Mixed QCD-EW corrections to W-pair production at electron-position colliders

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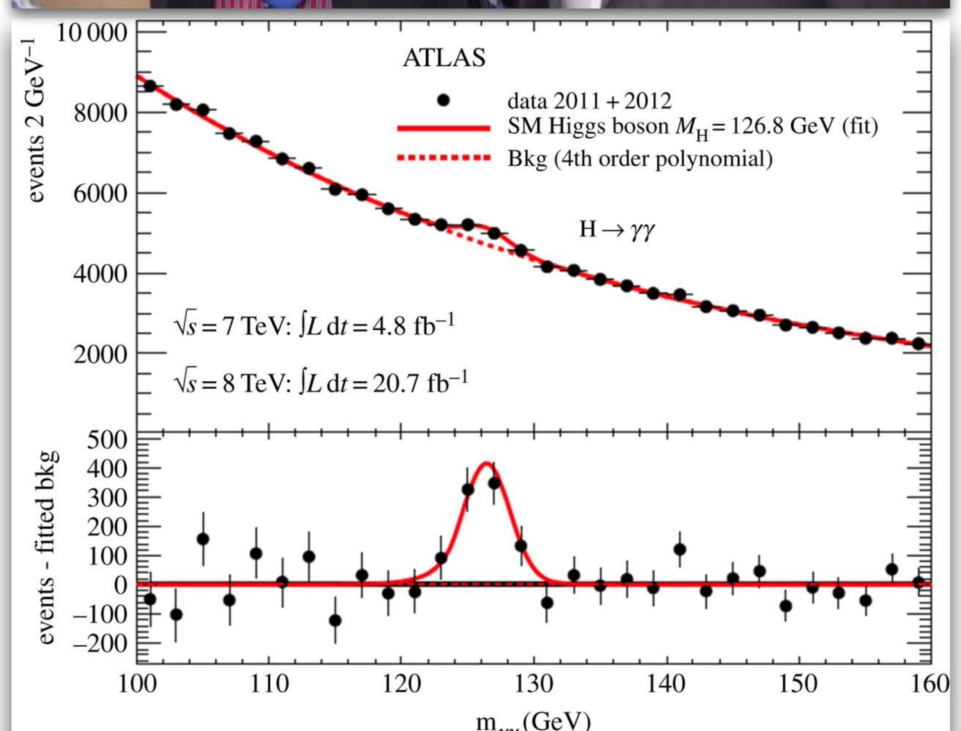
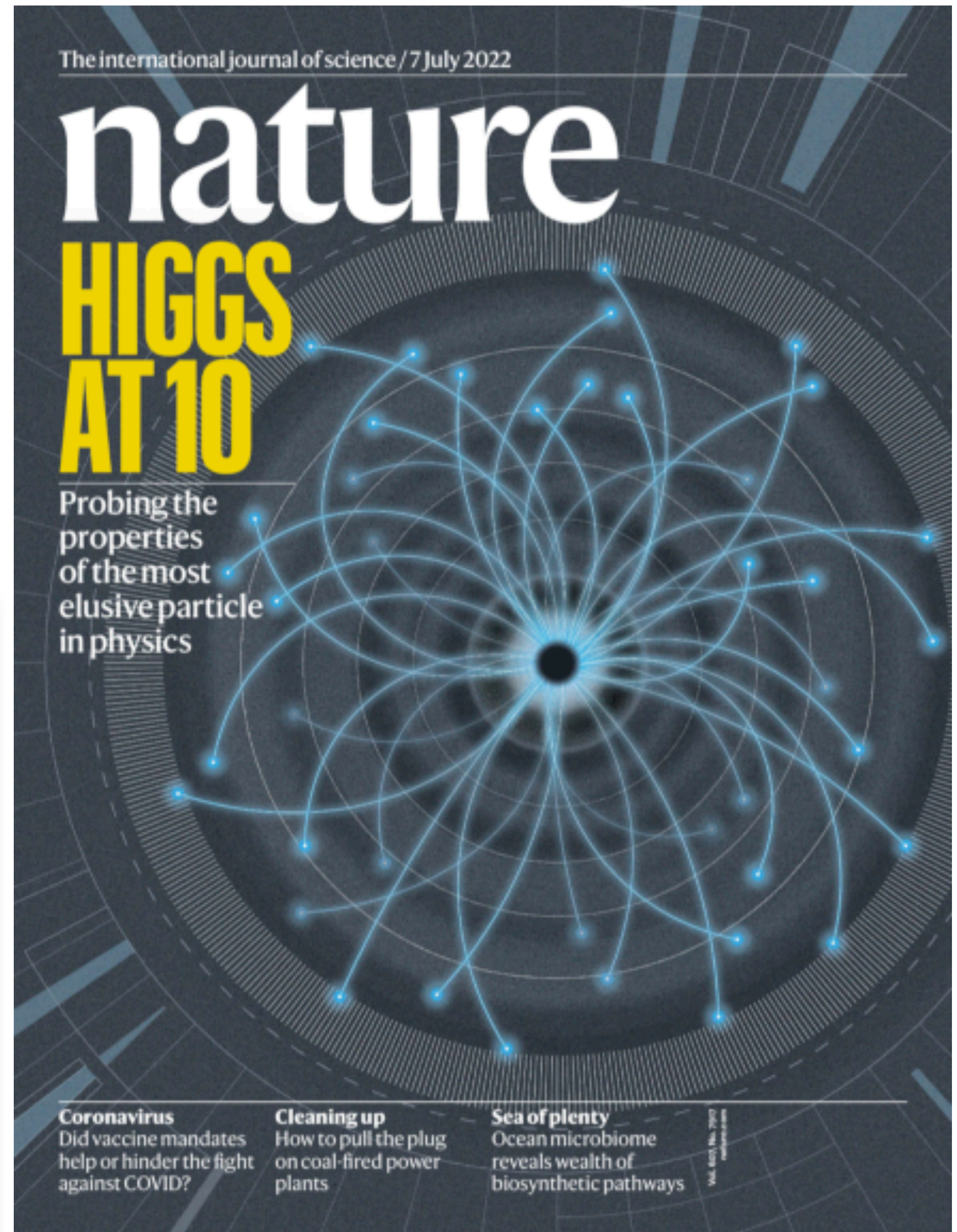
Based on 221* . * * * *

Outline

Discovery of Higgs

Ten Years



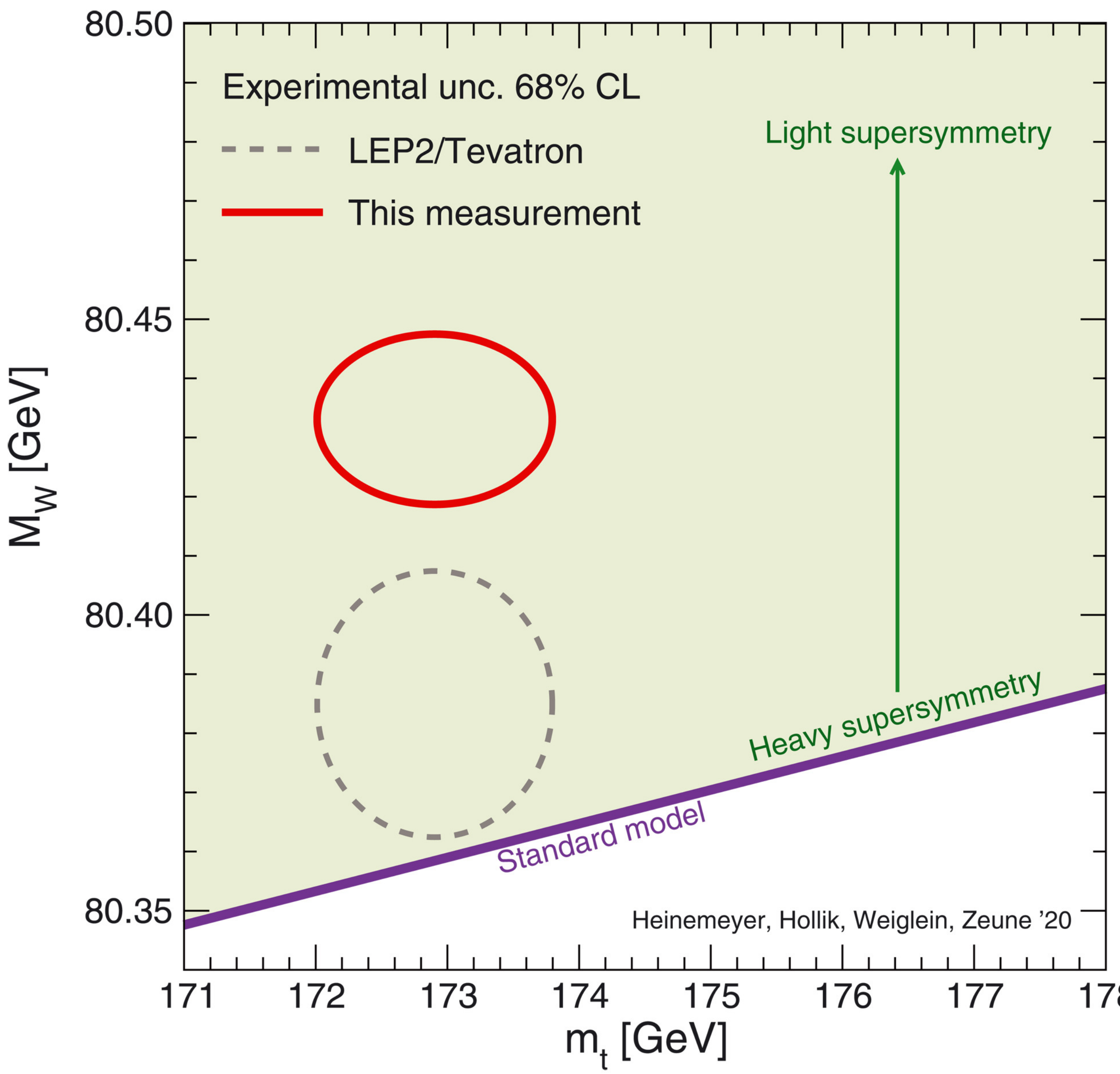


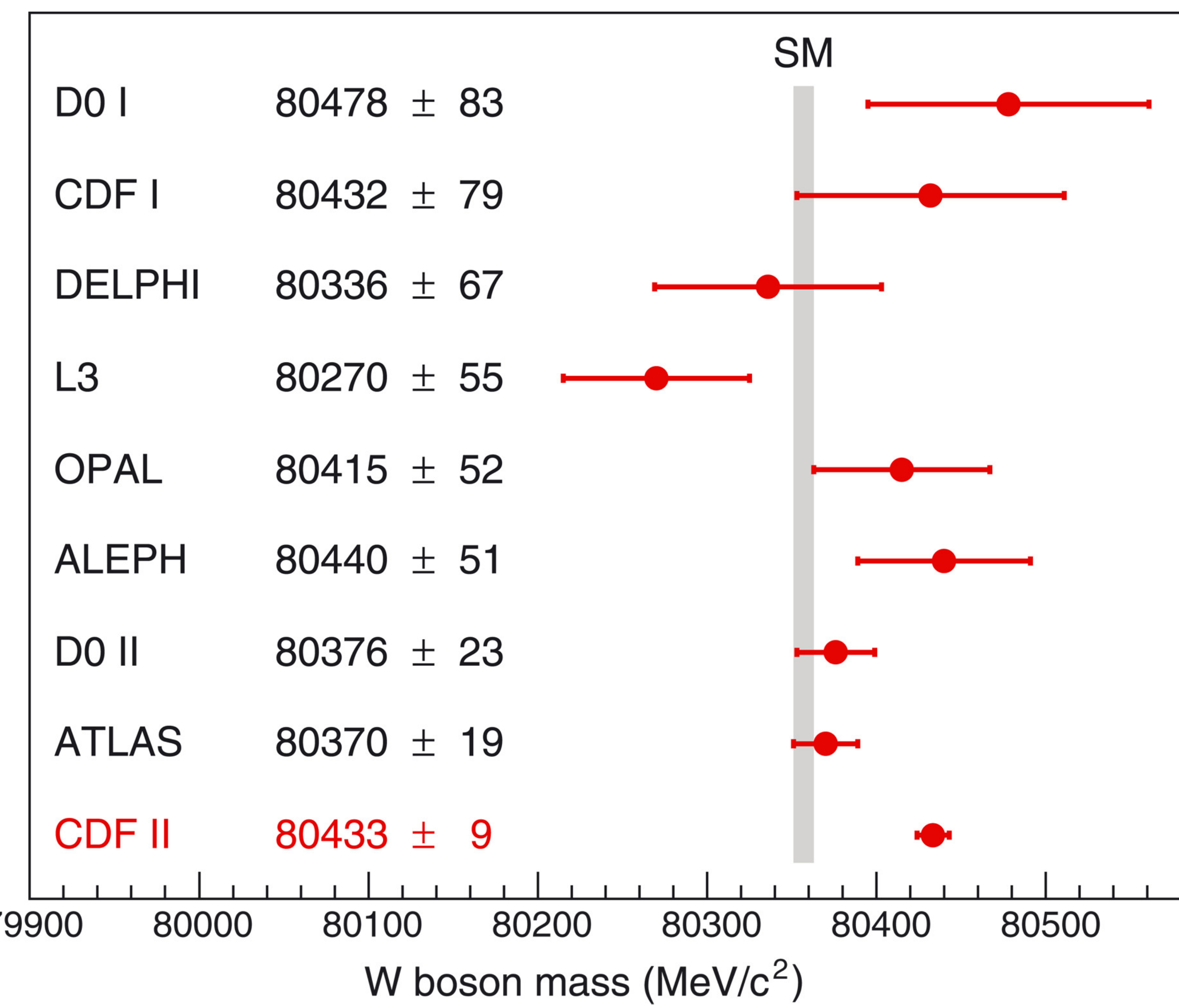
Background

W-mass anomaly

SM ⟷ CDF II ($7\ \sigma$)

$$m_W^{\rm CDF} = 80.4335 \pm 0.0094 \mathrm{GeV}$$





Background

W-mass anomaly

SM ⟷ CDF II (7 σ)

$$m_W^{\text{CDF}} = 80.4335 \pm 0.0094\text{GeV}$$

High-precision

Experimental Measurements

Theoretical Predictions

Background

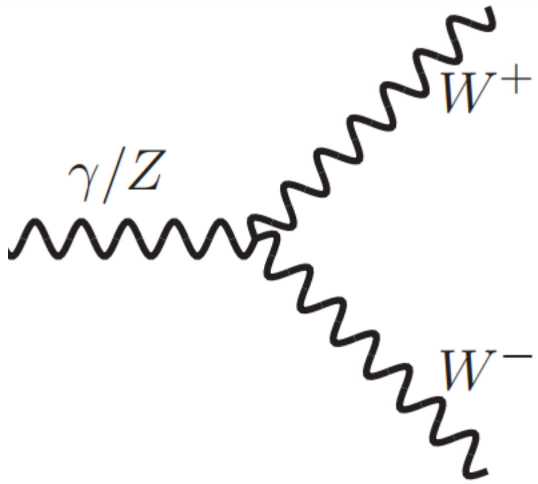
W-mass anomaly

SM ⟷ CDF II ($7\ \sigma$)

$$m_W^{\text{CDF}} = 80.4335 \pm 0.0094\text{GeV}$$

High-precision

W-pair production



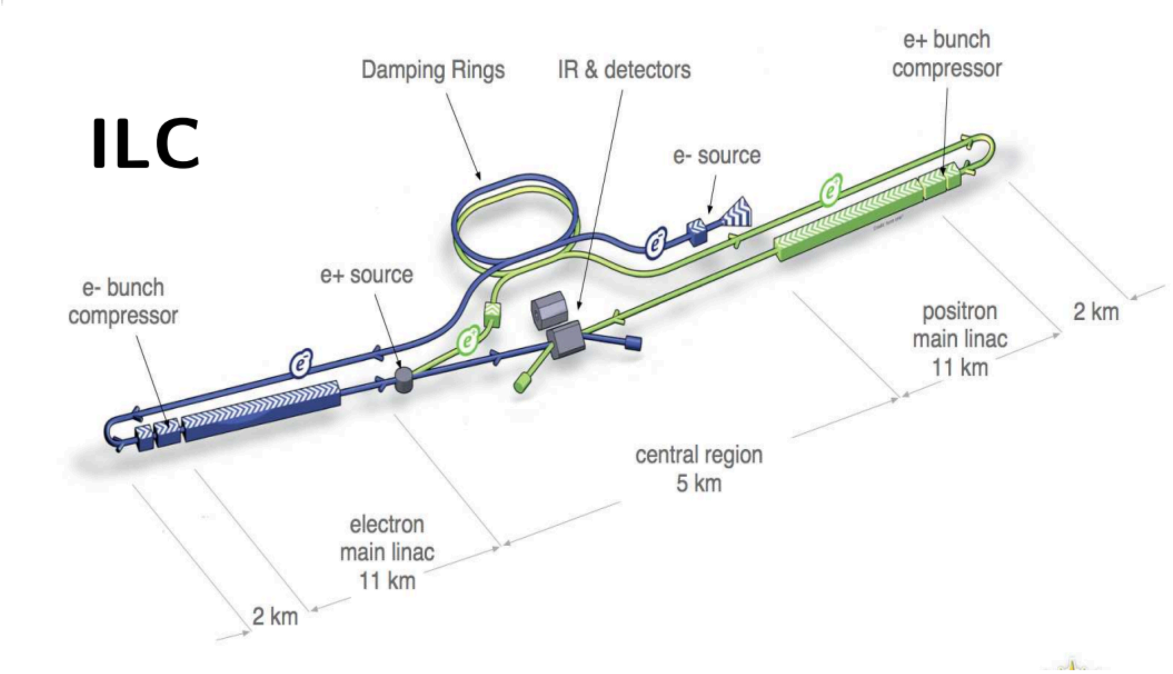
- Direct measurement of the W-mass
- Direct measurement of triple gauge boson couplings (TGCs)

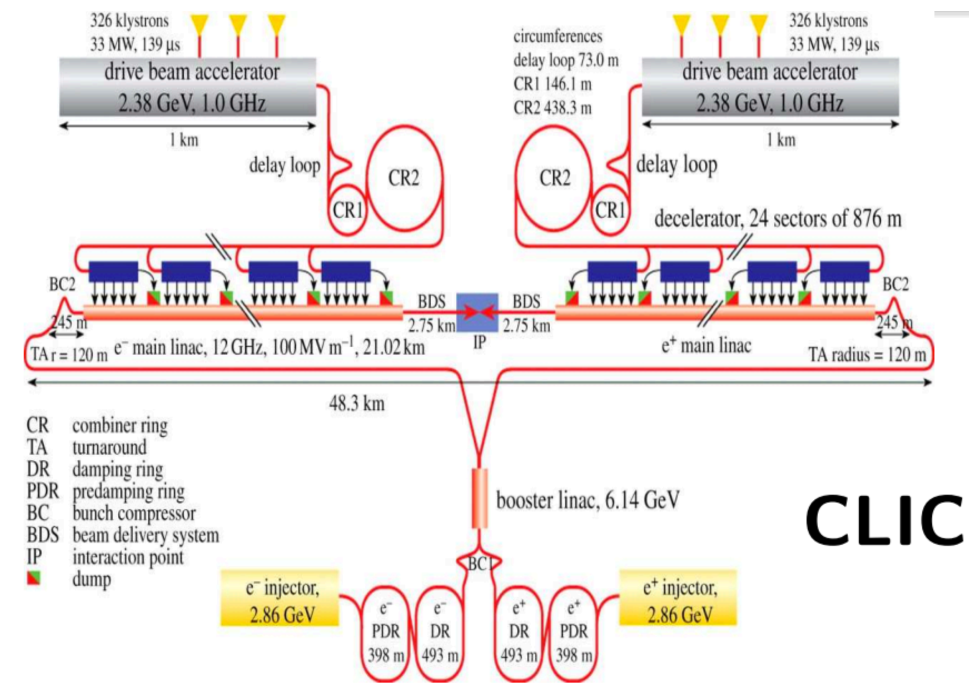
Background

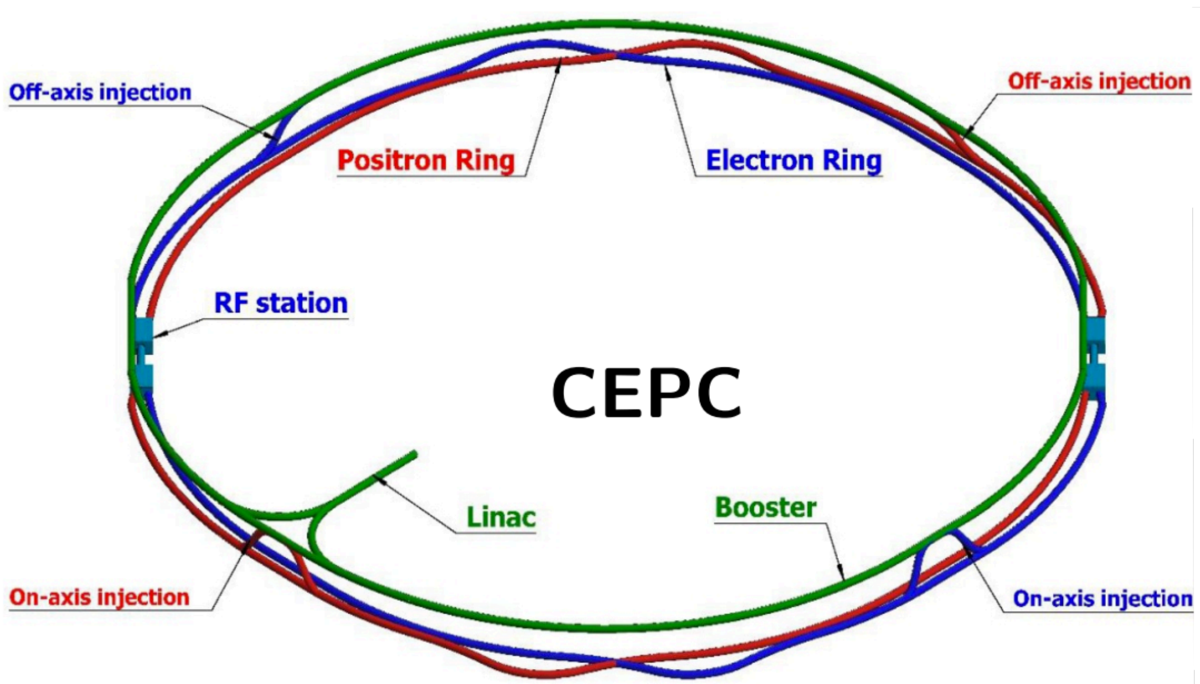
Lepton colliders

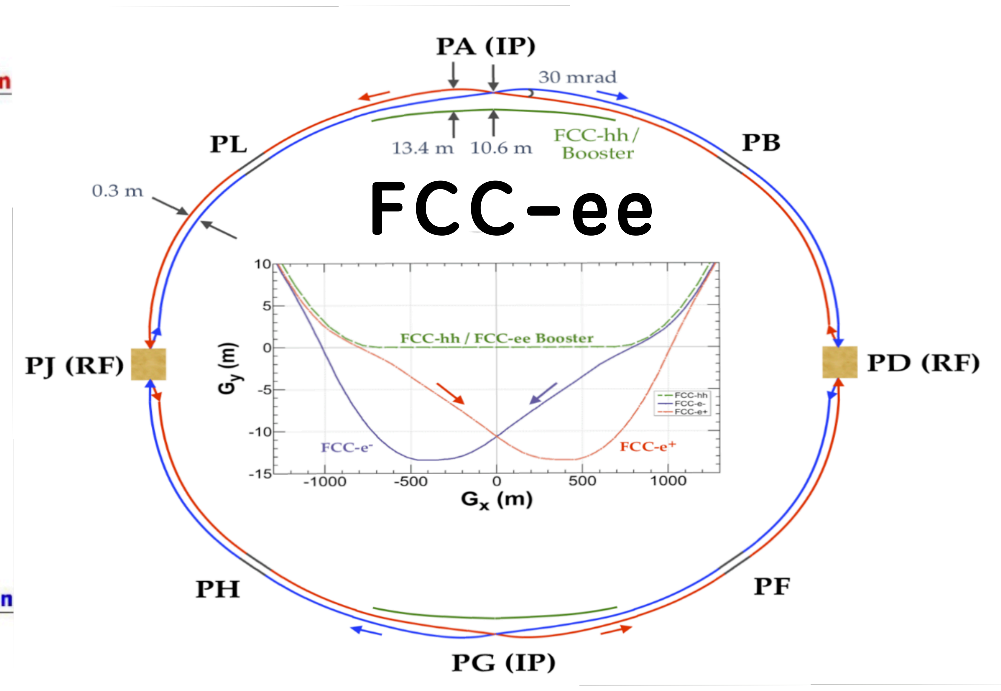
- Clean background ✓
- High luminosity ✓
- Polarized beams ✓

Lepton colliders

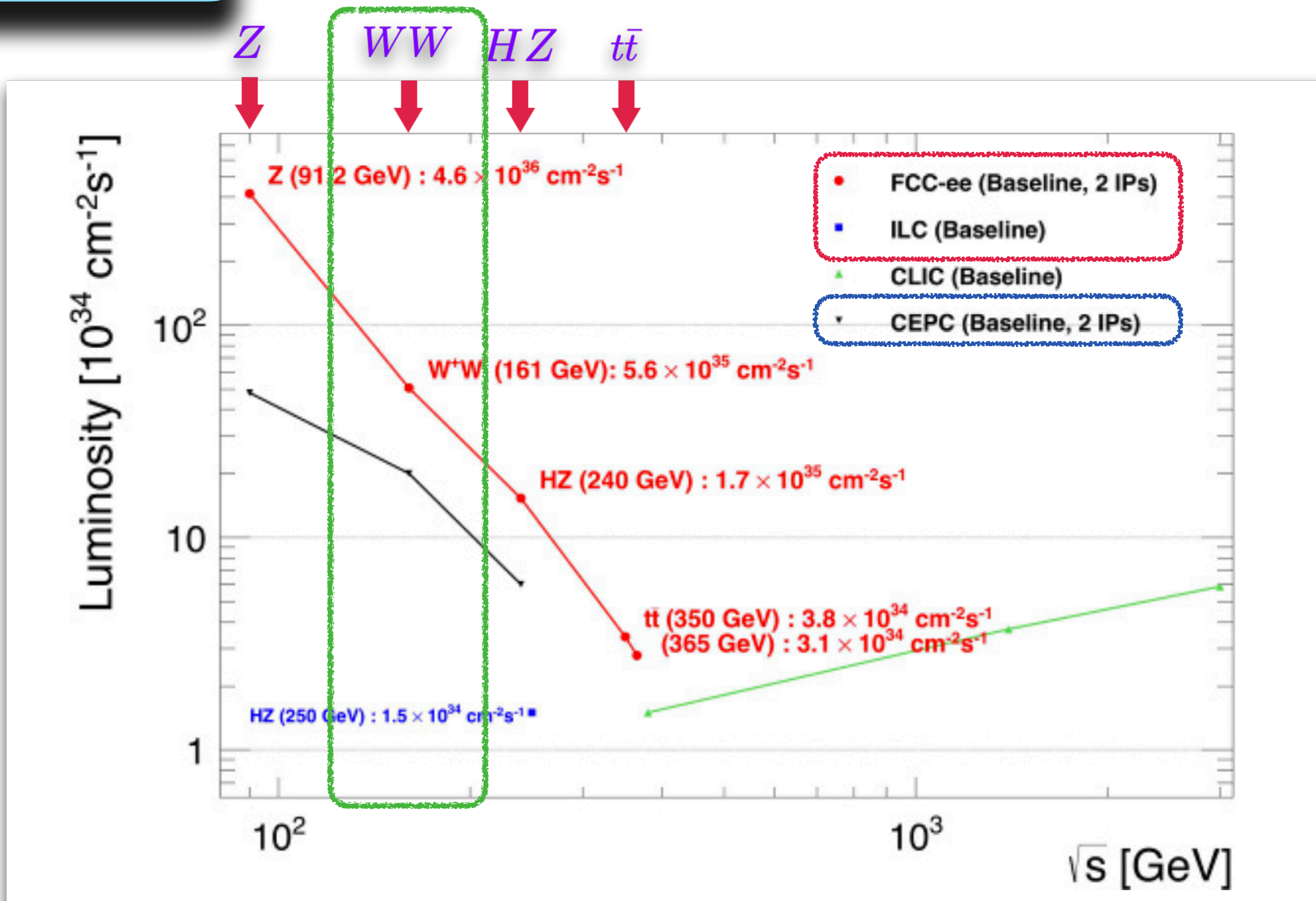








Lepton colliders



Background

W BOSON PRODUCTION IN e^+e^- COLLISIONS IN THE WEINBERG-SALAM MODEL

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PROBING THE WEAK BOSON SECTOR IN $e^+e^- \to W^+W^-$

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Received 23 June 1986

ELECTROWEAK RADIATIVE CORRECTIONS TO $e^+e^- \to W^+W^-$

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...d 23 December 1987

PHYSICAL REVIEW D VOLUME 26, NUMBER 7 1 OCTOBER 1982

***W*-pair production in electron-positron annihilation**

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(Received 19 Februa...

The cross section for $e^+e^- \to W^+$... ...hin the framework of quantum f...

PHYSICAL REVIEW D VOLUME 47, NUMBER 3 1 FEBRUARY 1993

***W* pair production in e^+e^- annihilation: Radiative corrections including hard bremsstrahlung**

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Background

NLO EW cross section

[0709.1075]

$\sqrt{s}$ [GeV]	$\sigma^{\rm LO}$ [pb]	$\sigma^{\rm NLO}$ [pb]	$\delta^{\rm NLO}$ [%]
161	4.411	2.556 ± 0.002	-42.1
165	11.761	8.553 ± 0.006	-27.3
170	15.465	12.264 ± 0.010	-20.7
200	19.354	17.796 ± 0.017	-8.1
250	16.406	16.033 ± 0.023	-2.3
300	13.473	13.543 ± 0.026	0.5
500	7.142	7.449 ± 0.019	4.3

Calculation setup

Consider the following process

$$e^+(p_1,-\sigma)+e^-(p_2,\sigma)\to W^+(p_3,\lambda_+)+W^-(p_4,\lambda_-)$$

The differential cross sections

$$\frac{d\sigma}{d\Omega}=\frac{\beta}{64\pi^2 s}\frac{1}{4}\sum_{\sigma,\lambda_+,\lambda_-}\left|\mathcal{M}(s,t,\sigma,\lambda_+,\lambda_-)\right|^2 .$$

Expand the amplitude

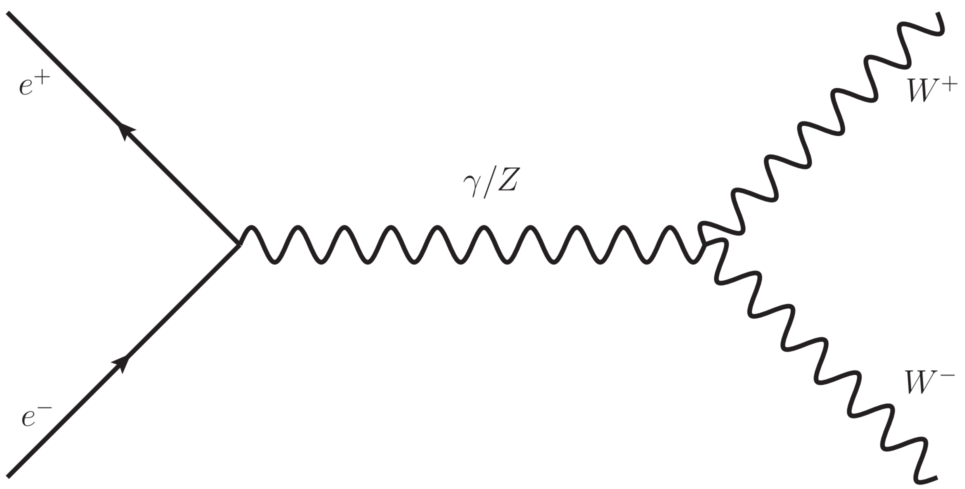
$$\mathcal{M}=\mathcal{M}_0+\delta\mathcal{M}^{\mathcal{O}(\alpha)}+\delta\mathcal{M}^{\mathcal{O}(\alpha\alpha_s)}+\delta\mathcal{M}^{\mathcal{O}(\alpha^2)}+\cdots$$

$\mathcal{M}_0+\delta\mathcal{M}^{\mathcal{O}(\alpha)}$ ⇓ **Well Studied**

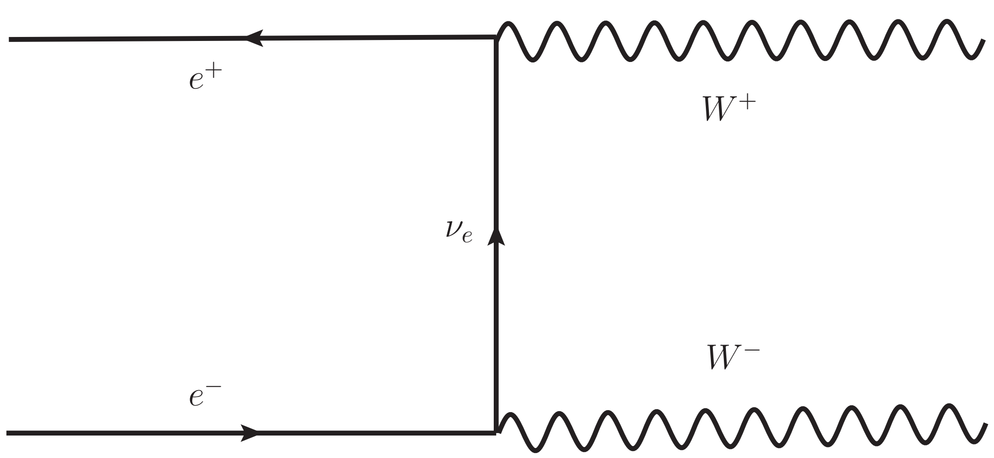
$\delta\mathcal{M}^{\mathcal{O}(\alpha\alpha_s)}+\delta\mathcal{M}^{\mathcal{O}(\alpha^2)}+\cdots$ ⇓ **Still Unknown**

Calculation setup

LO Feynman diagrams



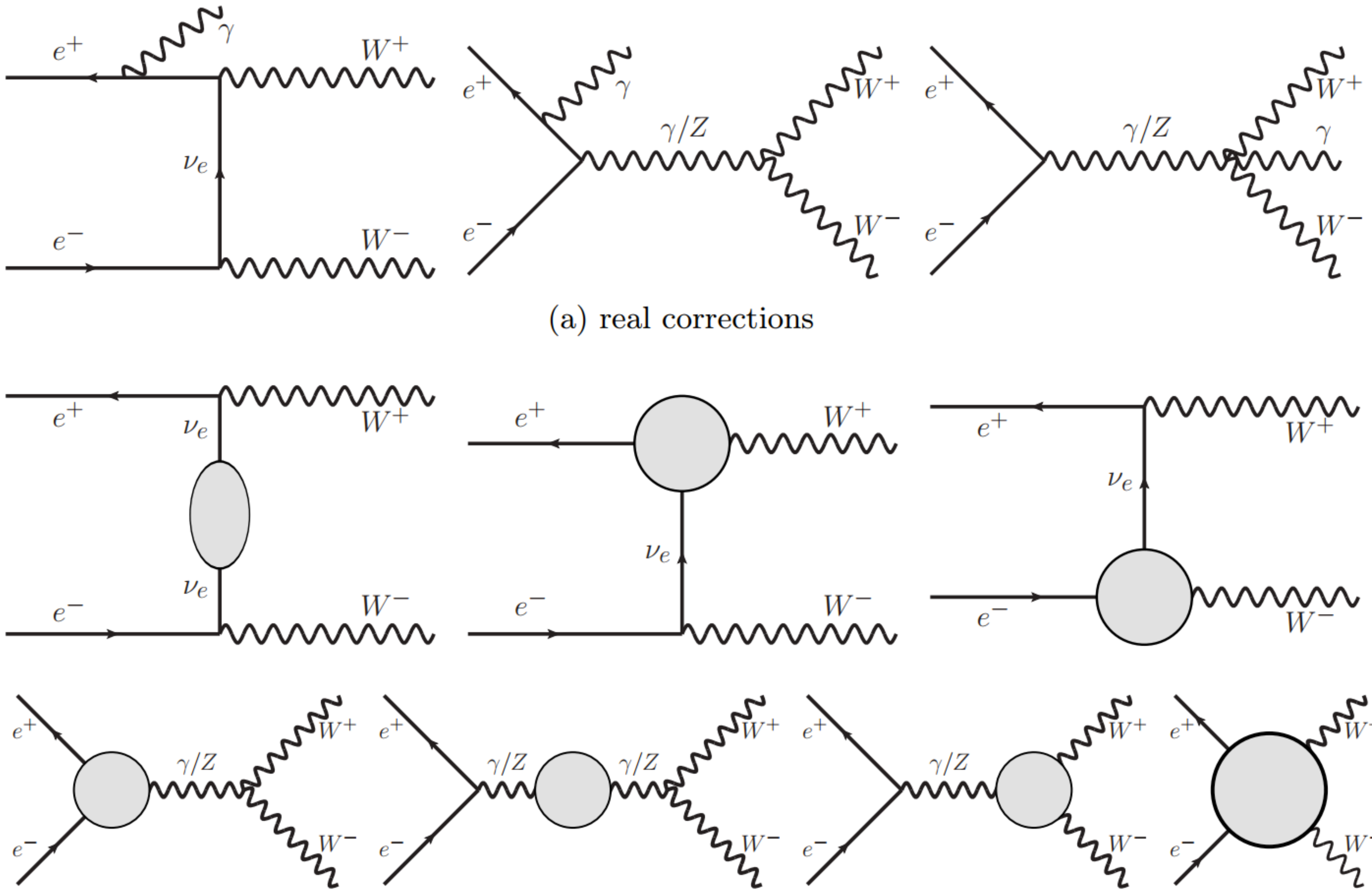
- Both left-handed and right-handed electrons contribute
- Triple gauge boson couplings (TGCs)



- Only left-handed electrons contribute
- Proportional to velocity β of W and dominates at threshold region

Calculation setup

NLO EW corrections



(a) real corrections

(b) virtual corrections

Calculation setup

NLO EW corrections

On-shell renormalization → UV-finite

Dipole subtraction scheme → IR-finite

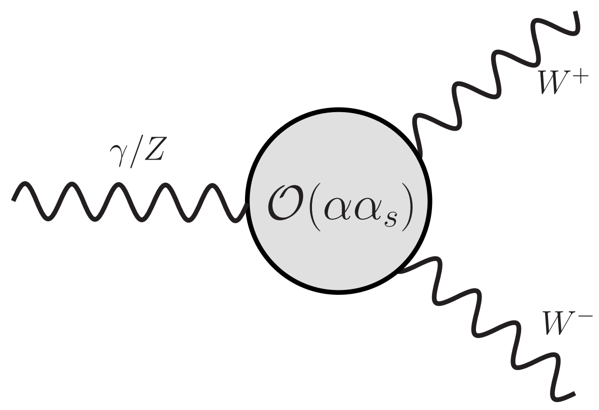
$$\mathrm{d}\sigma_{\mathrm{EW}} = \mathrm{d}\sigma_{\mathrm{virt}} + \mathrm{d}\sigma_{\mathrm{real}} + \mathrm{d}\sigma_{\mathrm{h.o.ISR}}$$

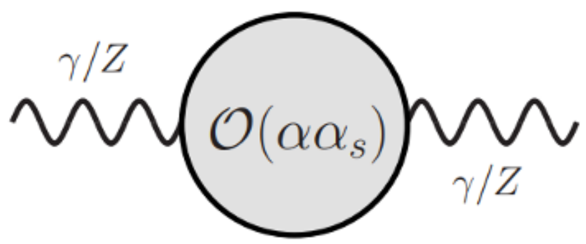
Calculation setup

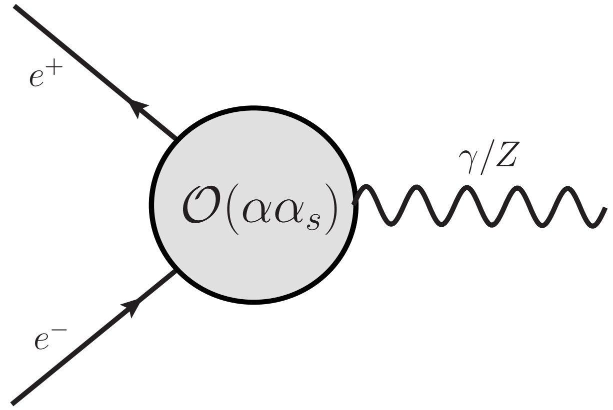
Mixed QCD-EW corrections

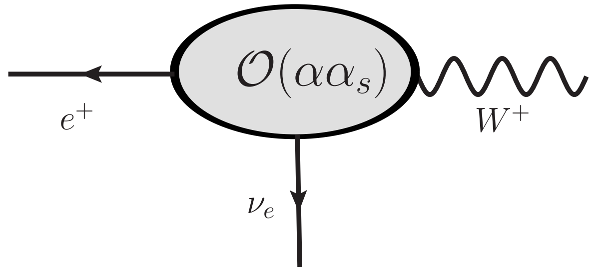
$\mathcal{O}(\alpha\alpha_s)$ corrections to the amplitude

$$\delta\mathcal{M}^{\mathcal{O}(\alpha\alpha_s)} = \delta\mathcal{M}^{\mathcal{O}(\alpha\alpha_s)}_{\mathrm{V^*WW}} + \delta\mathcal{M}^{\mathcal{O}(\alpha\alpha_s)}_{\gamma/\mathrm{Z\text{-}S.E.}} + \delta\mathcal{M}^{\mathcal{O}(\alpha\alpha_s)}_{\mathrm{eeV^*}} + \delta\mathcal{M}^{\mathcal{O}(\alpha\alpha_s)}_{\mathrm{e\nu_e W}}$$





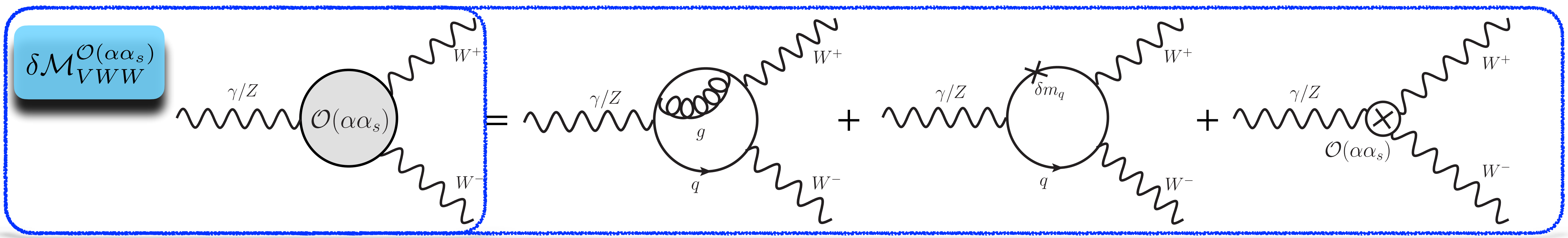


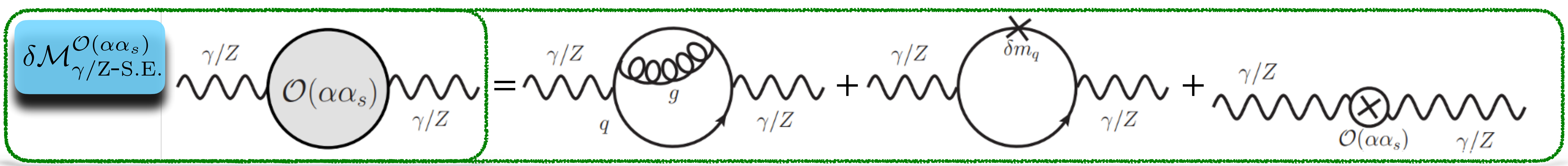


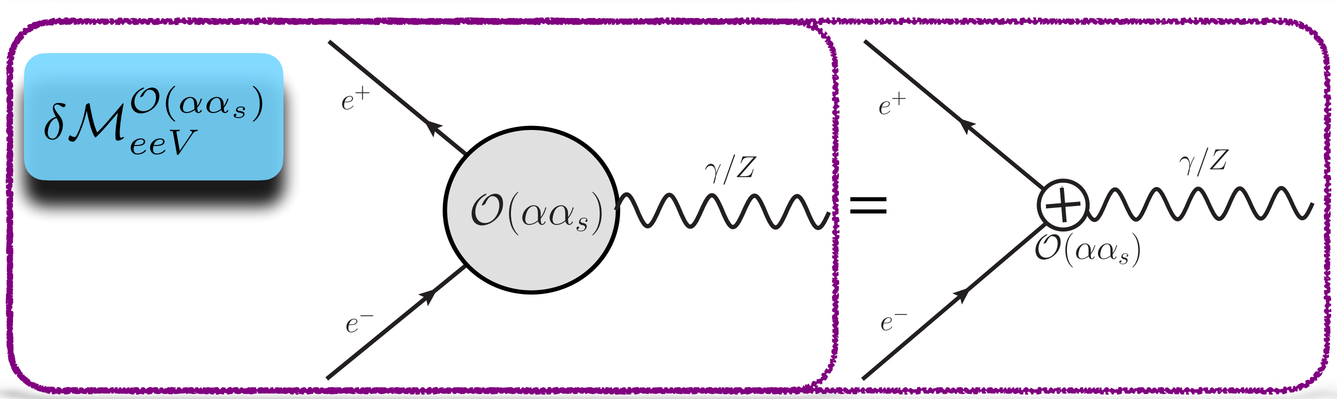
Calculation setup

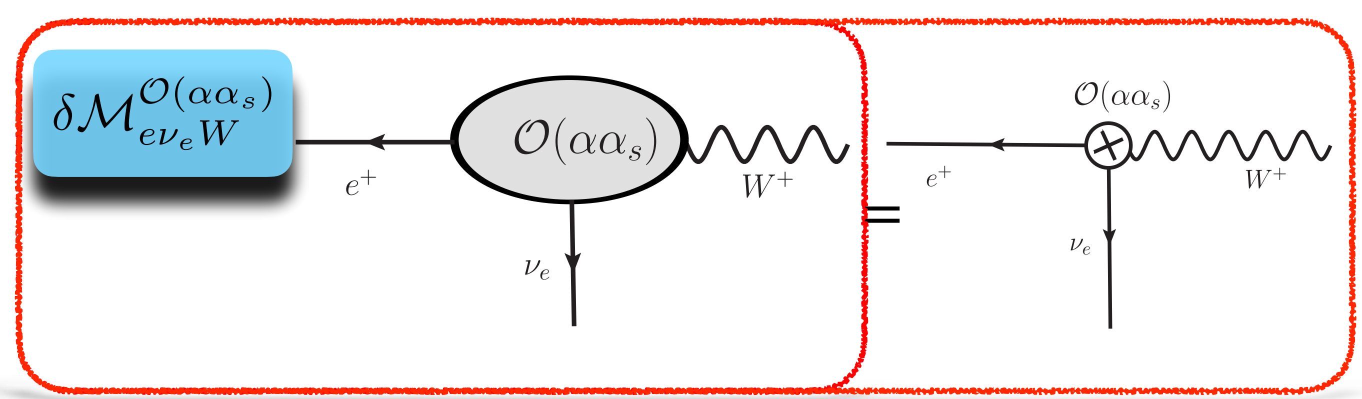
Mixed QCD-EW corrections

$\mathcal{O}(\alpha\alpha_s)$ corrections to the amplitude:









Calculation setup

Workflow of calculation

FeynArts : Generating Feynman diagrams and amplitudes

↓

FeynCalc : Tensor reduction and amplitudes reduction

↓

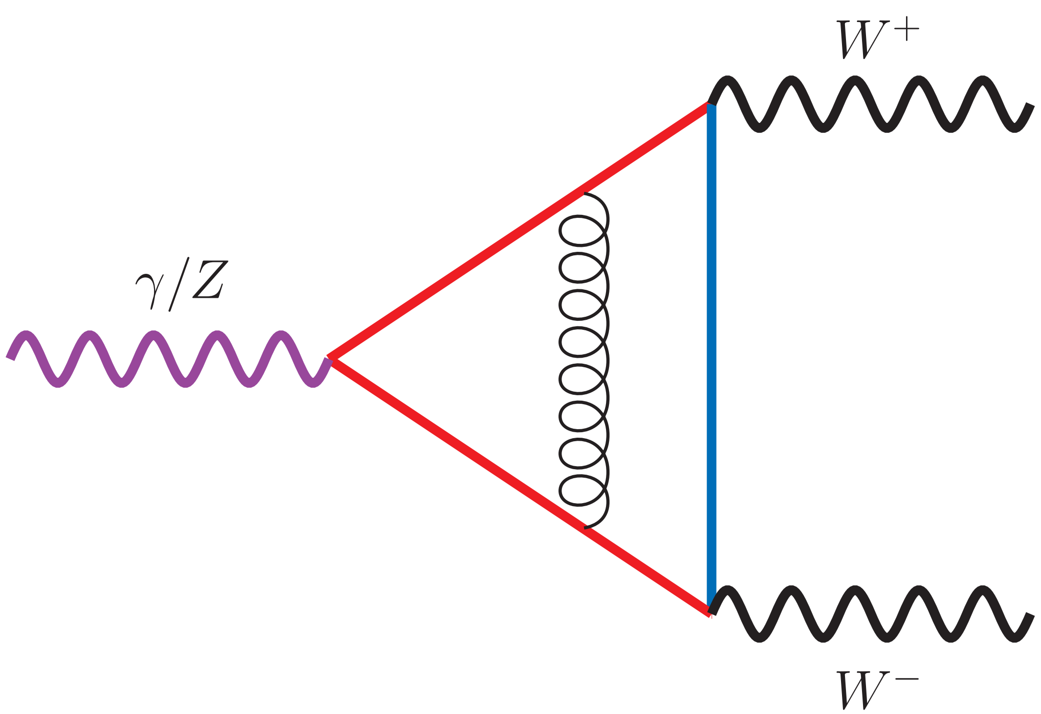
Fire/Kira/LiteRed : Integration by parts

↓

Canonical DEs : Calculation of master integrals

Canonical DEs

Master integrals	Self-energies	massless triangle	triangle with one massive flavor	triangle with two massive flavor
Status	✓	✓	✓	✗

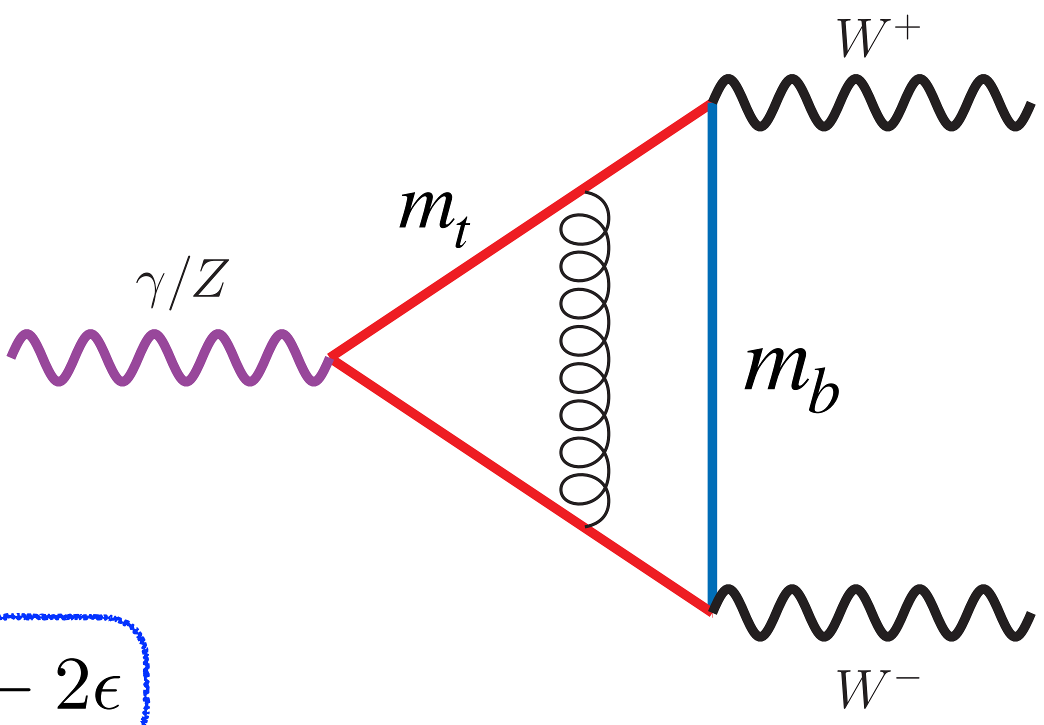


Complicated MIs with multiple scales

Notations

The two-loop triangle diagrams for

$$V^*(p_3+p_4) \to W^+(p_3) + W^-(p_4)\,,$$



The two-loop Feynman integrals

$d = 4 - 2\epsilon$

$$I(\alpha_1, \ldots, \alpha_7) = \int \mathcal{D}^d l_1 \mathcal{D}^d l_2 \frac{1}{D_1^{\alpha_1} \ldots D_7^{\alpha_7}}\,,$$

Propagators

$$D_1 = l_1^2 - m_t^2, \quad D_2 = l_2^2 - m_t^2, \quad D_3 = (l_1 - p_3)^2 - m_b^2, \quad D_4 = (l_2 - p_3)^2 - m_b^2$$
$$D_5 = (l_1 - p_3 - p_4)^2 - m_t^2, \quad D_6 = (l_2 - p_3 - p_4)^2 - m_t^2, \quad D_7 = (l_1 - l_2)^2\,.$$

Canonical DEs

Canonical basis

32 MIs in this family:

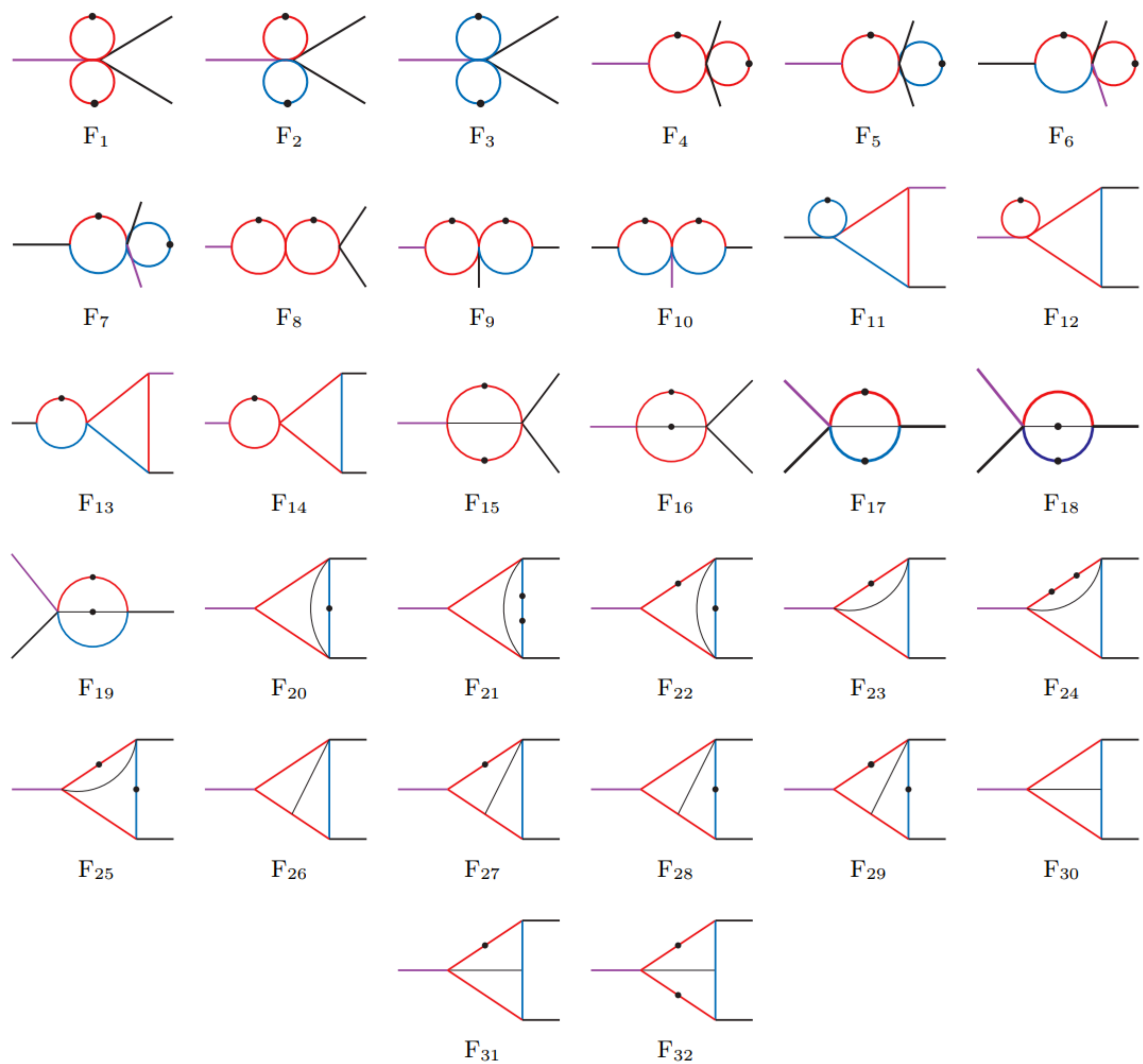
$$d\mathbf{F}(\vec{x},\epsilon) = d\tilde{\mathbb{A}}(\vec{x},\epsilon)\mathbf{F}(\vec{x},\epsilon)$$

$$\mathbf{I} = \mathbb{T}\mathbf{F}$$

[1304.1806]

$$d\mathbf{I}(\vec{x},\epsilon) = \epsilon d\mathbb{A}(\vec{x})\mathbf{I}(\vec{x},\epsilon)$$

ϵ factored out



General solutions

Path-order iterated integral:

$$\mathbf{I}(\vec{x},\epsilon) = \mathcal{P}\exp\left\{\epsilon\int_\gamma d\mathbb{A}\right\}\mathbf{I}(\vec{x}_0,\epsilon)$$

← **Boundary**

Order by order

$$\mathbf{I}^{(n)}(\vec{x}) = \begin{cases} \mathbf{I}^{(0)}(\vec{x}_0) & n=0, \\ \mathbf{I}^{(n)}(\vec{x}_0) + \int_\gamma d\mathbb{A}(\vec{x})\mathbf{I}^{(n-1)}(\vec{x}) & n>0. \end{cases}$$

Canonical DEs

Rationalization

Three square roots

$$\lambda_1 = \sqrt{s(s-4m_t^2)}\,, \qquad \lambda_2 = \sqrt{s(s-4m_W^2)}\,,$$

$$\lambda_3 = \sqrt{(m_W^2 - m_t^2 - m_b^2)^2 - 4\,m_t^2\,m_b^2}\,.$$

Change of variables

$$\frac{s}{m_t^2} = -\frac{(1-x)^2}{x}, \quad \frac{m_W^2}{m_t^2} = -\frac{(1-x)^2 z}{x(1+z)^2}, \quad \frac{m_b^2}{m_t^2} = \left(1+\frac{y}{z+1}\right)\left(1-\frac{(1-x)^2 z}{xy(z+1)}\right).$$

Rationalized simultaneously

$$\frac{\lambda_1}{m_t^2} = \frac{(1-x)(1+x)}{x}, \quad \frac{\lambda_2}{m_t^2} = \frac{(1-x)^2(1-z)}{x(1+z)}, \quad \frac{\lambda_3}{m_t^2} = \frac{(1-x)^2 z + xy^2}{xy(1+z)}.$$

Canonical DEs

Symbol letters

Dlog-form:

$$d\mathbf{I}(\vec{x},\epsilon) = \epsilon d\mathbb{A}(\vec{x})\mathbf{I}(\vec{x},\epsilon)\,, \quad d\mathbb{A}(\vec{x}) = \sum_i \mathbb{M}_i \mathrm{dlog}\eta_i$$

constant matrix

symbol letter

Letters:

$$\eta_1 = x, \qquad \eta_2 = y$$
$$\eta_3 = z, \qquad \eta_4 = 1-x$$
$$\eta_5 = 1+x, \qquad \eta_6 = 1+y$$
$$\eta_7 = 1-z, \qquad \eta_8 = 1+z$$
$$\eta_9 = x+z, \qquad \eta_{10} = y+z$$
$$\eta_{11} = 1+xz, \qquad \eta_{12} = 1-x+y$$
$$\eta_{13} = -1+x+xy, \qquad \eta_{14} = 1+y+z$$
$$\eta_{15} = y+z-xz, \qquad \eta_{16} = xy+xz-z$$
$$\eta_{17} = xy-(1-x)^2, \qquad \eta_{18} = xy-(1-x)^2 z$$
$$\eta_{19} = xy^2+(1-x)^2 z, \qquad \eta_{20} = xy(1+z)-(1-x)^2 z$$

Multiple polylogarithms

$$d\mathbb{A}(\vec{x}) = \sum_i \mathbb{M}_i d\log \eta_i \qquad \mathbf{I}^{(n)}(\vec{x}) = \mathbf{I}^{(n)}(\vec{x}_0) + \int_\gamma d\mathbb{A}(\vec{x})\mathbf{I}^{(n-1)}(\vec{x})$$

rational

Generalized polylogarithms

$$G(a_1, \ldots, a_n; z) = \int_0^z \frac{1}{t-a_1} G(a_2, \ldots, a_n; t)\, \mathrm{d}t\,,$$

with

$$G(a_1; z) = \int_0^z \frac{1}{t-a_1} \mathrm{d}t \quad a_1 \neq 0\,, \qquad G(\underbrace{0, \ldots, 0}_{n \text{ times}}; z) = \frac{\log^n(z)}{n!}$$

Goncharov, `98,`01

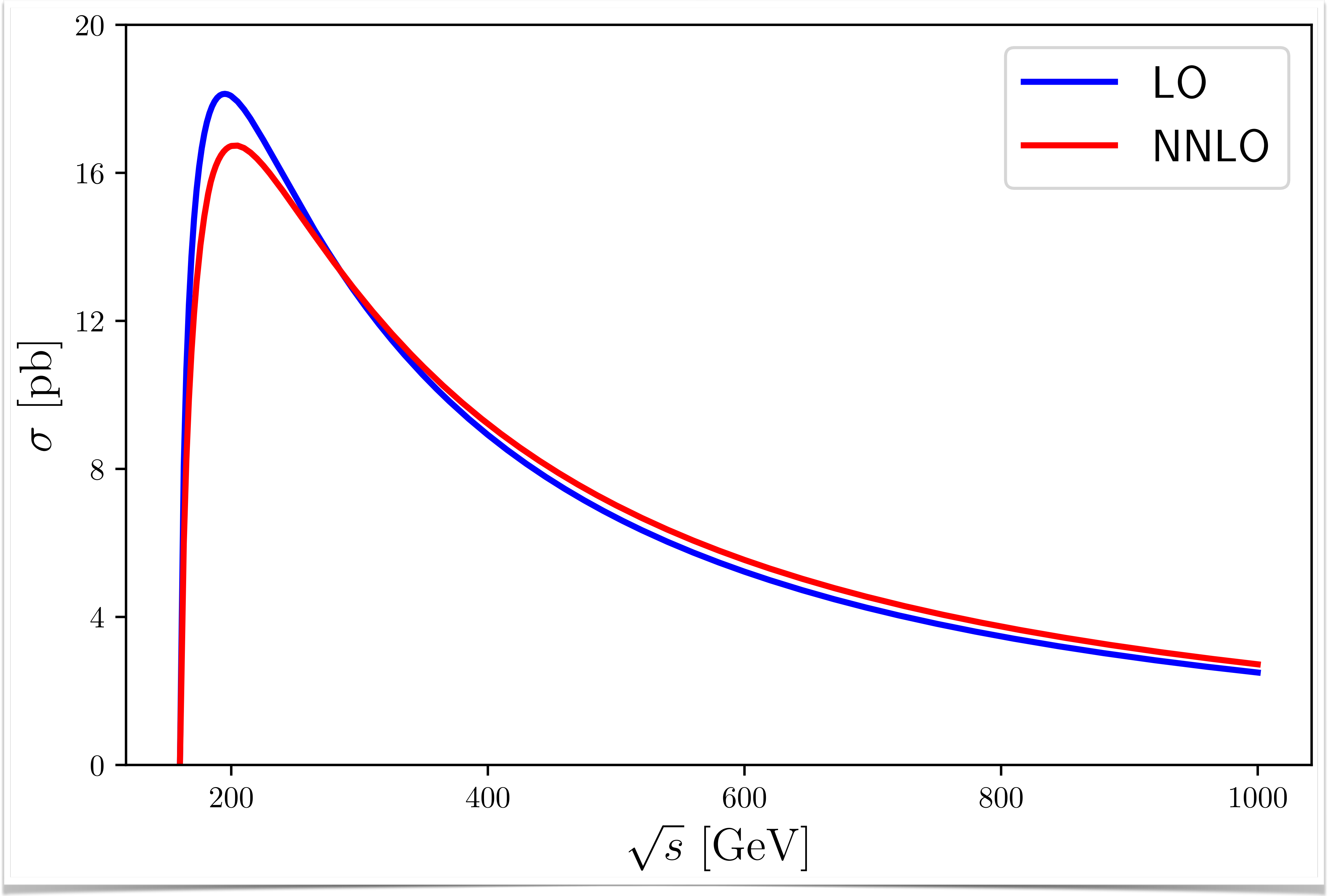
Canonical DEs

Boundary

- Known MIs from published paper
- Regularity of MIs at some kinematic limits
- Direct evaluation MIs at some special point
- ……

Numerical Results

NNLO results



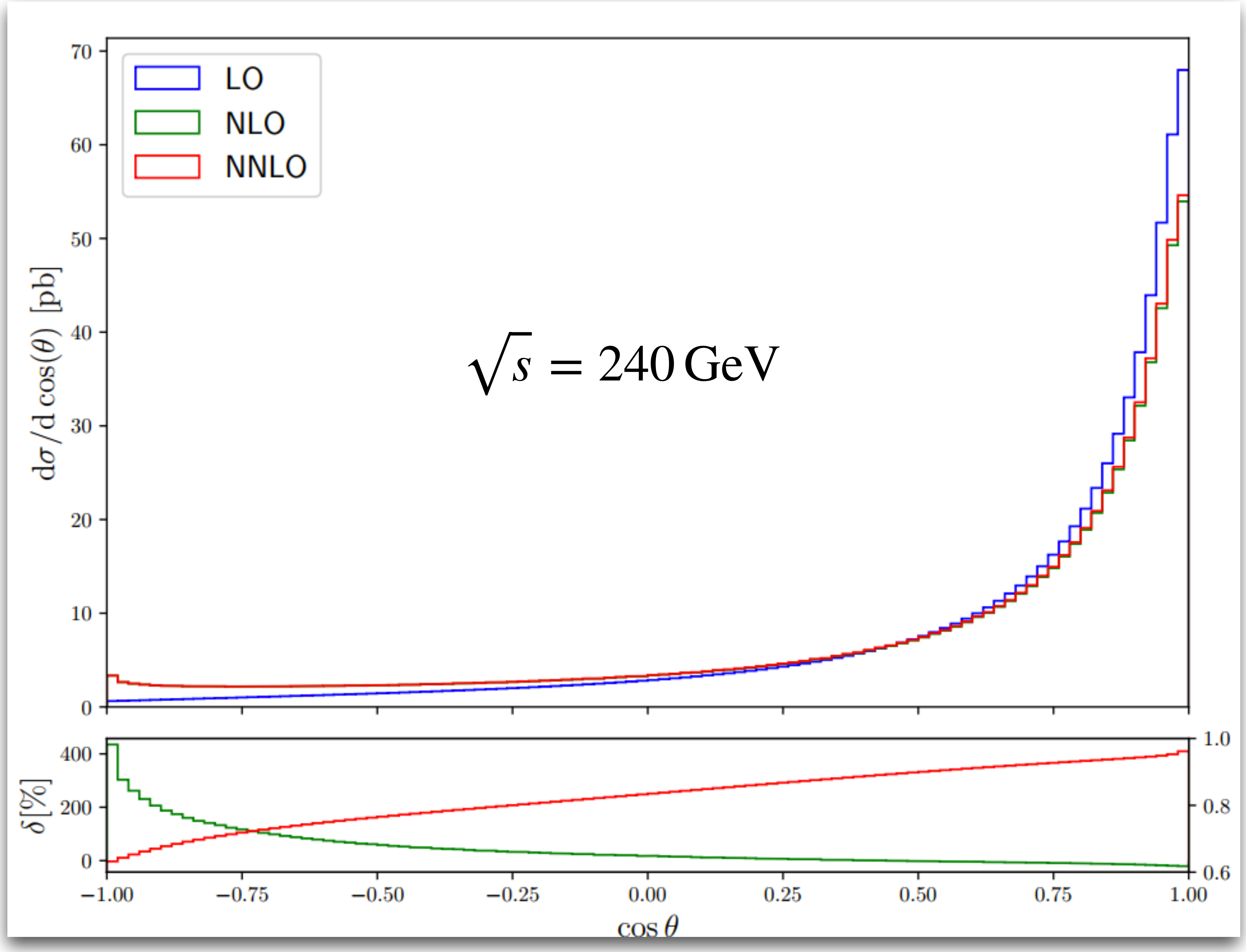
Numerical Results

NNLO results

$\sqrt{s}$ (GeV)	$\sigma^{(0)}$(pb)	$\delta\sigma^{(\alpha)}$(%)	$\sigma^{(\alpha)}$(pb)	$\delta\sigma^{(\alpha\alpha_s)}$(%)	$\sigma^{(\alpha\alpha_s)}$(pb)
161	**2.76674**	-27.46209	**2.00694**	0.93373	**2.03277**
240	**15.96118**	-3.48032	**15.4057**	0.91243	**15.5513**
250	**15.34263**	-2.73091	**14.9236**	0.91221	**15.0636**
500	**6.68469**	4.19535	**6.9651**	0.91329	**7.0262**
1000	**2.49752**	8.14203	**2.70086**	0.92485	**2.72396**

Numerical Results

NNLO angle distribution



Summary&Outlook

- W-pair production are important for W-mass measurements
- Complete computations of the NNLO QCD-EW corrections
- Analytical calculations of the MIs with Canonical DEs
- Complete computations of the NNLO EW corrections
- ……

Thanks