



ResBos2 and the CDF W Mass measurement

CTEQ

C.-P. Yuan

Michigan State University
Wu-Ki Tung Endowed Professor

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25th Min-Workshop on the frontier of LHC

In collaboration with **Josh Isaacson** and **Yao Fu** arXiv:2205.02788

CTEQ – Tung et al. (TEA)
in memory of Prof. Wu-Ki Tung



Outline

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- What's ResBos for?
- ResBos2 is the Version 2 of ResBos
- Higher order effects to the measurement of W mass at CDF II
- Conclusions and outlook



ResBos

(Resummation for Bosons)

Initial state QCD soft gluon resummation
and
Final state QED corrections

In collaboration with

Csaba Balazs, Alexander Belyaev, Ed Berger,
Qing-Hong Cao, Chuan-Ren Chen, Yao Fu,
Josh Isaacson, Zhao Li, Steve Mrenna,
Pavel Nadolsky, Jian-Wei Qiu, Carl Schmidt,
Peng Sun, Bin Yan and Feng Yuan

hep-ph/9704258
hep-ph/0401026
hep-ph/1205.4311
arXiv:2205.02788
etc.



What's **ResBos** for?

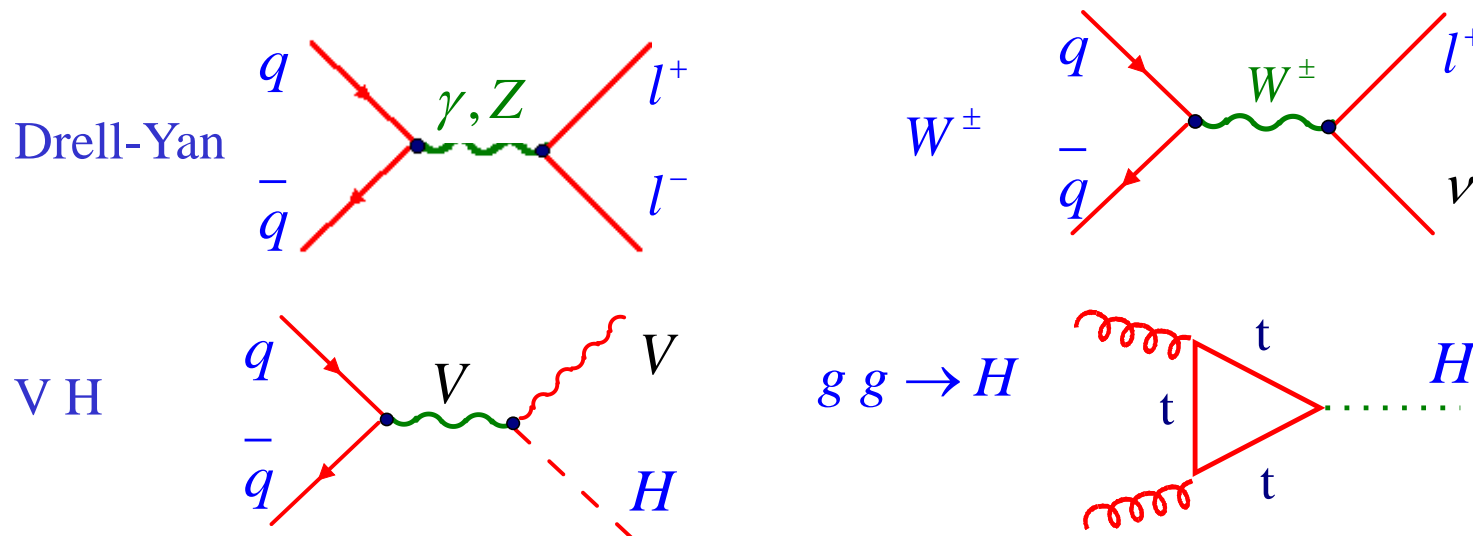
Precision Electroweak Physics
at Hadron Colliders

Physics of
Drell-Yan, W , Z and Higgs Bosons



What's it for? An Example

- Transverse momentum of



including initial state QCD Resummation
(and final state QED radiation)

- Kinematics of Leptons from the decays
(Spin correlation included)



W-boson physics

- ① W-boson production and decay at hadron collider
- ② How to measure W-boson mass and width?
- ③ High order radiative corrections:
 - ☞ QCD (NLO, NNLO, Resummation)
 - ☞ EW (QED-like, NLO)
- ④ ResBos and ResBos-A



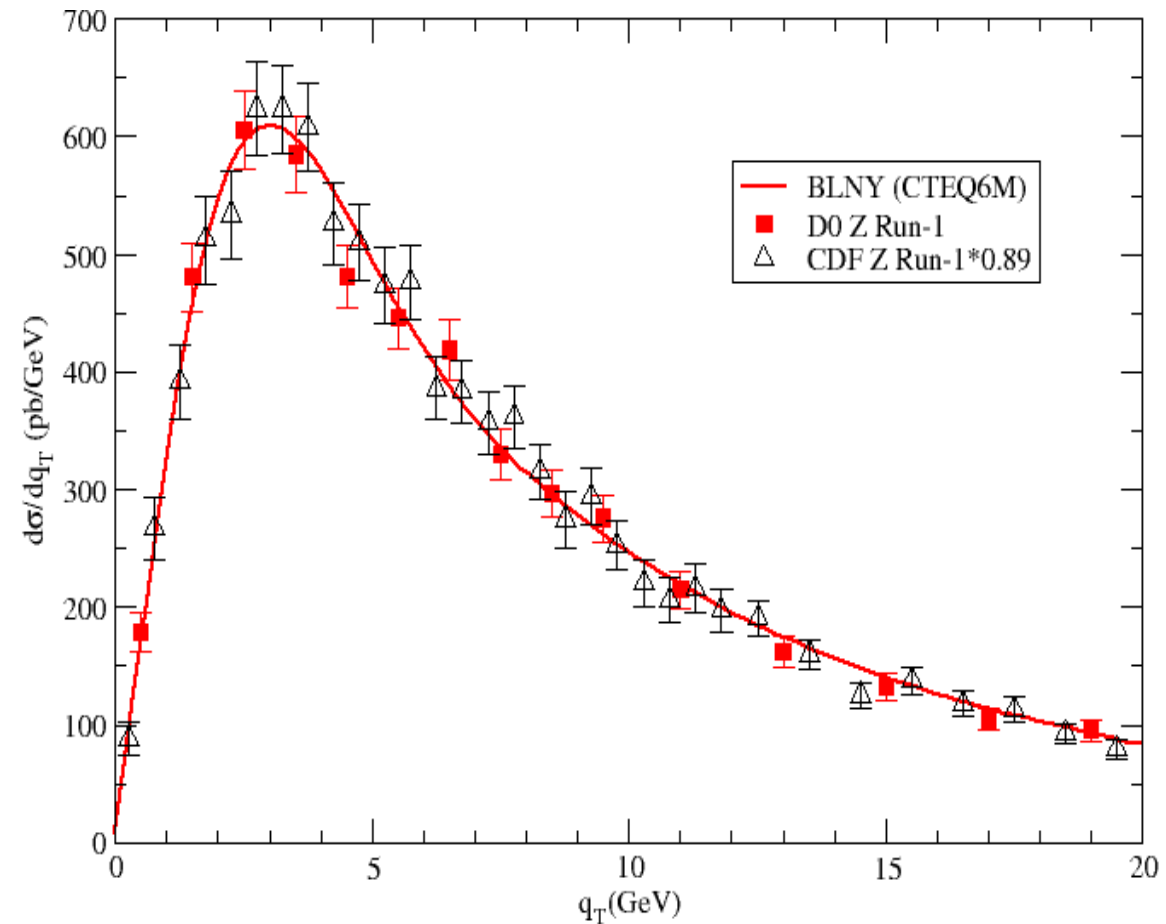
Resummation calculations agree well with data

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$$P\bar{P} \rightarrow Z \text{ @ Tevatron}$$

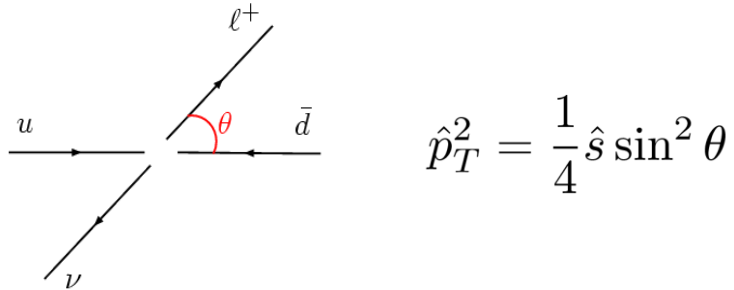
Predicted by **ResBos**:

A program that includes the effect of multiple soft gluon emission on the production of W and Z bosons in hadron collisions.



Transverse momentum of the charged lepton

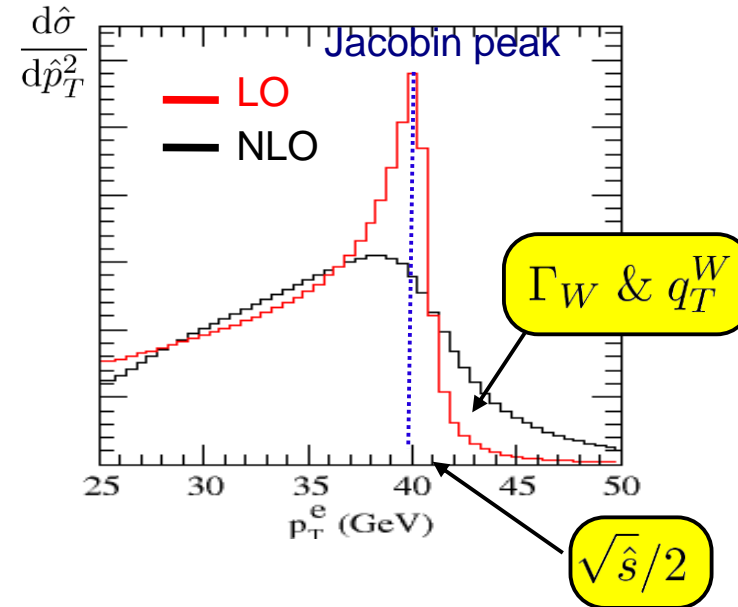
- In (ud) c.m. system,



Jacobian factor

$$\frac{d \cos \theta}{d \hat{p}_T^2} = -\frac{2}{\hat{s}} \frac{1}{\sqrt{1 - \frac{4 \hat{p}_T^2}{\hat{s}}}}$$

$$\Rightarrow \frac{d \hat{\sigma}}{d \hat{p}_T^2} \sim \frac{d \hat{\sigma}}{d \cos \theta} \times \frac{1}{\sqrt{1 - 4 \hat{p}_T^2 / \hat{s}}}$$



sensitive region for measuring

M_W : $p_T^e \sim 30 - 45$ GeV

Γ_W : not a good observable



Transverse mass of the W-boson

- Definition:

$$m_T^2(\ell, \nu) = 2 p_T^\ell p_T^\nu (1 - \cos \phi_{\ell\nu})$$

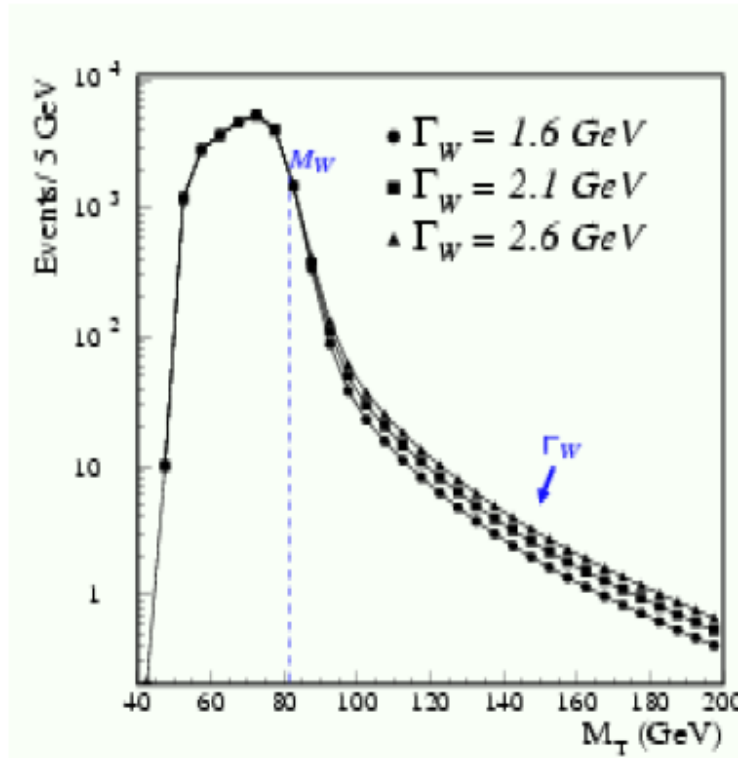
↓
from overall p_T imbalance

$$\Rightarrow \frac{d\hat{\sigma}}{dm_T^2} \sim \frac{1}{\sqrt{1 - m_T^2/\hat{s}}}$$

👉 unaffected by longitudinal boosts of $\ell\nu$ system

👉 not sensitive to q_T^W

👉 tail knows about Γ_W (direct measurement)



sensitive region:

$$M_W : M_T \sim 60 - 100 \text{ GeV}$$

$$\Gamma_W : M_T > 100 \text{ GeV}$$



What's QCD Resummation?

- Perturbative expansion

$$\frac{d\hat{\sigma}}{dq_T^2} \sim \alpha_s \left\{ 1 + \alpha_s + \alpha_s^2 + \dots \right\}$$

- The singular pieces, as $\frac{1}{q_T^2}$ (1 or log's)

$$\begin{aligned} \frac{d\hat{\sigma}}{dq_T^2} &\sim \frac{1}{q_T^2} \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_s^{(n)} \ln^{(m)} \left(\frac{Q^2}{q_T^2} \right) \\ &\sim \frac{1}{q_T^2} \left\{ \alpha_s (\underline{L+1}) \right. \\ &\quad + \alpha_s^2 (\underline{L^3 + L^2 + L+1}) \\ &\quad + \alpha_s^3 (\underline{L^5 + L^4 + L^3 + L^2 + L+1}) \\ &\quad \left. + \dots \right\} \end{aligned} \quad L \equiv \ln \left(\frac{Q^2}{q_T^2} \right)$$

Resummation is to reorganize the results in terms of the large Log's.



Resummed results:

$$\frac{d\sigma}{dq_T^2} \sim \frac{1}{q_T^2} \left\{ \begin{array}{l} \text{Determined by } A^{(1)} \text{ and } B^{(1)} \\ [\alpha_s(L+1) + \alpha_s^2(L^3 + L^2) + \alpha_s^3(L^5 + L^4) + \dots] \\ + [\text{Determined by } A^{(2)} \text{ and } B^{(2)} \\ + \alpha_s^2(L+1) + \alpha_s^3(L^3 + L^2) + \dots] \\ + [\\ + \alpha_s^3(L+1) + \dots] \\ + \dots \end{array} \right\}$$

Determined by $A^{(3)}$ and $B^{(3)}$

QCD Resummation

In the formalism by Collins-Soper-Sterman, in addition to these perturbative results, the effects from physics beyond the leading twist is also implemented as

[non-perturbative functions].



CSS Resummation Formalism

$$\frac{d\sigma}{dq_T^2 dy dQ^2} = \frac{\pi}{S} \sigma_0 \delta(Q^2 - M_W^2).$$

$$\left\{ \frac{1}{(2\pi)^2} \int d^2b e^{iq_T \cdot \tilde{b}} \tilde{W}(b, Q, x_A, x_B) \cdot [\text{Non-perturbative functions}] \right.$$

$$\left. + Y(q_T, y, Q) \right\}$$

$$\sum_j \int_{x_A}^1 \frac{d\xi_A}{\xi_A} C_{qj} \left(\frac{x_A}{\xi_A}, b, \mu \right) \cdot f_{j/A}(\xi_A, \mu)$$

$$\tilde{W} = e^{-S(b)} \cdot C \otimes f(x_A) \cdot C \otimes f(x_B)$$

$$\sum_k \int_{x_B}^1 \frac{d\xi_B}{\xi_B} C_{qk} \left(\frac{x_B}{\xi_B}, b, \mu \right) \cdot f_{k/B}(\xi_B, \mu)$$

Sudakov form factor $S(b) = \int_{(\frac{b_0}{b})^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\ln \left(\frac{Q^2}{\bar{\mu}^2} \right) A(\bar{\mu}) + B(\bar{\mu}) \right]$

[Non-perturbative functions] are functions of (b, Q, x_A, x_B) which include QCD effects beyond Leading Twist.



[non-perturbative function] is a function of (b, Q, x_A, x_B) , implemented to include effects beyond Leading Twist.

Until we know how to calculate QCD non-perturbatively, (Lattice Gauge Theory?), these functions can only be parameterized. However, the same functions should describe Drell-Yan, W^\pm , Z^0 data.

- ➔
- Test QCD in problems involving multiple scales.
 - Measuring these non-perturbative functions may help in understanding the non-perturbative part of QCD.

[non-perturbative functions], dependent of Q, b, x_A, x_B , is necessary to describe q_T – distribution of Drell-Yan, W^\pm, Z^0 events.

$$\exp \left[-g_1 b^2 - g_2 b^2 \ln \left(\frac{Q}{2Q_0} \right) - g_1 g_3 b^2 \ln (100 x_A x_B) \right]$$

New term with
x-dependence

BLNY parametrization

hep-ph/0212159

The coefficients g_1, g_2, g_3 need to be determined by existing data.



- Example: for W^\pm

$$\sigma_0 = \left(\frac{4\pi^2 \alpha}{3} \sum_{jj'} Q_{jj'}^{(W)} \right), \quad Q_{jj'}^{(W)} = \frac{1}{4 \sin^2 \theta_W} (kM)_{jj'}^2$$

The couplings of gauge bosons to fermions are expressed in the way to include the dominant **electroweak radiative corrections**. The propagators of gauge bosons also contain **energy-dependent width**, as done in LEP precision data analysis.

Note:

$$A \equiv \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n \cdot A^{(n)}, \quad B \equiv \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n \cdot B^{(n)},$$
$$C \equiv \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n \cdot C^{(n)}$$



As $q_T \rightarrow 0$

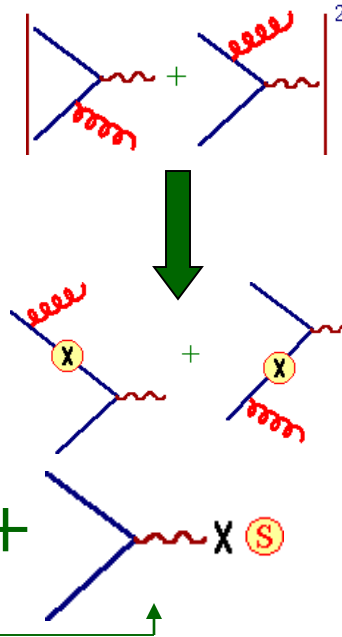
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$$\frac{d\sigma}{dq_T^2 dy dQ^2} \Big|_{q_T \rightarrow 0}$$

$$= \left(\frac{\pi}{s} \sigma_0 \right) \cdot \delta(Q^2 - M_W^2) \cdot \left(\frac{1}{2\pi q_T^2} \right) \left(\frac{\alpha_s(Q)}{\pi} \right)$$

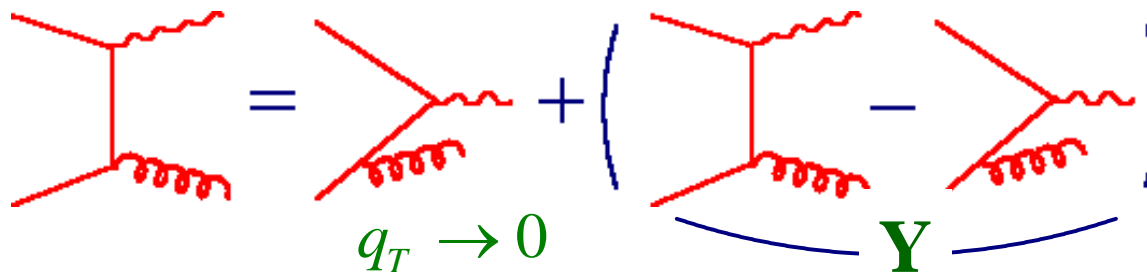
$$\cdot \left\{ \begin{aligned} & f_{q/A}(x_A, Q) [P_{\bar{q} \leftarrow \bar{q}} \otimes f_{\bar{q}}]_{x_B, Q} \\ & + [P_{q \leftarrow q} \otimes f_q]_{x_A, Q} f_{\bar{q}/B}(x_B, Q) \\ & + f_{q/A}(x_A, Q) f_{\bar{q}/B}(x_B, Q) \cdot \left[2 \left(\frac{4}{3} \right) \ln \left(\frac{Q^2}{q_T^2} \right) + 2(-2) \right] \end{aligned} \right\}$$

Diagrammatically,



Exponentiate

To preserve transverse momentum conservation, we have to go to the impact parameter space (**b-space**) to perform resummation.



The **Y**-term is added to include the full fixed-order contribution.



Diagrammatically, **Resummation** is doing



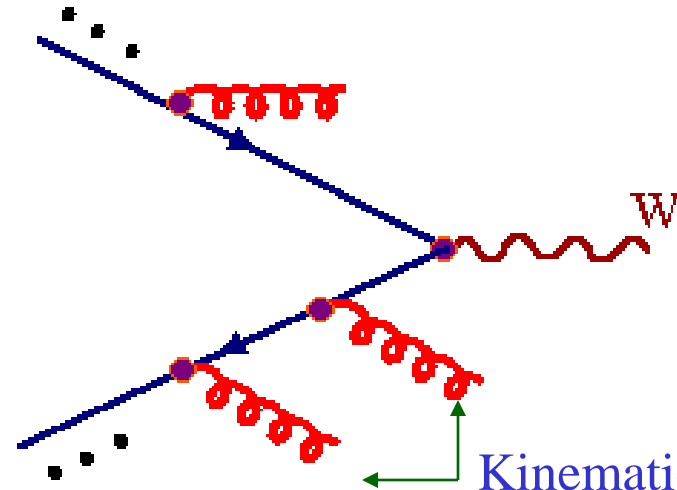
→ Resum large $\alpha_s^n \ln^m \left(\frac{Q^2}{q_T^2} \right)$ terms

$$\left. \frac{d\sigma}{dq_T^2 dy} \right|_{q_T \rightarrow 0} \sim \frac{1}{q_T^2} \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_s^n \ln^m \left(\frac{Q^2}{q_T^2} \right) \cdot C_m^n$$

Monte-Carlo programs **ISAJET**, **PYTHIA**, **HERWIG** contain these physics.

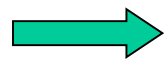
(Note: Arbitrary cut-off scale in these programs to affect the amount of **Backward radiation** , i.e. **Initial state radiation** .)

Monte-Carlo Approach



Backward Radiation
(Initial State Radiation)

Kinematics of the radiated gluon, controlled by Sudakov form factor with some arbitrary cut-off. (In contrast to perform integration in impact parameter space, i.e., **b space**.)



The shape of $q_T(w)$ is generated. But, the integrated rate remains the same as at Born level (**finite virtual correction is not included**).



Recently, there are efforts to include part of higher order effect in the event generator.



Event Generators (PYTHIA, HERWIG)

Note that the integrated rate is the same as the **Born level rate** ($\alpha_s^{(0)}$) even though the q_T – distribution is different (i.e., not $\delta(q_T^2)$ any more).

$$\begin{aligned}\sigma &= \int dq_T^2 \frac{d\sigma}{dq_T^2} \sim \int d^2b \underbrace{\int e^{i\vec{q}_T \cdot \vec{b}} d^2q_T \sigma_0}_{\delta^2(b)} e^{-S(b)} \\ &= \int d^2b \delta^2(b) \cdot \sigma_0 \cdot e^{-S(b)} \xrightarrow{\text{1 at } b=0} \\ &= \sigma_0\end{aligned}$$

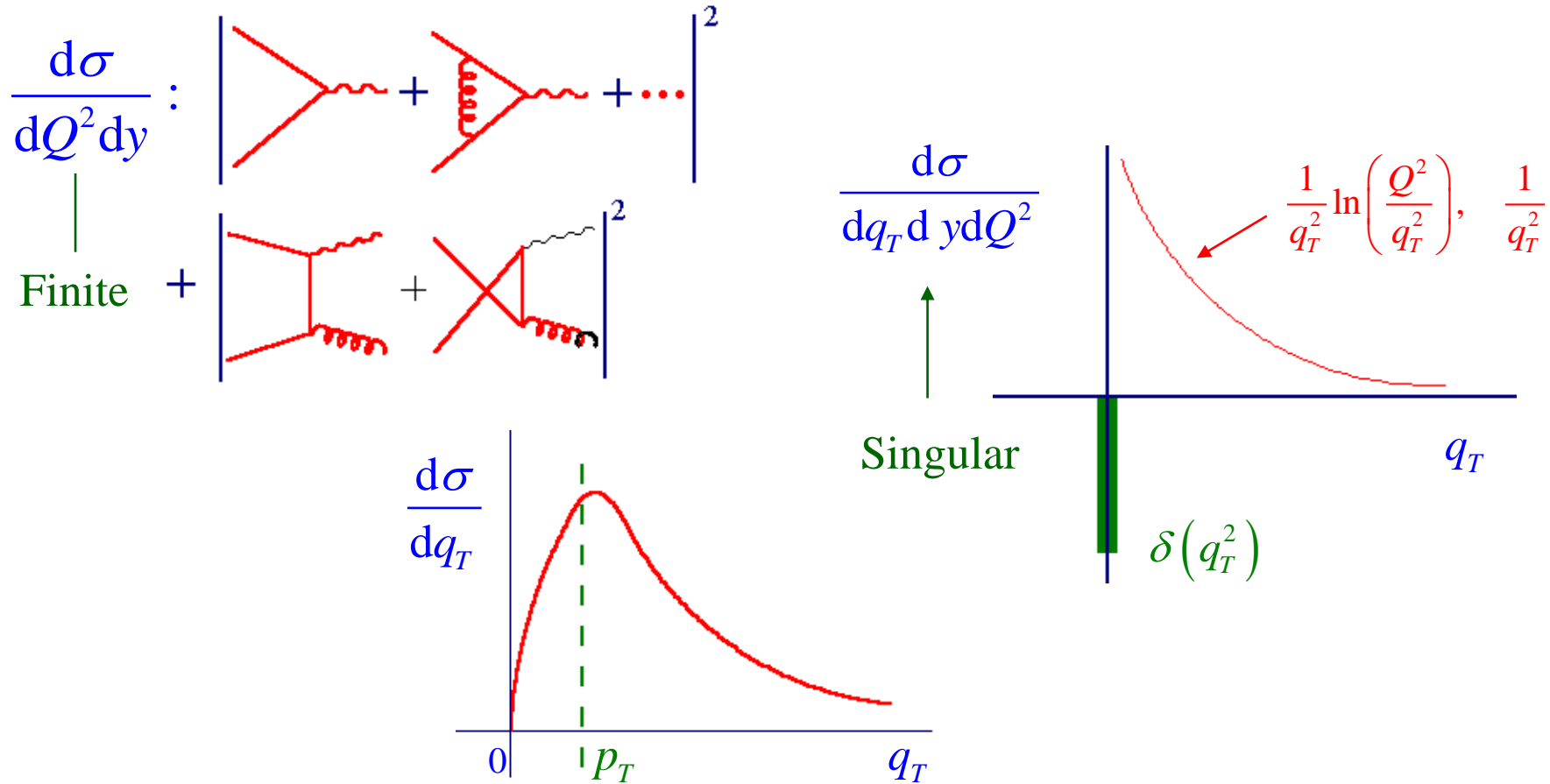
For C-Function = $\delta\left(1 - \frac{x_A}{\xi_B}\right)$



To recover the “K-factor” in the NLO total rate



To include the C-Functions



The area under the q_T -curve will reproduce the total rate at the order $\alpha_s^{(1)}$ if Y term is calculated to $\alpha_s^{(1)}$ as well.



Include NNLO in high q_T region

- To improve prediction in high q_T region
- To speed up the calculation, it is implemented through K-factor table which is a function of (Q, q_T, y) of the boson, not just a constant value.



ResBos predicts both rate and shape of distributions.



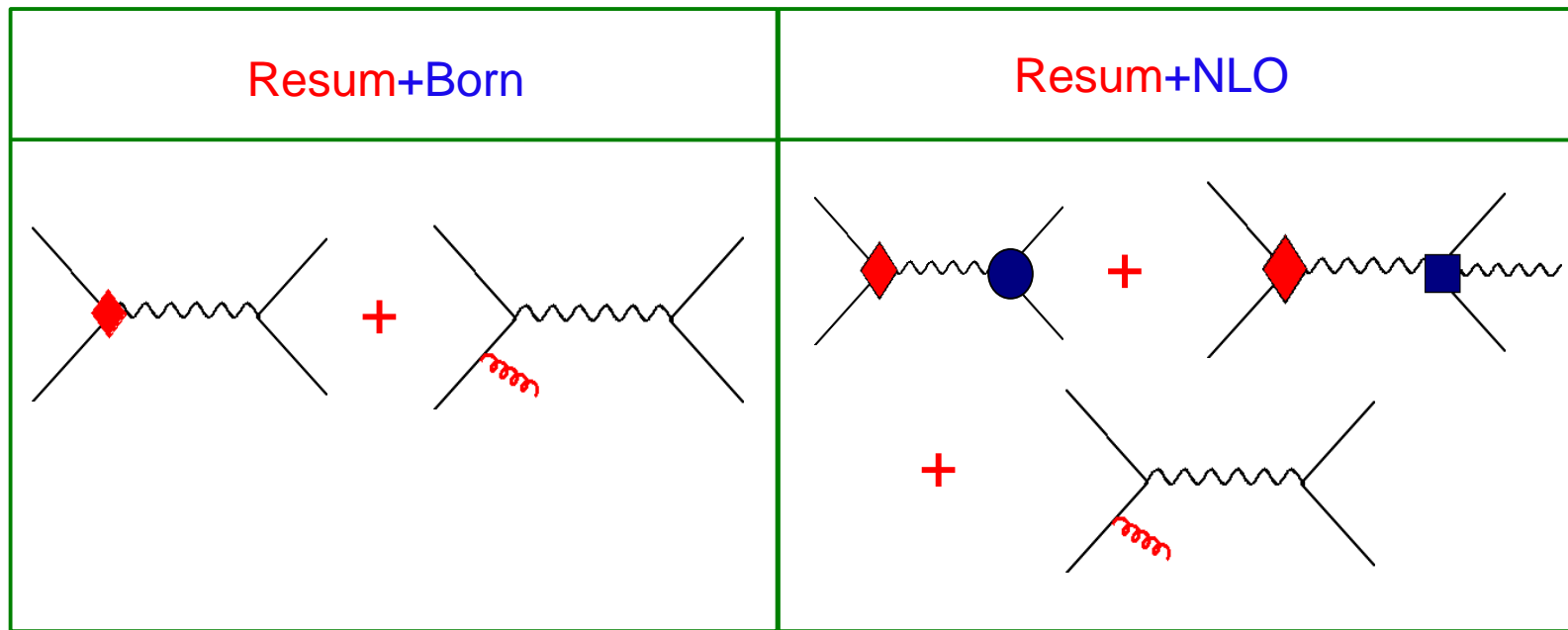
Precision measurements require accurate theoretical predictions



- **ResBos-A**: improved **ResBos** by including **final state NLO QED** corrections to **W** and **Z** production and decay

hep-ph/0401026

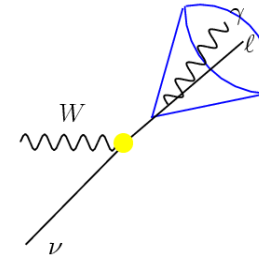
Qing-Hong Cao and CPY



● and ■ denote **FQED** radiation corrections, which dominates the **W** mass shift.

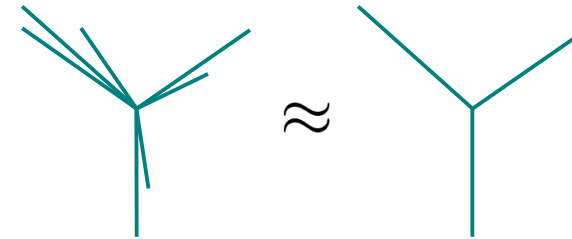
Need to consider the recombination effect

- Experimental: difficult to discriminate between electrons and photons with a small opening angle
- Theoretical: to define infra-safe quantities which are independent of long-distance physics




Essential feature of a general IRS physical quantity:

The observable must be such that it is insensitive to whether n or $n+1$ particles contributed if the $n+1$ particles has n -particle kinematics.



- Procedure @ Tevatron (for electron)

 $p'_e = p_e + p_\gamma$

- $\Delta R(e, \gamma) < 0.2$

- $E_\gamma < 0.15 E_e$ for

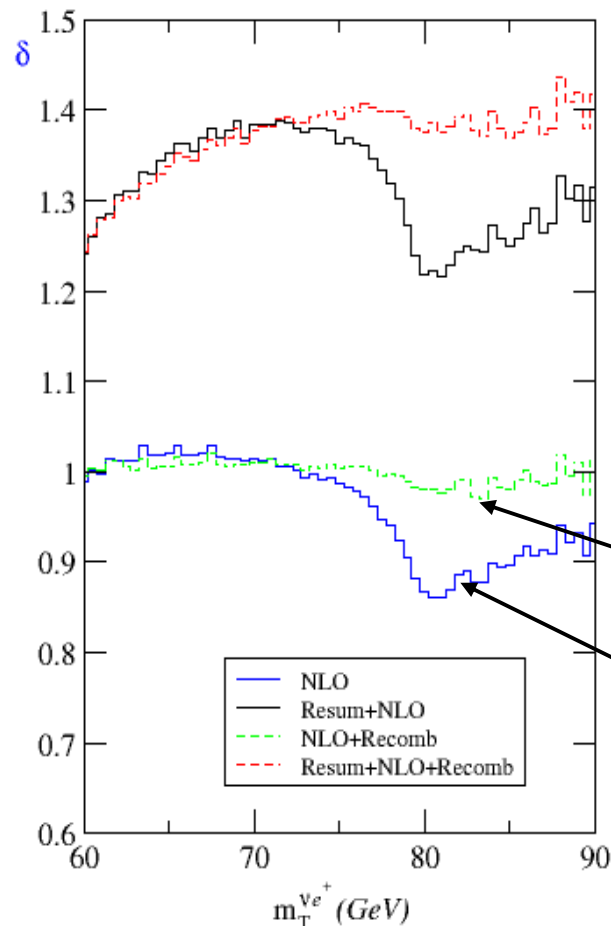
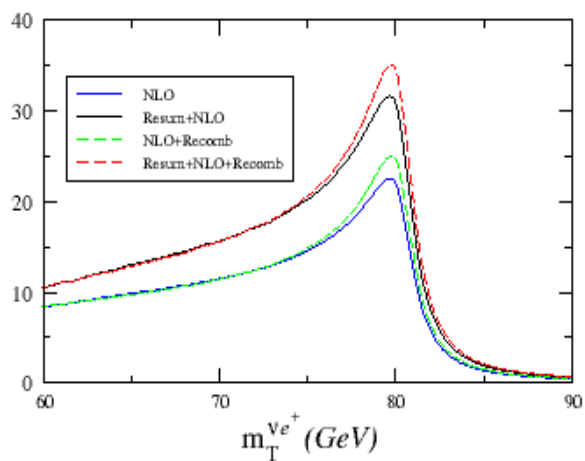
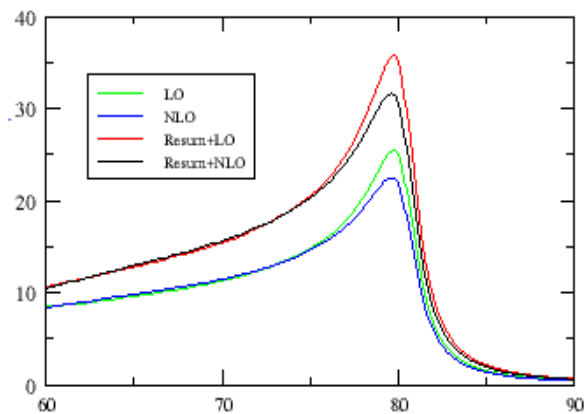
$$0.2 < \Delta R(e, \gamma) < 0.3$$

 rejection

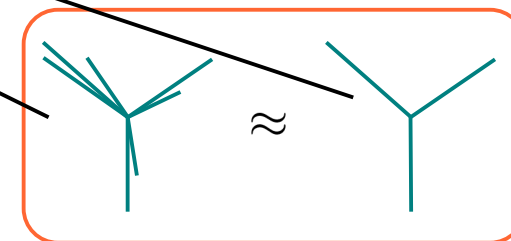
- $E_\gamma > 0.15 E_e$ for
 $0.2 < \Delta R(e, \gamma) < 0.4$



Recombination Effects for detecting electrons



Effects of QED correction decrease significantly after recombination.



infrared-safe



ResBos2

Version 2 of ResBos
(matched to NNLO in total inclusive rate)

Josh Isaacson, Yao Fu and CPY; arXiv:2205.02788

Collins-Soper-Sterman Formalism

Resummation

$$\frac{d\sigma_{\text{res}}}{dQ^2 d^2\vec{q}_T dy d\Omega} = \sigma \int \frac{d^2b}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}} \tilde{W},$$

$$\tilde{W} = e^{-S(b)} C \otimes f(x_A, C_3/b) C \otimes f(x_B, C_3/b)$$

$$S(b) = \int_{\frac{c_1^2}{b^2}}^{C_2^2 Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\ln \left(\frac{C_2^2 Q^2}{\bar{\mu}^2} \right) A(\bar{\mu}) + B(\bar{\mu}) \right]$$

- Sudakov factor
- Collinear factors
- Perturbative Coefficients (A, B, C)

[Collins, Soper, Sterman, '85] [...]



ResBos vs. ResBos2

	Order	Boundary Condition (C)	Anomalous Dimension		Fixed Order Matching (Y)
			γ_i (B)	Γ_{cusp} (A)	
	LL	1	-	1-loop	-
	NLL	1	1-loop	2-loop	-
ResBos	NLL' (+ NLO)	α_s	1-loop	2-loop	α_s
→	NNLL (+ NLO)	α_s	2-loop	3-loop	α_s
	NNLL' (+ NNLO)	α_s^2	2-loop	3-loop	α_s^2
→	N ³ LL (+ NNLO)	α_s^2	3-loop	4-loop	α_s^2
ResBos2	N ³ LL' (+ N ³ LO)	α_s^3	3-loop	4-loop	α_s^3
	N ⁴ LL (+ N ³ LO)	α_s^3	4-loop	5-loop	α_s^3

TABLE I. The definitions for the accuracy of the resummation calculation. The accuracy used by CDF was NNLL + NLO, while the state-of-the-art is N³LL + NNLO.



CDF W mass measurement

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SM prediction: $80,358.1 \pm 5.2$ MeV

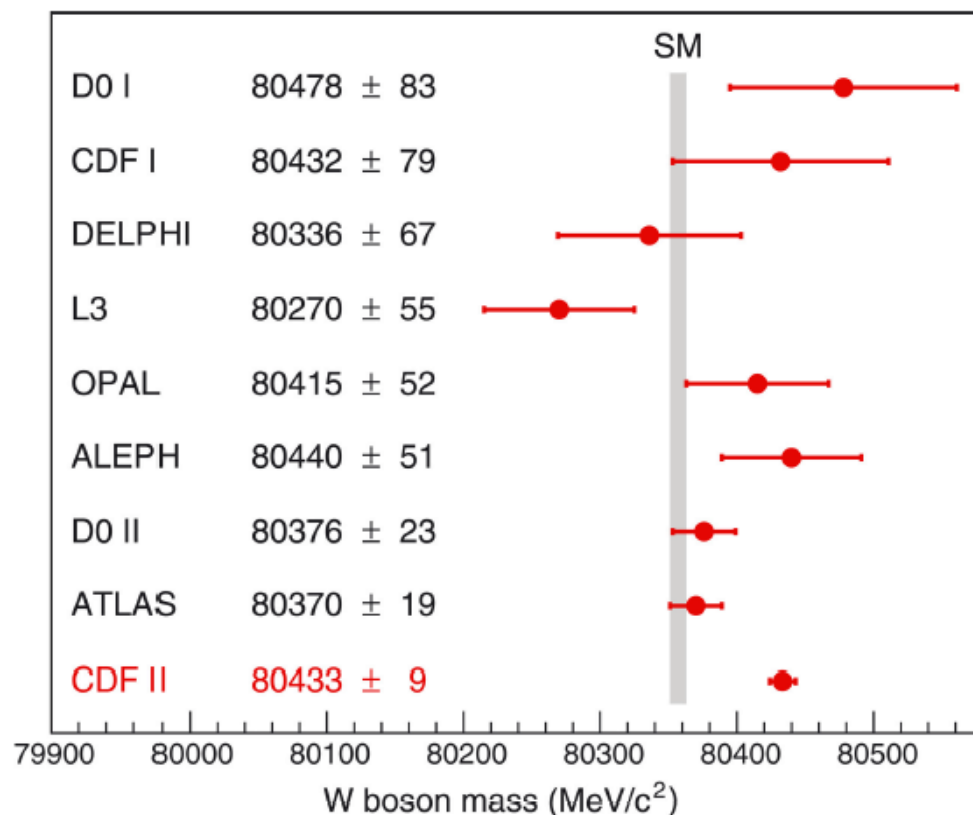


Figure reproduced from CDF-II measurement (Science 376, 170).

Quoted from CDF paper (Science 367, 170)

Simulated experiments are used to evaluate the statistical correlations between fits, which are found to be 69% (68%) between m_T and p_T^l (p_T^v) fit results and 28% between p_T^l and p_T^v fit results (43). The six individual M_W results are combined (including correlations) by means of the best linear unbiased estimator (66) to obtain $M_W = 80,433.5 \pm 9.4$ MeV, with $\chi^2/\text{dof} = 7.4/5$ corresponding to a probability of 20%. The m_T , p_T^l , and p_T^v fits in the electron (muon) channel contribute weights of 30.0% (34.2%), 6.7% (18.7%), and 0.9% (9.5%), respectively. The combined result is shown in Fig. 1, and its associated systematic uncertainties are shown in Table 2.

Also, **LHCb** result: $80,354 \pm 32$ MeV



Study the impact of higher order effects:
from NNLL+NLO
to NNNLL+NNLO

FROM RESBOS TO RESBOS2
FROM W(321)+Y TO W(432)+YK(R)

Shorthand notation:

$W(321)=W(321)+Y$, with full lepton angular correlations to α_s order.

$W(432)=W(432)+YK(R)$, with full lepton angular correlations to α_s^2 order.



from NNLL+NLO to NNNLL+NNLO

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- Generate pseudodata, including $p_T(Z)$, $p_T(W)$, m_T , $p_T(e)$, $p_T(\nu)$, using $W(432)$ and CT18 NNLO central set PDF.

$\alpha_s = 0.118$
CT18NNLO.00 PDF set

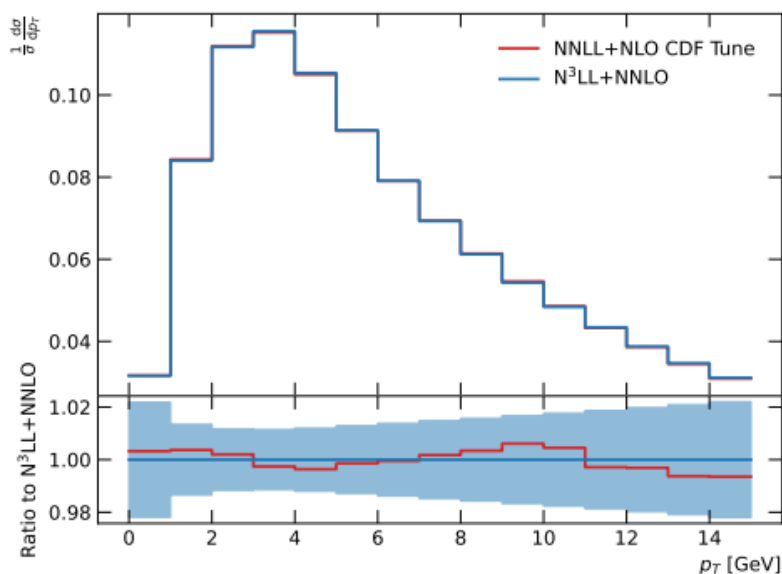
- Fit the normalized $p_T(Z)$ pseudodata with $W(321)$ calculation and CT18 NNLO α_s series PDFs, in which the g_2 and α_s values are the fitting parameters. This is called tuned $W(321)$ prediction.

$\alpha_s = 0.120$
CT18NNLO_as_0120 PDF set



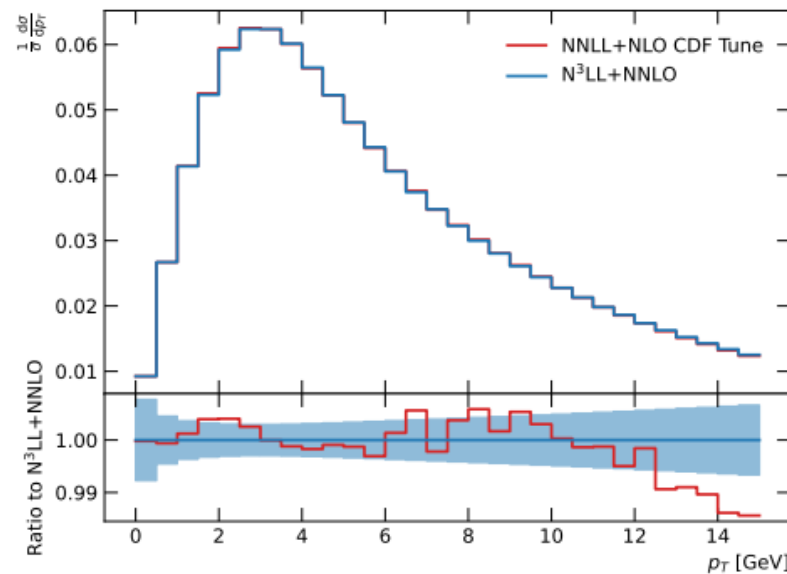
Comparison of tuned W(321) and pseudodata W(432)

$$\frac{1}{\sigma} \frac{d\sigma}{dp_T(Z)}$$



Fit to normalized pT(Z) pseudodata W(432)

$$\frac{1}{\sigma} \frac{d\sigma}{dp_T(W)}$$



Prediction of normalized pT(W) distribution from the tuned W(321)

The blue band represents the statistical uncertainty of the CDF measurement.



M_W template

- **Generate M_W template using the tuned W(321)**

Template: From 80.336 to 80.435 GeV; step is 1 MeV

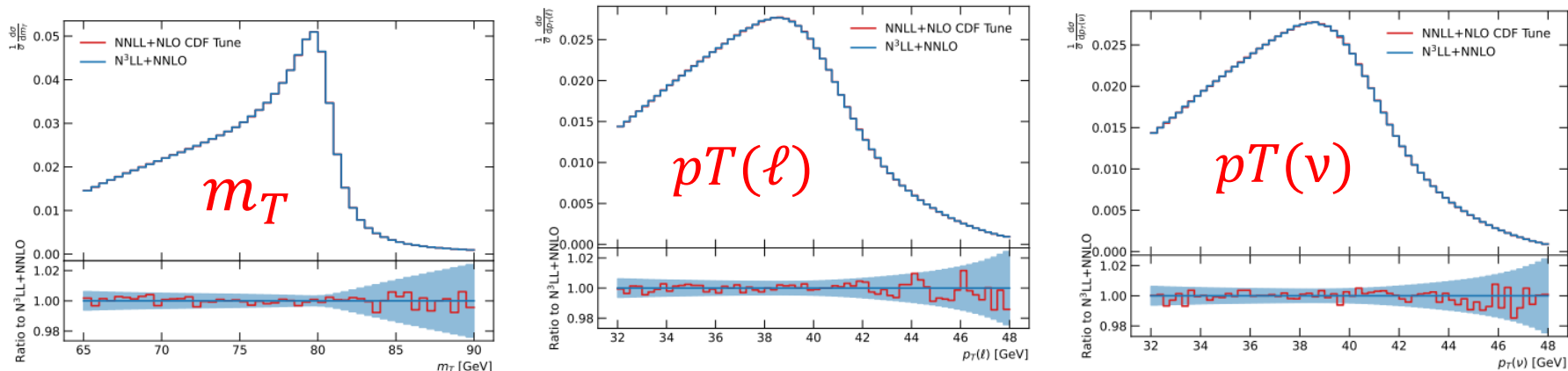
- **Shift in M_W from W(432) to tuned W(321)**

Do χ^2 fit to the normalized M_T , $p_T(l)$, $p_T(\nu)$ distributions to find M_W

Shift: *Fitted M_W – input M_W (80.385)*



Shift in M_W , when using the tuned W(321)



Observable	Mass Shift [MeV]	
	RESBOS2	+Detector Effect+FSR
m_T	1.5 ± 0.5	$0.2 \pm 1.8 \pm 1.0$
$p_T(\ell)$	3.1 ± 2.1	$4.3 \pm 2.7 \pm 1.3$
$p_T(\nu)$	4.5 ± 2.1	$3.0 \pm 3.4 \pm 2.2$

The blue band represents the statistical uncertainty of the CDF measurement.

Unc1: statistical uncertainty of the generated samples
 Unc2: uncertainty from different random seed of Gaussian smearing. It is estimated by generating 100 different smeared pseudodata with different random seed, using the mean value to determine the average shift, and the RMS to determine its uncertainty.

Another simple smearing model was also used:
 5% smearing on $p_T(l)$ and 11% on $p_T(\nu)$, the main conclusion does not change.



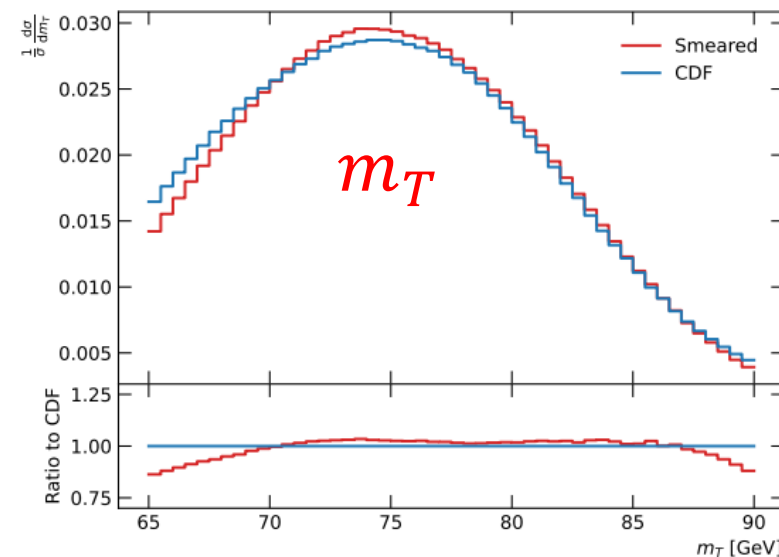
Detector Resolution effect and FSR

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- Smearing the momentum of $p_T(l)$ and $p_T(\nu)$

$$\frac{\sigma}{E} = \frac{a}{\sqrt{E}} \oplus b \oplus \frac{c}{E},$$

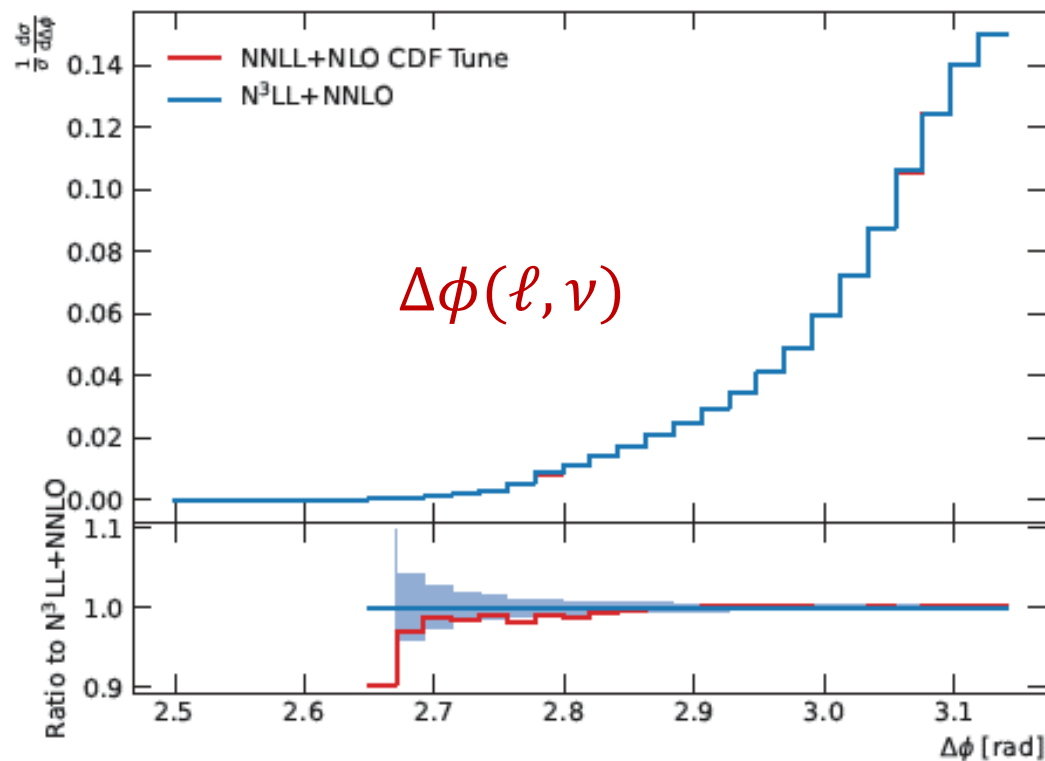
Gaussian smearing effect applied on $p_T(l)$ and $p_T(\nu)$



Same smearing was applied to both M_W template and tuned W(321) predictions.

Consider the electron channel, for its smaller background and less final state QED radiation (FSR) correction, as compared to the muon channel.

Angular correlation



W(432) has the correct lepton angular correlations at NNLO, and W(321) at NLO.

$$m_T^2 = 2(p_T(\ell)p_T(\nu) - \vec{p}_T(\ell) \cdot \vec{p}_T(\nu))$$

$$= 2p_T(\ell)p_T(\nu)(1 - \cos \Delta\phi(\ell, \nu))$$

FIG. 7. Comparison of the generated pseudodata for $\Delta\phi$ using the N³LL+NNLO calculation compared to the CDF tuned prediction at NNLL+NLO. The blue band represents the statistical uncertainty associated with the CDF measurement.



M_W template for studying the width effect

➤ **Generate M_W template using the W(432)**

Template: From 80.336 to 80.435 GeV, step is 1 MeV

Width: 2.0895 GeV (used in the CDF paper)

➤ **Changing the width of W boson**

According to the uncertainty of the W boson width reported by PDG, which is 0.042 GeV

Three pseudodata are generated for:

$$M_W = 80.385 \text{ GeV}, \Gamma_W = 2.0475 \text{ GeV}$$

$$M_W = 80.385 \text{ GeV}, \Gamma_W = 2.1315 \text{ GeV}$$

$$M_W = 80.385 \text{ GeV}, \Gamma_W \text{ is determined by NLO calculation, which is proportional to } M_W^3$$

$$\Gamma_W \sim g^2 M_W, \text{ with } g^2 \sim G_F M_W^2$$

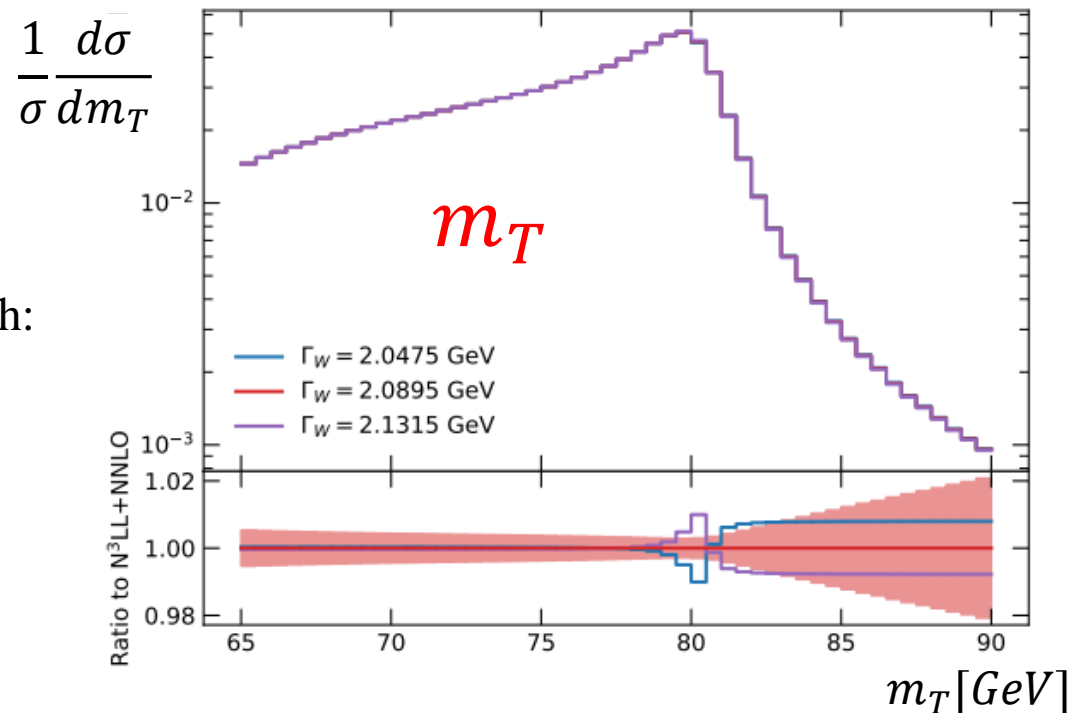


M_W template for studying the width effect

- Generate M_W template using the W(432)

Shift in M_W due to different W boson width:

Width	Mass Shift [MeV]
2.0475 GeV	2.0 ± 0.5
2.1315 GeV	0.3 ± 0.5
NLO	1.2 ± 0.5



The red band represents the statistical uncertainty of the CDF measurement.

Propagator takes the Breit-Wigner resonance form, with energy-dependent width.



PDF-induced shift in W boson mass



M_W template for studying the shift due to various PDFs

➤ **Generate M_W template using the W(432)**

Template: From 80.336 to 80.435 GeV; step is 1 MeV, CT18NNLO central set PDF.

Study the shift due to various PDFs in higher order calculation

➤ **Pseudodata generated by using W(432) + other PDFs**

Do χ^2 fit to the normalized M_T , $p_T(l)$, $p_T(\nu)$ distributions to find M_W

Shift: *Fitted M_W – input M_W (80.385)*



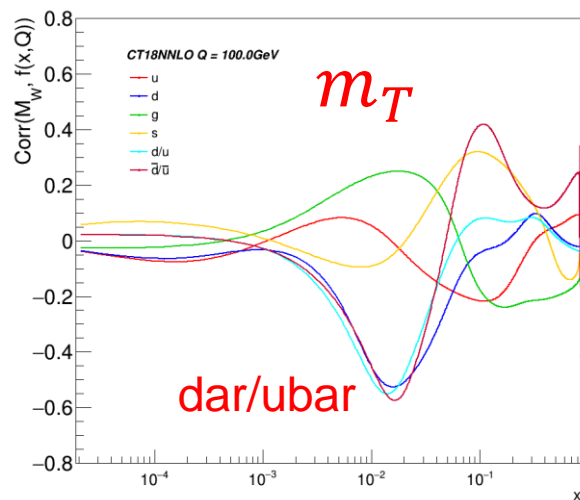
M_W template for studying the shift in M_W due to various PDFs

PDF Set	m_T		$p_T(\ell)$		$p_T(\nu)$	
	NNLO	NLO	NNLO	NLO	NNLO	NLO
CT18	0.0 ± 1.3	1.8 ± 1.2	0.0 ± 15.9	2.0 ± 14.3	0.0 ± 15.5	2.9 ± 14.2
MMHT2014	1.0 ± 0.6	2.6 ± 0.6	6.2 ± 7.8	36.7 ± 7.0	3.9 ± 7.5	36.0 ± 6.7
NNPDF3.1	1.1 ± 0.3	2.1 ± 0.4	2.1 ± 3.8	13.5 ± 4.9	5.4 ± 3.7	10.0 ± 4.9
CTEQ6M	N/A	2.8 ± 0.9	N/A	19.0 ± 10.4	N/A	20.9 ± 10.2

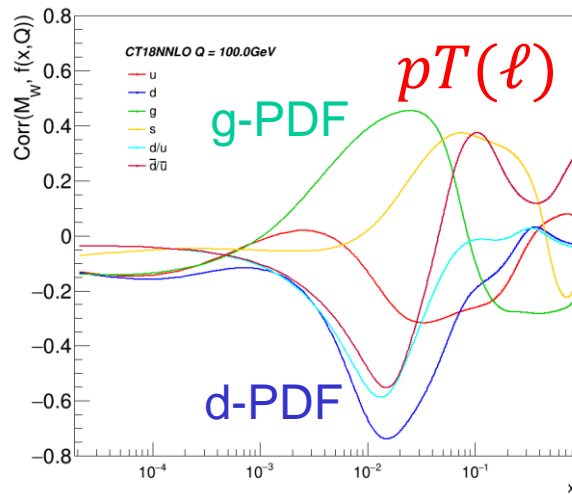
- The errors were generated by its own error PDF sets.
- Larger shifts are found when using the normalized $p_T(e)$ or $p_T(\nu)$ distributions.
- Large shifts are found when using NLO PDFs.
- Gluon PDF can contribute to NLO and NNLO predictions.
- $p_T(e)$ and $p_T(\nu)$ are more sensitive to **gluon** PDF errors than $m_T(e,\nu)$, hence, generate more shift in M_W .

Correlation cosine between the uncertainty of the extracted M_W and that of parton density

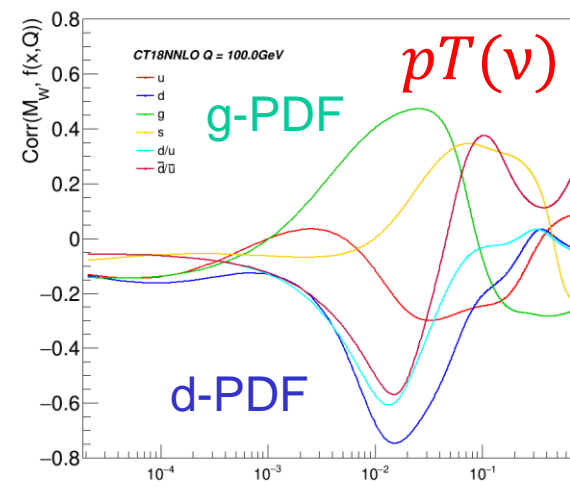
Extracted from $\frac{1}{\sigma} \frac{d\sigma}{dm_T}$



Extracted from $\frac{1}{\sigma} \frac{d\sigma}{dp_T(\ell)}$



Extracted from $\frac{1}{\sigma} \frac{d\sigma}{dp_T(\nu)}$



- Gluon PDF can contribute to NLO and NNLO predictions.
- $p_T(e)$ and $p_T(\nu)$ are more sensitive to **gluon** PDF errors than $m_T(e, \nu)$, hence, generate more shift in M_W .

W(432) predictions

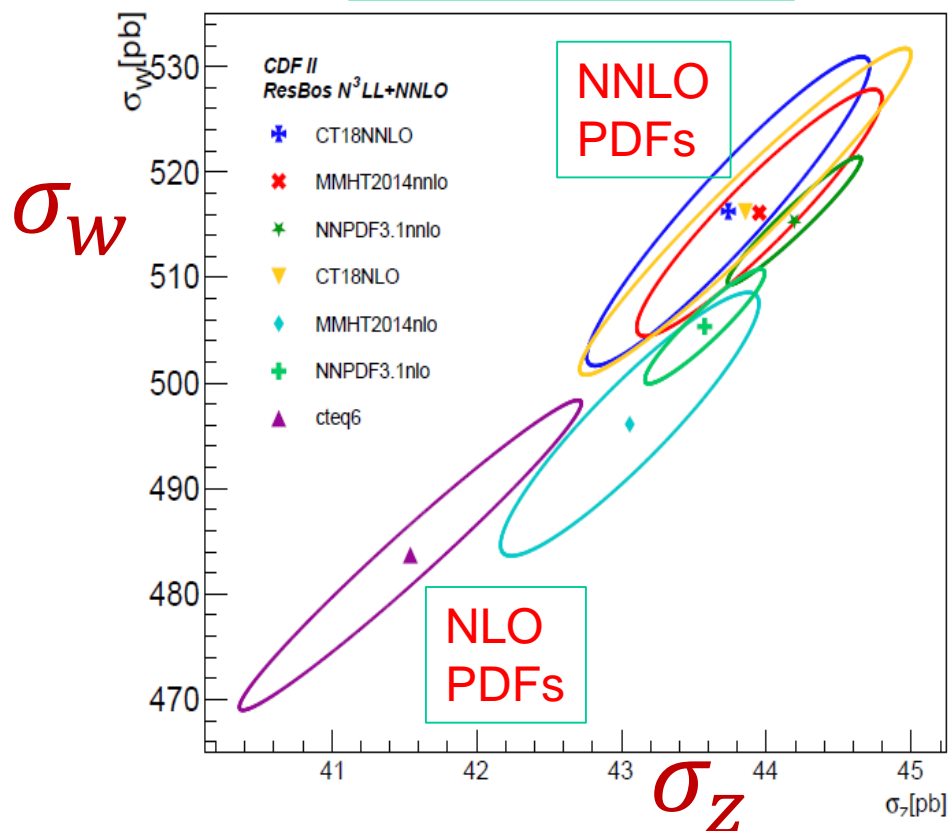


FIG. 10. PDF-induced correlation ellipses, at the 68% confidence level (C.L.), between the fiducial cross sections of W and Z boson production at the Tevatron Run II.

Kinematic cuts and event rates:

The Z boson events are generated satisfying $p_T(Z) < 15$ GeV, $30 < p_T(\ell) < 55$ GeV, $|\eta(\ell)| < 1$, and $66 < M_{\ell\ell} < 116$ GeV. The W boson events are generated satisfying $p_T(W) < 15$ GeV, $30 < p_T(\ell) < 55$ GeV, $30 < p_T(\nu) < 55$ GeV, $|\eta(\ell)| < 1$, and $60 < m_T < 100$ GeV. Here, m_T is the transverse mass and is defined as

$$m_T^2 = 2(p_T(\ell)p_T(\nu) - \vec{p}_T(\ell) \cdot \vec{p}_T(\nu)), \quad (4)$$

where $\vec{p}_T(\ell, \nu)$ is the vector transverse momentum and the dot product is related to the difference in ϕ angle between the two vectors. These selection criteria are consistent with those used by CDF. After the selection criteria, CDF had a selection of 1,811,700 (66,180) $W \rightarrow e\nu$ ($Z \rightarrow ee$) events and 2,424,486 (238,534) $W \rightarrow \mu\nu$ ($Z \rightarrow \mu\mu$) events [2].



ePump-optimization

arXiv: 1806.07950; 1907.12177

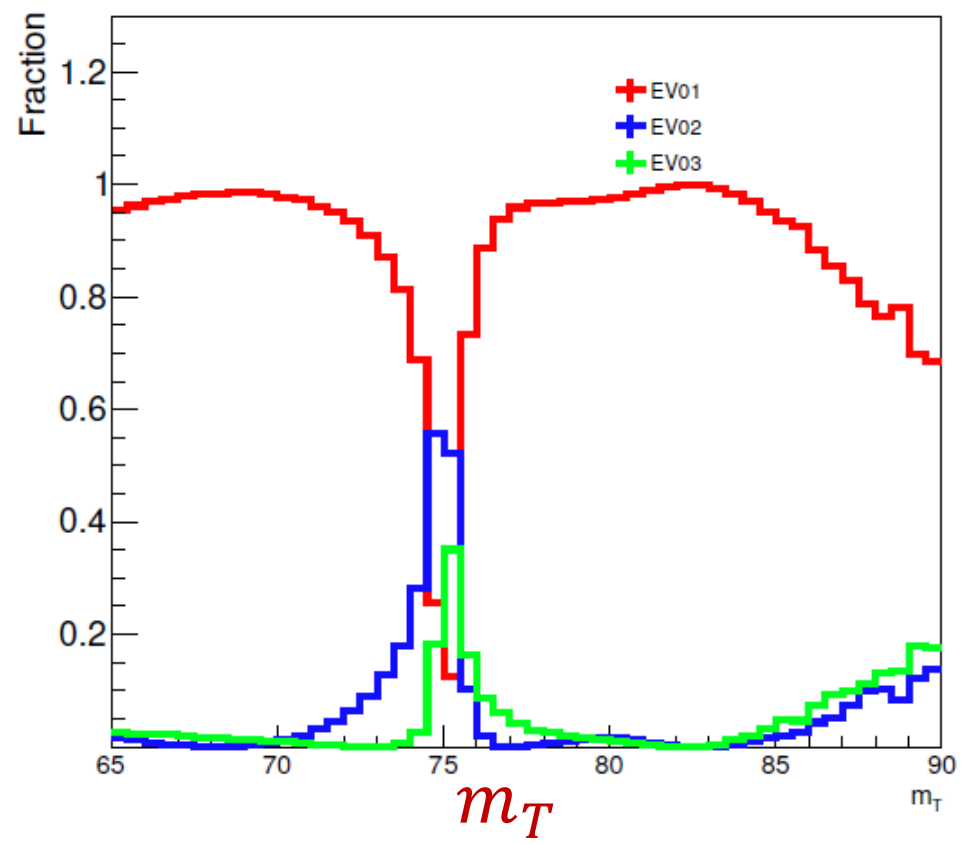


FIG. 12. Fractional contribution of the three leading optimized eigenvector PDFs (EV01, EV02 and EV03) to the variance of the m_T distribution, normalized to each bin, obtained from the ePump-optimization analysis.

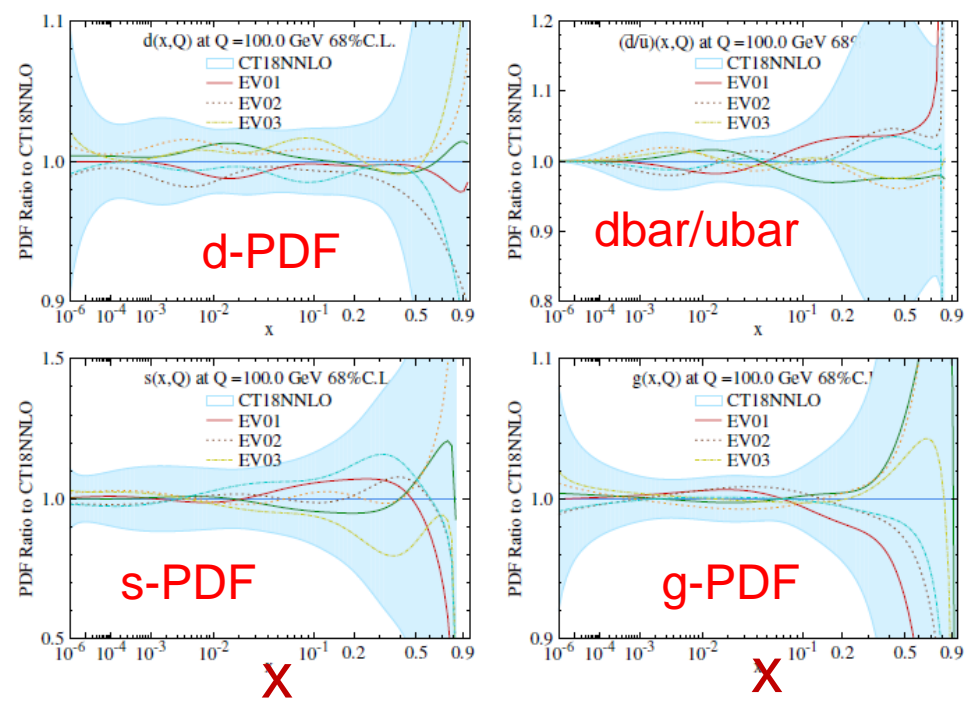


FIG. 13. Ratios of the top three pairs of eigenvector PDFs and the original CT18 NNLO error PDFs, at $Q = 100$ GeV, to the CT18 NNLO central value of d , \bar{d}/\bar{u} , s and g PDFs. These eigenvector PDFs were obtained after applying the ePump-optimization to the original CT18 NNLO PDFs with respect to the m_T distribution.

The three eigenvalues are 44.5, 3.0 and 2.4, respectively, with 50 bins in the m_T distribution.



ePump-optimization

arXiv: 1806.07950; 1907.12177

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The three eigenvalues are 44.5, 3.0 and 2.4, respectively, with 50 bins in the m_T distribution.

The combination of those top three optimized error PDFs contributes up to 99.6% in the total PDF variance of the 50 given data points, *i.e.*, with 50 bins in m_T distribution. This ePump-optimization allows us to conveniently use these three leading new eigenvectors (with a total of six error sets), in contrast to applying the full 58 error sets of the CT18 NNLO PDFs, to study the PDF-induced uncertainty of the m_T observable.

Only need to use these $2*3+1=7$, not $2*29+1=59$, PDF sets to study detector effects, etc. via Monte Carlo simulation.

Note that the first leading eigenvector set already accounts for $44.5/50= 89\%$ of the PDF variance.

Ratios of normalized pT(Z) and pT(W) due to QCD scale variation

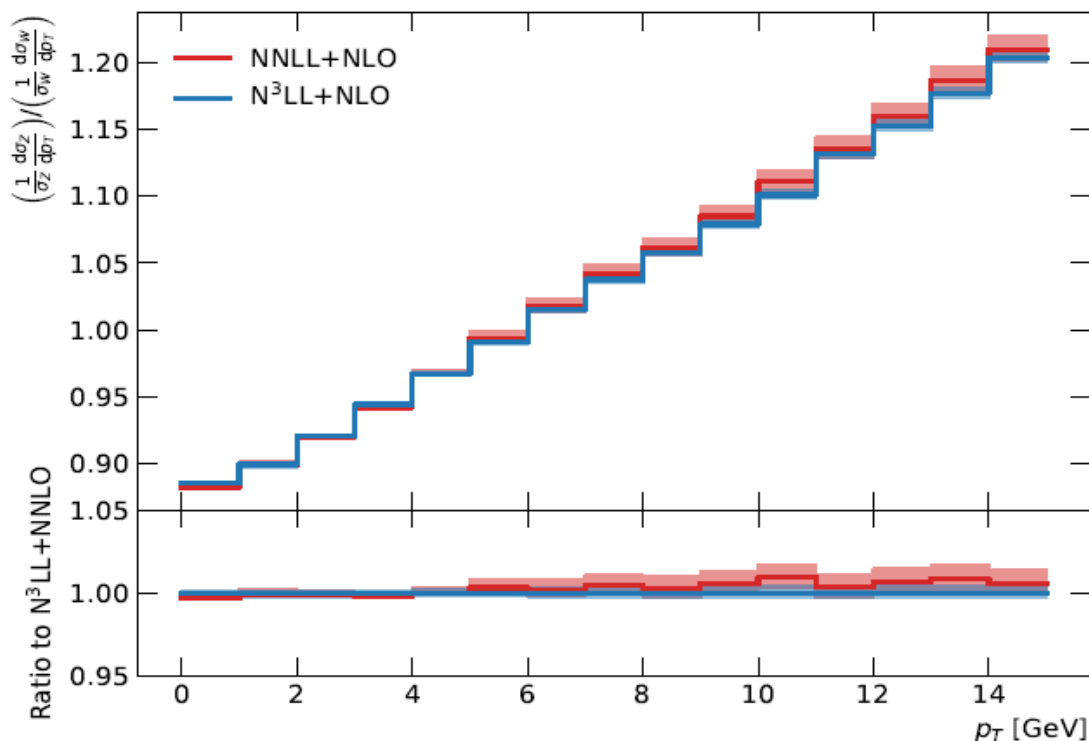


FIG. 3. The ratio for the normalized Z transverse momentum to the normalized W transverse momentum in the region used by the CDF experiment. The NNLL+NLO (red) prediction and the N^3 LL+NNLO (blue) prediction are consistent with each other over this region. The scale uncertainty is treated in a fully correlated manner.

- CDF did not use **ResBos** code to study the impact on the M_W measurement from using uncorrelated QCD scales to model the pT(W) distribution, after using their pT(Z) distribution to fix g_2 and α_s values. Instead, they used **DYQT** code. [arXiv: 0812.2862;1007.2351](https://arxiv.org/abs/0812.2862;1007.2351)
- In this study, we have assumed a fully correlated scale uncertainty between pT(W) and pT(Z) when varying the QCD scales in the **ResBos** calculation.
- We find this ratio is not very sensitive to higher order corrections even with this more aggressive estimate.



Learned from Prof. Joey Huston @ MSU

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==== my answers to questions from Dr. Natascia Vignaroli =====

(1)

> What do you think about the CDF anomaly?

=>

Our paper only discussed the impact of higher order contributions to the extraction of M_W , based on CDF's data-driven method. We cannot answer the question about the difference observed by CDF between their data and SM prediction.

If it is not due to new physics effect, then one could ask:

— Could there be some common systematic(s) among all six of the CDF analyses?

— Would it be worthwhile to do a W -mass analysis of $Z \rightarrow ee, \mu\mu$, though it will be statistics limited?

(2)

> How about the ATLAS measurements?

=>

ATLAS has a much better detector, but as compared to CDF, it suffers from being “too energetic” — most W bosons are boosted (to both longitudinal and transverse directions)! CDF has smaller PDF uncertainties, smaller QCD radiation (Sudakov) effects, and smaller pileup, etc.

=====



Conclusions and outlook

- Higher order effect in the **ResBos** calculation can bring the discrepancy from 7σ down to about 6σ , a shift around **10 MeV toward** the Standard Model (SM) prediction.
- LHC will further improve M_W measurement.
- A combined analysis of LHC and Tevatron M_W measurements will come in near future.
- If it is due to New Physics (NP), similar effect may also affect the measurement of **weak-mixing angle $\sin^2\theta_w$** via the forward-backward charged asymmetry (A_{FB}) of Drell-Yan pair production at the high luminosity LHC. In this case, it is crucial to be able to factorize the effect of PDFs in the A_{FB} measurement from the genuine electroweak physics (in either SM or NP). arXiv: 2202.13628
- More collaborations among experimentalists and theorists are needed!

Lessons learned from W mass measurements

Experimentalists



Theorists

2017 Featured Story #1: Million-dollar gift establishes endowed professorship in honor of the late Dr. Wu-Ki Tung



Michigan State University
(1992-2009)

<http://www.pa.msu.edu/node/5921>

- Co-founder of CTEQ (**The Coordinated Theoretical-Experimental Project on QCD**) in 1989 – present
- Nowadays, many, like this Mini-Workshop, are doing precisely that.



Backup slides



Where is it?

- **ResBos:** <http://hep.pa.msu.edu/resum/>
- **Plotter:** <http://hep.pa.msu.edu/wwwlegacy>

ResBos-A (including final state NLO QED corrections)

<http://hep.pa.msu.edu/resum/code/resbosa/>

has not been updated.

Why? Because it was not used for Tevatron experiments.

The plan is to include final state QED resummation inside ResBos2.

Sorry, the website is temporary down and will be restored later.



Physical processes included in ResBos

W^\pm

γ, Z

including gauge invariant set amplitude

H

Including the full NNLO contribution

$\gamma\gamma, ZZ, WW$

New physics: $W', Z', H^+, A^0, H^0 \dots$



Limitations of ResBos

- Any perturbative calculation is performed with some approximation, hence, with limitation.
- To make the best use of a theory calculation, we need to know what it is good for and what the limitations are.

It does not give any information about the hadronic activities of the event.



It could be used to reweight the distributions generated by (PYTHIA) event generator, by comparing the boson (and its decay products) distributions to ResBos predictions.

This has been done for W-mass analysis by CDF and D0



Conclusion

- ResBos is a useful tool for studying electroweak gauge bosons and Higgs bosons at the Tevatron and the LHC.
- It includes not only QCD resummation for low q_T region but also higher order effect in high q_T region, with spin correlations included via gauge invariant set of matrix elements.



PDF-induced uncertainty

Hessian Method

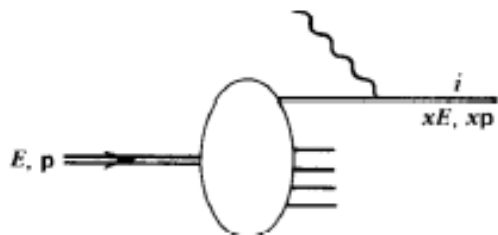
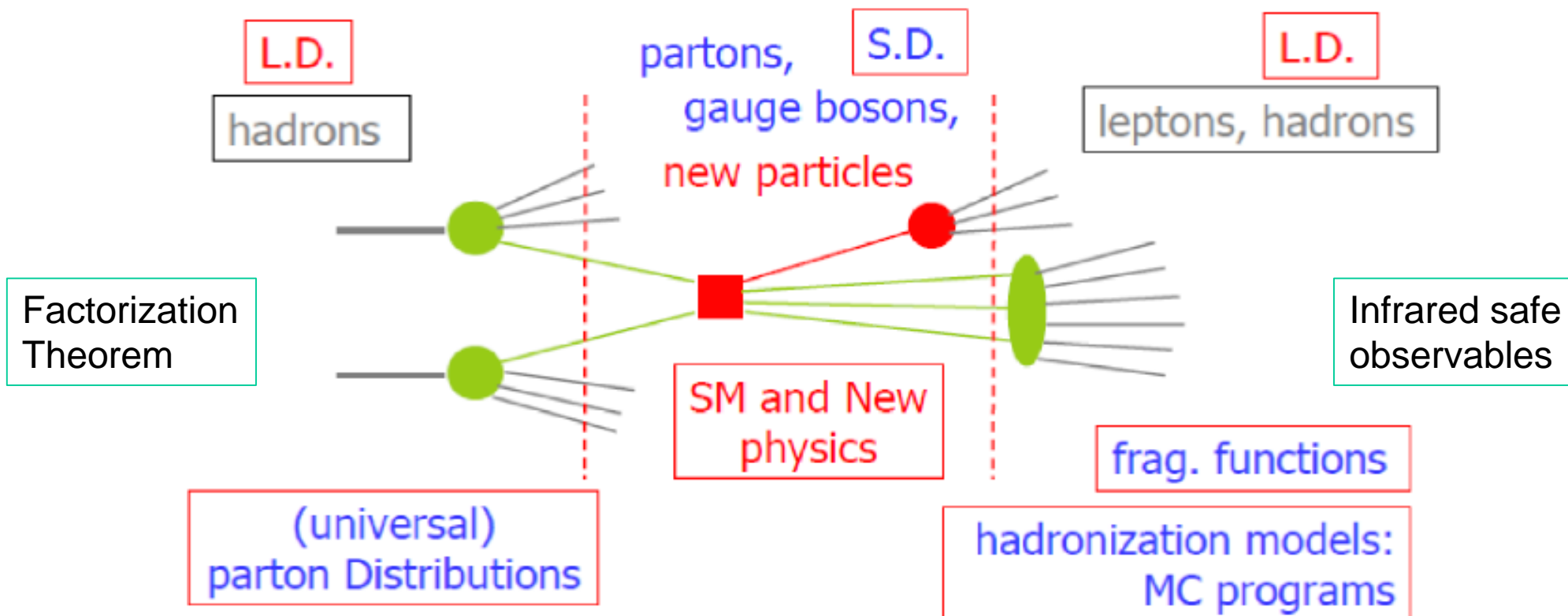
Hessian error PDF sets

such as CT18 and MSHT20 PDFs



QCD improved parton model

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Parton distribution function (PDF) $f_{j/A}(x, Q)$ describe the possibility to find a parton j , i.e. quark and gluon, in a nucleon A .



PDF-induced uncertainty

Let $X = X(\{a_i\})$ to be the observable as a function of fitting parameter. Using the linear approximation of parameter $\{z_i\}$, the symmetry uncertainty of X is,

$$\Delta X = \frac{1}{2} \left(\sum_{i=1}^{N_p} [X(\{z_i^+\}) - X(\{z_i^-\})]^2 \right)^{1/2},$$

Where $\{z_1^\pm\} = \{\pm T, 0, \dots\}$, $\{z_2^\pm\} = \{0, \pm T, 0, \dots\}$ and so on. The asymmetry uncertainty of X is,

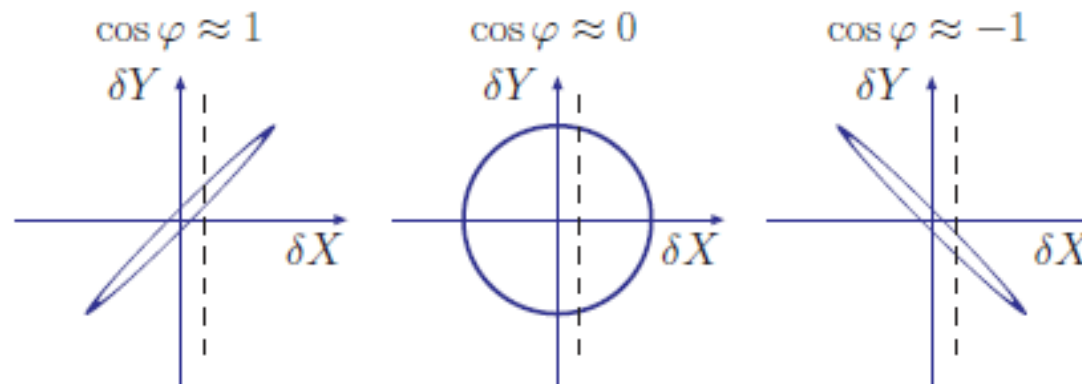
$$\delta^+ X = \sqrt{\sum_{i=1}^{N_a} \left[\max \left(X_i^{(+)} - X_0, X_i^{(-)} - X_0, 0 \right) \right]^2},$$

$$\delta^- X = \sqrt{\sum_{i=1}^{N_a} \left[\max \left(X_0 - X_i^{(+)}, X_0 - X_i^{(-)}, 0 \right) \right]^2},$$



PDF-induced correlations

Correlation ellipse for observables X and Y



In the framework of Hessian method, the correlation between two observables X and Y, which are function of PDFs, can be worked out as:

Correlation cosine



$$\cos \varphi = \frac{\vec{\nabla} X \cdot \vec{\nabla} Y}{\Delta X \Delta Y} = \frac{1}{4\Delta X \Delta Y} \sum_{\alpha=1}^N \left(X_{\alpha}^{(+)} - X_{\alpha}^{(-)} \right) \left(Y_{\alpha}^{(+)} - Y_{\alpha}^{(-)} \right)$$

Where the ΔX and ΔY are their symmetric uncertainties. By this correlation angle φ , the tolerance ellipse is defined by

$$X = X_0 + \Delta X \cos \theta, Y = Y_0 + \Delta Y \cos(\theta + \varphi),$$



ePump

(error PDF Updating Method Package)

- A tool to examine **the impact of a new data set** to further constrain the existing PDFs without using a global analysis code.
- A tool to **reduce the total number of error PDF sets** relevant to specific experimental observables.
- A tool to perform **a simultaneously fit to the parameter of New Physics model and PDFs.**

<http://hep.pa.msu.edu/epump/>

arXiv: 1806.07950

arXiv: 1907.12177

Sorry, the website is temporary down and will be restored later.



Motivation for ePump

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- UpdatePDFs: With many data sets and NNLO calculations, global fitting can be time consuming.
 - Need for fast and efficient method to estimate effects of new data before doing global fit.
 - Can estimate effects of different data set choices in real time.
- OptimizePDFs: Experimental analyses may require many MC calculations, using PDF error sets. Again, it's time consuming.
 - Optimize Hessian error PDFs to the observables, so irrelevant error PDFs may be discarded, while PDF-dependence is still maintained to desired precision.



ePump Updating: Hessian Profiling

- Plenty of new data from LHC need careful study on the impact to PDFs. However, a **complete global fitting take 1 to 2 days** after the implementation of applgrid/fastNLO and parallelization.
- **Hessian reweighting/profiling method** predict the updated PDFs and observables after including new data in global analysis of given Hessian eigenvector sets.

$$\Delta\chi^2(Z) = \Delta\chi_{old}^2(Z) + (X_i^E - X_i(Z))C_{ij}^{-1}(X_j^E - X_j(Z))$$

$$\text{Updated best-fit PDF} : f_{new}^0 = f^0 + \Delta f \cdot Z$$

$$\text{Updated error PDFs} : f^{\pm(r)} = f_{new}^0 \pm \Delta f \cdot U^{(r)} / \sqrt{1 + \lambda^{(r)}}$$

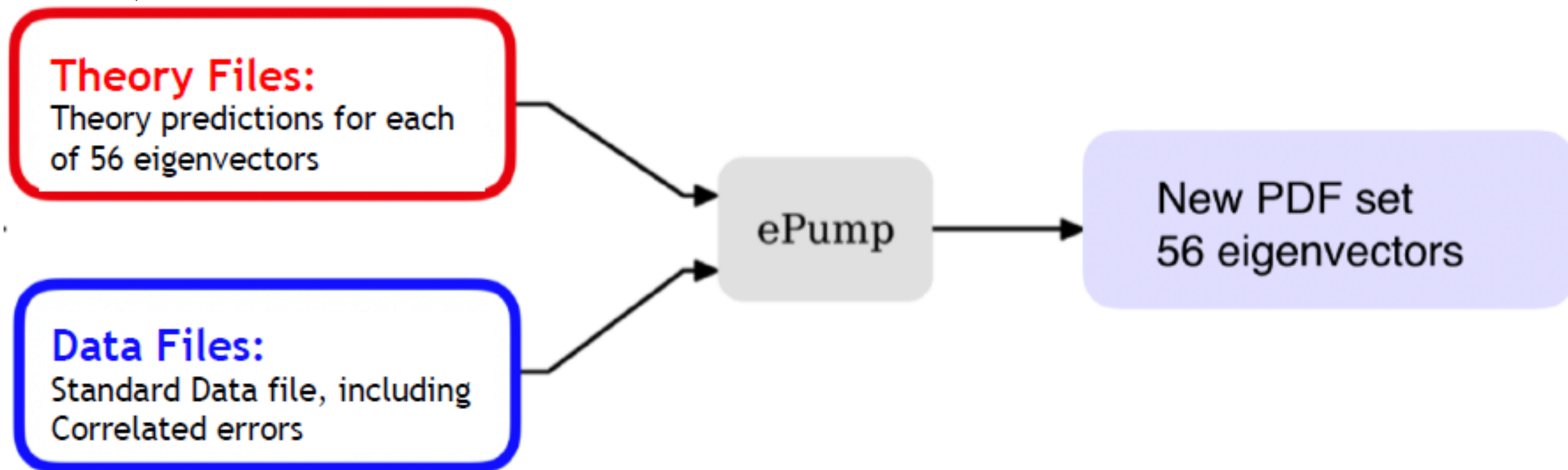
$$\text{Updated observables} : Y_{new}^0 = Y^0 + \Delta Y \cdot Z$$

- The Hessian reweighting method resulting as the program of **ePump** (error PDF updating method package) (PRD98,094005(2018)



How to use ePump

It could be a theory prediction of New Physics model, such as the Standard Model Effective Field Theory (SMEFT).

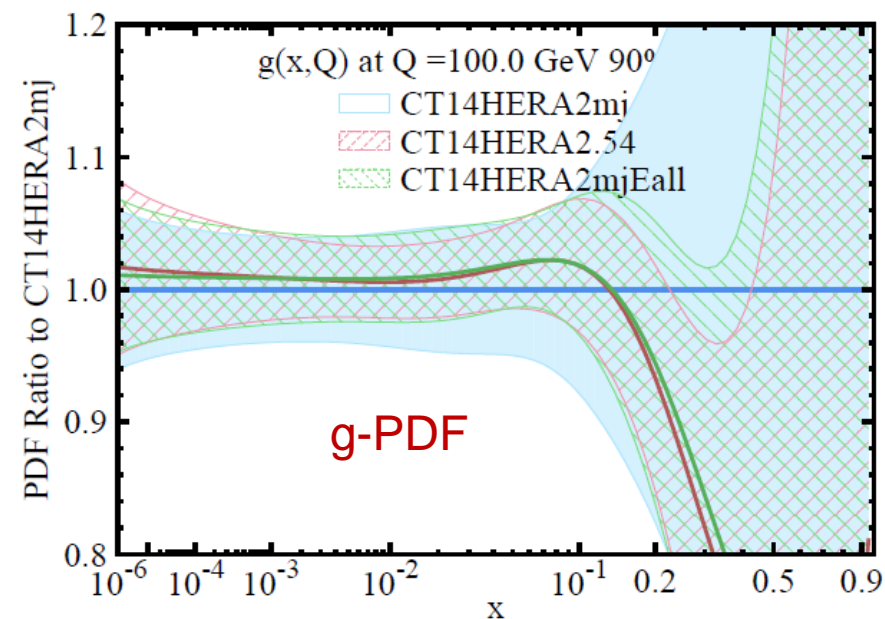
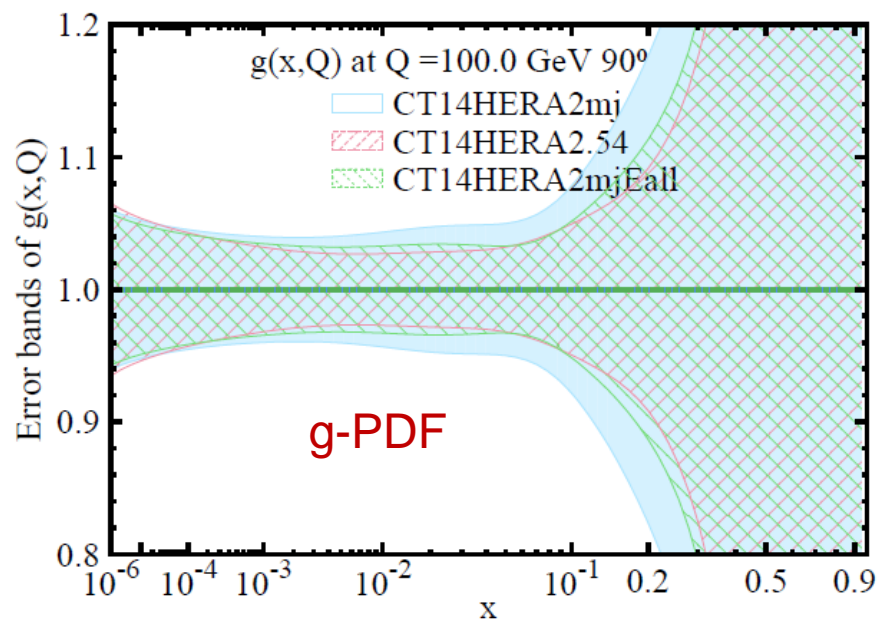


(Auxiliary Theory Files may also be included to update predictions for observables not included in fit.)

ePump-updating

An example to show the impact of jet data to constrain gluon PDF in the relevant x region.

- Remove all CDF, D0, ATLAS 7TeV, CMS TeV jet data from CT14HERA2 and refit \rightarrow CT14HERA2mj.
- Add back the 4 data sets to CT14HERA2mj by ePump and compare with CT14HERA2.





Optimized PDFs

- Based on Data Set Diagonalization [Pumplin, PRD 80 \(2009\) 034002](#)

Maximize: $\sum (X_\alpha(\mathbf{z}) - X_\alpha(\mathbf{0}))^2 / |\Delta X_\alpha|^2$ subject to constraint $\mathbf{z}^2 = 1$

Using Hessian approximation, $X_\alpha(\mathbf{z}) = X_\alpha(\mathbf{0}) + \Delta X_\alpha \cdot \mathbf{z}$

leads to matrix $M^{ij} = \sum \Delta X_\alpha^i \Delta X_\alpha^j / |\Delta X_\alpha|^2$ with eigenvalues/vectors, $\lambda^{(r)}$ and $\mathbf{U}^{(r)}$

⇒ New error PDFs: $f^{\pm(r)} = f^0 \pm \Delta f \cdot \mathbf{U}^{(r)}$

- Order PDFs by eigenvectors (note: $\sum \lambda^{(r)} =$ number of observables)
- Full set of optimized PDFs reproduces Hessian symmetric errors
- Eigenvalue $\lambda^{(r)}$ gives (sum of) fractional contribution of $f^{\pm(r)}$ to variances
- Depending on precision required, keep reduced set of error PDFs, based on eigenvalues.



Summary

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- The ePump package contains two functionalities
 - UpdatePDFs is a fast & efficient method to estimate the effect of new data on the a current set of best-fit and Hessian error PDFs.
 - OptimizePDFs can be used to find optimized set of Hessian error PDFs for specialized experimental analyses. It gives a simple method for reducing the number of optimized error PDFs, while maintaining a specified precision.

- A tool to examine **the impact of a new data set** to further constrain the existing PDFs without using a global analysis code.
- A tool to **reduce the total number of error PDF sets** relevant to specific experimental observables.
- A tool to perform **a simultaneously fit to the parameter of New Physics model and PDFs.**