Status of Resummation and Electroweak Corrections to Drell-Yan Production

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25th Mini-workshop on the frontier of LHC
W mass workshop: Uncertainties and Opportunities
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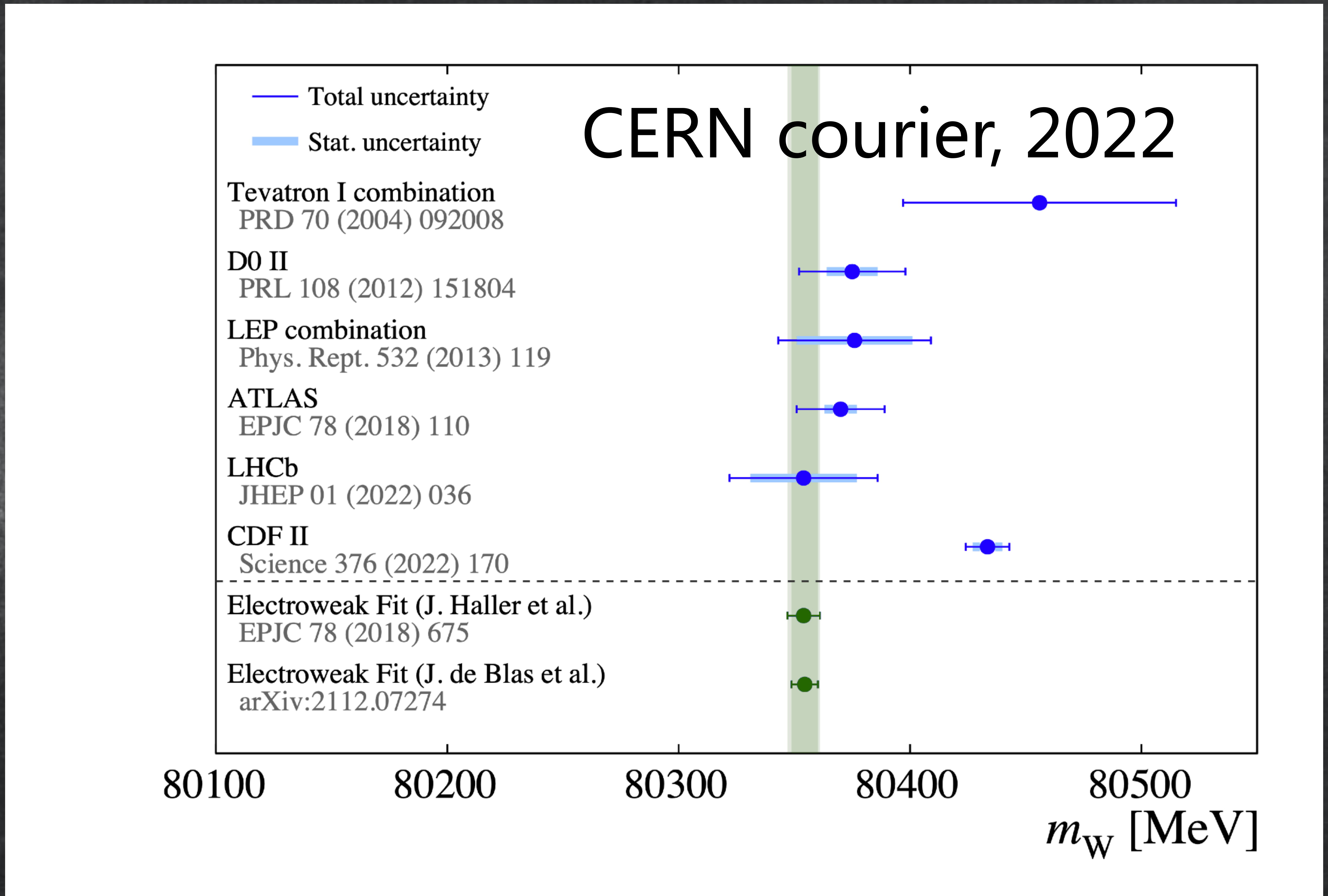
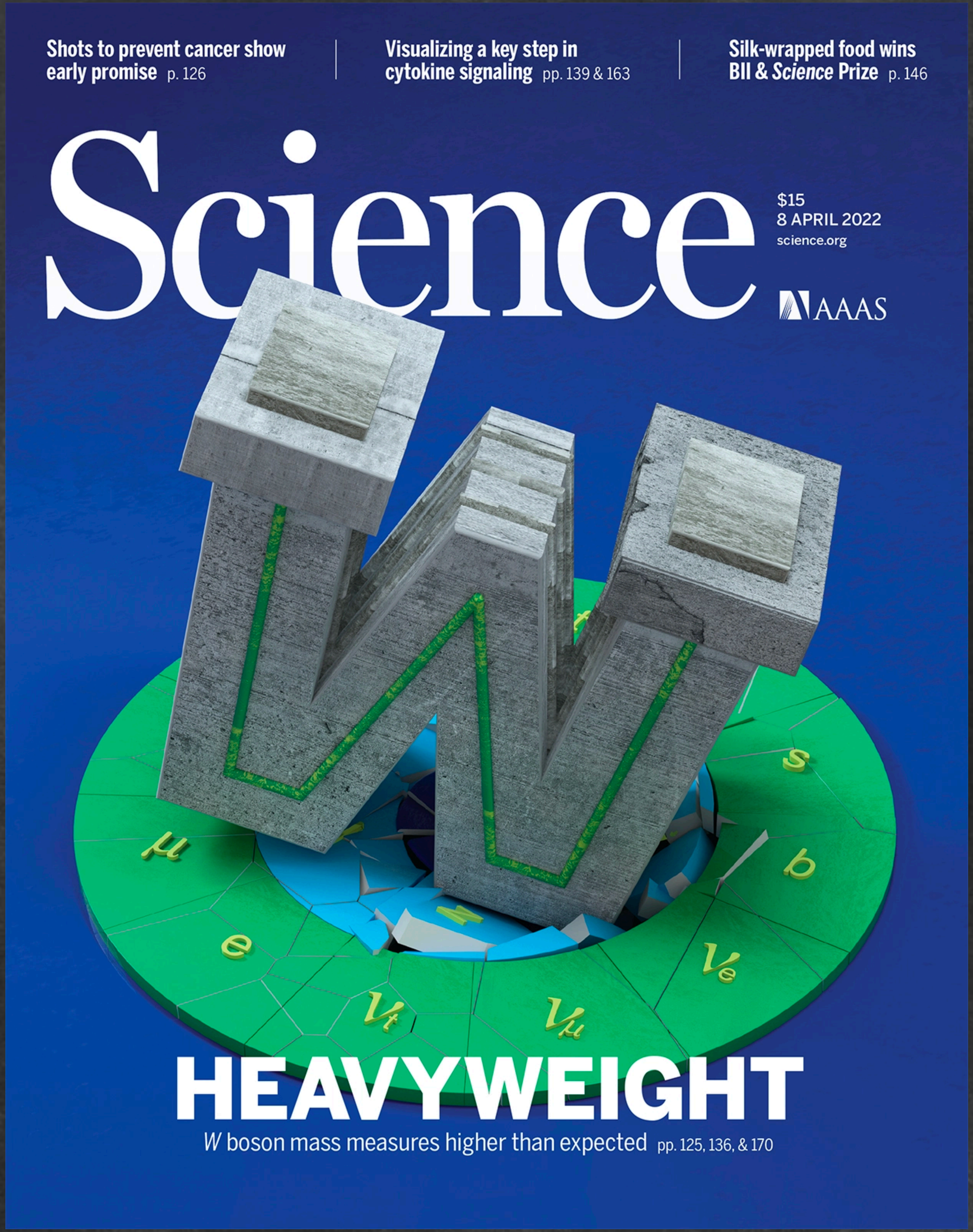
Outline of this talk

- Role of precision theory modeling in W mass measurement
- Overview of status qT resummation and uncertainties
  - Brief review of qT resummation
  - Accuracy frontier: current status
  - Impact to mW measurement at hadron colliders
- Status of EW corrections and uncertainties


CERN courier, 2022
Total uncertainty
Stat. uncertainty
Tevatron I combination
PRD 70 (2004) 092008
D0 II
PRL 108 (2012) 151804
LEP combination
Phys. Rept. 532 (2013) 119
ATLAS
EPJC 78 (2018) 110
LHCb
JHEP 01 (2022) 036
CDF II
Science 376 (2022) 170
Electroweak Fit (J. Haller et al.)
EPJC 78 (2018) 675
Electroweak Fit (J. de Blas et al.)
arXiv:2112.07274
80100
80200
80300
80400
80500
m_W [MeV]


Shots to prevent cancer show early promise p. 126
Visualizing a key step in cytokine signaling pp. 139 & 163
Silk-wrapped food wins BII & Science Prize p. 146
Science
$15
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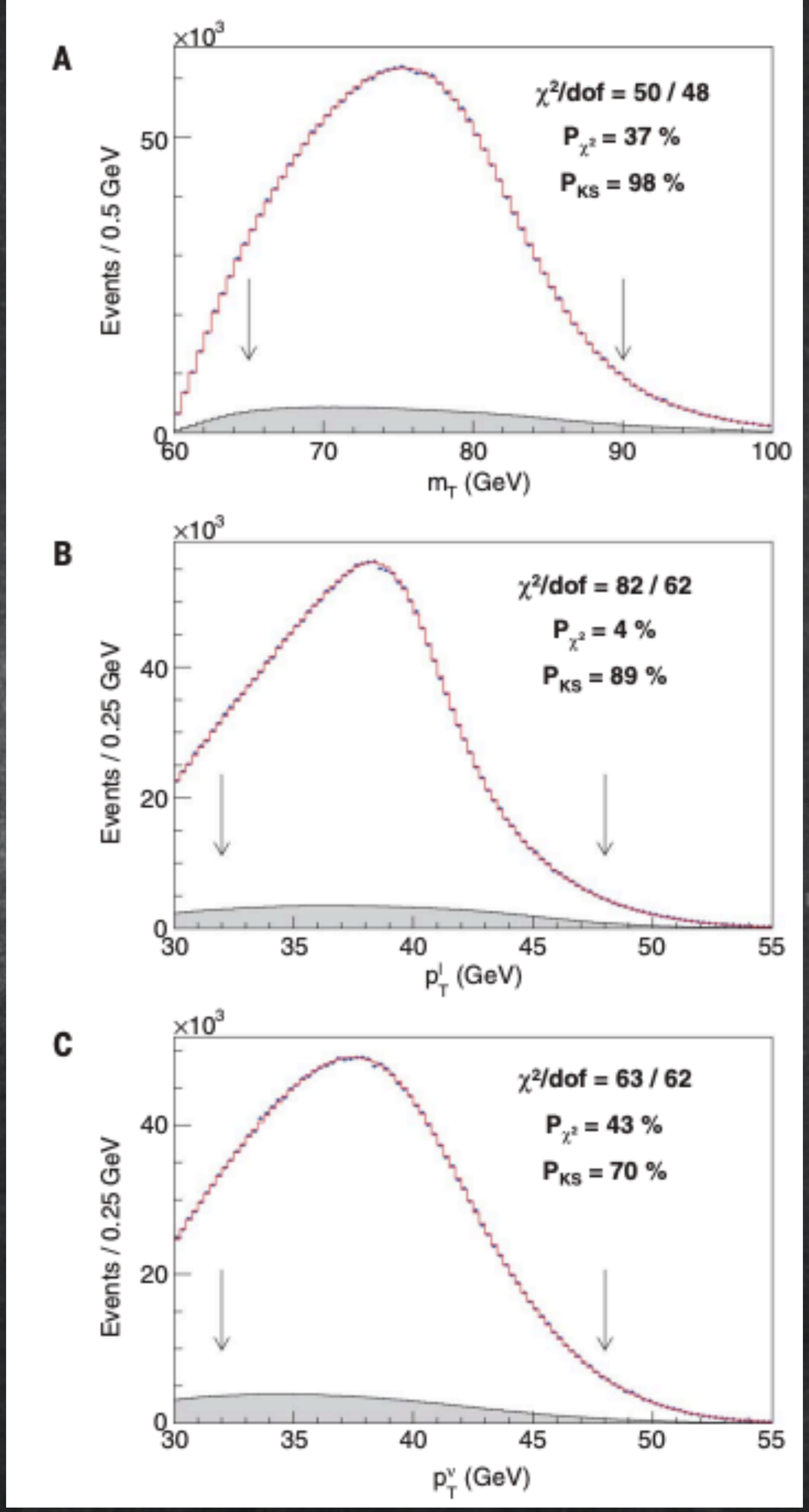
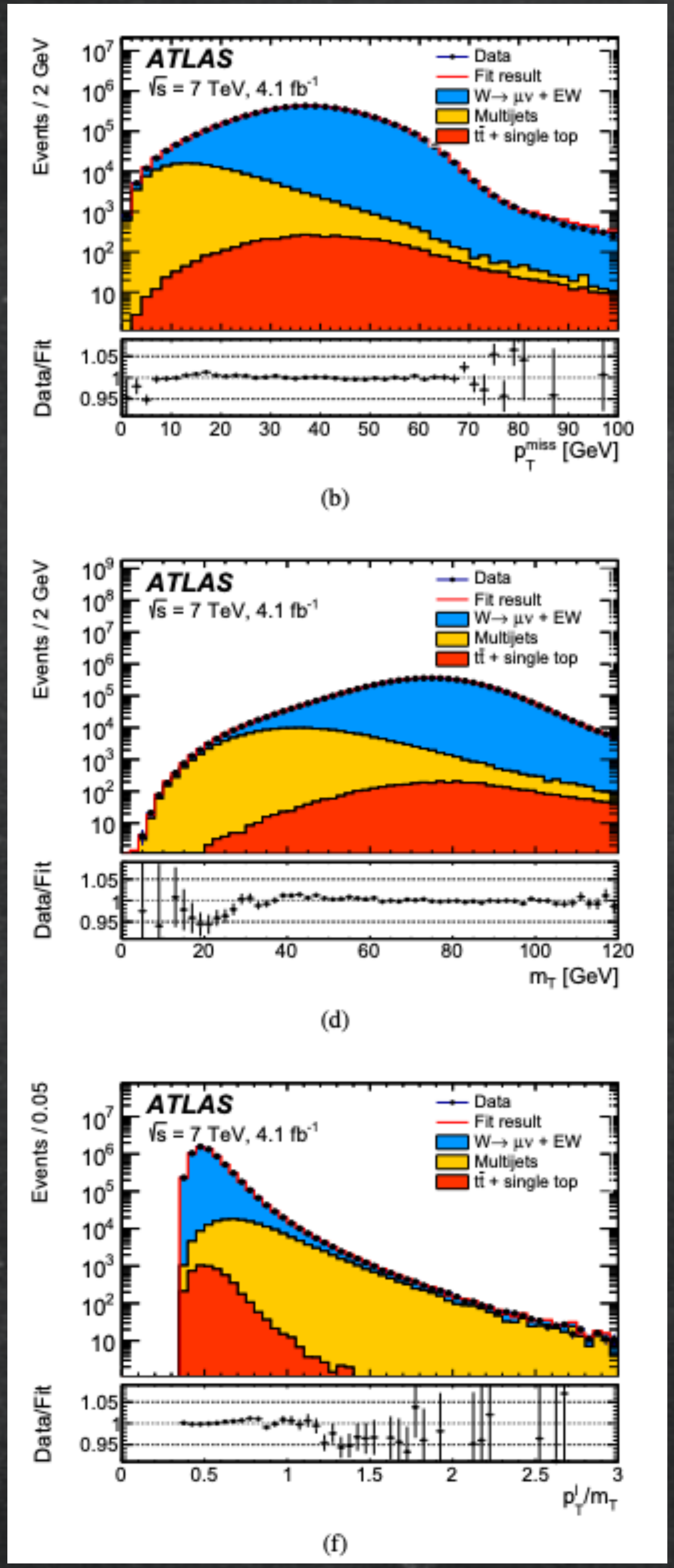
science.org
AAAS
HEAVYWEIGHT
W boson mass measures higher than expected pp. 125, 136, & 170

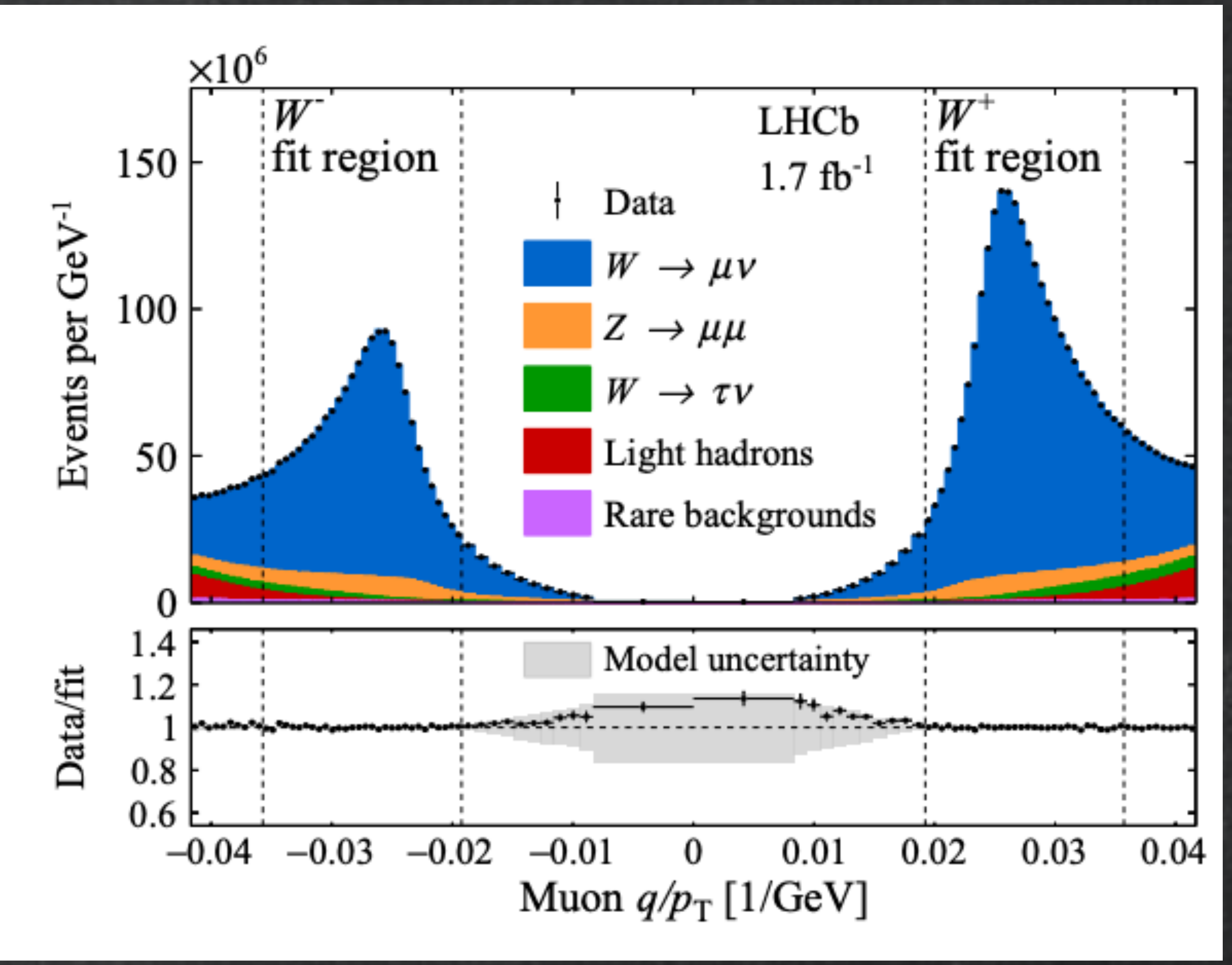
Experimental determination of mW at hadron collider

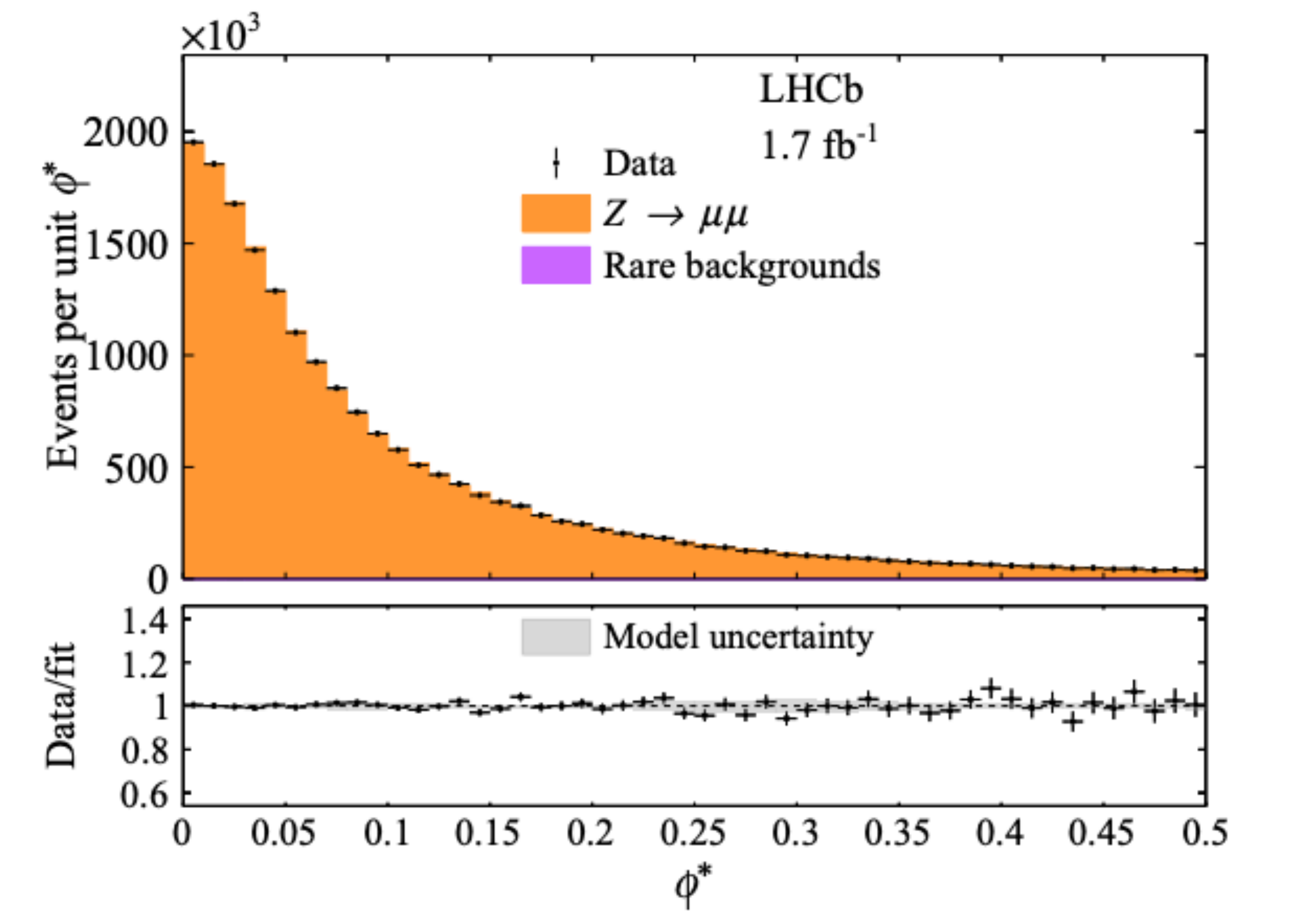


CDF II, science, 2022



ATLAS, EPJC, 2017





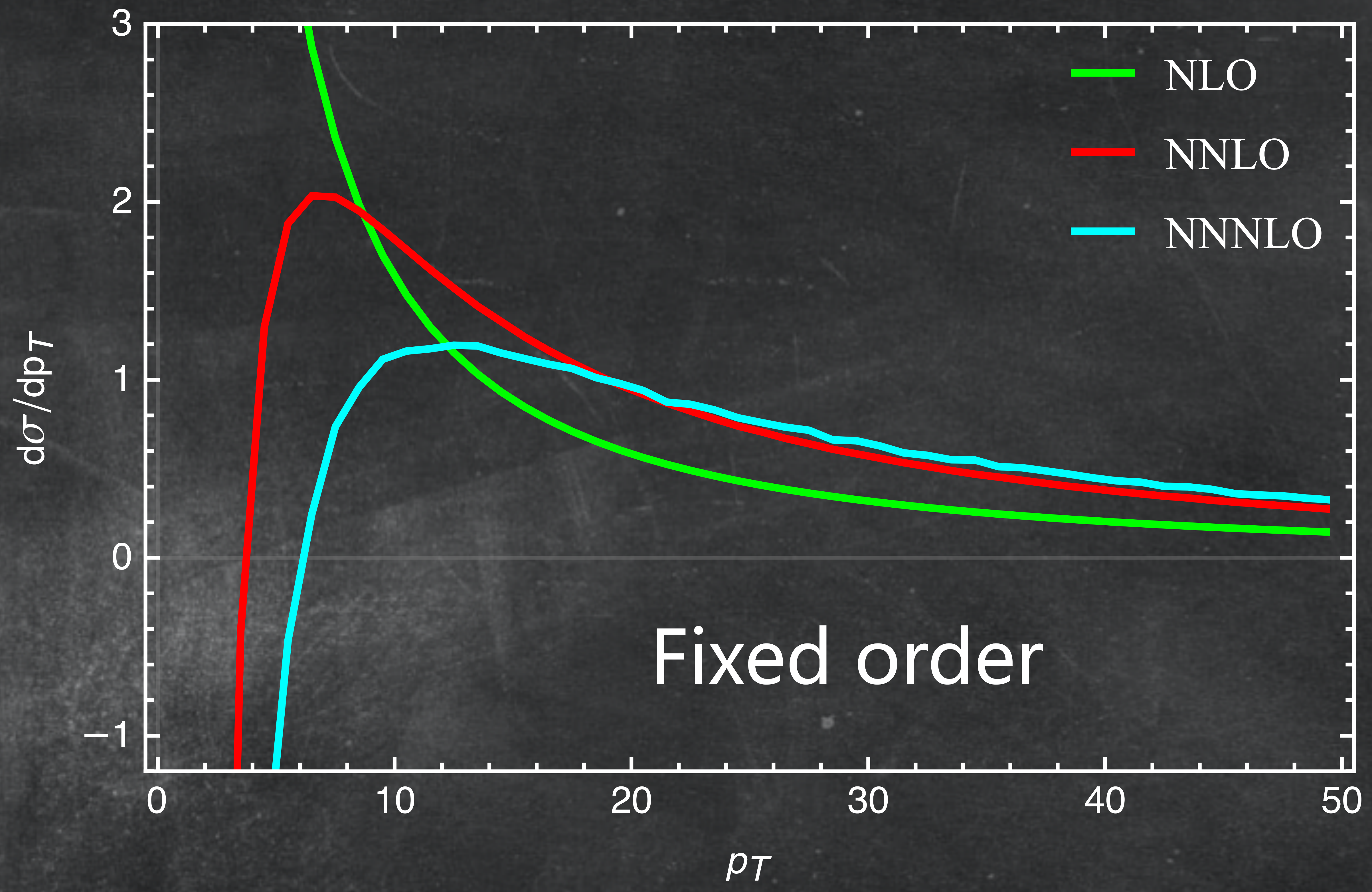
LHCb, JHEP, 2020

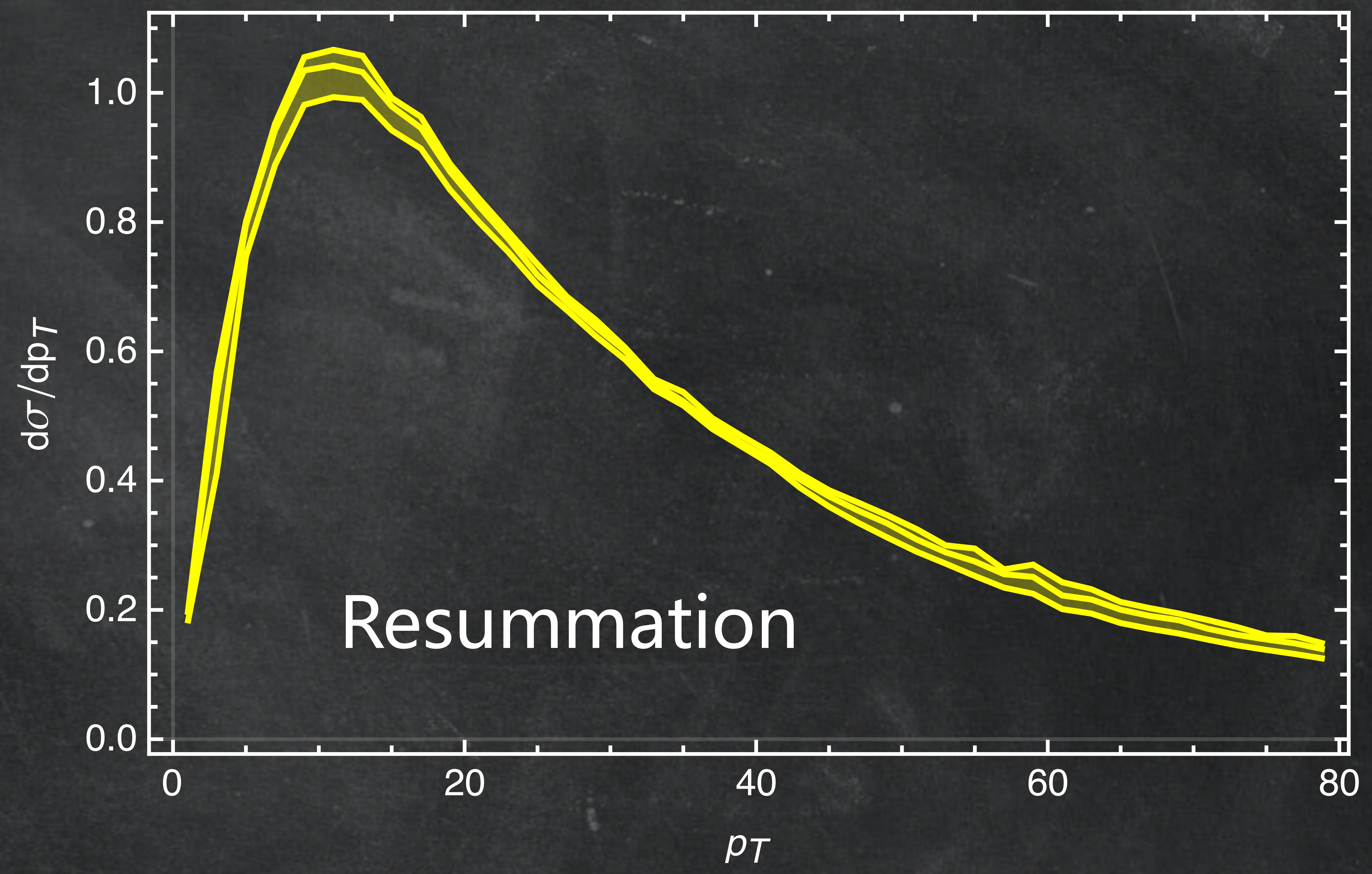
Source	Uncertainty (MeV)
Lepton energy scale	3.0
Lepton energy resolution	1.2
Recoil energy scale	1.2
Recoil energy resolution	1.8
Lepton efficiency	0.4
Lepton removal	1.2
Backgrounds	3.3
p_T^Z model	1.8
p_T^W/p_T^Z model	1.3
Parton distributions	3.9
QED radiation	2.7
W boson statistics	6.4
Total	9.4

Source	Size [MeV]
Parton distribution functions	9
Theory (excl. PDFs) total	17
Transverse momentum model	11
Angular coefficients	10
QED FSR model	7
Additional electroweak corrections	5
Experimental total	10
Momentum scale and resolution modelling	7
Muon ID, trigger and tracking efficiency	6
Isolation efficiency	4
QCD background	2
Statistical	23
Total	32

W-boson charge	W^+		W^-		Combined	
Kinematic distribution	p_T^ℓ	m_T	p_T^ℓ	m_T	p_T^ℓ	m_T
δm_W [MeV]						
Fixed-order PDF uncertainty	13.1	14.9	12.0	14.2	8.0	8.7
AZ tune	3.0	3.4	3.0	3.4	3.0	3.4
Charm-quark mass	1.2	1.5	1.2	1.5	1.2	1.5
Parton shower μ_F with heavy-flavour decorrelation	5.0	6.9	5.0	6.9	5.0	6.9
Parton shower PDF uncertainty	3.6	4.0	2.6	2.4	1.0	1.6
Angular coefficients	5.8	5.3	5.8	5.3	5.8	5.3
Total	15.9	18.1	14.8	17.2	11.6	12.9

The necessity of resummation





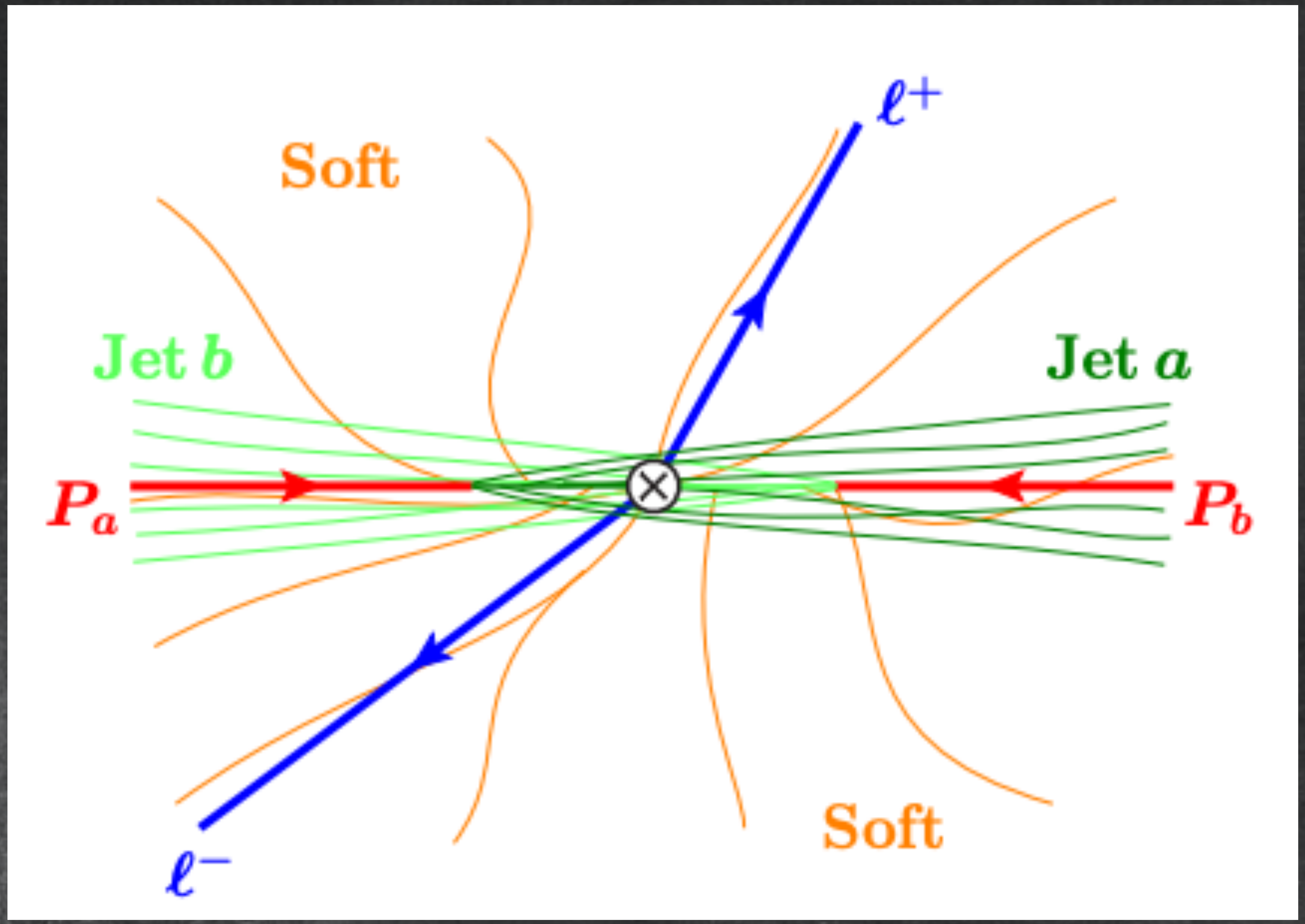
- Fixed order perturbation theory is valid at large pT ~ Q
- At small qT logarithms of the form log(Q/pT) render fixed order unphysical
- Resummation of these large logarithms recover physical prediction

Different formalisms to qT resummation

- Collins-Soper-Sterman formalism [Collins, Soper, Sterman, 1985; ResBos: C.-P. Yuan + Q.H. Cao, Z. Li, P. Sun, B. Yan, F. Yuan, et.al.; ...]
- TMD formalism [X.D. Ji, J.P. Ma, F. Yuan, 2004; ...]
- Soft-Collinear Effective Theory [Stewart et.al., 2001; Y. Gao, C.S. Li, J.J. Liu, 2005; ...]
- Parton Shower-like/Branching [Monni et.al., 2016; ...]

Different formalism are equivalent for the resummed logarithmic terms, with difference suppressed by power of qT.

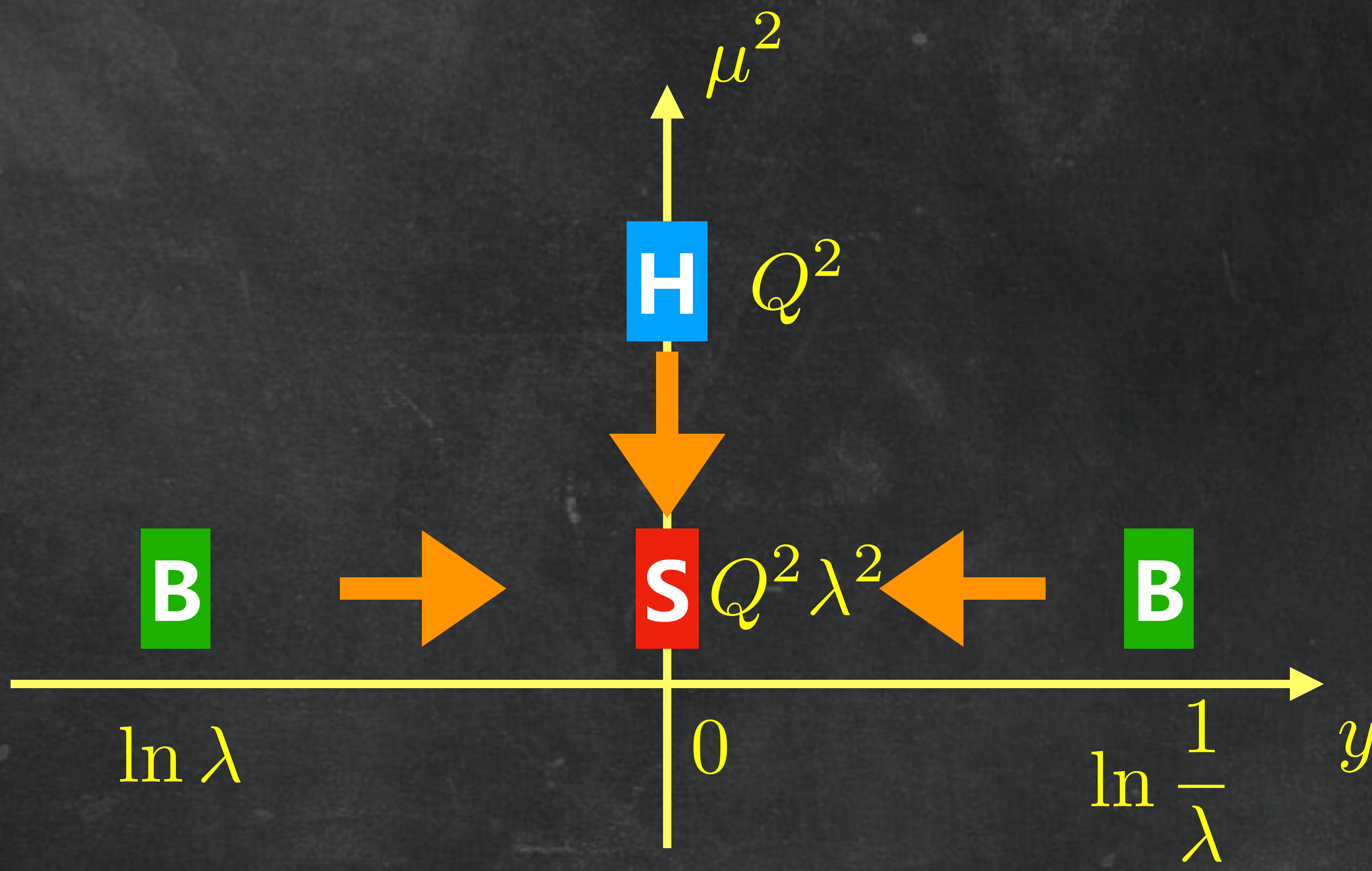
Resummation using renormalization group evolution



$$\frac{d\sigma_{\rm res}}{dQ\,dY\,dq_T} = \sum_q H_{q\bar{q}} q_T \int_0^\infty db\, b J_0(q_T b) \times B_q(x_a, b, \mu, \nu) B_{\bar{q}}(x_b, b, \mu, \nu) S(b, \mu, \nu)$$

$p^\mu = (p^-, p^+, p_\perp)$ $\qquad \lambda \sim \frac{q_T}{Q} \ll 1$

	mode	virtuality	rapidity
Hard	$Q(1,1,1)$	Q^2	0
Collinear	$Q(\lambda^2, 1, \lambda)$	$\lambda^2 Q^2$	$\ln\frac{1}{\lambda}$
anti-coll.	$Q(1, \lambda^2, \lambda)$	$\lambda^2 Q^2$	$\ln\lambda$
Soft	$Q(\lambda, \lambda, \lambda)$	$\lambda^2 Q^2$	0



2D renormalization group flow

Interplay between fixed-order and resummation

$$L = \ln \frac{Q}{p_T}$$

1-loop	L^2	$+$	L^1	$+$	1										
2-loop	L^4	$+$	L^3	$+$	L^2	$+$	L	$+$	1						
3-loop	L^6	$+$	L^5	$+$	L^4	$+$	L^3	$+$	L^2	$+$	L	$+$	1		
4-loop	L^8	$+$	L^7	$+$	L^6	$+$	L^5	$+$	L^4	$+$	L^3	$+$	$\cdots$		
5-loop	L^{10}	$+$	L^9	$+$	L^8	$+$	L^7	$+$	L^6	$+$	L^5	$+$	$\cdots$		

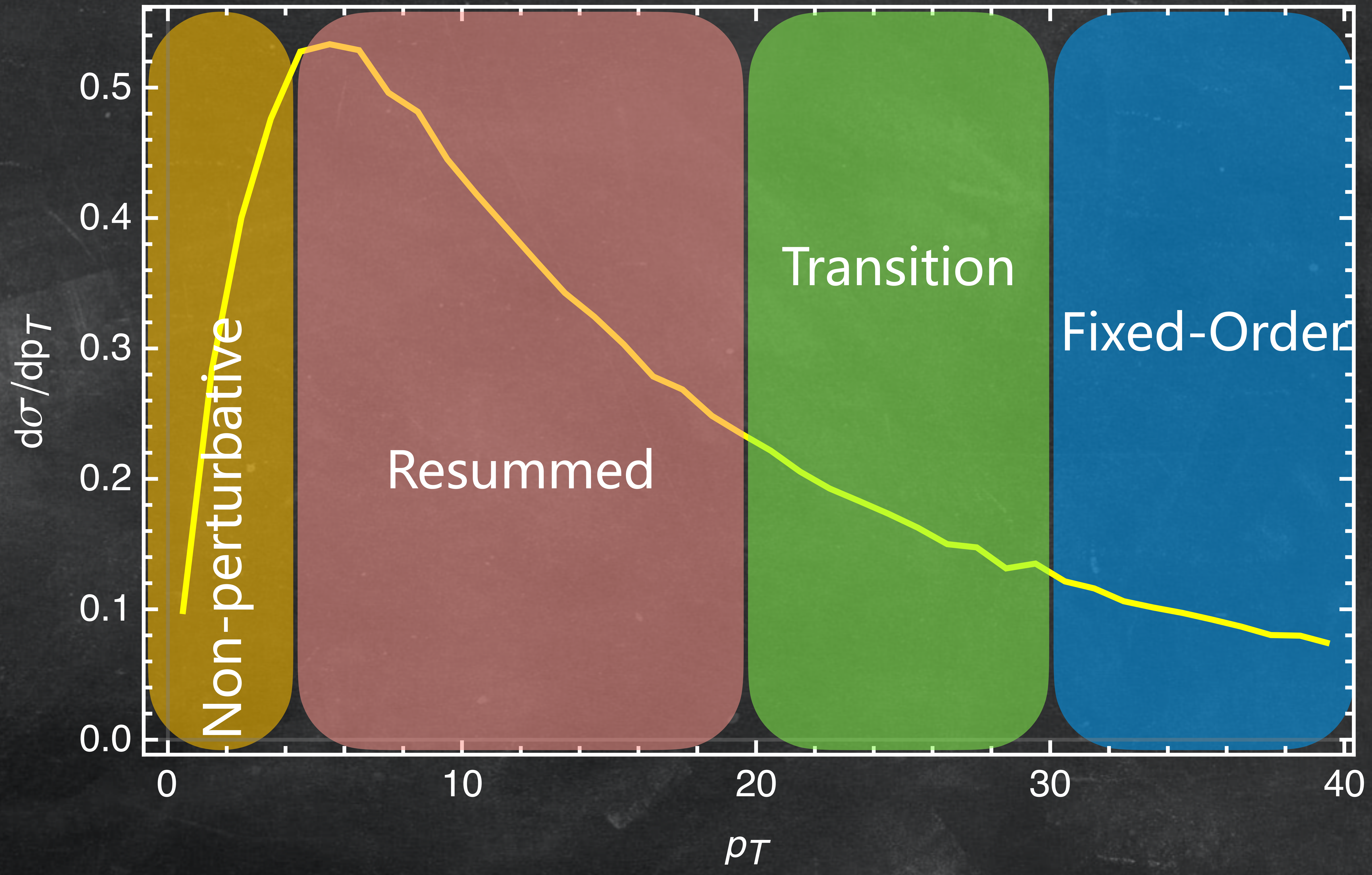
Fixed-order

Resummation

N3LL

- Currently the best available fixed order predictions is at 3-loop, see Xuan's talk
- Status of art on resummation is N3LL, all based on the following ingredients:
  - Two-loop beam function: Gehrmann, Lubbert, L.L. Yang, 2012, PRL
  - Three-loop rapidity anomalous dimension (Collins-Soper kernel): Y. Li, H.X.Z, 2016, PRL

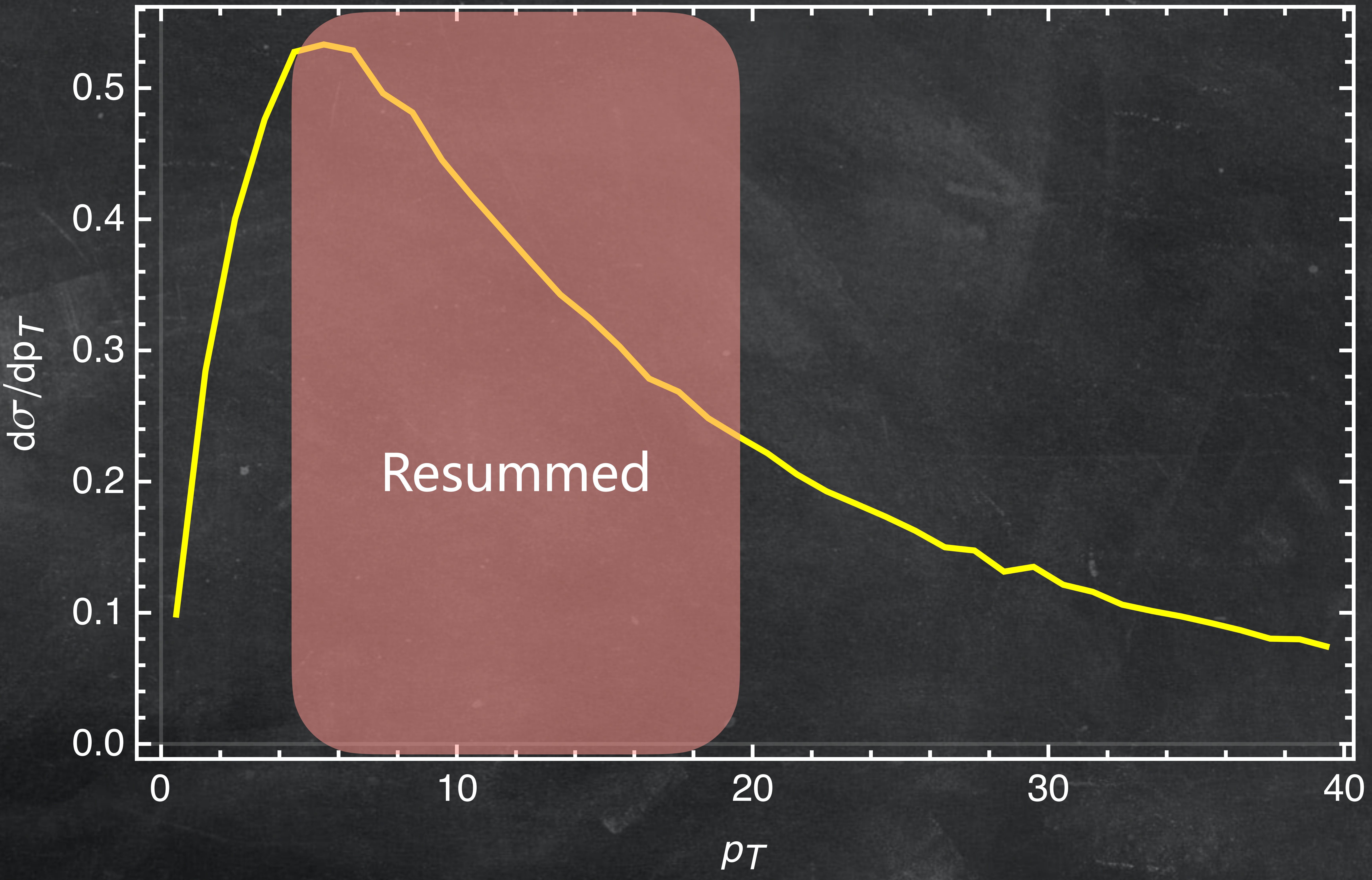
qT distribution as a multi-scale problem



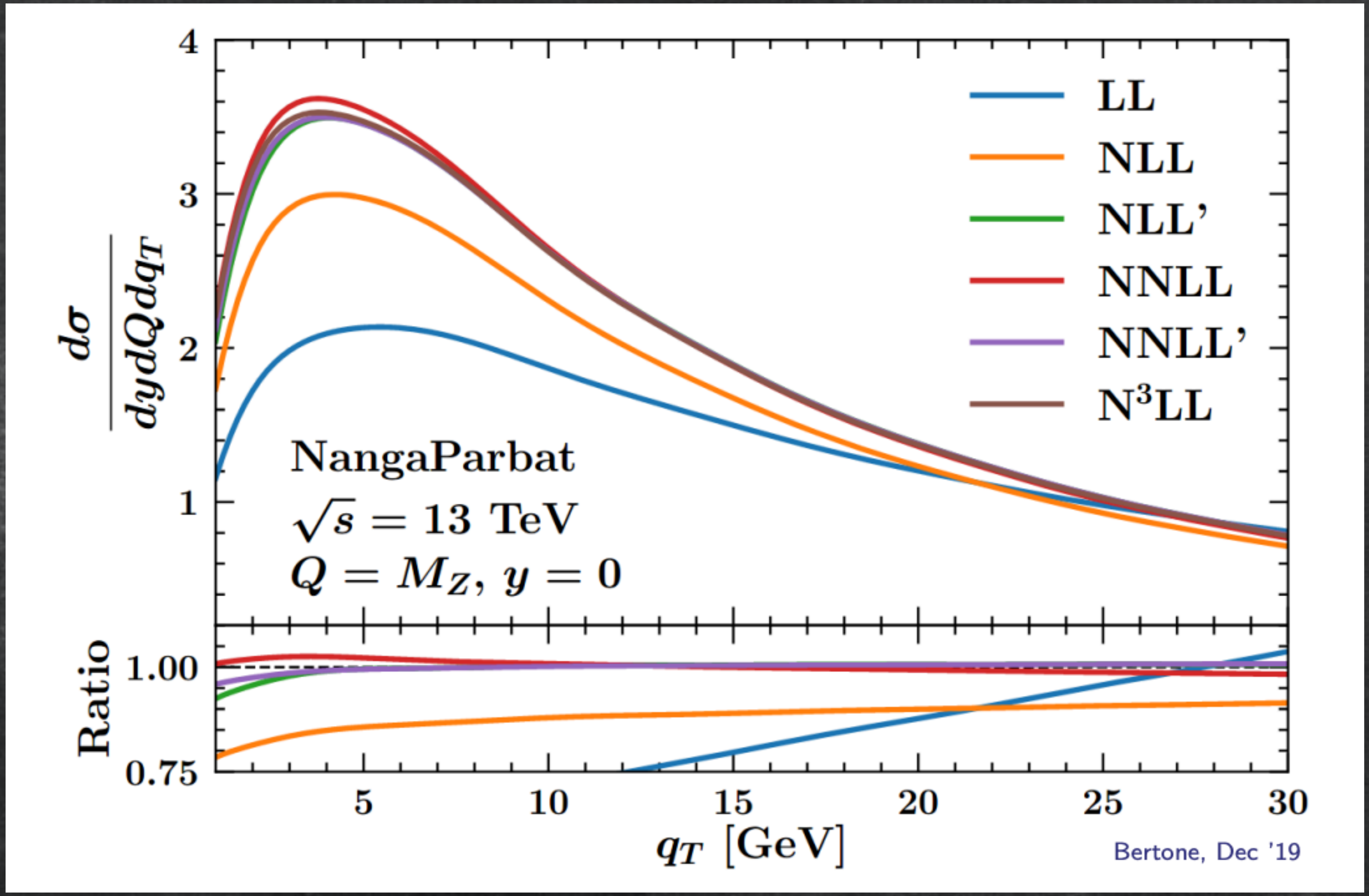
- In the following, I will summarize the best available theory predictions on resummed qT spectrum

- Whenever possible, comparison between predictions using different formalisms will be presented

Resummation region

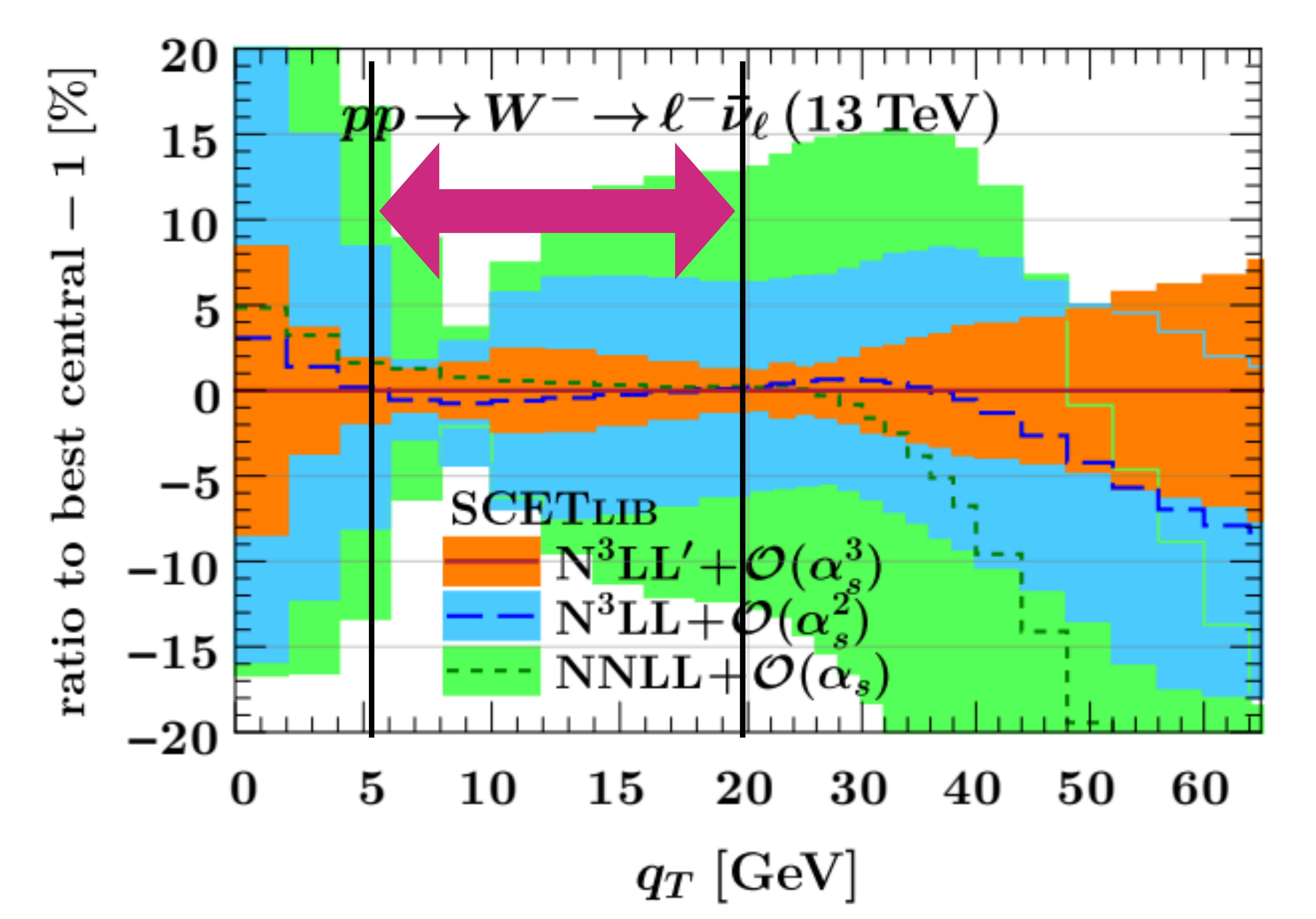


From LL to N3LL: effects of higher order resummation



- Resummation based on TMD formalism
- Pure perturbative resummation, no NP factor
- Large corrections from LL to NLL and NNLL: higher-order resummation necessary
- Good convergence starting from N3LL

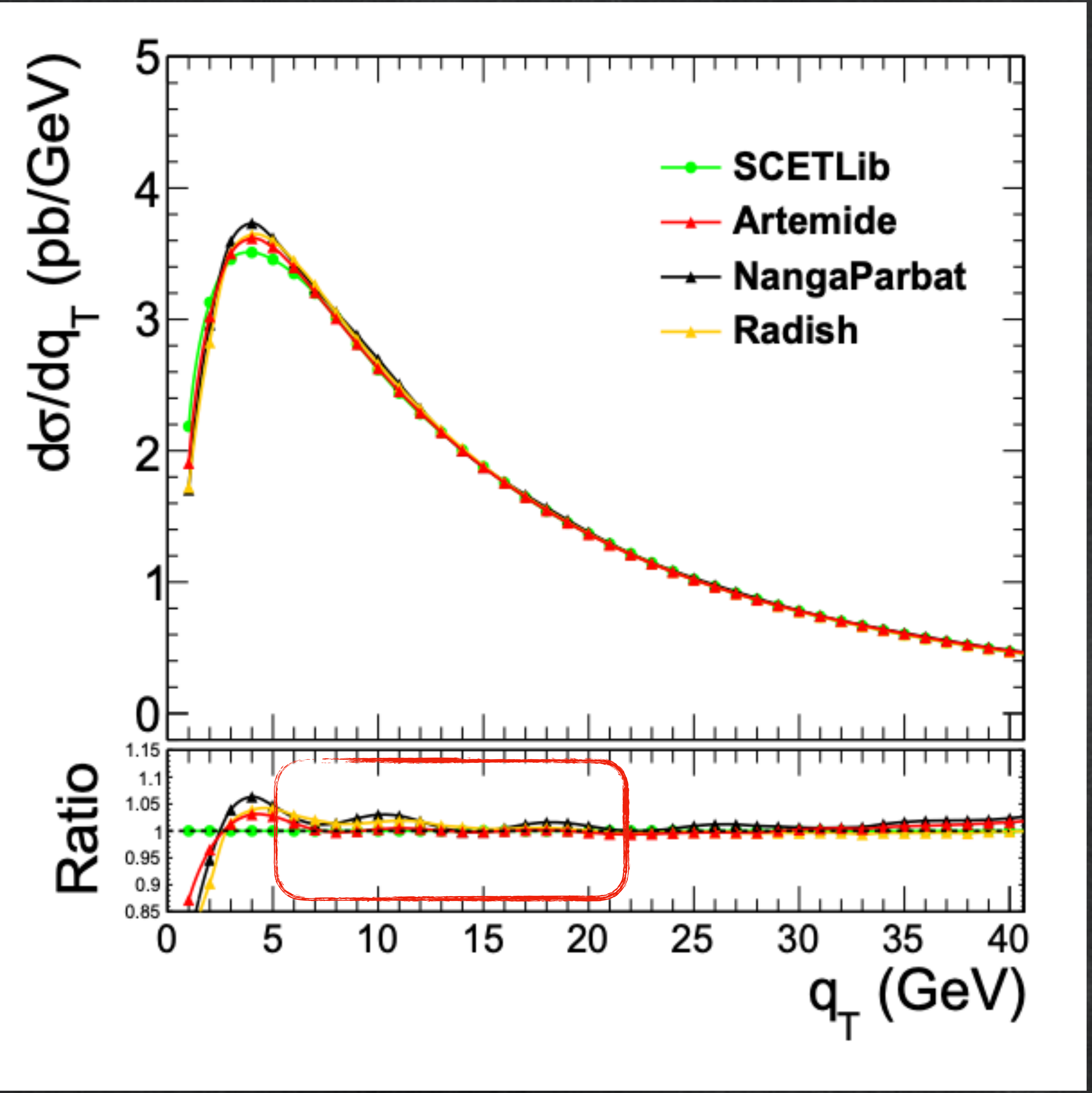
N3LL' resummation from SCET



Billis, Ebert, Michel, Tackmann, SCET 2022

- Resummation based on SCET formalism
- Pure perturbative resummation, no NP factor
- 2-3% relative uncertainties in the resummed region
- Scale uncertainty band overlap between NNLL and N3LL, indicating good perturbative convergence

Difference between resummation scheme



Cridge, 2020, Resummation Sub-group meeting

$$\frac{d\sigma_{\rm res}}{dQdY\,dq_T} = \sum_q H_{q\bar{q}} q_T \int_0^\infty db\, b J_0(q_T b) \times B_q(x_a, b, \mu, \nu) B_{\bar{q}}(x_b, b, \mu, \nu) S(b, \mu, \nu)$$

$$\exp\left[2\int_{\mu_s}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \left(\Gamma_{\rm cusp}[\alpha_s(\bar{\mu})] \ln \frac{b^2\bar{\mu}^2}{b_0^2} - \gamma_s[\alpha_s(\bar{\mu})]\right)\right]$$

- Different resummation formalism use different scheme in implementation, leading to numerical difference
- Mellin space v.s. x space
- Laudau pole prescription
- Global v.s. local b* prescription
- Modified logs: $\log(Q^2b^2)$ -> $\mathrm{Log}(1+Q^2b^2)$
- Additive matching v.s. multiplicative matching
- 5% difference between different codes in the resummed region, larger than perturbative scale uncertainties

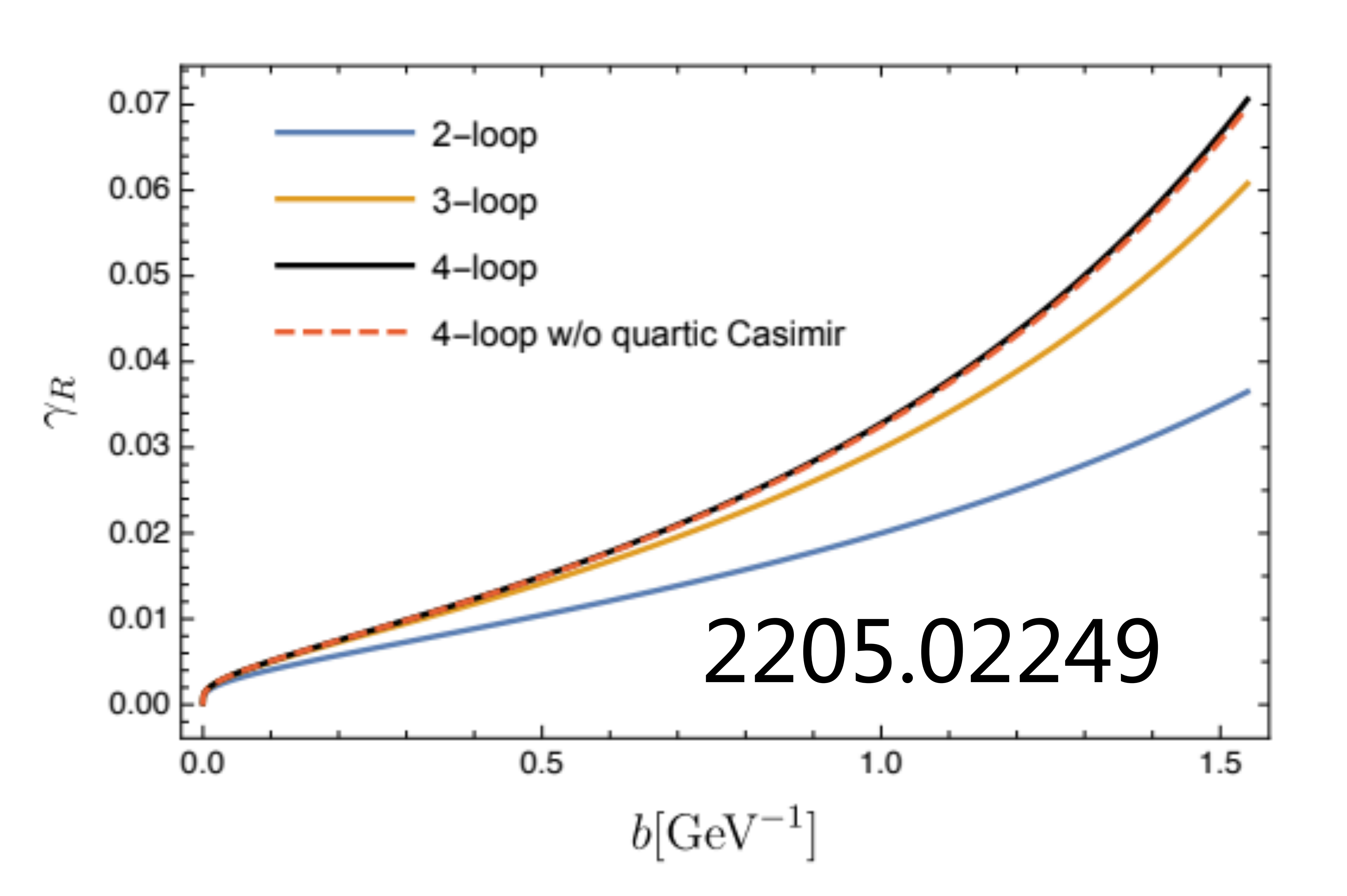
Towards N4LL for qT resummation

- 3-loop soft function: Y.Li, H.X.Z, 2016, PRL
- 3-loop beam function: M.X. Luo, T.Z. Yang, H.X.Z., Y.J. Zhu, 2019, PRL; Ebert, Mistlberger, Vita, 2020, JHEP
- Newly available: 4-loop rapidity anomalous dimension:

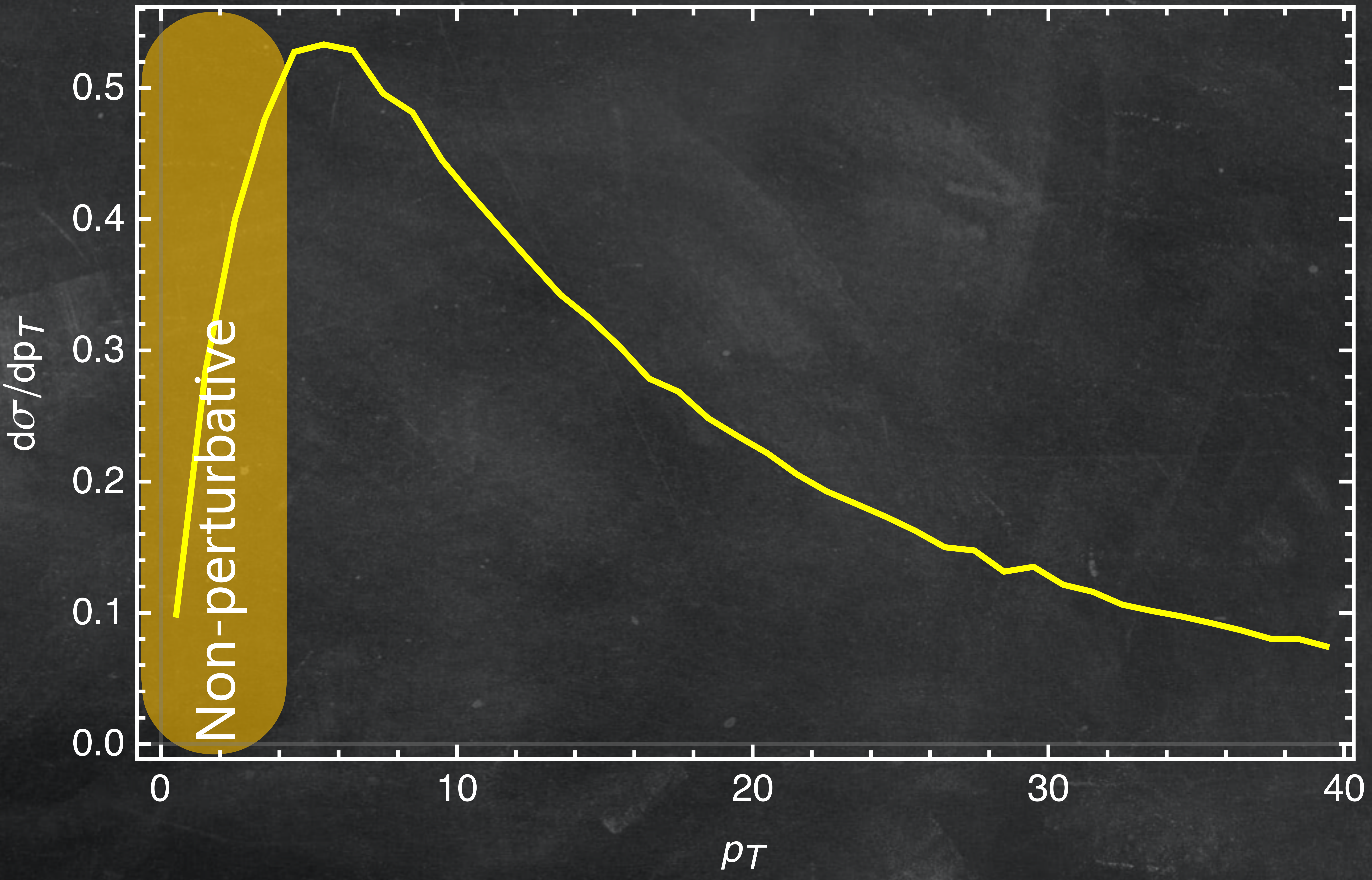
Accuracy	Hard, Beam, Soft function	$\Gamma_{\text{cusp}}(\alpha_s)$	$\gamma_i(\alpha_s)$	$\beta(\alpha_s)$	splitting function
LL	Tree level	1-loop	–	1-loop	–
NLL	Tree level	2-loop	1-loop	2-loop	1-loop
NNLL	1-loop	3-loop	2-loop	3-loop	2-loop
N^3LL	2-loop	4-loop	3-loop	4-loop	3-loop
N^4LL	3-loop	5-loop	4-loop	5-loop	4-loop ?

[4] arXiv:2205.02242 [pdf, other]
The Four-Loop Rapidity Anomalous Dimension and Event Shapes to Fourth Logarithmic Order
Claude Duhr, Bernhard Mistlberger, Gherardo Vita
Comments: 5 pages, 3 figures, 2 ancillary files

[5] arXiv:2205.02249 [pdf, other]
The Four Loop QCD Rapidity Anomalous Dimension
Ian Moult, Hua Xing Zhu, Yu Jiao Zhu
Comments: 16 pages, 2 figures + Appendices

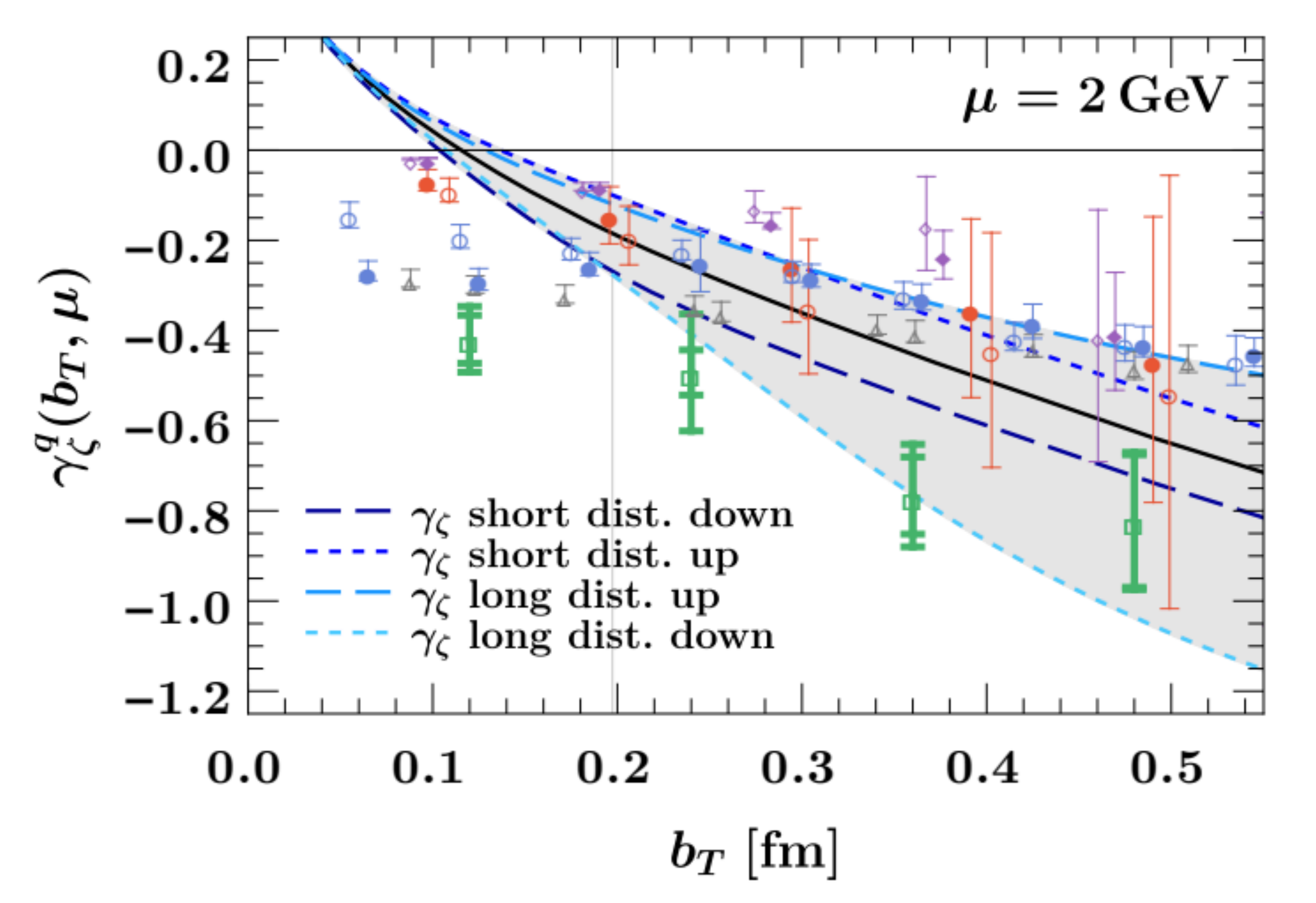


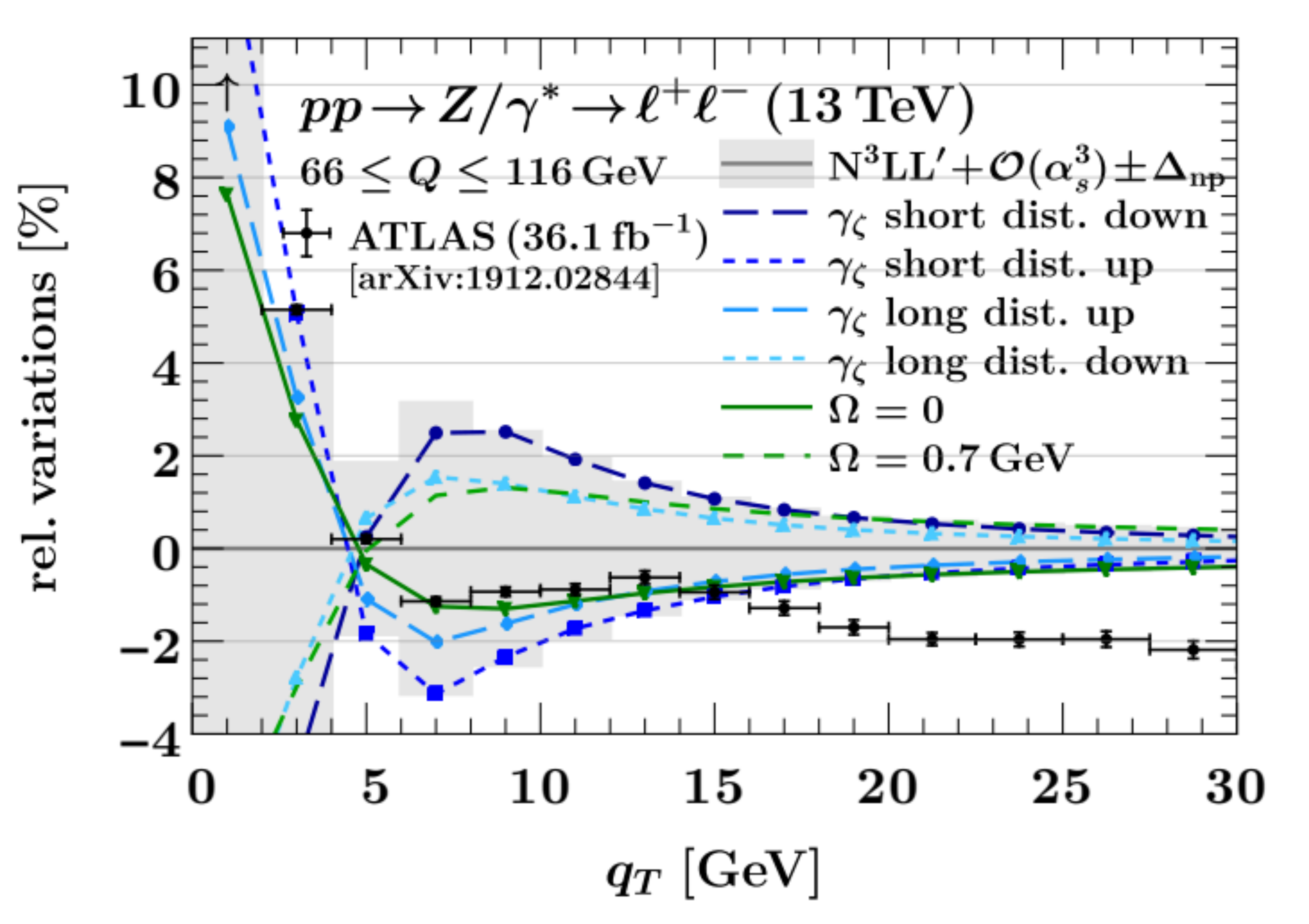
Non-perturbative region



Analytic non-perturbative parameterization

- Rapidity anomalous dimension using input from Lattice data (Shanahan, Wagman, Y. Zhao, 2021):

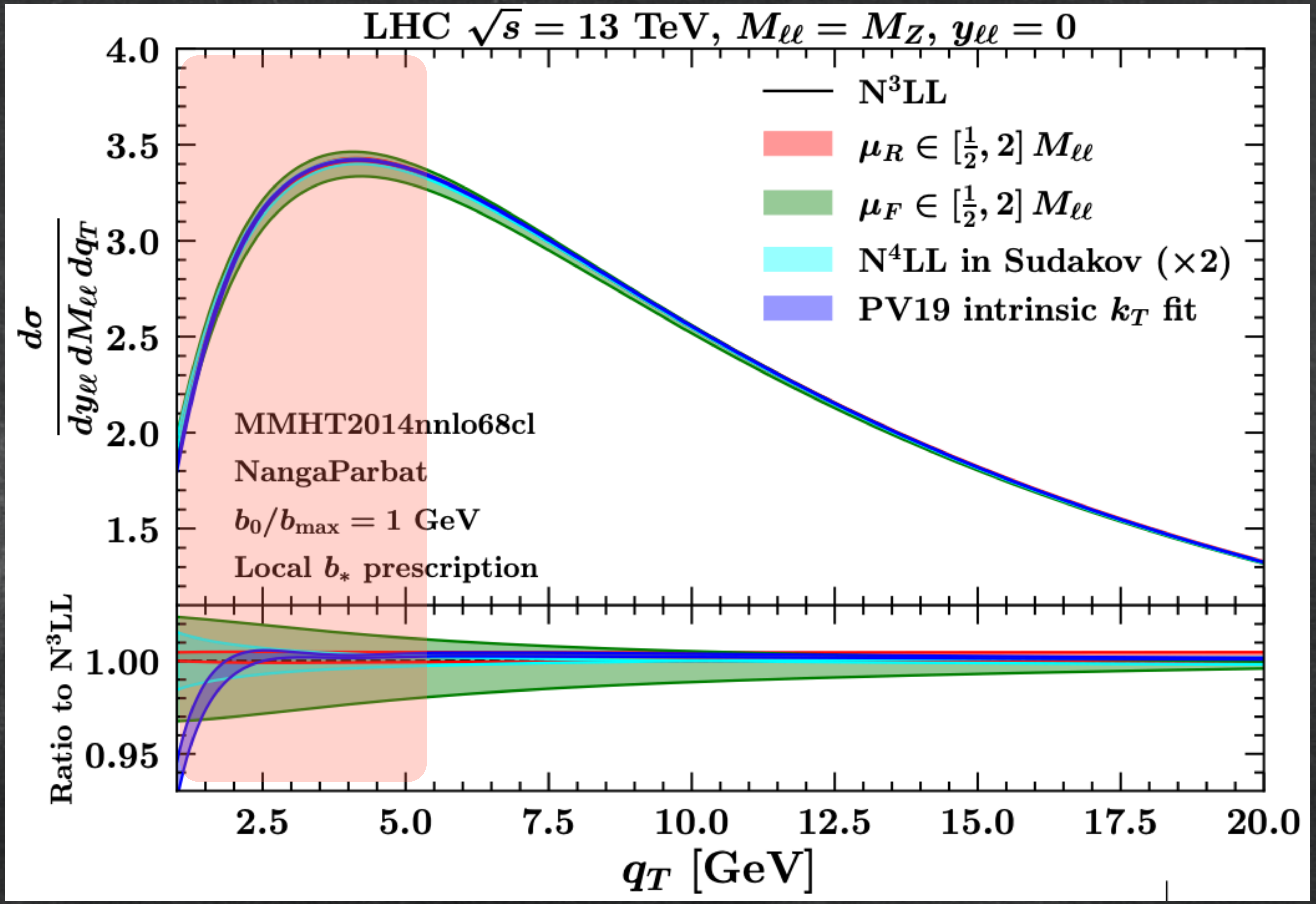




First principle, but so far large uncertainty

- $f_{NP} = \exp(-\Omega^2 b^2)$ for TMD PDFs

Non-perturbative parameterization from fitting data



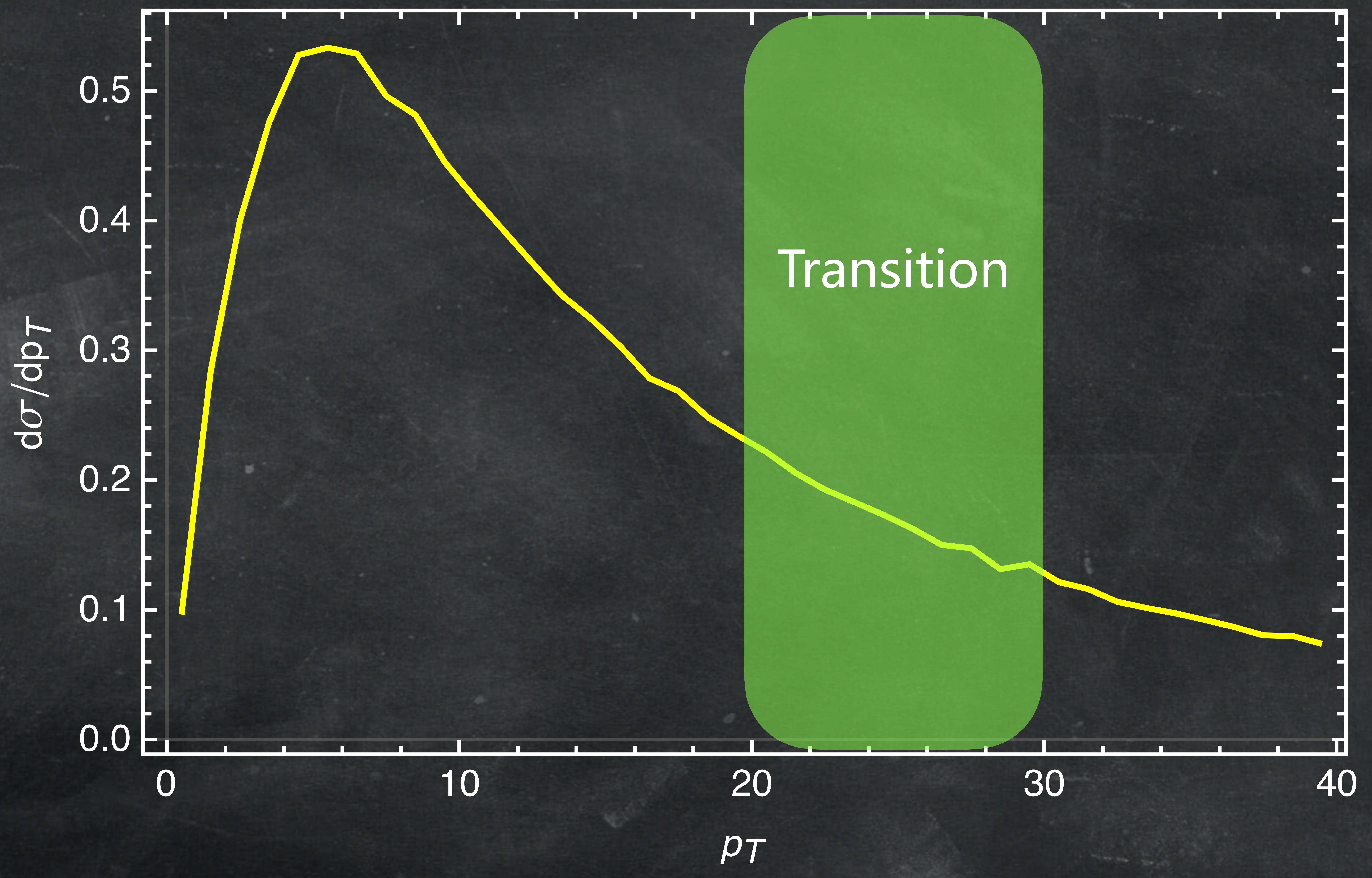
- Sophisticated 9 parameter model, fits to data from older Fermilab, Tevatron, LHC

$$f_{\rm NP}(x, b_T, \zeta) = \left[\frac{1-\lambda}{1 + g_1(x)\frac{b_T^2}{4}} + \lambda \exp\left(-g_{1B}(x)\frac{b_T^2}{4}\right)\right]$$
$$\times \exp\left[-\left(g_2 + g_{2B} b_T^2\right) \ln\left(\frac{\zeta}{Q_0^2}\right)\frac{b_T^2}{4}\right] ,$$

$$g_1(x) = \frac{N_1}{x\sigma} \exp\left[-\frac{1}{2\sigma^2} \ln^2\left(\frac{x}{\alpha}\right)\right] ,$$
$$g_{1B}(x) = \frac{N_{1B}}{x\sigma_B} \exp\left[-\frac{1}{2\sigma_B^2} \ln^2\left(\frac{x}{\alpha_B}\right)\right] .$$

Data driven, smaller uncertainties

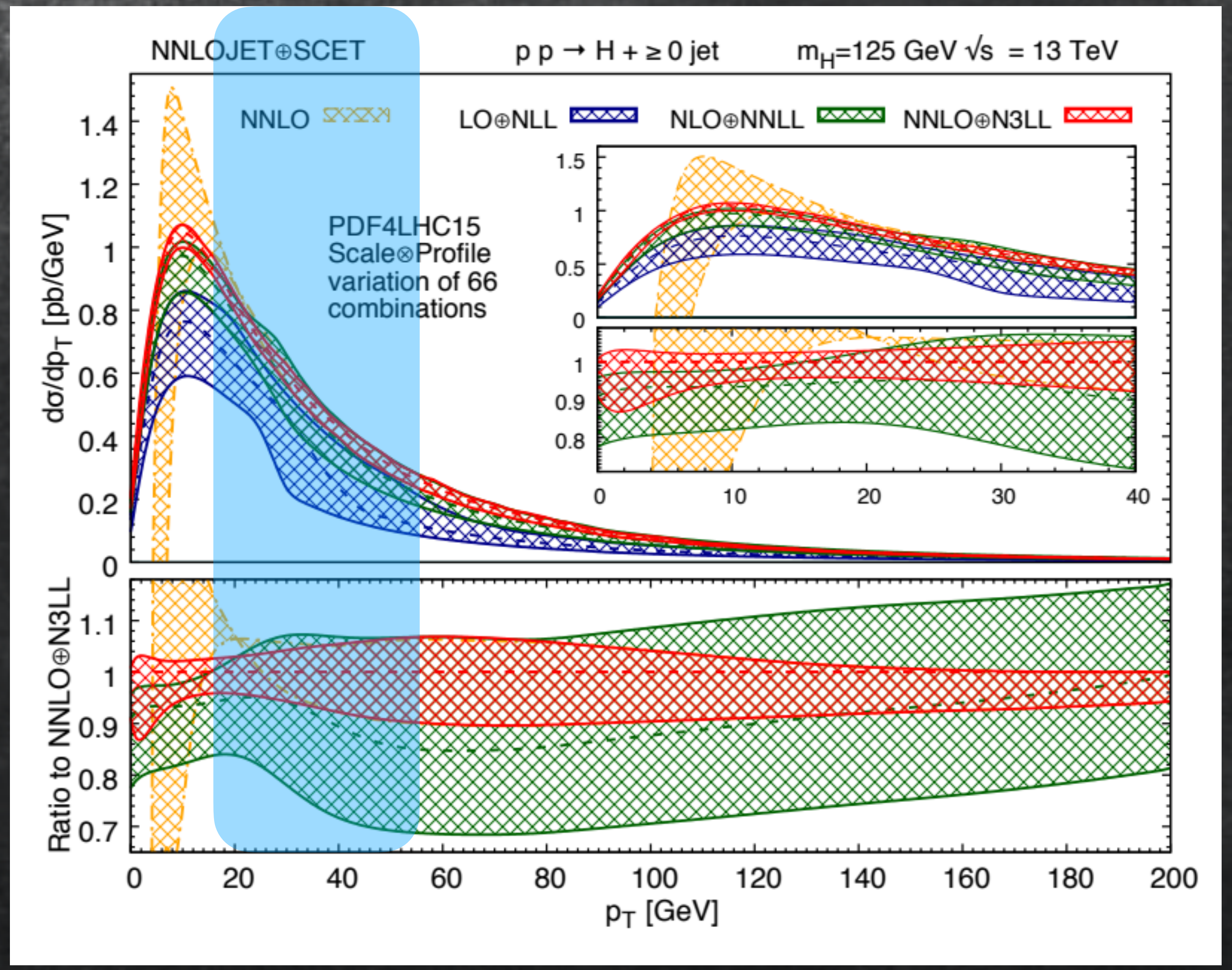
Transition region



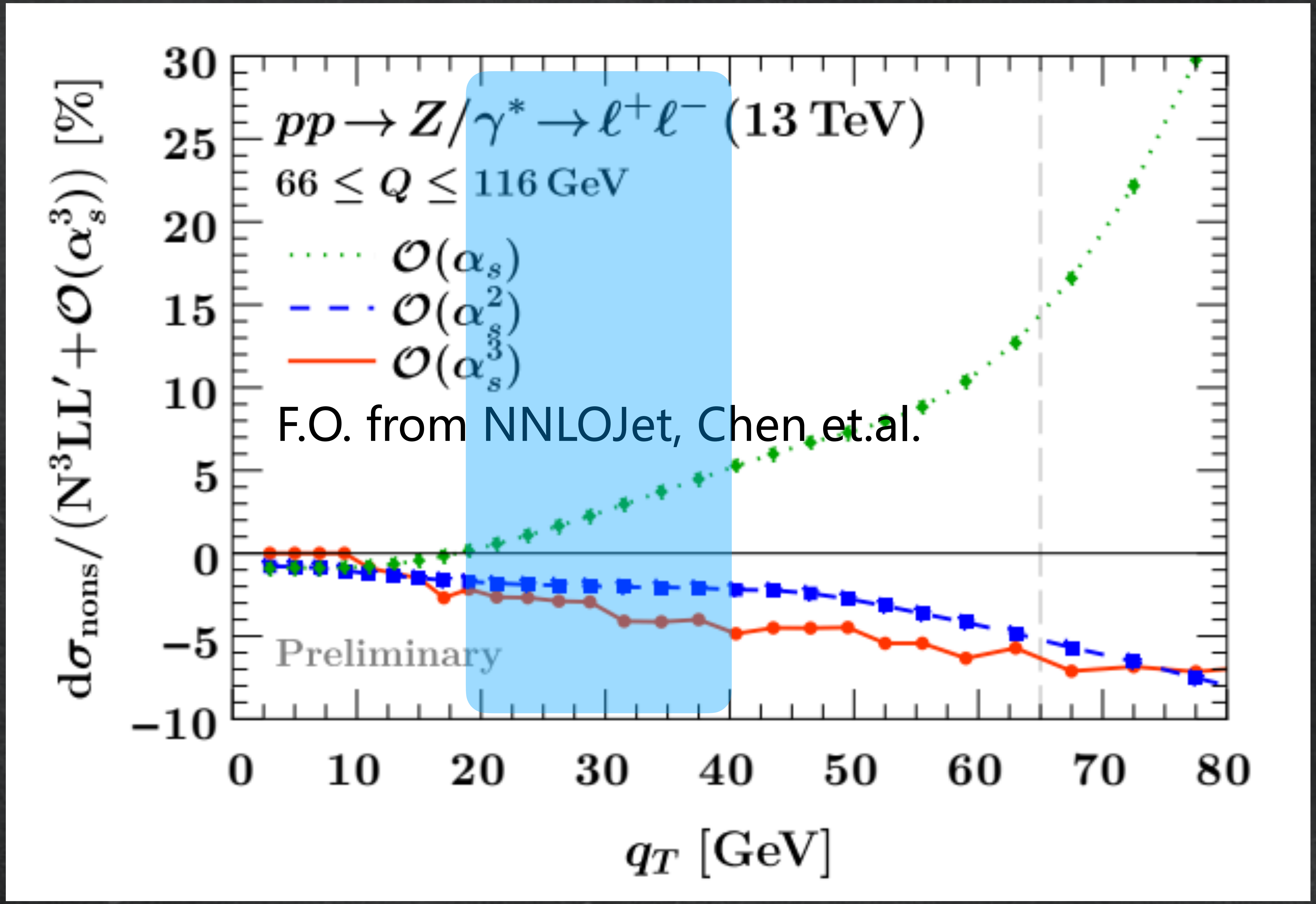
Matching corrections

$$\frac{d\sigma}{dq_T} = \frac{d\sigma}{dq_T}\bigg|_{\rm res} + \left(\frac{d\sigma}{dq_T}\bigg|_{\rm f.O.} - \frac{d\sigma}{dq_T}\bigg|_{\rm truncated}\right)$$

- However, resummation effects must be turned off at large pT
- In SCET this is achieved using profile scale. However, this should not be considered as the unique treatment

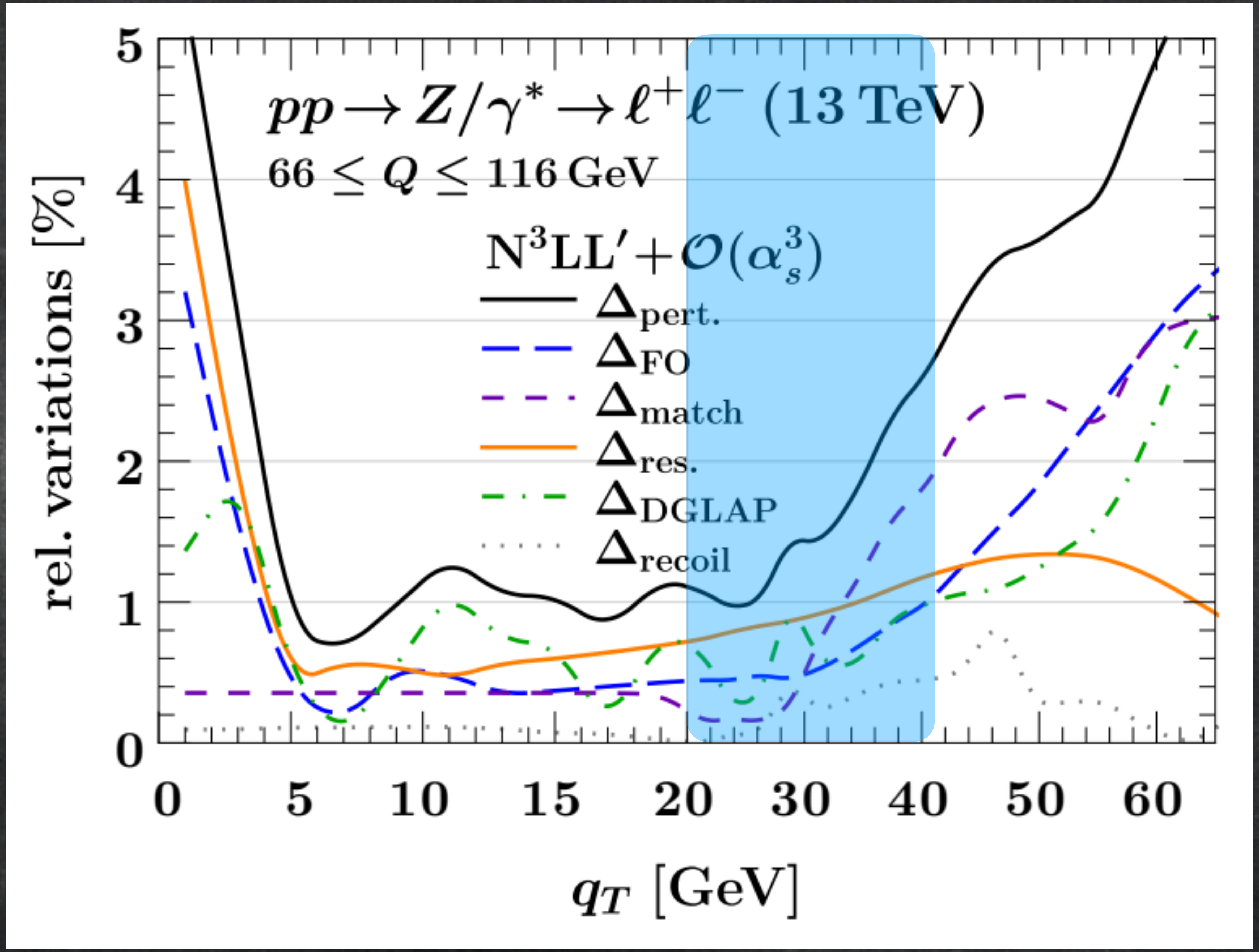


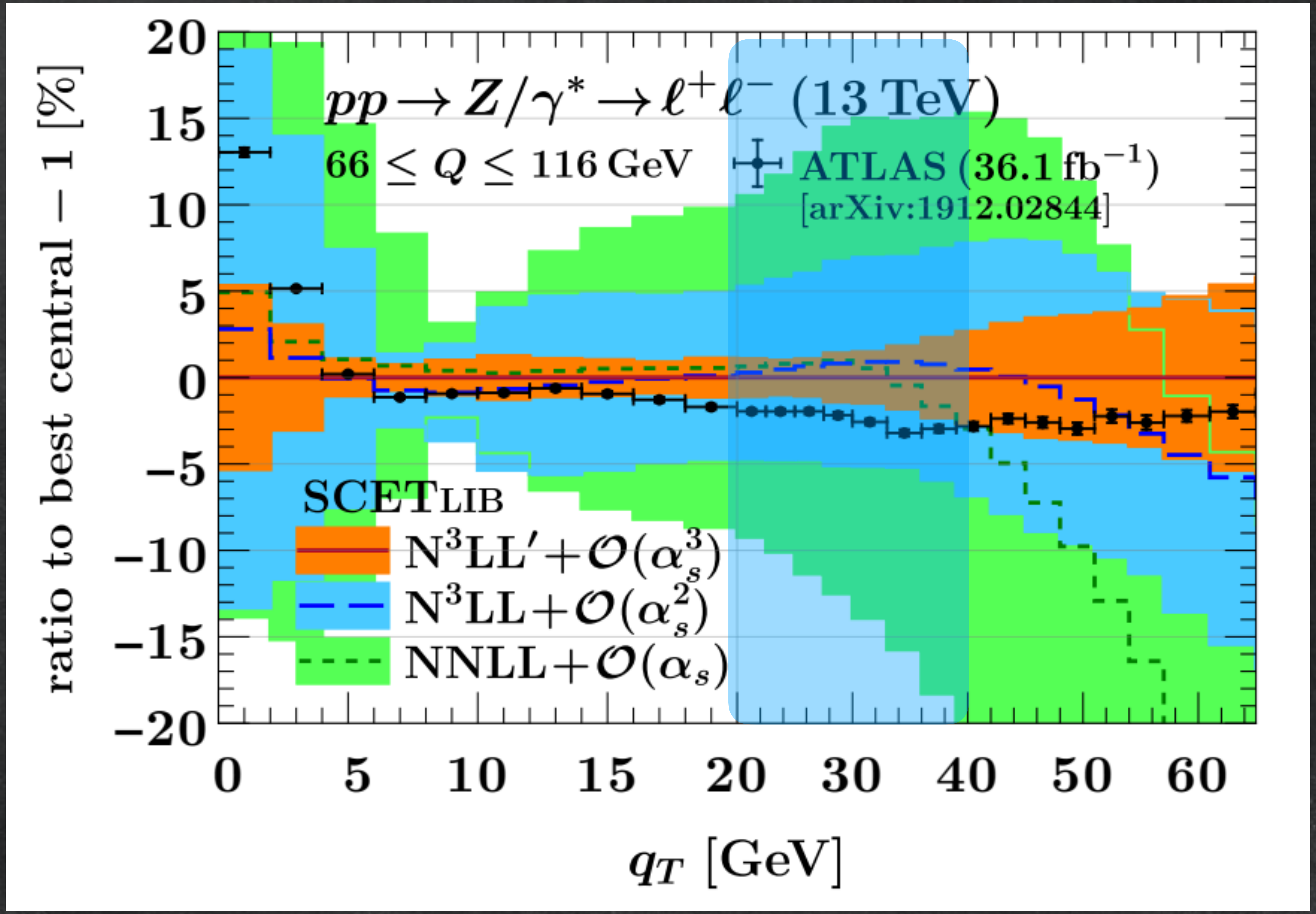
X. Chen et.al., 1805.00736



Billis, Ebert, Michel, Tackmann, SCET 2022

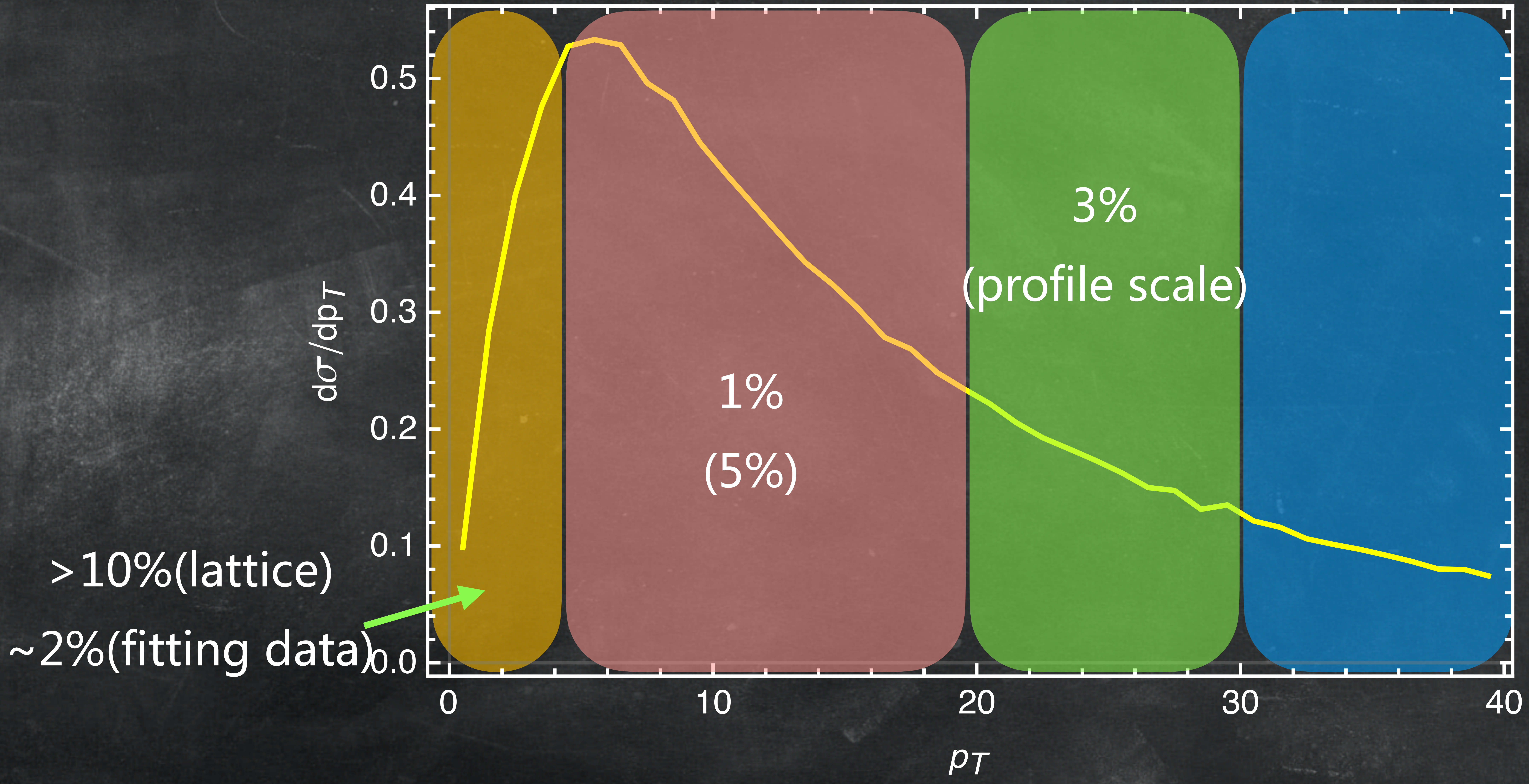
Matching uncertainty and compare to ATLAS





Billis, Ebert, Michel, Tackmann, SCET 2022

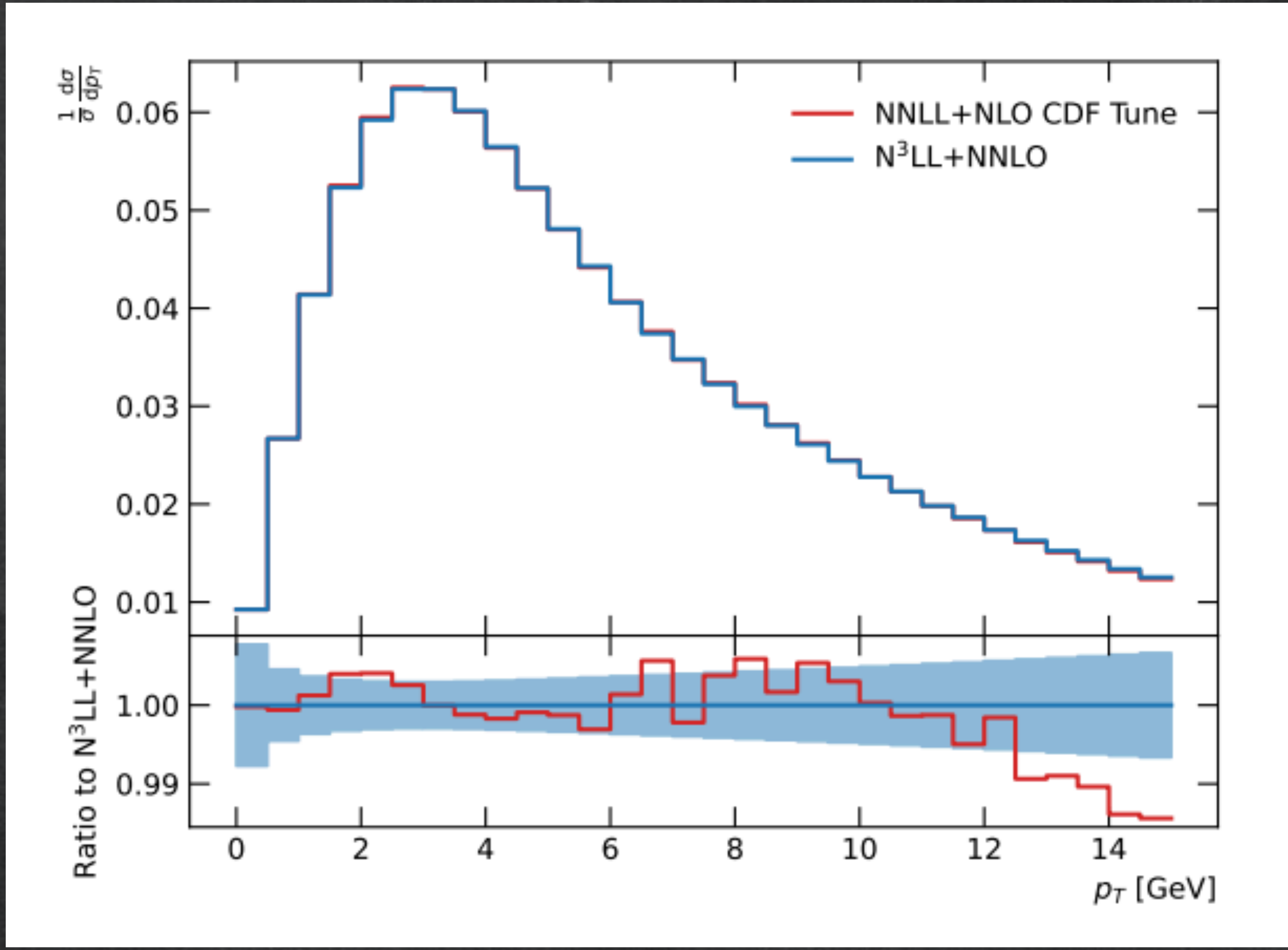
Summarize of the best available results



CDF

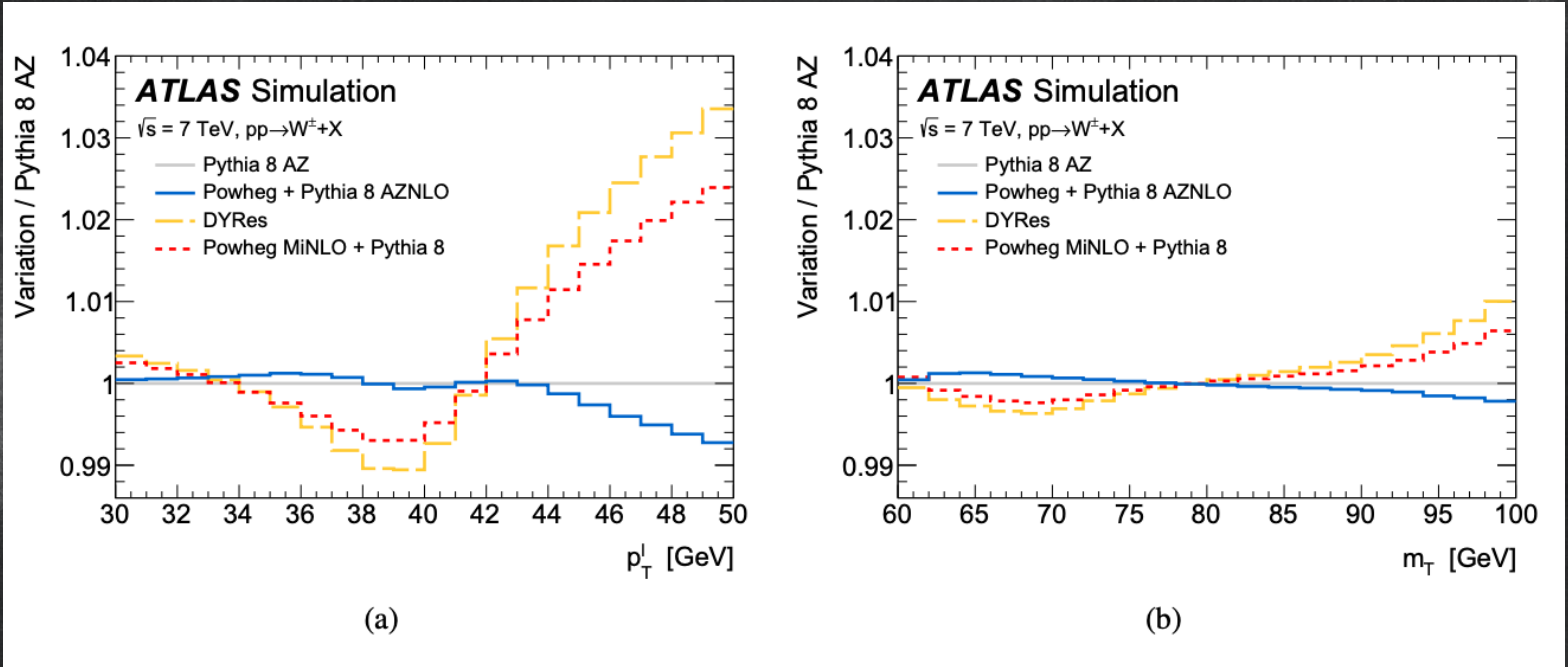
- All three experiments relies on calibrating theory prediction with Z boson pT
- CDF use ResBos (NNLL+ NLO) as theory input
- DYQT (NNLO) to estimate uncertainties in the W pT/Z pT ratio
- Very recently, ResBos2 (N3LL + NNLO) has been used to estimate missing higher order effects in CDF analysis

"We determine that the data-driven techniques used by CDF capture most of the higher order corrections, and using higher order corrections would result in a decrease in the value reported by CDF by at most 10 MeV."



Isaacson, Y. Fu, C.-P. Yuan, 2205.02788

ATLAS

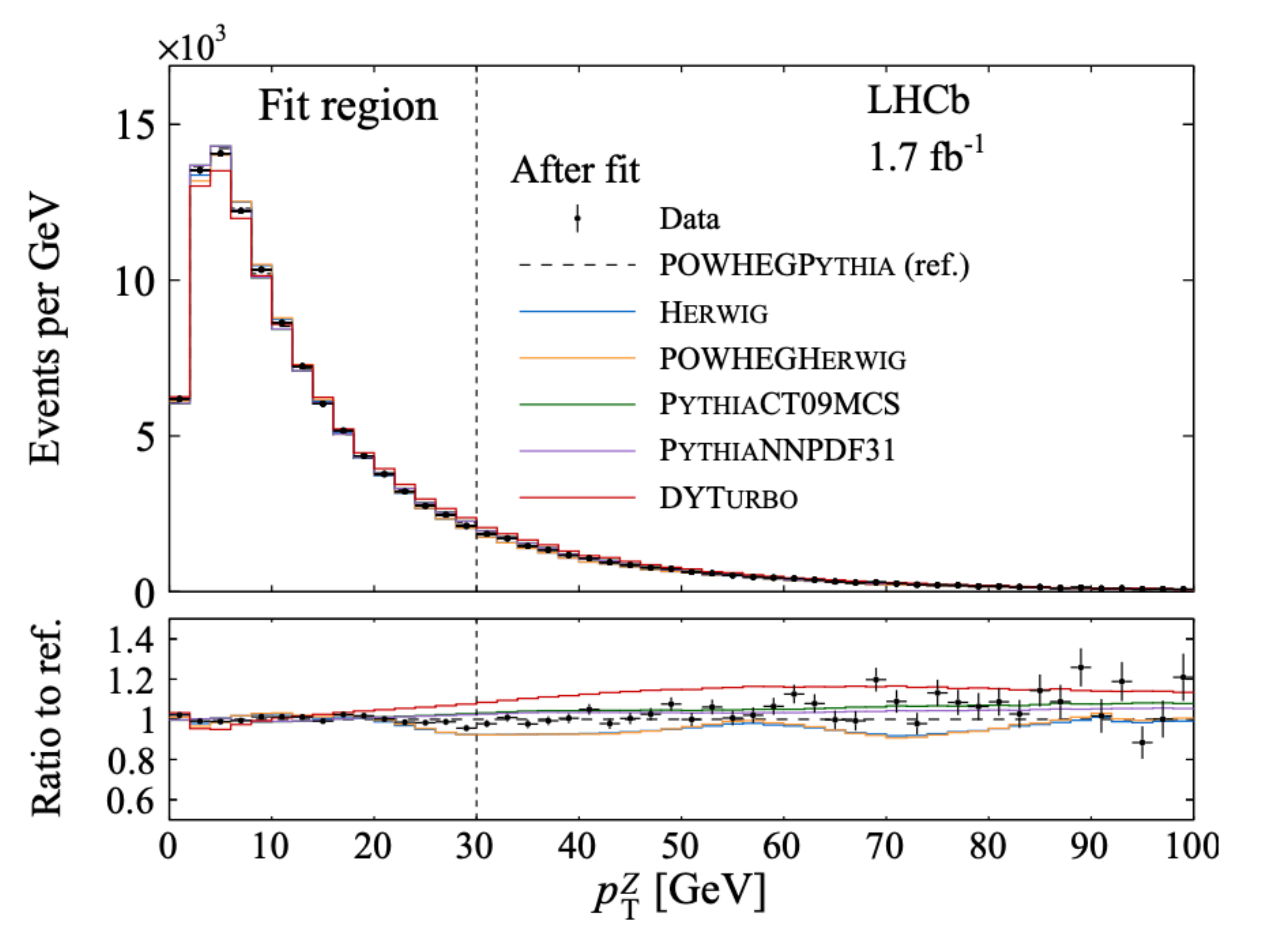


- Pythia 8 AZ tune to Z pT data. Resummation predictions (DYRes) differs substantially from Pythia 8 AZ, therefore not used in theory modeling

LHCb

Program	χ^2/ndf	α_s	
DYTurbo	208.1/13	0.1180	$g = 0.523 \pm 0.047\,\text{GeV}^2$
POWHEGPythia	30.3/12	0.1248 ± 0.0004	$k_T^{\text{intr}} = 1.470 \pm 0.130\,\text{GeV}$
POWHEGHerwig	55.6/12	0.1361 ± 0.0001	$k_T^{\text{intr}} = 0.802 \pm 0.053\,\text{GeV}$
Herwig	41.8/12	0.1352 ± 0.0002	$k_T^{\text{intr}} = 0.753 \pm 0.052\,\text{GeV}$
Pythia, CT09MCS	69.0/12	0.1287 ± 0.0004	$k_T^{\text{intr}} = 2.113 \pm 0.032\,\text{GeV}$
Pythia, NNPDF31	62.1/12	0.1289 ± 0.0004	$k_T^{\text{intr}} = 2.109 \pm 0.032\,\text{GeV}$

- DYTurbo is an analytic resummation code based on CSS formalism
- LHCb uses a variation between different codes listed above to estimate modeling uncertainties



Summary for resummation uncertainties

- LHC aims for a mW uncertainty at O(10 MeV) in the long term
- Currently ATLAS uses Pythia tuned to Z pT as theory model, LHCb use different code: Pythia, Herwig, DYTurbo to estimate modeling uncertainties
- In the future, will be important to use best available resummation+fixed order predictions for theory modeling. Current predictions not fully agree with LHC data. Furthermore, to match to required O(10 MeV), theory uncertainties requires to have <1% uncertainties

Theory opportunities

- Sizable difference in the resummed region between different codes. Can be reduced by pushing to N4LL accuracy. Many ingredients available, 4-loop splitting function calculation in progress

- Better treatment of non-perturbative corrections. Better lattice accuracy? How to incorporate new Lattice results, such as lattice TMD soft function [Q.-A. Zhang et.al., 2020, PRL; Y. Li et.al., 2021, PRL]?

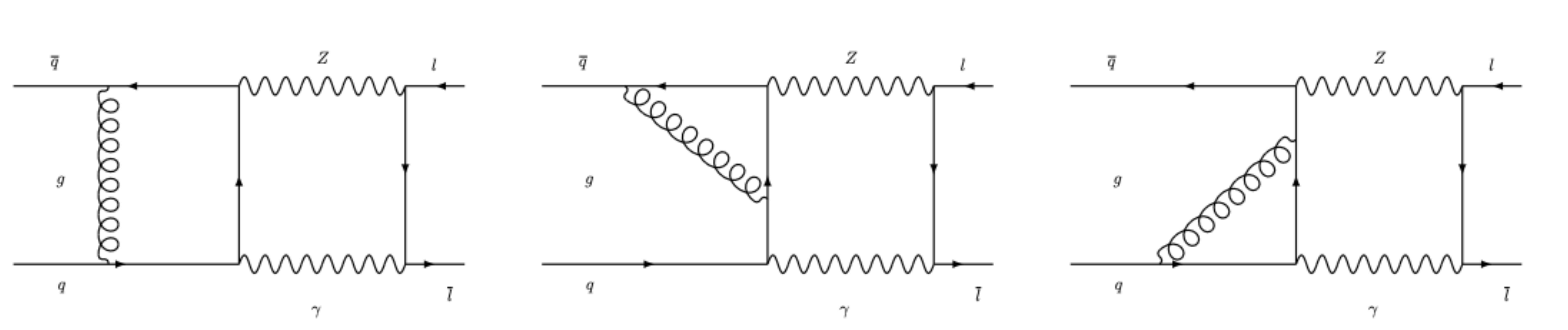
- Matching between resummed region and fixed-order region is not unique, leading to large uncertainty and discrepancy with LHC data

- Initial-state heavy quark effects, b, c

EW and mixed QCD-EW corrections

- Although QCD radiations is the main mechanism for Z/W pT distribution, EW corrections might be important as they can affect W and Z differently [Wackeroth, Hollik, 1997; …]

- Recently, mixed QCD-EW corrections have also been calculated, thanks to many remarkable progress in perturbation calculation



Bonciani, et.al., 2016; Heller, von Manteuffel, Schabinger 2019; Hasan, Schubert, 2020

- Mixed QCD-EW corrections to average lepton pT has been calculated [Behring, et.al., 2021] to estimate impact to W mass measurement

  - The magnitude of EW and QCD-EW corrections to the average lepton pT are comparable

	$V = Z$			$V = W^+$		
	$\mu = m_Z/4$	$\mu = m_Z/2$	$\mu = m_Z$	$\mu = m_W/4$	$\mu = m_W/2$	$\mu = m_W$
$F_V(0,0;1)$, [pb]	1273	1495	1700	7434	8810	10083
$F_V(1,0;1)$, [pb]	570.2	405.4	246.9	3502	2533	1580
$F_V(0,1;1)$, [pb]	$-5810 \cdot 10^{-3}$	$-6146 \cdot 10^{-3}$	$-6073 \cdot 10^{-3}$	$-1908 \cdot 10^{-3}$	$3297 \cdot 10^{-3}$	$10971 \cdot 10^{-3}$
$F_V(1,1;1)$, [pb]	$-2985 \cdot 10^{-3}$	$-2033 \cdot 10^{-3}$	$-1236 \cdot 10^{-3}$	$-8873 \cdot 10^{-3}$	$-7607 \cdot 10^{-3}$	$-7556 \cdot 10^{-3}$
$F_V(0,0;p_\perp^e)$ [GeV · pb]	42741	50191	57073	220031	260772	298437
$F_V(1,0;p_\perp^e)$ [GeV · pb]	23418	17733	12221	124487	95132	66090
$F_V(0,1;p_\perp^e)$ [GeV · pb]	−182.85	−192.77	−189.11	74.53	243.54	484.82
$F_V(1,1;p_\perp^e)$ [GeV · pb]	−163.87	−125.22	−92.05	−553.87	−482.0	−448.0

- [Behring, et.al., 2021, 2103.02671]

  - Significant correlations between Z and W

  - Correlation is stronger for pure EW corrections than mixed QCD-EW. In other words, mixed QCD-EW corrections have larger impact to W-mass measurement

For example, we estimate that the cuts employed by the ATLAS collaboration in their recent extraction of the W mass [5] may lead to a shift of about O(17) MeV due to unaccounted mixed QCD-electroweak effects in the production process.

Summary

- 7σ tension between CDF II and EW fit is unlikely to be completely resolved by higher order QCD/EW effects

- Current LHC measurement has O(20) MeV uncertainties. In the long term aim at O(10) MeV uncertainties, competitive with CDF II. This requires substantial progress in theory modeling

- Many opportunities for precision theory community:

  - Improving QCD resummation + fixed order predictions

  - Incorporating mixed QCD-EW corrections to QCD resummation

- An exciting era for precision calculation, world-wide competition, efforts from China should not be missing!