# Lattice-QCD calculation of TMD wave functions through Large-Momentum Effective Theory

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Lattice Parton Collaboration

# Outline



- Motivation

- Methodology
  - Lattice QCD
  - Large Momentum effective theory

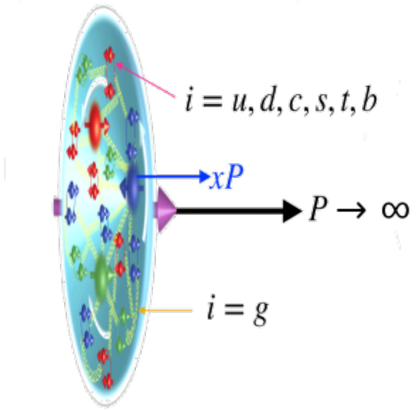
- Numerical results
  - Nonperturbative CS kernel calculation at 1 loop accuracy
  - *Intrinsic soft function determination

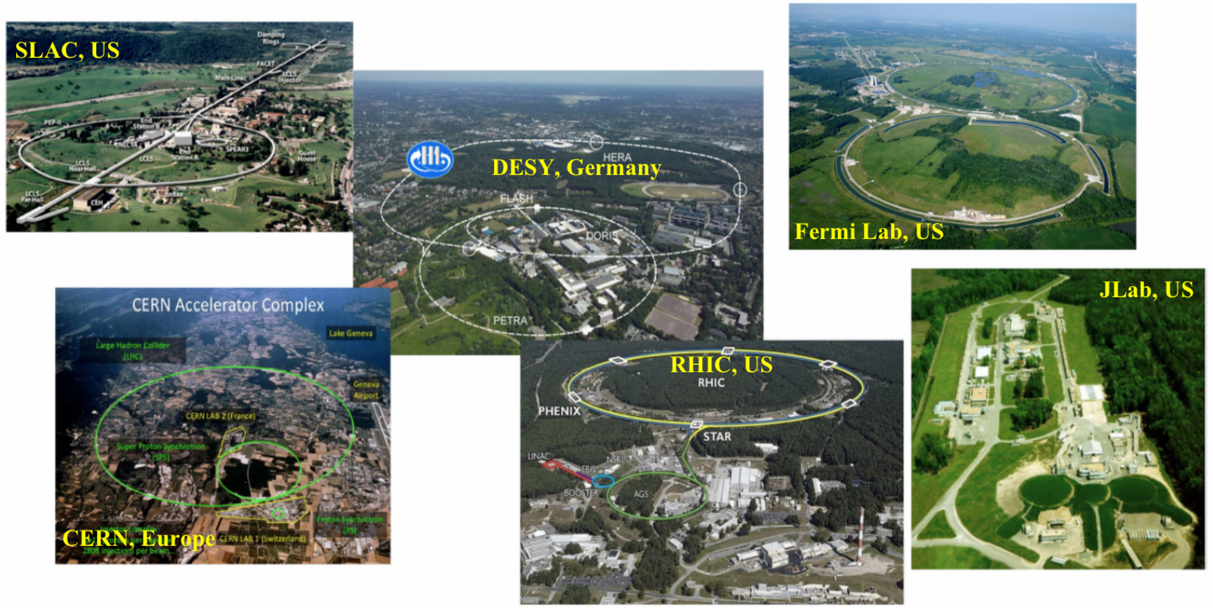
- Summary and outlook

# Motivation

- Feynman's parton model(1969)

1. Inner structure of nucleon through DIS process
2. Parton carries a portion of the longitudinal momentum of the nucleon.
3. Parton distribution functions (PDFs)
4. Distribution amplitude(DAs)



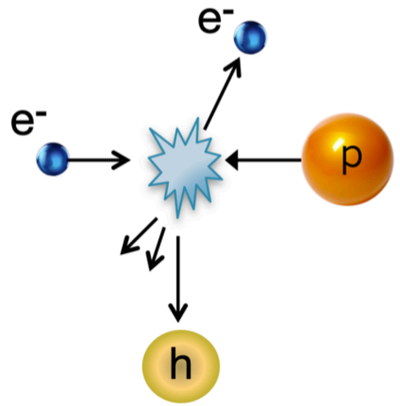

PDFs: the most important inputs for cross section in large hadron colliders.

DAs: important inputs for heavy hadron decays in detectors.

# Motivation

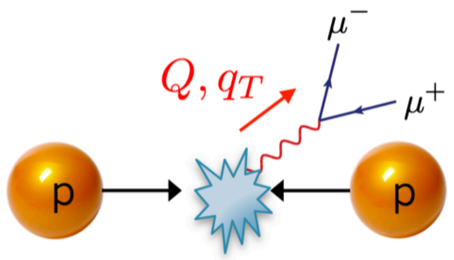
**Semi-Inclusive DIS**

$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$




**Drell-Yan**

$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$




*R. Angeles-Martinez et. al, arxiv 2507.05267 (2015)*

- TMDPDFs/TMDWFs as important inputs

- RG evolution ($\mu$)

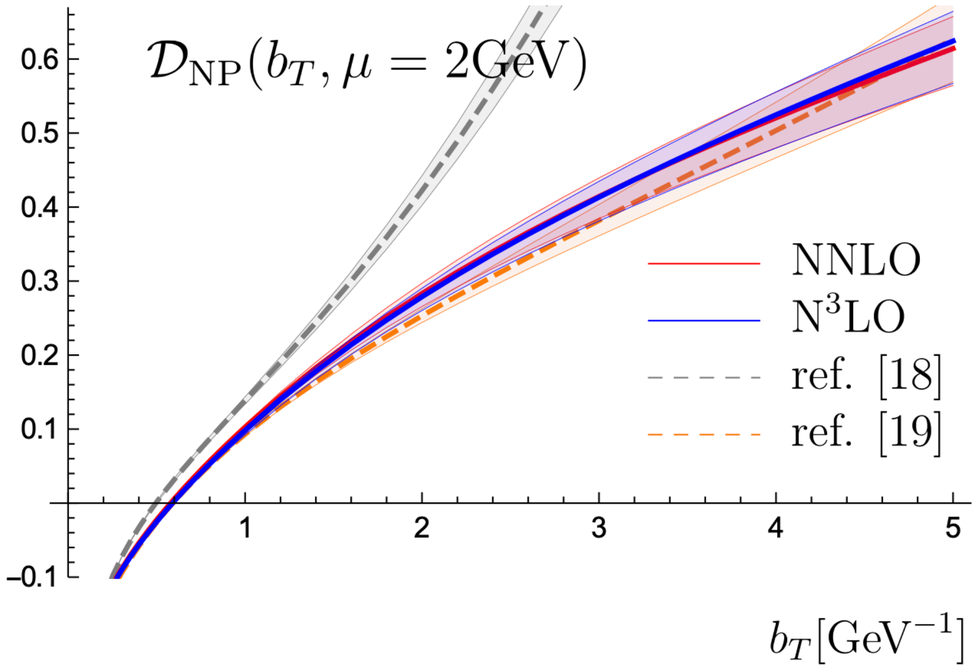
$$\mu \frac{d}{d\mu} f_{q/P}^{TMD}(x, b_\perp, \mu, \zeta) = \gamma(\mu, \zeta)$$

- Rapidity evolution ($\zeta$)

$$2\zeta \frac{d}{d\zeta} \ln f_{q/P}^{TMD}(x, b_\perp, \mu, \zeta) = K(b_\perp, \mu)$$

# Motivation

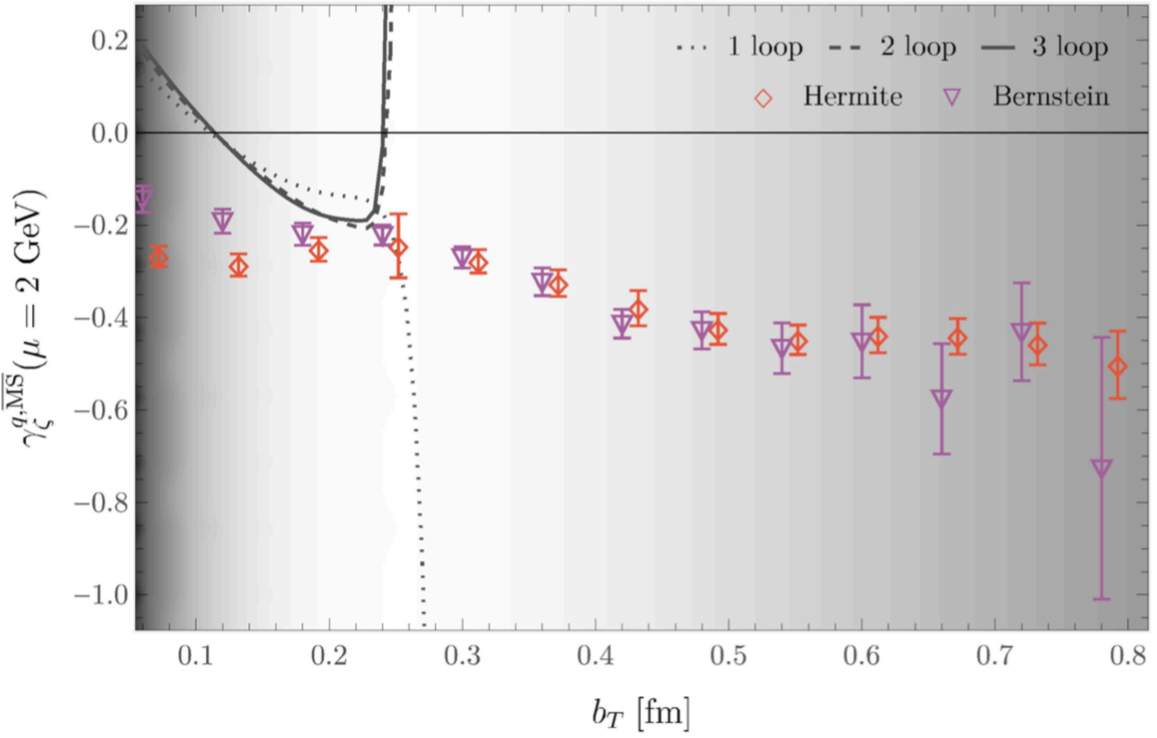
## phenomenological (traditional)




$$K(b_\perp, \mu) = -2(D_{NP} + \sum_{n=0}^{\infty} \alpha_s(\mu)^n d_n)$$

*I. Scimemi, A. Vadimirov, arxiv 1912.06532 (2020)*

## lattice




*P. Shanahan, M. Wangman, Y. Zhao, arxiv 2003.06063 (2020)*

From TMDPDFs

$$2\zeta\frac{d}{d\zeta}\ln f^{TMD}(x,b_\perp,\mu,\zeta)=K(b_\perp,\mu)$$

$$f^{TMD}(x,b_\perp,\mu,\zeta)\xrightarrow{\text{lattice}}\langle 0|\hat{O}(0)\hat{O}(b_\perp,z)\hat{O}(0)|0\rangle$$

$$C_3(b_\perp,z)$$

From TMDWFs

$$2\zeta\frac{d}{d\zeta}\ln \Psi^{TMD}(x,b_\perp,\mu,\zeta)=K(b_\perp,\mu)$$

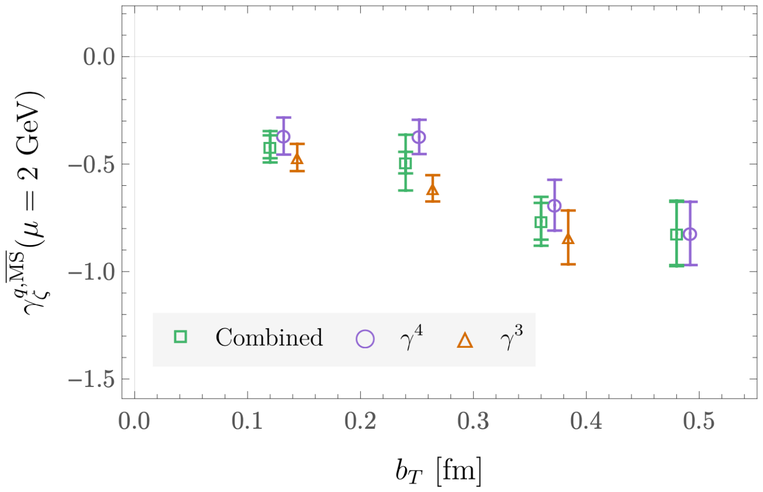
$$\Psi^{TMD}(x,b_\perp,\mu,\zeta)\xrightarrow{\text{lattice}}\langle 0|\hat{O}(b_\perp,z)\hat{O}(0)|0\rangle$$

$$C_2(b_\perp,z)$$

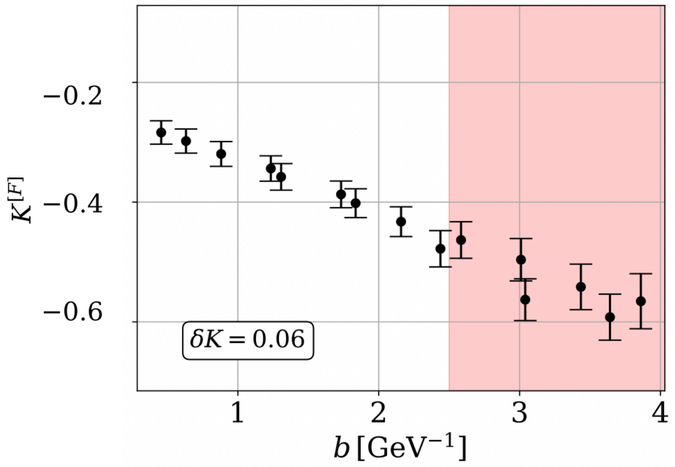
Simpler and has better signal for lattice calculation!

# Motivation

## $K(b_\perp, \mu)$ from TMDPDFs at **1-loop**

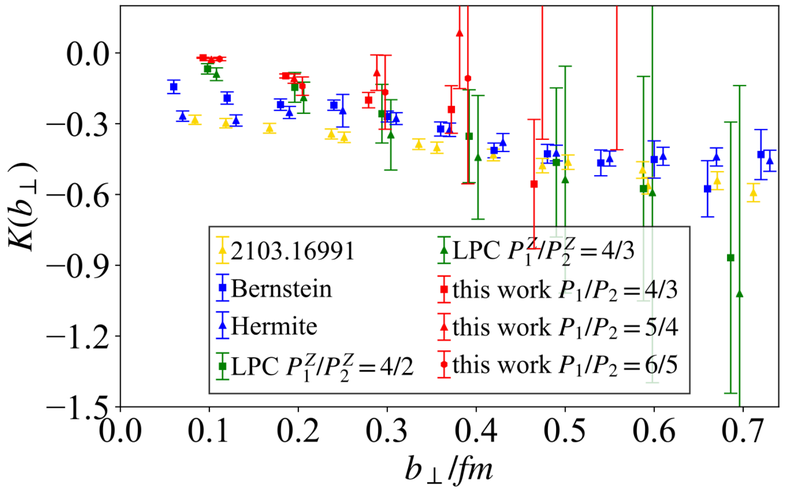



*P. Shanahan, M. Wangman, Y. Zhao, arxiv 2017.11903 (2021)*

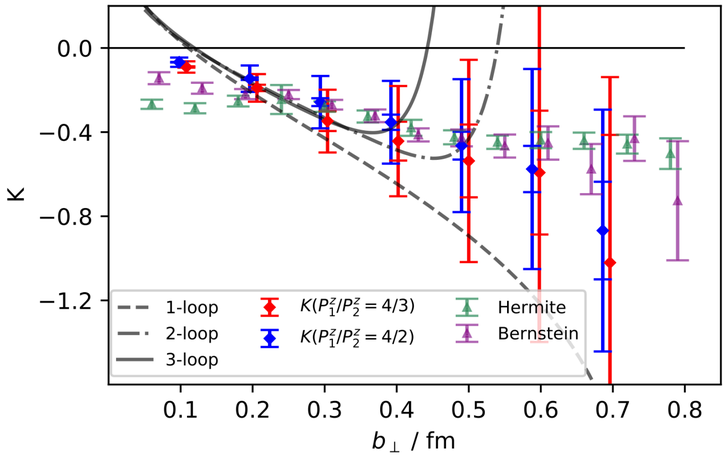



*M. Shlemmer et al, arxiv 2103.16991 (2021)*

## $K(b_\perp, \mu)$ from TMDWFs at **tree level**




*Y. Li et al, arxiv 2106.13027 (2021)*




*Q. Zhang et al, arxiv 2005.14572 (2020)*

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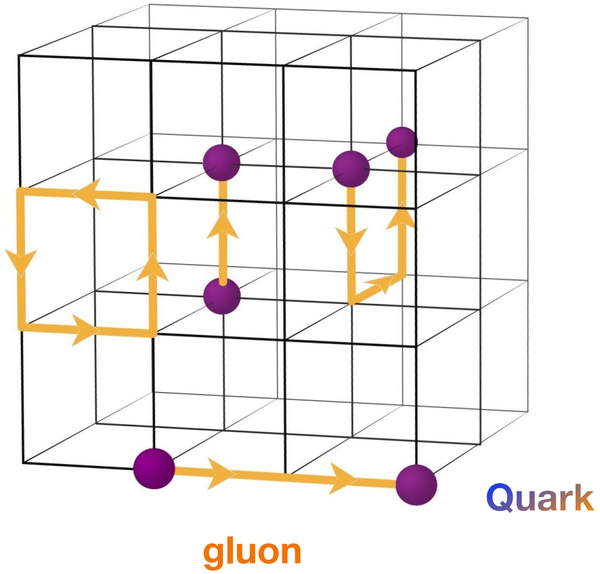
# Methodology: Lattice QCD

Numerical simulation on Euclidean space-time




Large Enough space (length $l \to \infty$), small enough resolving power (lattice spacing $a \to 0$) ;

Simulation of strong interaction (QCD) ;

Lattice QCD action

Gluon $$S_G^{\text{lat}}[U] = \frac{2}{g^2} \sum_{n \in \Lambda} \sum_{\mu < \nu} \text{Re tr}[1 - U_{\mu\nu}(n)]$$

Fermion $$S_F^{\text{lat}}[U] = a^4 \sum_{n \in \Lambda} \bar{\psi}(n)[\gamma^\mu D_\mu^{\text{lat}} + m]\psi(n)$$

Correlation functions on lattice

$$\langle \hat{O}(\bar{\psi}, \psi, U) \rangle = \frac{\int [\mathscr{D}U] e^{-S_G^{\text{lat}} - S_F^{\text{lat}}} \hat{O}(\bar{\psi}, \psi, U)}{\int [\mathscr{D}U] e^{-S_G^{\text{lat}} - S_F^{\text{lat}}}}$$

integral dimension

$$= L^3 \times T \times 4(spinor) \times 8(color) \sim 10^8$$

# Methodology: LaMET

## LaMET approach

X.D. Ji and Y.Z Liu, arxiv 2106.05310 (2021)

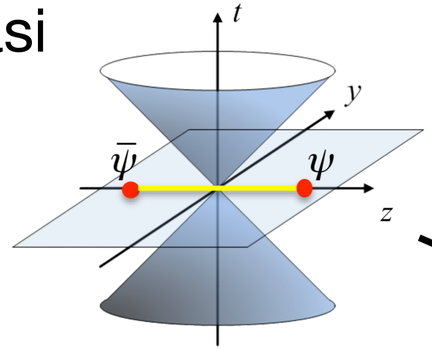
For a light-cone PDF

$$\langle \pi(P^+) | \bar{\psi}(\frac{\xi}{\sqrt{2}}\vec{n}^+)\Gamma\psi(-\frac{\xi}{\sqrt{2}}\vec{n}^+) | \pi(P^+)\rangle$$

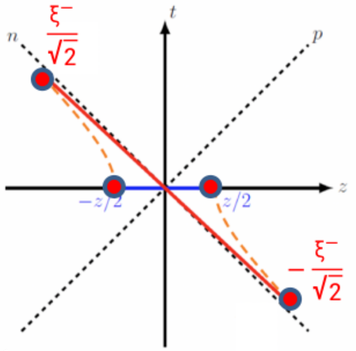
Define a lattice calculable, equal-time correlation: quasi-PDF

$$\langle \pi(P^z) | \bar{\psi}(-\frac{z}{2}\vec{n}^z)\Gamma\psi(\frac{z}{2}\vec{n}^z) | \pi(P^z)\rangle$$

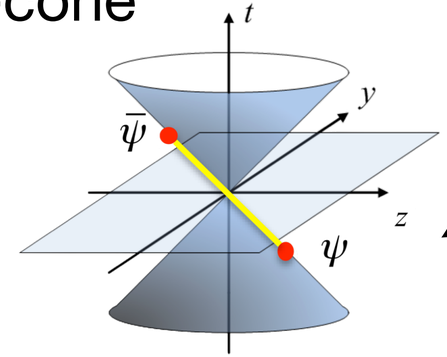
quasi




Boosting $P^z$




light-cone




EFT

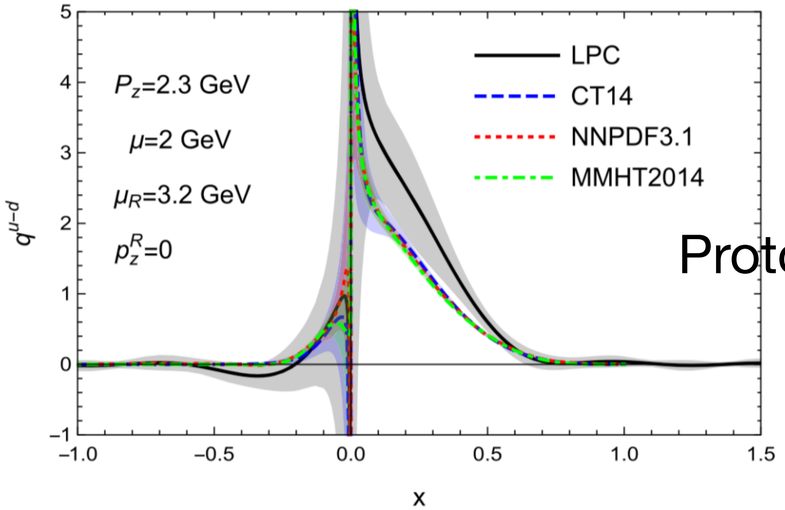
Instead of taking $P^z \to \infty$ limit, one can perform an expansion for large but finite $P^z$:

$$\tilde{f}(y, P^z, \tilde{\mu}) = \int_{-1}^{1} \frac{dx}{|x|} C\left(\frac{y}{x}, \frac{\mu}{xP^z}, \frac{\tilde{\mu}}{\mu}\right) f(x,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(yP^z)^2}, \frac{\Lambda^2_{\rm QCD}}{((1-y)P^z)^2}\right)$$

# Methodology: LaMET

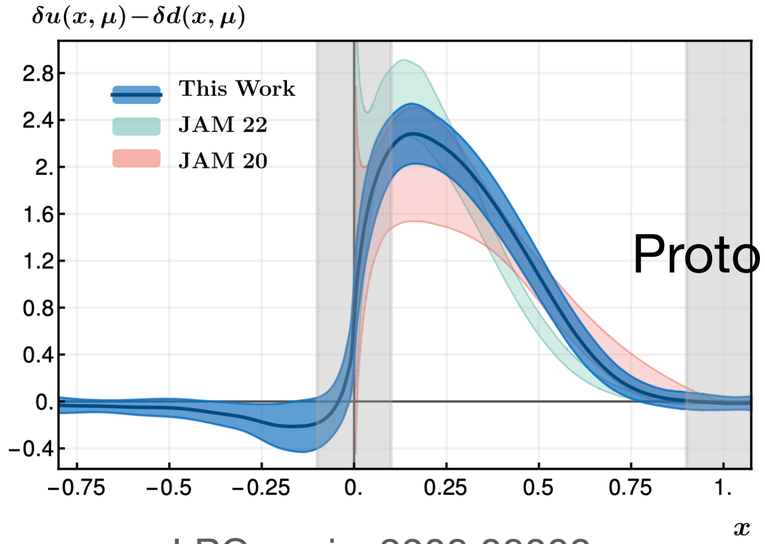
## LaMET achievement

*X.D. Ji and Y.Z Liu, arxiv 2106.05310 (2021)*



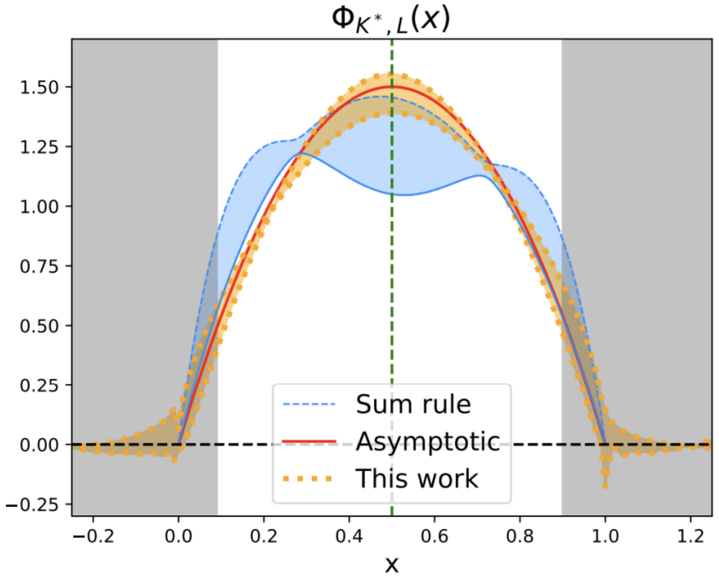

Proton unpolarized PDF

LPC, PRD101 (2020), 034020



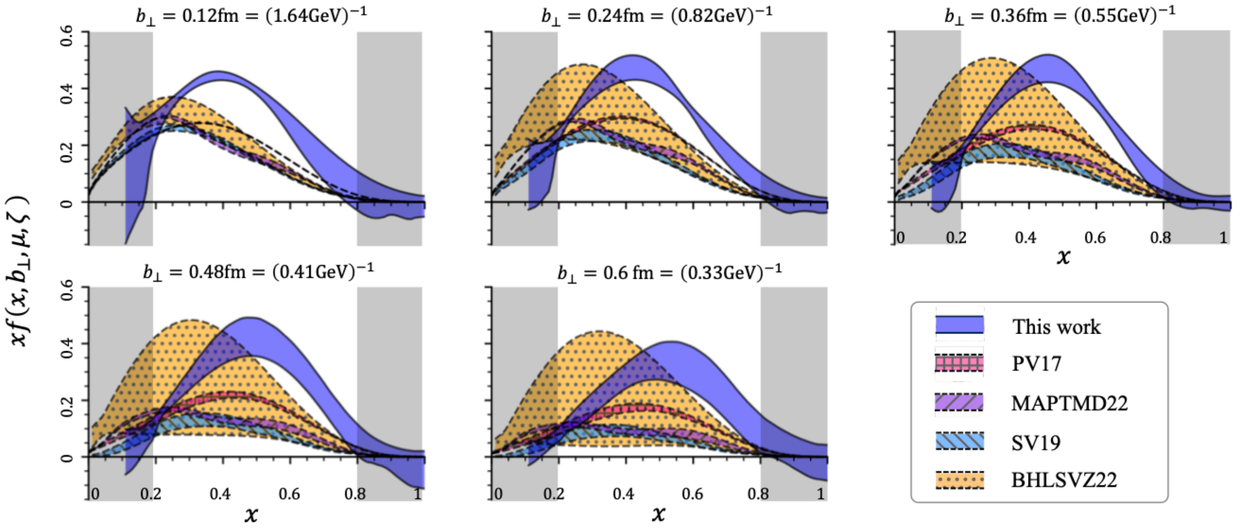

Proton transversity PDF

LPC, arxiv: 2208.08008




light meson LCDA

LPC, PRL127 (2021), 062002




LPC, arxiv: 2211.02340

Proton TMDPDF

# Methodology: LaMET

## TMD factorization

X.D. Ji and Y.Z Liu, arxiv 2106.05310 (2021)

$$\tilde{\Psi}^{\pm}\left(x, b_{\perp}, \mu, \zeta^{z}\right) S_{I}^{\frac{1}{2}}\left(b_{\perp}, \mu\right) = H^{\pm}\left(x, \zeta^{z}, \mu\right) e^{\left[\frac{1}{2} K\left(b_{\perp}, \mu\right) \ln \frac{\mp \zeta^{z}+i \epsilon}{\zeta}\right]} \Psi^{\pm}\left(x, b_{\perp}, \mu, \zeta\right)$$

$\Psi(x, b_\perp, \zeta)$: TMDWF, physical observable, nonperturbative.

$\tilde{\Psi}(x, b_\perp, \zeta^z)$: Quasi-TMDWF, calculable on lattice.

$S_r(b_\perp, \mu)$: intrinsic soft function, nonperturbative, need form factor on lattice.

$H(\zeta^z, \bar{\zeta}^z)$: Perturbative matching coefficient, up to 1-loop.

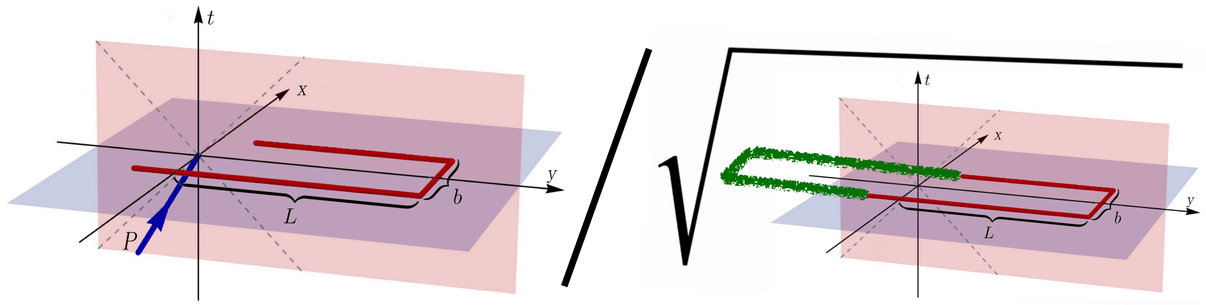
$K(b_\perp, \mu)$: Collins-Soper kernel, perturbative + nonperturbative.

## Collins-Soper kernel

$$K(b_{\perp}, \mu)=\frac{1}{\ln (P_{2}^{z} / P_{1}^{z})} \ln \left[\frac{H_{1}^{\pm}(\zeta_{1}^{z}, \bar{\zeta}_{1}^{z}) \tilde{\Psi}^{\pm}(x, b_{\perp} \zeta_{2}^{z})}{H_{1}^{\pm}(\zeta_{2}^{z}, \bar{\zeta}_{2}^{z}) \tilde{\Psi}^{\pm}(x, b_{\perp}, \zeta_{1}^{z})}\right]$$

## quasi-TMDWF

$$\tilde{\Phi}^{\pm}(z, b_{\perp}, \zeta^{z})=\frac{\langle 0|\bar{\psi}_{\mp n^{z}}(z n^{z} / 2+b_{\perp} n^{x}) \Gamma \psi_{\mp n^{z}}(-z n^{z} / 2)|\pi\rangle}{\sqrt{Z_{E}(2 L, b_{\perp})}}$$



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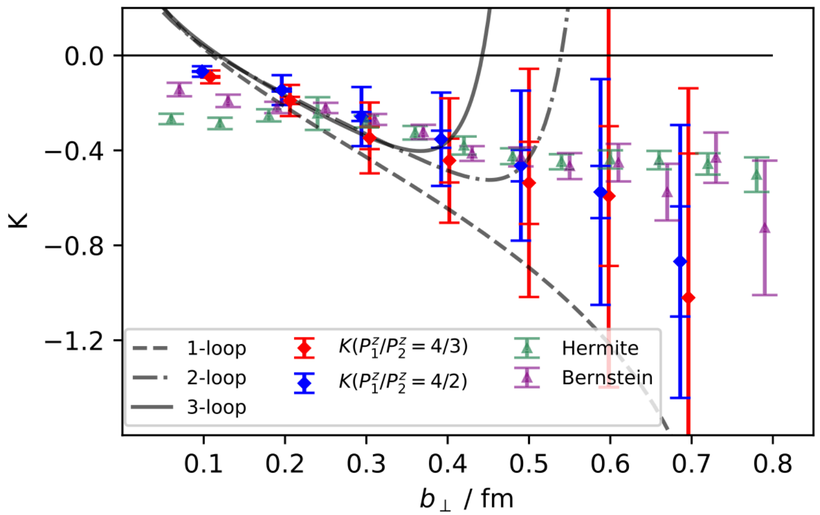
# Numerical Result: Lattice setup

## CLS configurations (previous calculation)

| $L^3 \times T$ | $a$ (fm) | $c_{sw}$ | $\kappa_l^{\text{sea}}$ | $m_\pi^{\text{sea}}$ (MeV) |
|---|---|---|---|---|
| $24^3 \times 48$ | 0.098 | 2.066 86 | 0.136 75 | 333 |
| 2.35 fm | | $N_{cfg}$ | $\kappa_l^v$ | $m_\pi^v$ (MeV) |
| | | 864 | 0.136 22 | 547 |



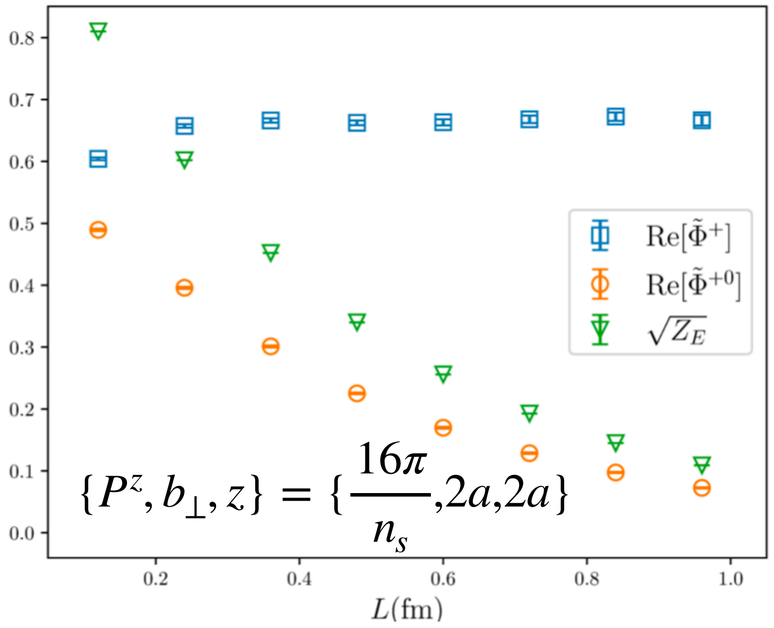

*Q. Zhang et al, arxiv 2005.14572 (2020)*

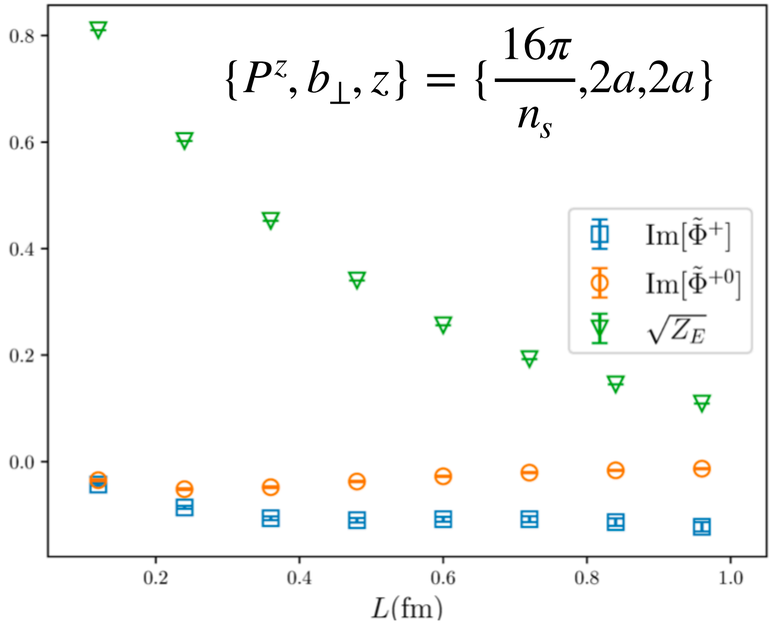
## MILC configurations (this work)

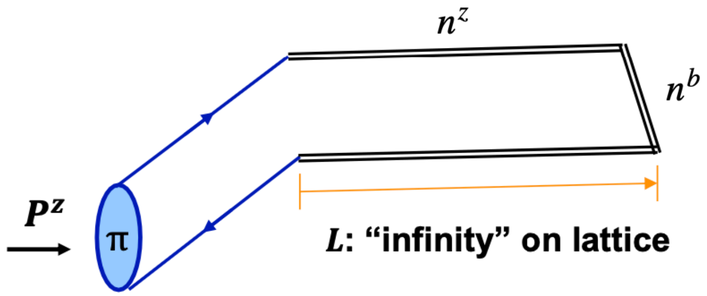
| $L^3 \times T$ | $a$ (fm) | $c_{sw}$ | $\kappa_l^{sea}$ | $m_\pi^{sea}$ (MeV) |
|---|---|---|---|---|
| $48^3 \times 64$ | 0.12 | 1.05088 | 0.12750 | 130 |
| 5.76fm | | $N_{cfg}$ | $\kappa_l^v$ | $m_\pi^v$ (MeV) |
| | | 382 | 0.12560 | 670 |

- 2+1+1 flavors of HISQ action (MILC)
- Momenta: 1.72GeV, 2.15GeV, 2.58GeV
- Gamma factor: 2.57, 3.21, 3.85

# Numerical Result: Large L limit




$$\tilde{\Psi}(z, b_\perp) = \lim_{L\to\infty} \tilde{\Psi}(L, z, b_\perp)$$

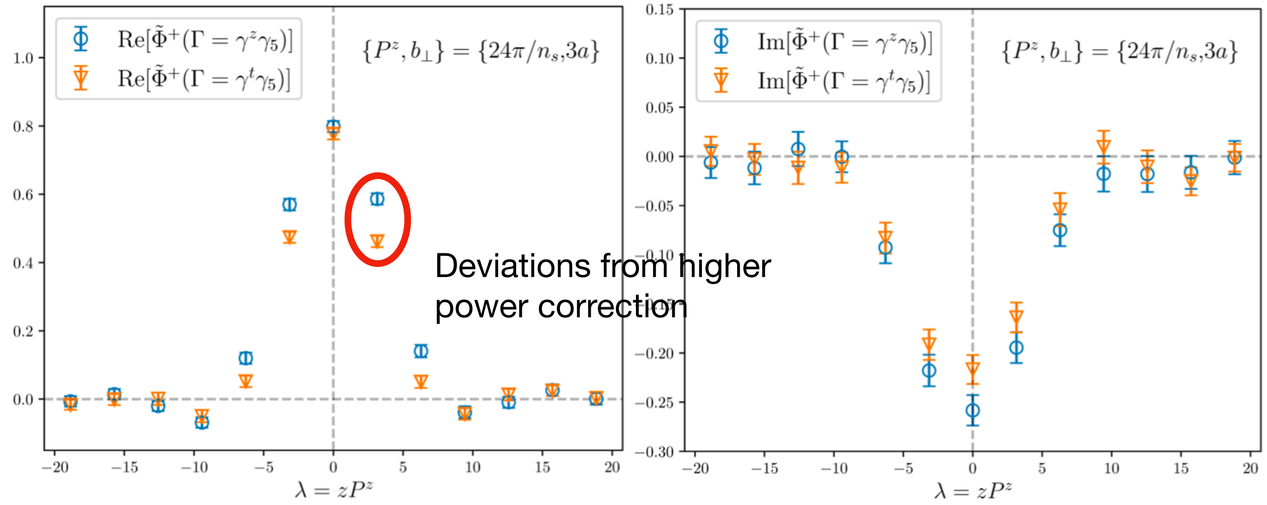
$$\tilde{\Phi}^{\pm}(z, b_\perp, \zeta^z) = \frac{\langle 0 \,|\, \overline{\psi}_{\mp n^z}(z n^z/2 + b_\perp n^x)\Gamma \psi_{\mp n^z}(-z n^z/2) \,|\, \pi\rangle}{\sqrt{Z_E(2L, b_\perp)}}$$

- $\tilde{\Phi}^{\pm 0}$ and $\sqrt{Z_E}$ contain linear divergence.
- linear divergence is cancelled at large $L$ by the ratio of $\tilde{\Phi}^{\pm 0}$ and $\sqrt{Z_E}$ .
- $L = L_0 = 6a = 0.72\mathrm{fm}$ as asymptotic

# Numerical Result: Quasi-TMDWFs in coordinate space



## Operator mixing



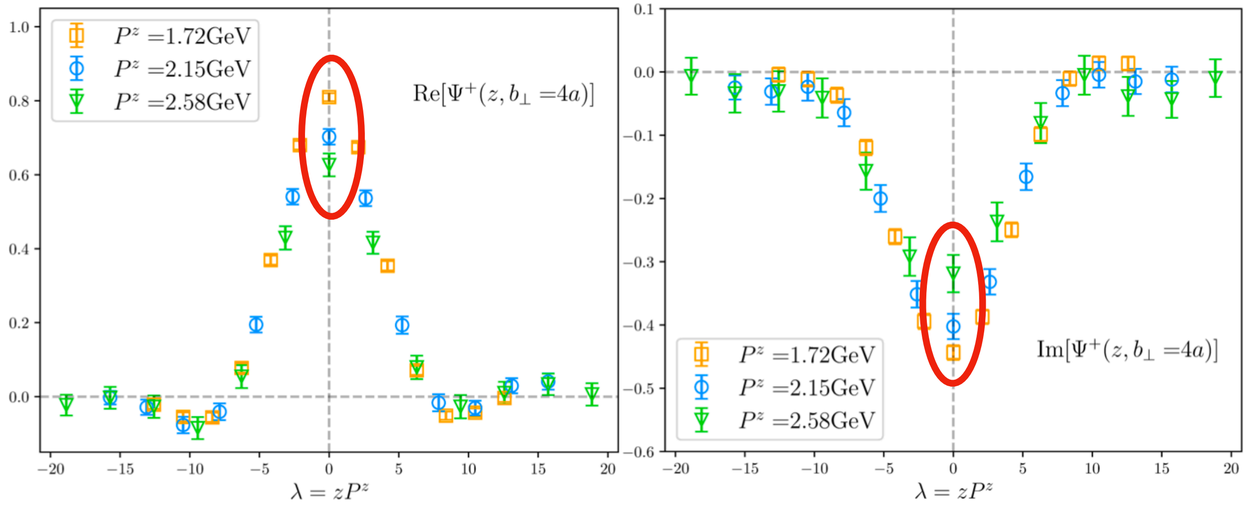

Deviations from higher power correction

- $\gamma^t\gamma_5$ and $\gamma^z\gamma_5$ give similar results. (Consistent with perturbative calculation)

*Zhi-Fu Deng et al, arxiv 2207.07280 (2022)*

- $\tilde{\Psi} = \frac{1}{2}[\tilde{\Psi}(\Gamma=\gamma^t\gamma_5) + \tilde{\Psi}(\Gamma=\gamma^z\gamma_5)]$

## Rapidity dependence




- $\tilde{\Psi}$ has small differences between rapidity, due to CS evolution kernel.

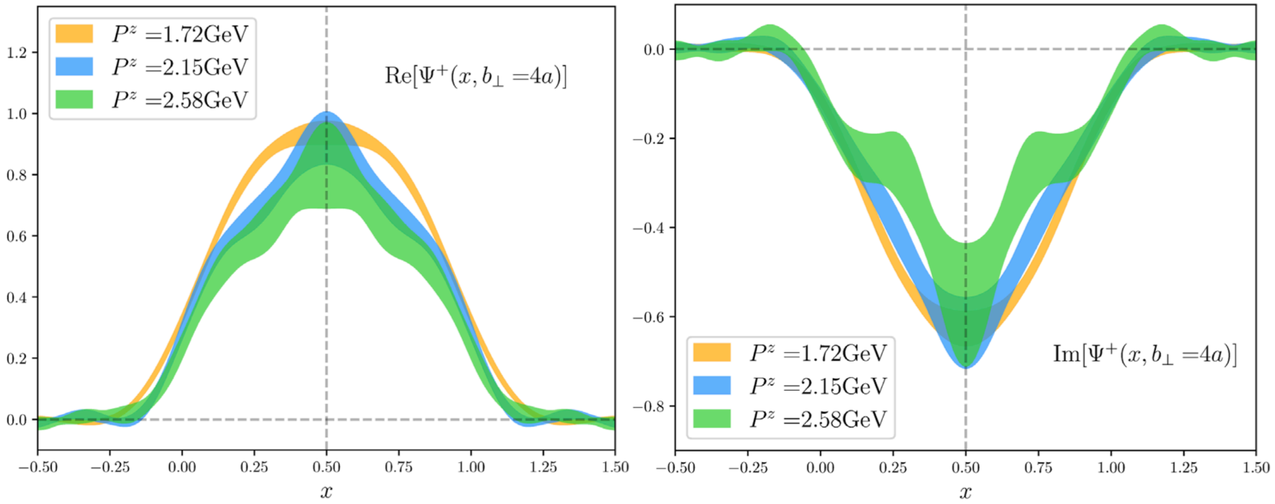
$$K(b_\perp,\mu) = \frac{1}{\ln(P_2^z/P_1^z)} \ln\left[\frac{H_1^\pm(\zeta_1^z,\bar{\zeta}_1^z)\tilde{\Psi}^\pm(x,b_\perp\zeta_2^z)}{H_1^\pm(\zeta_2^z,\bar{\zeta}_2^z)\tilde{\Psi}^\pm(x,b_\perp,\zeta_1^z)}\right]$$

* Tree level kernel $H_1^\pm(\zeta_1^z,\bar{\zeta}_1^z) = 1$

$$K(b_\perp,\mu) = \frac{1}{\ln(P_2^z/P_1^z)} \ln\frac{\int dx\tilde{\Psi}^\pm(x,b_\perp\zeta_2^z)}{\int dx\tilde{\Psi}^\pm(x,b_\perp,\zeta_1^z)} = \frac{1}{\ln(P_2^z/P_1^z)} \ln\frac{\tilde{\Psi}^\pm(z=0,b_\perp\zeta_2^z)}{\tilde{\Psi}^\pm(z=0,b_\perp,\zeta_1^z)}$$

# Numerical Result: Quasi-TMDWFs in momentum space

## Rapidity dependence



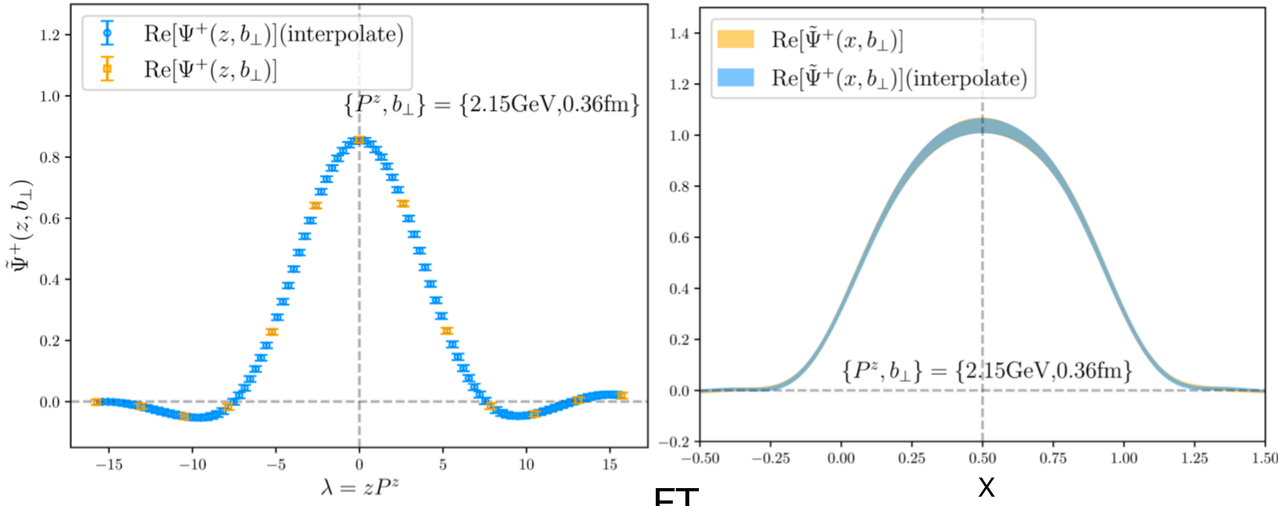

Fourier transformation

$$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta^z) = \lim_{L\to\infty} \int \frac{P^z dz}{2\pi} e^{i(x-1/2)P^z z} \tilde{\Phi}^{\pm}(z, b_\perp, \zeta^z)$$

$$= \sum_{-z_{max}}^{z_{max}} \frac{P^z \Delta z}{2\pi} e^{i(x-1/2)P^z z} \tilde{\Phi}^{\pm}(L = L_0, z, b_\perp, \zeta^z)$$

quasi-WF $\tilde{\Psi}(x, b_\perp, \mu, \zeta^z)$ is complex !

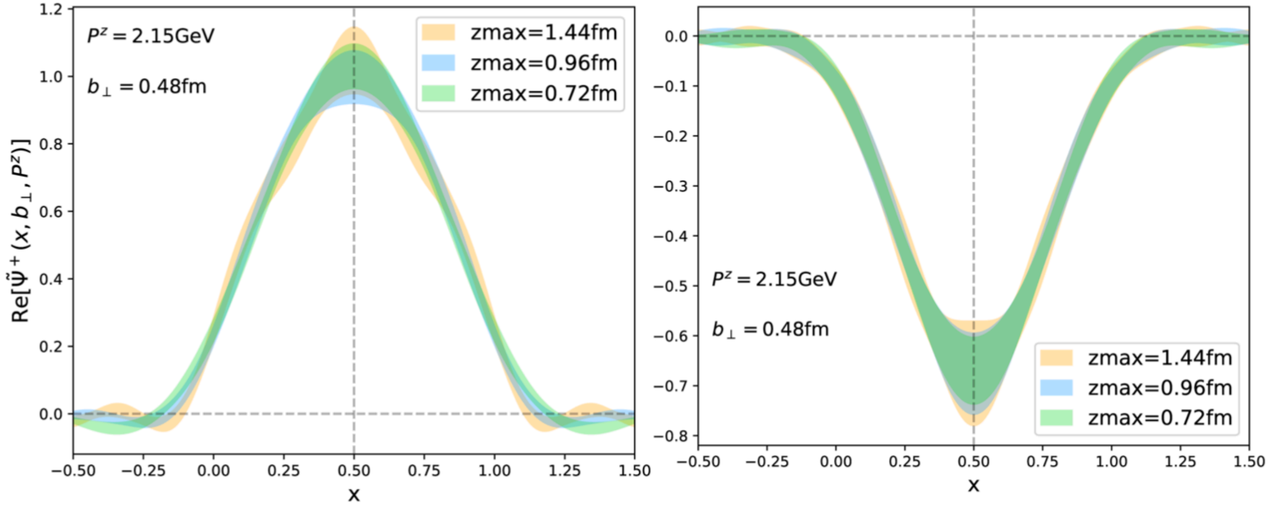
## Discrete effect




FT

$\tilde{\Psi}(z, b_\perp, \mu, \zeta^z) \longrightarrow \tilde{\Psi}(x, b_\perp, \mu, \zeta^z)$

## $z_{max}$ influence

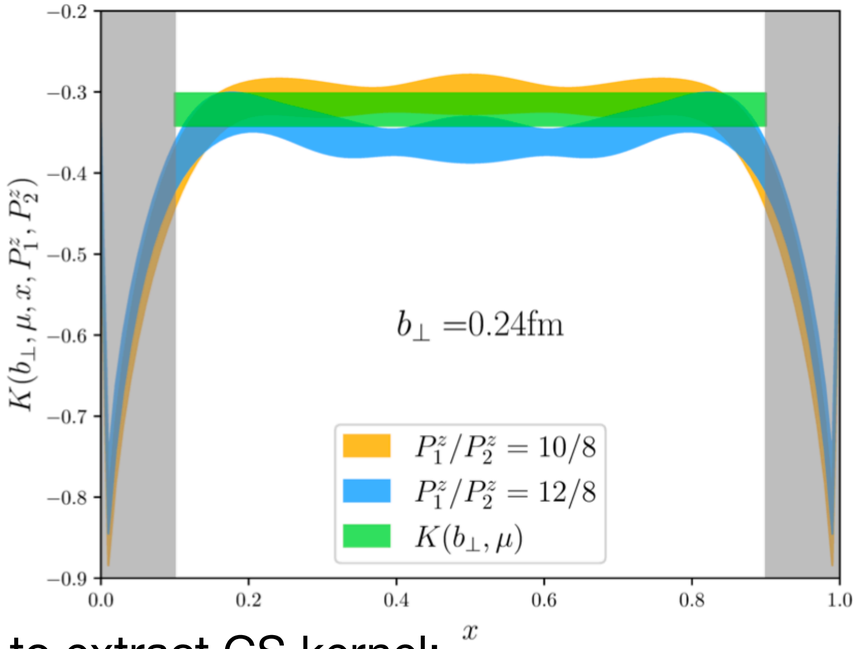


# Numerical Result: Collins-Soper Kernel

## Power correction

$$K(b_\perp,\mu)=\frac{1}{\ln(P_2^z/P_1^z)}\ln\left[\frac{H_1^{\pm}(\zeta_1^z,\bar{\zeta}_1^z)\tilde{\Psi}^{\pm}(x,b_\perp\zeta_2^z)}{H_1^{\pm}(\zeta_2^z,\bar{\zeta}_2^z)\tilde{\Psi}^{\pm}(x,b_\perp,\zeta_1^z)}\right]$$

Power correction contributions: $\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2(P^z)^2},\frac{\Lambda_{QCD}^2}{(1-x)^2(P^z)^2}\right)$

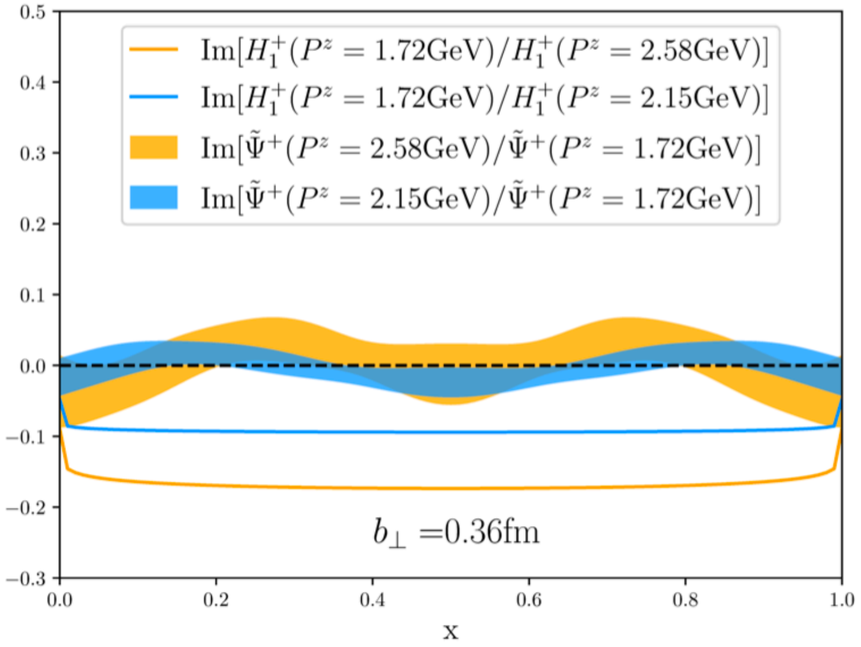



Joint fit to extract CS kernel:

$$K(b_\perp,\mu,x,P_1^z,P_2^z)=K(b_\perp,\mu)+A\left[\frac{1}{x^2(1-x)^2(P_1^z)^2}-\frac{1}{x^2(1-x)^2(P_2^z)^2}\right]$$

## Unexpected imaginary part

$$Im[K(b_\perp)]=\frac{1}{\ln(P_1^z/P_2^z)}\left(Im\left[\ln\frac{H_1^{\pm}(xP_2^z,\mu)}{H_1^{\pm}(xP_1^z,\mu)}\right]+Im\left[\ln\frac{\tilde{\Psi}^{\pm}(x,b_\perp,\mu,P_1^z)}{\tilde{\Psi}^{\pm}(x,b_\perp,\mu,P_2^z)}\right]\right)$$




The imaginary part mainly comes from the ratio of 1 loop matching coefficient.

# Numerical Result: Collins-Soper Kernel

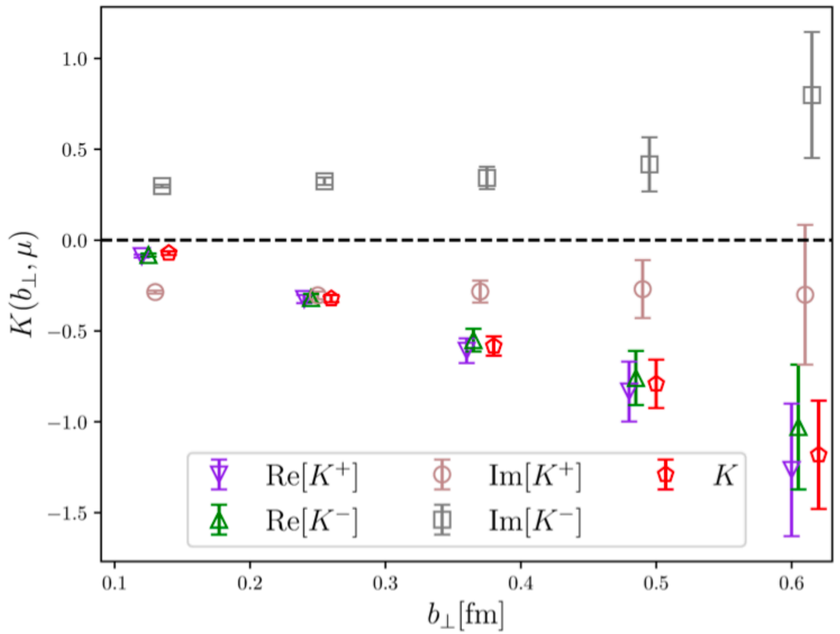
1 loop matching coefficient

$$H^{\pm}\left(\zeta_z,\bar{\zeta}_z,\mu\right)=1+\frac{\alpha_s C_F}{4\pi}\left[-\frac{5\pi^2}{6}-4+\ln\frac{-\zeta_z\pm i\epsilon}{\mu^2}+\ln\frac{-\bar{\zeta}_z\pm i\epsilon}{\mu^2}-\frac{1}{2}\left(\ln^2\frac{-\zeta_z\pm i\epsilon}{\mu^2}+\ln^2\frac{-\bar{\zeta}_z\pm i\epsilon}{\mu^2}\right)\right]$$

$$=1+\frac{\alpha_s C_F}{4\pi}\left[-\frac{5\pi^2}{6}-4+\left(\ln\frac{\zeta_z}{\mu^2}+\ln\frac{\bar{\zeta}_z}{\mu^2}\right)-\frac{1}{2}\left(\ln^2\frac{\zeta_z}{\mu^2}+\ln^2\frac{\bar{\zeta}_z}{\mu^2}\right)+\pi^2\pm 2i\pi\mp i\pi\left(\ln\frac{\zeta_z}{\mu^2}+\ln\frac{\bar{\zeta}_z}{\mu^2}\right)\right]$$

CS kernel from two gauge line directions

$$K^{\pm}(b_\perp,\mu)=\frac{1}{\ln(P_2^z/P_1^z)}\ln\left[\frac{H_1^{\pm}(\zeta_2^z,\mu)\tilde{\Psi}^{\pm}(b,x,P_1^z)}{H_1^{\pm}(\zeta_1^z,\mu)\tilde{\Psi}^{\pm}(b,x,P_2^z)}\right]$$




The imaginary part

$$Im\left[\ln\frac{H_1^{+}(\zeta_2^z,\mu)}{H_1^{+}(\zeta_1^z,\mu)}\right]=-\,Im\left[\ln\frac{H_1^{-}(\zeta_2^z,\mu)}{H_1^{-}(\zeta_1^z,\mu)}\right]$$

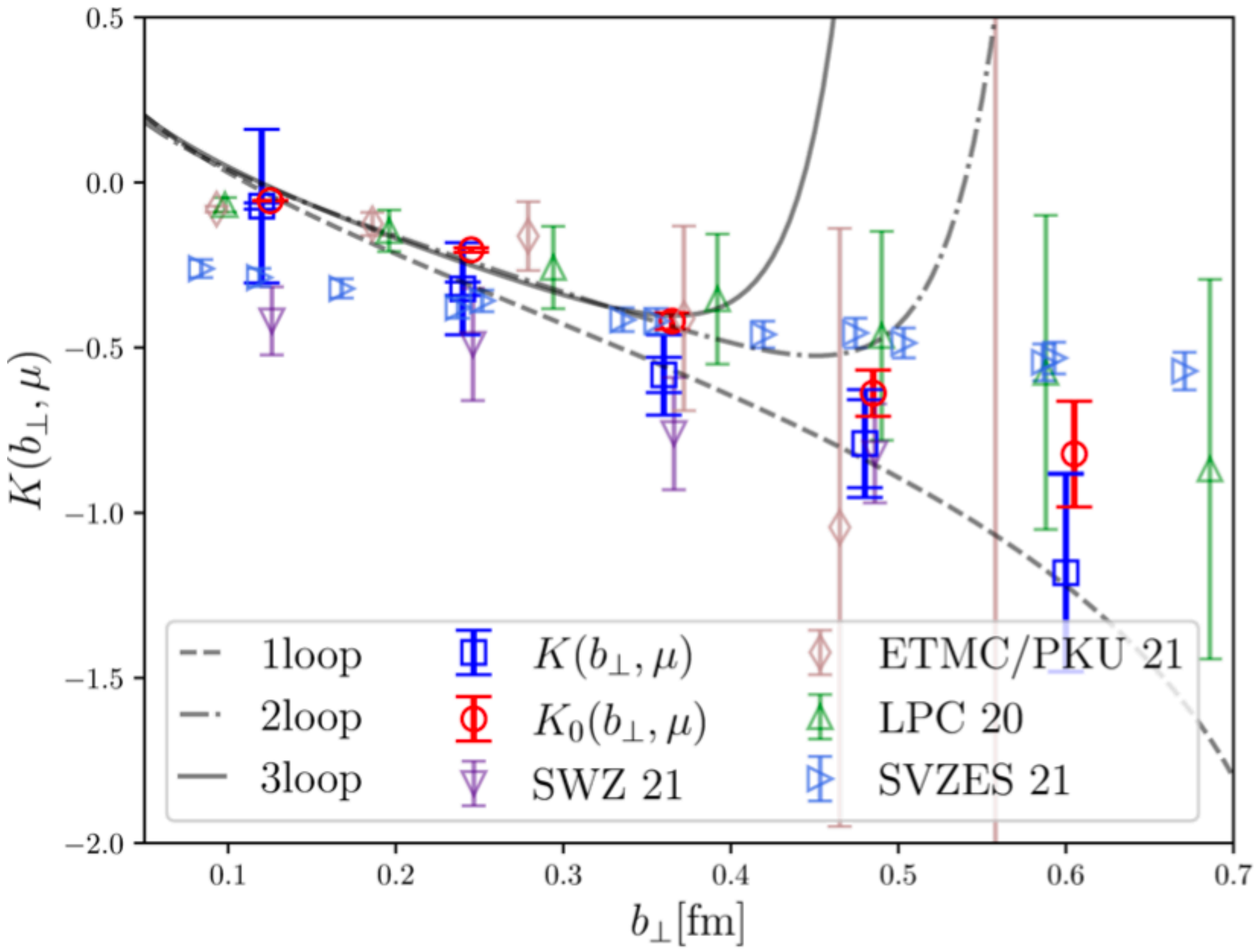
Redefine CS kernel

$$K(b_\perp,\mu)=\frac{1}{2}[K^{+}(b_\perp,\mu)+K^{-}(b_\perp,\mu)]$$

Systematic uncertainty

$$\sigma_{sys}=\sqrt{K^2(b_\perp,\mu)+Im^2[K^{+}(b_\perp,\mu)]-K(b_\perp,\mu)}$$

# Numerical Result: Comparison with previous LQCD




- Perturbative calculation:

  1loop, 2loop, 3loop: Y. Li et al, arxiv 1604.01404 (2017)

- From TMDWFs at tree level:

  ETMC/PKU 21: Y. Li et al, arxiv 2106.13027 (2021)

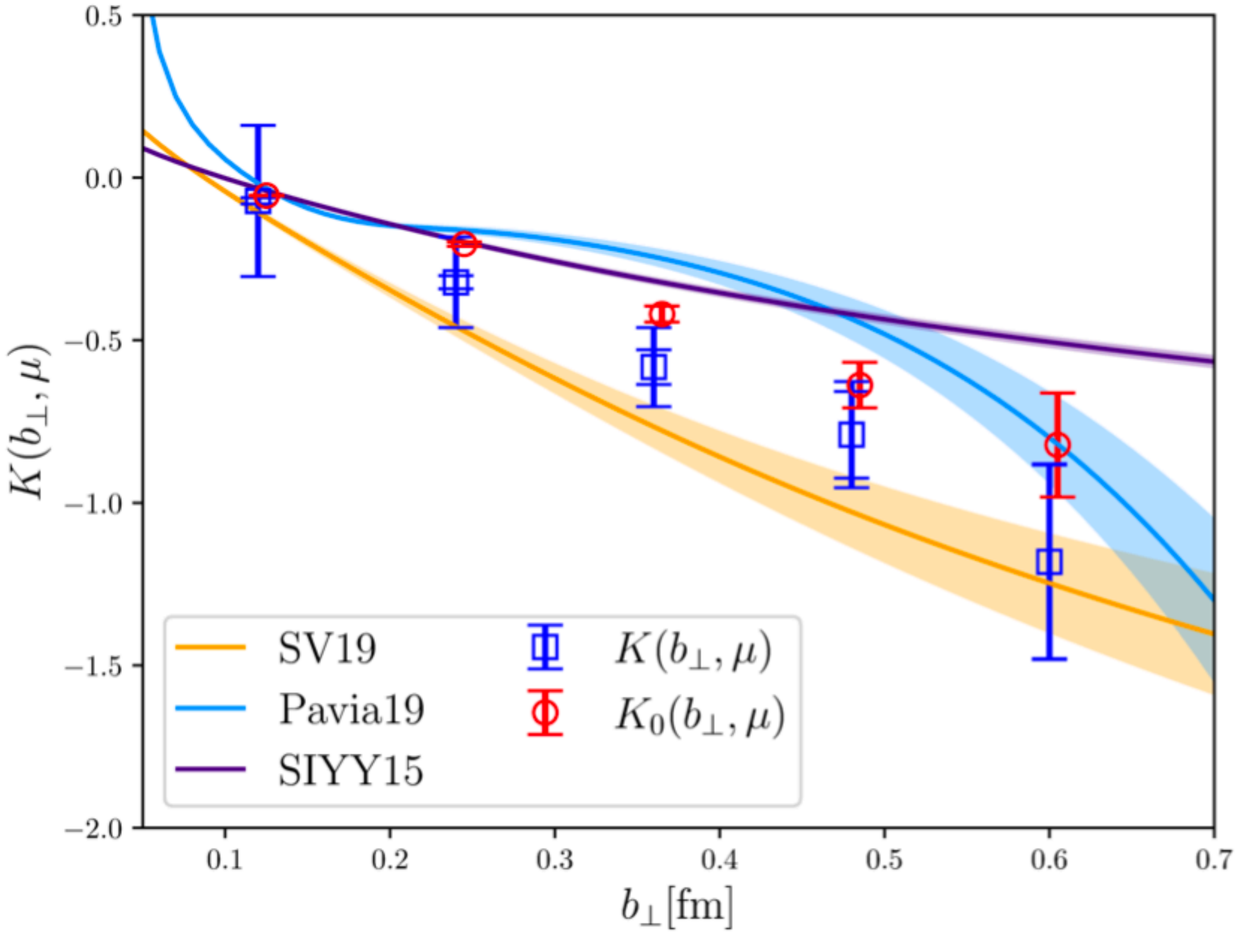
  LPC 20: Q. Zhang et al, arxiv 2005.14572 (2020)

- From TMDPDFs at 1-loop:

  SWZ 21: P. Shanahan et al, arxiv 2017.11903 (2021)

  SVZES 21: M. Schlemmer et al, arxiv 2103.16991 (2021)

# Numerical Result: Comparison with phenomenological




- Parameterization of $K(b_\perp,\mu)$:

  SV19: I. Scimemi et al, arxiv: 1912.06532 (2020)

- Parameterization of TMDPDFs:

  Pavia19: A. Bacchetta et al, arxiv: 1912.07550 (2020)

- Parameterization of TMDPDFs:

  SIYY15: Sun Peng et al, arxiv: 1406.3073 (2015)

# Numerical Result: Intrinsic soft function

## TMD factorization

soft gluon radiation

$$\tilde{\Psi}^{\pm}\left(x, b_{\perp}, \mu, \zeta^{z}\right) S_{I}^{\frac{1}{2}}\left(b_{\perp}, \mu\right)$$

$$= H^{\pm}\left(x, \zeta^{z}, \mu\right) e^{\left[\frac{1}{2} K\left(b_{\perp}, \mu\right) \ln \frac{\mp \zeta^{z}+i \epsilon}{\zeta}\right]} \Psi^{\pm}\left(x, b_{\perp}, \mu, \zeta\right)$$

*X.D. Ji and Y.Z Liu, arxiv 2106.05310 (2021)*

## soft function matching

$$S_{I}\left(b_{\perp}, \mu\right)=\frac{F\left(b_{\perp}, P_{1}, P_{2}, \Gamma, \mu\right)}{\int dx_{1} dx_{2} H\left(x_{1}, x_{2}, \Gamma\right) \tilde{\Psi}^{\pm *}\left(x_{2}, b_{\perp}, \zeta^{z}\right) \tilde{\Psi}^{\pm}\left(x_{1}, b_{\perp}, \zeta^{z}\right)}$$

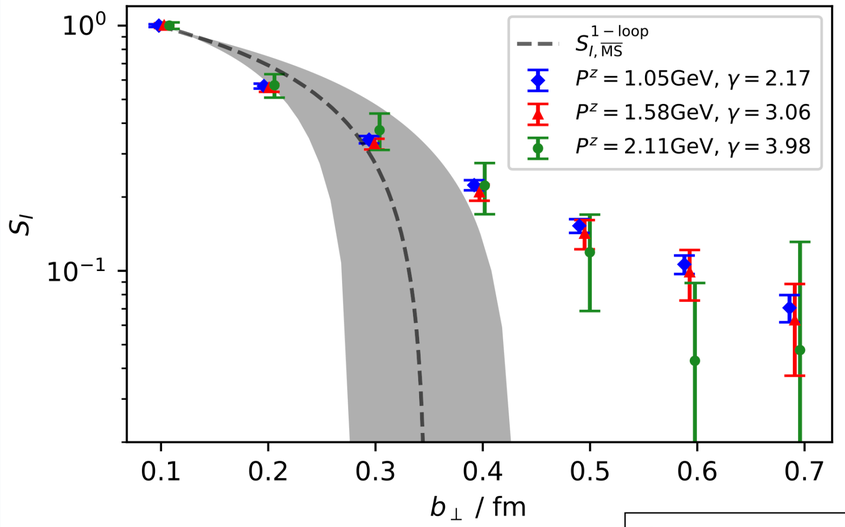
*Z.F. Deng et al, arxiv 2207.07280(2022)*

$F\left(b_{\perp}, P, \Gamma, \mu\right)$: four quark form factor, nonperturbative.

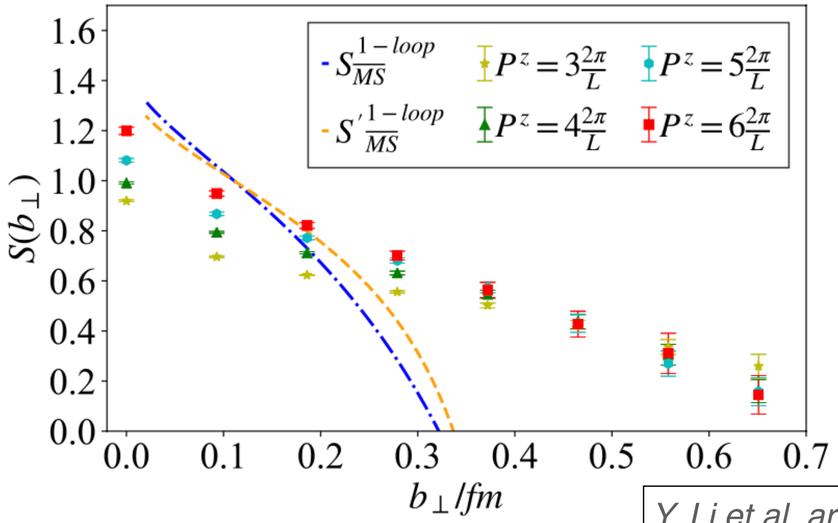
$H\left(x_{1}, x_{2}, \Gamma\right)$: Perturbative matching coefficient.

$\tilde{\Psi}^{\pm}(x, b_{\perp}, \zeta)$: quasi-TMDWF, nonperturbative.

## Tree level soft function results




*Q. Zhang et al, arxiv 2005.14572 (2020)*




*Y. Li et al, arxiv 2106.13027 (2021)*

# Numerical Result: Intrinsic soft function

## normalization and Dirac structure

$$F(b_\perp, \Gamma, P^z) = \frac{\langle \pi(P_2) | (\bar{q}\Gamma q)|_{b_\perp} (\bar{q}\Gamma q)|_0 | \pi(P_1)\rangle}{\langle \pi(P_2) | (\bar{q}\Gamma' q)|_0 |0\rangle\langle 0| (\bar{q}\Gamma' q)|_0 | \pi(P_1)\rangle}$$

$$\Gamma' = \gamma^\mu\gamma_5,\ P_2 = (P^z,0,0,-P^z), P_1 = (P^z,0,0,P^z)$$

*Z.F. Deng et al, arxiv 2207.07280(2022)*

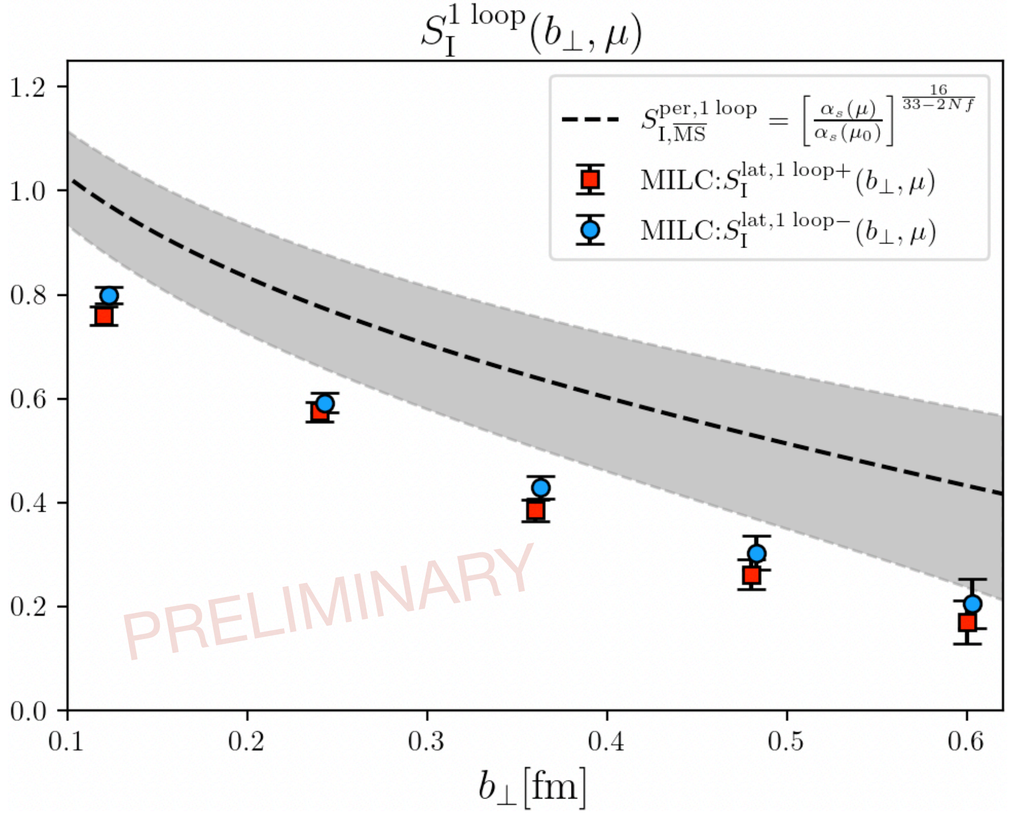
Fierz rearrangement

UV divergence

$$F(\Gamma) = -F(\gamma^x) - F(\gamma^y) - F(\gamma^x\gamma_5) - F(\gamma^y\gamma_5)$$

## 1 loop soft function

$$S_I(b_\perp, \mu) = \frac{F(b_\perp, P_1, P_2, \Gamma, \mu)}{\int dx_1 dx_2 H(x_1, x_2, \Gamma)\, \tilde{\Psi}^{\pm *}(x_2, b_\perp, \zeta^z)\, \tilde{\Psi}^{\pm}(x_1, b_\perp, \zeta^z)}$$




Based on MILC a12m310

# Outline

- Motivation

- Methodology
  - Lattice QCD
  - Large Momentum effective theory

- Numerical results
  - Nonperturbative CS kernel calculation at 1 loop accuracy
  - *Intrinsic soft function determination

- Summary and outlook

# Summary and outlook

- Collins-Soper kernel describes the rapidity evolution of TMDPDFs/ TMDWFs, which has added evidences for the extraction of partonic structure from LQCD in LaMET.

- This is the first attempt for extracting Collins-Soper kernel from TMDWFs at 1-loop level accuracy.

- More progress on calculation of soft function and light cone TMDWF at 1 loop accuracy….

# Thank you for your attention!