$ theories with $\mathfrak{g}_F=\mathfrak{e}_{N+1}$ (Seiberg 96')

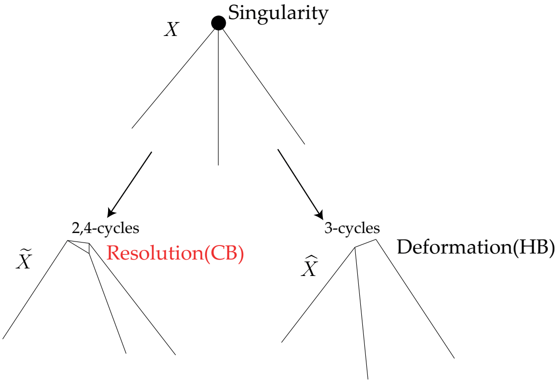
Geometric construction of 5d SCFTs

- 11d M-theory on canonical threefold singularity (Xie, Yau 17')
- Classification of canonical threefold singularities $X \to$ partial classification of 5d $\mathcal{N}=1$ SCFT $\mathcal{T}_{\mathbf{X}}^{\mathrm{5d}}$!



Geometric construction of 5d SCFTs

- Singularity X and the 5d SCFTs are hard to study, one considers instead the desingularization of X:



- Resolution $\leftrightarrow$ Coulomb branch of $\mathcal{T}_{\mathbf{X}}^{5\mathrm{d}}$, $SU(N) \to U(1)^{N-1}$!

CB of 5d SCFT in M-theory

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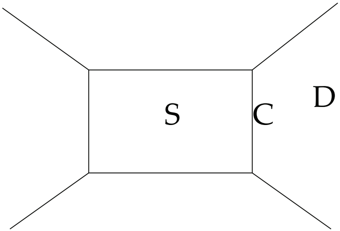
- Fundamental objects: M2, M5 branes, coupled to C_3 gauge field.
- M-theory on the resolved space $\widetilde{X}$, with new cycles:

(1) compact 4-cycles (complex surfaces) S_j $(j=1,\ldots,r)$

(2) non-compact 4-cycles D_α $(\alpha=1,\ldots,f)$

(3) compact 2-cycles (complex curves) C_i



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- M2-brane wrapping compact 2-cycles C_i: Charged particle with charge $q_{i,j} = C_i \cdot S_j$ under $U(1)_j$ gauge group.

1-form symmetry from CB

- From the charge matrix $q_{i,j}$, compute the electric 1-form symmetry $\Gamma_e^{(1)}$ on the CB from Smith decomposition! (Morrison, Schafer-Nameki, Willett 19')
- Technically, the resolution of X is difficult to compute in many cases.

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• An elegant mathematical structure: McKay correspondence.

(1) Conjugacy classes of Γ $\leftrightarrow$ topological cycles of $\widetilde{X}$

(2) Irreps ρ_i of Γ $\leftrightarrow$ vector bundles on $\widetilde{X}$.

1-form symmetry from CB

• Applying a version of 3d McKay correspondence (Ito, Nakajima 00'), proved in (Bridgeland, King, Reid 01')

Take the McKay quiver defined by (π is the natural 3-dim. rep. of Γ)

$$\rho_i \otimes \pi = \bigoplus_i a_{ji}\rho_j \, , \tag{33}$$

define the antisymmetric adjacency matrix $A(\Gamma) = \{A_{ij}\}$ by

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then $A(\Gamma)$ has the same Smith normal form as the larger intersection matrix $M(\Gamma)$:

$$M(\Gamma) = \left(\begin{array}{c|c|c} 0 & 0 & 0 \\ \hline 0 & 0 & (q^T) \\ \hline 0 & q & 0 \end{array}\right) , \tag{35}$$

- From q-matrix, one can compute the 1-form symmetry $\Gamma_e^{(1)}$!

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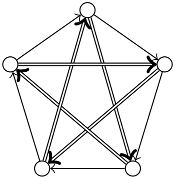

(36)

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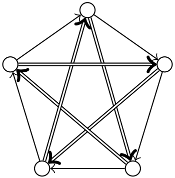

(36)

- The antisymmetric adjacency matrix of the Mckay quiver:

$$A(\mathbb{Z}_5) = \begin{pmatrix} 0 & 1 & 2 & -2 & -1 \\ -1 & 0 & 1 & 2 & -2 \\ -2 & -1 & 0 & 1 & 2 \\ 2 & -2 & -1 & 0 & 1 \\ 1 & 2 & -2 & -1 & 0 \end{pmatrix} \quad (37)$$

1-form symmetry from CB

- The Smith normal form of $A(\mathbb{Z}_5)$:

$$\begin{pmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{38}$$

- Hence $\Gamma_e^{(1)} = \mathbb{Z}_5$
- A full analysis: (Del Zotto, Heckman, Meynet, Moscrop, Zhang 22')

Geometric engineering of 4d $\mathcal{N}=2$ SCFTs

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- E.g. the original Argyres-Douglas theory from special points in the CB of pure $SU(3)$ gauge theory (Argyres, Douglas 95')
- String theory construction: IIB superstring on CY3 singularity (Shapere, Vafa 99')(Xie, Yau 15')(Wang, Xie, Yau, Yau 16')...

Geometric engineering of 4d $\mathcal{N}=2$ SCFTs

- More specifically, we consider the subcases of isolated hypersurface singularity (IHS) $X \in \mathbb{C}^4$:

$$F(x_1,x_2,x_3,x_4)=0\ ,\ \frac{\partial F}{\partial x_i}=0 \text{ if and only if } x_i=0\,. \tag{39}$$

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- Original Argyres-Douglas theory: IIB on

$$F(x)=x_1^2+x_2^2+x_3^2+x_4^3\,. \tag{40}$$

- Generalized Argyres-Douglas theory of type (G,G'):

$$F(x)=f_G(x_1,x_2)+f_{G'}(x_3,x_4)\,. \tag{41}$$

$$f_G(x,y)=\begin{cases} x^2+y^{n+1} & G=A_n \\ x^2y+y^{n-1} & G=D_n \\ x^3+y^4 & G=E_6 \\ x^3+y^3x & G=E_7 \\ x^3+y^5 & G=E_8\,. \end{cases} \tag{42}$$

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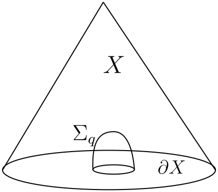

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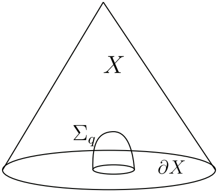


- The "defect group" $\mathfrak{h}^q$ is given by

$$\mathfrak{h}^q = \mathrm{Tor}\left(\frac{H_q(X,\partial X)}{H_q(X)}\right) \hookrightarrow H_{q-1}(\partial X)\,. \tag{43}$$

- $(p+q-1)$-brane wrapping Σ_q → charged under p-form symmetry!

Higher-form symmetry from geometry

- Define the link L_5 of X: take

$$B_\epsilon = \{x \in \mathbb{C}^4 : \|x\| \leq \epsilon\}\,, \tag{44}$$

$S_\epsilon = \partial B_\epsilon \cong S^7$, then the link

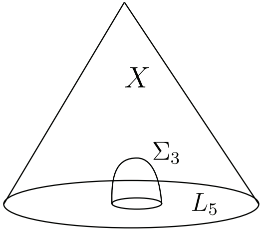
$$L_5 \cong S_\epsilon \cap X\,. \tag{45}$$

If X has a Ricci-flat metric, then L_5 is Sasaki-Einstein. The homology classes (computations see e. g. (Caibar, 99')):

$$\begin{aligned} &H_0(L_5,\mathbb{Z})=\mathbb{Z}, \quad H_1(L_5,\mathbb{Z})=0, \quad H_2(L_5,\mathbb{Z})=\mathbb{Z}^f\oplus(\mathfrak{f})^2 \\ &H_3(L_5,\mathbb{Z})=\mathbb{Z}^f, \quad H_4(L_5,\mathbb{Z})=0, \quad H_5(L_5,\mathbb{Z})=\mathbb{Z} \end{aligned} \tag{46}$$

Higher-form symmetry from geometry

The torsion part $(\mathfrak{f})^2$ of $H_2(L_5,\mathbb{Z})$ gives rise to torsional non-compact 3-cycles Σ_3 in $\mathbf{X}$



• In the 4d $\mathcal{N}=2$ theory, two polarization choices:

(1) "Electric theory": D3 branes wrapping $\Sigma_3 \to$ Wilson line in 4d,
Non-trivial electric 1-form symmetry $\Gamma_e^{(1)}=\mathfrak{f}$

(2) "Magnetic theory": D3 branes wrapping $\Sigma_3 \to$ 't Hooft line in 4d,
Non-trivial magnetic 1-form symmetry $\Gamma_m^{(1)}=\mathfrak{f}$

Higher-form symmetry from geometry

- The first computation of the 1-form symmetry of all generalized AD (G, G') theories! (Closset, Schafer-Nameki, Wang 20')(Del Zotto, Exteberria, Hosseini 20')

$[A_k, D_m]$:

$\Gamma^{(1)} = \mathfrak{f}$	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	D_{14}	D_{15}
A_1	0	0	0	0	0	0	0	0	0	0	0	0
A_2	$\mathbb{Z}_2$	0	0	$\mathbb{Z}_2$	0	0	$\mathbb{Z}_2$	0	0	$\mathbb{Z}_2$	0	0
A_3	0	$\mathbb{Z}_2$	0	0	0	$\mathbb{Z}_2$	0	0	0	$\mathbb{Z}_2$	0	0
A_4	0	0	$\mathbb{Z}_2^2$	0	0	0	0	$\mathbb{Z}_2^2$	0	0	0	0
A_5	0	0	0	$\mathbb{Z}_2^2$	0	0	0	0	0	$\mathbb{Z}_2^2$	0	0
A_6	0	0	0	0	$\mathbb{Z}_2^3$	0	0	0	0	0	0	$\mathbb{Z}_2^3$
A_7	0	0	0	0	0	$\mathbb{Z}_2^3$	0	0	0	0	0	0
A_8	$\mathbb{Z}_2$	0	0	$\mathbb{Z}_2$	0	0	$\mathbb{Z}_2^4$	0	0	$\mathbb{Z}_2$	0	0

5d/4d correspondence

• Similarly, one can also consider the 5d $\mathcal{N}=1$ theory $\mathcal{T}_{\mathbf{X}}^{\mathrm{5d}}$ from M-theory on the same X, related to $\mathscr{T}_{\mathbf{X}}^{\mathrm{4d}}$ by 5d/4d correspondence (Closset, Schafer-Nameki, Wang 20', 21')(Closset, Giacomelli, Schafer-Nameki, Wang 20')

• In $\mathcal{T}_{\mathbf{X}}^{\mathrm{5d}}$, two choices:

(1) M2 brane wrapping $\Sigma_3 \to$ Electric 0-form symmetry $\Gamma_e^{(0)} = \mathfrak{f}$, corresponding to the 4d magnetic theory

(2) M5 brane wrapping $\Sigma_3 \to$ Magnetic 3-form symmetry $\Gamma_m^{(3)} = \mathfrak{f}$, corresponding to the 4d electric theory

• $\mathcal{T}_{\mathbf{X}}^{\mathrm{5d}}$ from IHS X does not have any 1-form symmetry!

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- For $p = 0$ this does not hold, and the 0-form global symmetry can be non-abelian.