Theory of Resonant Depolarization

Summary of what I've learned from a paper on RD of VEPP-4M

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Resonant depolarization

- A depolarizer is a device with an external oscillating longitudinal or radial magnetic field of frequency f_d. Its frequency can be scanned so that

$$\nu_0 \pm \frac{f_d}{f_0} = k + \epsilon(t)$$

f_0 is the revolution frequency of the storage ring, ν_0 is the closed orbit spin tune, k is an integer. ϵ is the detuning from the "forced" spin resonances

$$\nu_0 \pm \frac{f_d}{f_0} = k \qquad (1)$$

- When one of the spin resonances Eq.(1) is crossed under certain conditions, depolarization can be observed.
- The exact resonant value f_d can be measured and the fractional part of ν_0 is deduced.
- $\nu_0 = a < \gamma > + \Delta\nu$, the systematic error $\Delta\nu$ is a function of energy and field perturbations.

Theory: single crossing

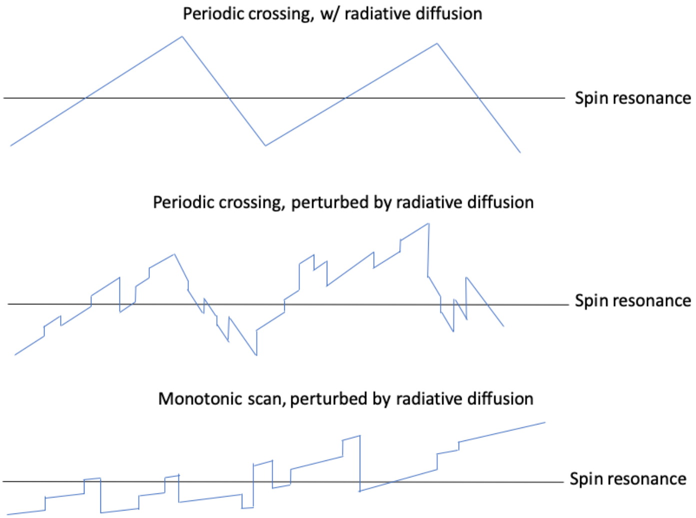
The depolarization after a single crossing of a spin resonance is

$$S_y = S_{y0}[2\exp(-\frac{\pi|\omega_k|^2}{2\alpha_0}) - 1] \tag{2}$$

with the acceleration rate $\alpha_0 = \dot{\epsilon}\omega_0$, $\omega_0 = 2\pi f_0$, ω_k is the spin resonance strength.

- Depolarization occurs when $\dot{\epsilon} \sim |\omega_k|^2\omega_0$.
- Rapid crossing ($\dot{\epsilon} \gg |\omega_k|^2\omega_0$) leads to a tiny polarization loss $\frac{\pi\omega_k|^2\omega_0}{\dot{\epsilon}}$.
- Adiabatic crossing ($\dot{\epsilon} \ll |\omega_k|^2\omega_0$) leads to a spin flip.

Resonance crossing and radiative diffusion



Theory: successive uncorrelated fast crossings

Assume the depolarizer frequency f_d is close to a spin resonance, if there is an additional modulation of ϵ with a low frequency f_m and an amplitude ϵ_{max}, the spin resonance could be periodically crossed with a rate $2\epsilon_{\text{max}} f_m$.

- Each of such crossing can be fast, so that only a tiny depolarization occurs.
- Successive crossings can be uncorrelated due to quantum fluctuation, if

$$\nu_0^2 \geq \tau_p f_m^3 / f_0^2 \tag{3}$$

- Then these consecutive perturbations can accumulate diffusively, and lead to a depolarization time τ_d

$$\tau_d \approx \frac{\epsilon_{\text{max}}}{|\omega_k|^2 \omega_0} \tag{4}$$

- The fast crossing condition is $f_m \tau_d \gg 1$.

Key beam parameters

Symbol	Parameter	Expression
σ_δ	rms relative energy spread	$\sigma_\delta = \frac{<\Delta E^2>}{E_0}$
ν_z, λ_z	synchrotron tune, long. damping decrement	$\lambda_z = T_{\text{rev}}/\tau_z$
ν_0	closed orbit spin tune	$\nu_0 = a\gamma_0$
σ_ν	instantaneous spread of spin frequency	$\sigma_\nu = \nu_0 \sigma_\delta$
ϵ_ν	intrinsic linewidth of spin frequency	$\frac{\epsilon_\nu}{\nu_0} \approx \sqrt{(\frac{\alpha_c {\sigma_\delta}^2}{2})^2 + < B'' \sigma_x^2 >^2}$
ϵ_{diff}	diffusive lindwidth of spin frequency	$\frac{\sigma_\nu}{\nu_z} \frac{\lambda_z}{2\pi}$

- Typically, $\sigma_\nu \gg \epsilon_\nu$.
- A forced RD process takes a much longer time compared to orbital oscillations.
- The intrinsic linewidth corresponds to average over orbital oscillations.
- The error of RD is much less than σ_ν, but is limited by $\sqrt{\epsilon_\nu^2 + \epsilon_{\text{diff}}^2}$.

Key depolarizer parameters

Symbol	Parameter	Expression
ϕ	orbital rotation angle	$\phi = Bl/B\rho$
$\lvert\omega_k\rvert$	spin resonance strength	$\lvert\omega_k\rvert = \nu\lvert F^\nu\rvert\frac{\phi}{2\pi}$ for TEM kicker
f_d	depolarizer scan frequency	$\nu_0 \pm f_d/f_0 = k + \epsilon(t)$
δf_d	spectral linewidth	$\delta f_d \approx \sqrt{df_d/dt}$

Table: Key depolarizer parameters

- For a longitudinal field depolarizer, $\lvert\omega_k\rvert = \frac{\phi}{4\pi}$.
- For a standing wave transverse field depolarizer, $\lvert\omega_k\rvert = \nu\lvert F^\nu\rvert\frac{\phi}{4\pi}$.
- δ_{f_d} is determined by the thermal noise of its signal generator under stationary condition.
- At uniform frequency tuning during scanning, additional broadening occurs as a consequence of the uncertainty relation, and usually is the main contribution to δ_{f_d} as shown in the table.

Depolarization in the noise band of depolarizer

- In fact, ω_k due to the depolarizer is generally very weak, while the frequency scan is relatively fast, a monotonic scan of the depolarizer frequency to cross a spin resonance is in the "fast crossing" domain.
- When the depolarizer frequency is very close to the spin resonance, the successive uncorrelated fast crossings occur due to the noise band δf_d of the depolarizer, and could lead to "in-band depolarization".
- This requires $max\left\{|\epsilon|, \epsilon_\nu\right\} < \frac{\delta f_d}{f_0}$ and $\nu_0^2 \geq \tau_p(\delta f_d)^3/f_0^2$.
- The depolarization time is then

$$\tau_d \approx \frac{\delta f_d}{4\pi|\omega_k|^2 f_0^2} \tag{5}$$

- "In-band depolarization" is possible if each resonance crossing in the noise band is fast: $(\delta f_d/f_0)^2 \gg |\omega_k|^2$.

Modulation resonances

- Modulation resonances $\nu + l\nu_{\mathrm{mod}} \pm \nu_d = k$ are "solitary" when $\sigma_\nu \ll \nu_\gamma$.
- Crossings of spin resonances due to synchrotron oscillations are "correlated" when $\nu_0^2 \ll \tau_p {\nu_\gamma}^3 / f_0$.
- Under these conditions, resonant depolarization might occur near each modulation resonance.
- The ratio of τ_d at 1st modulation (l=1) and the main resonance (l=0) is

$$\frac{\tau_d^{(1)}}{\tau_d^{(0)}} = \frac{I_0(A)}{I_1(A)} \tag{6}$$

- Synchrotron oscillations: $\nu(\theta) = \nu_0 + \Delta_\gamma \cos(\nu_\gamma \theta)$, $A = \frac{\sigma_\nu^2}{\nu_\gamma^2}$.
- Regular field pulsations: $\nu(\theta) = \nu_0 + \Delta_B \cos(\nu_B \theta)$, $\Delta_B = \nu\sqrt{< (\frac{\Delta B}{B})^2 >}$, $A = \frac{\Delta_B^2}{\nu_B^2}$.

Depolarization band

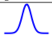
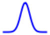
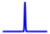
- Depolarization band: the difference bettween the depolarization frequencies during scan "up" and "down".
- Initially, a larger depolarization band could be due to depolarization at modulation resonances.
- This ambiguity can be suppressed by
  - Synchrotron sideband: scan synchrotron tune to check the resonance position.
  - Regular field pulsation: reduce the level of power supply ripple, at 50 Hz and harmonics.
- Then the depolarization band is determined by
  - frequency scan step
  - depolarization linewidth
  - the energy drift between two successive calibrations
  - at a minimum, the spin linewidth

RD operation modes

RD operation modes differ in the ratio of the depolarizer depolarizer and the spin linewidth.

- "CLUB": fast preliminary energy calibration with an accuracy of about 10^{-5}.
- "J/ψ": most precise calibrations (10^{-6}) in narrow resonance peaks.
- "CPT": precise comparison of the spin frequencies of electron and positron.

Table 2. Three main RD operation modes with the same depolarizer plates (Depolarizer 3).

Type	w_k	dE/dt keV/c	δ_d keV	τ_d s	ΔE keV	Line width depolarizer	spin
"CLUB"	$\sim 10^{-6}$	10	2.3	~ 1	10		
"J/ψ"	5×10^{-7}	0.3	0.4	~ 1	2		
"CPT"	4×10^{-8}	0.005	0.05	~ 100	0.002		

- In the table, $\delta_d = \frac{E}{a\gamma}\frac{\sqrt{\dot{f}_d}}{f_0}$, i.e., $\delta_d[keV] = 1000\sqrt{\frac{dE/dt[keV/s]*0.4406}{f_0}}$
- The intrinsic spin linewidth is $\sim$1 keV (2 Hz).

Experiment setup and typical measurements

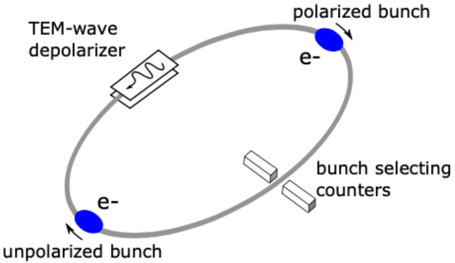
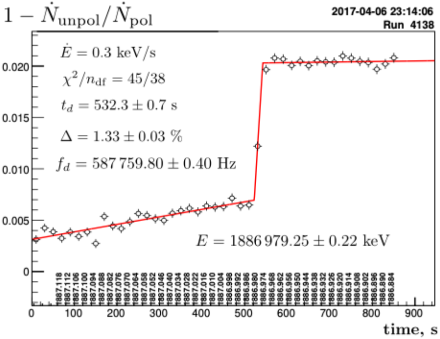


Fig. 5. Two-bunch scheme with Touschek polarimeter.



- The depolarizer scan rate and amplitude are set up to provide a depolarization time of about 1 s.
- The typical time for collecting statistics at one point is 20 s.
- The jumps occur between two measurement points, which ensures the accuracy of the absolute energy value at a level of 10^{-6}.

Comparison between different machines

Table 3. Comparison of parameters affecting the spin spectrum in different colliders

Collider	E, GeV	σ_ν	ν_γ	τ_p, min	f_0, kHz	σ_ν/ν_γ	Γ	ϵ_ν	$\lambda_\gamma/2\pi$, rad^{-1}	$\varepsilon_{\rm diff}$
VEPP-4M	1.85	0.0015	~ 0.01	4200	820	0.15	5×10^{-5}	$\sim 4\times10^{-6}$	1.8×10^{-6}	2.7×10^{-7}
	4.73	0.0098	0.015	45		≈ 0.7	~ 0.01	$\sim 10^{-4}$	3.0×10^{-5}	2.1×10^{-5}
LEP	45.6	0.061	0.083	300	11	0.73	0.054	—	4.7×10^{-4}	3.4×10^{-4}
FCC-ee	45.6	0.039	0.0250	15000	3	1.56	0.155	$\sim 7.3\times10^{-5}$	1.25×10^{-4}	2×10^{-4}

- CEPC-Z, $\sigma_\nu = 0.041$,$\nu_\gamma = 0.035$,$\lambda_\gamma/2\pi = 1.36e-4$, $\epsilon_{\text{diff}} = 1.6e-4$
- CEPC-W, $\sigma_\nu = 0.127$,$\nu_\gamma = 0.061$,$\lambda_\gamma/2\pi = 7.13e-4$, $\epsilon_{\text{diff}} = 1.5e-3$

If naively consider this limits the accuracy of beam energy measurement by RD, then this corresponds to 70 keV @ Z, 660 keV @ W.