Semileptonic hyperon decays at BESIII

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BESIII

Institute of High Energy Physics
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Introduction

- **Motivation**:
  - General modular method for the semileptonic hyperon decays
    - Determination of spin correlations and hyperon polarization
    - Test of the SM of flavour-changing weak decays [PRL10 (1963) 531]
    - Test of CP conservation in semileptonic hyperon decays
  - Extraction of decay parameters using provided modular method
    - Last measurement of some decay observables performed > 30 years ago
    - One-step: $\Lambda \to pe^-\bar{\nu}_e$, etc.
    - Two-step: $\Xi^- \to \Lambda e^-\bar{\nu}_e$, $\Lambda_c^+ \to \Lambda e^+\nu_e$, etc.
- Theoretical work is based on:
  - Polarization observables in e^+e^- annihilation to a $B\bar{B}$ pair [PRD 99 (2019) 056008]
  - Helicity analysis for $\Xi^0 \to \Sigma^+(\to p\pi^0)l^-\bar{\nu}_l$ $(l = e^-, \mu^-)$ [EPJ C59 (2009) 27]
- Helicity method allows:
  - Compact calculations of the angular decay distributions
  - Analyze the semileptonic decays of polarized hyperons
  - Take into account the lepton mass effects $\Rightarrow$ vector and axial-vector currents

Production process of two spin-$\frac{1}{2}$ baryons

- General framework of the $e^+e^- \to J/\psi \to \Lambda\bar{\Lambda}$ [PRD99 (2019) 056008]
- Spin density matrix of the production process:

$$\rho_{B_1,\bar{B}_2} = \frac{1}{4}\sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}}(\theta_1)\sigma_{\mu}^{B_1}\otimes\sigma_{\bar{\nu}}^{\bar{B}_2}$$

$$C_{\mu\bar{\nu}} = \begin{pmatrix} 1+\alpha_\psi\cos^2\theta_1 & 0 & \beta_\psi\sin\theta_1\cos\theta_1 & 0 \\ 0 & \sin^2\theta_1 & 0 & \gamma_\psi\sin\theta_1\cos\theta_1 \\ -\beta_\psi\sin\theta_1\cos\theta_1 & 0 & \alpha_\psi\sin^2\theta_1 & 0 \\ 0 & -\gamma_\psi\sin\theta_1\cos\theta_1 & 0 & -\alpha_\psi-\cos^2\theta_1 \end{pmatrix}$$

$\sigma_0^B = \mathbf{1}_2, \sigma_1^\Lambda = \sigma_x, \sigma_2^\Lambda = \sigma_y$ and $\sigma_3^\Lambda = \sigma_z$

$$\beta_\psi = \sqrt{1-\alpha_\psi^2}\sin(\Delta\Phi) \qquad \gamma_\psi = \sqrt{1-\alpha_\psi^2}\cos(\Delta\Phi)$$

- Main parameters of $C_{\mu\bar{\nu}}(\theta_1)$:
  θ_Λ - scattering angle of Λ baryon
  $\alpha_\psi \in [-1,+1]$ - baryon angular distribution parameters
  $\Delta\Phi \in [-\pi,+\pi]$ - relative phase between the two transitions

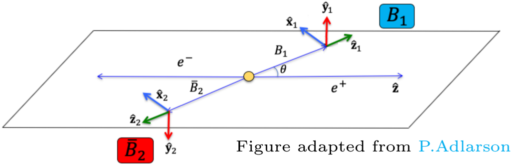

Figure adapted from P.Adlarson's slides

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$$\sigma_0^B = \mathbf{1}_2, \sigma_1^\Lambda = \sigma_x, \sigma_2^\Lambda = \sigma_y \text{ and } \sigma_3^\Lambda = \sigma_z \qquad \beta_\psi = \sqrt{1-\alpha_\psi^2}\sin(\Delta\Phi) \qquad \gamma_\psi = \sqrt{1-\alpha_\psi^2}\cos(\Delta\Phi)$$

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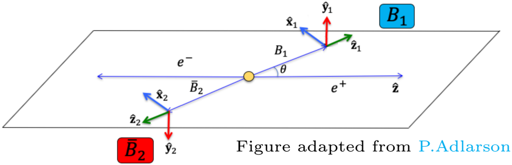
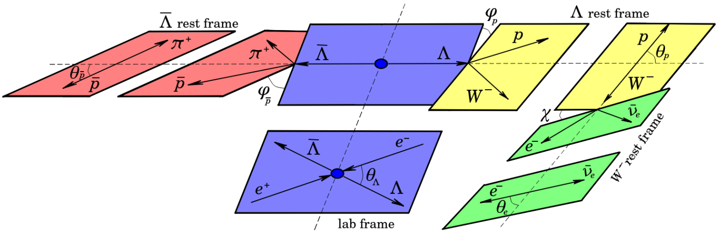


Figure adapted from P.Adlarson's slides

- Production process doesn't depend on the final states. It is the same for:
  - $e^+e^- \to J/\psi \to (\Lambda \to p\pi^-)(\bar{\Lambda} \to \bar{p}\pi^+)$
  - $e^+e^- \to J/\psi \to (\Lambda \to pe^-\bar{\nu}_e)(\bar{\Lambda} \to \bar{p}\pi^+)$ and c.c. process

Semileptonic-hadronic decay chain (1)



- Decay matrix or transition matrix:

$b_{\mu\nu}$ for $\{\frac{1}{2} \to \frac{1}{2} + \{0, \pm 1\}\}$

$$\sigma_\mu \to \sum_{\nu=0}^{3} b_{\mu\nu}\sigma_\nu^d$$

$a_{\mu\nu}$ for $\{\frac{1}{2} \to \frac{1}{2} + 0\}$

$$\sigma_\mu \to \sum_{\nu=0}^{3} a_{\mu\nu}\sigma_\nu^d$$

- Helicity amplitudes: [PRD99 (2019) 056008]

$H_{\frac{1}{2}0}, H_{-\frac{1}{2}0}, H_{\frac{1}{2}1}, H_{-\frac{1}{2}-1}$

$B_{\frac{1}{2}}, B_{-\frac{1}{2}}$

- Kinematic variables and parameters:

$\Omega = \{\phi_p, \theta_p, 0\}$, $\Omega' = \{\chi, \theta_e, 0\}$

$q^2 \in (m_e^2, (M_\Lambda - M_p)^2)$, $\alpha_\Lambda^{sl}(\theta_e, q^2)$

$\Omega = \{\phi_{\bar{p}}, \theta_{\bar{p}}, 0\}$

$\alpha_{\bar{\Lambda}}$

Semileptonic Λ decay

- Momenta and masses: $\Lambda(p_1, M_1) \to p(p_2, M_2) + e^-(p_e, m_e) + \bar{\nu}_e(p_{\bar{\nu}_e}, 0)$
- FF for the weak current-induced baryonic $1/2^+ \to 1/2^+$ transitions [EPJ C59 (2009) 27]:

$$\langle p(p_2)|J_\mu^{V+A}|\Lambda(p_1)\rangle = \bar{u}(p_2)\left[\gamma_\mu\left(F_1^V(q^2) + F_1^A(q^2)\gamma_5\right) + \frac{i\sigma_{\mu\nu}q^\nu}{M_1}\left(F_2^V(q^2) + F_2^A(q^2)\gamma_5\right) + \frac{q_\mu}{M_1}\left(F_3^V(q^2) + F_3^A(q^2)\gamma_5\right)\right]u(p_1)$$

where $q_\mu = (p_1 - p_2)_\mu$

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$\mathbf{0}$

where $q_\mu = (p_1 - p_2)_\mu$

- For $\Lambda \to pe^-\bar{\nu}_e$ at $\mathcal{O}(\frac{m_e^2}{2q^2}) \to 0 \Rightarrow F_3^{V,A}(q^2) \to 0$
- $H_{\lambda_2\lambda_W} = (H^V_{\lambda_2\lambda_W} + H^A_{\lambda_2\lambda_W})$ with $(\lambda_2 = \pm 1/2; \lambda_W = 0, \pm 1)$: $H^{V,A}_{\lambda_2\lambda_W} \equiv H^{V,A}_{\lambda_2\lambda_W}(F^{V,A}_{1,2}(q^2))$

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0

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vector:

$$H^V_{\frac{1}{2}1} = \sqrt{2M_-}\left(-F_1^V - \frac{M_1 + M_2}{M_1}F_2^V\right),$$
$$H^V_{\frac{1}{2}0} = \frac{\sqrt{M_-}}{\sqrt{q^2}}\left((M_1 + M_2)F_1^V + \frac{q^2}{M_1}F_2^V\right),$$

axial-vector:

$$H^A_{\frac{1}{2}1} = \sqrt{2M_+}\left(F_1^A - \frac{M_1 - M_2}{M_1}F_2^A\right),$$
$$H^A_{\frac{1}{2}0} = \frac{\sqrt{M_+}}{\sqrt{q^2}}\left(-(M_1 - M_2)F_1^A + \frac{q^2}{M_1}F_2^A\right).$$

where $M_\pm = (M_1 \pm M_2)^2 - q^2$; $\quad H^{V,A}_{-\lambda_2,-\lambda_W} = \pm H^{V,A}_{\lambda_2\lambda_W}$

Intermediate step

- $\alpha_{\Lambda}^{sl}(\theta_e, q^2) \Rightarrow \{\alpha, \alpha', \alpha'', \beta_{1,2}, \gamma_{1,2}\}(q^2)$ and $g_{av}^{\Lambda}(q^2)$, $g_{w}^{\Lambda}(q^2)$
- $(\alpha,\ \alpha',\ \alpha'',\ \beta_{1,2}$ and $\gamma_{1,2})/n \in [-1, +1]$

$$n = |H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2 + 2(|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2), \qquad \alpha' = |H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2 - 2(|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2),$$

$$\alpha = |H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2 + 2(|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2), \qquad \alpha'' = |H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2 - 2(|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2),$$

$$\beta_1 = 4\Im(H^*_{-\frac{1}{2}0} H_{-\frac{1}{2}-1}), \qquad \gamma_1 = 4\Re(H^*_{-\frac{1}{2}0} H_{-\frac{1}{2}-1}),$$

$$\beta_2 = 4\Im(H^*_{\frac{1}{2}1} H_{\frac{1}{2}0}), \qquad \gamma_2 = 4\Re(H^*_{\frac{1}{2}1} H_{\frac{1}{2}0}),$$

where

$$\beta_{1.2} = \left(\frac{(n \mp \alpha'')^2}{2} - \frac{(\alpha \mp \alpha')^2}{2}\right)^{1/2} \sin\phi, \quad \gamma_{1.2} = \left(\frac{(n \mp \alpha'')^2}{2} - \frac{(\alpha \mp \alpha')^2}{2}\right)^{1/2} \cos\phi$$

- $\alpha^2 + (\alpha')^2 - (\alpha'')^2 + \sum_{i=1}^{2}(\gamma_i^2 + \beta_i^2) = n^2$
- Main parameters to describe semileptonic hyperon decays are:

  - $F_1^V(0)$, $F_2^V(0)$, $F_1^A(0)$

  or

  - $g_{av}^D(0) = \dfrac{F_1^A(0)}{F_1^V(0)}$, $\quad g_w^D(0) = \dfrac{F_2^V(0)}{F_1^V(0)}$

- Last measurement of g_{av} and g_w in $\Lambda \to p e^- \bar{\nu}_e$ by E-555 (Fermilab) [PRD41 (1990) 780]
  - $g_{av} = 0.731 \pm 0.016$ and $g_w = 0.15 \pm 0.30$
  - $g_{av} = 0.719 \pm 0.016$ with constraint $g_w \approx 0.97$ (CVC)

Exclusive joint angular distribution

- Process $e^+e^- \to (\Lambda \to pe^-\bar{\nu}_e)(\bar{\Lambda} \to \bar{p}\pi^+)$

$$\mathrm{Tr}\rho_{pW\bar{p}} \propto \mathcal{W}(\boldsymbol{\xi};\boldsymbol{\omega}) = \sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}} b^{\Lambda}_{\mu 0} a^{\bar{\Lambda}}_{\bar{\nu}0}$$

- $C_{\mu\bar{\nu}}(\theta_\Lambda;\alpha_\psi,\Delta\Phi)$
- $b^{\Lambda}_{\mu 0} \equiv b_{\mu 0}(\theta_p,\varphi_p,\theta_e,q^2;g^{\Lambda}_{av},g^{\Lambda}_{w})$
- $a^{\bar{\Lambda}}_{\bar{\nu}0} \equiv a_{\bar{\nu}0}(\theta_{\bar{p}},\varphi_{\bar{p}};\alpha_{\bar{\Lambda}})$

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- $a^{\bar{\Lambda}}_{\bar{\nu}0} \equiv a_{\bar{\nu}0}(\theta_{\bar{p}}, \varphi_{\bar{p}}; \alpha_{\bar{\Lambda}})$

$$\begin{aligned}
\mathrm{d}\Gamma \propto \mathcal{W}(\boldsymbol{\xi}; \alpha_\psi, \Delta\Phi, g^{\Lambda}_{av}, g^{\Lambda}_{w}, \alpha_{\bar{\Lambda}}) = \\
\sigma^{sl}_{\Lambda}(\theta_l, q^2) \big[1 + \alpha_\psi \cos^2\theta_\Lambda \quad & \text{Cross section} \\
+ a^{sl}_{\Lambda}(\theta_l, q^2)\alpha_{\bar{\Lambda}}(\sin^2\theta_\Lambda(n_{1,x}n_{2,x} - \alpha_\psi n_{1,y}n_{2,y}) + (\cos^2\theta_\Lambda + \alpha_\psi)n_{1,z}n_{2,z}) \\
+ a^{sl}_{\Lambda}(\theta_l, q^2)\alpha_{\bar{\Lambda}}\sqrt{1-\alpha_\psi{}^2}\cos(\Delta\Phi)\sin\theta_\Lambda\cos\theta_\Lambda(n_{1,x}n_{2,z} + n_{1,z}n_{2,x}) \quad & \text{Spin correlations} \\
+ \sqrt{1-\alpha_\psi{}^2}\sin(\Delta\Phi)\sin\theta_\Lambda\cos\theta_\Lambda(a^{sl}_{\Lambda}(\theta_l, q^2)n_{1,y} + \alpha_{\bar{\Lambda}}n_{2,y}) \big] \quad & \text{Polarization}
\end{aligned}$$

where $$a^{sl}_{\Lambda}(\theta_e, q^2) = \frac{\alpha^{sl}_{\Lambda}(\theta_e, q^2)}{\sigma^{sl}_{\Lambda}(\theta_e, q^2)} = \frac{\alpha + \alpha''\cos^2\theta_e \mp (n + \alpha')\cos\theta_e}{n + \alpha'\cos^2\theta_e \mp (\alpha + \alpha'')\cos\theta_e}$$

- $\Delta\Phi \neq 0 \Rightarrow$ independent determination of α^{sl}_{Λ} and $\alpha_{\bar{\Lambda}}$
- Same expression for $e^+e^- \to (\Lambda \to p\pi^-)(\bar{\Lambda} \to \bar{p}\pi^+)$ [PLB772(2017)16]: $\alpha^{sl}_{\Lambda} \Leftrightarrow \alpha_\Lambda$
- Possible measurement of g_{av} and g_w using BESIII data

Summary and outline

- Description and implementation of helicity formalism of semileptonic-hadronic decay chain is at the advantage stage
- Introduction of a general modular method for the full process will allow to
  - describe semileptonic baryon/hyperon and cascade decays
  - extract spin correlations and hyperon polarizations
  - test assumptions of Cabibbo theory: CVC hypothesis, 2nd-class currents,...
  - test the CP conservation
- **Ongoing steps**:
  - Measurement of g_{av}^{Λ} and $g_{av}^{\bar{\Lambda}}$ for $J/\psi \to \Lambda\bar{\Lambda}$ using BESIII data + memo
  - Verification of formalism for $\Xi^- \to \Lambda(\to p\pi^-)e^-\bar{\nu}_e$ + c.c. decays
  - Preparation of phenomenology paper

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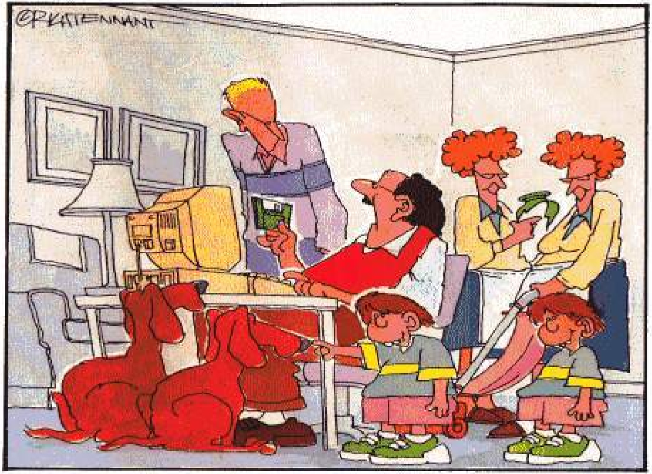
Thank you for your attention!

Contributions

- Conference talks:
  - *"Light hyperon physics at BESIII"*, 32nd Recontres de Blois, France, 17-22 October 2021
  - *"CP violation of hyperon-antihyperon pairs at BESIII"*, BEACH2022, Poland, 5-11 June 2022
- Publications:
  - *"Light hyperon physics at the BESIII experiment"*, *Proceedings of 32nd Recontres de Blois conference*, submitted
  - *"Study of CP violation in hyperon decays at super-charm-tau factories with a polarized electron beam"*, [PRD 105 (2022) 116022]
  - *"First Measurement of the Absolute Branching Fraction for $\Lambda \to p\mu^- \bar{\nu}_\mu$"*, [PRL 127 (2021) 121802]

Backups



"I ALWAYS BACK UP EVERYTHING."

Decay chain: $\frac{1}{2} \to \frac{1}{2} + 0$

[PRD99(2019)056008]

- Relation between helicity amplitudes and decay parameters

$$\text{normalization } |A_S|^2 + |A_P|^2 = |B_{-\frac{1}{2}}|^2 + |B_{\frac{1}{2}}|^2 = 1,$$

$$\alpha_D = -2\Re(A_S^* A_P) = |B_{1/2}|^2 - |B_{-1/2}|^2,$$

$$\beta_D = -2\Im(A_S^* A_P) = 2\Im(B_{1/2} B_{-1/2}^*),$$

$$\gamma_D = |A_S^*|^2 - |A_P|^2 = 2\Re(B_{1/2} B_{-1/2}^*),$$

$$\text{where } \beta_D = \sqrt{1-\alpha_D^2}\sin\varphi_D \text{ and } \gamma_D = \sqrt{1-\alpha_D^2}\cos\varphi_D$$

- Non-zero elements of the decay matrix $a_{\mu\nu}$:

$$a_{00} = 1,$$
$$a_{03} = \alpha_D,$$
$$a_{10} = \alpha_D \cos\varphi \sin\theta,$$
$$a_{11} = \gamma_D \cos\theta \cos\varphi - \beta_D \sin\varphi,$$
$$a_{12} = -\beta_D \cos\theta \cos\varphi - \gamma_D \sin\varphi,$$
$$a_{13} = \sin\theta \cos\varphi,$$
$$a_{20} = \alpha_D \sin\theta \sin\varphi,$$
$$a_{21} = \beta_D \cos\varphi + \gamma_D \cos\theta \sin\varphi,$$
$$a_{22} = \gamma_D \cos\varphi - \beta_D \cos\theta \sin\varphi,$$
$$a_{23} = \sin\theta \sin\varphi,$$
$$a_{30} = \alpha_D \cos\theta,$$
$$a_{31} = -\gamma_D \sin\theta,$$
$$a_{32} = \beta_D \sin\theta,$$
$$a_{33} = \cos\theta$$

- Main parameters: $\theta \equiv \theta_{p/\bar{p}}$, $\varphi \equiv \varphi_{p/\bar{p}}$, $\alpha_D \equiv \alpha_{\Lambda/\bar{\Lambda}}$, $\varphi_D \equiv \varphi_{\Lambda/\bar{\Lambda}}$

Decay chain: $\frac{1}{2} \to \frac{1}{2} + \{0, \pm 1\}$

- Relation between helicity amplitudes and decay parameters

$$\sigma_D^{sl} = \frac{1}{4}(1 \mp \cos\theta_l)^2 |H_{\frac{1}{2}1}|^2 + \frac{1}{4}(1 \pm \cos\theta_l)^2 |H_{-\frac{1}{2}-1}|^2 + \frac{1}{2}\sin^2\theta_l (|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2),$$
$$\alpha_D^{sl} = \frac{1}{4}(1 \mp \cos\theta_l)^2 |H_{\frac{1}{2}1}|^2 - \frac{1}{4}(1 \pm \cos\theta_l)^2 |H_{-\frac{1}{2}-1}|^2 + \frac{1}{2}\sin^2\theta_l (|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2),$$
$$\beta_D^{sl} = \frac{1}{\sqrt{2}}\sin\theta_l ((1 \pm \cos\theta_l)\Im(H^*_{-\frac{1}{2}0} H_{-\frac{1}{2}-1}) + (1 \mp \cos\theta_l)\Im(H^*_{\frac{1}{2}1} H_{\frac{1}{2}0})),$$
$$\gamma_D^{sl} = \frac{1}{\sqrt{2}}\sin\theta_l ((1 \pm \cos\theta_l)\Re(H^*_{-\frac{1}{2}0} H_{-\frac{1}{2}-1}) + (1 \mp \cos\theta_l)\Re(H^*_{\frac{1}{2}1} H_{\frac{1}{2}0}))$$

- Non-zero elements of the decay matrix $b_{\mu\nu}$:

$$\boxed{b_{00} = \sigma_D^{sl},}$$
$$b_{03} = \alpha_D^{sl},$$
$$\boxed{b_{10} = \alpha_D^{sl}\cos\phi_p \sin\theta_p,}$$
$$b_{11} = \mp(\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\cos\theta_p\cos\phi_p \pm (\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\sin\phi_p,$$
$$b_{12} = \pm(\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\cos\theta_p\cos\phi_p \pm (\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\sin\phi_p,$$
$$b_{13} = \sigma_D^{sl}\sin\theta_p\cos\phi_p,$$
$$\boxed{b_{20} = \alpha_D^{sl}\sin\theta_p\sin\phi_p,}$$
$$b_{21} = \mp(\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\cos\theta_p\sin\phi_p \mp (\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\cos\phi_p,$$
$$b_{22} = \pm(\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\cos\theta_p\sin\phi_p \mp (\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\cos\phi_p,$$
$$b_{23} = \sigma_D^{sl}\sin\theta_p\sin\phi_p,$$
$$\boxed{b_{30} = \alpha_D^{sl}\cos\theta_p,}$$
$$b_{31} = \pm(\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\sin\theta_p,$$
$$b_{32} = \mp(\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\sin\theta_p,$$
$$b_{33} = \sigma_D^{sl}\cos\theta_p$$

- Main parameters:
  $\sigma_D^{sl} \equiv \sigma_{\Lambda/\bar{\Lambda}}^{sl}(\theta_e, q^2)$, $\alpha_D^{sl} \equiv \alpha_{\Lambda/\bar{\Lambda}}^{sl}(\theta_e, q^2)$, $\beta_D^{sl} \equiv \beta_{\Lambda/\bar{\Lambda}}^{sl}(\theta_e, q^2)$, $\gamma_D^{sl} \equiv \gamma_{\Lambda/\bar{\Lambda}}^{sl}(\theta_e, q^2)$
- Upper and lower signs refer to $(e^-, \bar{\nu}_e)$ and (e^+, ν_e) configurations, respectively

Form factors

$$F_i^{V,A}(q^2) = F_i^{V,A}(0)\prod_{n=0}^{n_i}\frac{1}{1-\frac{q^2}{m_{V,A}^2+n\alpha'^{-1}}} \approx F_i^{V,A}(0)\left(1+q^2\sum_{n=0}^{n_i}\frac{1}{m_{V,A}^2+n\alpha'^{-1}}\right) \equiv F_i^{V,A}(0)c_i^{V,A}(q^2)$$

	$F_i^{V,A}(0)(\Lambda\to p)$	$m_{V,A}$	α' [GeV^{-2}]	n_i
$F_1^V(q^2)$	$-\sqrt{\frac{3}{2}}$ [1]	$m_{K^*(892)^0}$ $(J^P=1^-)$	0.9	$n_1=1$
$F_2^V(q^2)$	$\frac{M_\Lambda\mu_p}{2M_p}F_1^V(0)$ [2]			$n_2=2$
$F_3^V(q^2)$	0 [4]	-		$n_3=2$
$F_1^A(q^2)$	$0.719F_1^V(0)$ [3]	$m_{K^*(1270)^0}$ $(J^P=1^+)$		$n_1=1$
$F_2^A(q^2)$	0 [4]	–		$n_2=2$
$F_3^A(q^2)$	$\frac{M_\Lambda(M_\Lambda+M_p)}{(m_{K^-})^2}F_1^A(0)$ [4]	m_K $(J^P=0^-)$		$n_3=2$

- [1] [PR135(1964)B1483], [PRL13(1964)264]
- [2] $\mu_p = 1.793$ [Lect.NotesPhys.222(1985)1], [Ann.Rev.Nucl.Part.Sci.53(2003)39], [JHEP0807(2008)132]
- [3] [PRD41(1990)780]
- [4] Vanish in the $SU(3)$ symmetry limit; Goldberger-Treiman relation [PR110(1958)1178], [PR111(1958)354]

Helicity amplitudes of the lepton pair $h^l_{\lambda_l \lambda_\nu}$

- Lepton and antineutrino spinors

$$\bar{u}_{l^-}(\mp\frac{1}{2}, p_{l^-}) = \sqrt{E_l + m_l}\left(\chi^\dagger_\mp, \frac{\pm|\vec{p}_{l^-}|}{E_l + m_l}\chi^\dagger_\mp\right),$$
$$v_{\bar{\nu}}(\frac{1}{2}) = \sqrt{E_\nu}\begin{pmatrix}\chi_+ \\ -\chi_+\end{pmatrix},$$

where $\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}$ and $\chi_- = \begin{pmatrix}0\\1\end{pmatrix}$ are Pauli two-spinors

- SM form of the lepton current ($\lambda_W = \lambda_{l^-} - \lambda_{\bar{\nu}}$)

$$h^l_{\lambda_{l^-}=\mp 1/2, \lambda_{\bar{\nu}}=1/2} = \bar{u}_{l^-}(\mp\frac{1}{2})\gamma^\mu(1+\gamma_5)v_{\bar{\nu}}(\frac{1}{2})\begin{Bmatrix}\epsilon_\mu(-1)\\ \epsilon_\mu(0)\end{Bmatrix}$$

where $\epsilon^\mu(0) = (0;0,0,1)$ and $\epsilon^\mu(\mp 1) = (0;\mp 1, -i, 0)/\sqrt{2}$

- Moduli squared of the helicity amplitudes [EPJ C59 (2009) 27]

$$\text{nonflip}(\lambda_W = \mp 1) : |h^l_{\lambda_l=\mp\frac{1}{2}, \lambda_\nu=\pm\frac{1}{2}}|^2 = 8(q^2 - m_l^2),$$

$$\text{flip}(\lambda_W = 0) : |h^l_{\lambda_l=\pm\frac{1}{2}, \lambda_\nu=\pm\frac{1}{2}}|^2 = 8\frac{m_l^2}{2q^2}(q^2 - m_l^2)$$

- Upper and lower signs refer to the configurations $(l^-, \bar{\nu}_l)$ $(\lambda_\nu = 1/2)$ and (l^+, ν_l) $(\lambda_\nu = -1/2)$, respectively
- In case of the e-mode only nonflip transition remains under assumption $\frac{m_e^2}{2q^2} \to 0$