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Supersymmetric Alignment Models for $(g-2)_\mu$

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Based on YN, M. Reece and M. Suzuki (Harvard), JHEP 2021.

Muon g-2 anomaly

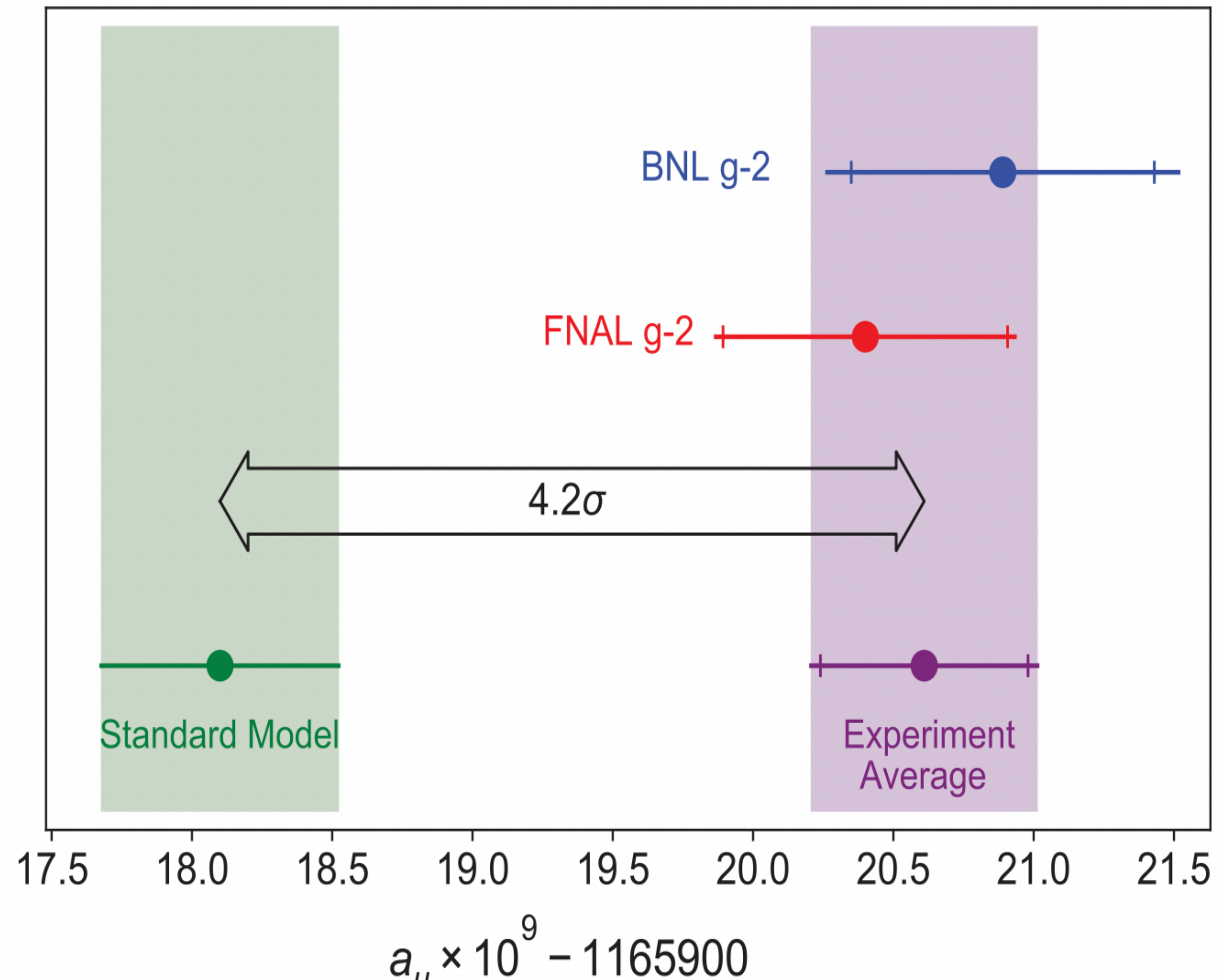
Anomalous magnetic moment of the muon $a_\mu \equiv (g-2)/2$

shows discrepancy between theory and experiment :

$$\Delta a_\mu^{\rm obs} = a_\mu^{\rm exp} - a_\mu^{\rm theory}$$
$$= (25.1 \pm 5.9) \times 10^{-10}$$

The discrepancy is of the order of the SM electroweak contribution :

$$a_\mu({\rm EW}) = (15.4 \pm 0.1) \times 10^{-10}$$



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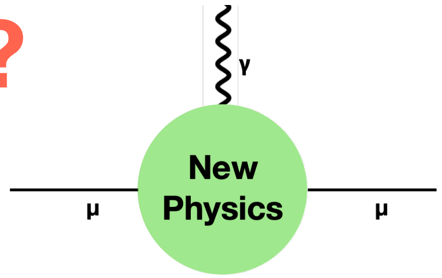
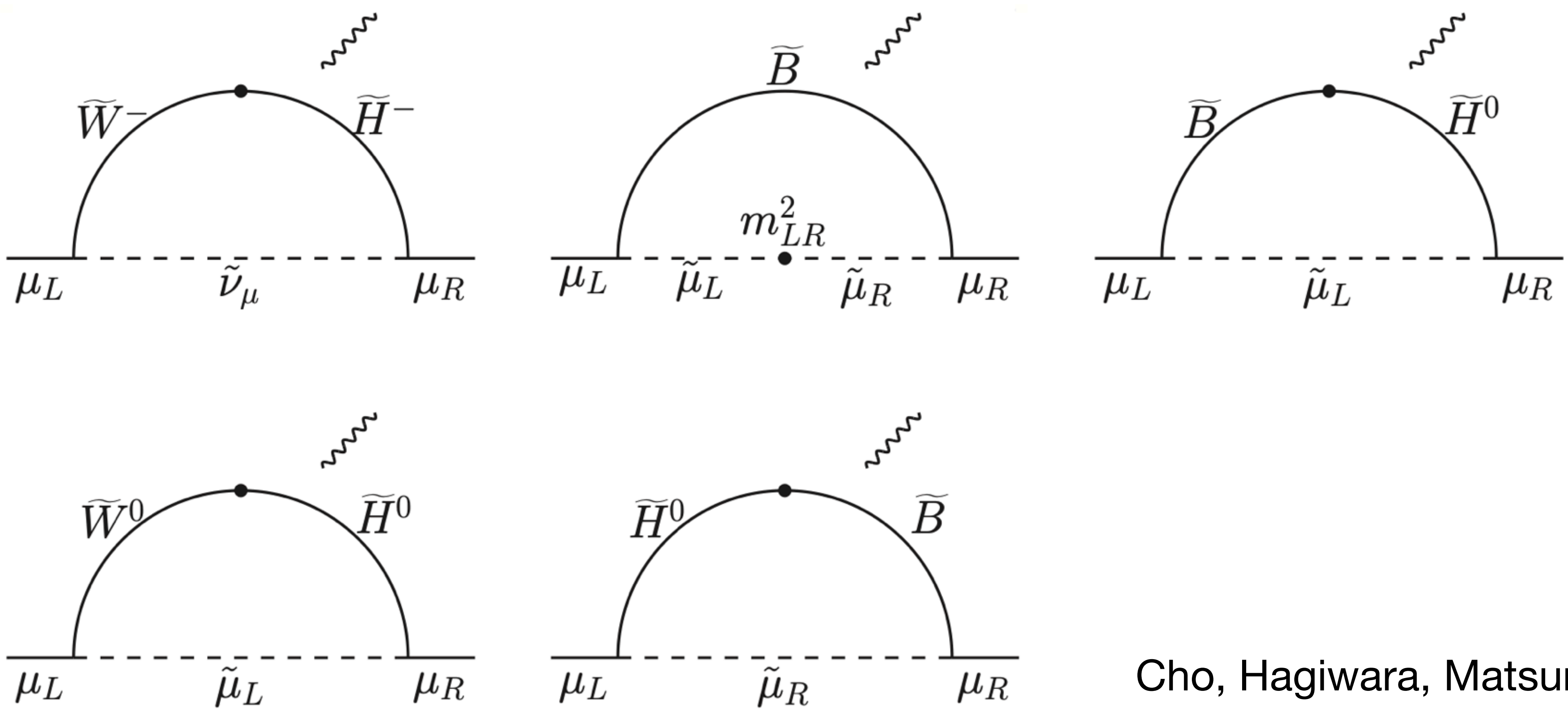
A hint of TeV scale new physics ?



SUSY explanation

Low-energy SUSY is an attractive candidate to explain the muon g-2 anomaly !

<u>Loops of sleptons and electroweakinos</u> can generate a new contribution with the correct size.

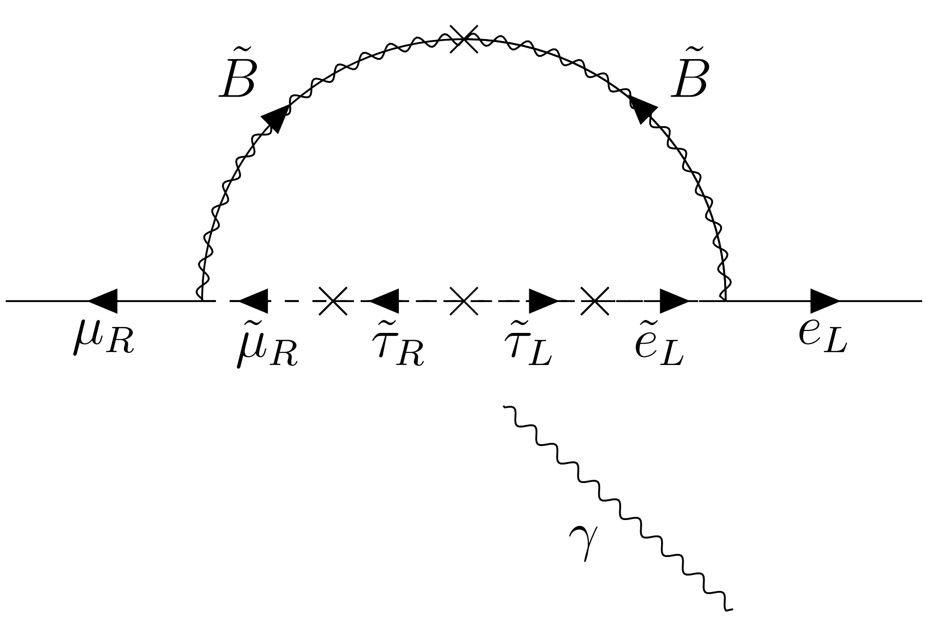


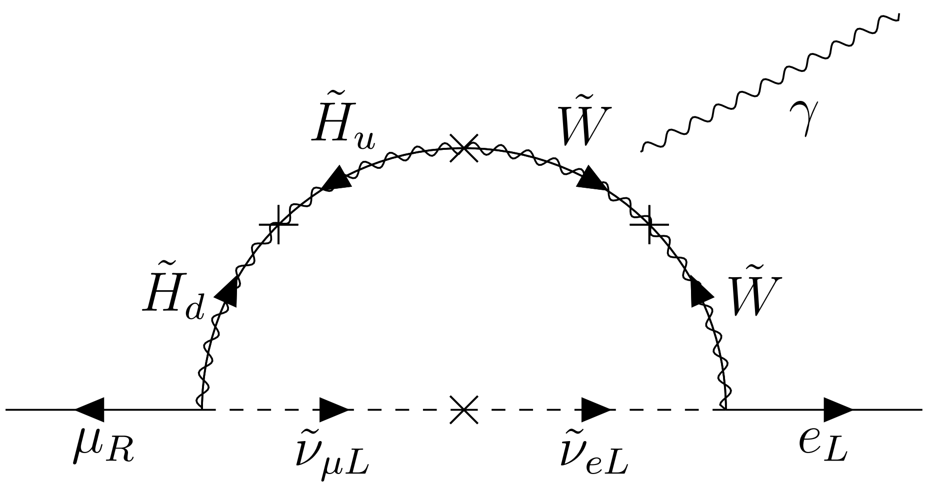
Cho, Hagiwara, Matsumoto, Nomura (2011)

CP & Flavor

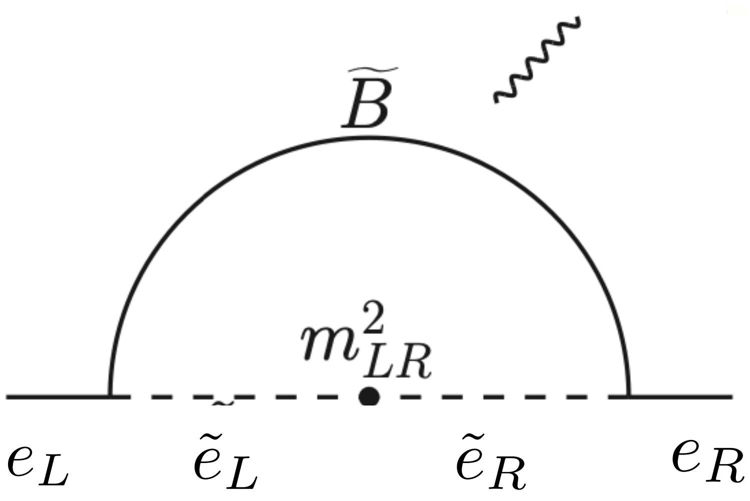
However …

Light sleptons and electroweakinos generally lead to dangerously large **lepton flavor violation (LFV)** such as $\mu \to e + \gamma$.





An arbitrary CP violation (CPV) induces a large **electric dipole moment (EDM) of the electron**.



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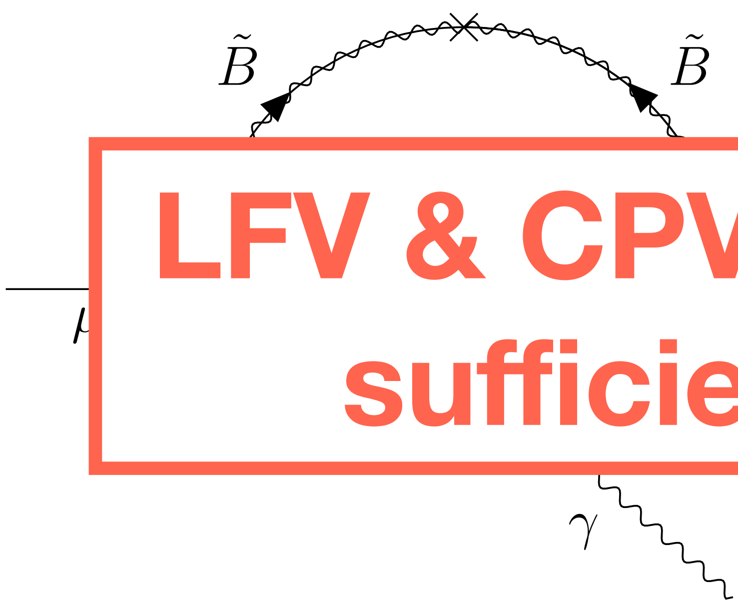

LFV & CPV processes must be sufficiently suppressed.

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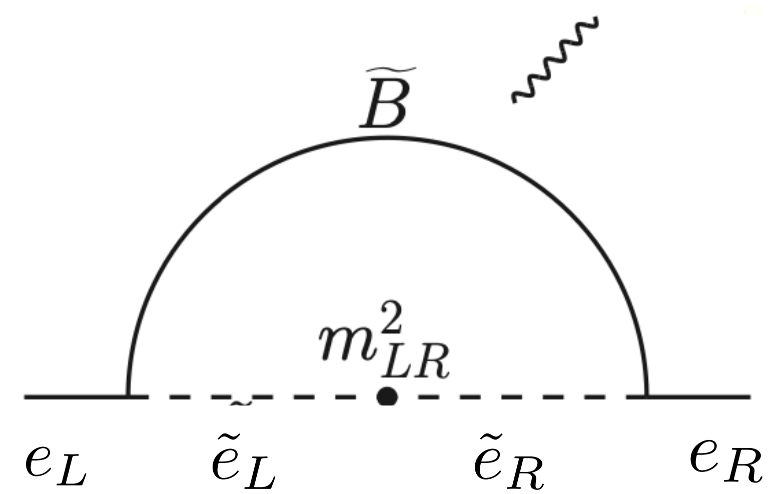

Alignment & SCPV

✓ Alignment

Introduce U(1) horizontal symmetries which can explain the hierarchical masses of quarks and leptons.

Froggatt, Nielsen (1979)

Horizontal symmetries also control the structure of sfermion masses and suppress flavor violating processes.

Leurer, Nir, Seiberg (1993)

✓ Spontaneous CP violation

Makes it possible to suppress contributions to EDMs and at the same time accommodate the correct CKM phase.

Nir, Rattazzi (1996)

➡ **A viable solution to the muon g-2 anomaly !**

Flavor Structure

- Quark mass ratios are parametrized by powers of $\lambda \sim 0.2$

$$m_c/m_t \sim \lambda^3 \ , \quad m_u/m_t \sim \lambda^6 - \lambda^7,$$

$$m_b/m_t \sim \lambda^2 \ , \quad m_s/m_b \sim \lambda^2 \ , \quad m_d/m_b \sim \lambda^4$$

- Orders of magnitude of different CKM entries

$$|V_{12}^{\rm CKM}| \sim \lambda \ , \quad |V_{23}^{\rm CKM}| \sim \lambda^2 \ , \quad |V_{13}^{\rm CKM}| \sim \lambda^3$$

- The CPV effect is parametrized by a phase $\delta_{\rm CKM} \simeq 1.2$

Flavor Structure

- Charged lepton mass ratios are parametrized by

$$m_\mu/m_\tau \sim \lambda^2 \,, \quad m_e/m_\tau \sim \lambda^5 \,, \quad m_\tau/m_t \sim \lambda^3$$

- Neutrino mass ratios (normal ordering is assumed)

$$m_{\nu_1}/m_{\nu_3} \lesssim \lambda \,, \quad m_{\nu_2}/m_{\nu_3} \sim 1$$

- Orders of magnitude of different PMNS entries

$$|V_{12}^{\rm PMNS}| \sim \lambda \,, \quad |V_{13}^{\rm PMNS}| \sim \lambda \,, \quad |V_{23}^{\rm PMNS}| \sim 1$$

Horizontal symmetry

Consider $U(1)_H$ symmetry under which quark and lepton supermultiplets have nontrivial charges.

A **flavon** chiral superfield S_1

$U(1)_H$ charge : $H(S_1) = -1$

$\langle S_1 \rangle = \lambda\Lambda$ ($\Lambda = 1$: some UV mass scale)

<u>The fermion mass ratios and mixing matrices are explained</u> with appropriate charge assignments of quarks and leptons.

The superpotential for quarks & leptons in the gauge eigenbasis :

$$W_{\text{Yukawa}} = Y_{u\,ij} Q_i \bar{u}_j H_u + Y_{d\,ij} Q_i \bar{d}_j H_d + Y_{e\,ij} L_i \bar{e}_j H_d + Y_{\nu\,ij} \frac{(H_u L_i)(H_u L_j)}{M_N}$$

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CKM & PMNS mixing matrices :

$$|V_{ij}^{\mathrm{CKM}}| \sim \lambda^{|H(Q_i)-H(Q_j)|} \,, \qquad |V_{ij}^{\mathrm{PMNS}}| \sim \lambda^{|H(L_i)-H(L_j)|}$$

Alignment

- The horizontal symmetry also determines the scaling of the soft scalar mass-squared matrices.

e.g. $H(Q_1), H(Q_2), H(Q_3)$ are chosen to realize the CKM entries

$$M^2_{\widetilde{Q}} \sim \underbrace{\widetilde{m}^2_q}_{\text{Typical mass scale of squarks}} \begin{pmatrix} 1 & \boxed{\lambda} & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

Off-diagonal components lead to FCNCs.

- Trilinear soft SUSY breaking terms have <u>a similar structure to the Yukawa matrices</u> due to the horizontal symmetry.

Realistic Models

To achieve light sleptons and electroweakinos to explain the muon g-2 anomaly …

Introduce $U(1)_{H_1} \times U(1)_{H_2}$ symmetry

Two **flavon** chiral superfields

	S_1	S_2
Charges :	$(-1, 0)$	$(0, -1)$

VEVs : $\langle S_1 \rangle \sim \lambda, \langle S_2 \rangle \sim \lambda^2$

Quarks and leptons carry charges (H_1, H_2)

Define $H = H_1 + 2H_2$

Realistic Models

- Fermion mass ratios

$$H(Q_3)+H(\bar{u}_3)=0,\quad H(Q_2)+H(\bar{u}_2)=3,\quad H(Q_1)+H(\bar{u}_1)=7,$$

$$H(Q_3)+H(\bar{d}_3)=0,\quad H(Q_2)+H(\bar{d}_2)=2,\quad H(Q_1)+H(\bar{d}_1)=4,$$

$$H(L_3)+H(\bar{e}_3)=0,\quad H(L_2)+H(\bar{e}_2)=2,\quad H(L_1)+H(\bar{e}_1)=5$$

- CKM & PMNS mixing matrices

$$H(Q_1)-H(Q_2)=1,\quad H(Q_1)-H(Q_3)=3,\quad H(Q_2)-H(Q_3)=2,$$

$$H(L_1)-H(L_2)=1,\quad H(L_1)-H(L_3)=1,\quad H(L_2)-H(L_3)=0$$

$$H=H_1+2H_2 \qquad H(H_u)=H(H_d)=0 \qquad \tan\beta\sim 50$$

Realistic Models

A working example of charge assignments

Q_1	Q_2	Q_3	$\bar{u}_1$	$\bar{u}_2$	$\bar{u}_3$
$(3,0)$	$(0,1)$	$(0,0)$	$(-2,3)$	$(1,0)$	$(0,0)$

$\bar{d}_1$	$\bar{d}_2$	$\bar{d}_3$
$(-3,2)$	$(2,-1)$	$(0,0)$

L_1	L_2	L_3	$\bar{e}_1$	$\bar{e}_2$	$\bar{e}_3$
$(5,0)$	$(0,2)$	$(0,2)$	$(-4,2)$	$(2,-2)$	$(0,-2)$

$$\Rightarrow \quad Y_u \sim \begin{pmatrix} \lambda^7 & \lambda^4 & \lambda^3 \\ 0 & \lambda^3 & \lambda^2 \\ 0 & \lambda & 1 \end{pmatrix}, \qquad Y_d \sim \begin{pmatrix} \lambda^4 & 0 & \lambda^3 \\ 0 & \lambda^2 & \lambda^2 \\ 0 & 0 & 1 \end{pmatrix},$$

$$Y_e \sim \begin{pmatrix} \lambda^5 & 0 & 0 \\ 0 & \lambda^2 & 1 \\ 0 & \lambda^2 & 1 \end{pmatrix}, \qquad Y_\nu \sim \begin{pmatrix} \lambda^{10} & \lambda^9 & \lambda^9 \\ \lambda^9 & \lambda^8 & \lambda^8 \\ \lambda^9 & \lambda^8 & \lambda^8 \end{pmatrix}.$$

Flavor structure

- The soft mass-squared matrices for squarks & sleptons

$$M^2_{\widetilde{Q}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^5 & \lambda^3 \\ \lambda^5 & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad M^2_{\widetilde{\bar{u}}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^9 & \lambda^8 \\ \lambda^9 & 1 & \lambda \\ \lambda^8 & \lambda & 1 \end{pmatrix},$$

$$M^2_{\widetilde{\bar{d}}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^{11} & \lambda^7 \\ \lambda^{11} & 1 & \lambda^4 \\ \lambda^7 & \lambda^4 & 1 \end{pmatrix},$$

$$M^2_{\widetilde{L}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^9 & \lambda^9 \\ \lambda^9 & 1 & 1 \\ \lambda^9 & 1 & 1 \end{pmatrix}, \quad M^2_{\widetilde{\bar{e}}} \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^{14} & \lambda^{12} \\ \lambda^{14} & 1 & \lambda^2 \\ \lambda^{12} & \lambda^2 & 1 \end{pmatrix}.$$

Off-diagonal components are very suppressed !

- The trilinear soft SUSY breaking terms have the same structure as the Yukawa matrices

Spontaneous CPV

Makes it possible to suppress contributions to EDMs and at the same time accommodate the correct CKM phase.

Another **flavon** S_N

$U(1)_H$ charge : $(-N, 0)$

$$W_S = Z(aS_N^2 + bS_N S_1^N + cS_1^{2N})$$

$$\frac{\langle S_N \rangle}{\langle S_1 \rangle^N} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow |\langle S_N \rangle| \sim \lambda^N$$

$$b^2 - 4ac < 0 \Rightarrow \text{A complex } \langle S_N \rangle$$

CP phases are introduced to the Yukawa matrices through couplings with S_N

Realistic Models

A working example of charge assignments

Q_1	Q_2	Q_3	$\bar{u}_1$	$\bar{u}_2$	$\bar{u}_3$
$(3,0)$	$(0,1)$	$(0,0)$	$(-2,3)$	$(1,0)$	$(0,0)$
$\bar{d}_1$	$\bar{d}_2$	$\bar{d}_3$			
$(-3,2)$	$(2,-1)$	$(0,0)$			
L_1	L_2	L_3	$\bar{e}_1$	$\bar{e}_2$	$\bar{e}_3$
$(5,0)$	$(0,2)$	$(0,2)$	$(-4,2)$	$(2,-2)$	$(0,-2)$

$$N = 3$$

CP phase

$$\Rightarrow \quad Y_u \sim \begin{pmatrix} \lambda^7 & \lambda^4 & \lambda^3 \\ 0 & \lambda^3 & \lambda^2 \\ 0 & \lambda & 1 \end{pmatrix}, \quad Y_d \sim \begin{pmatrix} \lambda^4 & 0 & \lambda^3 \\ 0 & \lambda^2 & \lambda^2 \\ 0 & 0 & 1 \end{pmatrix}$$

Realistic Models

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Q_1	Q_2	Q_3	$\bar{u}_1$	$\bar{u}_2$	$\bar{u}_3$
$(3,0)$	$(0,1)$	$(0,0)$	$(-2,3)$	$(1,0)$	$(0,0)$

$\bar{d}_1$	$\bar{d}_2$	$\bar{d}_3$
$(-3,2)$	$(2,-1)$	$(0,0)$

L_1	L_2	L_3	$\bar{e}_1$	$\bar{e}_2$	$\bar{e}_3$
$(5,0)$	$(0,2)$	$(0,2)$	$(-4,2)$	$(2,-2)$	$(0,-2)$

$$N = 4$$

CP phase

$$\Rightarrow \quad Y_u \sim \begin{pmatrix} \lambda^7 & \lambda^4 & \lambda^3 \\ 0 & \lambda^3 & \lambda^2 \\ 0 & \lambda & 1 \end{pmatrix}, \qquad Y_d \sim \begin{pmatrix} \lambda^4 & 0 & \lambda^3 \\ 0 & \lambda^2 & \lambda^2 \\ 0 & 0 & 1 \end{pmatrix}$$

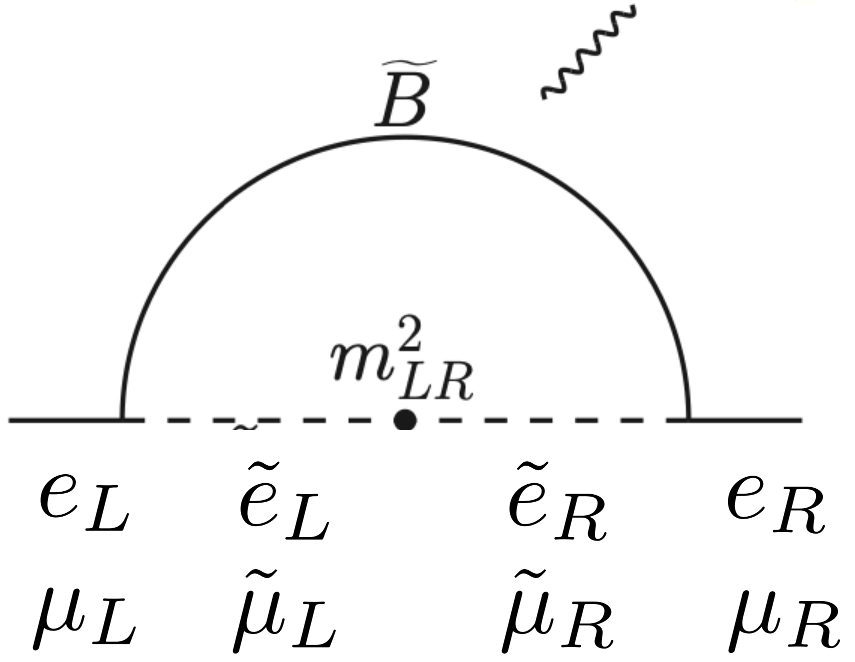
Electron EDM

- Contributions from off-diagonal entries of soft mass-squared matrices are suppressed.

- The dominant contribution is obtained from flavor diagonal entries with the phase of μ

The contribution is related to that of the muon g-2 :

$$|d_e^{\rm SUSY}| \simeq 10^{-24} \left(\frac{a_\mu}{2 \times 10^{-9}} \right) |\arg(\mu)|\, e\,{\rm cm}$$



$$N = 3 \qquad \mu = |\mu|(1 + i\,\kappa\lambda^6)$$

$$N = 4 \qquad \mu = |\mu|(1 + i\kappa\,\lambda^8) \longleftarrow S_1^N S_N^\dagger$$

Flavor & CP observables

SUSY contributions to flavor & CP observables are sufficiently suppressed !

$$\widetilde{m}_q = M_3 = 5\,\mathrm{TeV}, \quad \widetilde{m}_\ell = M_{1,2} = \mu = 500\,\mathrm{GeV}, \quad \tan\beta = 50$$

Observable	Experimental bound	Model with S_3	Model with S_4
$\lvert\epsilon_K\rvert$	$2.228(11) \times 10^{-3}$	$\sim 10^{-3}$	$\sim 10^{-7}$
$\lvert\Delta M_D\rvert$	$0.63^{+0.27}_{-0.29} \times 10^{-14}\,\mathrm{GeV}$	$\sim 5 \times 10^{-15}\,\mathrm{GeV}$	$\sim 5 \times 10^{-15}\,\mathrm{GeV}$
nEDM	$\leqslant 10^{-26}\; e\,\mathrm{cm}$	$\sim 10^{-28}\; e\,\mathrm{cm}$	$\sim 10^{-28}\; e\,\mathrm{cm}$
$\mathrm{Br}(\mu \to e + \gamma)$	$\leqslant 4.2 \times 10^{-13}$	$\sim 10^{-16}$	$\sim 10^{-16}$
eEDM	$\leqslant 1.1 \times 10^{-29}\; e\,\mathrm{cm}$	$\sim 5 \times 10^{-29}\; e\,\mathrm{cm}$	$\sim 10^{-30}\; e\,\mathrm{cm}$

$$\mathrm{Br}(\mu \to e + \gamma) \simeq 5 \times 10^{-4} \left(\frac{\tan\beta}{50}\right)^2 \left(\frac{500\,\mathrm{GeV}}{\widetilde{m}_\ell}\right)^4 \left(\frac{\mu\, M_1}{\widetilde{m}_\ell^2}\right)^2 \times \left(\left|0.5(\delta^\ell_{21})_{LL}\right|^2 + \left|(\delta^\ell_{23})_{RR}(\delta^\ell_{31})_{LL}\right|^2\right)$$

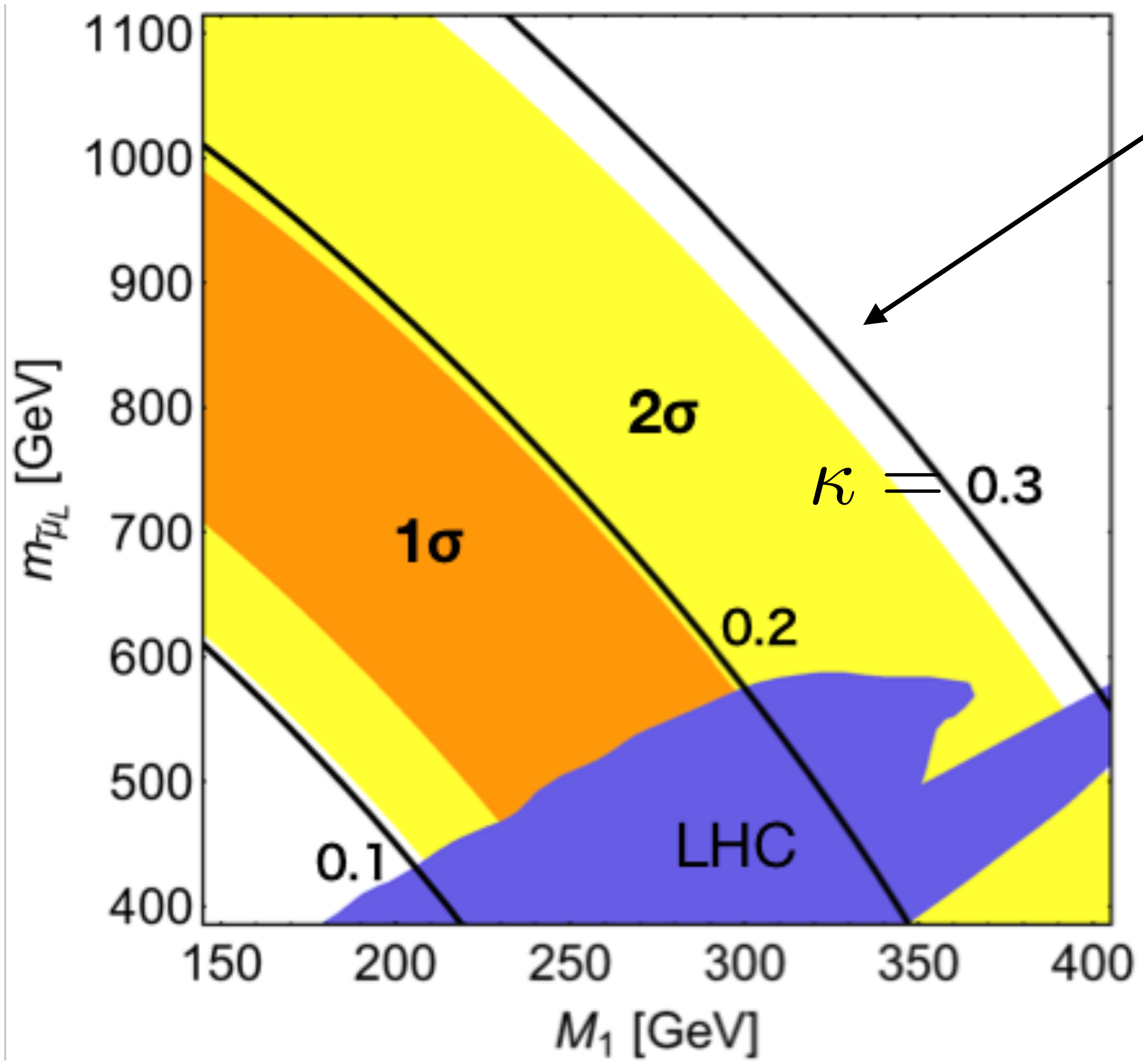
$$(\hat{M}^2_{\widetilde{X}})_{ij} \equiv \widetilde{m}^2(\delta_{ij} + (\delta^X_{ij})_{HH})$$ (Yukawa diagonal basis)

$$(\delta^\ell_{21})_{LL} \approx \lambda^{8.7}, \quad (\delta^\ell_{23})_{RR}(\delta^\ell_{31})_{LL} \approx \lambda^{9.7}$$

Results

$$N = 3 \qquad \mu = |\mu|(1 + i\kappa\,\lambda^6)$$

$$\tan\beta = 50,\ \ \mu = M_2 = 2M_1$$



Current bound on eEDM :

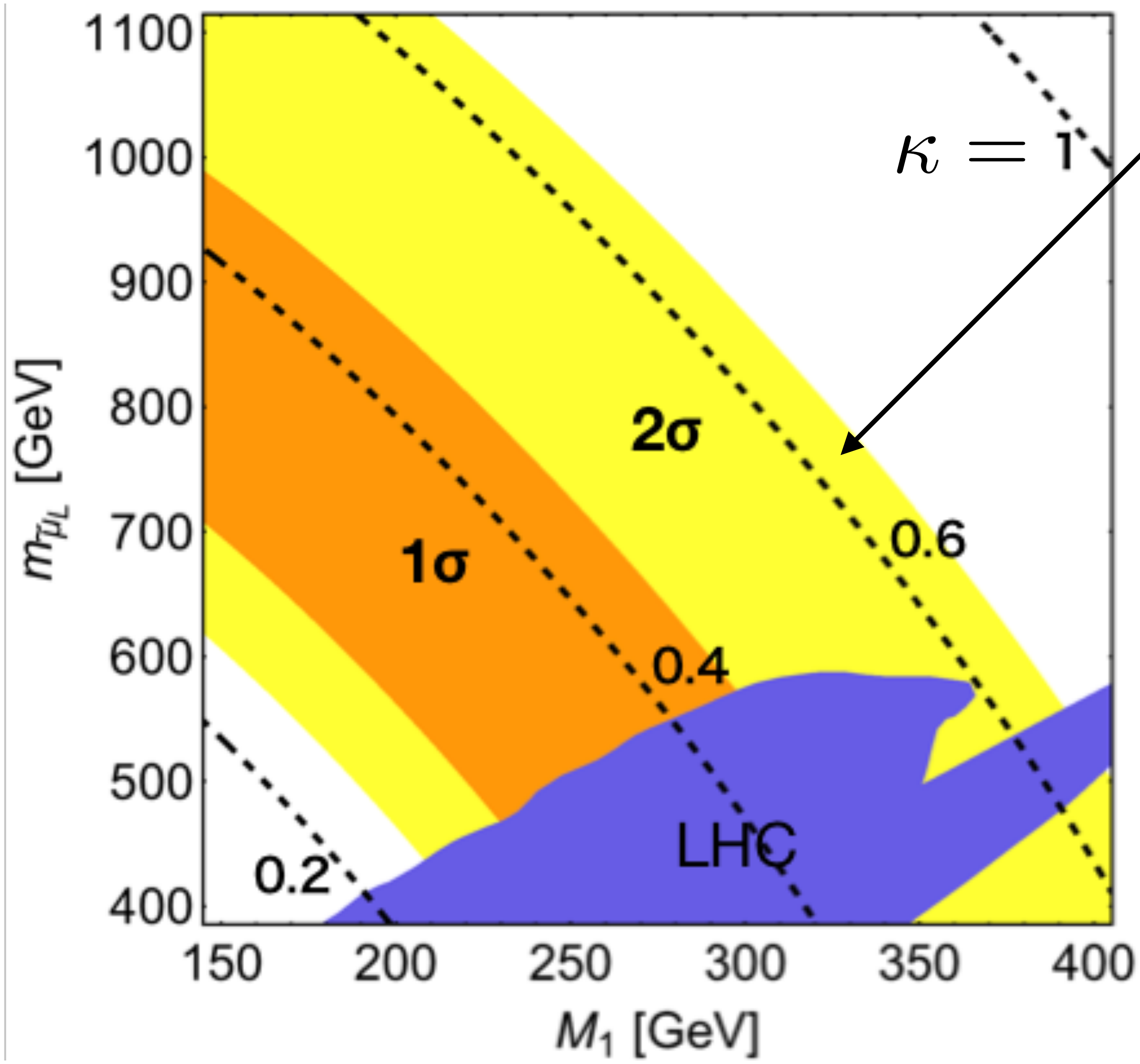
$$|d_e| = 1.1 \times 10^{-29}\, e\,\mathrm{cm}$$

Most of the parameter space is excluded without small κ.

Results

$$N = 4 \qquad \mu = |\mu|(1 + i\kappa\,\lambda^8)$$

$$\tan\beta = 50,\ \ \mu = M_2 = 2M_1$$



Future reach of eEDM :

$$|d_e| = 10^{-30}\, e\,\mathrm{cm}$$

Future eEDM measurement will cover the favored region !

Summary

- SUSY provides an attractive possibility to explain the muon g-2 anomaly, but <u>LFV and CPV processes must be suppressed</u>.

- <u>U(1) horizontal symmetries</u> address hierarchical masses of quarks and leptons and also control the structure of sfermion masses (**Alignment**).

- **Spontaneous CPV** suppresses contributions to EDMs and accommodates the correct CKM phase.

- Favored parameter space to address the muon g-2 anomaly will be <u>extensively investigated by future electron EDM experiments</u> !

Thank you.