Spin-light polarimeter

Beam Polarization Measurement Using Synchrotron Radiation

26 April 2022

Outline & Reference

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  - Classic Theory of SR
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- summary

Ref: [1] Karabekov I P, Rossmanith R. Measurement of longitudinal beam polarization by synchrotron radiation[C]//Proceedings of International Conference on Particle Accelerators. IEEE, 1993: 457-459.
[2] Mohanmurthy P, Dutta D. A Spin-Light Polarimeter for Multi-GeV Longitudinally Polarized Electron Beams[J]. IEEE Transactions on Nuclear Science, 2014, 61(1): 528-537.
[3] Mohanmurthy P, Dutta D. An update on the developmental status of the Spin-Light Polarimeter for the Electron Ion Collider[J]. arXiv preprint arXiv:1401.6744, 2014.
[4] Mohanmurthy P, Dipangkar D. Feasibility of the spin-light polarimetry technique for longitudinally polarized electron beams[C]//EPJ Web of Conferences. EDP Sciences, 2014, 66: 11025.
[5] Belomesthnykh, S. A., et al. "An observation of the spin dependence of synchrotron radiation intensity." Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 227.1 (1984): 173-181.
[6] Mohanmurthy P. A novel spin-light polarimeter for the Electron Ion Collider[J]. arXiv preprint arXiv:1310.6340, 2013.

Spin-light polarimeter

➢ Motivation

- The longitudinal polarization of the electron beam is one of the dominant systematic uncertainties in parity violating electron scattering (PVES) experiment.
- The polarization of the electron beam must be monitored continuously with an uncertainty of 0.5%.

➢ Method

The spin dependence synchrotron radiation (SR), called “spin-light”, can be used to monitor the electron beam polarization.

Compare to other methods

Compton	Spin-Light	Møller
non-invasive, continuous	non-invasive, continuous	invasive
analyzing power energy dependent	analyzing power energy dependent	analyzing power energy independent
high currents	moderately high currents	low currents
target is 100% polarized (requires stable laser)	no target needed	target is $< 10\%$ polarized
electron & photon detection are two independent measurements	beam left & right detectors provide two independent measurements	no independent measurements possible
high precision absolute polarimeter	high precision relative polarimeter	high precision absolute polarimeter
Best reported [3] instrumental uncertainty: 0.4%	expected instrumental uncertainty: 0.6%	Best reported [1] instrumental uncertainty: 0.47%
Best reported [3] absolute uncertainty: 0.5%	estimated absolute uncertainty: $\sim$2.5%	Best achieved [2] absolute uncertainty: 0.85%

Classical theory of SR

单个电子在磁场中运动的瞬时辐射功率:
The total radiative power is given by Larmor formula

$$P_{clas} = \frac{2}{3}\frac{e^2\gamma^4 c}{R^2}$$

The angular distribution of the radiated power is given by

$$\frac{dP_{clas}}{d\boldsymbol{\Omega}} = \frac{dP_{clas}}{d\theta d\phi} = \frac{e^2\gamma^4 c}{4\pi R^2}\frac{(1-\beta cos\theta)^2 - (1-\beta^2)sin^2\theta cos^2\phi}{(1-\beta cos\theta)^5}$$

Angle (θ, ϕ) are measured with respect to the direction of electron's motion.

➢ Properties of SR

- SR is strongly linearly polarized.
- For highly relativistic electrons, the radiation with an opening angle $\theta \approx 1/\gamma$.
- Critical frequency:

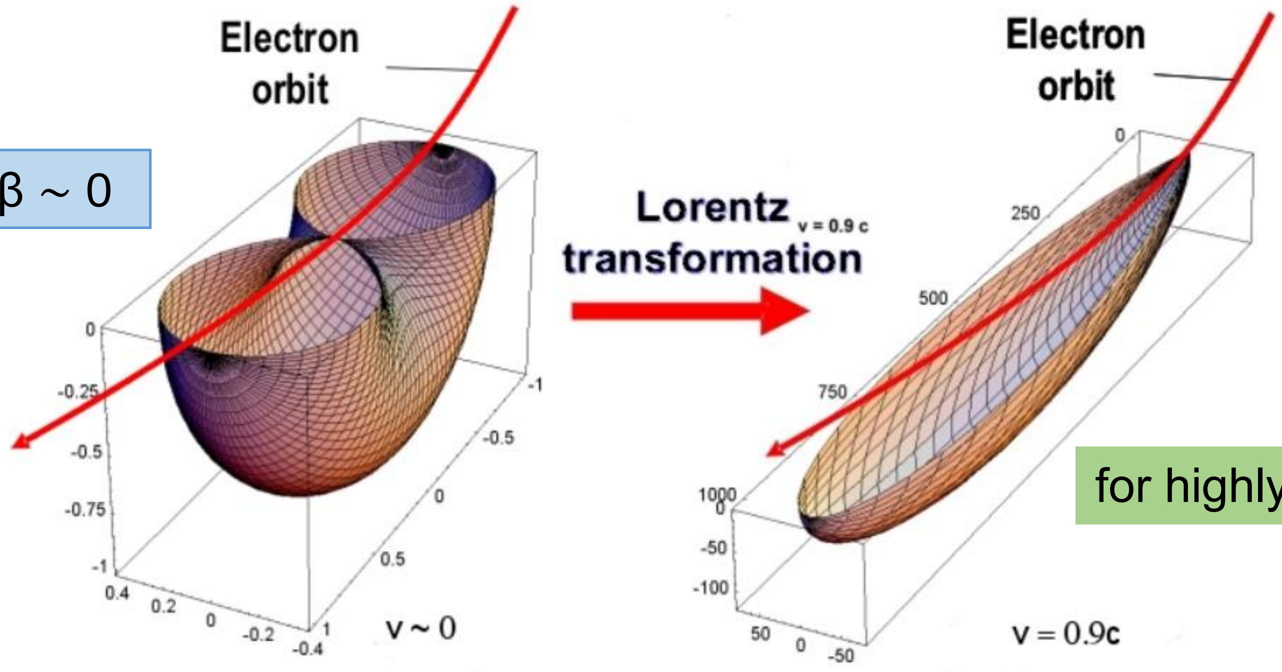
$$w_c = \frac{3}{2}\gamma^3 c/R$$

- Critical energy:

$$E_c = m_e c^2\sqrt{\frac{m_e cR}{\hbar}}$$

Classical theory of SR

for slow electrons, $\beta \sim 0$



for highly relativistic electrons, $\beta \sim 1$

Angular distribution of synchrotron radiation shown for the bottom half of the electron's orbital plane.

Quantum Theory of SR

- QED corrections give electrons spin dependence in the radiated power.
- Ternov and et. al. provide the Dirac equation.

Taylor expanded:

$$P = P^{clas}\left[\left(1 - \frac{55\sqrt{3}}{24}\xi + \frac{64}{3}\xi^2\right) - \left(\frac{1+jj'}{2}\right)\left(j\xi + \frac{5}{9}\xi^2 \frac{245\sqrt{3}}{48}j\xi^2\right) + \left(\frac{1-jj'}{2}\right)\left(\frac{4}{3}\xi^2 + \frac{315\sqrt{3}}{432}j\xi^2\right) + \ldots\right]$$

- Classic SR
- Thomas Precession
- Larmor precession
- Interaction between Larmor and Thomas precession
- Radiation from intrinsic magnetic moment.

Spin dependent term

Spin-flip dependent term

j: spin orientations with respect to the magnetic field

- 极化的电子束流与无极化的电子束流辐射功率之差为：which is called “spin-light”

$$P^{spin} = P^{pol} - P^{unpol} = -j\xi P^{cals}\int_0^\infty \frac{9\sqrt{3}}{8\pi} y^2 K_{1/3}(y)dy$$

Quantum Theory of SR

- Polarized electron beams have longitudinal (P_z), transverse horizontal (P_y) and transverse vertical (P_x) components relative to the beam direction.

➢ Transverse polarization

ignoring spin flip terms and other terms of order ξ^2

$$P_\gamma(tran) = \frac{9n_e}{16\pi^3}\frac{ce^2}{R^2}\gamma^5\int_0^\infty \frac{y^2dy}{(1+\xi y)^4}\oint d\Omega(1+\alpha^2)^2 \times\left[K^2_{2/3}(z)+\frac{\alpha^2}{1+\alpha^2}K^2_{1/3}(z) - (+)p_{x(y)}\xi y\frac{1}{\sqrt{1+\alpha^2}}K_{1/3}(z)K_{2/3}(z)\right], \quad (6)$$

公式中，$\alpha = \gamma\varphi$

上述表达式中的极化相关项是φ角的偶函数，因此，当在所有角度上积分时，它使总SR功率与自旋相关。因此，通过测量总辐射功率中的自旋依赖性，可以测量横向极化电子束的极化。

➢ Longitudinal polarization

ignoring spin flip terms and other terms of order ξ^2

$$P_\gamma(long) = \frac{9n_e}{16\pi^3}\frac{ce^2}{R^2}\gamma^5\int_0^\infty \frac{y^2dy}{(1+\xi y)^4}\oint d\Omega(1+\alpha^2)^2 \times\left[K^2_{2/3}(z)+\frac{\alpha^2}{1+\alpha^2}K^2_{1/3}(z) + p_z\xi y\frac{\alpha}{\sqrt{1+\alpha^2}}K_{1/3}(z)K_{2/3}(z)\right], \quad (7)$$

上述表达式中的极化相关项是φ角的奇函数，在所有的角度上积分等于0，总的SR辐射对于纵向极化而言是自旋无关的。然而，空间上方($0 < \varphi < \pi/2$)的辐射功率和空间下方($-\pi/2 < \varphi < 0$)的辐射功率在电子的轨道平面是不同的，依赖于自旋。因此通过测量spatial asymmetry,就可以测量电子束流的纵向极化。

Quantum Theory of SR

➢ Longitudinal polarization

辐射到有限水平角的光子总数可表示为:

$$N_\gamma = \frac{3}{4\pi^2}\frac{1}{137}\frac{I_e}{e}\gamma\Delta\theta\int_{y_1}^{y_2} y dy \int_{-\alpha}^{\alpha}(1+\alpha^2)^{3/2} \times \left[K_{2/3}^2(z) + \frac{\alpha^2}{1+\alpha^2}K_{1/3}^2(z)\right] d\alpha, \quad (8)$$

电子轨道上方和下方空间中辐射的光子通量之差由下式给出:

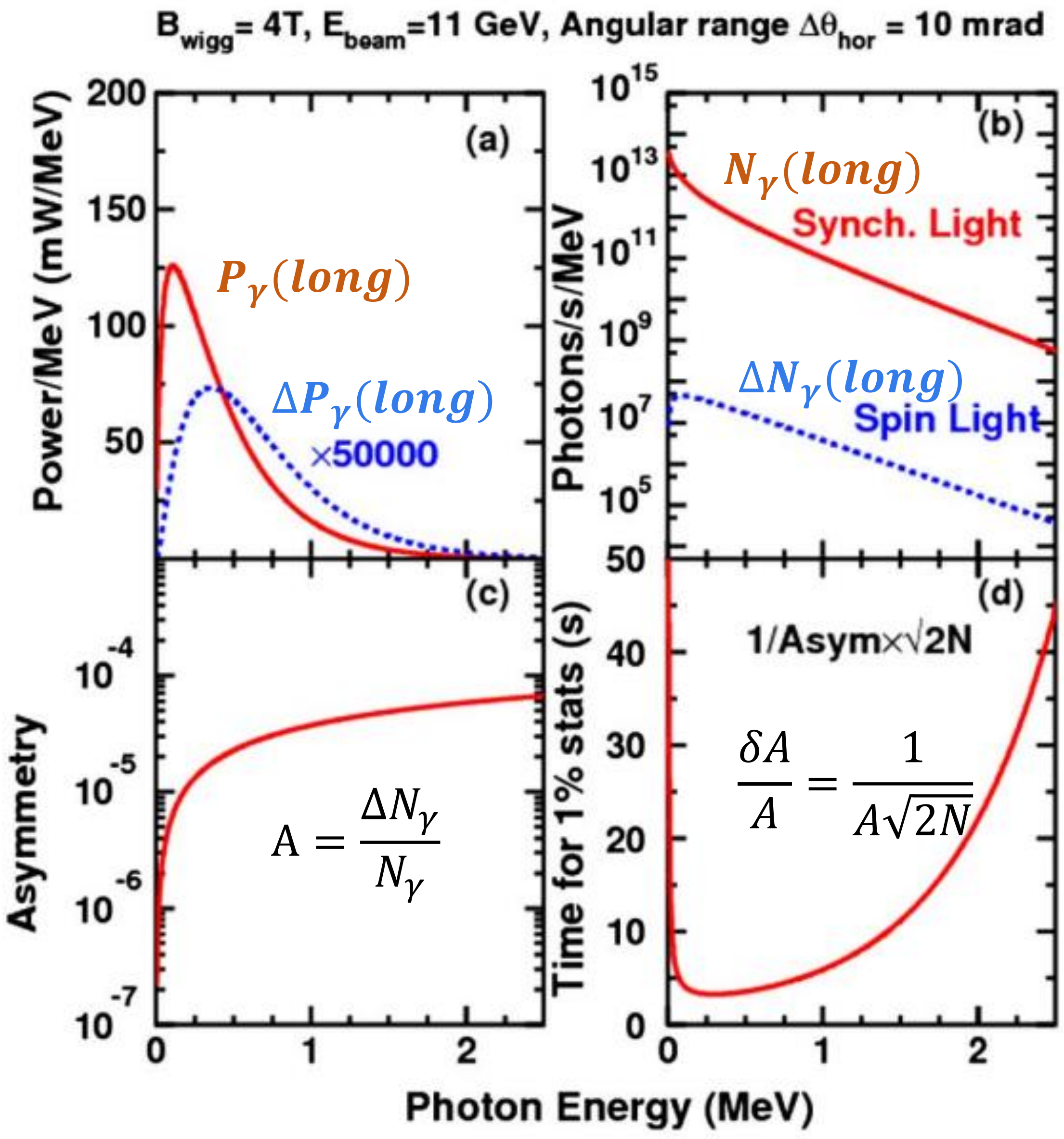
$$\Delta N_\gamma(p_z) = \frac{3}{\pi^2}\frac{1}{137}\frac{I_e}{e}p_z\xi\gamma\Delta\theta\int_{y_1}^{y_2} y^2 dy \int_{0}^{\alpha}\alpha(1+\alpha^2)^{3/2} \times K_{1/3}(z)K_{2/3}(z)d\alpha \quad (9)$$

The asymmetry is defined:

$$A = \frac{\Delta N_\gamma}{N_\gamma}$$

Performance of spin-light polarimeter

(a)The total SR power and the spin-dependence power. Integrated over a horizontal angular acceptance of $\Delta\theta$=10 mrad

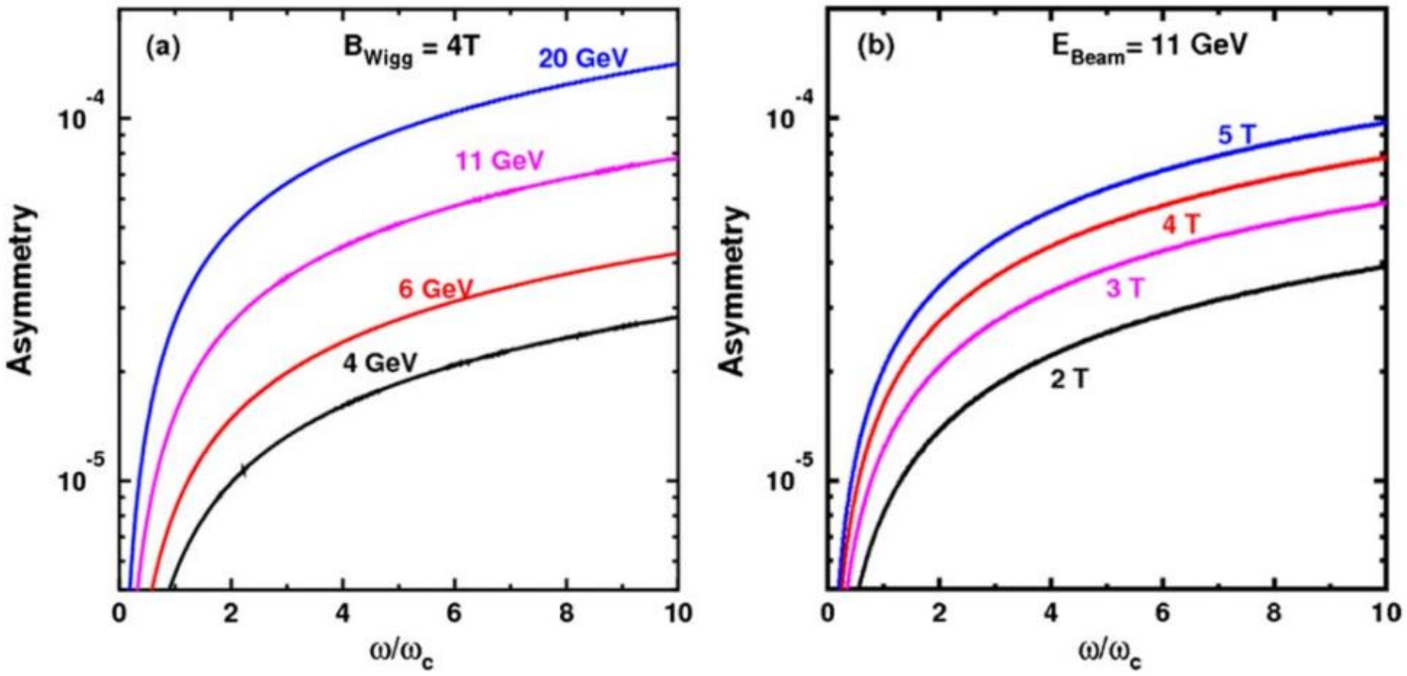


(b) The number of SR photons N_γ, and the spin-light photons ΔN_γ

(c) One should measure the hard tail of the SR spectrum ($E_\gamma > 500keV$)

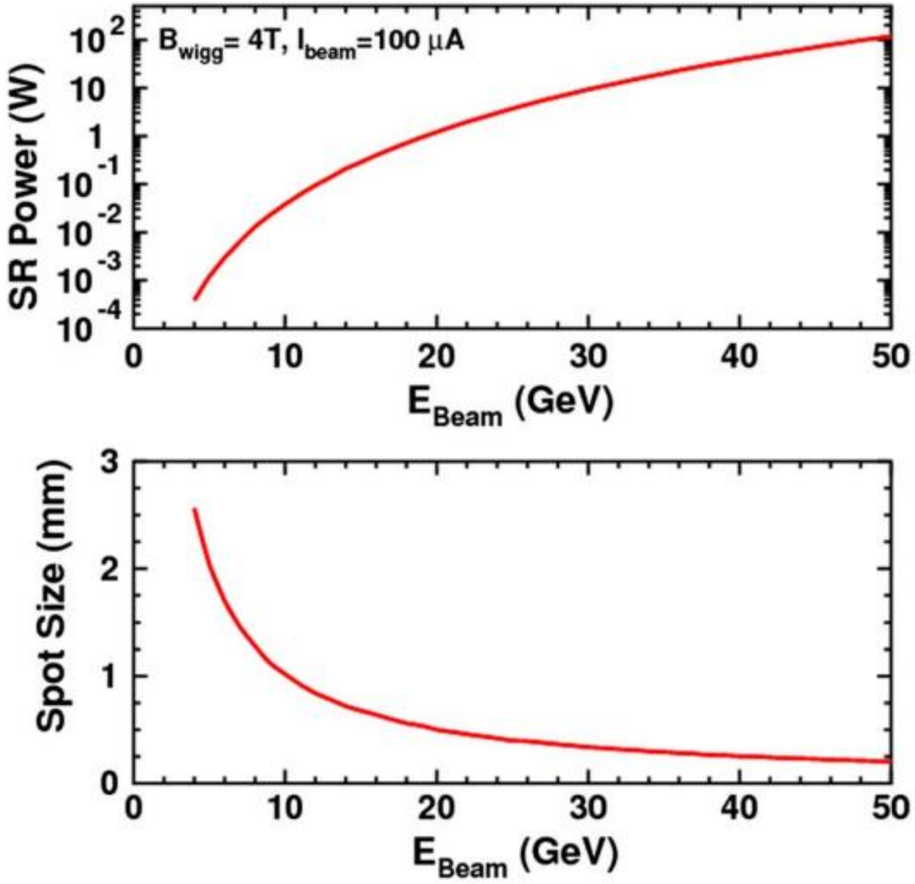
(d) 1% statistical error within few tens of seconds.

Performance of spin-light polarimeter



(a) The energy dependence of the asymmetry for beam energy is 4-20 GeV
(b) The spin dependent asymmetry for magnetic field B = 2-5 T

Summary: it is best suited for the 4 - 20 GeV energy range for currents less than 10 mA

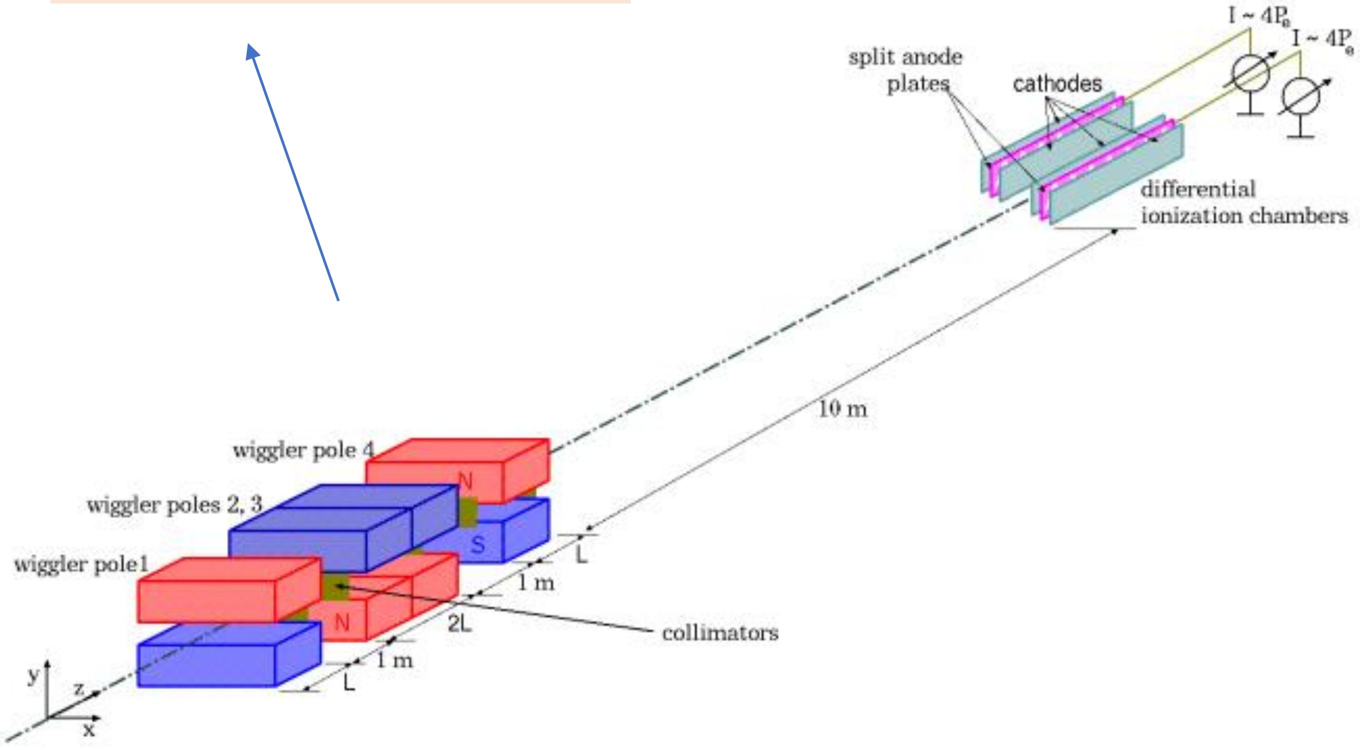


(top) The total SR power as a function of the electron beam energy
(bottom) The vertical size of the SR beam spot vs the electron beam energy

Spin-light polarimeter

layout of the spin-light polarimeter include
- a 3-pole wiggler magnet
- collimators
- a split plane ionization chamber (IC)

Produce SR photons



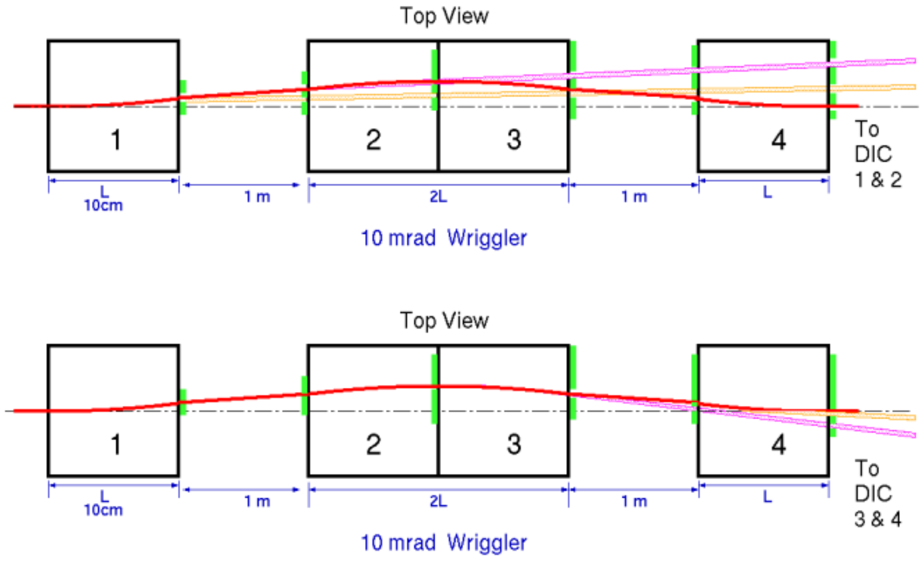


Figure 5: (Top - Bottom): A. Schematic diagram showing the location of collimators (yellow strips) on the wiggler pole faces which guide the SR photons produced at the wiggler magnet to the beam left ionization chamber.; B. Schematic diagram showing the location of collimators on the wiggler pole faces which guide the SR photons produced at the wiggler magnet to the beam right ionization chamber.

backup

results

- Spin-light polarimeter can achieve statistical precision of in measurement cycles of less than 10 minutes for 4 - 20 GeV electron beams with beam currents of ~100μA.