

Probing heavy Majorana neutrinos and the Weinberg operator through $pp \rightarrow \mu^\pm \mu^\pm jj$ at CMS



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*The 8th China LHC Physics Workshop
23-27 Nov, Nanjing*



The dim-5 Weinberg operator

1957

1970s

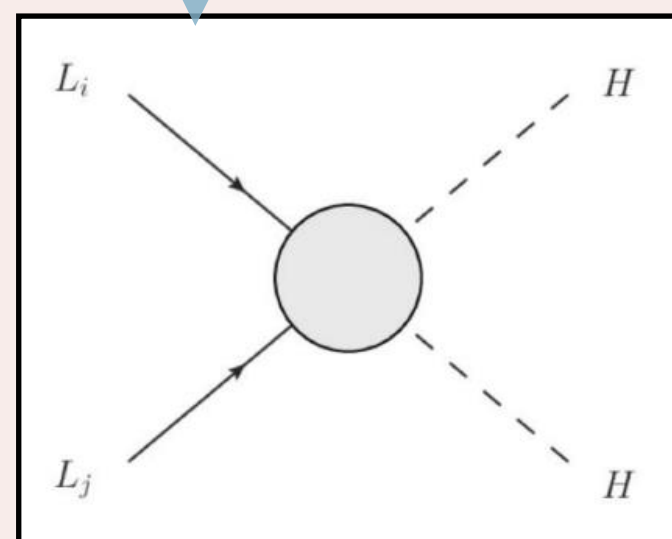
2002-2005

Neutrino oscillation [B. Pontecorvo \('57\)](#)

An effective field theory (EFT) solution

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}}^{(4)} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

The only gauge-invariant operator at dim-5
Weinberg Operator [Weinberg \('79\)](#)

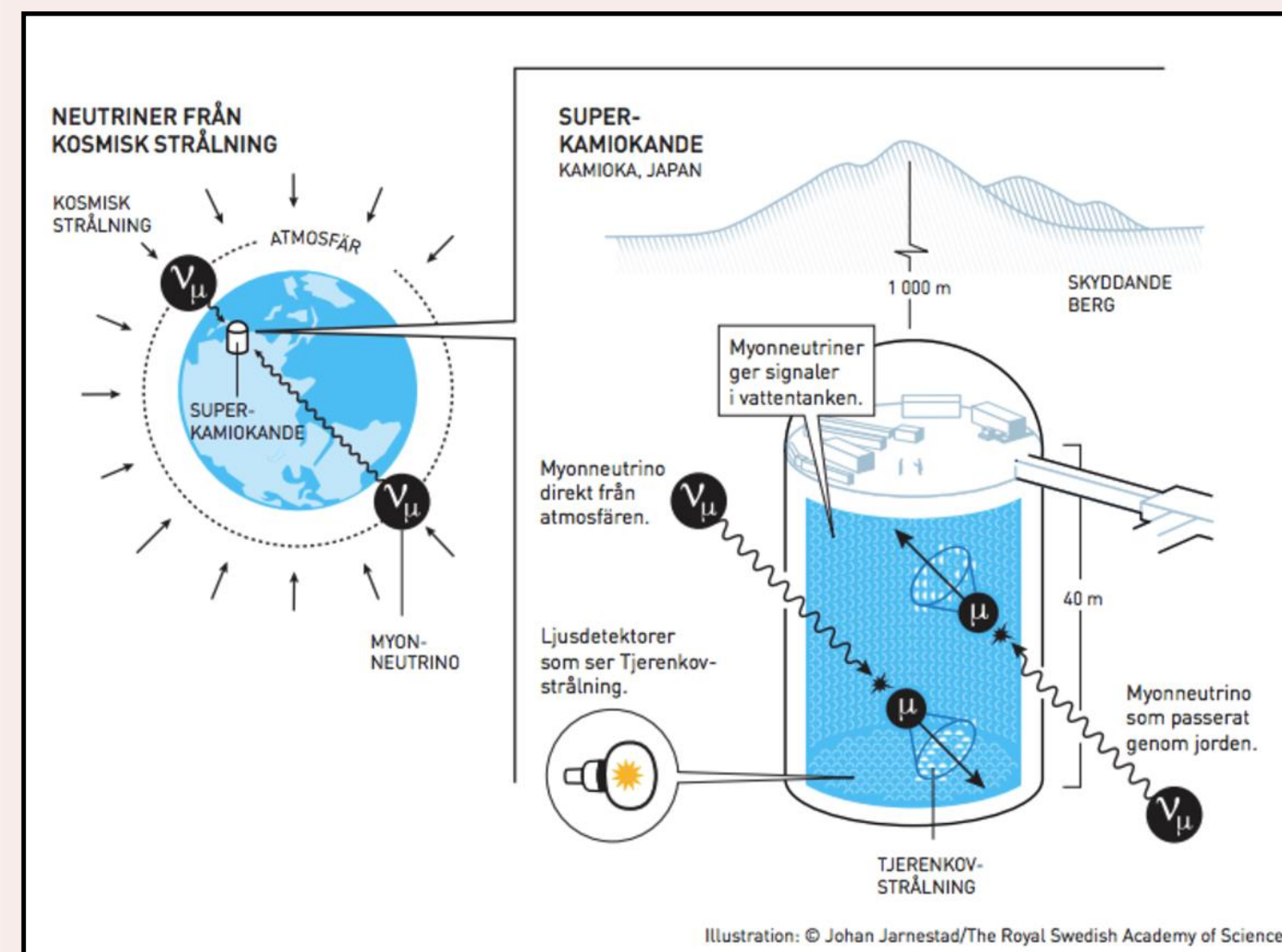


- Adding this operator will give SM neutrinos **Majorana masses**
- EFT doesn't describe the new physics in detail. Need UV complete models.

Observation of neutrino oscillations

- SNO: [PhysRevLett.89.011301](#)
- Super-Kamiokande: [PhysRevD.71.112005](#)

SM neutrino masses are **non-zero**



Seesaw models

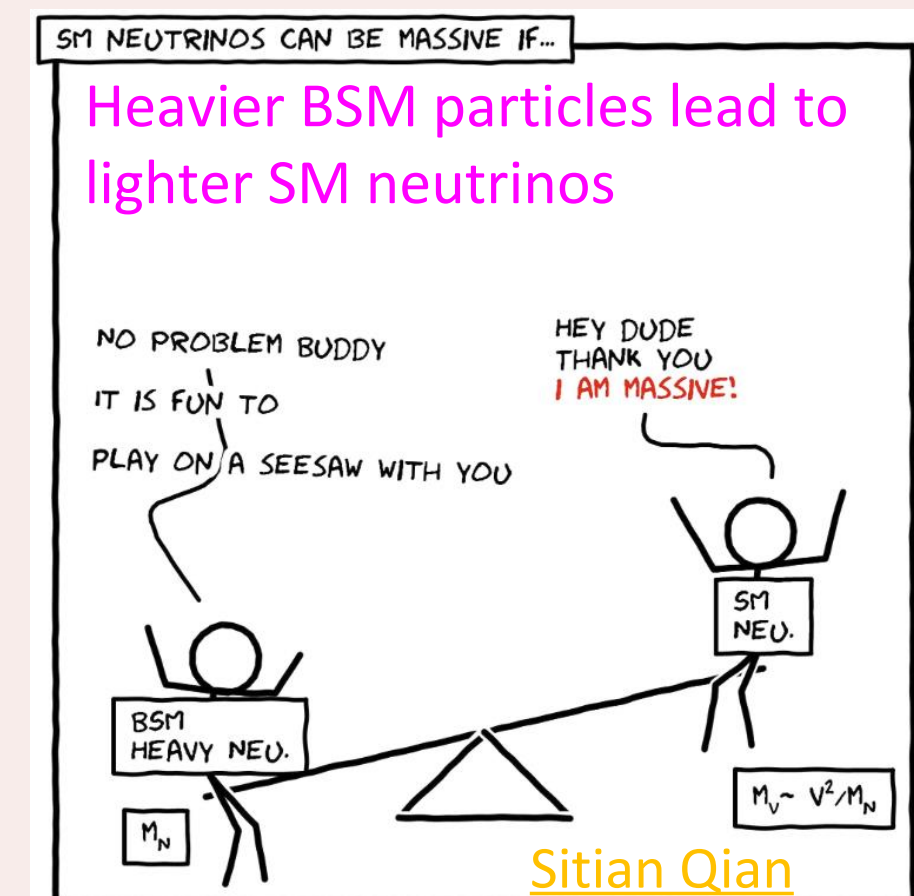
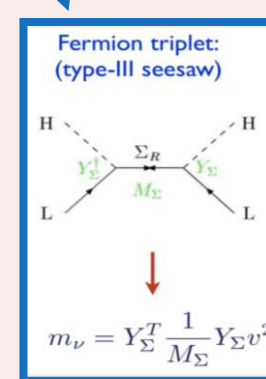
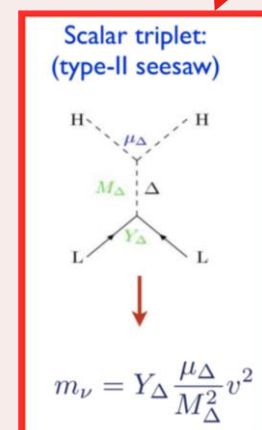
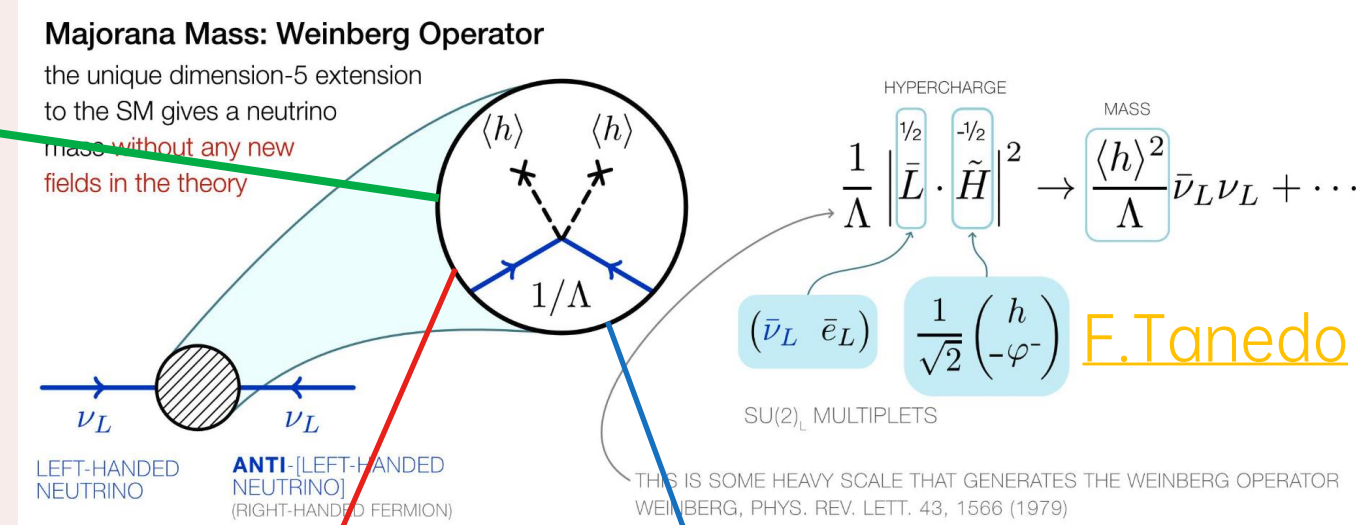
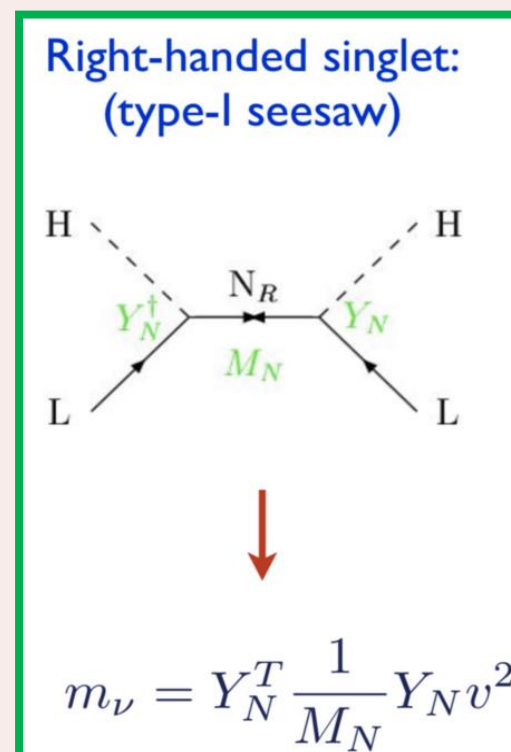
1937

1970s

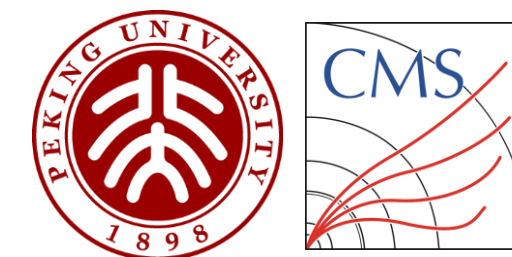
- Majorana fermion **E. Majorana ('37)**
 - A fermion that is its own antiparticle

Seesaw mechanism

- Opening the black box of **Weinberg operator** requires a "seesaw"
- Only three different kinds of realization at Born-level are allowed

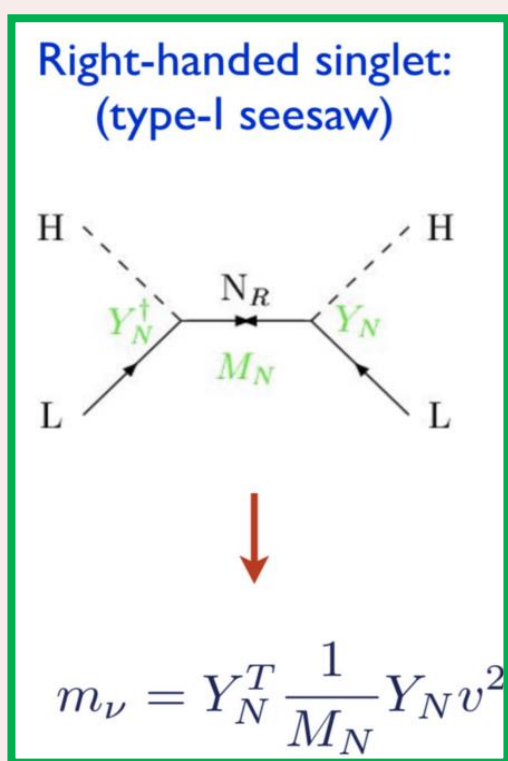


Analyses of the Type-I Seesaw Model

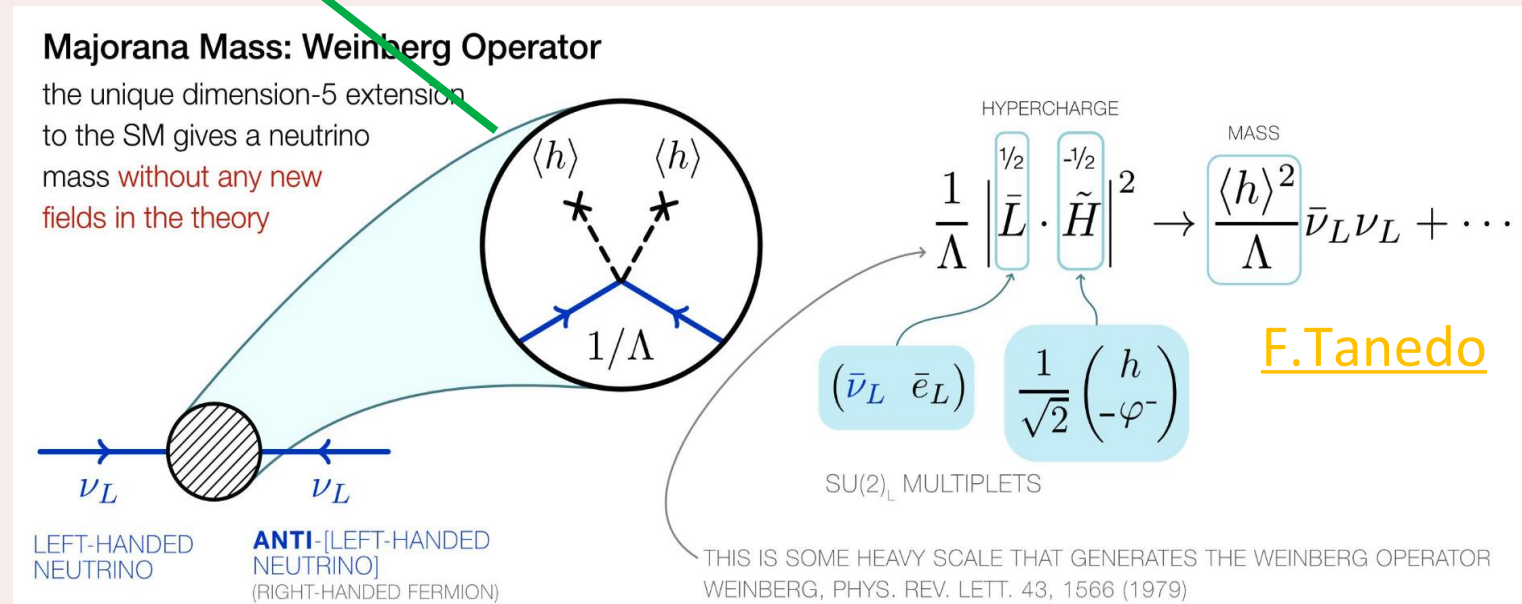


2018-2019

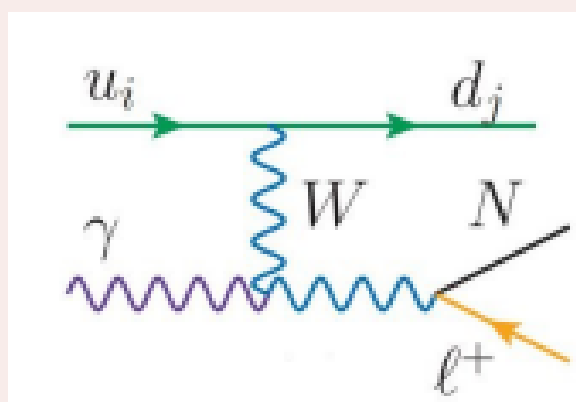
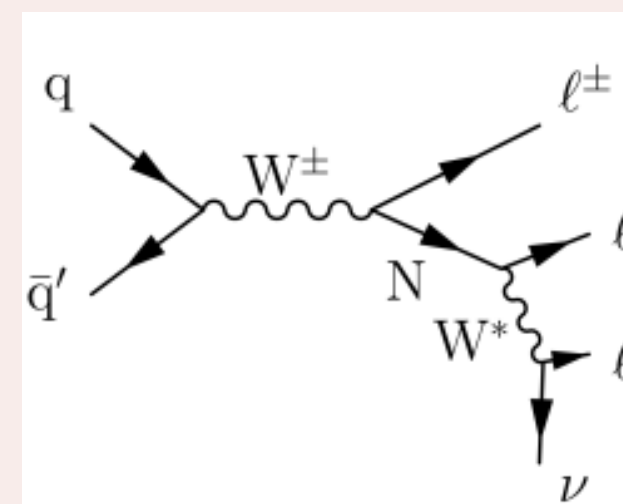
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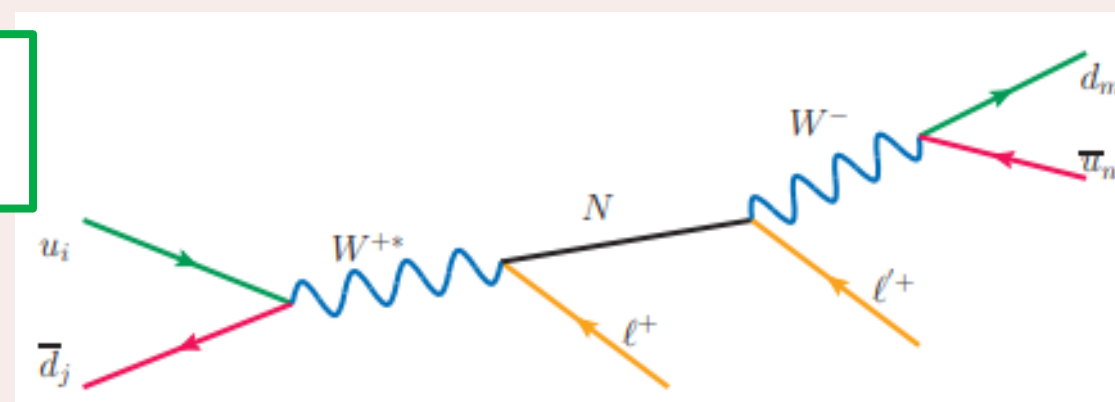
[PhysRevLett.120.221801: Trilepton](#)
[JHEP01\(2019\)122: Same-sign dilepton](#)



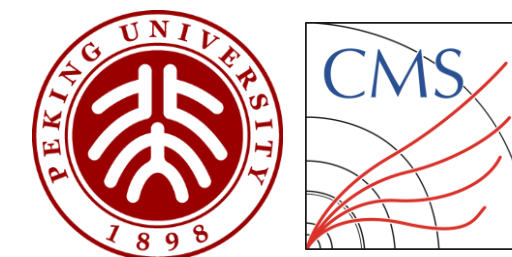
[PhysRevLett.120.221801: Trilepton](#)



[JHEP01\(2019\)122: Same-sign dilepton](#)

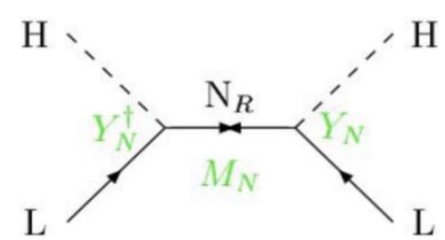


Analyses of the Type-I Seesaw Model



2018-2019

Right-handed singlet:
(type-I seesaw)

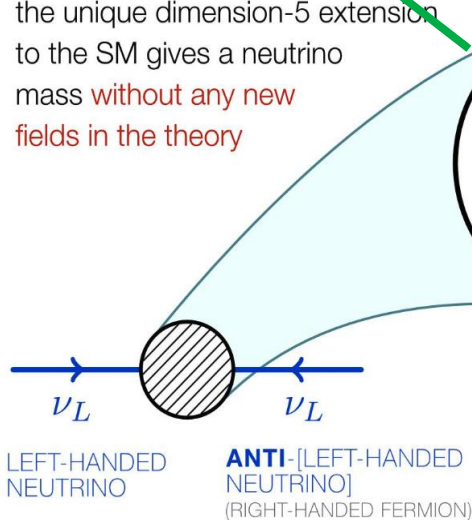


$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

[PhysRevLett.120.221801: Trilepton](#)
[JHEP01\(2019\)122: Same-sign dilepton](#)

Majorana Mass: Weinberg Operator

the unique dimension-5 extension to the SM gives a neutrino mass without any new fields in the theory

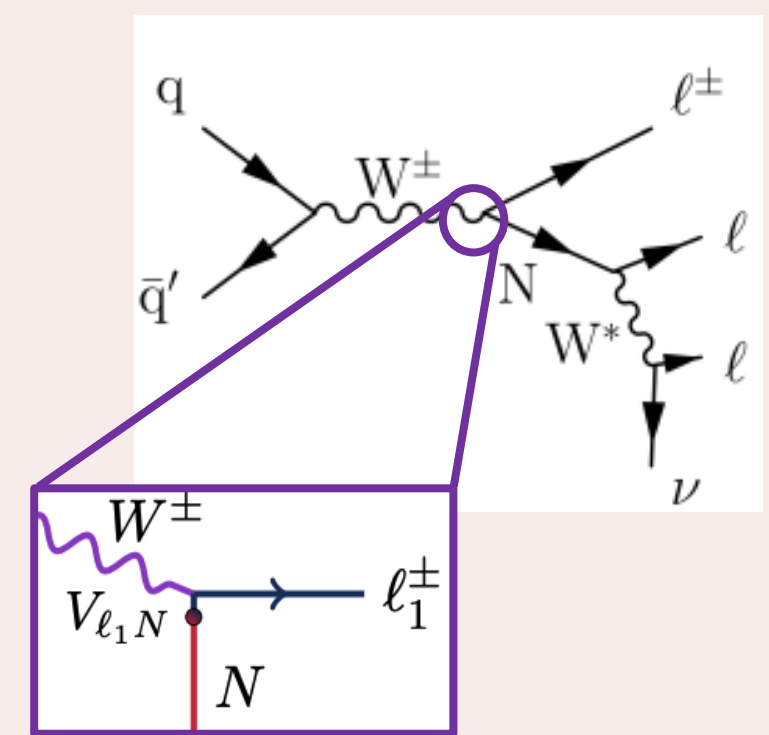


$$\frac{1}{\Lambda} \left[\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix} \cdot \tilde{H} \right]^2 \rightarrow \frac{\langle h \rangle^2}{\Lambda} \bar{\nu}_L \nu_L + \dots$$

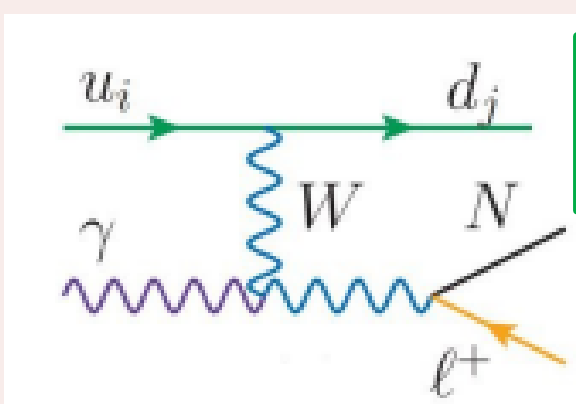
HYPERCHARGE
MASS
SU(2)_L MULTIPLETS

THIS IS SOME HEAVY SCALE THAT GENERATES THE WEINBERG OPERATOR
WEINBERG, PHYS. REV. LETT. 43, 1566 (1979)

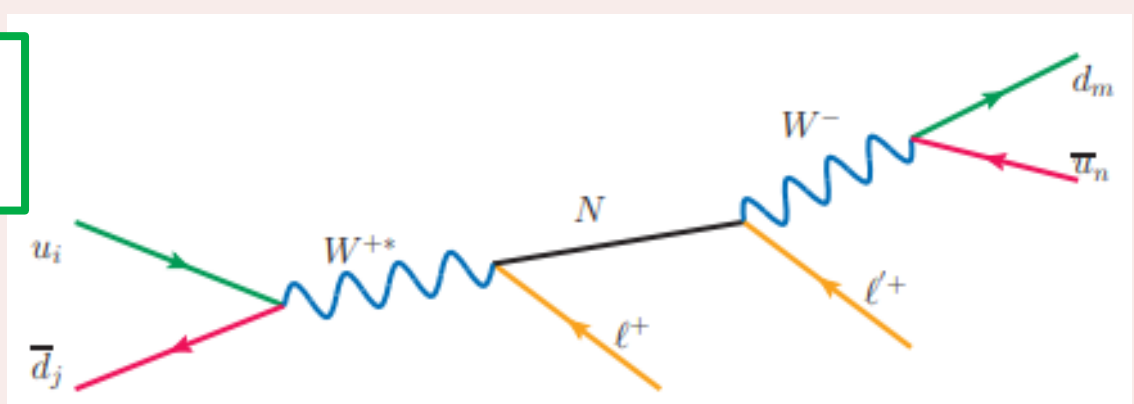
[PhysRevLett.120.221801: Trilepton](#)



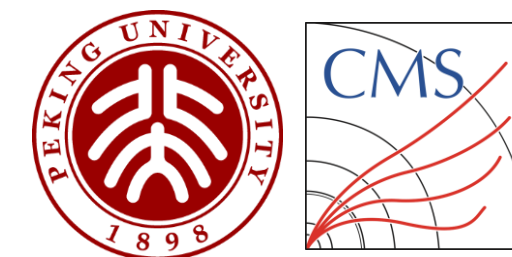
$$\sigma(pp \rightarrow N \ell^\pm + X) \equiv |V_{\ell N}|^2 \times \sigma_0(pp \rightarrow N \ell^\pm + X)$$



[JHEP01\(2019\)122: Same-sign dilepton](#)

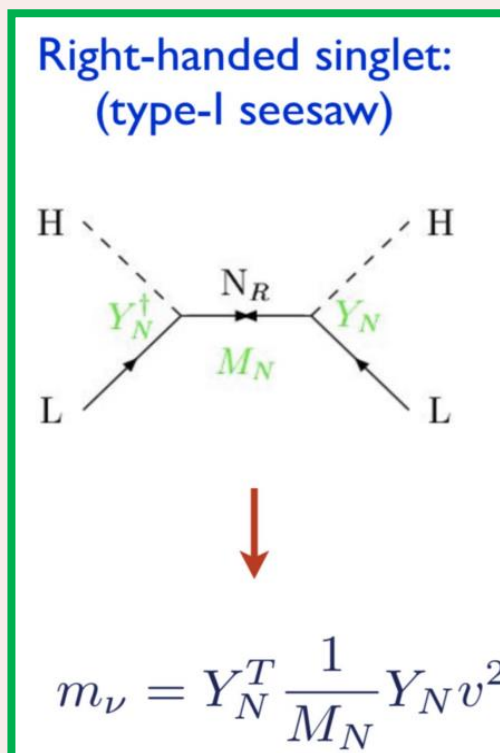


Analyses of the Type-I Seesaw Model

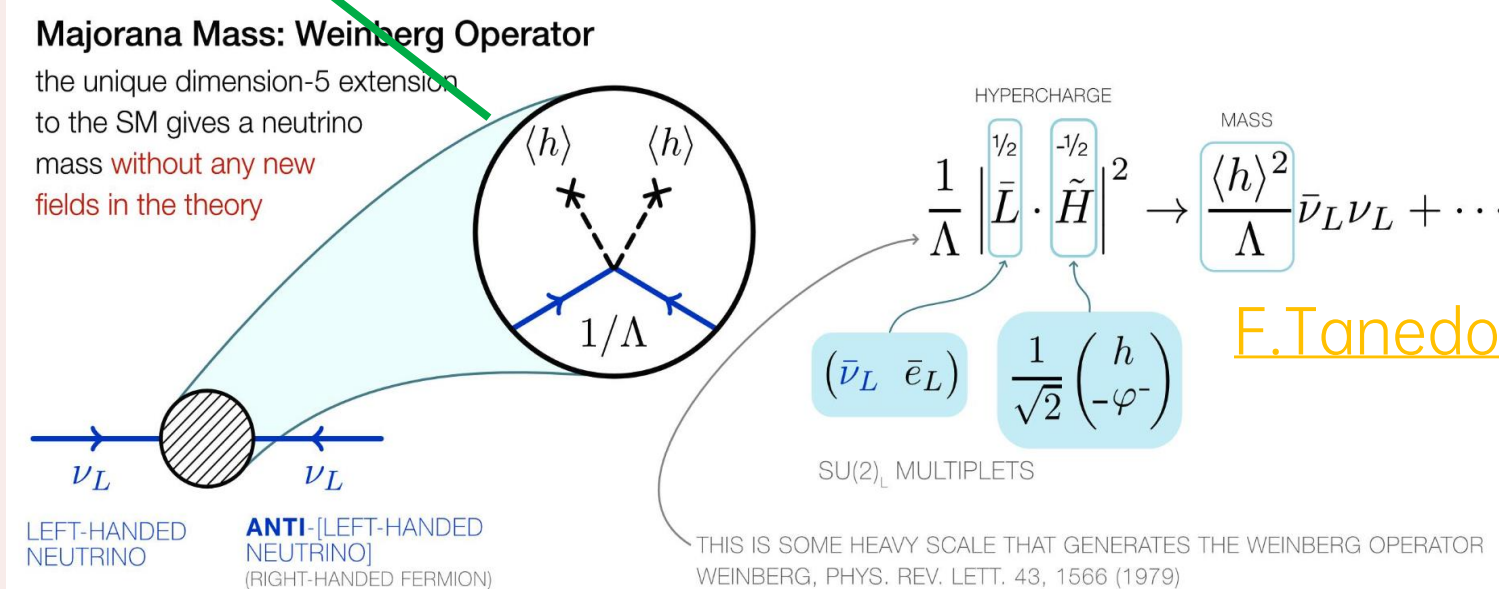


2018-2019

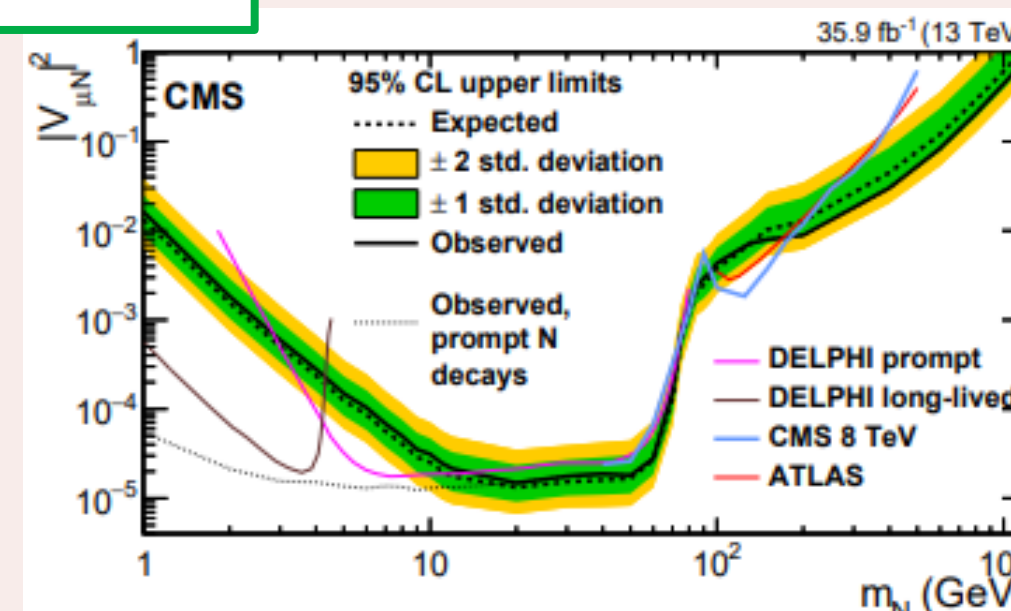
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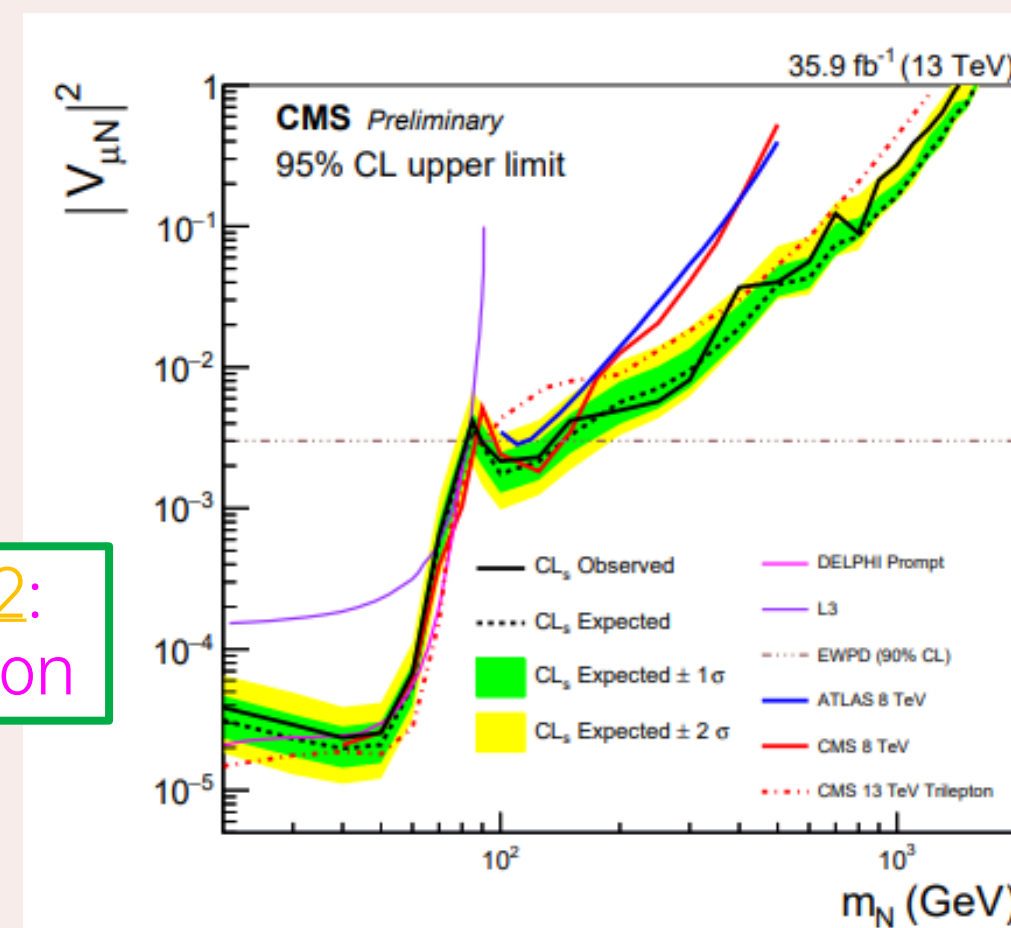
PhysRevLett.120.221801: Trilepton
JHEP01(2019)122: Same-sign dilepton



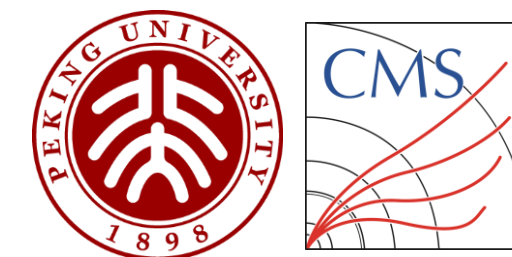
PhysRevLett.120.221801:
Trilepton



JHEP01(2019)122:
Same-sign dilepton



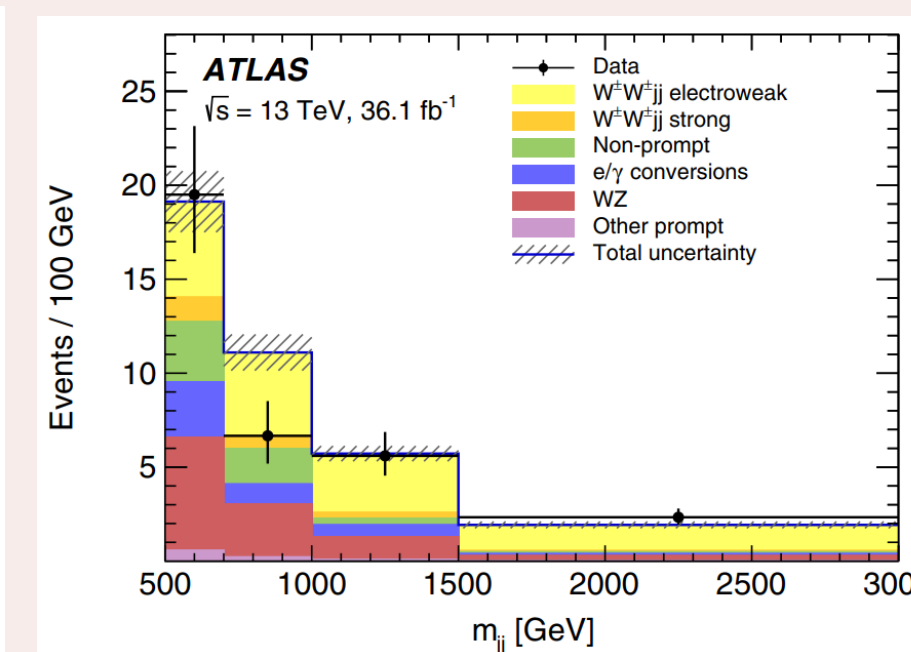
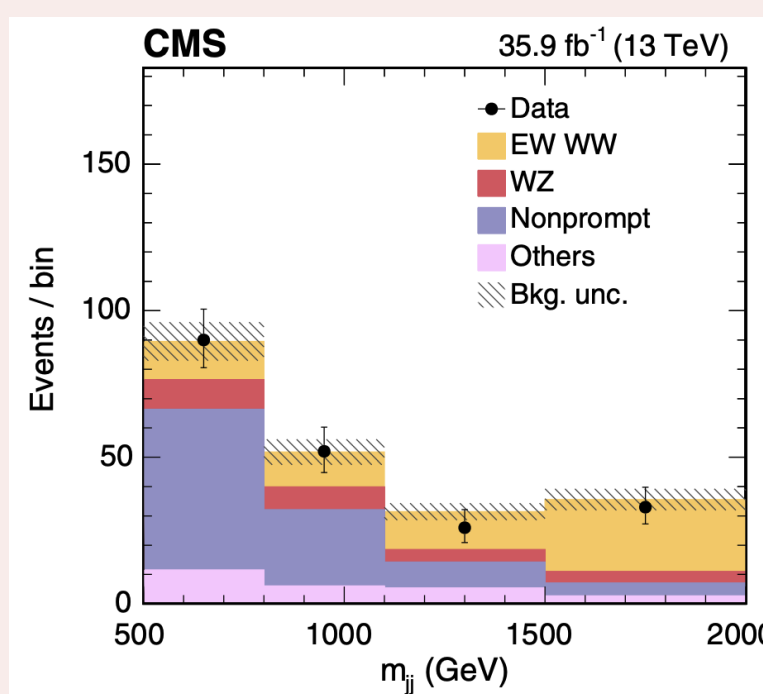
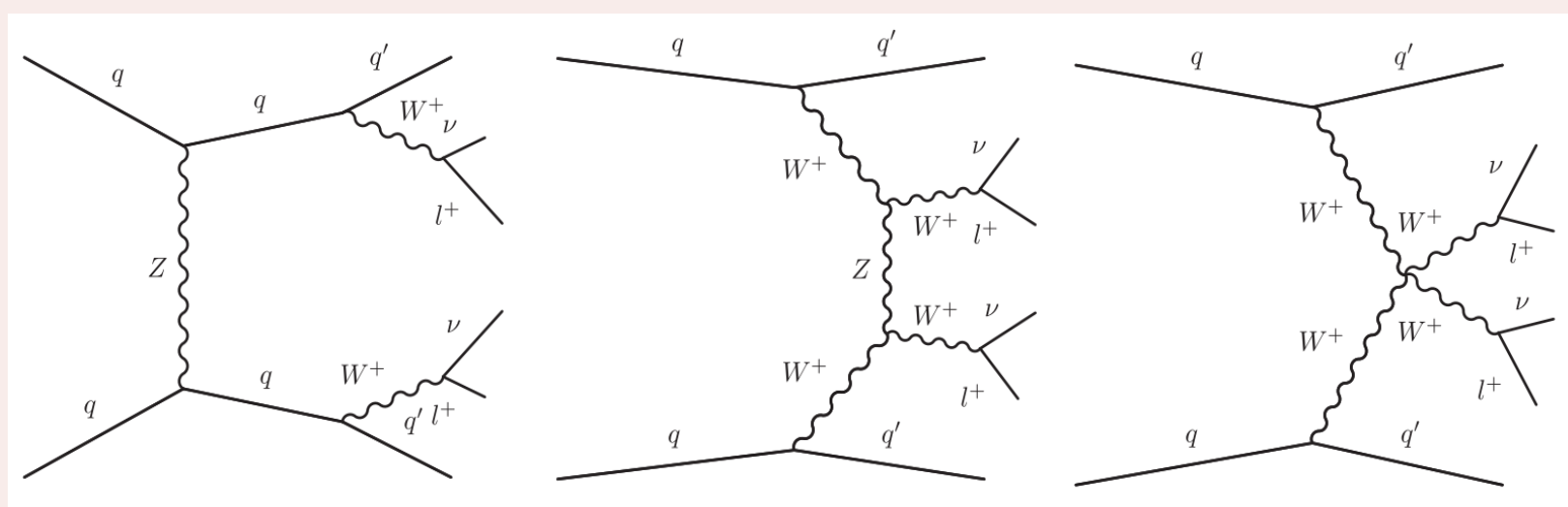
Introduction on same-sign WW scattering



2018-2019

Large Boson Collider

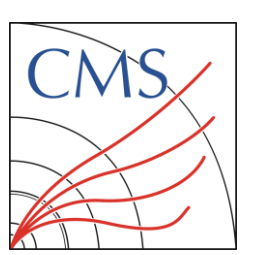
- EW production of $W^\pm W^\pm jj$: Through Vector Boson Scattering (VBS)
 - VBS processes are the key to reveal the nature of Electroweak Symmetry Breaking (EWSB)
- EW production of $W^\pm W^\pm jj$ has its own advantages:
 - Dominant EW production over QCD
 - Clear event topology: two same-sign leptons + back-to-back VBS dijet



The first observed VBS process: same-sign WW scattering

- CMS: [PhysRevLett.120.081801](https://arxiv.org/abs/1208.4094)
- ATLAS: [PhysRevLett.123.161801](https://arxiv.org/abs/1208.4094)

Heavy Majorana neutrinos in $pp \rightarrow \mu^\pm \mu^\pm jj$

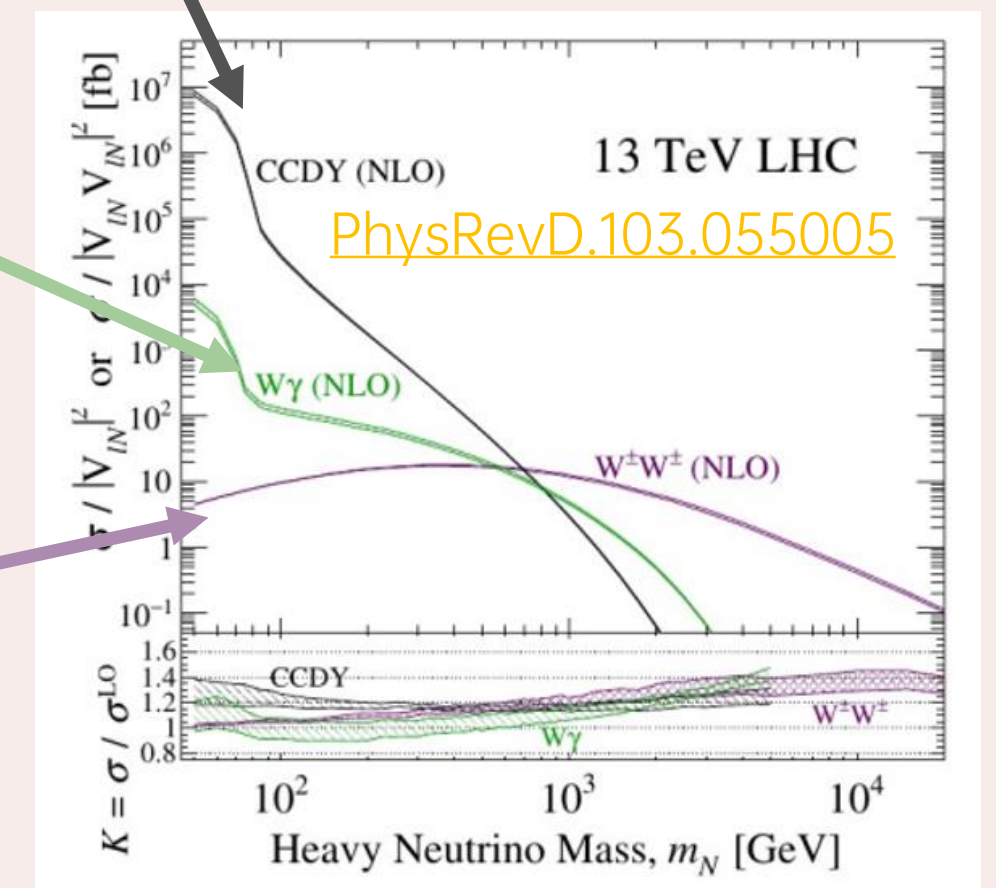
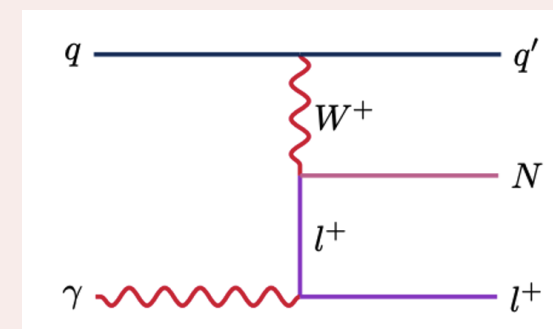
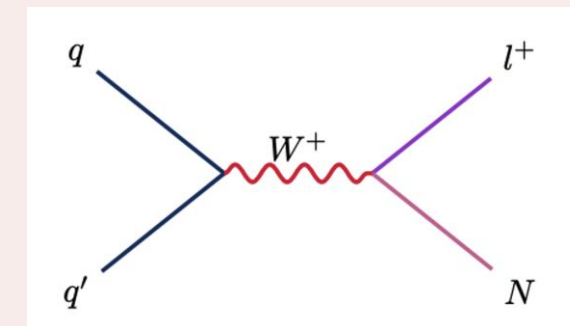
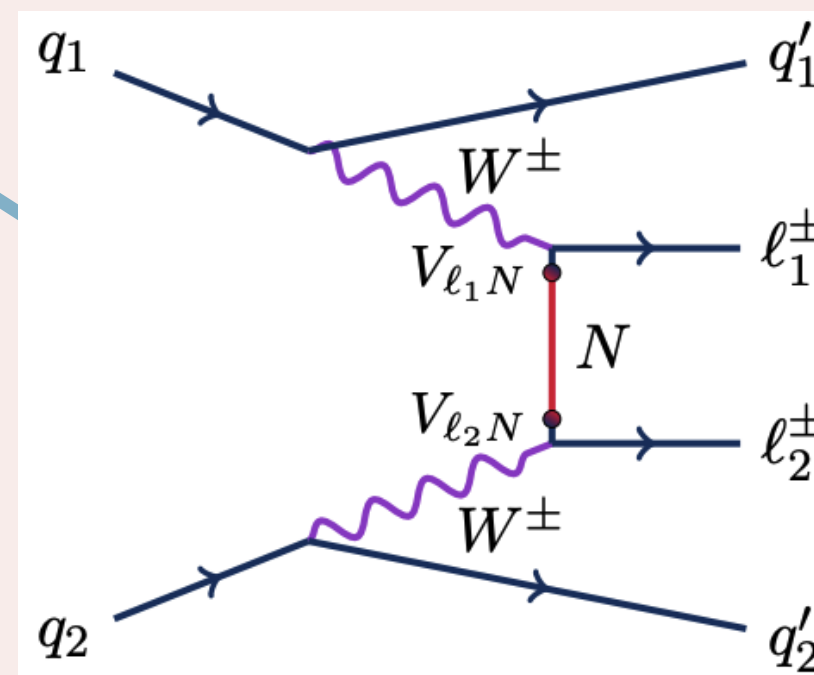


2021

- T-channel exchange of a heavy Majorana neutrino [PhysRevD.103.055005](#)
- The “neutrinoless double- β decay” version of the LHC
- Consider the dimuon channel

$$V_{lN} = \begin{pmatrix} V_{eN_1} & V_{eN_2} & V_{eN_3} \\ V_{\mu N_1} & V_{\mu N_2} & V_{\mu N_3} \\ V_{\tau N_1} & V_{\tau N_2} & V_{\tau N_3} \end{pmatrix}$$

$$\sigma(pp \rightarrow \ell_i^\pm \ell_j^\pm + X) \equiv |V_{e_i N} V_{e_j N}|^2 \times \sigma_0(pp \rightarrow \ell_i^\pm \ell_j^\pm + X)$$



- T-channel process extends the probing mass range to ~ 20 TeV

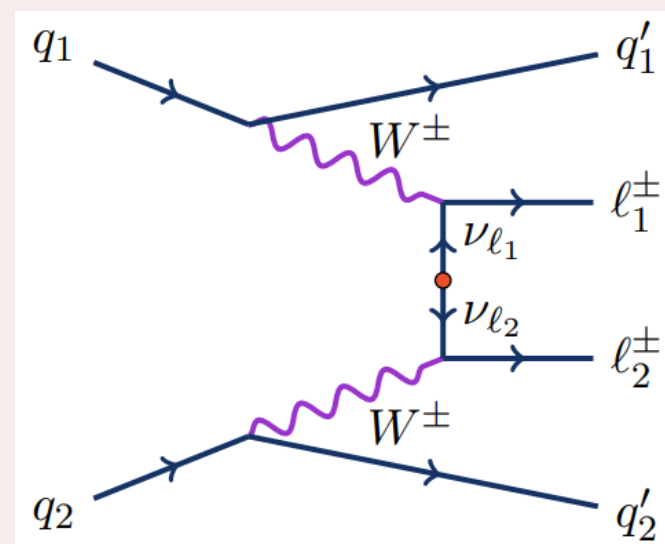
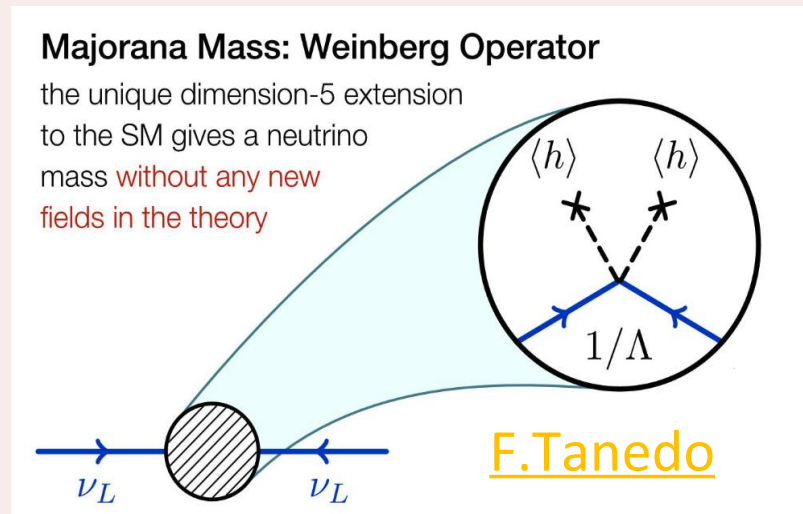
Weinberg operator in $pp \rightarrow \mu^\pm \mu^\pm jj$

2021

Take an EFT approach:

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}}^{(4)} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

The Weinberg operator is the only gauge-invariant operator at dimension-5: [Weinberg \('79\)](#)



Wilson Coefficients

$$\mathcal{L}_5 = \frac{C_5^{\ell\ell'}}{\Lambda} [\Phi \cdot \bar{L}_\ell^c] [L_{\ell'} \cdot \Phi] + \text{H.c.}$$

SM Higgs Doublet
SM Lepton
EFT Scale

$v = \sqrt{2}\langle\Phi\rangle \approx 246 \text{ GeV}$ Higgs vev

$$m_{\ell\ell'} = C_5^{\ell\ell'} v^2 / \Lambda$$

Effective Majorana Mass

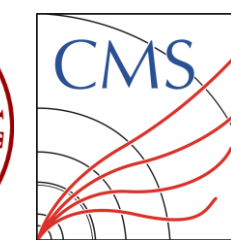
[PhysRevD.103,115014](#)

$$\sigma \sim |m_{\ell\ell'}|^2 \propto |C_5^{\ell\ell'} / \Lambda|^2$$

$$\frac{\nu_\ell(p) \nu_{\ell'}^c(-p)}{p^2} = \frac{i\not{p}}{p^2} \frac{-iC_5^{\ell\ell'} v^2}{\Lambda} \frac{i\not{p}}{p^2} = \frac{im_{\ell\ell'}}{p^2}$$

$$\hat{\sigma}(W^+W^+ \rightarrow \ell^+\ell'^+) = \frac{(2 - \delta_{\ell\ell'})}{2\pi 3^2} \left| \frac{C_5^{\ell\ell'}}{\Lambda} \right|^2 + \mathcal{O}\left(\frac{m_W^2}{M_{WW}^2}\right)$$

Signal simulation



- Generator: [Madgraph5_aMC@NLO](#)
- Focus on same-sign dimuon channel: $pp \rightarrow \mu^\pm \mu^\pm jj$
- Heavy Majorana neutrino process
 - UFO model: [SM_HeavyN_NLO](#)

```
import model SM_HeavyN_NLO
define p = g u c d s u~ c~ d~ s~
define j = p
generate p p > mu+ mu+ j j QED=4 QCD=0 $$ w+ w- / n2 n3 [QCD]
add process p p > mu- mu- j j QED=4 QCD=0 $$ w+ w- / n2 n3 [QCD]
```

- HMN: Parameters refer to [PhysRevD.103.055005](#)
 - Only consider lightest HMN N_1
 - Additional heavy mass eigenstates are decoupled $m_{N_2}=m_{N_3}=10^{10}$ GeV
 - Only mixing element $|V_{\mu N_1}|^2=1$

$$V_{lN} = \begin{pmatrix} V_{eN_1} & V_{eN_2} & V_{eN_3} \\ V_{\mu N_1} & V_{\mu N_2} & V_{\mu N_3} \\ V_{\tau N_1} & V_{\tau N_2} & V_{\tau N_3} \end{pmatrix}$$

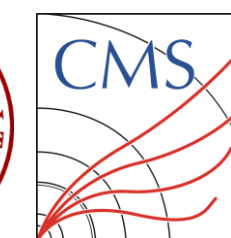
- Weinberg op. process
 - UFO model: [SMWeinbergNLO](#)

```
import model SMWeinbergNLO
generate p p > mu+ mu+ j j QED=4 QCD=0 $$ w+ w- [QCD]
add process p p > mu- mu- j j QED=4 QCD=0 $$ w+ w- [QCD]
```

- Weinberg op. : Parameters refer to [PhysRevD.103.115014](#)
 - Focus on $c_{\mu\mu}^5 = 1$ and $\Lambda = 200$ TeV

- Generator Cuts $p_T^j > 20$ GeV and $|\eta^j| < 5.5$

Background estimation



- Main backgrounds
 - Non-prompt muon
 - SM $W^\pm W^\pm jj$

Category	Estimation
$W^\pm W^\pm$ EW	From MC simulation
$W^\pm W^\pm$ QCD	
WW DPS	
WZ/γ^* QCD	
WZ EW	
Tribosons & ttV	
ZZ	From data-driven
Non-prompt muon	

- Non-prompt muon

- Heavy flavor decayed leptons, misidentified charge pions, etc.
- A data-driven approach based on matrix method

p_1 : probability for a real lepton passes the loose selection to also pass the tight selection

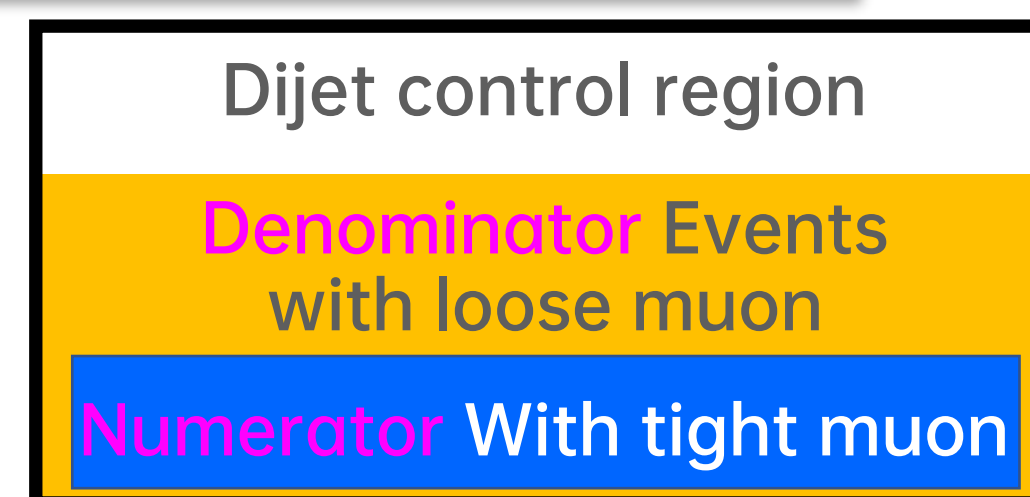
m_{XY}
X: # real leptons
Y: # fake leptons

$$\begin{pmatrix} m_{20} \\ m_{11} \\ m_{02} \end{pmatrix} = \frac{p_2 - p_1}{-(p_1 - p_2)^3} \begin{pmatrix} p_2^2 & -p_2(1-p_2) & (1-p_2)^2 \\ -2p_2p_1 & p_1(1-p_2) + p_2(1-p_1) & -2(1-p)(1-p_2) \\ p_1^2 & -p_1(1-p_1) & (1-p_1)^2 \end{pmatrix} \begin{pmatrix} n_{20} \\ n_{11} \\ n_{02} \end{pmatrix}$$

n_{XY}
X: # tight leptons
Y: # loose-but-not-tight leptons

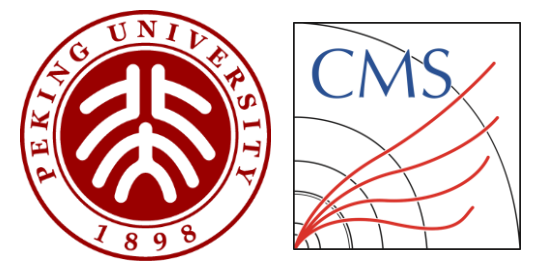
p_2 : probability for a fake lepton passes the loose selection to also pass the tight selection

$p_2 = \text{Numerator} / \text{Denominator}$



- n_{20}, n_{11}, n_{02} are observed yields measured in control regions
- p_1, p_2 are probabilities of interest measured in Z mass and QCD enriched dijet control regions respectively
- m_{20}, m_{11}, m_{02} are thus calculated based on measured n_{20}, n_{11}, n_{02} and p_1, p_2

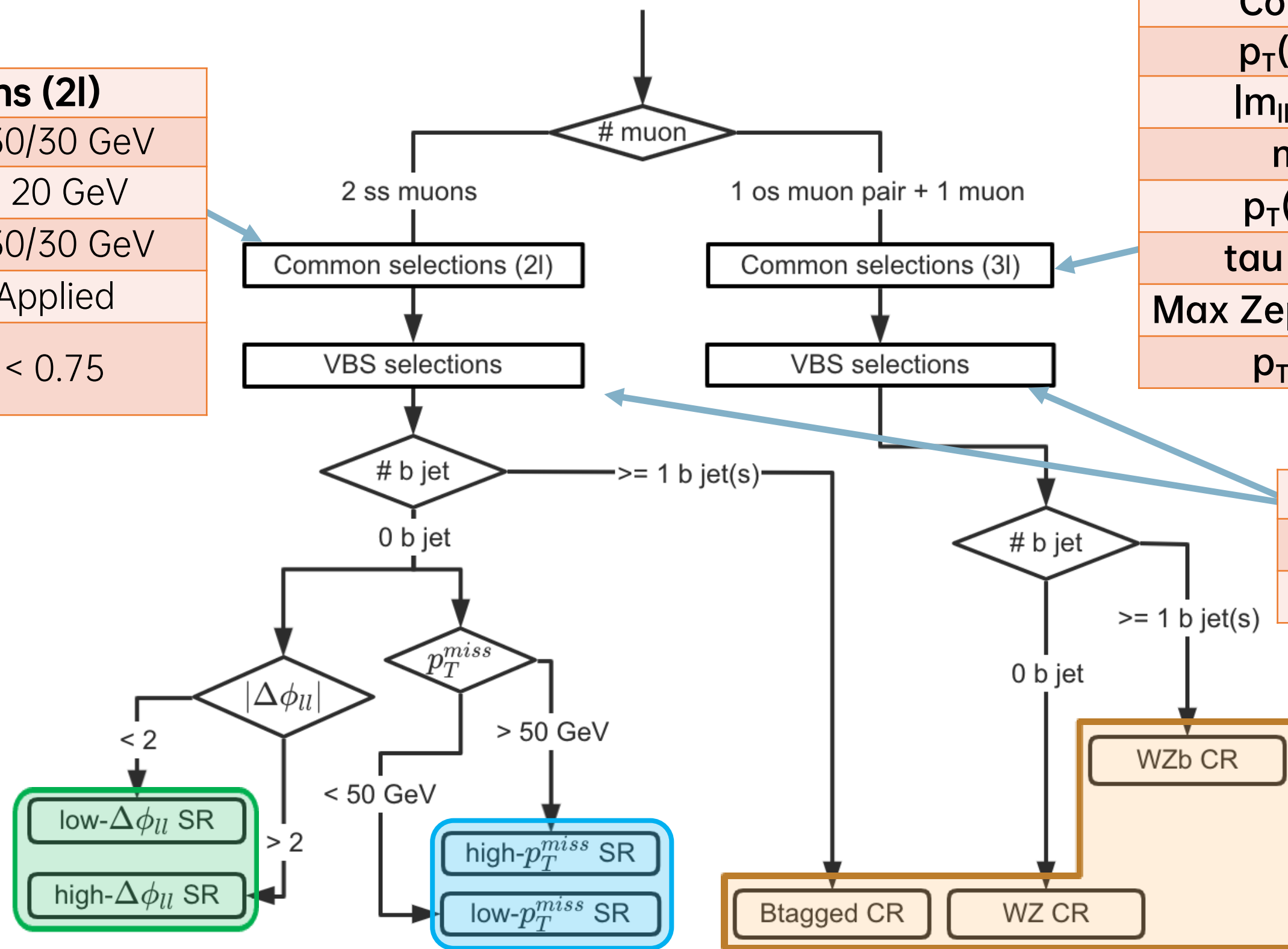
Event selection



Signal regions and background enriched control region selections are defined

Common selections (2l)	
$p_T(\text{lep})$	$> 30/30 \text{ GeV}$
m_{ll}	$> 20 \text{ GeV}$
$p_T(\text{jet})$	$> 30/30 \text{ GeV}$
tau veto	Applied
Max Zeppenfeld	< 0.75

Common selections (3l)	
$p_T(\text{lep})$	$> 25/10/25 \text{ GeV}$
$ m_{ll}-m_{ZZ} $	$< 15 \text{ GeV}$
m_{lll}	$> 100 \text{ GeV}$
$p_T(\text{jet})$	$> 30/30 \text{ GeV}$
tau veto	Applied
Max Zeppenfeld	< 1.0
p_T^{miss}	$> 30 \text{ GeV}$

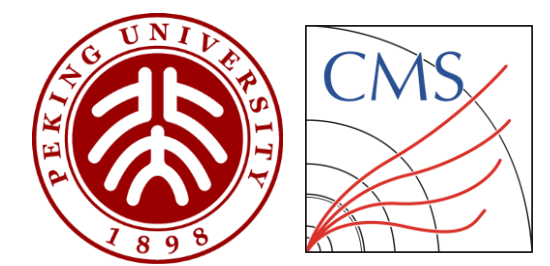


VBS selections	
m_{jj}	$> 750 \text{ GeV}$
$ \Delta\eta_{jj} $	> 2.5

Low- $\Delta\phi_{ll}$ SR and high- $\Delta\phi_{ll}$ SR are used for heavy Majorana neutrino processes

High- p_T^{miss} SR and low- p_T^{miss} SR are used for Weinberg op. process

Fit Strategy



□ Via a simultaneous fit of signal regions and control regions

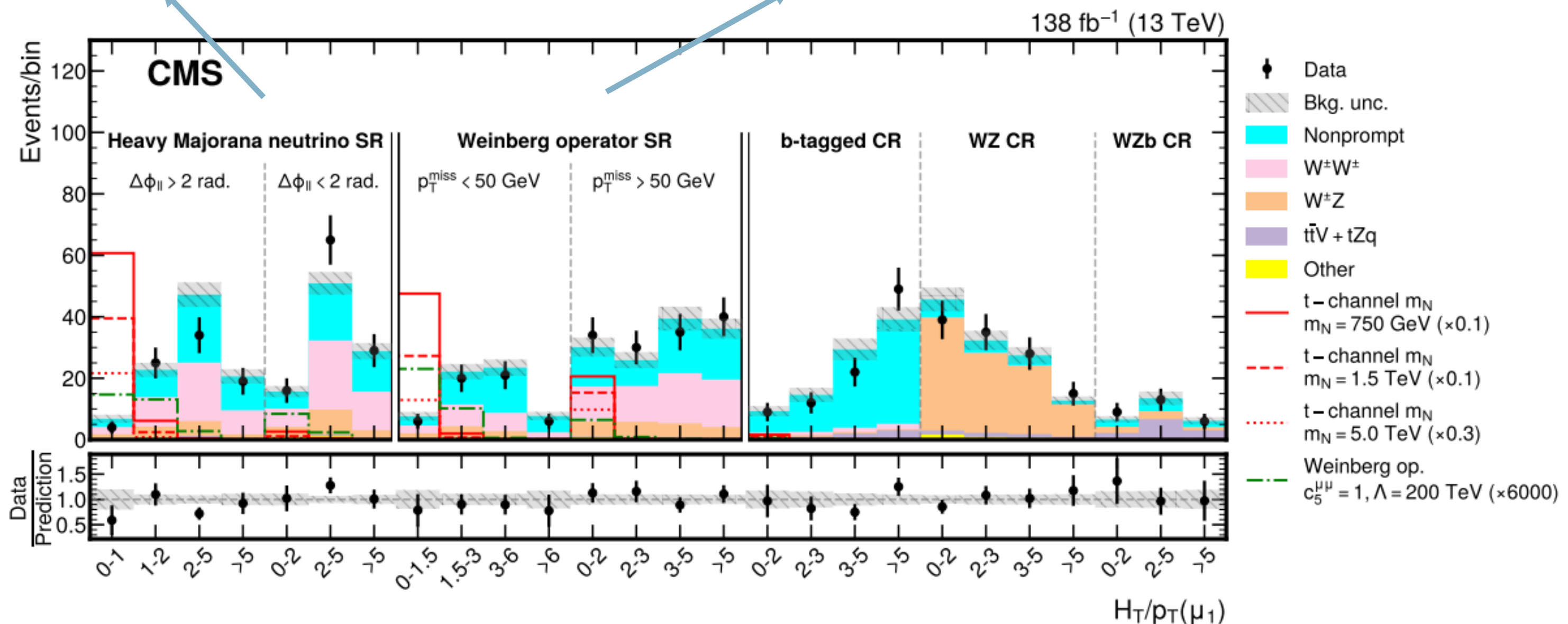
- Sub signal regions ($\Delta\phi_{ll}/p_T^{\text{miss}}$) are considered
- The fitted distribution is $H_T/p_{\mu 1}$ where $H_T = \sum p_T^i(\text{jet}), (i \in p_T(\text{jet}) > 30\text{GeV})$

Region	Variable	variable binning	Region	Variable	variable binning
high $\Delta\phi_{\ell\ell}$ bin in $W^\pm W^\pm$ SR	$H_T/p_T^{\ell 1}$	[0, 1, 2, 5, ∞]	low p_T^{miss} bin in $W^\pm W^\pm$ SR	$H_T/p_T^{\ell 1}$	[0, 1.5, 3, 6, ∞]
low $\Delta\phi_{\ell\ell}$ bin in $W^\pm W^\pm$ SR	$H_T/p_T^{\ell 1}$	[0, 2, 5, ∞]	high p_T^{miss} bin in $W^\pm W^\pm$ SR	$H_T/p_T^{\ell 1}$	[0, 2, 3, 5, ∞]
b-tagged CR	$H_T/p_T^{\ell 1}$	[0, 2, 3, 5, ∞]	b-tagged CR	$H_T/p_T^{\ell 1}$	[0, 2, 3, 5, ∞]
WZ CR	$H_T/p_T^{\ell 1}$	[0, 2, 3, 5, ∞]	WZ CR	$H_T/p_T^{\ell 1}$	[0, 2, 3, 5, ∞]
WZb CR	$H_T/p_T^{\ell 1}$	[0, 2, 5, ∞]	WZb CR	$H_T/p_T^{\ell 1}$	[0, 2, 5, ∞]

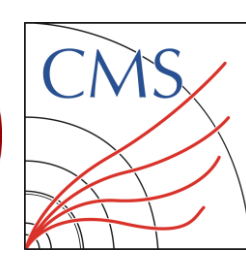
- Inverted signal regions
- Control non-prompt
- Control WZ
- Control tZq

HMN Fitting Strategy

Weinberg Op. Fitting Strategy



Result on Upper limits: Heavy Majorana Neutrinos



Interpretations of limits

- Parameter of interest (POI) in fit: signal strength $\hat{\mu}$
 - Can be converted to upper limits on cross section:

$$\hat{\sigma} = \sigma_{\text{benchmark}} \times \hat{\mu}$$

For HMN:
Unit mixing only for muon:

$$|V_{\mu N}|^2 = 1, \\ |V_{eN}|^2 = |V_{\tau N}|^2 = 0$$

For VBF production of HMN

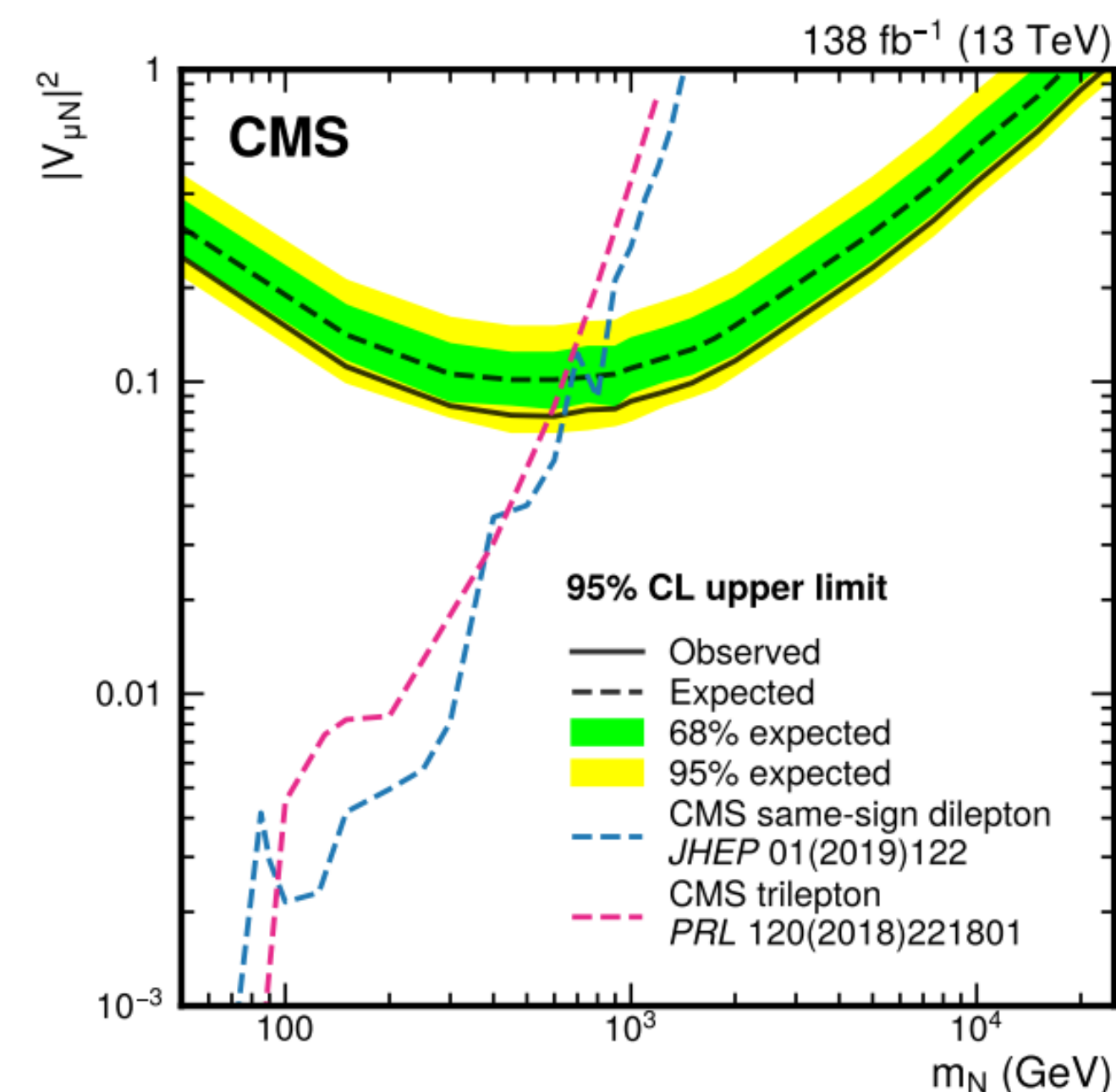
- The cross section dependence reads:

$$\sigma(pp \rightarrow \ell_i^\pm \ell_j^\pm + X) \equiv |V_{\ell_i N} V_{\ell_j N}|^2 \times \sigma_0(pp \rightarrow \ell_i^\pm \ell_j^\pm + X)$$

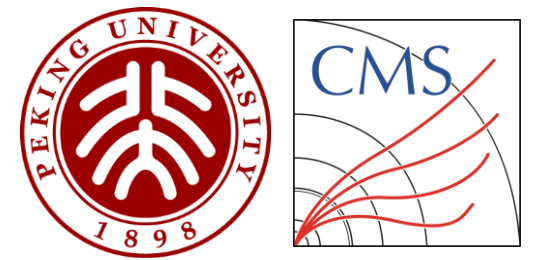
- Upper limits on signal strength can be translated to the squared mixing element $|V_{\mu N}|^2 = \sqrt{\hat{\mu}}$

Better constraints on $|V_{\mu N}|^2$ for $m_N \gtrsim 650$ GeV

Probing m_N up to 23 TeV



Result on Upper limits: Weinberg operator



□ Interpretations of limits

- POI in fit: signal strength $\hat{\mu}$
 - Can be converted to upper limits on cross section:

$$\hat{\sigma} = \sigma_{\text{benchmark}} \times \hat{\mu}$$

For Weinberg op. :

Unit Wilson Coefficient only for dimuon:

$$|C_5^{\mu\mu}|^2 = 1, \text{ others} = 0$$

And EFT Scale $\Lambda = 200$ GeV

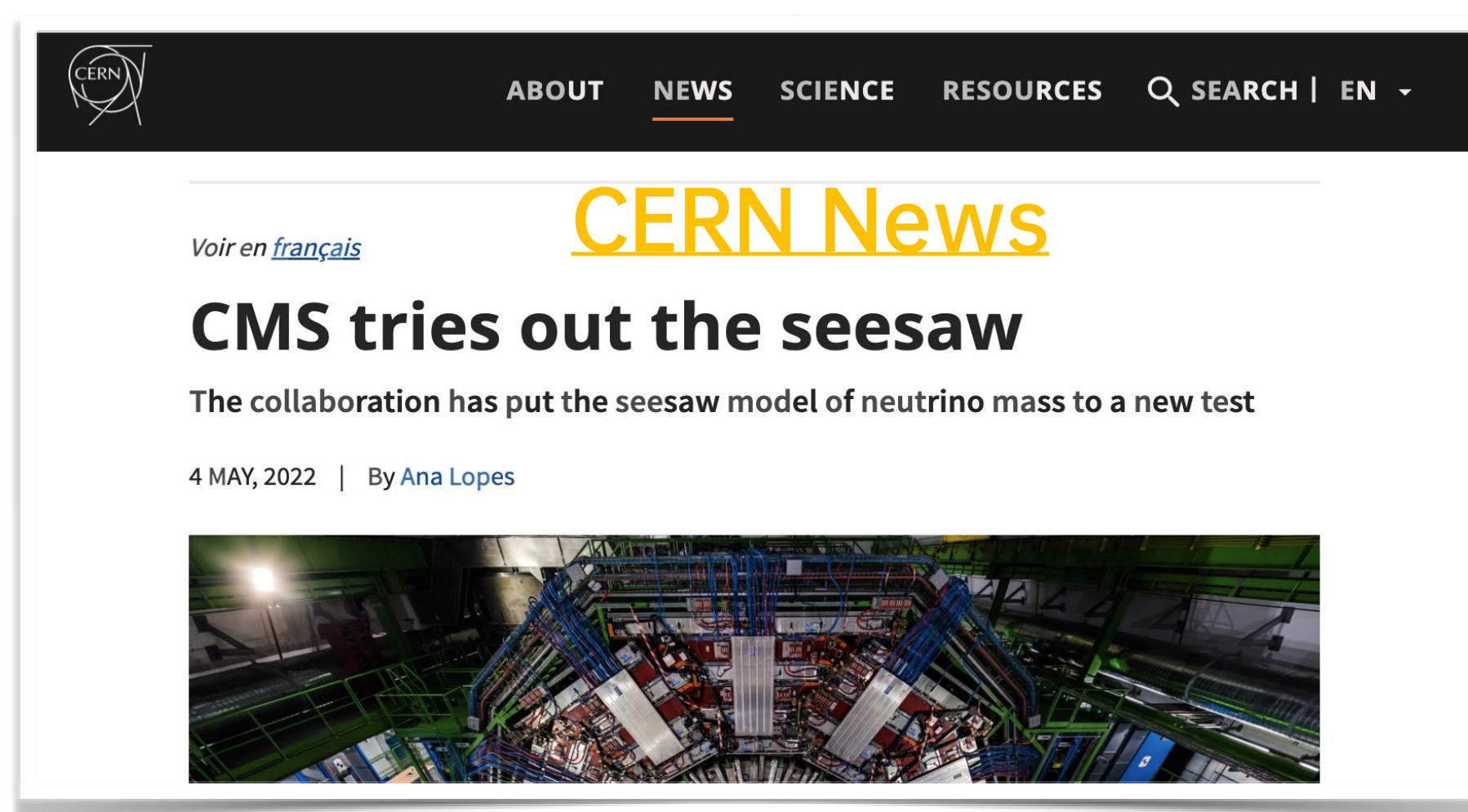
□ For Weinberg op. processes

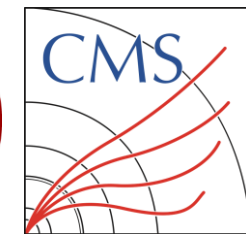
- The cross-section dependence reads:
$$\hat{\sigma}(W^+W^+ \rightarrow \ell^+\ell'^+) = \frac{(2 - \delta_{\ell\ell'})}{2\pi 3^2} \left| \frac{C_5^{\ell\ell'}}{\Lambda} \right|^2 + \mathcal{O}\left(\frac{m_W^2}{M_{WW}^2}\right)$$
- Effective Majorana Mass is given by:
$$m_{\ell\ell'} = C_5^{\ell\ell'} v^2 / \Lambda$$
- Interpretation: translate to EFT scale limit with Wilson coefficient fixed to unit, thus
$$\hat{\Lambda} = 200 \times \hat{\mu}^{-\frac{1}{2}} \text{ GeV}$$
, and translate to effective Majorana mass limit
$$m_{\mu\mu} = v^2 |C_5^{\mu\mu}| / \hat{\Lambda}$$

□ Results

- Observed (expected) lower bound on EFT scales Λ : 5.6 (4.7) TeV (assuming $C_5^{\mu\mu} = 1$)
- Observed (expected) upper limit of effective Majorana mass $|m_{\mu\mu}|$: 10.8 (12.8) GeV

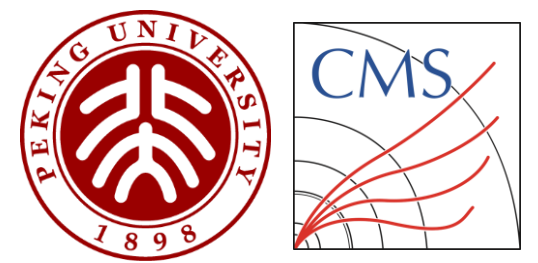
- Performed analysis on VBF production of same-sign muon pairs associated with two jets
 - Heavy Majorana neutrino from Type-I Seesaw Model
 - Upper limits on $|V_{\mu N}|^2$ for m_N up to around 23 TeV
 - Better constraints on $|V_{\mu N}|^2$ for $m_N \gtrsim 650$ GeV
 - First search at collider on dimension-5 Weinberg operator
 - Upper limit of effective Majorana mass $|m_{\mu\mu}|$, observed (expected): 10.8 (12.8) GeV
- [arXiv: 2206.08956](https://arxiv.org/abs/2206.08956) accepted by PRL as an Editors' Suggestion





Backup

Cross sections of signal samples



- Cross sections of **HMN** processes
 - Cross sections from around 0.1 to 18 fb

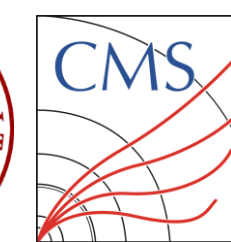
m_N (GeV)	50	150	300	450	600	750	900	1000	1250
XS(fb)	4.606	13.57	17.99	18.20	16.91	15.47	13.86	12.84	10.67
m_N (GeV)	1500	1750	2000	2500	5000	7500	10000	15000	20000
XS (fb)	8.918	7.612	6.425	4.811	1.598	0.7736	0.4480	0.2052	0.1165

- Cross sections of **Weinberg op.** processes
 - Focus with benchmark inputs: $c_{\mu\mu}^5 = 1$ and $\Lambda = 200$ TeV

$$\sigma \sim |m_{\ell\ell'}|^2 \propto |C_5^{\ell\ell'} / \Lambda|^2$$

EFT scale Λ (TeV)	10	100	200	400
Effective Majorana mass (GeV)	6	0.6	0.3	0.15
Cross Section (pb)	1.33×10^{-4}	1.50×10^{-4}	3.74×10^{-7}	9.35×10^{-8}

Systematics Uncertainties



□ Theoretical uncertainties:

- QCD scale uncertainties and PDF uncertainties considered for signal processes (HMN & WO) and $W^\pm W^\pm jj$ process
 - Renormalization and factorization QCD scale uncertainties
 - PDF uncertainties: standard deviation of PDF variations in sample production
- NLO corrections are considered for $W^\pm W^\pm jj$ and $W^\pm Z jj$ process

□ Data-Driven non-prompt muon background

- Two major sources of uncertainty:
 - Statistical uncertainty for non-prompt rate measurement: by most 5%
 - Flavor composition discrepancies of faking lepton jets between measurement region and signal region: estimated from varying p_T of "away-side" jets, by most 7%
 - The estimation approaches are however conservative, thus a 30% total uncertainty for non-prompt muon modeling is applied in addition.

□ Common experimental uncertainties

- Integrated luminosity
- Jet Energy Scale
- ...

□ Sample statistical uncertainties

- [Barlow-Beeston-lite method](#)

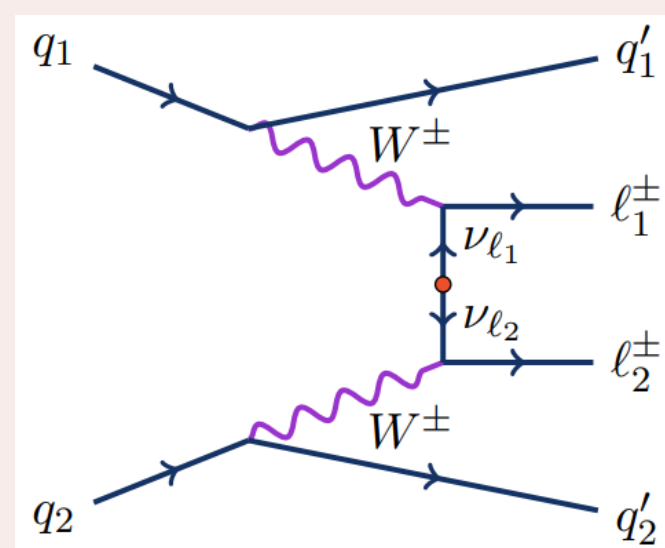
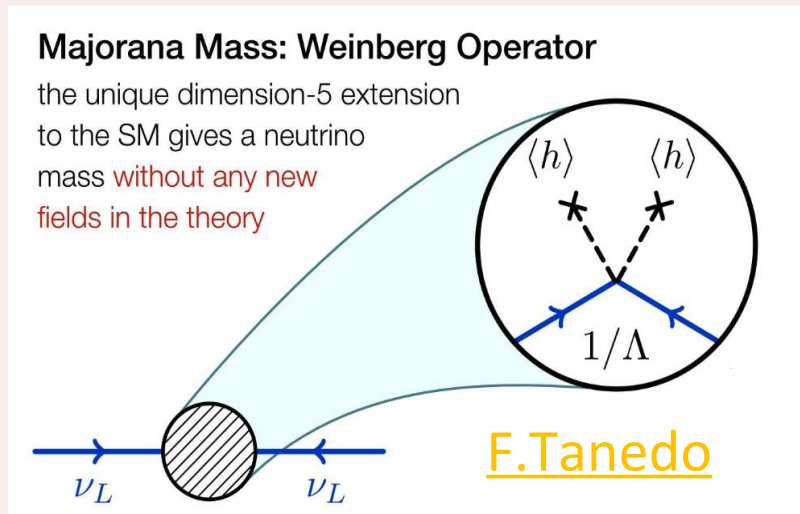
Weinberg operator in $pp \rightarrow \mu^\pm \mu^\pm jj$

2021

Take an EFT approach:

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}}^{(4)} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

The Weinberg operator is the only gauge-invariant operator at dimension-5: [Weinberg \('79\)](#)



Wilson Coefficients

$$\mathcal{L}_5 = \frac{C_5^{\ell\ell'}}{\Lambda} [\Phi \cdot \bar{L}_\ell^c] [L_{\ell'} \cdot \Phi] + \text{H.c.}$$

→ SM Higgs Doublet
→ SM Lepton
→ EFT Scale

$v = \sqrt{2}\langle\Phi\rangle \approx 246 \text{ GeV}$ Higgs vev

$$m_{\ell\ell'} = C_5^{\ell\ell'} v^2 / \Lambda$$

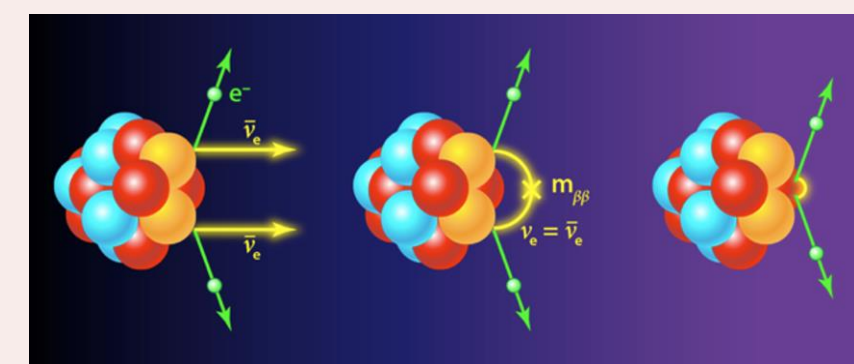
Effective Majorana Mass

[PhysRevD.103,115014](#)

$$\sigma \sim |m_{\ell\ell'}|^2 \propto |C_5^{\ell\ell'} / \Lambda|^2$$

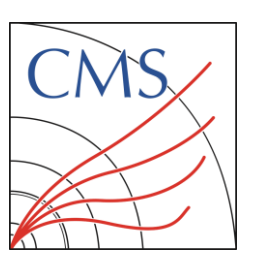
$$\nu_\ell(p) \nu_{\ell'}^c(-p) = \frac{i\not{p}}{p^2} \frac{-iC_5^{\ell\ell'} v^2}{\Lambda} \frac{i\not{p}}{p^2} = \frac{im_{\ell\ell'}}{p^2}$$

$$\hat{\sigma}(W^+W^+ \rightarrow \ell^+\ell'^+) = \frac{(2 - \delta_{\ell\ell'})}{2\pi^3^2} \left| \frac{C_5^{\ell\ell'}}{\Lambda} \right|^2 + \mathcal{O}\left(\frac{m_W^2}{M_{WW}^2}\right)$$

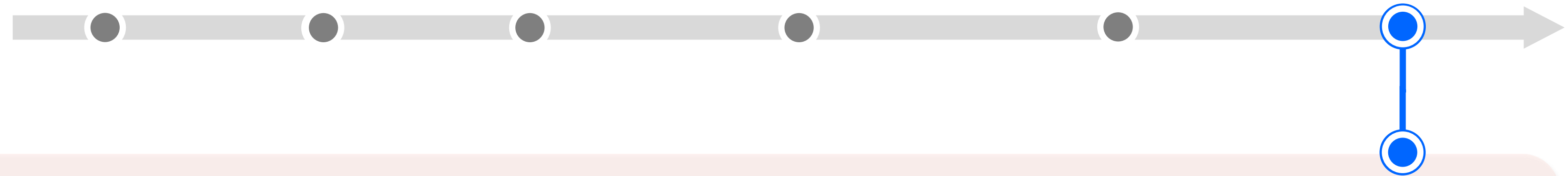


$0\nu\beta\beta$ @ [GERDA \[2009.06079\]](#)
 $|m_{ee}| < 79 - 180 \text{ meV}$ at 90% C.L.

Weinberg operator in $pp \rightarrow \mu^\pm \mu^\pm jj$



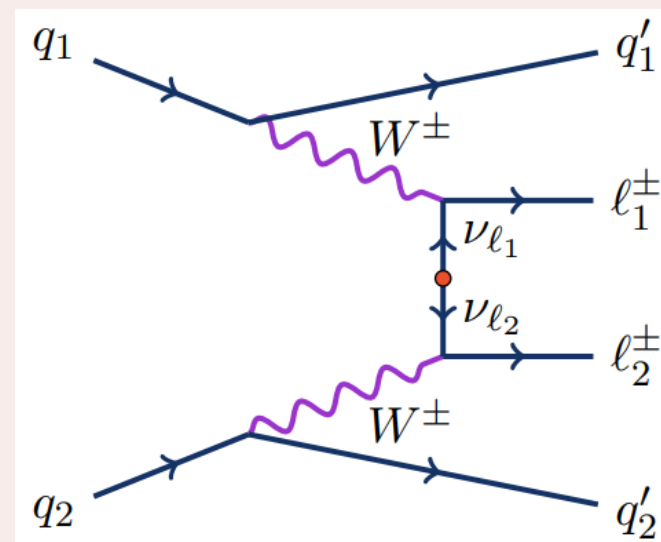
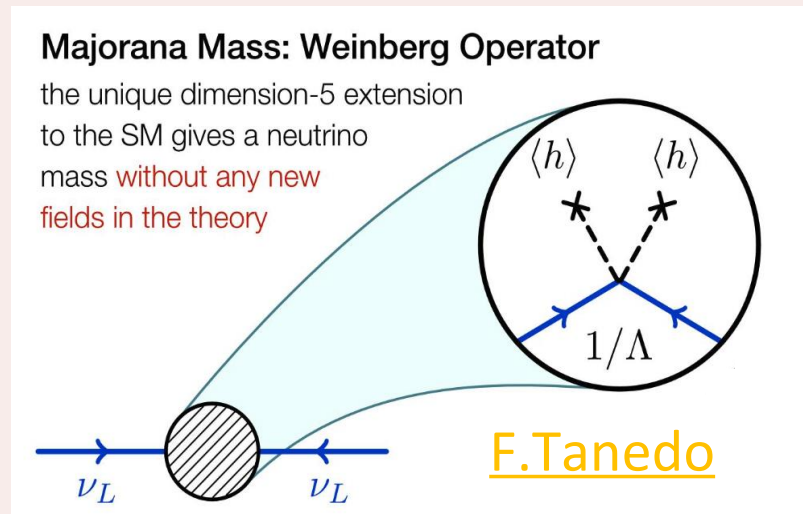
2021



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Effective Majorana Mass

[PhysRevD.103,115014](#)

$$\sigma \sim |m_{\ell\ell'}|^2 \propto |C_5^{\ell\ell'} / \Lambda|^2$$

$$\frac{\nu_\ell(p) \nu_{\ell'}^c(-p)}{p} = \frac{i \not{p} \frac{-i C_5^{\ell\ell'} v^2}{\Lambda} i \not{p}}{p^2} = \frac{i m_{\ell\ell'}}{p^2}$$

$$\hat{\sigma}(W^+ W^+ \rightarrow \ell^+ \ell'^+) = \frac{(2 - \delta_{\ell\ell'})}{2\pi 3^2} \left| \frac{C_5^{\ell\ell'}}{\Lambda} \right|^2 + \mathcal{O}\left(\frac{m_W^2}{M_{WW}^2}\right)$$

