光前夸克模型中重子弱衰变形状因子计算

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2022年03月28日

1. 研究目的
2. 标准光前夸克模型(SLF QM)
   - 重子束缚态
   - 重子弱衰变矩阵元
   - 重子弱衰变形状因子
3. 协变光前夸克模型(CLF QM)

研究目的

- 目前理论上，重子跃迁形状因子普遍采用标准光前夸克模型(SLF QM)计算，计算结果依赖于螺旋度的选取，具有很大的局限性。
- 受协变光前夸克模型(CLF QM)在介子弱衰变方面成功应用的启示，我们尝试着采用CLF QM 计算重子弱衰变形状因子。

SLF QM 计算$\mathcal{B}(1/2) \to \mathcal{B}(1/2)$ 弱衰变形状因子

运动学量：

$$
\begin{aligned}
M_0^2 &= \frac{m_1^2 + k_\perp^2}{x} + \frac{m_2^2 + k_\perp^2}{\bar{x}}, \quad k_{iz} = \frac{x_i M_0}{2} - \frac{m_i^2 + k_{i\perp}^2}{2x_i M_0}, \\
k_i &= \left(\frac{m_i^2 + k_{i\perp}^2}{x_i M_0}, x_i M_0, \vec{k}_{i\perp} \right) = \left(e_i - k_{iz}, e_i + k_{iz}, \vec{k}_{i\perp} \right), \\
e_i &= \sqrt{m_i^2 + k_{i\perp}^2 + k_{iz}^2} = \frac{x_i M_0}{2} + \frac{m_i^2 + k_{i\perp}^2}{2x_i M_0}, \\
M_0 &= e_1 + e_2.
\end{aligned} \tag{1}
$$

在$\bar{P} = k_1 + k_2 = (M_0, \overrightarrow{0})$ 质心坐标系下，

$$
k_1 = (e_1, \overrightarrow{k}), \qquad k_2 = (e_2, -\overrightarrow{k}) \tag{2}
$$

极化矢量:

$$
\begin{aligned}
\epsilon^\mu(0) &= (0, 0, 0, 1), \\
\epsilon^\mu(\pm) &= \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0).
\end{aligned} \tag{3}
$$

重子束缚态

在SLF QM中，我们采用夸克-di夸克模型，重子束缚态可表达为，

$$
\begin{aligned}
|\mathcal{B}(P,J,J_z)\rangle = & \int \frac{dxd^2k_{1\perp}}{2(2\pi^3)}\frac{dxd^2k_{2\perp}}{2(2\pi^3)}2(2\pi)^3\delta^3(P-k_1-k_2) \\
& \times \sum_{\lambda_1,\lambda_2,\alpha,\beta,\gamma,b,c} \Psi^{JJ_z}_{nLS_{[qq]}J_l}(k_1,k_2,\lambda_1,\lambda_2)\, C_{\alpha\beta\gamma}F^{bc} \\
& \times \left| Q^{\alpha}(k_1,\lambda_1)\left[q_b^{\beta}q_c^{\gamma}\right](k_2,\lambda_2)\right\rangle \qquad (4)
\end{aligned}
$$

其中，Q表示一个重夸克，两个轻夸克$[qq]$表示一个标量或矢量di-夸克；$S_{[qq]}$ 表示di-夸克自旋，L 表示重子轨道角动量，$J_l = L + S_{[qq]}, J$分别表示轻系统和重子总角动量。

系数$C_{\alpha\beta\gamma}$是归一化色因子，F^{bc} 是归一化味道因子，

$$
\begin{aligned}
& C_{\alpha'\beta'\gamma'}F^{b'c'}C_{\alpha\beta\gamma}F^{bc}\left\langle Q^{\alpha'}\left(k_1',\lambda_1'\right)\left[q_{b'}^{\beta'}q_{c'}^{\gamma'}\right]\left(k_2',\lambda_2'\right)\middle| Q^{\alpha}(k_1,\lambda_1)\left[q_a^{\beta}q_b^{\gamma}\right](k_2,\lambda_2)\right\rangle \\
& = 2^2(2\pi)^6\delta^3\left(k_1'-k_1\right)\delta^3\left(k_2'-k_2\right)\delta_{\lambda_1'\lambda_1}\delta_{\lambda_2'\lambda_2}. \qquad (5)
\end{aligned}
$$

束缚态归一化:

$$
\left\langle \mathcal{B}\left(P'',J'',J_z''\right)\middle|\mathcal{B}\left(P',J',J_z'\right)\right\rangle = 2(2\pi)^3P^+\delta^3(P''-P')\delta_{J'J''}\delta_{J_z'J_z''} \qquad (6)
$$

- **波函数(WF)**

$$\Psi^{JJ_z}_{nLS_{[qq]}J_l}(k_1,k_2,\lambda_1,\lambda_2)$$
$$= \sum_{s_1,s_2,L_z,J_{lz}} \left\langle \lambda_1 \left| \mathcal{R}_M^\dagger(k_1,m_1) \right| s_1 \right\rangle \left\langle \lambda_2 \left| \mathcal{R}_M^\dagger(k_2,m_2) \right| s_2 \right\rangle$$
$$\times \langle S_1 J_l; s_1 J_{lz} \mid S_1 J_l; JJ_z \rangle \left\langle LS_{[qq]}; L_z s_2 \mid LS_{[qq]}; J_l J_{lz} \right\rangle \phi_{nLL_z}(x_1,x_2,k_{1\perp},k_{2\perp}), \quad (7)$$

✠ $\mathcal{R}_M$是Melosh变换矩阵元

For spin $\frac{1}{2}$,

$$\langle s | \mathcal{R}_M(x,\mathbf{k}_\perp,m_i) | \lambda \rangle = \frac{\bar{u}_D(k_i,s)\, u(k_i,\lambda)}{2m_i}$$
$$= \frac{m_i + x_i M_0 + i\vec{\sigma}_{s\lambda} \cdot \vec{\mathbf{k}_\perp} \times \vec{n}}{\sqrt{(m_i + x_i M_0)^2 + \mathbf{k}_\perp^2}} \quad (8)$$

For spin 0 $\longrightarrow$ 标量di夸克,

$$\langle s | \mathcal{R}_M(x,\mathbf{k}_\perp,m_i) | \lambda \rangle = 1 \quad (9)$$

For spin 1 $\longrightarrow$ 矢量(轴矢) di夸克,

$$\langle s | \mathcal{R}_M(x,k_i,m_i) | \lambda \rangle = -\varepsilon^*_{LF}(k_i,\lambda) \cdot \varepsilon_I(k_i,s) \quad (10)$$

- ϕ_{nLL_z} 为径向轨道WF，通常采用高斯WF，

$$\phi_{nLL_z}(x, k_\perp) = \sqrt{\frac{dk_z}{dx}}\varphi_{nLm}(\mathbf{k}_\perp, \beta), \tag{11}$$

$$\varphi_{n00}(\vec{k}, \beta) = \varphi_{nS}(\vec{k}, \beta), \tag{12}$$

$$\varphi_{n1m}(\vec{k}, \beta) = k_m\varphi_{nP}(\vec{k}, \beta) = -\varepsilon(k_1 + k_2, m) \cdot k\varphi_{nP}(\vec{k}, \beta), \tag{13}$$

- C-G系数 $\langle S_1 J_l; s_1 J_{lz} \mid S_1 J_l; JJ_z\rangle$; $\langle LS_{[qq]}; L_z s_2 \mid LS_{[qq]}; J_l J_{lz}\rangle$

以量子数$(n, L, S^P_{[qq]}, J^P_l, J^P) = (n, 0, 0^+, 0^+, \frac{1}{2}^+)$的重子为例，$[qq]$是标量di-夸克，

$$J = J_l + S_Q = \frac{1}{2}, \qquad J_l = L + S_{[qq]} = 0$$

$$\left\langle LS_{[qq]} \mid J_l J_{lz}\right\rangle = \langle 00 \mid 00\rangle = 1$$

$$\begin{pmatrix} \langle\frac{1}{2}0 \mid \frac{1}{2}\frac{1}{2}\rangle & \langle-\frac{1}{2}0 \mid \frac{1}{2}\frac{1}{2}\rangle \\ \langle\frac{1}{2}0 \mid \frac{1}{2}-\frac{1}{2}\rangle & \langle-\frac{1}{2}0 \mid \frac{1}{2}-\frac{1}{2}\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I,$$

$$\begin{aligned} \langle s_1 J_l \mid JJ_z\rangle &= \chi^\dagger_{s1} \cdot I \cdot \chi_{J_z} \\ &= \frac{1}{\sqrt{(M_0 + m_1)^2 - m_2^2}}\bar{u}_D(k_1, s_1) \cdot u(k_1 + k_2, J_z) \end{aligned} \tag{14}$$

这里$u_{(D)}$表示Dirac旋量场。

- 顶角函数$\Gamma_{LS_{[qq]}J_l}$

为了计算方便，我们将$\Psi^{JJ_z}_{nLS_{[qq]}J_l}(k_1,k_2,\lambda_1,\lambda_2)$ 写为一种协变形式,

$$\Psi^{JJ_z}_{nLS_{[qq]}J_l}(k_1,k_2,\lambda_1,\lambda_2) = \frac{1}{\sqrt{(M_0+m_1)^2-m_2^2}}\bar{u}(k_1,\lambda_1)\cdot\Gamma_{LS_{[qq]}J_l}\cdot u(\bar{P},J_z)\cdot\phi_{nL}(x_1,x_2,k_{1\perp},k_{2\perp}) \quad (15)$$

其中，$\bar{P}=k_1+k_2$ 组分夸克$k_{1,2}$均在壳。

量子数$(n, L, S^P_{[qq]}, J^P_l, J^P) = (n, 0, 0^+, 0^+, \frac{1}{2}^+)$的重子的WF,

$$\begin{aligned}
&\Psi^{\frac{1}{2} J_z}_{ns00}(k_1, k_2, \lambda_1, \lambda_2) \\
&= \left\langle \lambda_1 \left| \mathcal{R}^\dagger_M (k_1, m_1) \right| s_1 \right\rangle \left\langle \lambda_2 \left| \mathcal{R}^\dagger_M (k_2, m_2) \right| s_2 \right\rangle \left\langle \frac{1}{2} 0; s_1 0 \mid \frac{1}{2} 0; \frac{1}{2} J_z \right\rangle \langle 00 \mid 00 \rangle \phi_{ns} \\
&= \frac{\bar{u}(k_1, \lambda_1) u_D(k_1, s_1)}{2m_1} \cdot \frac{1}{\sqrt{(M_0 + m_1)^2 - m_2^2}} \bar{u}_D(k_1, s_1) \cdot u(k_1 + k_2, J_z) \cdot \phi_{n00} \\
&= \frac{1}{\sqrt{(M_0 + m_1)^2 - m_2^2}} \bar{u}(k_1, \lambda_1) \cdot u(k_1 + k_2, J_z) \cdot \phi_{ns} \qquad (16)
\end{aligned}$$

与Eq. (15)匹配,

$$\begin{aligned}
&\Psi^{\frac{1}{2} J_z}_{ns00}(k_1, k_2, \lambda_1, \lambda_2) \\
&= \frac{1}{\sqrt{(M_0 + m_1)^2 - m_2^2}} \bar{u}(k_1, \lambda_1) \cdot \Gamma_{s00} \cdot u(\bar{P}, J_z) \cdot \phi_{ns}(x_1, x_2, k_{1\perp}, k_{2\perp}) \qquad (17)
\end{aligned}$$

很容易得到

$$\Gamma_{s00} = 1$$

重子弱衰变矩阵元

利用重子束缚态Eq. (7)，以及产生湮灭算符间对易关系，可以得到重子弱衰变矩阵元一般表达式，

$$\begin{aligned}&\left\langle\mathcal{B}'\left(P',J_z'\right)\left|\gamma^\mu\left(1-\gamma_5\right)\right|\mathcal{B}\left(P,J_z\right)\right\rangle\\&\quad=\int\frac{dxd^2\mathbf{k}_\perp}{2(2\pi)^3}\frac{\phi_{nL'}'^{*}\left(x',k_\perp'\right)\phi_{1L}\left(x,k_\perp\right)}{2\sqrt{k_1^+k_1'^+\left(k_1\cdot\bar{P}+m_1M_0\right)\left(k_1'\cdot\bar{P}'+m_1'M_0'\right)}}\\&\qquad\times\bar{u}\left(\bar{P}',J_z'\right)\bar{\Gamma}_{L'S_{[qq]}J_l'}\left(\not{k}_1'+m_1'\right)\gamma^\mu\left(1-\gamma_5\right)\left(\not{k}_1+m_1\right)\Gamma_{LS_{[qq]}J_l}u\left(\bar{P},J_z\right),\quad(18)\end{aligned}$$

重子弱衰变形状因子

$\mathcal{B}(1/2) \to \mathcal{B}(1/2)$弱衰变矩阵元可参数化为，

$$\left\langle \mathcal{B}'\left(P', J_z'\right)\left|\bar{c}\gamma_\mu b\right|\mathcal{B}\left(P, J_z\right)\right\rangle = \bar{u}\left(P', J_z'\right)\left[f_1^V\left(q^2\right)\gamma_\mu + i\frac{f_2^V\left(q^2\right)}{M+M'}\sigma_{\mu\nu}q^\nu + \frac{f_3^V\left(q^2\right)}{M+M'}q_\mu\right]u\left(P, J_z\right), \tag{19}$$

$$\left\langle \mathcal{B}'\left(P', J_z'\right)\left|\bar{c}\gamma_\mu\gamma_5 b\right|\mathcal{B}\left(P, J_z\right)\right\rangle = \bar{u}\left(P', J_z'\right)\left[g_1^A\left(q^2\right)\gamma_\mu + i\frac{g_2^A\left(q^2\right)}{M+M'}\sigma_{\mu\nu}q^\nu + \frac{g_3^A\left(q^2\right)}{M+M'}q_\mu\right]\gamma_5 u\left(P, J_z\right), \tag{20}$$

取$q^+ = 0$, $\mathbf{q}_\perp \neq 0$，上式Eqs. (19)和(20) 可写为，

$$\left\langle \mathcal{B}'\left(P', J_z'\right)\left|V^+\right|\mathcal{B}\left(P, J_z\right)\right\rangle = 2\sqrt{P^+P'^+}\left[f_1^V\left(q^2\right)\delta_{J_z'J_z} + \frac{f_2^V\left(q^2\right)}{M+M'}\left(\vec{\sigma}\cdot\vec{q}_\perp\sigma^3\right)_{J_z'J_z}\right] \tag{21}$$

$$\left\langle \mathcal{B}'\left(P', J_z'\right)\left|A^+\right|\mathcal{B}\left(P, J_z\right)\right\rangle = 2\sqrt{P^+P'^+}\left[g_1^A\left(q^2\right)\left(\sigma^3\right)_{J_z'J_z} + \frac{g_2^A\left(q^2\right)}{M+M'}\left(\vec{\sigma}\cdot\vec{q}_\perp\right)_{J_z'J_z}\right] \tag{22}$$

很容易可以得到形状因子，

$$f_1^V\left(q^2\right)=\int\frac{dxd^2\mathbf{k}_\perp}{2(2\pi)^3}\frac{\phi_{nL'}^{\prime *}\left(x',k'_\perp\right)\phi_{1L}\left(x,k_\perp\right)}{16P^+P'^+\sqrt{k_1'^+k_1^+\left(k'_1\cdot\bar{P}'+m'_1M'_0\right)\left(k_1\cdot\bar{P}+m_1M_0\right)}}$$
$$\times\ \mathrm{Tr}\left[\left(\bar{\not P}+M_0\right)\gamma^+\left(\bar{\not P}'+M'_0\right)\bar{\Gamma}_{L'S_{[qq]}J'_l}\left(\not k'_1+m'_1\right)\gamma^+\left(\not k_1+m_1\right)\Gamma_{LS_{[qq]}J_l}\right],\tag{23}$$

$$f_2^V\left(q^2\right)=\frac{-i(M+M')q_\perp^i}{\mathbf{q}_\perp^2}\int\frac{dxd^2\mathbf{k}_\perp}{2(2\pi)^3}\frac{\phi_{nL'}^{\prime *}\left(x',k'_\perp\right)\phi_{1L}\left(x,k_\perp\right)}{16P^+P'^+\sqrt{k_1'^+k_1^+\left(k'_1\cdot\bar{P}'+m'_1M'_0\right)\left(k_1\cdot\bar{P}+m_1M_0\right)}}$$
$$\times\ \mathrm{Tr}\left[\left(\bar{\not P}+M_0\right)\sigma^{i+}\left(\bar{\not P}'+M'_0\right)\bar{\Gamma}_{L'S_{[qq]}J'_l}\left(\not k'_1+m'_1\right)\gamma^+\left(\not k_1+m_1\right)\Gamma_{LS_{[qq]}J_l}\right],\tag{24}$$

这里可能用到，

$$\begin{aligned}\frac{1}{2}\sum_{J_z,J'_z}u\left(\bar{P},J_z\right)\delta_{J_zJ'_z}\bar{u}\left(\bar{P}',J'_z\right)&=\frac{1}{4\sqrt{P^+P'^+}}\left(\bar{P}+M_0\right)\gamma^+\left(\bar{P}'+M'_0\right)\\ \frac{1}{2}\sum_{J_z,J'_z}u\left(\bar{P},J_z\right)\left(\sigma^3\sigma_\perp^i\right)_{J_zJ'_z}\bar{u}\left(\bar{P}',J'_z\right)&=-\frac{i}{4\sqrt{P^+P'^+}}\left(\bar{P}+M_0\right)\sigma^{i+}\left(\bar{P}'+M'_0\right)\end{aligned}\tag{25}$$

CLF QM 计算$\mathcal{B}(1/2) \to \mathcal{B}(1/2)$ 弱衰变形状因子

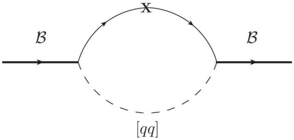

Figure: 矩阵元费曼图

根据费曼规则，重子矩阵元可以写为

$$\langle \mathcal{B}'\left(P', J_z'\right)|\gamma^{\mu}\left(1-\gamma_5\right)|\mathcal{B}\left(P, J_z\right)\rangle = \int \frac{d^2k}{(2\pi)^4} \frac{H_{M'}H_M}{N_1'N_1} \bar{u}\left(P', J_z'\right)\cdot \mathcal{S} \cdot u\left(P, J_z\right) \quad (26)$$

类费米圈的迹$\mathcal{S}$，

$$\mathcal{S} = \mathrm{Tr}\left[\bar{\Gamma}_{L'S_{[qq]}J_l'}\left(\not{k}_1' + m_1'\right)\gamma^{\mu}\left(1-\gamma_5\right)\left(\not{k}_1 + m_1\right)\Gamma_{LS_{[qq]}J_l}\right], \quad (27)$$

这里$P(M, \vec{p}) \neq \bar{P}(M_0, \vec{p})$

谢谢!

色允许b-重子（Ξ_b, Ω_b）两体非轻弱衰变的研究

汇报人：王丽婷

日期：2022年3月28日

文献：PRD 105,013003(2022)

◆研究动机

◆研究创新点

◆研究方法

◆数值结果

Restudy of the color-allowed two-body nonleptonic decays of bottom baryons Ξ_b and Ω_b supported by hadron spectroscopy

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文献：PRD 105,013003(2022)

研究动机

1. 受$b \to c$弱跃迁中存在的$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}e(\mu)\nu_{e(\mu)})}$反常的启发，研究重子弱衰变，如$\Xi_b \to \Xi_c \ell^- \nu_\ell, \Omega_b \to \Omega_c \ell^- \nu_\ell$，对寻找超出标准模型（SM）的新物理（NP）是有意义的，其中关键一点就是计算跃迁形状因子。
2. 随着实验数据的积累，LHCb实验展示出其在探索重子衰变方面的潜力。此外，SuperKEKB和Belle II 实验也为重味物理的研究提供了巨大的平台。
3. 研究重子弱衰变有很多方法：the Quark Models，the Flavor Symmetry Method，the Light-Front Approach，the QCD Sum Rules。

研究创新点

关键问题： 计算跃迁形状因子，通常采用quark-diquark 方案。

quark-diquark 方案：
重子空间波函数（WF）

↓

简谐振子WF：结果依赖于简谐振子WF参数。

$$\Psi_{J,J_M} = [\mathcal{R}_M^\dagger]_{k_{1,m_1}}[\mathcal{R}_M^\dagger]_{k_{2,m_2}}\langle S_Q J_l | J,J_M\rangle\langle LS_{[qq]} | J_l,J_{lM}\rangle\phi_{L,L_M}$$

→

三体光前夸克模型(The Three-body Light-Front Quark Model)：重子空间波函数（WF）

↓

利用与高斯展开方法（GEM）相关的半相对论夸克模型，求解三体薛定谔方程。

$$\mathcal{H}\Psi_{J,J_M} = E\Psi_{J,J_M}$$

$$\Psi_{J,J_M} = \chi^c\{\chi_{S,S_M}^s\psi_{L,L_M}^p\}_{J,J_M}\psi^f$$

文献：PRD 105,013003(2022)

研究创新点

a challenge. Usually, the quark-diquark scheme as an approximate treatment was widely used in previous theoretical works [25–29,35]. And the spatial wave functions of these hadrons involved in the bottom baryon weak decays are approximately taken as a simple harmonic oscillator wave function, which makes the results dependent on the parameter of the harmonic oscillator wave function. For avoiding the uncertainty from these approximate treatments mentioned above, in this work we calculate the weak transition form factors of the $\Xi_b \to \Xi_c^{(*)}$ and $\Omega_b \to \Omega_c^{(*)}$ transitions with emitting a pseudoscalar meson (π^-, K^-, D^-, and D_s^-) or a vector meson (ρ^-, K^{*-}, D^{*-} and D_s^{*-}) in the three-body light-front quark model. Here, $\Xi_c^{(*)}$ denotes

excited state $\Omega_c(2S)$. In the realistic calculation, we take the numerical spatial wave functions of these involved bottom and charmed baryons as input, where the semirelativistic potential model [30,36] associated with the Gaussian expansion method (GEM) [37–40] is adopted. By fitting the mass spectrum of these observed bottom and charmed baryons, the parameters of the adopted semirelativistic potential model can be fixed. Comparing with former approximation of taking a simple harmonic oscillator wave fu av sp

In this section, we illustrate how to obtain the concerned spatial wave functions by the semirelativistic quark model with the help of the GEM. Different from the meson system, baryon is a typical three-body system. Thus, its wave function can be extracted by solving the three-body Schrödinger equation. Here, the semirelativistic potentials were given in Refs. [36,42], which are applied to the

文献：PRD 105,013003(2022)

研究方法

一、三体LFQM 计算形状因子

重子束缚态: $$|\mathcal{B}_Q(P,J,J_z)\rangle = \int \frac{d^3\tilde{p}_1}{2(2\pi)^3}\frac{d^3\tilde{p}_2}{2(2\pi)^3}\frac{d^3\tilde{p}_3}{2(2\pi)^3}2(2\pi)^3 \times \sum_{\lambda_1,\lambda_2,\lambda_3}\Psi^{J,J_z}(\tilde{p}_i,\lambda_i)C^{\alpha\beta\gamma}\delta^3(\tilde{\bar{P}}-\tilde{p}_1-\tilde{p}_2-\tilde{p}_3) \times F_{nnQ}|n_\alpha(\tilde{p}_1,\lambda_1)\rangle|n_\beta(\tilde{p}_2,\lambda_2)\rangle|Q_\gamma(\tilde{p}_3,\lambda_3)\rangle.$$

$\mathcal{B}_Q(\bar{3}_f)$ WF: $$\Psi^{J,J_z}(\tilde{p}_i,\lambda_i) = A_0\bar{u}(p_1,\lambda_1)[(\bar{\not{P}}+M_0)\gamma_5]v(p_2,\lambda_2) \times \bar{u}_Q(p_3,\lambda_3)u(\bar{P},J,J_z)\phi(x_i,k_{i\perp}),$$

$\mathcal{B}_Q(6_f)$ WF: $$\Psi^{J,J_z}(\tilde{p}_i,\lambda_i) = A_1\bar{u}(p_1,\lambda_1)[(\bar{\not{P}}+M_0)\gamma_{\perp\alpha}]v(p_2,\lambda_2) \times \bar{u}_Q(p_3,\lambda_3)\gamma_\perp^\alpha\gamma_5 u(\bar{P},J,J_z)\phi(x_i,k_{i\perp}),$$

研究方法

强子矩阵元：

$$
\begin{aligned}
&\langle \mathcal{B}_c^{(*)}(\bar{3}_f, 1/2^+)(\bar{P}', J_z')|\bar{c}\gamma^\mu(1-\gamma_5)b|\mathcal{B}_b(\bar{3}_f, 1/2^+)(\bar{P}, J_z)\rangle \\
&= \int \left(\frac{dx_1 d^2\vec{k}_{1\perp}}{2(2\pi)^3}\right)\left(\frac{dx_2 d^2\vec{k}_{2\perp}}{2(2\pi)^3}\right)\frac{\phi(x_i, \vec{k}_{i\perp})\phi^*(x_i', \vec{k}_{i\perp}{}')}{16\sqrt{x_3 x_3' M_0{}^3 M_0'^3}}\frac{\mathrm{Tr}[(\bar{\not{P}}' - M_0')\gamma_5(\not{p}_1 + m_1)(\bar{\not{P}} + M_0)\gamma_5(\not{p}_2 - m_2)]}{\sqrt{(e_1+m_1)(e_2+m_2)(e_3+m_3)(e_1'+m_1')(e_2'+m_2')(e_3'+m_3')}} \\
&\quad \times \bar{u}(\bar{P}', J_z')(\not{p}_3{}' + m_3')\gamma^\mu(1-\gamma_5)(\not{p}_3 + m_3)u(\bar{P}, J_z),
\end{aligned}
$$

$$
\begin{aligned}
&\langle \mathcal{B}_c^{(*)}(6_f, 1/2^+)(\bar{P}', J_z')|\bar{c}\gamma^\mu(1-\gamma_5)b|\mathcal{B}_b(6_f, 1/2^+)(\bar{P}, J_z)\rangle \\
&= \int \left(\frac{dx_1 d^2\vec{k}_{1\perp}}{2(2\pi)^3}\right)\left(\frac{dx_2 d^2\vec{k}_{2\perp}}{2(2\pi)^3}\right)\frac{\phi(x_i, \vec{k}_{i\perp})\phi^*(x_i', \vec{k}_{i\perp}{}')}{48\sqrt{x_3 x_3' M_0{}^3 M_0'^3}}\frac{\mathrm{Tr}[\gamma_\perp^\alpha(\bar{\not{P}}' + M_0')(\not{p}_1 + m_1)(\bar{\not{P}} + M_0)\gamma_\perp^\beta(\not{p}_2 - m_2)]}{\sqrt{(e_1+m_1)(e_2+m_2)(e_3+m_3)(e_1'+m_1')(e_2'+m_2')(e_3'+m_3')}} \\
&\quad \times \bar{u}(\bar{P}', J_z')\gamma_{\perp\alpha}\gamma_5(\not{p}_3{}' + m_3')\gamma^\mu(1-\gamma_5)(\not{p}_3 + m_3)\gamma_{\perp\beta}\gamma_5 u(\bar{P}, J_z),
\end{aligned}
$$

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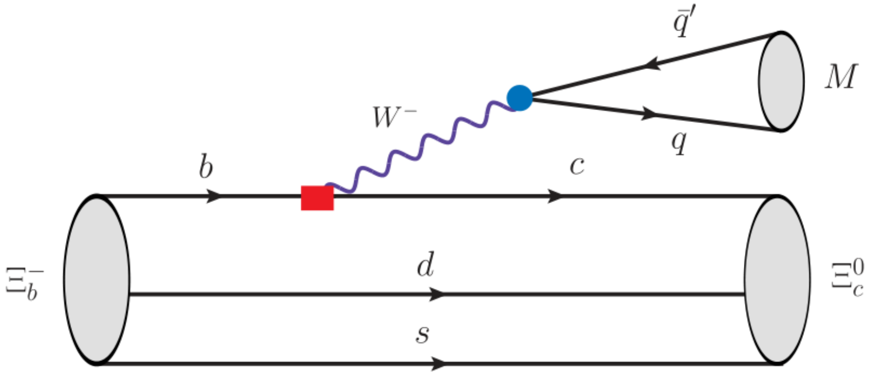
研究方法

二、简单因子化方法计算非轻弱衰变

$$\langle \mathcal{B}_c^{(*)}(P', J'_z) M^- | \mathcal{H}_{\rm eff} | \mathcal{B}_b(P, J_z) \rangle$$
$$= \frac{G_F}{\sqrt{2}} V_{cb} V^*_{qq'} \langle M^- | \bar{q}' \gamma_\mu (1-\gamma_5) q | 0 \rangle$$
$$\times \langle \mathcal{B}_c^{(*)}(P', J'_z) | \bar{c} \gamma^\mu (1-\gamma_5) b | \mathcal{B}_b(P, J_z) \rangle,$$



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数值结果

TABLE III. The form factors for the $\Xi_b \to \Xi_c^{(*)}$ transitions in the standard light front quark model. Here, we adopt the form defined in Eq. (4.1) for analyzing these form factors.

	$F(0)$	$F(q^2_{\max})$	b_1	b_2
		$\Xi_b \to \Xi_c$		
f_1^V	0.481	1.015	0.970	0.233
f_2^V	−0.127	−0.312	1.380	0.578
f_3^V	−0.046	−0.097	1.187	0.875
g_1^A	0.471	0.978	0.929	0.226
g_2^A	−0.026	−0.068	1.318	0.122
g_3^A	−0.154	−0.377	1.493	0.947
		$\Xi_b \to \Xi_c(2970)$		
f_1^V	0.214	0.200	−1.146	2.282
f_2^V	−0.072	−0.081	−0.356	1.600
f_3^V	−0.111	−0.221	1.444	0.168
g_1^A	0.204	0.186	−1.269	2.474
g_2^A	−0.087	−0.231	1.867	−0.907
g_3^A	−0.095	−0.113	−0.022	1.687

重夸克极限下：

- $f_1^V \approx g_1^A$
- $f_1^V(q^2_{max}) \approx g_1^A(q^2_{max}) \approx 1$

- $\Xi_b \to \Xi_c\ (2970)$

  $f_1^V(q^2_{max}) = g_1^A(q^2_{max}) = 0$

数值结果

TABLE V. The form factors for the $\Omega_b \to \Omega_c^{(*)}$ transitions in the standard light front quark model. We use a three parameter form defined in Eq. (4.1) for these form factors.

	$F(0)$	$F(q^2_{\max})$	b_1	b_2
		$\Omega_b \to \Omega_c$		
f_1^V	0.493	1.232	1.765	1.272
f_2^V	0.436	1.075	1.658	1.001
f_3^V	−0.255	−0.620	1.628	1.005
g_1^A	−0.161	−0.329	1.053	0.337
g_2^A	0.011	0.018	0.822	1.526
g_3^A	0.055	0.137	1.680	1.052
		$\Omega_b \to \Omega_c(2S)$		
f_1^V	0.180	0.163	−1.135	3.320
f_2^V	0.133	0.107	−1.727	4.270
f_3^V	−0.150	−0.215	0.481	0.239
g_1^A	−0.058	−0.047	−1.701	3.487
g_2^A	0.029	0.053	1.455	0.772
g_3^A	0.023	0.023	−0.671	2.407

重夸克极限下：

$$f_1^V(q^2_{\max}) = \frac{1}{3} + \frac{1}{3}\frac{M^2 + M'^2}{MM'} = 1.23,$$

$$f_2^V(q^2_{\max}) = \frac{1}{3}\frac{M + M'}{M'} = 1.08,$$

$$f_3^V(q^2_{\max}) = -\frac{1}{3}\frac{M - M'}{M'} = -0.41,$$

$$g_1^A(q^2_{\max}) = -\frac{1}{3},$$

$$g_2^A(q^2_{\max}) = g_3^A(q^2_{\max}) = 0,$$

数值结果

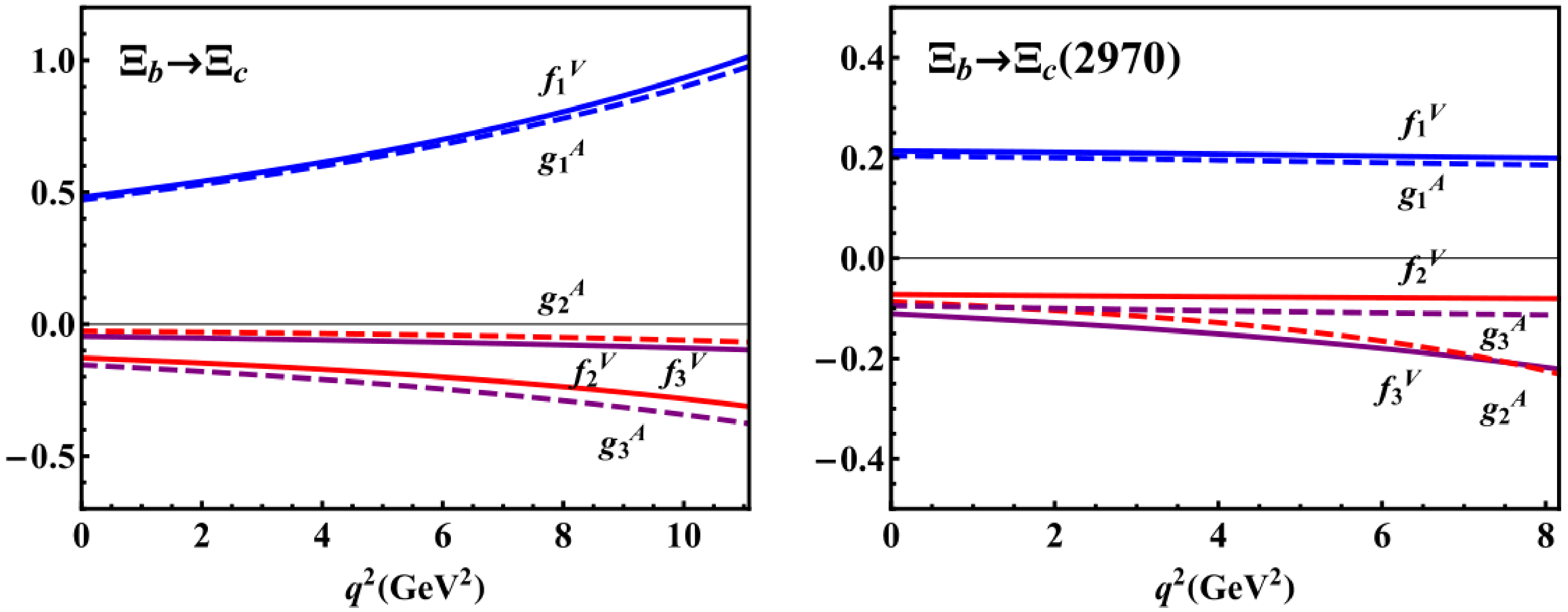

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TABLE VIII. Comparison of theoretical predictions for $\mathcal{B}(\Xi_b^{0,-} \to \Xi_c^{+,0} M^-)$ and $\mathcal{B}(\Omega_b^- \to \Omega_c^0 M^-)$. Here, all values should be multiplied by a factor of 10^{-3}.

	This work	Cheng [21]	Ivanov *et al.* [22,23]	Zhao [25]	Gutsche *et al.* [24]	Chua [27]
$\Xi_b^{0,-} \to \Xi_c^{+,0}\pi^-$	4.03 (4.29)	4.9 (5.2)	7.08 (10.13)	8.37 (8.93)	–	$3.66^{+2.29}_{-1.59}$ ($3.88^{+2.43}_{-1.69}$)
$\Xi_b^{0,-} \to \Xi_c^{+,0}\rho^-$	13.3 (14.1)	–	–	24.0 (25.6)	–	$10.88^{+6.83}_{-4.74}$ ($11.56^{+7.25}_{-5.04}$)
$\Xi_b^{0,-} \to \Xi_c^{+,0}K^-$	0.31 (0.33)	–	–	0.667 (0.711)	–	$0.28^{+0.17}_{-0.12}$ ($0.29^{+0.18}_{-0.13}$)
$\Xi_b^{0,-} \to \Xi_c^{+,0}K^{*-}$	0.71 (0.76)	–	–	1.23 (1.31)	–	$0.56^{+0.35}_{-0.24}$ ($0.60^{+0.37}_{-0.26}$)
$\Xi_b^{0,-} \to \Xi_c^{+,0}D^-$	0.58 (0.62)	–	–	0.949 (1.03)	0.45	$0.43^{+0.29}_{-0.20}$ ($0.45^{+0.31}_{-0.21}$)
$\Xi_b^{0,-} \to \Xi_c^{+,0}D^{*-}$	1.51 (1.60)	–	–	1.54 (1.64)	0.95	$0.77^{+0.50}_{-0.35}$ ($0.82^{+0.53}_{-0.37}$)
$\Xi_b^{0,-} \to \Xi_c^{+,0}D_s^-$	14.8 (15.7)	14.6	–	24.6 (26.2)	–	$10.87^{+7.51}_{-5.03}$ ($11.54^{+7.98}_{-5.34}$)
$\Xi_b^{0,-} \to \Xi_c^{+,0}D_s^{*-}$	32.4 (34.4)	23.1	–	36.5 (39.0)	–	$16.24^{+10.54}_{-7.25}$ ($17.26^{+11.2}_{-7.70}$)
$\Omega_b^- \to \Omega_c^0\pi^-$	2.82	4.92	5.81	4.00	1.88	$1.10^{+0.85}_{-0.55}$
$\Omega_b^- \to \Omega_c^0\rho^-$	7.92	12.8	–	10.8	5.43	$3.07^{+2.41}_{-1.53}$
$\Omega_b^- \to \Omega_c^0K^-$	0.22	–	–	0.326	–	$0.08^{+0.07}_{-0.04}$
$\Omega_b^- \to \Omega_c^0K^{*-}$	0.41	–	–	0.544	–	$0.16^{+0.12}_{-0.08}$
$\Omega_b^- \to \Omega_c^0D^-$	0.52	–	–	0.636	–	$0.15^{+0.14}_{-0.08}$
$\Omega_b^- \to \Omega_c^0D^{*-}$	0.48	–	–	0.511	–	$0.16^{+0.13}_{-0.08}$
$\Omega_b^- \to \Omega_c^0D_s^-$	13.5	17.9	–	17.1	–	$4.03^{+3.72}_{-2.21}$
$\Omega_b^- \to \Omega_c^0D_s^{*-}$	9.73	11.5	–	11.7	–	$3.18^{+2.69}_{-1.61}$

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谢谢！

数值结果

TABLE VI. The branching ratios and asymmetry parameters of the $\Xi_b \to \Xi_c^{(*)} M$ transitions with M denoting a pseudoscalar or vector meson, where the branching ratios out of or in brackets correspond to the $\Xi_b^0 \to \Xi_c^+$ and $\Xi_b^- \to \Xi_c^0$ transitions, respectively.

Mode	$\mathcal{B}(\times 10^{-3})$	α	Mode	$\mathcal{B}(\times 10^{-3})$	α
$\Xi_b^{0,-} \to \Xi_c^{+,0}\pi^-$	4.04 (4.29)	−1.000	$\Xi_b^{0,-} \to \Xi_c^{+,0}\rho^-$	13.3 (14.1)	−0.792
$\Xi_b^{0,-} \to \Xi_c^{+,0}K^-$	0.31 (0.33)	−1.000	$\Xi_b^{0,-} \to \Xi_c^{+,0}K^{*-}$	0.71 (0.76)	−0.737
$\Xi_b^{0,-} \to \Xi_c^{+,0}D^-$	0.58 (0.62)	−0.983	$\Xi_b^{0,-} \to \Xi_c^{+,0}D^{*-}$	1.51 (1.60)	−0.239
$\Xi_b^{0,-} \to \Xi_c^{+,0}D_s^-$	14.8 (15.7)	−0.978	$\Xi_b^{0,-} \to \Xi_c^{+,0}D_s^{*-}$	32.4 (34.4)	−0.206
$\Xi_b^{0,-} \to \Xi_c^{+,0}(2970)\pi^-$	1.78 (1.89)	−0.999	$\Xi_b^{0,-} \to \Xi_c^{+,0}(2970)\rho^-$	2.78 (2.95)	−0.763
$\Xi_b^{0,-} \to \Xi_c^{+,0}(2970)K^-$	0.04 (0.05)	−0.998	$\Xi_b^{0,-} \to \Xi_c^{+,0}(2970)K^{*-}$	0.09 (0.10)	−0.702
$\Xi_b^{0,-} \to \Xi_c^{+,0}(2970)D^-$	0.04 (0.05)	−0.952	$\Xi_b^{0,-} \to \Xi_c^{+,0}(2970)D^{*-}$	0.12 (0.12)	−0.181
$\Xi_b^{0,-} \to \Xi_c^{+,0}(2970)D_s^-$	1.05 (1.12)	−0.940	$\Xi_b^{0,-} \to \Xi_c^{+,0}(2970)D_s^{*-}$	2.30 (2.45)	−0.148

数值结果

TABLE VII. The branching rates and asymmetry parameters of $\Omega_b \to \Omega_c^{(*)} M$ transitions with M denoting a pseudoscalar or vector pmeson.

Mode	$\mathcal{B}(\times 10^{-3})$	α	Mode	$\mathcal{B}(\times 10^{-3})$	α
$\Omega_b^- \to \Omega_c^0 \pi^-$	2.82	0.59	$\Omega_b^- \to \Omega_c^0 \rho^-$	7.92	0.61
$\Omega_b^- \to \Omega_c^0 K^-$	0.22	0.58	$\Omega_b^- \to \Omega_c^0 K^{*-}$	0.41	0.62
$\Omega_b^- \to \Omega_c^0 D^-$	0.52	0.49	$\Omega_b^- \to \Omega_c^0 D^{*-}$	0.48	0.69
$\Omega_b^- \to \Omega_c^0 D_s^-$	13.5	0.47	$\Omega_b^- \to \Omega_c^0 D_s^{*-}$	9.73	0.70
$\Omega_b^- \to \Omega_c^0(2S) \pi^-$	0.30	0.58	$\Omega_b^- \to \Omega_c^0(2S) \rho^-$	0.70	0.60
$\Omega_b^- \to \Omega_c^0(2S) K^-$	0.02	0.57	$\Omega_b^- \to \Omega_c^0(2S) K^{*-}$	0.03	0.60
$\Omega_b^- \to \Omega_c^0(2S) D^-$	0.03	0.45	$\Omega_b^- \to \Omega_c^0(2S) D^{*-}$	0.02	0.65
$\Omega_b^- \to \Omega_c^0(2S) D_s^-$	0.62	0.43	$\Omega_b^- \to \Omega_c^0(2S) D_s^{*-}$	0.36	0.65

[3] The naïve factorization approach works well for the color-allowed dominated processes. But, there exists the case that the color-suppressed and penguin dominated processes can not be explained by the naïve factorization, which may show important nonfactorizable contributions to nonleptonic decays [29]. As indicated in Refs. [26,27,66], the nonfactorizable contributions in bottom baryon nonleptonic decays are considerable comparing with the factorized ones. Since a precise study of nonfactorizable contributions is beyond the scope of the present work, we still adopt the naïve factorization approximation.