Analytic constraints from genericity in supersymmetry (and improvements to the Nelson-Seiberg theorem)

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Outline

1. Formalizing the notion of "generic".
2. Brief review of R-symmetric Wess-Zumino models.
3. Apply (1) to (2) $\implies$ New analytic constraints for SUSY vacua in R-symmetric WZ models.
4. (If there's time) improvements to the Nelson-Seiberg theorem

1. Genericity

"Generic"

- Often used informally to mean "without fine-tuning"
- A small change to the parameters of the model should not alter any of the properties that we happen to be interested in.
- Implicitly: there is some underlying parameter space, and we are not at a special isolated point / lower-dimensional surface.
- E.g. systems where low-energy couplings are set dynamically
- Often related to naturalness arguments (hierarchy problem etc.), but not equivalent.

One can make this into a formal condition in several ways, only one of which is detailed here.

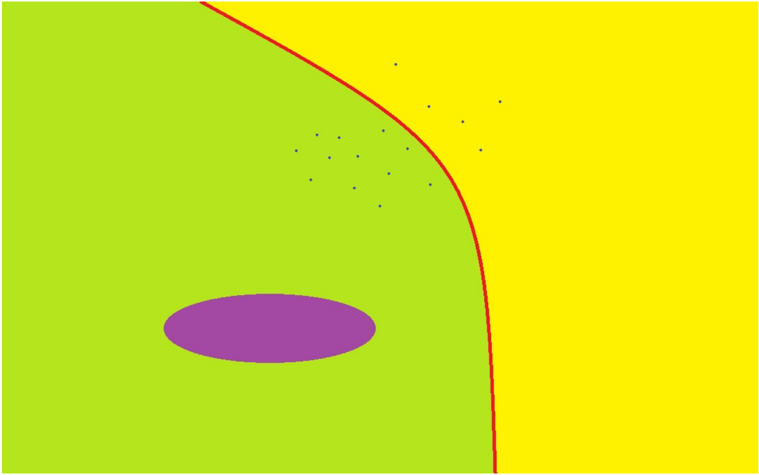
Take a model, with a set of continuous parameters $\{c_\alpha\}$ (e.g. the couplings in a QFT)

Often, a “generic” model is taken to be one that has all possible interactions allowed by some conditions (symmetry, renormalization) - we won’t need that assumption here.

We take a topological definition of genericity: we say that a model has some property P at a point c_α^0 in parameter space generically if P also holds in an open region about c_α^0.

(Note that we do not assume P holds almost everywhere. We may have P generically in some region and $\neg P$ generically in another. No fine-tuning, not no tuning.)

(c_1, c_2) plane:



A property the holds in the green, yellow or purple regions is generic by our definition.

Consider a property P that can be represented as a function $F(c_\alpha)$ on the parameter space, such that at points where the model possesses the relevant property the function takes some constant value (wlog 0).

$$P \text{ holds at } c_\alpha \implies F(c_\alpha) = 0$$

if $F(c_\alpha)$ is differentiable, and P holds at c_α^0 generically, we can consider an infinitesimal variation in the c_α and thus obtain:

$$\left.\frac{\mathrm{d}F}{\mathrm{d}c_\alpha}\right|_{c_\beta^{(0)}} = 0 \quad \forall\alpha.$$

(any higher derivatives that exist should also vanish)

If our model is a QFT, with some fields ϕ_i, then we may be interested in properties of the vacuum.

F may then depend on c_α through the vacuum expectations of the fields $\langle \phi_i \rangle = \phi_i(c_\alpha)$.
$F = F(c_\alpha, \phi_i(c_\alpha))$

Assuming everything is once differentiable

$$\left.\frac{\mathrm{d}F}{\mathrm{d}c_\alpha}\right|_{c_\beta^{(0)}} = \left.\left(\frac{\partial F}{\partial c_\alpha} + \frac{\partial F}{\partial \phi_i}\frac{\partial \phi_i}{\partial c_\alpha}\right)\right|_{c_\beta^{(0)}} = 0.$$

2. R-symmetric Wess-Zumino models

Wess-Zumino models:

- $\mathcal{N} = 1$ SUSY. Only chiral superfields Φ_i, with scalar components ϕ_i
- Superpotential $W(\phi_i)$, appears in action as

$$S_W = -\int \mathrm{d}^4x \left(\int \mathrm{d}\theta^\alpha \mathrm{d}\theta_\alpha \, W + \text{c.c.} \right).$$

  (We can ignore the Kähler potential here - won't affect our conclusions)

- SUSY-preserving vacuum iff there is a simultaneous solution to the F-term equations

$$\partial_i W = \frac{\partial W}{\partial \phi_i} = 0, \quad \forall i$$

- As the scalar potential is $V = |\partial_i W|^2$, solution to F-terms gives the v.e.vs for a SUSY vacuum at $V = 0$.

R-symmetry

- Continuous R-symmetry: a $U(1)$ symmetry that acts non-trivially on the grassman numbers θ^α. By convention θ has charge $r_\theta = 1$.
- Integration measure $\int \mathrm{d}\theta^\alpha \mathrm{d}\theta_\alpha$ not invariant.
- Superpotential W must have charge $r_W = 2$.
- Fields ϕ_i have charges r_i.

$\langle W \rangle = 0$ at a SUSY vacuum

(Dine et al, 2009)

Consider an R-symmetry transformation, parameterized by ζ:

$$\theta \to e^{i\zeta}\theta, \quad W \to e^{2i\zeta}W, \quad \phi_i \to e^{r_i i\zeta}\phi_i \ .$$

we have:

$$e^{2i\zeta}W(\phi_i) = W(e^{r_i i\zeta}\phi_i)$$

Considering the limit as $\zeta \to 0$

$$2W = \sum_i r_i \phi_i \partial_i W \ .$$

At a SUSY vacuum, we know $\partial_i W = 0, \forall i$, and so we must also have

$$\langle W \rangle = 0$$

3. Generic R-symmetric W-Z models

Consider a continuously parameterized family of R-symmetric superpotentials, given by some complex coefficients $\{c_\alpha\}$.

In realistic cases, this will often be constructed from a linear combination of permitted (by symmetry etc.) terms $\{p_\alpha(\phi_i)\}$, so that

$$W(\phi_i, c_\alpha) = \sum_\alpha c_\alpha p_\alpha(\phi_i).$$

We now ask whether a family of models as on the previous slide possesses a SUSY-preserving vacuum generically.

Fortunately, we know that $W = 0$ at a SUSY vacuum - we can use W itself as our indicative function F.

(From here on, we write W for $\langle W \rangle$ at the vacuum, considered as a function $W(\phi_i(c_\alpha), c_\alpha)$ of both the field v.e.vs and the c_α)

Putting our results together, we have that at any generic SUSY vacuum

$$\left.\frac{\mathrm{d}W}{\mathrm{d}c_\alpha}\right|_{c_\beta^{(0)}} = \left.\left(\frac{\partial W}{\partial c_\alpha} + \frac{\partial W}{\partial \phi_i}\frac{\partial \phi_i}{\partial c_\alpha}\right)\right|_{c_\beta^{(0)}} = 0 \;\; \forall \alpha.$$

But again, we know that $\frac{\partial W}{\partial \phi_i} = 0$ at a SUSY vacuum - the second term vanishes - leaving us with

$$\left.\frac{\partial W}{\partial c_\alpha}\right|_{c_\beta^{(0)}} = 0 \quad \forall \alpha$$

Assuming genericity thus gives us one extra analytic constraint for each free parameter!

W vanishes termwise

If we take the superpotential to be of the form $W(\phi_i, c_\alpha) = \sum_\alpha c_\alpha p_\alpha(\phi_i)$, we obtain

$$\left.\frac{\partial W}{\partial c_\alpha}\right|_{c_\beta^{(0)}} = p_\alpha = 0 \quad \forall\alpha$$

- the superpotential must vanish term-by-term!

(A non-trivial conclusion - the $\partial_i W$ are also zero at the vacuum and don't generically vanish termwise)

Improving the Nelson-Seiberg theorem

“Famous” result by Nelson and Seiberg (1994):

For generic* W-Z models, an R-symmetry is a necessary condition, and a *broken* R-symmetry is a sufficient condition, for supersymmetry breaking

(*in our sense, and also in the sense of “containing all permitted polynomial terms”)
Based on a simple counting argument. Uses field redefinitions that might be singular at interesting points

Immediate application of our results: further refinements to Nelson-Seiberg like results - necessary and sufficient conditions for generic SUSY breaking based on R-charges (papers in progress. . .)

Basic idea: look at the F-terms and count independent variables

Easy example: any superfield that appears in the superpotential on its own must vanish at a SUSY vacuum.
For instance, let X be an R-charge 2 field, so that

$$W = c_1 X + \dots$$

SUSY vacuum must have $X = 0$, $\implies$ X can't appear in solutions to F-term equations.

But fields of charge 2 are important...

Consider a generic polynomial R-symmetric superpotential of up to cubic order

$$\begin{aligned} W &= a_i X_i + b_{ij} X_i Y_j + c_{ijk} X_i Y_j Y_k + d_{(r)ijk} X_i P_{(r)j} Q_{(-r)k} \\ &+ \underbrace{\xi_{ijk} X_i X_j A_k}_{r_k=-2} \\ &+ \text{terms not containing } X_i\text{, e.g:} \underbrace{(\mu_{ij} + \nu_{ijk} Y_k) A_i A_j}_{r_i+r_j=2} + \underbrace{\lambda_{ijk} A_i A_j A_k}_{r_i+r_j+r_k=2} \end{aligned}$$

(again, the X_i have R-charge 2, Y_j have R-charge 0 etc.)

Only the F-term equations that come from X fields are inhomogeneous.

$$F_{X_i} = \partial_i W = a_i + b_{ij} Y_j + c_{ijk} Y_j Y_k + d_{(r)ijk} P_{(r)j} Q_{(-r)k} = 0$$

Where we've dropped the term $\xi_{ijk} X_j A_k$, as we know the X_i must be zero at a SUSY vacuum.

The other F-terms can be solved by setting all fields to zero.
Thus having at least one field of charge 2 is a necessary condition for SUSY breaking!

But we can do better: the only terms that can appear in the X_i F-terms are

- Uncharged fields Y_j
- *Pairs* of oppositely charged fields $P_{(r)j}Q_{(-r)k}$

Additionally, we know that the P and Q fields must not have charges such that terms like P^2, P^3 etc may appear in the superpotential - otherwise they must vanish at the vacuum.

If there are not as many of these variables as there are charge 2 X_i fields, then the F-term equations generically have no solution, and SUSY must be broken - a sufficient condition for (generic) SUSY breaking

We can also derive a necessary condition for SUSY breaking: we make an assumption that *two* fields in each term in W not containing X must vanish (thus solving both $W = 0$ termwise and the non-X F-terms). If, after that, we *still* have enough variables to solve the remaining X_i F-terms, there must be a SUSY vacuum.

(As of the time of writing, we haven't been able to reduce this to a unified necessary-and-sufficient condition)

Generalizations beyond SUSY?

- To apply exactly the same reasoning, we need another case where the integration measure in the action is non-invariant under a continuous symmetry.
- Non-SUSY example: scaling symmetry $x \to \lambda x$. (Measure $\mathrm{d}^d x$ is not invariant)
- Same reasoning applies: a potential V must be zero at the vacuum, hence $\frac{\partial V}{\partial c_\alpha} = 0$ generically.
- Possible applications in scale-invariant physical systems (phase transitions/ critical phenomena, CFT, fluid dynamics)
- $\frac{\mathrm{d}F}{\mathrm{d}c_\alpha} = 0$ still holds in other cases - can we get anything useful from these?

Conclusions

- Genericity can be formalized. Topologically - not a probability argument - but we do have to chose a parameterization.
- Leads to novel analytic constraints for models to generically possess some property.
- Demonstrably useful in W-Z models. Should have applications elsewhere.

Thanks for listening!