Final state interaction of antiproton-proton in hadron decays

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Outline

$\bar{p}p$ scattering measurements at LEAR

Measurement	Incoming $\bar{p}$ momentum (MeV/c)	Experiment
integrated cross sections		
$\sigma_{tot}(\bar{p}p)$	222-599 (74 momenta)	PS172
	181,219,239,261,287,505,590	PS173
$\sigma_{ann}(\bar{p}p)$	177-588 (53 momenta)	PS173
	38-174 (14 momenta)	PS201
$\bar{p}p$ elastic scattering		
ρ = Re $f(0)$/Im $f(0)$	233,272,550,757,1077	PS172
	181,219,239,261,287,505,590	PS173
$d\sigma/d\Omega$	679-1550 (13 momenta)	PS172
	181,287,505,590	PS173
	439,544,697	PS198
A_{0n}	497-1550 (15 momenta)	PS172
	439,544,697	PS173
D_{0n0n}	679-1501 (10 momenta)	PS172
$\bar{p}p$ charge exchange		
$d\sigma/d\Omega$	181-595 (several momenta)	PS173
	546,656,693,767,875,1083,1186,1287	PS199
	601.5,1202	PS206
A_{0n}	546,656,767,875,979,1083,1186,1287	PS199
D_{0n0n}	546,875	PS199
K_{n00n}	875	PS199

Revival of antinucleon-nucleon physics

- Near-threshold enhancement in the $\bar{p}p$ invariant-mass spectrum:
  $J/\psi \to \gamma\bar{p}p \;\; \to$ BES collaboration (2003, 2012, 2016)
  $B^+ \to K^+\bar{p}p \;\; \to$ BaBar collaboration (2005)
  $e^+e^- \to \bar{p}p \;\; \to$ FENICE (1998), BaBar (2006,13), BESIII (2019)
  ($\bar{p}p \to e^+e^- \;\; \to$ PS170 (1994))

  $\Rightarrow$ new resonances, $\bar{p}p$ bound states, exotic glueball states ?

- Facility for Antiproton and Ion Research (FAIR)
  - PANDA Project
    Study of the interactions between antiprotons and fixed target protons and nuclei in the momentum range of 1.5-15 GeV/c using the high energy storage ring HESR
  - PAX Collaboration
    experiments with a polarized antiproton beam
    transversity distribution of the valence quarks in the proton
    $\bar{N}N$ double-spin observables

$\bar{N}N$ partial-wave analysis

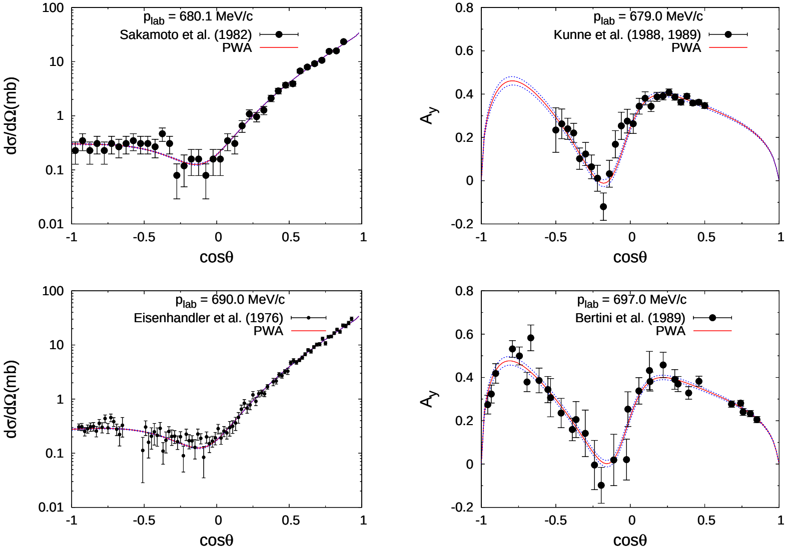
R. Timmermans et al., PRC 50 (1994) 48

- use a meson-exchange potential for the long-range part
- apply a strong absorption at short distances (boundary condition) in each individual partial wave ($\approx$ 1.2 fm)
- 30 parameters, fitted to a selection of $\bar{N}N$ data (3646!)
- However, resulting amplitudes are not explicitly given:
  no proper assessment of the uncertainties (statistical errors)
  phase-shift parameters for the $^1S_0$ and $^1P_1$ partial waves are not pinned down accurately

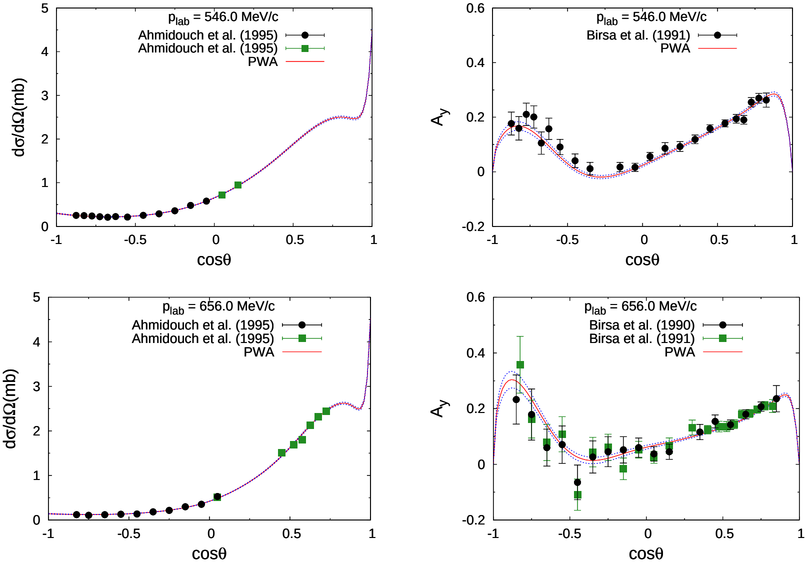
D. Zhou and R. Timmermans, PRC 86 (2012) 044003

- use now potential where the long-range part is fixed from chiral EFT (N^2LO)
- somewhat larger number of $\bar{N}N$ data (3749!)
- now, resulting amplitudes and phase shifts are given!
- lowest momentum: $p_{lab} = 100$ MeV/c ($T_{lab} = 5.3$ MeV)
- highest total angular momentum: $J = 4$

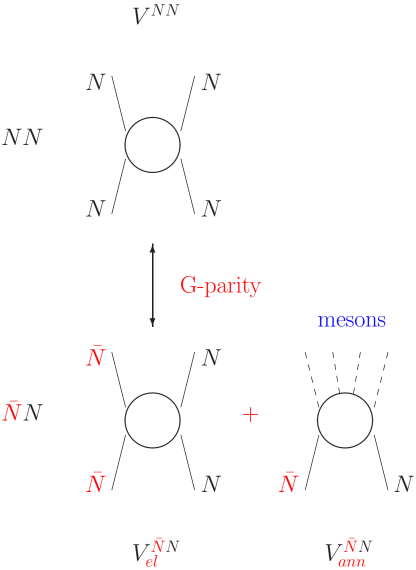
$\bar{N}N$ PWA: $\bar{p}p \to \bar{p}p$



$\bar{N}N$ PWA $\bar{p}p \to \bar{n}n$



The $\bar{N}N$ interaction



Traditional approach: meson-exchange

I) $V^{\bar{N}N}_{el}$ ... derived from an *NN* potential via G-parity

(Charge conjugation plus 180^o rotation around the *y* axis in isospin space)

$\Rightarrow$

$$
\begin{aligned}
V^{\bar{N}N}(\pi,\ \omega) &= -V^{NN}(\pi,\ \omega) \qquad \text{odd G} - \text{parity} \\
V^{\bar{N}N}(\sigma,\ \rho) &= +V^{NN}(\sigma,\ \rho) \qquad \text{even G} - \text{parity} \\
&\ldots
\end{aligned}
$$

II) $V^{\bar{N}N}_{ann}$

employ a phenomenological optical potential, e.g.

$V_{opt}(r) = (U_0 + iW_0)\, e^{-r^2/(2a^2)}$

with parameters U_0, W_0, a fixed by a fit to $\bar{N}N$ data

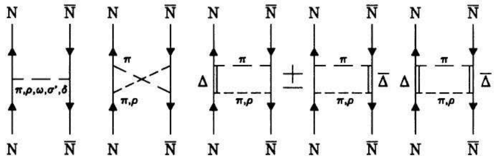
examples: Dover/Richard (1980,1982), Paris (1982,...,2009), Nijmegen (1984), Jülich (1991,1995), ...

Meson-exchange: Jülich $\bar{N}N$ model

I) V_{el}

starting point: Bonn NN potential

(R. Machleidt, K. Holinde, C. Elster, Phys. Rep. 149 (1986) 1)



(G-parity: Charge conjugation plus 180^o rotation around the y axis in isospin space)

$\Rightarrow$

$$
\begin{aligned}
V_{\bar{N}N}(\pi,\ \omega) &= -V_{NN}(\pi,\ \omega)\ \text{– odd G – parity} \\
V_{\bar{N}N}(\sigma,\ \rho) &= +V_{NN}(\sigma,\ \rho)\ \text{– even G – parity}
\end{aligned}
$$

well defined over whole range

no modification of short-range part is done

II) V_{ann}

- phenomenological optical potential (A)

  $V_{opt}(r) = (U_0 + iW_0)e^{-r^2/(2a^2)}$

  (state- and energy independent!)

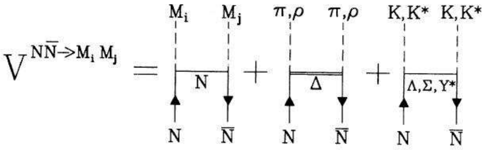
  Fit to $\bar{N}N$ data [σ_{tot}, σ_{el}, σ_{ann}] up to $p_{lab} \approx$ 800 MeV/c ($T_{lab} \approx$ 300 MeV)

  best fit:

  $a = 0.36$ fm, $U_0 = -0.63$ GeV, $W_0 = -4.567$ GeV

The Jülich $\bar{N}N$ model

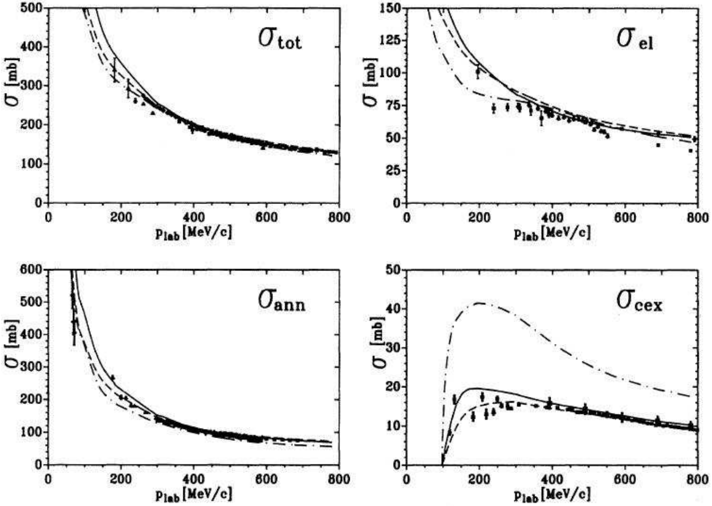
- microscopic annihilation model (for 2-meson channels) (D)



$M_{i,j} = \pi, \eta, \rho, \omega, f_0, a_0, f_1, a_1, f_2, a_2$

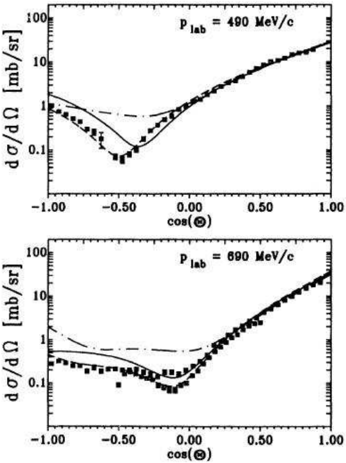
- T. Hippchen et al., PRC 44 (1991) 1323; V. Mull et al., PRC 44 (1991) 1337
- V. Mull & K. Holinde, PRC 51 (1995) 2360

$\bar{p}p$ integrated cross sections



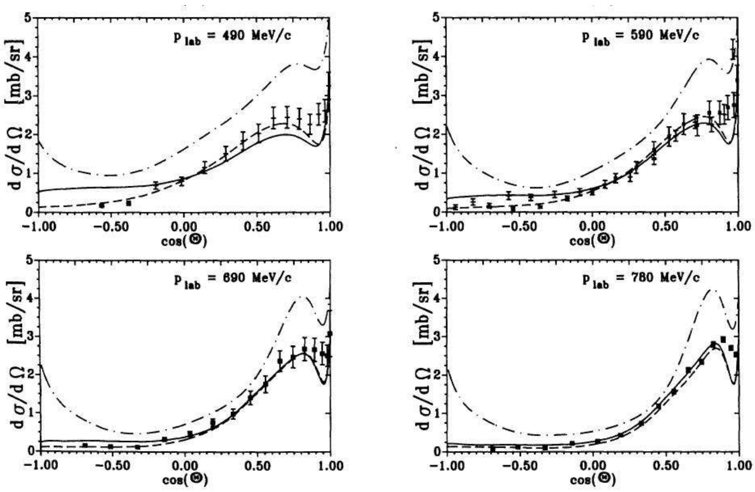
—— D (microscopic) – – A (phenomenological)

$\bar{p}p$ differential cross sections



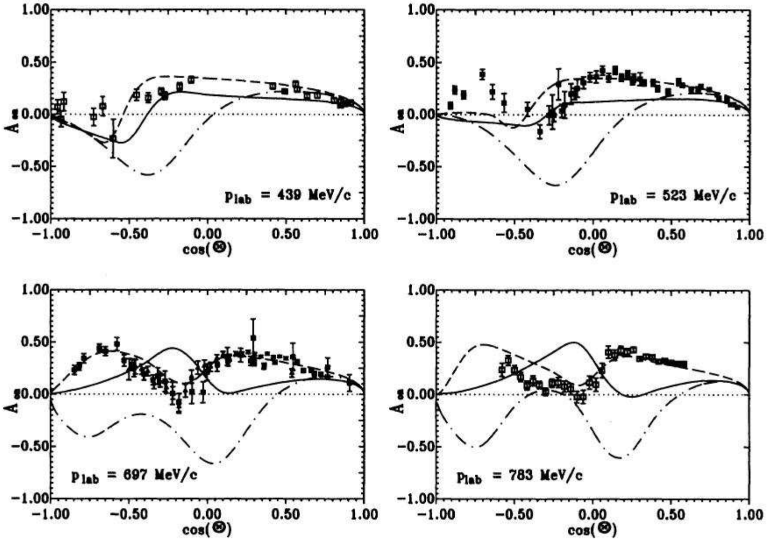
—— D (microscopic) – – A (phenomenological)

$\bar{p}p \to \bar{n}n$ differential cross sections



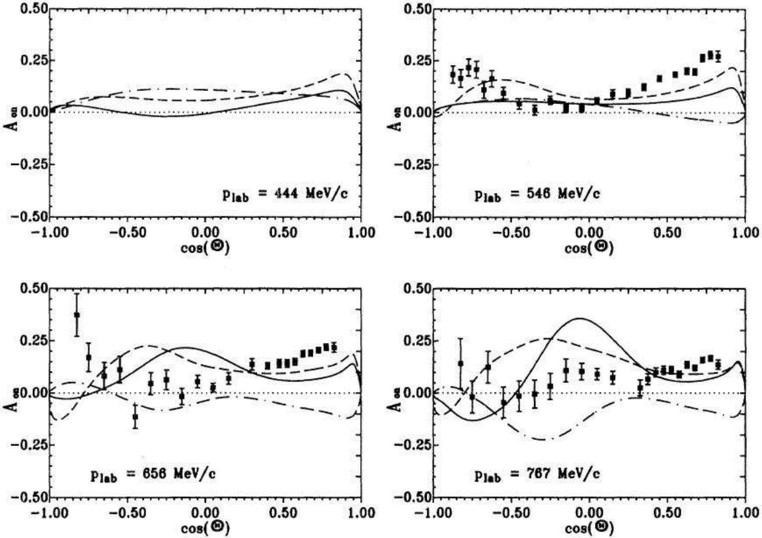
—— D (microscopic) – – A (phenomenological)

$\bar{p}p$ polarizations



—— D (microscopic) – – A (phenomenological)

$\bar{p}p \to \bar{n}n$ polarizations



—— D (microscopic) – – A (phenomenological)

Chiral Effective Field Theory

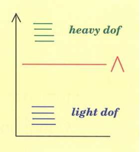
S. Weinberg, Physica 96A (1979) 327; PLB 251 (1990) 288

- Respect/exploit symmetries of the underlying QCD
- Different scales: Separation of low and high energy dynamics

  • low-energy dynamics is described in terms of the relevant degrees of freedom (e.g. pions)

  • high-energy dynamics remains unresolved

  $\rightarrow$ absorbed into contact terms



(U.-G. Meißner)

- Power counting

  Expand interaction in powers $Q^n = (q/\Lambda)^n$, $n = 0, 1, 2, ...$

  q ... soft scale (nucleon three-momentum, pion four-momentum, pion mass)

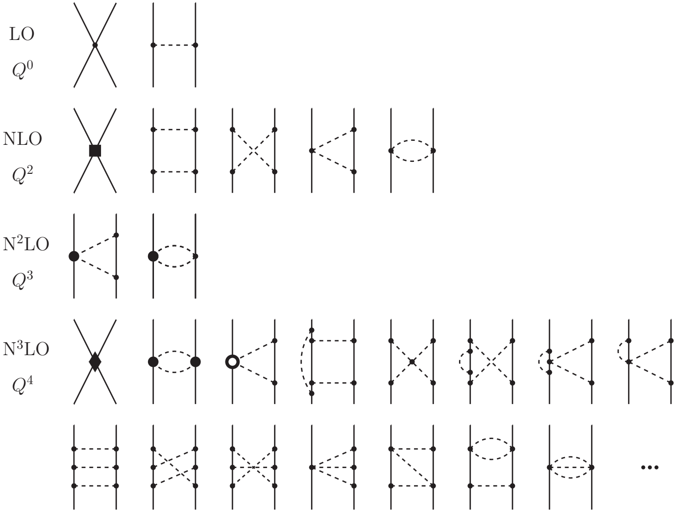
  Λ ... hard scale ($\approx$ 1 GeV ... m_ρ, M_N)

  $\Rightarrow$ systematic improvement of results by going to higher order (power)

  $\Rightarrow$ estimation of theoretical uncertainty

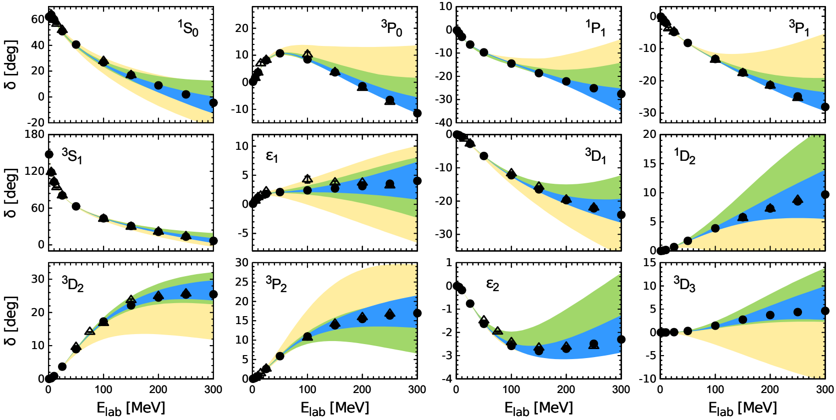
  expected to work for $q < \Lambda$

NN interaction in chiral effective field theory



- $4N$ contact terms involve low-energy constants (LECs) ... parameterize unresolved short-range physics

⇒ need to be fixed by fit to experiments



E. Epelbaum, H. Krebs, Ulf-G. Meißner (EKM), EPJA 51 (2015) 53

—— LO, —— NLO, —— N^3LO

(see Reinert, Epelbaum, Krebs, EPJA 54 (2018) 86, for present status (N^4LO, N^4LO+))

The $\bar{N}N$ interaction in chiral EFT

- $V^{NN} = \quad V_{1\pi} + V_{2\pi} + V_{3\pi} + ... + V_{cont}$
- $V^{\bar{N}N}_{el} = -V_{1\pi} + V_{2\pi} - V_{3\pi} + ... + V_{cont}$
- $V^{\bar{N}N}_{ann} = \sum_X V^{\bar{N}N\to X} \qquad X \hat{=} \pi, 2\pi, 3\pi, 4\pi, ...$

- $V_{1\pi}$, $V_{2\pi}$, ... can be taken over from chiral EFT studies of the *NN* interaction
- Xian-Wei Kang, J.H., Ulf-G. Meißner, JHEP 02 (2014) 113 (N^2LO)
  starting point: *NN* interaction by Epelbaum, Glöckle, Meißner, NPA 747 (2005) 362
- Ling-Yun Dai, J.H., Ulf-G. Meißner, JHEP 07 (2017) 078 (N^3LO)
  starting point: *NN* interaction by Epelbaum, Krebs, Meißner, EPJA 51 (2015) 53

- V_{cont} ... same structure as in *NN*: $V_{cont} = \tilde{C} + C\,(p^2 + p'^2) + ...$

However, now the LECs have to be determined by a fit to $\bar{N}N$ data (phase shifts, inelasticites)!

no Pauli principle $\rightarrow$ more partial waves, more contact terms

- $V^{\bar{N}N}_{ann}$ has no counterpart in *NN*

empirical information: annihilation is short-ranged and practically energy-independent

$$V^{\bar{N}N}_{ann;eff} = \sum_X V^{\bar{N}N\to X} G^0_X V^{X\to\bar{N}N}, \quad V^{\bar{N}N\to X}(p, p_X) \approx p^L\,(a + b\,p^2 + ...); \quad p_X \approx \text{const.}$$

regularized Lippmann-Schwinger equation

$$T^{L'L}(p',p) = V^{L'L}(p',p) + \sum_{L''} \int_0^\infty \frac{dp''p''^2}{(2\pi)^3} \frac{V^{L'L''}(p',p'')\, T^{L''L}(p'',p)}{2E_p - 2E_{p''} + i\eta}$$

- $\bar{N}N$ potential up to N^{2}LO (Kang et al., 2014)
  employ the non-local regularization scheme of EGM (NPA 747 (2005) 362)
  $(V(p',p) \to f^\Lambda(p')\, V(p',p)\, f^\Lambda(p);\;\; f^\Lambda(p) = e^{-(p/\Lambda)^4})$

- $\bar{N}N$ potential up to N^{3}LO (Dai et al., 2017)
  employ the regularization scheme of EKM (EPJA 51 (2015) 53)
  $(V_\pi(q) \to V_\pi(r) \times f_R(r) \to V_\pi^{reg}(q);\;\; f_R(r) = \left[1 - exp(-r^2/R^2)\right]^6)$
  $(V_{cont}\colon V(p',p) \to f^\Lambda(p')\, V(p',p)\, f^\Lambda(p);\;\; f^\Lambda(p) = e^{-(p/\Lambda)^2}$ $\quad R = 0.8\text{-}1.2$ fm; $\Lambda = 2/R)$

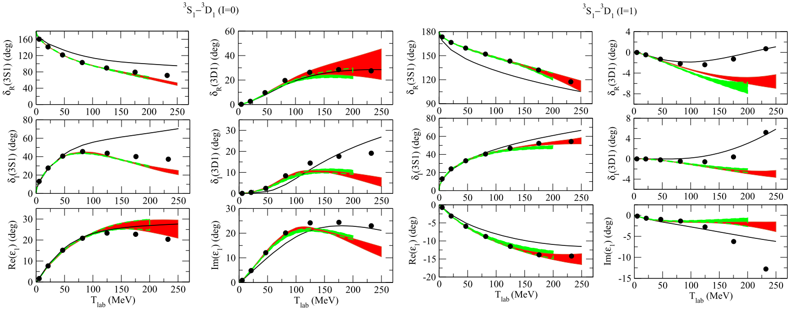
- Fit to phase shifts and inelasticity parameters in the isospin basis
  (D. Zhou, R.G.E. Timmermans, PRC 86 (2012) 044003)

- Calculation of observables is done in particle basis:
  - ⋆ Coulomb interaction in the $\bar{p}p$ channel is included
  - ⋆ the physical masses of p and n are used

Results for $^3S_1$-$^3D_1$ phase shifts (N^2LO)

Xian-Wei Kang, J.H., Ulf-G. Meißner, JHEP 02 (2014) 113 (N^2LO)

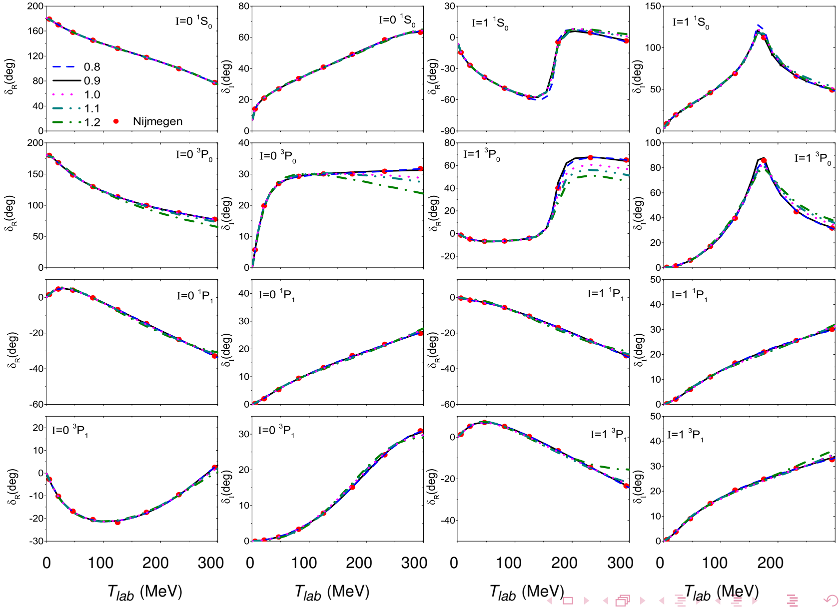
(bands represent cutoff variations!)



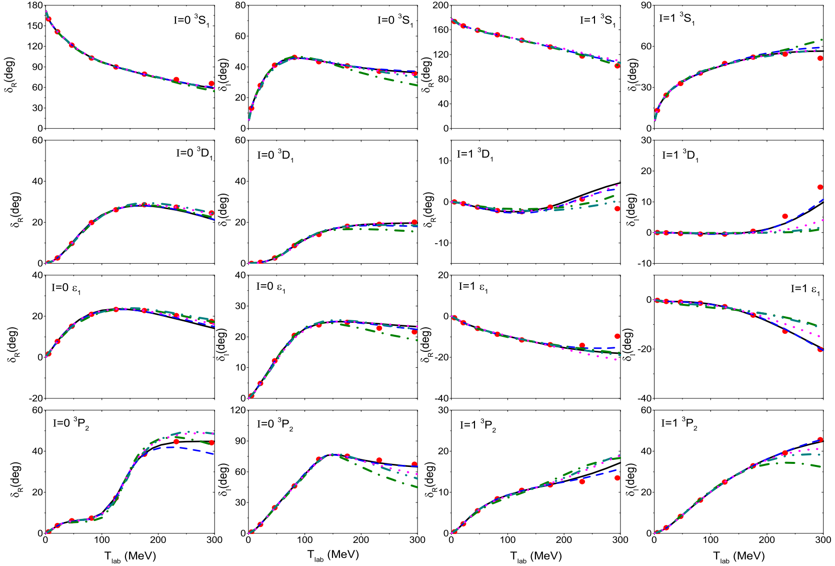
Jülich A (OBE); N2LO; NLO

- PWA of Zhou, Timmermans, PRC 86 (2012) 044003

$\bar{N}N$ phase shifts (Dai et al., 2017; N^{3}LO)



$\bar{N}N$ phase shifts (N^{3}LO)



Uncertainty

- Uncertainty for a given observable $X(p)$:
  (Epelbaum, Krebs, Meißner, EPJA 51 (2015) 53)
  (S. Binder et al. [LENPIC coll.], PRC 93 (2016) 044002)
- estimate uncertainty via
  - the expected size of higher-order corrections
  - the actual size of higher-order corrections

$$\Delta X^{LO} = Q^2|X^{LO}| \qquad (X^{NLO} \approx Q^2 X^{LO})$$

$$\Delta X^{NLO} = \max\left(Q^3|X^{LO}|, Q^1|\delta X^{NLO}|\right); \quad \delta X^{NLO} = X^{NLO} - X^{LO}$$

$$\Delta X^{N^2LO} = \max\left(Q^4|X^{LO}|, Q^2|\delta X^{NLO}|, Q^1|\delta X^{N^2LO}|\right); \quad \delta X^{N^2LO} = X^{N^2LO} - X^{NLO}$$

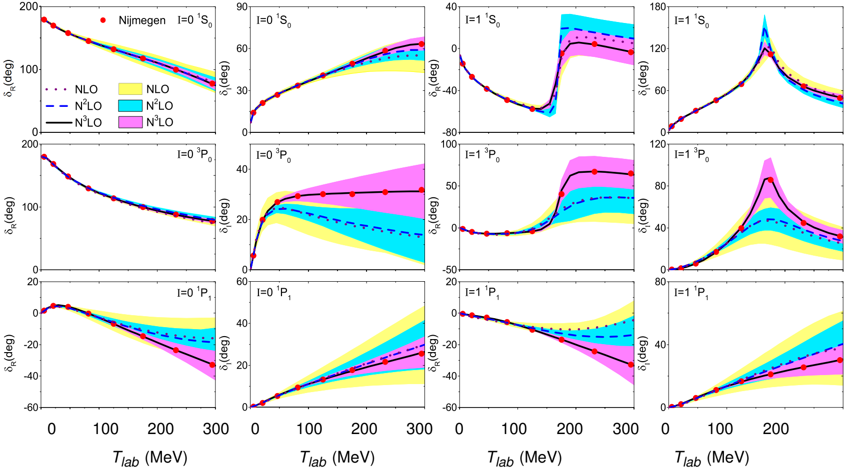
$$\Delta X^{N^3LO} = \max\left(Q^5|X^{LO}|, Q^3|\delta X^{NLO}|, Q^2|\delta X^{N^2LO}|, Q^1|\delta X^{N^3LO}|\right); \quad \delta X^{N^3LO} = X^{N^3LO} - X^{N^2LO}$$

- expansion parameter Q is defined by

$$Q = \max\left(\frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right); \qquad p \ldots \bar{N}N \text{ on}-\text{shell momentum}$$

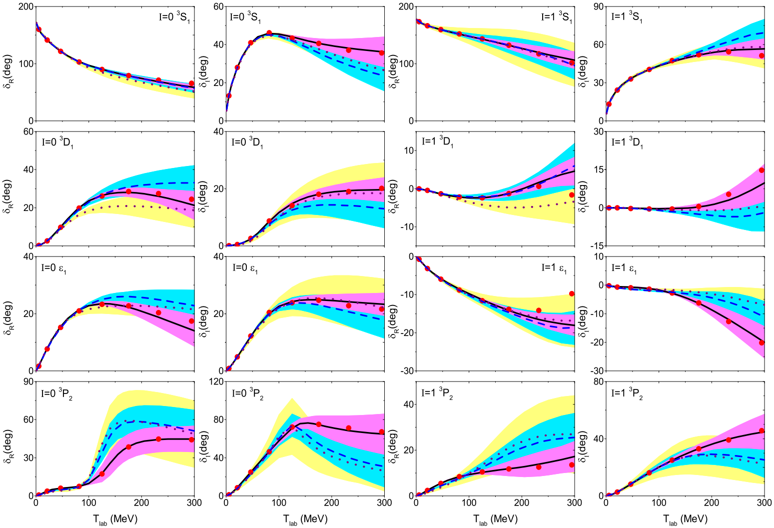
Λ_b ... breakdown scale $\rightarrow$ $\Lambda_b = 500 - 600$ MeV [for $R = 0.8 - 1.2$ fm] (EKM, 2015)

$\bar{N}N$ phase shifts

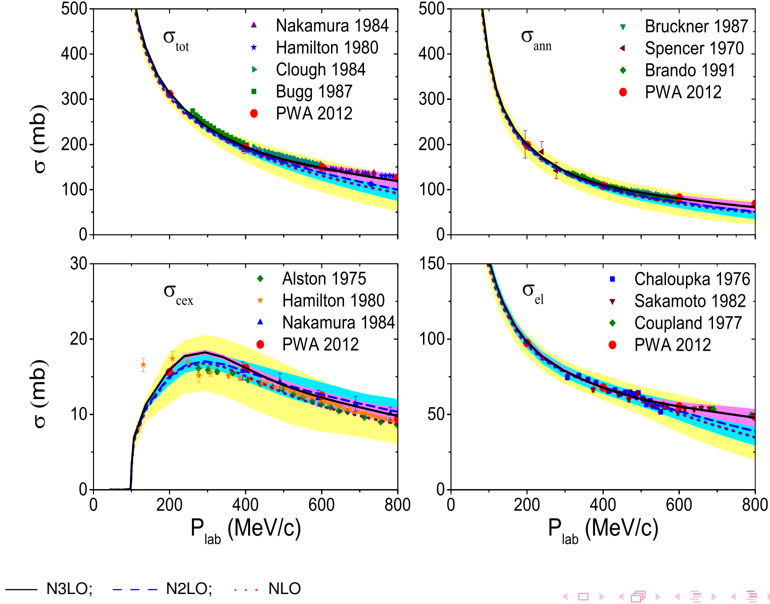


N3LO; N2LO; NLO (cutoff $R = 0.9$ fm)

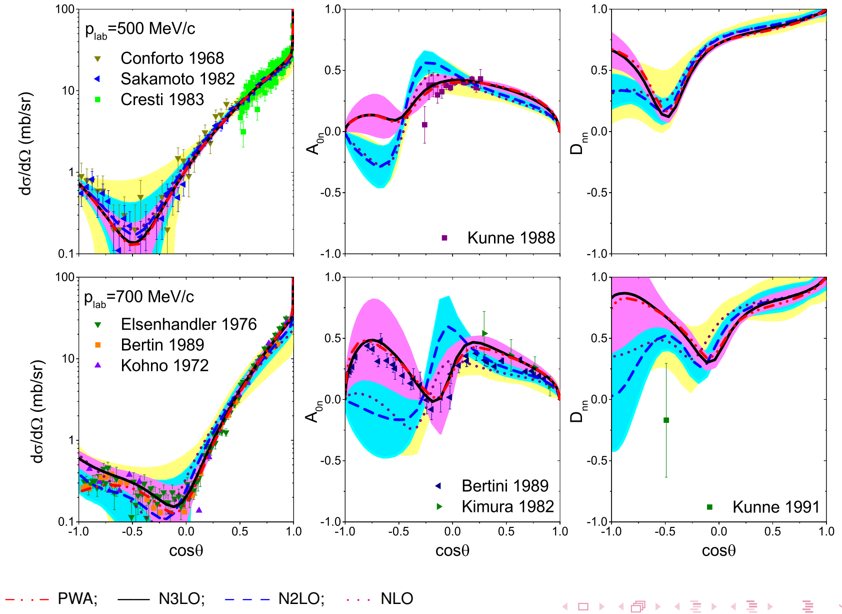
$\bar{N}N$ phase shifts



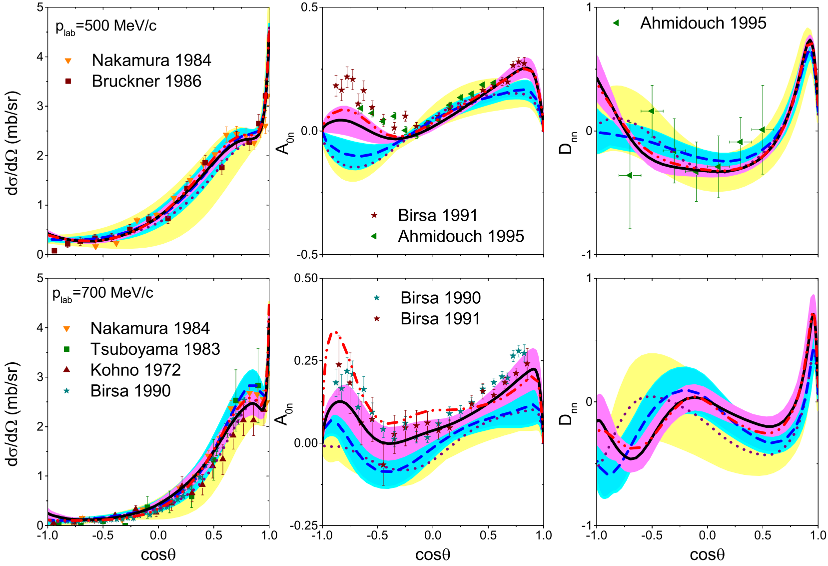
$\bar{p}p$ integrated cross sections



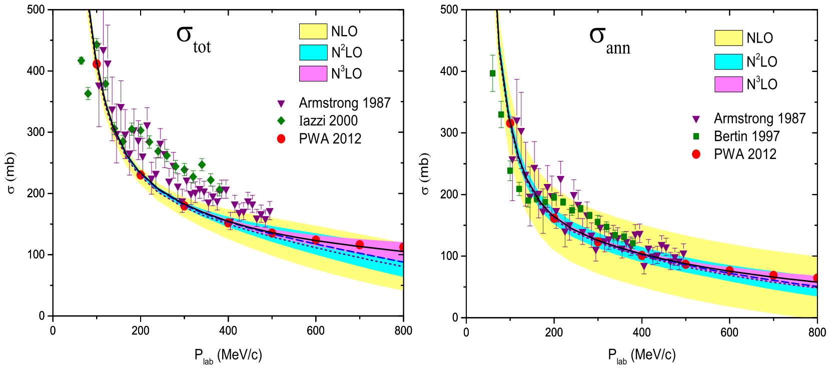
$\bar{p}p \to \bar{p}p$



$\bar{p}p \to \bar{n}n$



$\bar{n}p$ cross sections



$\bar{N}N$ interaction in the final state

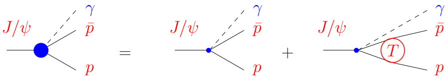
Treatment of the final-state interaction

- Migdal-Watson: $A \approx N \cdot A_0 \cdot T_{\bar{p}p}$

A_0 ... elementary production/reaction amplitude, N ... normalization factor

works reliably only for interactions with a rather large scattering length,
e.g. $^1S_0$ $np \to$ a = -23.5 fm
A. Gasparyan et al., PRC 72 (2005) 034006

- DWBA: $A = A_0 + A_0 G_{\bar{p}p} T_{\bar{p}p}$



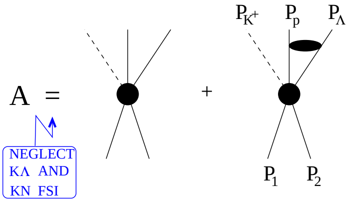
for a short-ranged production mechanism A_0 is only weakly momentum (energy) dependent

- Jost-function approach: $A \approx A_0[1 + G_{\bar{p}p} T_{\bar{p}p}] = A_0 \psi_q^{(-)*}(0) = A_0 J^{-1}(-q)$

(may be valid for excess energies $\lesssim$ 50 MeV)

MW used in the initial investigation: A. Sibirtsev et al., PRD 71 (2005) 054010
DWBA used in refined study: X.-W. Kang et al., PRD 91 (2015) 074003

Dispersion relations (Muskhelishvili, Omnes)



$$t=(P_K-P_1)^2$$
$$m^2=(P_p-P_\Lambda)^2$$
$$s=(P_1-P_2)^2$$

Assume point-like production operator (large momentum transfer)

- is practically constant with respect to variations in $m^2_{p\Lambda}$

Dispersion relation technique (A. Gasparyan et al., PRC 69 (2004) 034006)

$$A(s,t,m^2) = \exp\left[\frac{1}{\pi}\int_{m_0^2}^{\infty}\frac{\delta_{p\Lambda}(m'^2)}{m'^2-m^2-i0}dm'^2\right]\times\Phi(s,t,m^2)$$

model independent!!

$\Rightarrow$ theoretical uncertainty of extracted scattering length: ± 0.3 fm

However, valid only for elastic scattering, single-channel systems

State dependence of final-state interaction

Which $\bar{p}p$ partial waves can occur near threshold?

J/ψ $(\psi'(3686))$: $I^G(J^{PC}) = 0^-(1^{--})$

$J/\psi \to \gamma\bar{p}p$: J^{PC} is conserved

$\Rightarrow {}^1S_0$ (0^{-+}), ${}^3P_0$ (0^{++}), ...

BESIII Collaboration, PRL 108 (2012) 112003: PWA $\to 0^{-+}$

$J/\psi \to \omega\bar{p}p$, $\phi\bar{p}p$: I^G, J^{PC} is conserved

$\Rightarrow {}^{11}S_0$, ${}^{13}P_0$, ...

$J/\psi \to \pi^0\bar{p}p$: I^G, J^{PC} is conserved

$\Rightarrow {}^{33}S_1$, ${}^{33}P_1$, ...

$J/\psi \to \eta\bar{p}p$: I^G, J^{PC} is conserved

$\Rightarrow {}^{13}S_1$, ${}^{13}P_1$, ...

$B^+ \to K^+\bar{p}p$, $B \to D\bar{p}p$

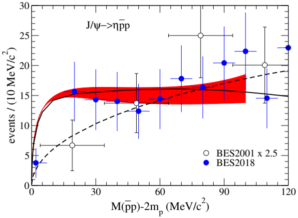
Parity is not conserved $\to$ more partial waves possible

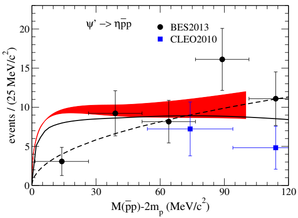
${}^{(2I+1)(2S+1)}L_J$

$\bar{p}p$ in final state

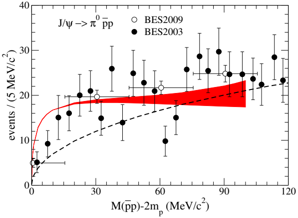
X.-W. Kang, JH, U.-G. Meißner, PRD 91 (2015) 074003 (N^{2}LO)

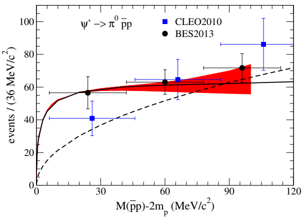
$\bar{N}N$ FSI in $^3S_1$ state is relevant (bands represent cutoff variations!)





$J/\psi \to \eta\bar{p}p$ (left)
$\psi' \to \eta\bar{p}p$ (right)



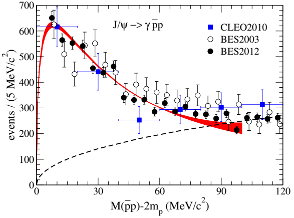


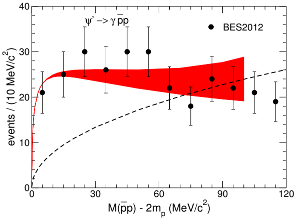
$J/\psi \to \pi^0\bar{p}p$ (left)
$\psi' \to \pi^0\bar{p}p$ (right)

$\bar{p}p$ in final state

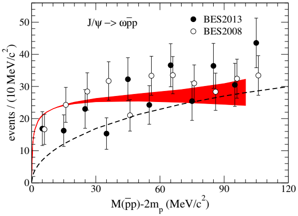
X.-W. Kang, JH, U.-G. Meißner, PRD 91 (2015) 074003 (N^2LO)

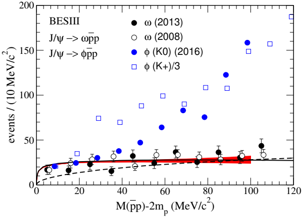
$\bar{N}N$ FSI in ${}^1S_0$ state is relevant (bands represent cutoff variations!)





$J/\psi \to \gamma\bar{p}p$ (left)
$\psi' \to \gamma\bar{p}p$ (right)



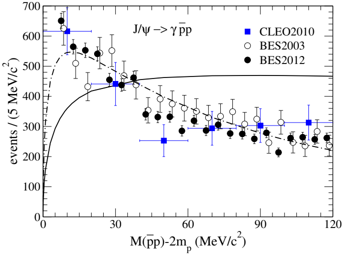


$J/\psi \to \omega\bar{p}p$ (left)
$J/\psi \to \phi\bar{p}p$ (right)

Migdal-Watson versus DWBA

Results for Jülich meson-exchange $\bar{N}N$ potential A(OBE):

– · – ... in Migdal-Watson approach (A. Sibirtsev (2005))
——- ... in DWBA (X.-W. Kang (2015); L.-Y. Dai (2018))



⇒ Migdal-Watson approach should not be trusted!

$\bar{p}p$ in final state

- in principle, same $\bar{p}p$ FSI effects in the same final states
- no large effect in $J/\psi \to \omega\bar{p}p \Rightarrow$ no large effect in $I = 0$

  caveat: $\bar{p}p$ could be produced predominantly in the $^{13}P_0$ state in the decay into $\omega\bar{p}p$ ($\phi\bar{p}p$) but in the $^{11}S_0$ in case of $\gamma\bar{p}p$
- no large effect in $\psi' \to \gamma\bar{p}p$
  $\Rightarrow$ different isospin combinations must be relevant in J/ψ decay
- $\bar{p}p$ FSI predicted by the meson-exchange models, but also the one suggested by the $\bar{p}p$ PWA (χEFT potentials) does not reproduce the $J/\psi \to \gamma\bar{p}p$ invariant mass spectrum

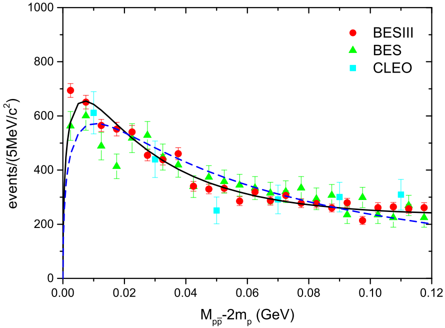
  however: no $\bar{p}p$ scattering data near threshold
  contribution of the (spin-singlet) $^1S_0$ partial wave is small
  $\Rightarrow$ possibly the interaction in the $^1S_0$ is not well constrained

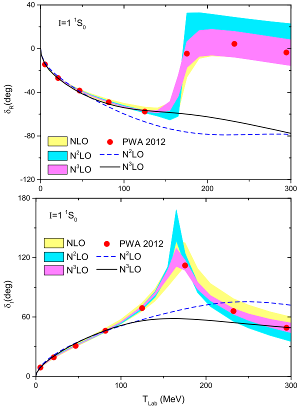
  $\Rightarrow$ it is possible to readjust the $^{31}S_0$ ($I = 1$) interaction so that the $\gamma\bar{p}p$ data are reproduced - without spoiling the $\bar{p}p$ results!

  - X.-W. Kang, JH, U.-G. Meißner, PRD 91 (2015) 074003
  - L.-Y. Dai, JH, U.-G. Meißner, PRD 98 (2018) 014005

$X(1835)$: $J/\psi \to \gamma \bar{p}p$

L.-Y. Dai, JH, U.-G. Meißner, PRD 98 (2018) 014005





refitted $I = 1$ $^1S_0$: — N^{3}LO - - - N^{2}LO

reproduces the $\bar{p}p$ data with same quality as the original χEFT potentials

- BESIII (M. Ablikim et al.), PRL 117 (2016) 042002

Evidence for $\bar{N}N$ bound states?

	N^2LO [1]	N^3LO [2]	El-Bennich [3]	Entem [4]	Milstein [5]
$^{11}S_0$	-	-	-4.8-i 26	-	22-i 33
$^{31}S_0$	-37-i 47*	-51-i 41* (-2.1-i 94)[†] *	-	-	-
$^{13}S_1$	$+(5.6 \cdots 7.7) - \mathrm{i}\,(49.2 \cdots 60.5)$	-	-	-	-
$^{11}P_1$	-	-	1877±i 13	-	-
$^{13}P_0$	$-(3.7 \cdots 0.2) - \mathrm{i}\,(22.0 \cdots 26.4)$	?	1876±i 5	1895±i 17	-
$^{33}P_0$	-	-	1871±i 11	-	-
$^{13}P_1$	-	-	1872±i 10	-	-
$^{33}P_1$	-	-	-4.5-i 9	-	-

$M_p + M_{\bar{p}} = 1876.574$ MeV, $\quad E_B$, M_R in MeV

[1] Xian-Wei Kang et al., JHEP 02 (2014) 113 $\quad$ * needed for $J/\psi \to \gamma\bar{p}p$
[2] Ling-Yun Dai, JHEP 07 (2017) 078 ([†] N^2LO) $\quad$ * needed for $J/\psi \to \gamma\bar{p}p$
[3] B. El-Bennich et al., PRC 79 (2009) 054001
[4] D.R. Entem & F. Fernández, PRC 73 (2006) 045214
[5] A.I. Milstein & S.G. Salnikov, NPA 966 (2017) 54

BES 2005; BESIII 2011,2016: $X(1835)$ $\quad (J^{PC} = 0^{-+}, I = 0)$

seen in $J/\psi \to \gamma\pi^+\pi^-\eta'$: $M_R = 1836.5 \pm 3^{+5.6}_{-2.1}$ MeV, $\Gamma = 190 \pm 9^{+38}_{-36}$ MeV

evidence (?) in $J/\psi \to \gamma\bar{p}p$: $M_R = 1832^{+19}_{-5}{}^{+18}_{-17}$ MeV, $\Gamma < 76$ MeV (90 % C.L.)

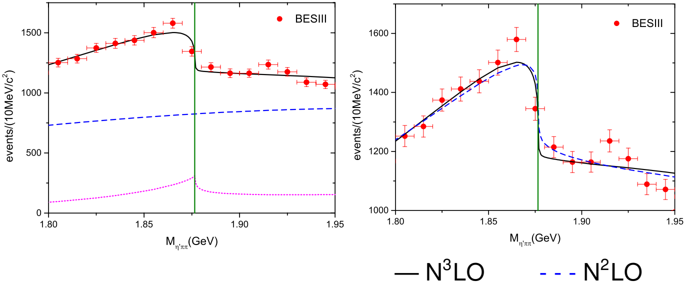
L.-Y. Dai, JH, U.-G. Meißner, PRD 98 (2018) 014005

$$A_{J/\psi\to\gamma\pi^+\pi^-\eta'} = A^0_{J/\psi\to\gamma\pi^+\pi^-\eta'} + A_{J/\psi\to\gamma\bar{N}N} G^0_{\bar{N}N} V_{\bar{N}N\to\pi^+\pi^-\eta'}$$

$V_{\bar{N}N\to\pi^+\pi^-\eta'} \propto \tilde{C} + C\, p^2_{\bar{N}N}$ constrained from BR($\bar{p}p \to \pi^+\pi^-\eta'$)

$A^0_{J/\psi\to\gamma\pi^+\pi^-\eta'} \propto \tilde{C}_{\eta'} + C_{\eta'}\, Q_{\pi^+\pi^-\eta'}$ smooth background: $\tilde{C}_{\eta'}$, $C_{\eta'}$ ... free parameters

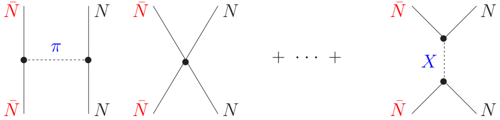
$J/\psi \to \gamma\pi^+\pi^-\eta'$



- BESIII (M. Ablikim et al.), PRL 117 (2016) 042002

What about a genuine resonance?

A genuine $X(1835)$ resonance would contribute to the $\bar{N}N$ interaction too!



$$V^{\bar{N}N} \Rightarrow V^{\bar{N}N} + \gamma_0^{\bar{N}N} \frac{1}{E_{\bar{N}} + E_N - m_X^0} \gamma_0^{\bar{N}N}$$

m_X^0 ... bare mass of a possible $X(1835)$ resonance

$\gamma_0^{\bar{N}N}$ ... bare $\bar{N}NX$ vertex

one needs to determine m_X^0 and the parameters of the bare $\bar{N}NX$ vertex in a combined fit to $\bar{N}N$ data and the $J/\psi \to \gamma\bar{p}p$ invariant mass spectrum

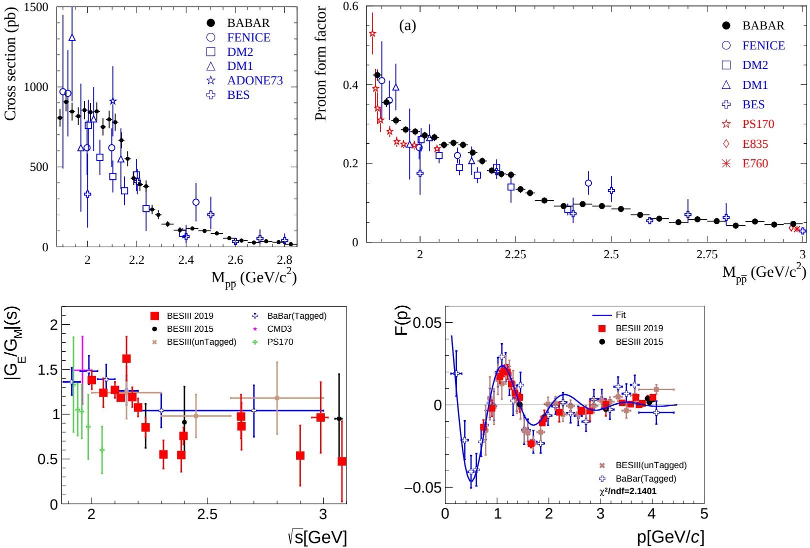
is done by us for

$e^+e^- \to \Lambda_c^+\bar{\Lambda}_c^-$ $(X(4630))$ $\to$ L.-Y.Dai, JH, U.-G. Meißner, PRD 96 (2017) 116001

$\bar{p}p \to \bar{D}D$ $(\psi(3770))$ $\to$ JH, G. Krein, PRD 91 (2015) 114022

essential difference: resonances are above threshold!

The reaction $e^+e^- \to \bar{p}p$: experimental situation



BaBar: J.P. Lees et al., PRD 87 (2013) 092005, BESIII: M. Ablikim et al., PRL 124 (2020) 042001

theory: Y.-H. Lin, H.-W. Hammer, U.-G. Meißner, PRL 128 (2022) 052002

The reaction $e^+e^- \to \bar{p}p$: formulae

$$\sigma_{e^+e^-\to\bar{p}p} = \frac{4\pi\alpha^2\beta}{3s}\, C_p(s)\, \left[|G_M(s)|^2 + \frac{2M_p^2}{s}\,|G_E(s)|^2\right]$$

$$|G_{\rm eff}(s)| = \sqrt{\frac{\sigma_{e^+e^-\to\bar{p}p}(s)}{\frac{4\pi\alpha^2\beta}{3s}\, C_p(s)\left[1+\frac{2M_p^2}{s}\right]}}$$

$\sqrt{s} = M_{\bar{p}p} = q^2, \quad \beta = k_p/k_e \approx 2\,k_p/\sqrt{s}$

Sommerfeld-Gamov factor: $C_p(s) = y/(1-exp(-y))$; $y = \pi\alpha\sqrt{s}/(2\,k_p)$ (for $\bar{p}p$, etc.)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta}{4s}\, C_p(s)\, |G_M(s)|^2 \left[(1+\cos^2\theta) + \frac{4M_p^2}{s}\left|\frac{G_E(s)}{G_M(s)}\right|^2 \sin^2\theta\right]$$

$$P_y = \frac{2M_p \sin 2\theta}{\sqrt{s}D}{\rm Im} G_E^* G_M = -\frac{2M_p \sin 2\theta}{\sqrt{s}D}\,|G_E(s)|\,|G_M(s)|\sin\Phi; \quad \Phi = \arg(\frac{G_E}{G_M})$$

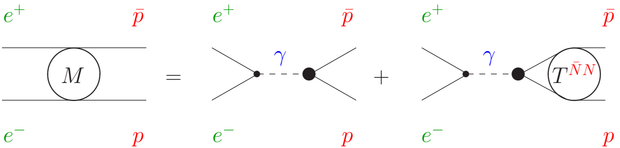
$C_{xx}, C_{yy}, C_{zz}, C_{xz}, C_{zy}$ ... involve other combinations of $G_E(s)$, $G_M(s)$

$$D = \sin^2\theta\,\frac{4M_p^2}{s}|G_E(s)|^2 + (1+\cos^2\theta)|G_M(s)|^2$$

- P_y, C_{xx}, etc. ... difficult to measure for $\bar{p}p$
  easier for $\Lambda\bar{\Lambda}$, etc. (self-analyzing weak decay of hyperons)

$e^+e^- \to \bar{p}p$ in DWBA

one-photon exchange $\Rightarrow$ $\bar{N}N$, e^+e^- are in the $^3S_1$, $^3D_1$ partial waves



$$M_{L,L'} \propto f_L^{e^+e^-} \cdot f_{L'}^{\bar{p}p}$$

$$f_{L=0}^{e^+e^-} = \left[1 + \frac{m_e}{\sqrt{s}}\right]; \quad f_{L=2}^{e^+e^-} = \left[1 - \frac{2m_e}{\sqrt{s}}\right]$$

$$f_{L=0}^{\bar{p}p} = \left[G_M + \frac{M_p}{\sqrt{s}} G_E\right]; \quad f_{L=2}^{\bar{p}p} = \left[G_M - \frac{2M_p}{\sqrt{s}} G_E\right]$$

$$f_{L=2}^{\bar{p}p}(k_p = 0) = 0 \;\rightarrow\; G_M(k_p = 0) = G_E(k_p = 0)$$

$$f_{L'}^{\bar{p}p}(k; E_k) = f_{L'}^{\bar{p}p;0}(k) + \sum_L \int_0^\infty \frac{dp p^2}{(2\pi)^3} f_L^{\bar{p}p;0}(p) \frac{1}{2E_k - 2E_p + i0^+} T_{LL'}^{\bar{p}p}(p, k; E_k)$$

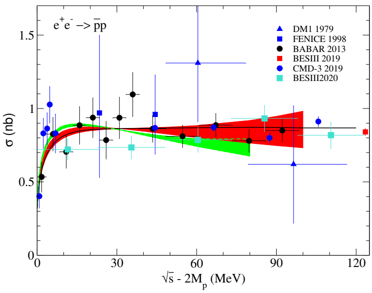
$f_{L'}^{\bar{p}p;0}$ ... bare vertex with bare form factors G_M^0 and G_E^0

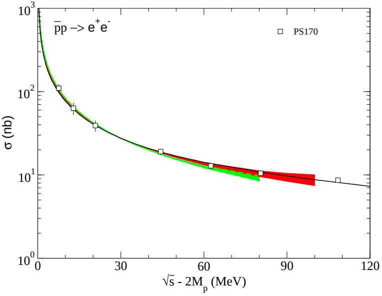
- assume $G_M^0 \equiv G_E^0 = \text{const.}$ ... only single parameter (overall normalization)

Results for $e^+e^- \leftrightarrow \bar{p}p$

J.H., X.-W. Kang, U.-G. Meißner, NPA 929 (2014) 102 (N^{2}LO)

(bands represent cutoff variations!)





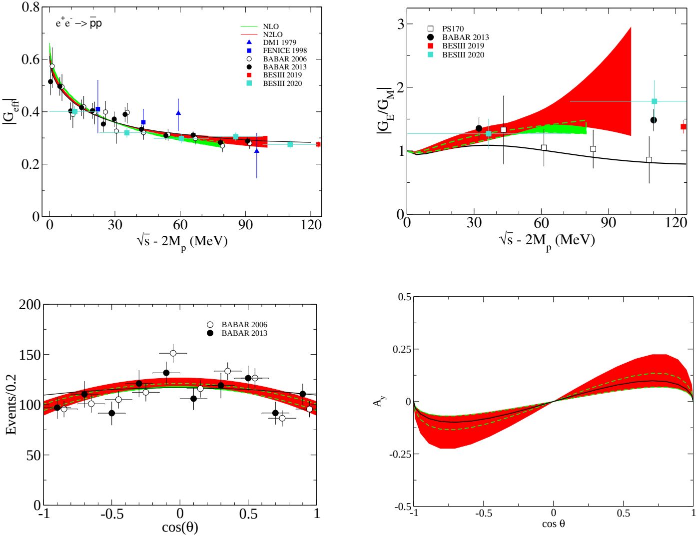
—- Jülich A (OBE) [meson-exchange; T. Hippchen et al., PRC 44 (1991) 1323]

PS170: G. Bardin et al., NPB 411 (1994) 3

($\sigma_{\bar{p}p\to e^+e^-} \propto \frac{k_e^2}{k_p^2}\sigma_{e^+e^-\to\bar{p}p}$; but there is a systematic overall difference of $\approx$ 1.47)

Note: $\sigma_{e^+e^-\to\bar{p}p} \neq 0$ at threshold because of attractive Coulomb interaction in $\bar{p}p$!

Results for $e^+e^- \to \bar{p}p$

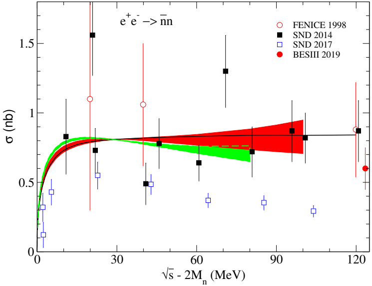


$\epsilon = \sqrt{s} - 2M_p = 36.5$ MeV

Results for $e^+e^- \to \bar{n}n$

J.H., C. Hanhart, X.-W. Kang, U.-G. Meißner, PRD 92 (2015) 054032 (N^2LO)

(bands represent cutoff variations!)



—- Jülich A (OBE) [meson-exchange; T. Hippchen et al., PRC 44 (1991) 1323]

FENICE: A. Antonelli et al., NPB 517 (1998) 3
SND 2014: M.M. Achasov et al., PRD 90 (2014) 112007
SND 2017: K.I. Belobodorov et al., EPJ WoC 199 (2019) 02026
BESIII 2019: preliminary !!

Summary I

- $\bar{N}N$ interaction up to N^3LO in chiral effective field theory
- new local regularization scheme is used for pion-exchange contributions
- new uncertainty estimate suggested by Epelbaum, Krebs, Meißner

- excellent description of $\bar{N}N$ amplitudes is achieved
- nice agreement with $\bar{p}p$ observables for $T_{lab} \leq 250$ MeV is achieved
- predictions are made for low energies ($T_{lab} \leq 5.3$ MeV):
  - low-energy annihilation cross section
  - level shifts of antiprotonic atoms

$\Rightarrow$ approach works not only for NN but also very well for $\bar{N}N$

Summary II

- Our analysis does not exclude that something exotic is seen in $J/\psi \to \gamma\bar{p}p$. However, it strongly suggests that FSI effects are a plausible and likely explanation for the enhancement in the $\bar{p}p$ invariant mass distribution. This conjecture explains also the energy dependence of $J/\psi \to \omega\bar{p}p$, $\psi(3686) \to \gamma\bar{p}p$, $e^+e^- \to \bar{p}p$, etc.
- The particularly strong enhancement seen in $J/\psi \to \gamma\bar{p}p$ could be indeed an evidence for a $\bar{p}p$ bound state (baryonium):
  In our analysis it is in the isospin $I = 1$ ${}^1S_0$ state.
- However, it is not an unambiguous signal for a bound state
  near-threshold bound state –> strong FSI effects
  strong FSI effects $-\not>$ near-threshold bound state
- Reliable conclusions from the $\bar{p}p$ invariant mass spectrum on the sub-threshold region are difficult to draw. One cannot avoid sizable variations/uncertainties in such an extrapolation.
  Alternative: $\bar{p}p$ annihilation into selective four-meson channels, e.g.
  $\bar{p}p \to \pi^0\bar{p}p_{bound} \to \pi^0\pi^-\pi^+\eta'$
- Differences in the distributions for $J/\psi \to \gamma\bar{p}p$, $J/\psi \to \omega\bar{p}p$, and $\psi' \to \gamma\bar{p}p$ have to be expected. They are simply sign of different reaction mechanisms.

Backup slides

Annihilation potential

- experimental information:
  • annihilation occurs dominantly into 4 to 6 pions (two-body channels like $\bar{p}p \to \pi^+\pi^-$, $\rho^\pm\pi^\mp$ etc. contribute in the order of $\approx 1\%$)
  • thresholds: for 5 pions: $\approx$ 700 MeV for $\bar{N}N$: 1878 MeV
  • produced pions have large momenta $\to$ annihilation process depends very little on energy
  • annihilation is a statistical process: properties of the individual particles (mass, quantum numbers) do not matter
- phenomenlogical models: bulk properties of annihilation can be described rather well by simple energy-independent optical potentials
- range associated with annihilation is around 1 fm or less
  $\to$ short-distance physics

$\Rightarrow$ describe annihilation in the same way as the short-distance physics in $V_{el}^{\bar{N}N}$, i.e. by contact terms (LECs)

$\Rightarrow$ describe annihilation by a few effective (two-body) annihilation channels (unitarity is preserved!)

$$V^{\bar{N}N} = V_{el}^{\bar{N}N} + V_{ann;eff}^{\bar{N}N}; \qquad V_{ann;eff}^{\bar{N}N} = \sum_X V^{\bar{N}N\to X} G_X^0 V^{X\to\bar{N}N}$$

$$V^{\bar{N}N\to X}(p_{\bar{N}N}, p_X) \approx p_{\bar{N}N}^L (a + b\, p_{\bar{N}N}^2 + \dots); \quad p_X \approx \text{const.}$$

Contributions of V_{cont} for $\bar{N}N$ up to N^{3}LO

$V_{el}^{\bar{N}N}$

$$
\begin{aligned}
V^{L=0} &= \tilde{C}_\alpha + C_\alpha (p^2 + p'^2) + D^1_\alpha p^2 p'^2 + D^2_\alpha (p^4 + p'^4) \\
V^{L=1} &= C_\beta \, p\, p' + D_\beta \, p\, p' (p^2 + p'^2) \\
V^{L=2} &= D_\gamma \, p^2 p'^2
\end{aligned}
$$

$\tilde{C}_i$ ... LO LECs [4], C_i ... NLO LECs [+14], D_i ... N^{3}LO LECs [+30], $p = |\mathbf{p}\,|;\ p' = |\mathbf{p}\,'|$

$V_{ann;eff}^{\bar{N}N}$

$$
\begin{aligned}
V^{L=0}_{ann} &= -i\,(\tilde{C}^a_\alpha + C^a_\alpha p^2 + D^a_\alpha p^4)\,(\tilde{C}^a_\alpha + C^a_\alpha p'^2 + D^a_\alpha p'^4) \\
V^{L=1}_{ann} &= -i\,(C^a_\beta p + D^a_\beta p^3)\,(C^a_\beta p' + D^a_\beta p'^3) \\
V^{L=2}_{ann} &= -i\,(D^a_\gamma)^2 p^2 p'^2 \\
V^{L=3}_{ann} &= -i\,(D^a_\delta)^2 p^3 p'^3
\end{aligned}
$$

α ... $^1S_0$ and $^3S_1$
β ... $^3P_0$, $^1P_1$, and $^3P_1$
γ ... $^1D_2$, $^3D_2$ and $^3D_3$
δ ... $^1F_3$, $^3F_3$ and $^3F_4$

- unitarity condition: higher powers than what follows from Weinberg power counting appear!
- same number of contact terms (LECs)

effective χ square

Fit to phase shifts and inelasticity parameters in the isospin basis

$\tilde{\chi}^2 \approx |S_{LL'} - S^{PWA}_{LL'}|^2/\Delta^2$ ... $S_{LL'}$ are S-matrix elements

(no uncertainties for the PWA given $\rightarrow$ Δ^2 ... simple scaling parameter)

	R=0.8 fm	R=0.9 fm	R=1.0 fm	R=1.1 fm	R=1.2 fm
$T_{lab} \leq 25$ MeV	0.003	0.004	0.004	0.019	0.036
$T_{lab} \leq 100$ MeV	0.023	0.025	0.036	0.090	0.176
$T_{lab} \leq 200$ MeV	0.106	0.115	0.177	0.312	0.626
$T_{lab} \leq 300$ MeV	2.012	2.171	3.383	5.531	9.479

- minimum around $R = 0.8 \sim 0.9$ fm ($R = 0.9 \sim 1.0$ fm in the NN case)

Calculation of observables is done in particle basis:

- Coulomb interaction in the $\bar{p}p$ channel is included
- the physical masses of p and n are used

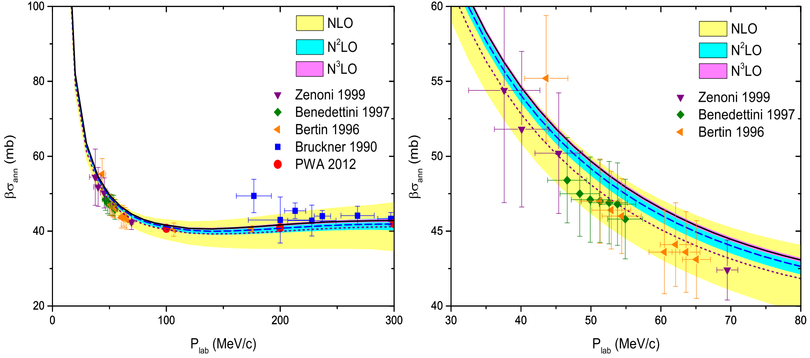
$\bar{n}n$ channels opens at $p_{lab} = 98.7$ MeV/c ($T_{lab} = 5.18$ MeV/c)

$\bar{N}N$ partial-wave cross sections [in mb]

		$\bar{p}p \to \bar{p}p$				$\bar{p}p \to \bar{n}n$			
	p_{lab} (MeV/c)	200	400	600	800	200	400	600	800
	T_{lab} (MeV)	21.1	81.7	175	295	21.1	81.7	175	295
$^1S_0$	N^{3}LO	15.9	8.0	4.1	2.0	0.7	0.1		
	PWA	15.7	7.9	4.1	2.1	0.7	0.1		
$^3S_1$	N^{3}LO	66.6	25.9	13.1	8.0	2.9	0.9	0.5	0.3
	PWA	66.1	26.0	13.2	8.8	3.0	1.0	0.5	0.2
$^3P_0$	N^{3}LO	4.9	5.4	5.1	3.6	1.5	0.8	0.1	
	PWA	4.9	5.4	5.0	3.5	1.5	0.8	0.1	
$^1P_1$	N^{3}LO	1.0	2.5	4.4	5.6	0.8	0.1		
	PWA	0.9	2.5	4.5	5.6	0.8	0.1		
$^3P_1$	N^{3}LO	1.8	5.0	4.1	3.6	5.1	3.0	0.2	0.1
	PWA	1.8	4.9	4.0	3.5	4.9	2.9	0.2	0.1
$^3P_2$	N^{3}LO	7.0	17.1	14.1	9.9	1.0	1.5	0.4	0.1
	PWA	7.0	17.0	13.9	9.6	0.9	1.4	0.4	0.1

(N^{3}LO with $R = 0.9$ fm)

$\bar{p}p$ annihilation cross section



$\beta = \frac{v_{\bar{p}}}{c}$

- anomalous threshold behavior due to attractive Coulomb interaction

Hadronic level shifts in hyperfine states of $\bar{p}$H

Deser-Trueman formula:

$$\Delta E_S + \mathrm{i}\,\frac{\Gamma_S}{2} = -\frac{4}{M_p r_B^3} a_S^{sc} \left(1 - \frac{a_S^{sc}}{r_B}\beta\right)$$

$$\Delta E_P + \mathrm{i}\,\frac{\Gamma_P}{2} = -\frac{3}{8 M_p r_B^5} a_P^{sc}$$

r_B ... Bohr radius ... 57.6 fm; $\beta = 2(1 - \Psi(1)) \approx 3.1544$
a^{sc} ... Coulomb-distorted $\bar{p}p$ scattering length

Carbonell, Richard, Wycech, ZPA 343 (1992) 343:
works well once Coulomb and p-n mass difference is taken into account

NOTE:

different sign conventions for scattering lengths in $\bar{N}N$ and $\bar{K}N$!

$\Delta E < 0 \Leftrightarrow$ repulsive shift

Hadronic level shifts in hyperfine states of $\bar{p}H$

	NLO	N^{2}LO	N^{3}LO	N^{2}LO*	Experiment
$E_{^1S_0}$ (eV)	−448	−446	−443	−436	−440(75) [1]
					−740(150) [2]
$\Gamma_{^1S_0}$ (eV)	1155	1183	1171	1174	1200(250) [1]
					1600(400) [2]
$E_{^3S_1}$ (eV)	−742	−766	−770	−756	−785(35) [1]
					−850(42) [3]
$\Gamma_{^3S_1}$ (eV)	1106	1136	1161	1120	940(80) [1]
					770(150) [3]
$E_{^3P_0}$ (meV)	17	12	8	16	139(28) [4]
$\Gamma_{^3P_0}$ (meV)	194	195	188	169	120(25) [4]
E_{1S} (eV)	−670	−688	−690	−676	−721(14) [1]
Γ_{1S} (eV)	1118	1148	1164	1134	1097(42) [1]
E_{2P} (meV)	1.3	2.8	4.7	2.3	15(20) [4]
Γ_{2P} (meV)	36.2	37.4	37.9	27	38.0(2.8) [4]

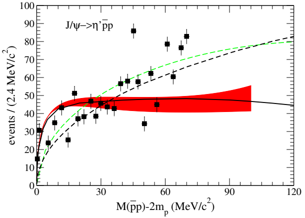
[1] Augsburger et al., NPA 658 (1999) 149; [2] Ziegler et al., PLB 206 (1988) 151;
[3] Heitlinger et al., ZPA 342 (1992) 359; [4] Gotta et al., NPA 660 (1999) 283

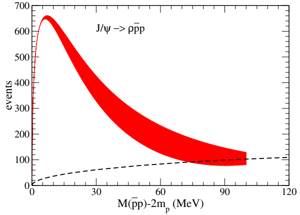
* Xian-Wei Kang et al., JHEP 02 (2014) 113

$\bar{p}p$ in final state

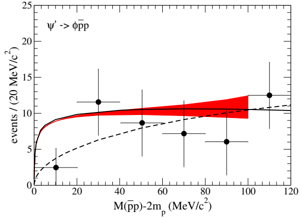
X.-W. Kang, JH, U.-G. Meißner, PRD 91 (2015) 074003 (N^{2}LO)

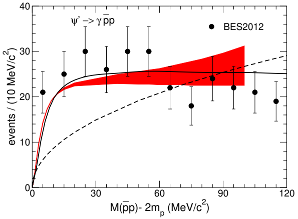
$\bar{N}N$ FSI in additional channels (bands represent cutoff variations!)





$J/\psi \to \eta\bar{p}p$ (left)
$J/\psi \to \rho\bar{p}p$ (right)





$\psi' \to \phi\bar{p}p$ (left)
$\psi' \to \gamma\bar{p}p$ (right)