

N3LO CORRECTIONS TO NEUTRAL AND CHARGED CURRENT AT THE LHC

Karlsruher Institut für Technologie

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W质量研讨会: UNCERTAINTIES AND OPPORTUNITIES



"THREE CLOUDS" IN PARTICLE PHYSICS SINCE 2021

Test of lepton universality in beauty-quark decays #1 LHCb Collaboration • Roel Aaij (NIKHEF, Amsterdam) et al. (Mar 22, 2021) Published in: Nature Phys. 18 (2022) 3, 277-282 • e-Print: 2103.11769 [hep-ex] ♪ pdf ② links ③ DOI ⊡ cite ☐ datasets ① 313 citations	≻ Fitt		
Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46#1ppmMuon g-2 Collaboration • B. Abi (Oxford U.) et al. (Apr 7, 2021)Published in: Phys.Rev.Lett. 126 (2021) 14, 141801 • e-Print: 2104.03281 [hep-ex]	$\alpha S = \alpha T$		
P pdf ∂ links ∂ DOI ⊡ cite			
High-precision measurement of the W boson mass with the CDF II detector #1 CDF Collaboration • T. Aaltonen (Helsinki U. and Helsinki Inst. of Phys.) et al. (Apr 8, 2022) Published in: <i>Science</i> 376 (2022) 6589, 170-176			
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Statistics from iNSPIRE-HEP by 19-04-2022

Further experimental confirmation

- ► Fermilab Run 2 ~ Run 5 analysis
- ► LHCb Upgrade I (2025) and II (2030)
- ATLAS, LHCb, CMS all have on-going analysis of W mass.

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N3LO corrections to neutral and charged current at the LHC

ting the elephant with BSM free parameters "he "oblique corrections" S-T-U in vacuum polarisation:

$$= 4e^{2}[\Pi'_{33}(0) - \Pi'_{3Q}(0)]$$

= $\frac{e^{2}[\Pi_{11}(0) - \Pi_{33}(0)]}{\sin^{2}(\theta_{W})\cos^{2}(\theta_{W})m_{Z}^{2}}$
= $4e^{2}[\Pi'_{11}(0) - \Pi'_{33}(0)]$

Peskin and Takeuchi `92





"THREE CLOUDS" IN PARTICLE PHYSICS SINCE 2021

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`22

Carpenter, Murphy, Smylie

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utral and charged current at the LHC



- Challenge experiment with better/alternative predictions
 - Lattice prediction of HVP in g-2 Borsanyi, Fodor et. al. 21 ► Improve template fit in CDFII (ResBos@NLO+NNLL)







W MASS IN CDFILMEASUREMENT



Table 1. Individual fit results and uncertainties for the M_W **measurements.** The fit ranges are 65 to 90 GeV for the $m_{\rm T}$ fit and 32 to 48 GeV for the p_T^{ℓ} and p_T^{ν} fits. The χ^2 of the fit is computed from the expected statistical uncertainties on the data points. The bottom row shows the combination of the six fit results by means of the best linear unbiased estimator (66).

Distribution	W boson mass (MeV)	χ² /dof
$\overline{m_{\mathrm{T}}(e, \mathbf{v})}$	$80,429.1 \pm 10.3_{stat} \pm 8.5_{syst}$	39/48
$p_{\mathrm{T}}^{\ell}(e)$	$80,\!411.4\pm10.7_{stat}\pm11.8_{syst}$	83/62
$p_{\mathrm{T}}^{\mathrm{v}}(e)$	$80,426.3 \pm 14.5_{stat} \pm 11.7_{syst}$	69/62
$m_{\mathrm{T}}(\mu, \nu)$	$80,\!446.1\pm9.2_{stat}\pm7.3_{syst}$	50/48
$p_{\mathrm{T}}^{\ell}(\mu)$	$80,\!428.2\pm9.6_{stat}\pm10.3_{syst}$	82/62
$p_{\mathrm{T}}^{\mathrm{v}}(\mathrm{\mu})$	$80,\!428.9\pm13.1_{stat}\pm10.9_{syst}$	63/62
Combination	$80,\!433.5\pm6.4_{stat}\pm6.9_{syst}$	7.4/5

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 $p_T^{l(\nu)} = \sqrt{(p_x^{l(\nu)})^2 + (p_y^{l(\nu)})^2}$

$$E_T^{l(\nu)} = \sqrt{m^2 + (p_x^{l(\nu)})^2 + (p_y^{l(\nu)})^2} \approx p_T^{l(\nu)}$$

$$m_T^W = \sqrt{2E_T^l E_T^\nu (1 - \cos \Delta \phi)}$$

ResBos, DYqT, PHOTOS, HORACE N3LO corrections to neutral and charged current at the LHC

► PDG world average: $m_W = 80379 \pm 12 MeV$ (Particle Data Group `20)

► CDFII latest result: $m_W = 80433 \pm 9 MeV$ (CDF `22)

> Indirect measurement of m_T^W , p_T^l , p_T^ν distributions

Template fit to best best parameter values ► Full error = Experiment + Theory model ► Experiment statistics: ±6.4 *MeV*

► Experiment systematic: ±5.3 MeV

Theory model: $\pm 5.2 MeV \pm ?? MeV$

Table 2.	Uncertainties	on the	combine
M _w resu	lt.		

Source	Uncertainty
Lepton energy scale	3.0
Lepton energy resolution	1.2
Recoil energy scale	1.2
Recoil energy resolution	1.8
Lepton efficiency	0.4
Lepton removal	1.2
Backgrounds	3.3
p _T ^z model	1.8
$p_{\rm T}^W/p_{\rm T}^Z$ model	1.3
Parton distributions	3.9
QED radiation	2.7
W boson statistics	6.4
Total	9.4







W MASS IN CDFILMEASUREMENT

$\rightarrow d\sigma/dm_T^W$ Template fit to best best parameter values:



CDFII: 0.2 MeV granularity of m_W with fixed Γ_W

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W MASS IN CDFILMEASUREMENT

- $\rightarrow d\sigma/dm_T^W$ Template fit to best best parameter values:
 - A Relativistic Breit-Wigner form: $(s^2 - m_W^2 + is^2 \Gamma_W / m_W)^{-1}$ with fixed $\Gamma_W \gtrsim 50$
 - ► Binned maximum-likelihood fit: (Poisson distribution cross bins)

$$-\ln \mathscr{L}_{b}(m_{W}) = -\sum_{b} \left(n_{b} \ln \left(\Delta \sigma_{b}(m_{W}) \right) - \Delta \sigma_{b}(m_{W}) \right)$$

 $\succ \chi^2$ analysis for the best fit of each obs.

> The best linear unbiased estimator to combine each obs.:

≻
$$\chi^2$$
/dof = 7.4/5 → p-value = 20%

► Weight distribution:

 $m_T^W \sim 64.2\%, p_T^l \sim 25.4\%, p_T^\nu \sim 10.4\%$ Xuan Chen (KIT)

Events / 0.5

0 ⊑ 60

∧əg 40

20

0 30

Events / 0.25

CDFII: Best fitted results for m_T^W , p_T^l



N3LO corrections to neutral and charged current at the LHC



PRECISION PREDICTIONS AT THE LHC

QCD improved parton model



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 ± 10 % level! N3LO corrections to neutral and charged current at the LHC

 $(1)^{(1)} + (1)^{(1)} + (1)^{(1)}$ $\hat{\sigma}_{ab}$ $J_{b|B}(x_b)$ P_B $\frac{\mathrm{d}x_b}{f_{a|A}(x_a)} \frac{f_{b|B}(x_b)}{f_{ab}(x_b)} \frac{\hat{\sigma}_{ab}(x_a, x_b)}{\left(1 + \mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)\right)}$ non-perturbative effects Hard scattering

(*Perturbative quantum field theory*)

(Fragmentation, hadronisation)

 $\pm 1.2 \text{ GeV}/13 \text{ TeV}$









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VV N3LO corrections to neutral and charged current at the LHC



- ► Assemble each $\hat{\sigma}_{ab}(x_a, x_b)$ at N3LO
 - Integration of QCD radiation with unitarity cuts
 - > Standard treatment of multi-loop calculations except elliptic integrals with $\tau = m^2/\hat{s}$ where $\hat{s} = x_a x_b s$

Application of ggF Higgs production

- Remarkable precision of the first N3LO XS (Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger 15 to 18)
- ► Available in public code iHixs 2 (Dulat, Lazopoulos, Mistlberger 18)
- ► Further application to bbF Higgs (Dulat, Lazopoulos, Mistlberger 19)

► VBF to Higgs and HH using DIS structure function (Dreyer, Karlberg 17 19)

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-0.2

MSTW08 68cl





> Use threshold expansion at different region of τ and truncate at sufficiently high orders. (Mistlberger 18) ► Use generalised power series ansatz to test the approximation and match coeff. of overlapping regions.

Not exact analytical solution of elliptic integrals but numerically precise enough for phenomenology







Differential N3LO accuracy

- Projection to Born
 - ► Jet production in DIS (NNLOJET) Currie, Gehrmann, Glover, Huss, Niehues `18
 - ► Higgs decay to bb (MCFM) Mondini, Schiavi, Williams `19
- ► Higgs production via ggF (RapidiX+NNLOJET) XC, Gehrmann, Glover, Huss, Mistlberger, Pelloni `21 $d\sigma_{N^{k}LO}^{F} = \mathcal{H}_{N^{k}LO}^{F} \otimes d\sigma_{LO}^{F} \Big|_{\delta(\tau)} + \left[d\sigma_{N^{k-1}LO}^{F+jet} - d\sigma_{N^{k}LO}^{F CT} \right]_{\tau > \tau_{cut}} + \mathcal{O}(\tau_{cut}^{2}/Q^{2})$ ►qT slicing ► Higgs production via ggF (HN3LO+NNLOJET) Cieri, XC, Gehrmann, Glover, Huss `18 ► Higgs pair production via ggF (with modified iHixs2) Chen, Li, Shuo, Wang `19 ► Drell-Yan production (NNLOJET) XC, Gehrmann, Glover, Huss, Yang, Zhu `21

► Combined with resummation (N3LL at small qT)

Drell-Yan production (DYTurbo) Camarda, Cieri, Ferrera `21 (RadISH+NNLOJET) XC, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli 22

► Higgs production via ggF (SCETlib) Billis, Dehnadi, Ebert, Michel, Tackmann `21 Xuan Chen (KIT)





- N3LO corrections to neutral and charged current at the LHC





- Differential N3LO predictions for neutral current production
 - Fully differential N3LO Drell-Yan production (XC, Gehrmann, Glover, Huss, Yang, Zhu `21)
 - > Apply qt-slicing at N3LO with SCET factorisation and expand to N3LO:

$$egin{split} rac{d^3\sigma}{dQ^2d^2ec{q}_Tdy} &= \int rac{d^2b_{\perp}}{(2\pi)^2} e^{-iq_{\perp}\cdot b_{\perp}} \sum_q \sigma_{
m LO}^{\gamma^*} H_{qar{q}} iggl[\sum_k \int_{x_1}^1 rac{dz_1}{z_1} \mathcal{I}_{qk}\left(z_1, b_T^2, \mu
ight) f_{k/h_1}(x_1/z_1, \mu) \ & imes \sum_j \int_{x_2}^1 rac{dz_2}{x_2} \mathcal{I}_{ar{q}j}\left(z_2, b_T^2, \mu
ight) f_{j/h_2}(x_2/z_2, \mu) \mathcal{S}\left(b_{\perp}, \mu
ight) + (q \leftrightarrow ar{q}) iggr] + \mathcal{O}\left(rac{q_T^2}{Q^2}
ight) \end{split}$$

> All factorised functions are recently known up to N3LO: 1) 3-loop hard function $H_{a\bar{a}}^{(3)}$ (Gehrmann, Glover, Huber, Ikizlerli, Studerus `10) 2) Transverse-momentum-dependent (TMD) soft function $S(b_{\perp}, \mu)$ at α_s^3 (Li, Zhu `16) 3) Matching kernel of TMD beam function I_{qk} at α_s^3 (Luo, Yang, Zhu, Zhu `19, Ebert, Mistlberger, Vita `20) > Apply qt cut to factorise N3LO contribution into two parts: $\mathrm{d}\sigma_{N^{3}LO}^{\gamma^{*}} = \left[\mathscr{H}^{\gamma^{*}} \otimes \mathrm{d}\sigma^{\gamma^{*}} \right]_{N^{3}LO} \Big|_{\delta(I)}$

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$$+ \left[\mathrm{d}\sigma_{NNLO}^{\gamma^* + jet} - \mathrm{d}\sigma_{N^3LO}^{\gamma^* CT} \right]_{p_{T,\gamma^*} > q_T^{cut}} + \mathcal{O}((q_T^{cut}/Q)^2)$$





>Differential N3LO predictions for neutral and charged current production $O(\alpha \alpha_s^3)$

- ► Computational setup for $pp \rightarrow \gamma^* \rightarrow l^+l^-$ (identical setup in the inclusive calculation by Durh, Dulat and Mistlberger in Phys. Rev. Lett. 125 (2020) 17, 172001)
 - Fix Q value for γ^* at 100 GeV (NNLO and N3LO scale variations deviate)
 - Use central value of PDF4LHC15_nnlo_mc as benchmark input
 - $\mu_R = \mu_F = 100 \text{ GeV}$ for central QCD scale and use 7-point variations for uncertainty estimation
 - > Apply p_{T,γ^*} > 0.25 GeV constrain for NNLO γ^* + *Jet* without jet definition
- > Computational setup for $pp \to W^{\pm} \to l^{\pm}\nu$
 - ► Dynamic QCD scale $\mu_R = \mu_F = m_{l\nu}$ with 7 variations and $m_{l\nu} \in [0, +\infty]$
 - ► Use NNPDF31_nnlo PDFs and $p_{T,l\nu} > 0.5$ GeV
- Common setup
 - ► Consider LO decay with $m_e = m_\mu = 0$
 - ► $\alpha_s(m_Z) = 0.118$, G_{μ} EW-scheme with fixed α value

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► Differential N3LO predictions for neutral and charged current production



XC, Gehrmann, Glover, Huss, Yang, Zhu Phys.Rev.Lett. 128 (2022) 5 XC, Gehrmann, Glover, Huss, Yang, Zhu in preparation *Xuan Chen (KIT)* N3LO corrections to neutral and charged current at the LHC



► Differential N3LO predictions for neutral current production

 $d\sigma_{FO}^{\gamma^*}/d|y_{\gamma^*}|$



XC, Gehrmann, Glover, Huss, Yang, Zhu in preparation XC, Gehrmann, Glover, Huss, Yang, Zhu Phys.Rev.Lett. 128 (2022) 5 N3LO corrections to neutral and charged current at the LHC *Xuan Chen (KIT)*



 $d\sigma_{FO}^{W^-}/d|y_{W^-}|$





Differential N3LO predictions for charged current production

 $m_T = (E_T^l + E_T^\nu)^2 - (\overrightarrow{p}_T^l)^2$

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$$+\overrightarrow{p}_{T}^{\nu})^{2} = \sqrt{2E_{T}^{l}E_{T}^{\nu}(1-\cos\phi)}$$

Breit-Wigner form (running decay width):

$$s^2 - m_W^2 + is^2 \Gamma_W / m_W$$

(MeV)	Wmass	Wwidth
PDG (2020)	80379 ± 12	2085 ± 42
CDFII	80433 ± 9	2089.5 ± 0.6
L3	80270 ± 55	2180 ± 14





Differential N3LO predictions for charged current production

$$m_T = (E_T^l + E_T^{\nu})^2 - (\overrightarrow{p}_T^l + \overrightarrow{p}_T^{\nu})^2 = \sqrt{2E_T^l E_T^{\nu} (1 - \cos\phi)}$$



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ATLAS `17

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Differential N3LO predictions for charged current production

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 $_{T} + \overrightarrow{p}_{T}^{\nu})^{2} = \sqrt{2E_{T}^{l}E_{T}^{\nu}(1 - \cos\phi)}$







- Differential N3LO predictions for neutral current production with fiducial cuts
 - Apply ATLAS fiducial cuts at 13 TeV
 - > Dynamical scale $\mu_F = \mu_R = \sqrt{m_{ll}^2 + p_T^{ll^2}}$

► $m_{ll} \in [66, 116] \text{ GeV}, |\eta^{l^{\pm}}| < 2.5$

- Symmetric cuts: $|p_T^{l^{\pm}}| > 27 \text{ GeV}$ Introduce power correction at $O(q_T^{cut}/m_{ll})$
- ► Solution:

> Apply Lorentz Boost below q_T^{cut} Buonocore, Rottoli, Kallweit, Wiesemann 21 Camarda, Cieri, Ferrera 21 ► Product cuts: $\sqrt{p_T^{l^+} p_T^{l^-}} > 27 \text{ GeV}$

Salam, Slade `21 $\min\{p_T^{l^+}, p_T^{l^-}\} > 20 \text{ GeV}$ > Typical fiducial cuts for m_T^V , p_T^V in DY production > Large log terms appear in $p_T^l \sim m_V/2$, $m_T^V \sim 2 \times \min[p_T^l]$, $p_T^V \sim 0$ N3LO corrections to neutral and charged current at the LHC *Xuan Chen (KIT)*

[dd]

Δσ





 $p_T^{\rm cut}$ [GeV] XC, Gehrmann, Glover, Huss, Monni, Rottoli, Re, Torrielli 22



► Differential N3LO predictions for neutral current production with fiducial cuts

Resum all order contributions at N3LL using RadISH and matched to N3LO



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CONCLUSION AND OUTLOOK

- > Precision phenomenology could be the key to reveal new physics principles. > For theory predictions of LHC observables, there has been rapid progress in perturbative QCD calculations at NNLO and N3LO accuracy.
- Differential N3LO precision is now available for neutral and charged current production at the LHC.
- > Our standard methodology to estimate theoretical uncertainties via scale variation is challenged at N3LO.
- Resumed N3LO+N3LL predictions are essential to compare with data and help to stabilise the convergence of scale variations.
- > EW, QCD-EW corrections are not included in this talk but equivalent important at the level of accuracy. How to combine different source of corrections/uncertainties is the key to make accurate interpretation of experiment data.

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- or Your Attention
- N3LO corrections to neutral and charged current at the LHC



BACKUP SLIDES

- production with fiducial cuts
 - and matched to N3LO



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BACKUP SLIDES

Differential N3LL +NNLO predictions for charged current production with fiducial cuts

- ► Precise W measurement with calibration against Z.
- Improved QCD uncertainties through out pT.
- ► Different EW and QCD-EW correction between Z and W are not yet considered.

Bizon, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Walker `19

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N3LO corrections to neutral and charged current at the LHC



