北京航空航天大學
BEIHANG UNIVERSITY

# Recent Progress on heavy quark on the Lattice

*A personal prospective*

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# OUTLOOK

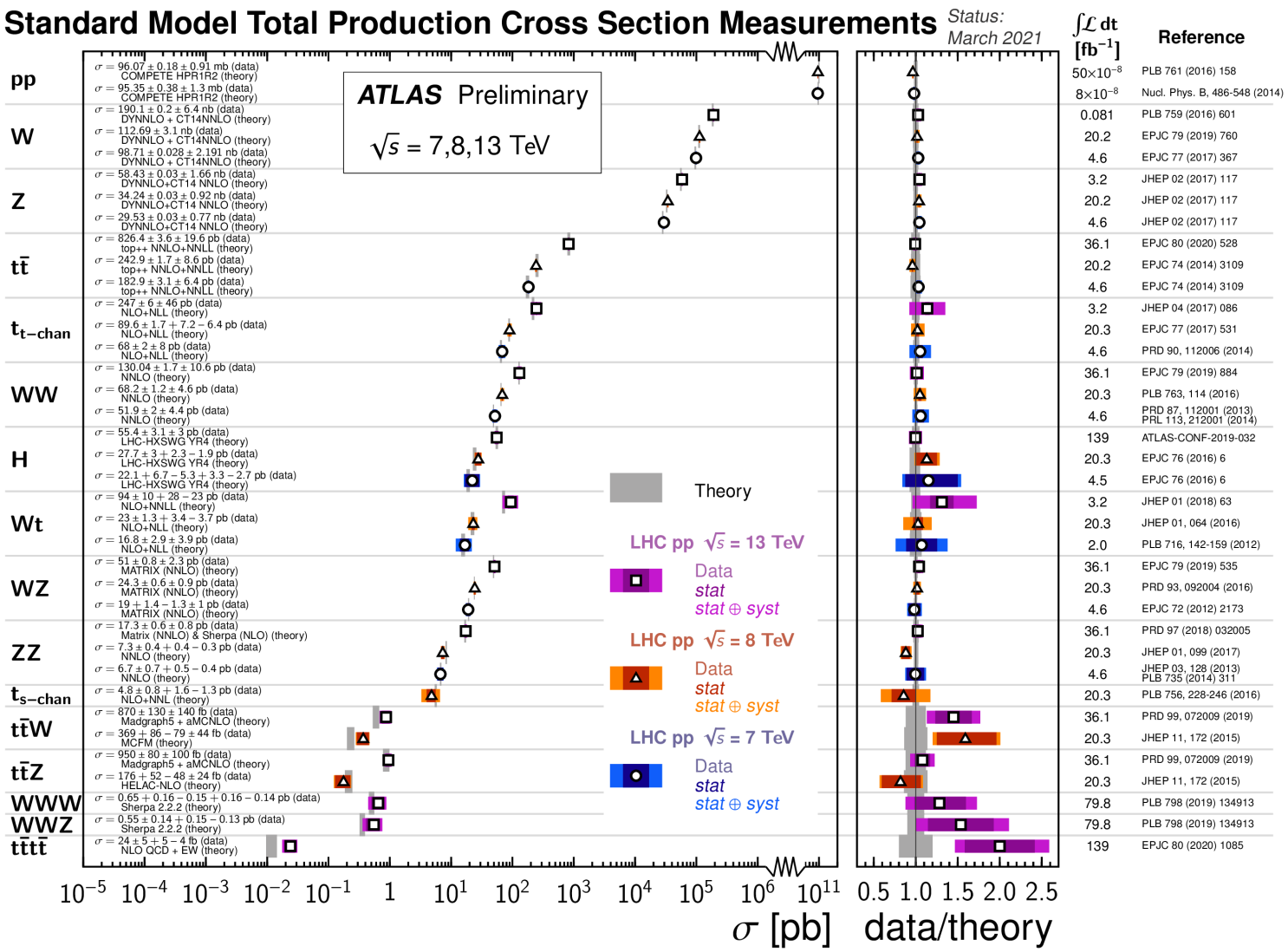
- Motivation
- Brief introduction of Lattice QCD
- Flavor Phenomenology from Lattice QCD
- Distribution amplitudes from Lattice QCD
- Challenges and opportunities

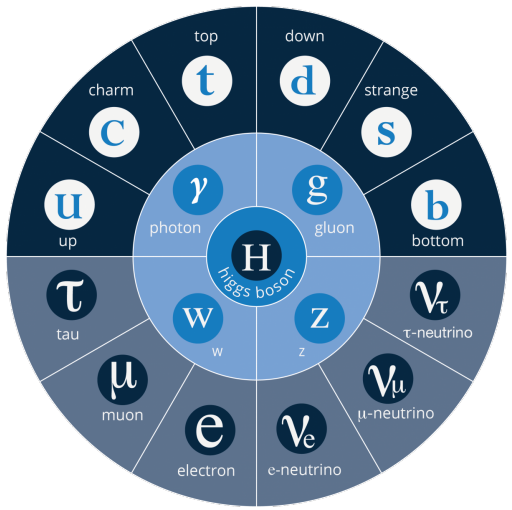
# Motivation

What can we learn from flavor physics?

And what kind of help can LQCD provide?

# Great success of Standard Model




Great success of EW and QCD!

# Searching for new physics beyond SM

## Dark clouds on SM?

- ➢ Neutrino: Mass, Dirac or Majorana Fermion?
- ➢ Dark matter, dark energy?
- ➢ ……
- ➢ Hierarchy problem?
- ➢ Landau pole v.s. Triviality?

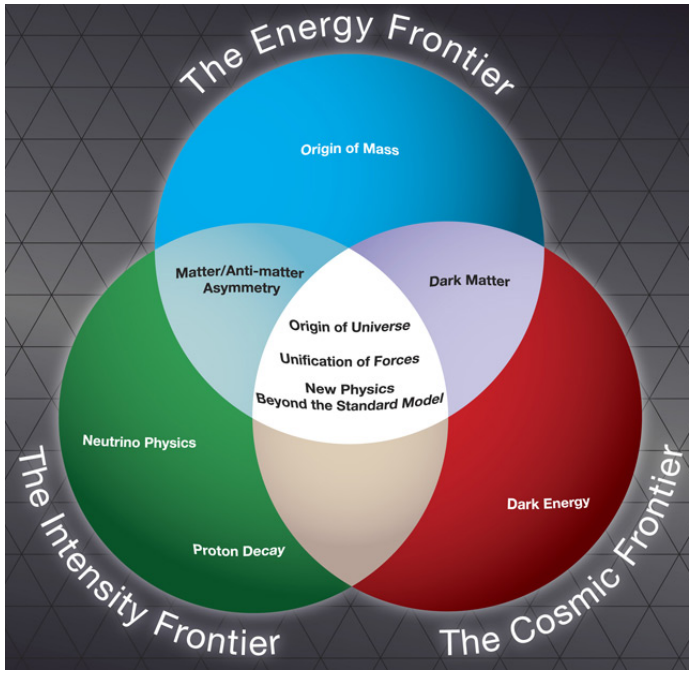
## Beyond SM: Three Frontiers

✓ Direct search: LHC, SPPC, ……

✓ Indirect search:

The flavor sector ⇒
- g-2
- B factories
- Tau/charm factories
- Precision measurements

……



# More opportunities from FLAVOR physics

**The flavor sector is sensitive to NP at very high energy scales:**

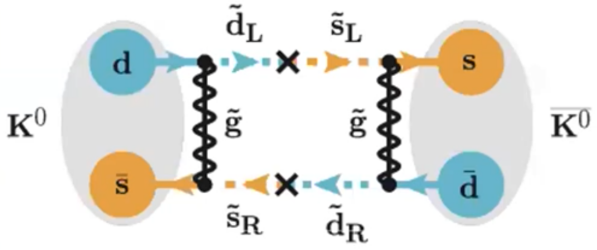
- New particles may contribute to SM processes via loops, leading to deviations from SM expectations
- We may see (or perhaps already seen) evidence from NP from anomalies in flavor sector, before we directly produce new particles in colliders

### Muon anomalous magnetic moment

Sensitive to SUSY, dark photon, ……


### Flavor-changing neutral currents (FCNC)

Sensitive to SUSY, leptoquarks, flavor-changing Z', composite Higgs, ……



# Lattice QCD

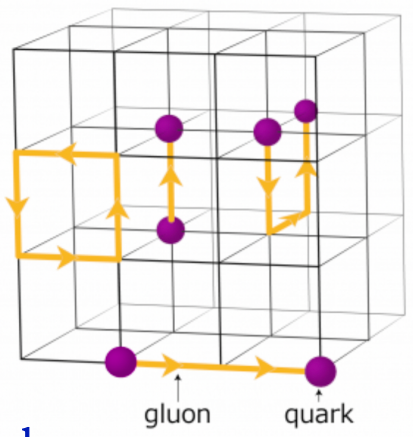
## Capabilities and Limitations

# Lattice QCD in a nutshell

**LQCD is formulated as a Feynman path integral on a 4D Euclidean grid.**

**Simulations provide a stochastic computation follows QCD Lagrangian:**

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi - \frac{1}{4}G^{a}_{\mu\nu}G^{a,\mu\nu}$$




- Gluon fields on links of a hypercube;
- Quark fields on sites: approaches to fermion discretization – Wilson, Staggered, Overlap;

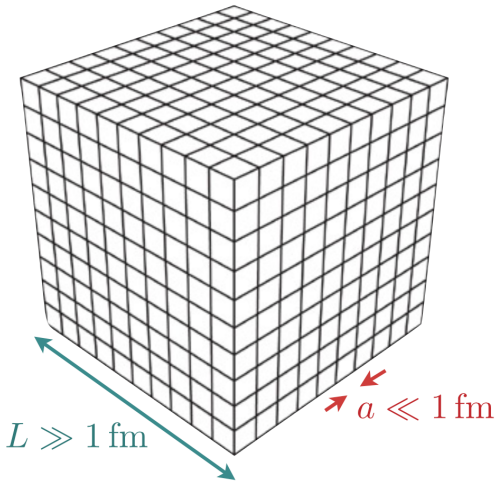
☞ **Discrete: lattice spacing $a$ → UV regulator; box length $L$ → IR regulator;**

☞ **Derivatives: discretization errors ($a \to 0$); $\mathcal{O}(a)$ improved actions; ……**

☞ **Finite volume ($M_{\pi}L \to \infty$): FV errors exponentially small for $M_{\pi}L > 4$;**

☞ **Chiral extrapolation ($M_{\pi} \to 135\text{MeV}$);**

☞ **Numerical importance sampling of path integral: statistical errors.**



# Lattice QCD in a nutshell

**LQCD Observables:**

- Building blocks: ensembles of gauge configurations; quark propagators
- Hadron & interactions put in as external probes: N-point correlation function

**LQCD Methodology:**

☞ Generate gauge configurations;

☞ Calculate quark propagators on the gauge configurations;

☞ Formulate operators that best probe the physics:

- Low energy effective operators encapsulating SM/BSM physics;

☞ Construct hadronic correlation functions by the building blocks;

☞ Extract hadron ground states by reduction formula;

☞ Evaluate the hadronic matric elements.

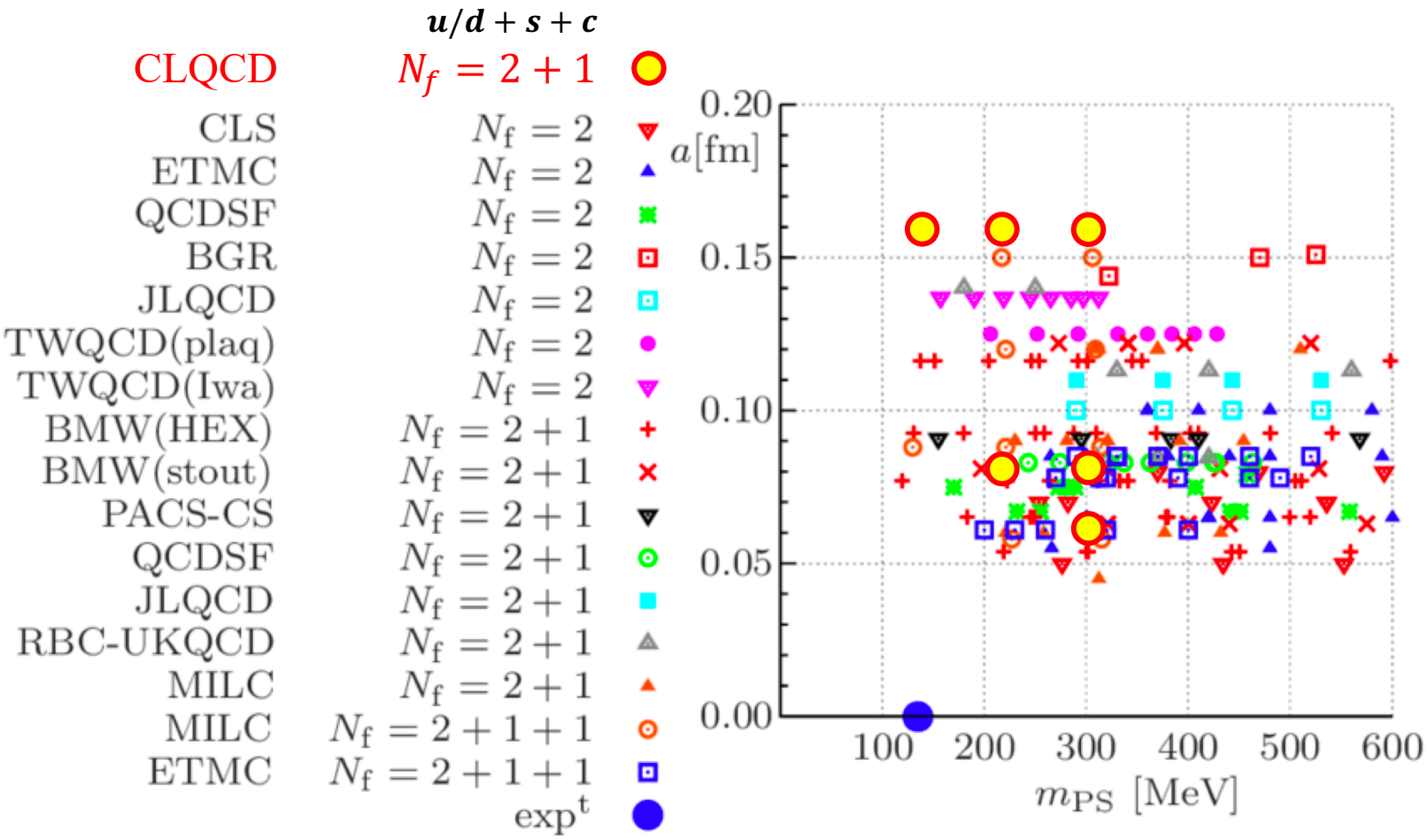
# Recover to continuum physics

| Lattice v.s. Continuum | |
|---|---|
| We simulate: | We want: |
| 😀 At finite lattice spacing $a$ | 🤔 $a \to 0$ |
| 😀 In finite volume $L^3$ | 🤔 $L \to \infty$ |
| 😀 Euclidean space | 🤔 Minkowski space ⇨ **Lost the real time information!** |
| 😀 Lattice regularization | 🤔 Some continuum scheme |
| 😀 Some bare input quark masses: $am_l,\ am_s,\ am_c,\ am_b$ In general, $m_\pi^{\rm lat} \neq m_\pi^{\rm phy}$ | 🤔 $m_q^{\rm lat} = m_q^{\rm phy}$ |

⇨ **Need to <u>control all limits</u>:** particularly simultaneously control FV and discretization

⇨ **<u>Universality</u>:** different input parameters **must** give converge results.

# Recover to continuum physics




*Source image from K. Jansen et al, added CLQCD (in preparation) in extras.*

# Flavor Phenomenology from LQCD

Leptonic & semileptonic decays of heavy hadron

CKM matrix elements & unitarity

# The CKM quark-mixing matrix

The Cabibbo-Kobayashi-Maskawa (CKM) matrix parameterizes the mixing of quark flavors under weak interactions.

- ☞ 3×3 unitary matrix ⇒ three mixing angle & one CP violation phase
- ☞ Elements largest along the diagonal ⇒ hierarchical structure as expansion in power of $\lambda \equiv |V_{us}| \sim 0.22$
- ☞ The CKM matrix elements are fundamental SM parameters that must be obtained by matching theory with experiment

*PDG2020*

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$= \begin{pmatrix} 0.97401 \pm 0.00011 & 0.22650 \pm 0.00048 & 0.00361^{+0.00011}_{-0.00009} \\ 0.22636 \pm 0.00048 & 0.97320 \pm 0.00011 & 0.04053^{+0.00083}_{-0.00061} \\ 0.00854^{+0.00023}_{-0.00016} & 0.03978^{+0.00082}_{-0.00060} & 0.999172^{+0.000024}_{-0.000035} \end{pmatrix}$$

$0.9985 \pm 0.0005$ ⇒ $10^{-4}$ accuracy

$1.025 \pm 0.022$ ⇒ $10^{-2}$ accuracy

$0.9970 \pm 0.0018$ ⇒ $10^{-3}$ accuracy

$1.026 \pm 0.022$ ⇒ $10^{-2}$ accuracy

# LQCD inputs to the CKM matrix

**Combing experiments + LQCD calculations of "Golden channels" provide the testing of CKM unitary.**

Single hadron in initial state and at most one hadron in final state, both hadrons are stable in QCD

Indirect searching for BSM

$$V_{CKM} = \begin{pmatrix} \boldsymbol{V_{ud}} & \boldsymbol{V_{us}} & \boldsymbol{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu, \pi\ell\nu & B \to \ell\nu, \pi\ell\nu \\ & & \Lambda_b \to \pi\ell\nu \\ \boldsymbol{V_{cd}} & \boldsymbol{V_{cs}} & \boldsymbol{V_{cb}} \\ D \to \ell\nu, \pi\ell\nu & D_s \to \ell\nu, D \to K\ell\nu & B \to D\ell\nu, D^*\ell\nu \\ & \Lambda_c \to \Lambda\ell\nu, \Xi_c \to \Xi\ell\nu & \Lambda_b \to \Lambda_c\ell\nu \\ \boldsymbol{V_{td}} & \boldsymbol{V_{ts}} & \boldsymbol{V_{tb}} \\ \langle B_d | \bar{B}_d \rangle & \langle B_s | \bar{B}_s \rangle & \\ B \to \pi\ell\ell & B \to K\ell\ell & \end{pmatrix}$$

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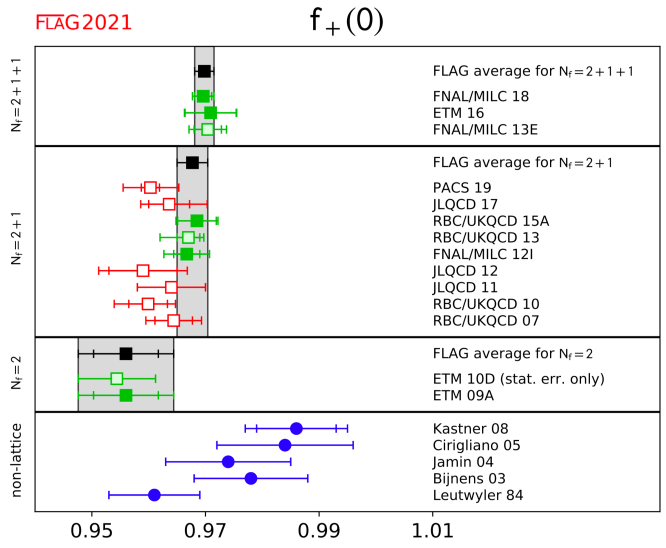
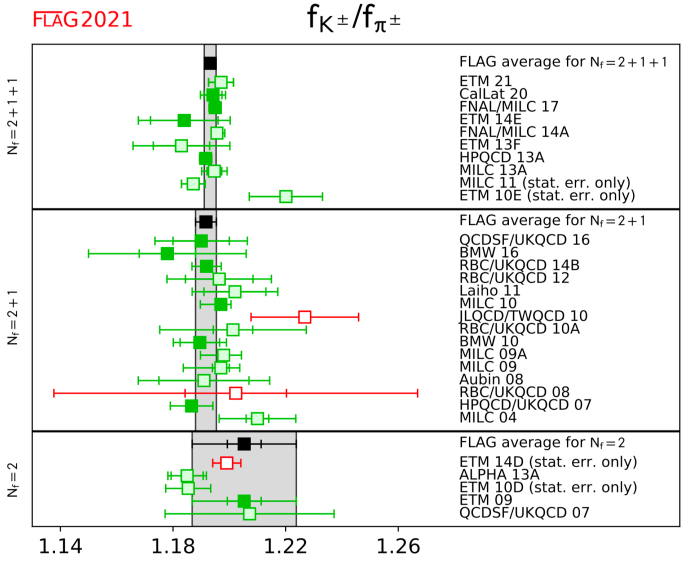
# A quick glance of testing first-row CKM matrix elements




⇨ $f_+^{K\pi}(0)$: direct, $N_f = 2+1+1$: $f_+(0) = 0.9698(17)$ **⇒ 0.175% error**

FNAL/MILC, PRD99(2019); ETMC, PRD93(2016)

direct, $N_f = 2+1$: $f_+(0) = 0.9677(27)$

RBC/UKQCD, JHEP1506(2015); FNAL/MILC, PRD87(2013)

direct, $N_f = 2$: $f_+(0) = 0.9560(57)(62)$ ETMC, PRD80(2009)

⇨ $f_{K^\pm}/f_{\pi^\pm}$: pure QCD including SU(2) isospin-breaking correction:

direct, $N_f = 2+1+1$: $f_{K^\pm}/f_{\pi^\pm} = 1.1932(21)$ **⇒ 0.176% error**

ETMC, PRD104(2021); CalLat, PRD102(2020); FNAL/MILC, PRD98(2018); HPQCD, PRD88(2013); ETMC, PRD91(2015)

direct, $N_f = 2+1$: $f_{K^\pm}/f_{\pi^\pm} = 1.1917(37)$

RBC/UKQCD, PRD93(2016); HPQCD/UKQCD, PRL100(2008); MILC, 1012.0868; BMW, PRD81(2010); S. Dürr *et al.*, PRD95(2017); QCDSF/UKQCD, PLB767(2017)

direct, $N_f = 2$: $f_{K^\pm}/f_{\pi^\pm} = 1.205(18)$ ETMC, JHEP0907(2009)

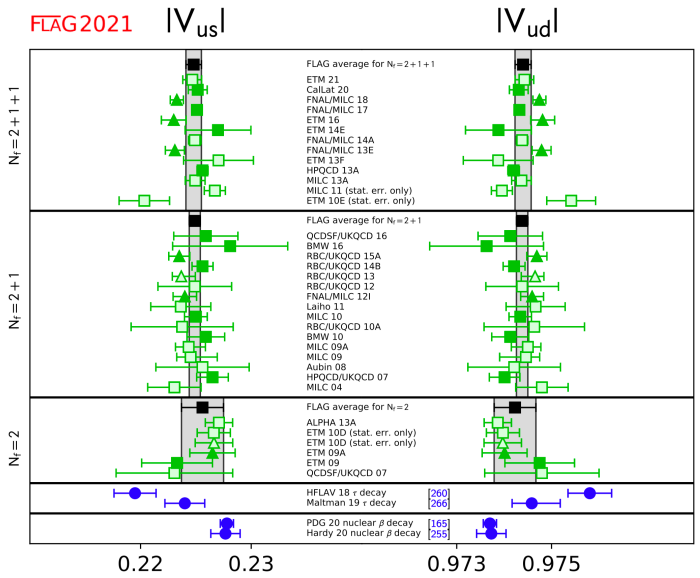
# A quick glance of testing first-row CKM matrix elements

## ⇨ $|V_{ud}|$ and $|V_{us}|$:

Combing experiments data, the $N_f = 2+1+1$ FLAG21 average:

$$|V_{us}| = 0.2232(6)$$

$$|V_{us}|/|V_{ud}| = 0.2313(5)$$



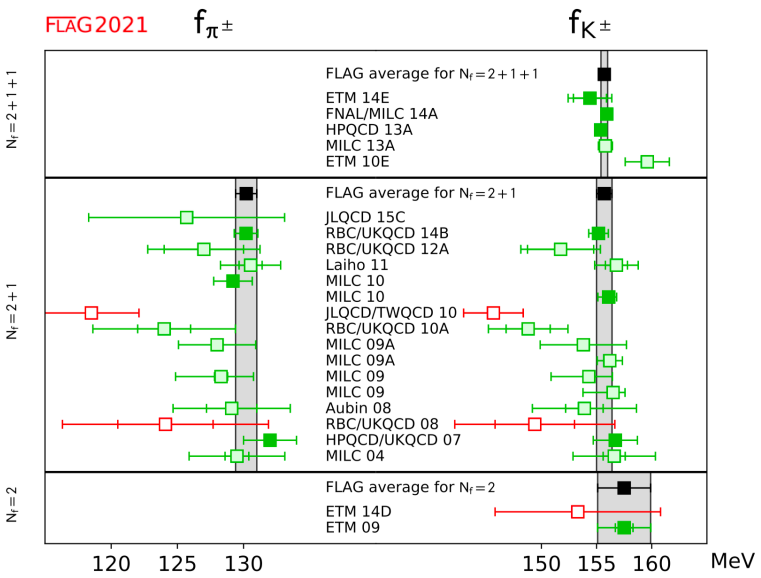

## ⇨ $f_\pi$ and $f_K$:

$N_f = 2+1$ : $f_{\pi^\pm} = 130.2\ (0.8)$ MeV

$N_f = 2+1+1$ : $f_{K^\pm} = 155.7\ (0.3)$ MeV

$N_f = 2+1$ : $f_{K^\pm} = 155.7\ (0.7)$ MeV

$N_f = 2$ : $f_{K^\pm} = 157.5\ (2.4)$ MeV

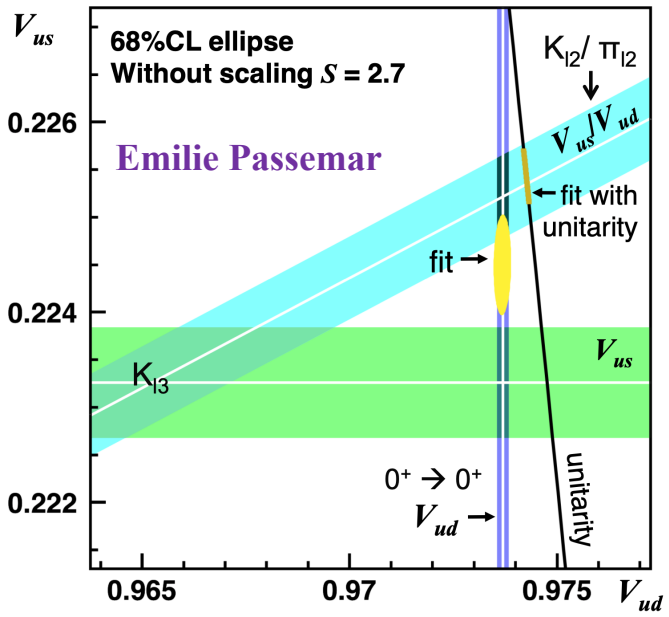



## ⇨ First-row CKM unitarity

$$\Delta_u = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$$

- $f_+(0) + |V_{ud}|$ from PDG: $\Delta_u \sim 4.3\sigma$
- $f_+(0) + f_K/f_\pi$ and $|V_{ud}^{\beta-\text{decays}}|$ :

$$\Delta_u = 2.3 - 2.6\sigma$$

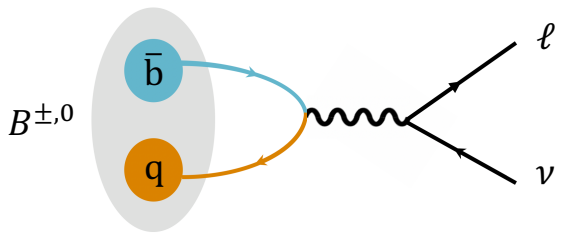


# Now let's turn to the heavy flavor parts……

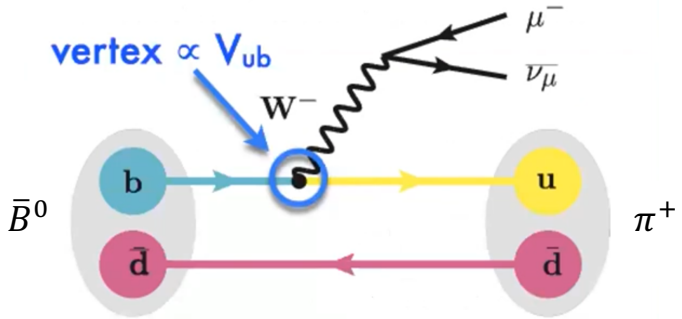
$$V_{CKM} = \begin{pmatrix} \boldsymbol{V_{ud}} & \boldsymbol{V_{us}} & \boldsymbol{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu, \pi\ell\nu & B \to \ell\nu, \pi\ell\nu \\ & & \Lambda_b \to \pi\ell\nu \\ \boldsymbol{V_{cd}} & \boldsymbol{V_{cs}} & \boldsymbol{V_{cb}} \\ D \to \ell\nu, \pi\ell\nu & D_s \to \ell\nu, D \to K\ell\nu & B \to D\ell\nu, D^*\ell\nu \\ & \Lambda_c \to \Lambda\ell\nu, \Xi_c \to \Xi\ell\nu & \Lambda_b \to \Lambda_c\ell\nu \\ \boldsymbol{V_{td}} & \boldsymbol{V_{ts}} & \boldsymbol{V_{tb}} \\ \langle B_d | \bar{B}_d \rangle & \langle B_s | \bar{B}_s \rangle & \\ B \to \pi\ell\ell & B \to K\ell\ell & \end{pmatrix}$$

"Golden channels"

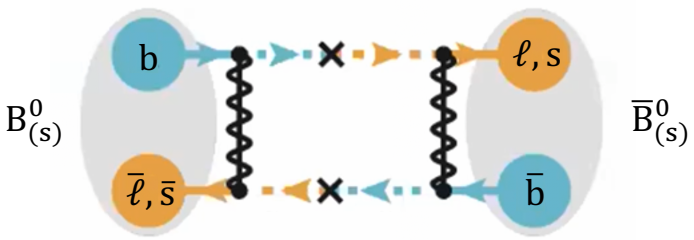
✓ **Decay constant: $\langle 0|\mathcal{J}|1\rangle$**




✓ **Form factors: $\langle 1|\mathcal{J}|1'\rangle$**




✓ **Mixing parameter: $\langle \bar{1}|\mathcal{J}^{\Delta F=2}|1\rangle$**

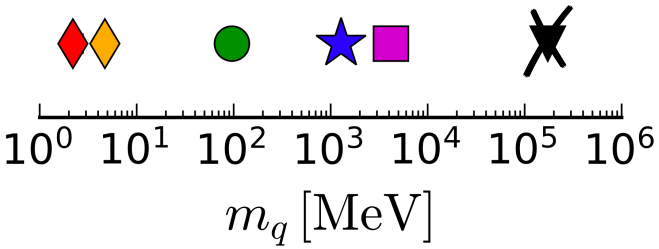


# LQCD realization of heavy quarks

🤔 **Problems of heavy quarks on discrete lattice:**

Care about both **IR (finite volume)** and **UV (discretization)** **regulators**:

$m_\pi L \gtrsim 4$, and $a^{-1} \gg$ mass scale of interest




More flavors, need finer lattice

➪ For $m_\pi = m_\pi^{\text{phy}} \sim 140\text{MeV}$, and $m_c \simeq 1.3\text{GeV}$, $m_b \simeq 4.2\text{GeV}$, that needs:

- $L \gtrsim 5.6\text{fm}$,

  **and** $a^{-1} \gg 1.3\text{GeV} \simeq (0.15\text{fm})^{-1}$ **for charm**

  **and** $a^{-1} \gg 4.2GeV \simeq (0.05fm)^{-1}$ **for bottom** $\Rightarrow$ $\boldsymbol{N \equiv L/a \gg 120,\ N^4 \gg 10^8}$ **lattice sites!**

VERY EXPENSIVE to satisfy both constraints simultaneously……

# LQCD realization of heavy quarks

**Simulate $b$-quark on lattice: expand by $1/m_b$**

| Effective theory approaches | HQET-inspired extrapolation method |
|---|---|
| • Need to include multiple operators matched to full QCD (NRQCD, HQET, static);<br>• Suitable for relativistic heavy-quark physics calculations;<br>• Come with systematic errors which are hard to estimate/reduce. | • Same formula for light and charm, start to be extended to the bottom region;<br>• Theoretically cleaner and systematically improvable;<br>• Need small a to control extrapolation in heavy quark mass. |

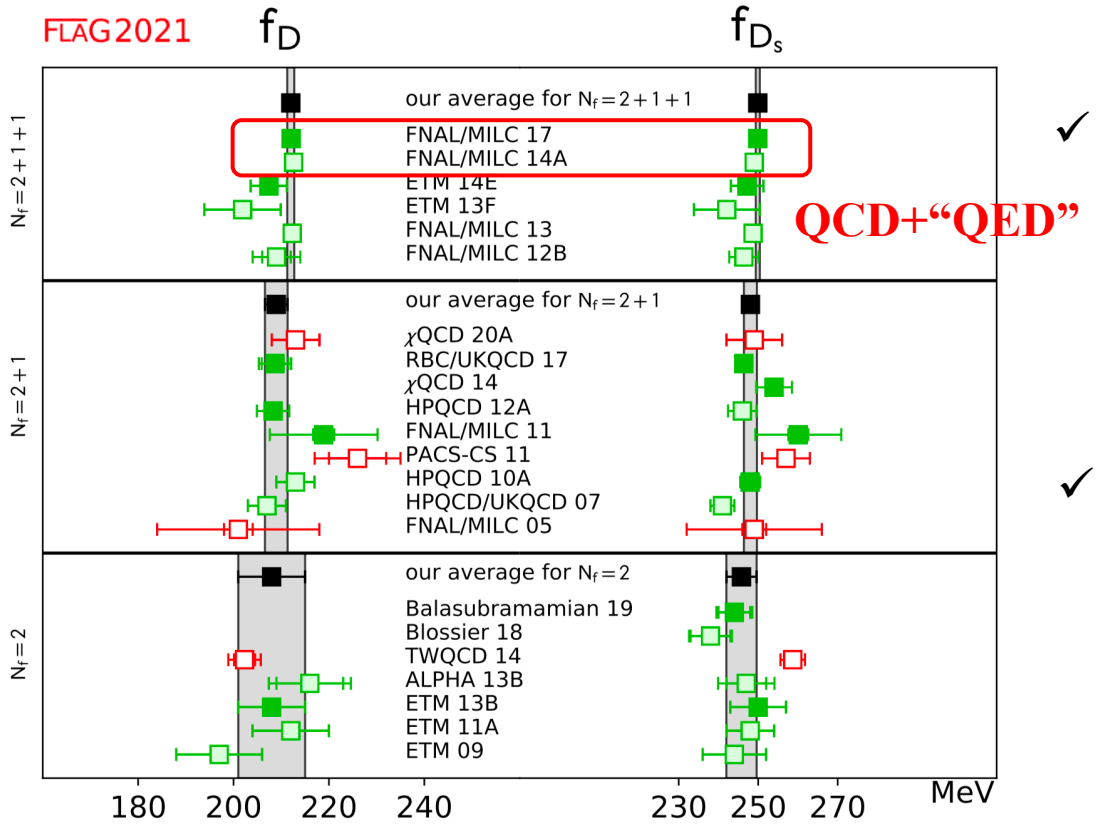
## Very finer lattice on the road……

Lots of efforts to produce **very fine lattice spacings**,

Discrete simulation at $\boldsymbol{m_b^{\mathrm{phy}}}$ **scale** will become possible soon!

# Decay constant

**Decay constant $f_D$ and $f_{D_s}$**




QCD+"QED"

Can be extracted from Euclidean matrix elements of axial current:

$$\langle 0|A^{\mu}_{cq}|D_q(p)\rangle = i f_{D_q}\ p^{\mu}_{D_q},$$

✓ $N_f = 2+1+1$:

$$f_D = 212.0(0.7)\ \text{MeV} \qquad f_{D_s} = 249.9(0.5)\ \text{MeV}$$

$$\frac{f_{D_s}}{f_D} = 1.1783(0.0016)$$

FNAL/MILC, PRD98(2018);

ETMC, PRD91(2015)

✓ $N_f = 2+1$:

$$f_D = 209.0(2.4)\ \text{MeV}$$

$$f_{D_s} = 248.0(1.6)\ \text{MeV}$$

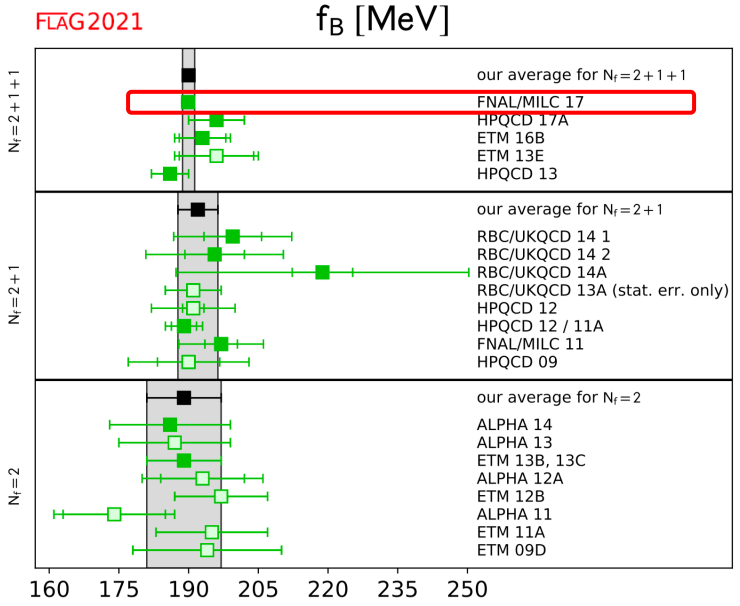
$$\frac{f_{D_s}}{f_D} = 1.174(0.007)$$

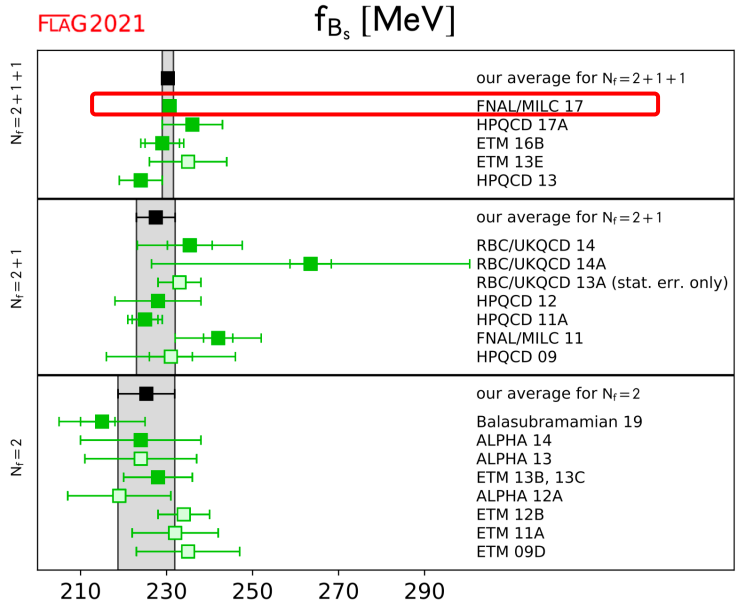
RBC/UKQCD, JHEP12(2017);

χQCD, PRD92(2015);

HPQCD, PRD86(2012), PRD82(2010);

FNAL/MILC, PRD85(2012);




- Experiment measurements:

$$B(B_d \to \mu^+\mu^-) < (1.9) \times 10^{-10} \text{ at 95\% CL},$$
$$B(B_s \to \mu^+\mu^-) = (2.69^{+0.37}_{-0.35}) \times 10^{-9},$$

LHCb, CMS, Nature522(2015)

Sensitive to NP

- Lattice calculations:

$$\langle 0|A^\mu|B_q(p)\rangle = ip_B^\mu f_{B_q}$$

✓ $N_f = 2+1+1$: **~1% error**

$$f_B = 190.0(1.3) \text{ MeV}$$
$$f_{B_s} = 230.3(1.3) \text{ MeV}$$
$$\frac{f_{B_s}}{f_B} = 1.209(0.005)$$

FNAL/MILC, PRD98(2018);
ETMC, PRD93(2016);
HPQCD, PRL110(2013), PRD97(2018)

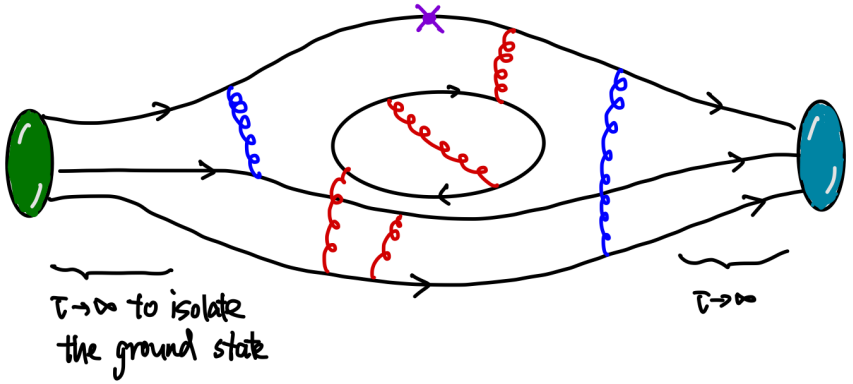
✓ $N_f = 2+1$:

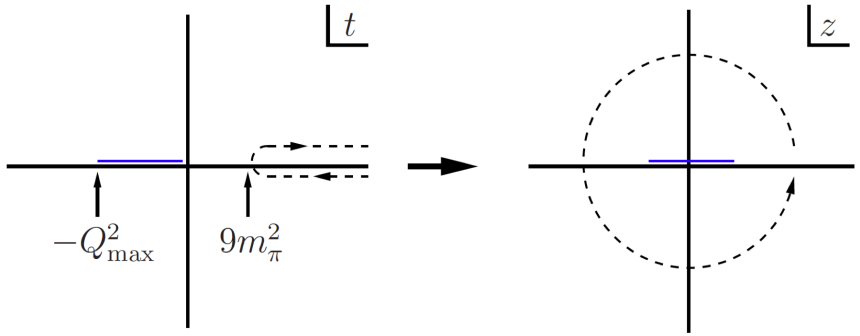
$$f_B = 192.0(4.3) \text{ MeV}$$
$$f_{B_s} = 228.4(3.7) \text{ MeV}$$
$$\frac{f_{B_s}}{f_B} = 1.201(0.016)$$

RBC/UKQCD, PRD91(2015), 1812.08791;
FNAL/MILC, PRD85(2012);
HPQCD, PRD85(2012), PRD86(2012);

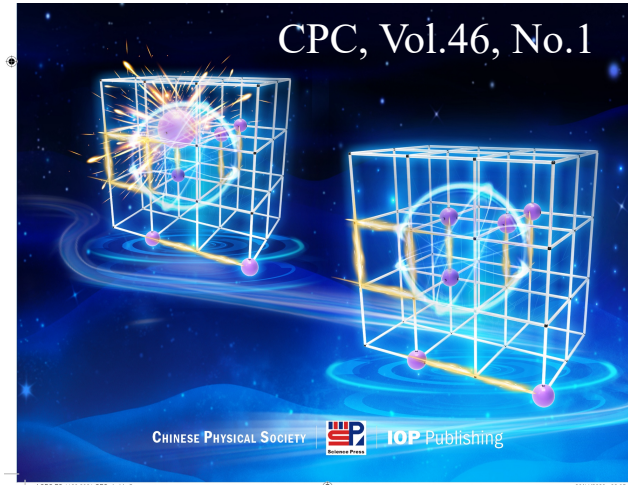
# Form factors

- ✓ **Significantly more information (functions v.s. numbers)**
- ✓ **LQCD calculations of 2-point & 3-point function**
- ✓ **Conformal mapping: z-expansion → wider kinematic range**



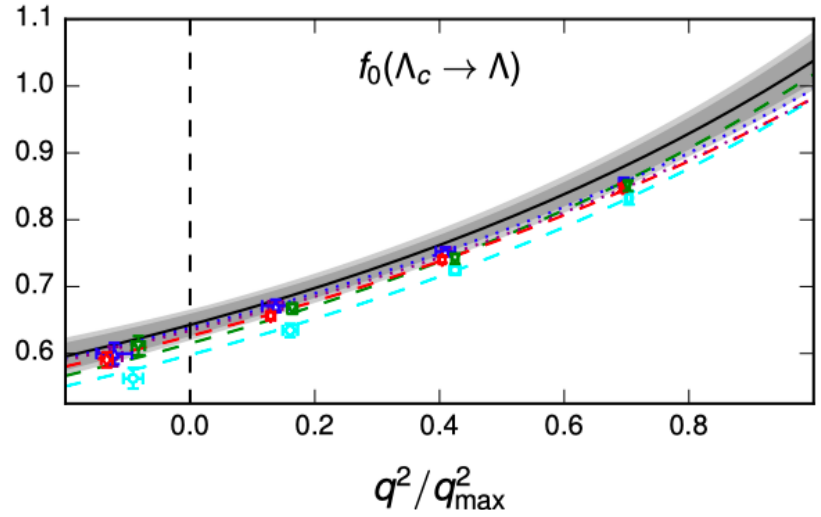

- ✓ **LQCD + experiment → CKM matrix elements**
- ✓ **Better precision needed for BESIII, LHCb and Belle II……**

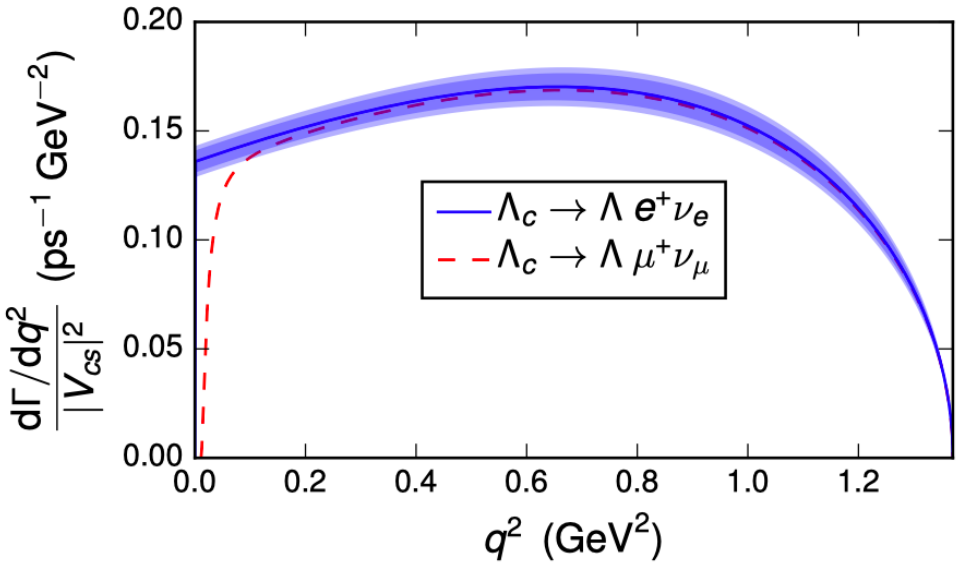


# Form factors for heavy baryon semileptonic decays

| Process | $a$[fm] | $m_\pi$[MeV] | Ref. |
|---|---|---|---|
| $\Lambda_c \to \Lambda$ | 0.08，0.11 | 140-360 | *S.Meinel, PRL118,028001(2017)* |
| $\Lambda_c \to \Lambda(1520)$ | 0.08，0.11 | 300-430 | *S.Meinel, G.Rendon, PRD105,054511(2022), PRD105,L051505(2022)* |
| $\Lambda_c \to n$ | 0.08，0.11 | 230-360 | *S. Meinel, PRD97,034511(2018)* |
| $\Xi_c \to \Xi$ | 0.08，0.108 | 290, 300 | *Q.A. Zhang, et.al, CPC46,011002(2022)* |
| $\Lambda_c \to \Lambda$ | 0.1555 | 550 | *H. Bahtiyar, Turk.J.Phys.45,(2021)* |

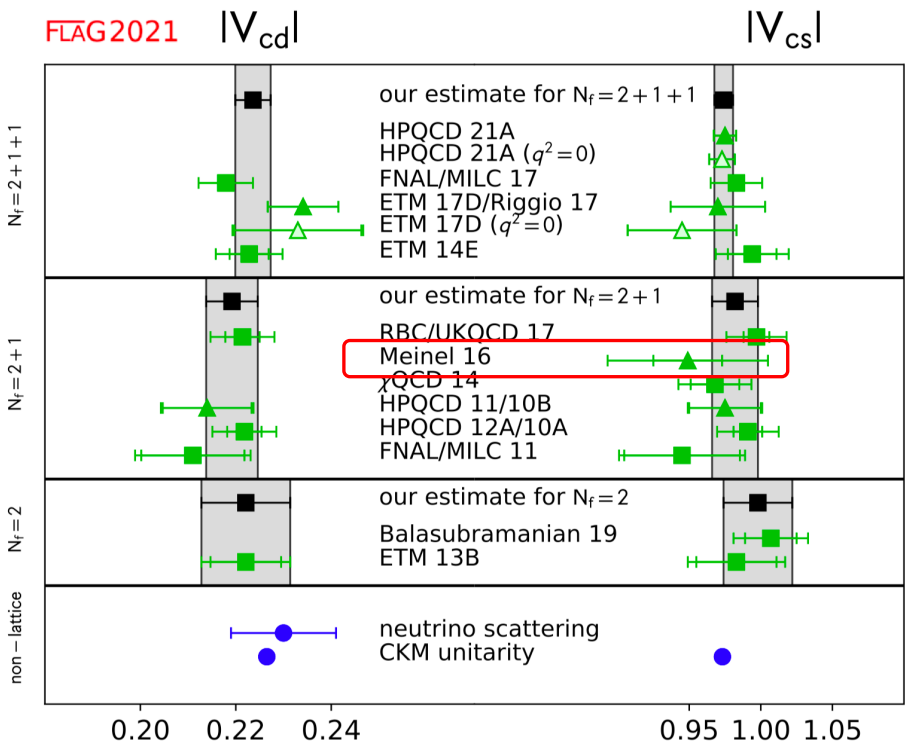
# $\Lambda_c \to \Lambda$ form factors from LQCD




*S.Meinel, PRL(118)2017*

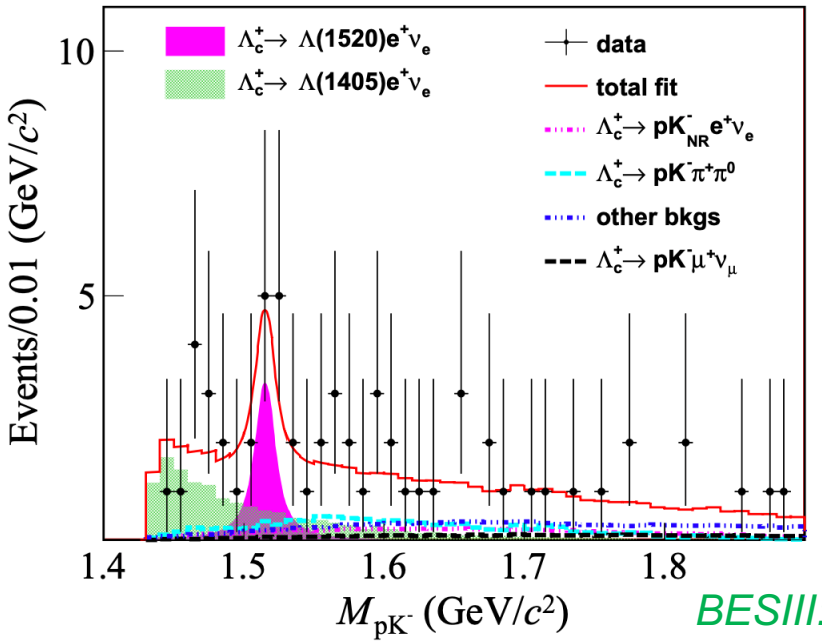
$$\frac{\Gamma(\Lambda_c \to \Lambda \ell^+ \nu_\ell)}{|V_{cs}|^2} = \begin{cases} 0.2007(71)(74)\ \mathrm{ps}^{-1}, & \ell = e, \\ 0.1945(69)(72)\ \mathrm{ps}^{-1}, & \ell = \mu. \end{cases}$$



# $\Lambda_c \to \Lambda(1520)$ decays from LQCD

**BESIII first measurement $\Lambda_c \to \Lambda(1520)$, $\Lambda_c \to \Lambda(1405)$**

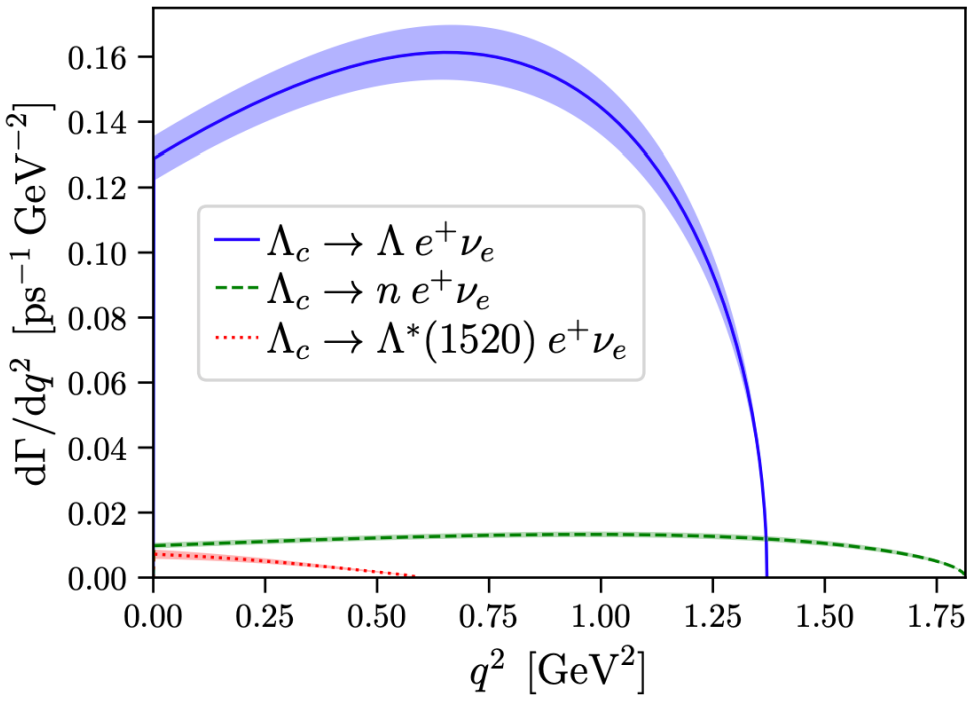
| | $\mathcal{B}(\Lambda_c^+ \to \Lambda(1520)e^+\nu_e)$ | $\mathcal{B}(\Lambda_c^+ \to \Lambda(1405)e^+\nu_e)$ |
|---|---|---|
| Constituent quark model [7] | 1.01 | 3.04 |
| Molecular state [8] | −− | 0.02 |
| Nonrelativistic quark model [9] | 0.60 | 2.43 |
| Lattice QCD [11, 12] | $0.512 \pm 0.082$ | −− |
| Measurement | $0.99 \pm 0.51 \pm 0.10$ | $1.69 \pm 0.76 \pm 0.16$ |



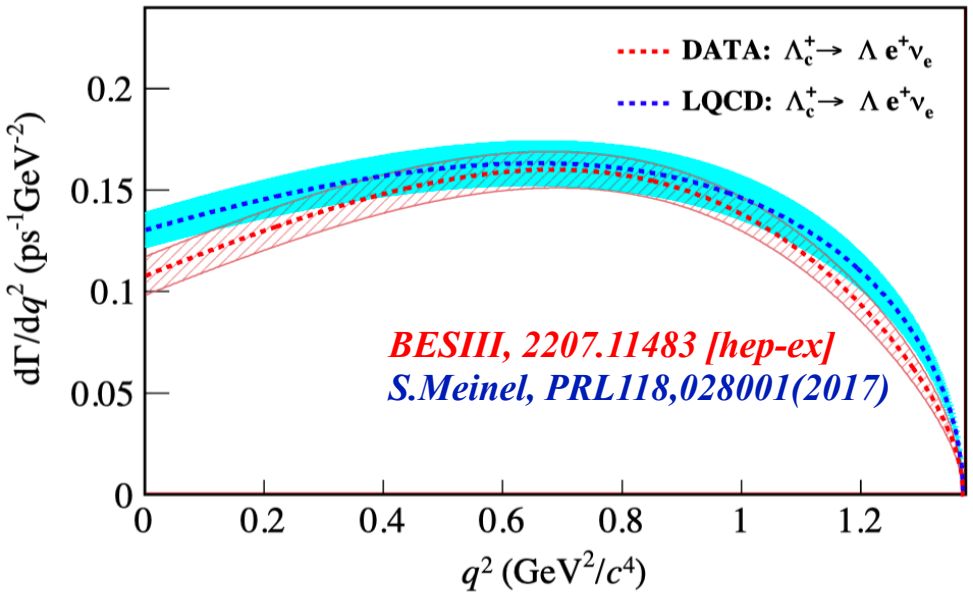

*PRD105,054511(2022), PRD105,L051505(2022)*

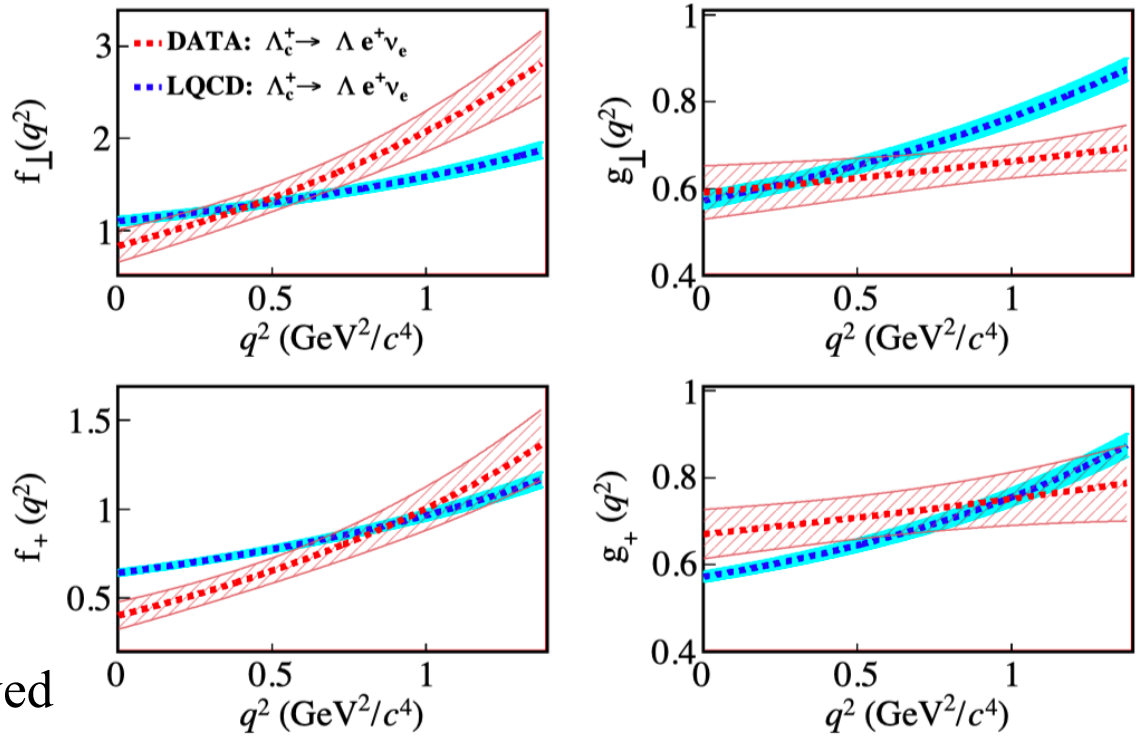
*BESIII: 2207.11483 [hep-ex]*

**Exotic $\Lambda(1405)$?**



# $\Lambda_c \to \Lambda$: LQCD v.s. Experiments?




- ✓ No clear difference in decay widths is observed within uncertainties;
- ✓ Measured FFs show different kinematic behavior compared to LQCD calculations.

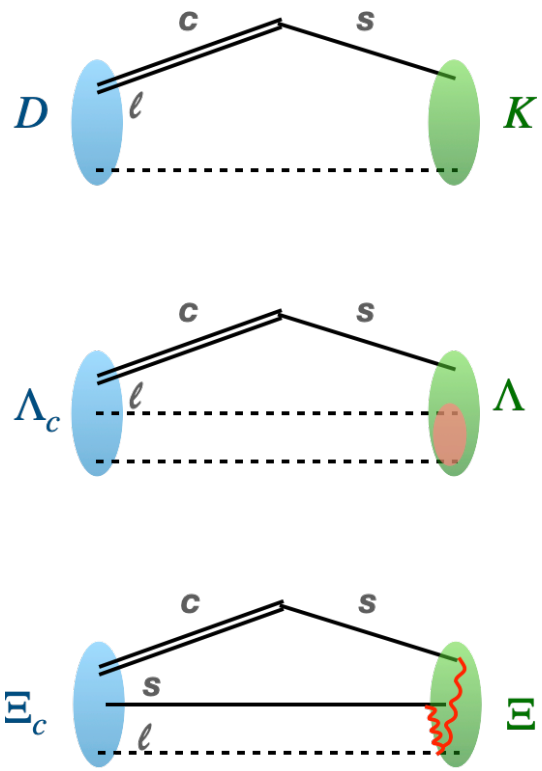
# $\Xi_c \to \Xi$ form factors from LQCD

🤔 A different pattern between inclusive and exclusive decays of $\Lambda_c$ and $D$

$$\frac{\mathcal{B}(\Lambda_c^+ \to X e^+ \nu_e) = (3.95 \pm 0.34 \pm 0.09)\,\%}{\mathcal{B}(\Lambda_c^+ \to \Lambda e^+ \nu_e) = (3.63 \pm 0.38 \pm 0.20)\,\%} \boxed{\sim 1}$$

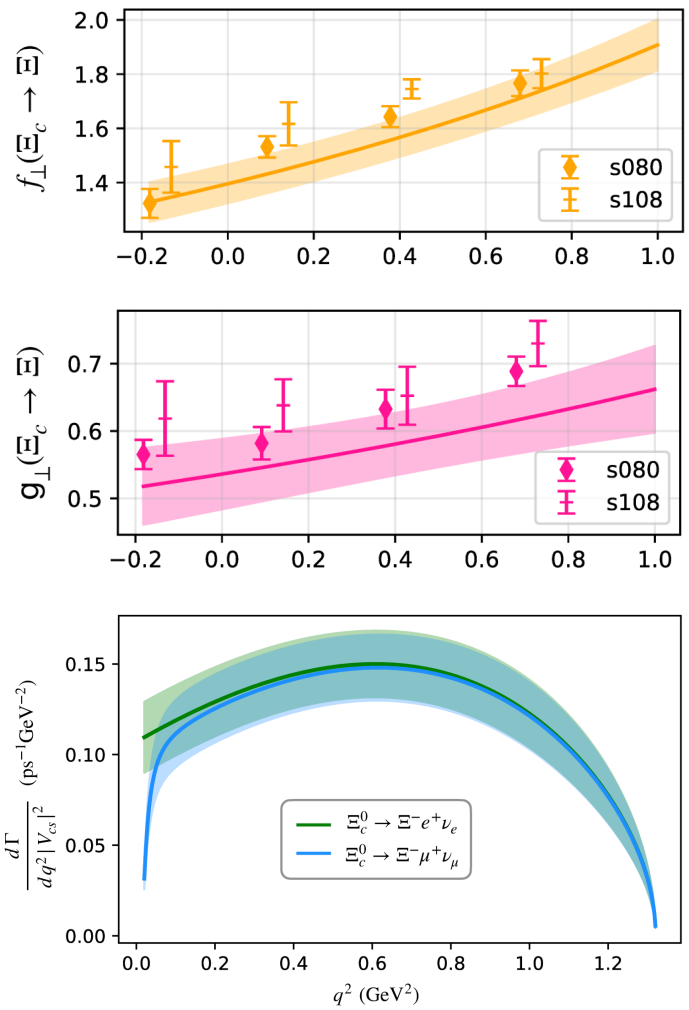
*BESIII, PRL121, 251801(2018)*

$$\frac{\mathcal{B}(D^0 \to X e^+ \nu_e) = (6.49 \pm 0.11)\,\%}{\mathcal{B}(D^0 \to K^- e^+ \nu_e) = (3.542 \pm 0.035)\,\%} \boxed{\sim 2}$$




- $\Xi_c$ contains more versatile decay modes, will reveal more QCD dynamics
- $\Xi_c - \Xi_c{}'$ mixing effect

# $\Xi_c \to \Xi$ form factors from LQCD




✓ Branching fractions:

$$\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = 2.38(0.30)(0.32)(0.07)\,\%$$

$$\mathscr{B}\left(\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu\right) = 2.29(0.29)(0.30)(0.06)\,\%$$

$$\mathscr{B}\left(\Xi_c^+ \to \Xi^0 e^+ \nu_e\right) = 7.18(0.90)(0.96)(0.20)\,\%$$

$$\mathscr{B}\left(\Xi_c^+ \to \Xi^0 \mu^+ \nu_\mu\right) = 6.91(0.87)(0.91)(0.19)\,\%$$

| | |
|---|---|
| PDG | $\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (1.8 \pm 1.2)\,\%$ |
| Belle | $\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50)\,\%$ |
| ALICE | $\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (2.43 \pm 0.25 \pm 0.35 \pm 0.72)\,\%$ |
| QCD SR | $\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (3.4 \pm 1.7)\,\%$ |
| LF QM | $\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (3.49 \pm 0.95)\,\%$ |
| LCSR | $\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (2.4^{+0.9}_{-1.0})\,\%$ |

✓ Determination of $|V_{cs}|$:

⇒ from ALICE results:

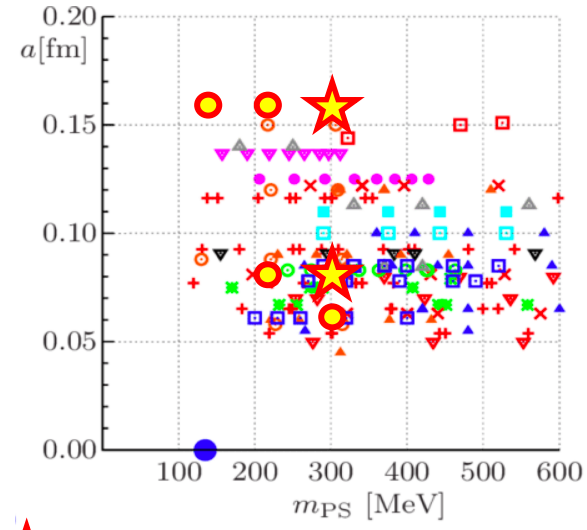
$$|V_{cs}| = 0.983(0.060)_{\text{stat.}}(0.065)_{\text{syst.}}(0.167)_{\text{exp.}}$$

⇒ from Belle results:

$$|V_{cs}| = 0.834(0.051)_{\text{stat.}}(0.056)_{\text{syst.}}(0.127)_{\text{exp.}}$$

⇒ PDG average:

$$|V_{cs}| = 0.97320 \pm 0.00011$$




★ First work on CLQCD ensembles

# $\Xi_c - \Xi_c{}'$ mixing from LQCD

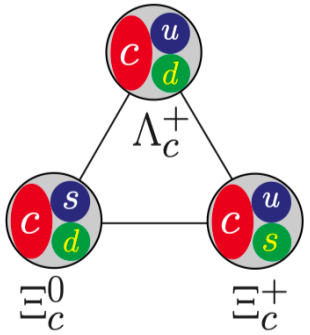
🤔 **Mixing:**

$$\begin{pmatrix} \Xi_c \\ \Xi_c' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \Xi_c^{\bar{3}} \\ \Xi_c^{6} \end{pmatrix}$$

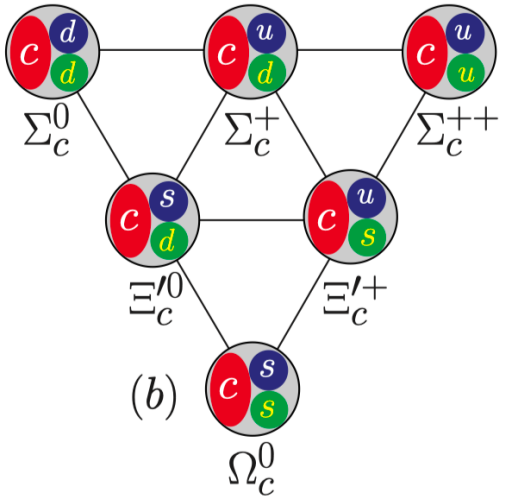
*He, et al., PLB823,136765(2021)*

| channel | branching ratio(%) | | |
|---|---|---|---|
| | experimental data | fit data(pole model) | fit data(constant). |
| $\Lambda_c^+ \to \Lambda^0 e^+ \nu_e$ | $3.6 \pm 0.4$ | $3.61 \pm 0.32$ | $3.62 \pm 0.32$ |
| $\Lambda_c^+ \to \Lambda^0 \mu^+ \nu_\mu$ | $3.5 \pm 0.5$ | $3.48 \pm 0.30$ | $3.45 \pm 0.30$ |
| $\Xi_c^+ \to \Xi^0 e^+ \nu_e$ | $2.3 \pm 1.5$ | $3.89 \pm 0.73$ | $3.92 \pm 0.73$ |
| $\Xi_c^0 \to \Xi^- e^+ \nu_e$ | $1.54 \pm 0.35$ | $1.29 \pm 0.24$ | $1.31 \pm 0.24$ |
| $\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu$ | $1.27 \pm 0.44$ | $1.24 \pm 0.23$ | $1.24 \pm 0.23$ |
| fit parameter (pole model) | $f_1 = 1.01 \pm 0.87,\ \delta f_1 = -0.51 \pm 0.92$<br>$f_1' = 0.60 \pm 0.49,\ \delta f_1' = -0.23 \pm 0.41$ | | $\chi^2/d.o.f = 1.6$ |
| fit parameter (constant) | $f_1 = 0.86 \pm 0.92,\ \delta f_1 = -0.25 \pm 0.88$<br>$f_1' = 0.85 \pm 0.36,\ \delta f_1' = -0.43 \pm 0.50$ | | $\chi^2/d.o.f = 1.9$ |

***More details see Fei Huang's talk……***




Triplet




Sixtet

$$T_{c\bar{3}} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix} \quad T_{c6} = \begin{pmatrix} \Sigma_c^{++} & \frac{\Sigma_c^+}{\sqrt{2}} & \frac{\Xi_c^{+\prime}}{\sqrt{2}} \\ \frac{\Sigma_c^+}{\sqrt{2}} & \Sigma_c^0 & \frac{\Xi_c^{0\prime}}{\sqrt{2}} \\ \frac{\Xi_c^{+\prime}}{\sqrt{2}} & \frac{\Xi_c^{0\prime}}{\sqrt{2}} & \Omega_c^0 \end{pmatrix}$$

# $\Xi_c - \Xi_c{}'$ mixing from LQCD

🤔 **Mixing:**

$$\begin{pmatrix} \Xi_c \\ \Xi_c' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \Xi_c^{\bar{\mathbf{3}}} \\ \Xi_c^{\mathbf{6}} \end{pmatrix}$$

- Determine the mixing angle:

✓ Ke, Li, PRD105,9(2022)

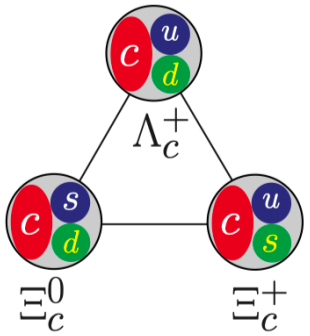
$\theta = 16.27^\circ \pm 2.30^\circ$ or $\theta = 85.54^\circ \pm 2.30^\circ$

✓ Geng, *et al*, 2210.07211, 2212.02971

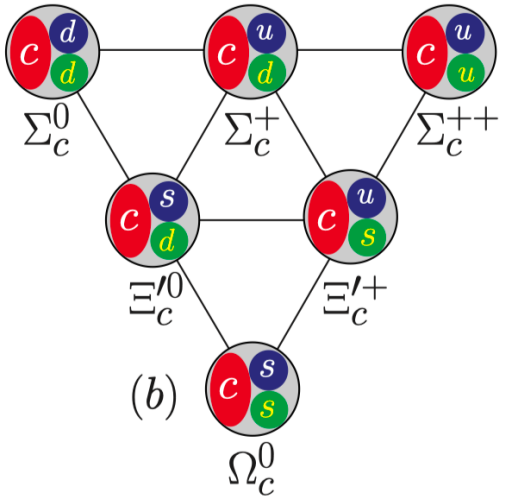
$\theta = 0.137(5)\pi = 24.66(9)^\circ$

✓ Aliev, PRD83, 016008(2011)

$\theta = 5.5^\circ \pm 1.8^\circ$




Triplet

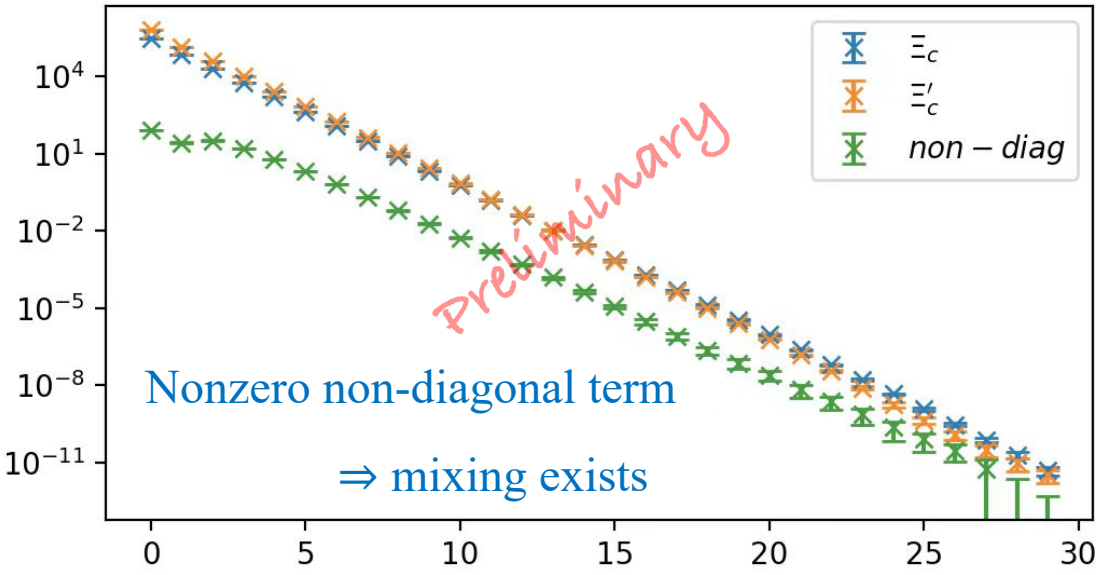


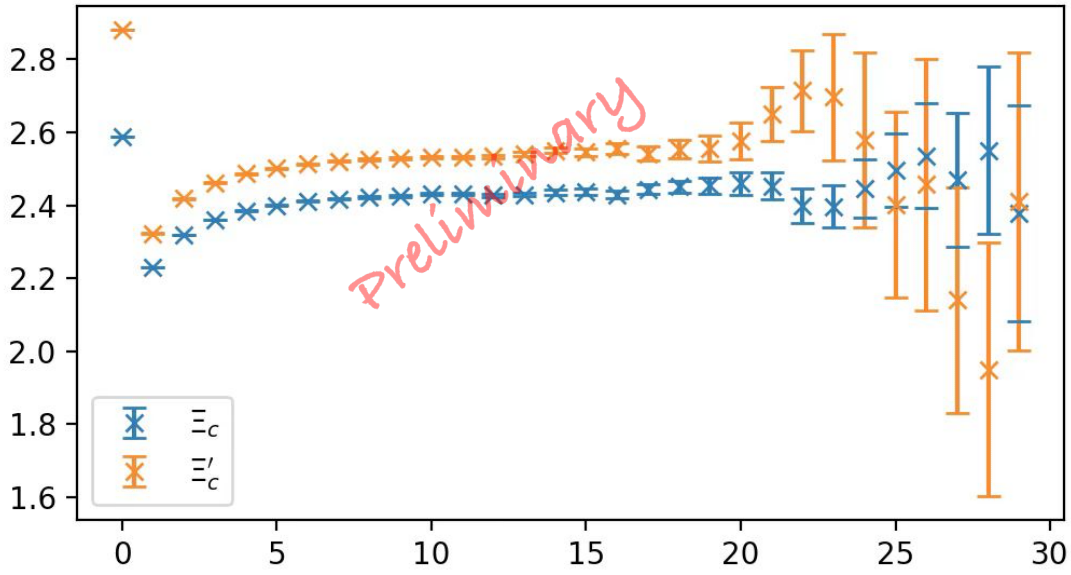

Sixtet

# $\Xi_c - \Xi_c{}'$ mixing from LQCD

🤔 **$\Xi_c - \Xi_c{}'$ mixing and SU(3) breaking:**

$$\begin{pmatrix} \left\langle O_{\Xi_c^{\bar{3}}} \bar{O}_{\Xi_c^{\bar{3}}} \right\rangle & \left\langle O_{\Xi_c^{\bar{3}}} \bar{O}_{\Xi_c^{6}} \right\rangle \\ \left\langle O_{\Xi_c^{6}} \bar{O}_{\Xi_c^{\bar{3}}} \right\rangle & \left\langle O_{\Xi_c^{6}} \bar{O}_{\Xi_c^{6}} \right\rangle \end{pmatrix}$$

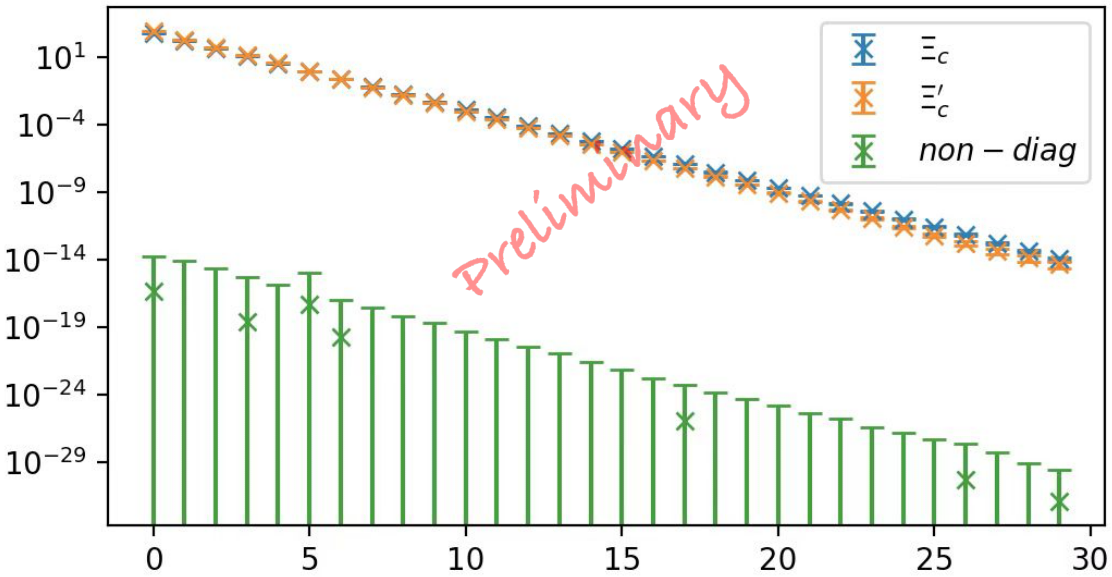


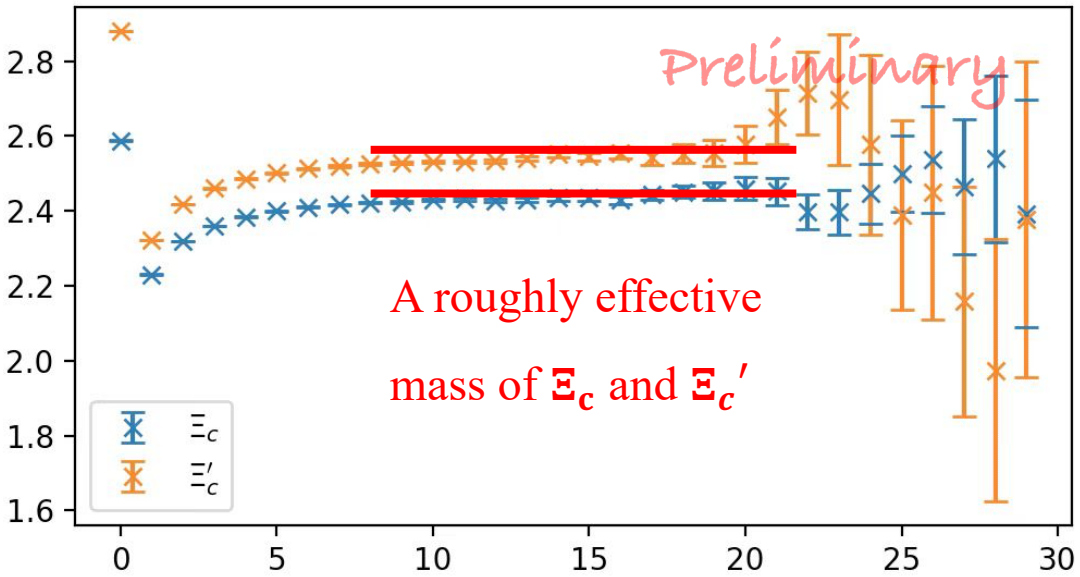



# $\Xi_c - \Xi_c{}'$ mixing from LQCD

🤔 $\Xi_c - \Xi_c{}'$ mixing and SU(3) breaking:

$$\begin{pmatrix} \langle O_{\Xi_c^{\bar{3}}} \bar{O}_{\Xi_c^{\bar{3}}} \rangle & \langle O_{\Xi_c^{\bar{3}}} \bar{O}_{\Xi_c^{6}} \rangle \\ \langle O_{\Xi_c^{6}} \bar{O}_{\Xi_c^{\bar{3}}} \rangle & \langle O_{\Xi_c^{6}} \bar{O}_{\Xi_c^{6}} \rangle \end{pmatrix} \xrightarrow{\text{diagonalize}} \approx \begin{pmatrix} \langle O_{\Xi_c} \bar{O}_{\Xi_c} \rangle & 0 \\ 0 & \langle O_{\Xi_c'} \bar{O}_{\Xi_c'} \rangle \end{pmatrix}$$

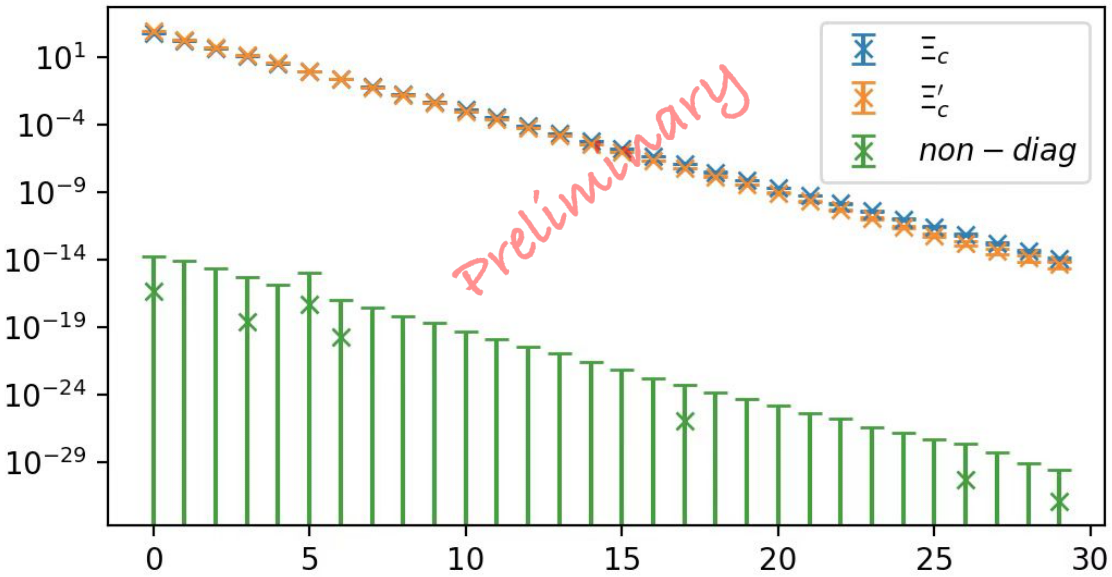


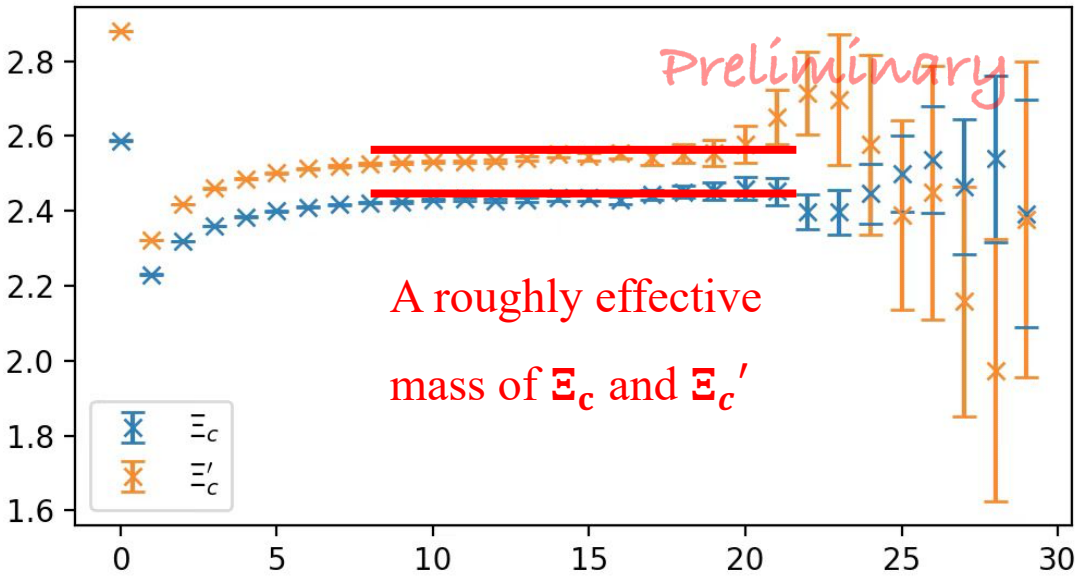



# $\Xi_c - \Xi_c{}'$ mixing from LQCD

🤔 $\Xi_c - \Xi_c{}'$ mixing and SU(3) breaking:

$$\begin{pmatrix} \langle O_{\Xi_c^{\bar{3}}} \bar{O}_{\Xi_c^{\bar{3}}} \rangle & \langle O_{\Xi_c^{\bar{3}}} \bar{O}_{\Xi_c^{6}} \rangle \\ \langle O_{\Xi_c^{6}} \bar{O}_{\Xi_c^{\bar{3}}} \rangle & \langle O_{\Xi_c^{6}} \bar{O}_{\Xi_c^{6}} \rangle \end{pmatrix} \xrightarrow{\text{diagonalize}} \approx \begin{pmatrix} \langle O_{\Xi_c} \bar{O}_{\Xi_c} \rangle & 0 \\ 0 & \langle O_{\Xi_c'} \bar{O}_{\Xi_c'} \rangle \end{pmatrix} \xrightarrow[\text{Joint fit}]{\text{Prior param.}} \boxed{\text{Exact values}}$$

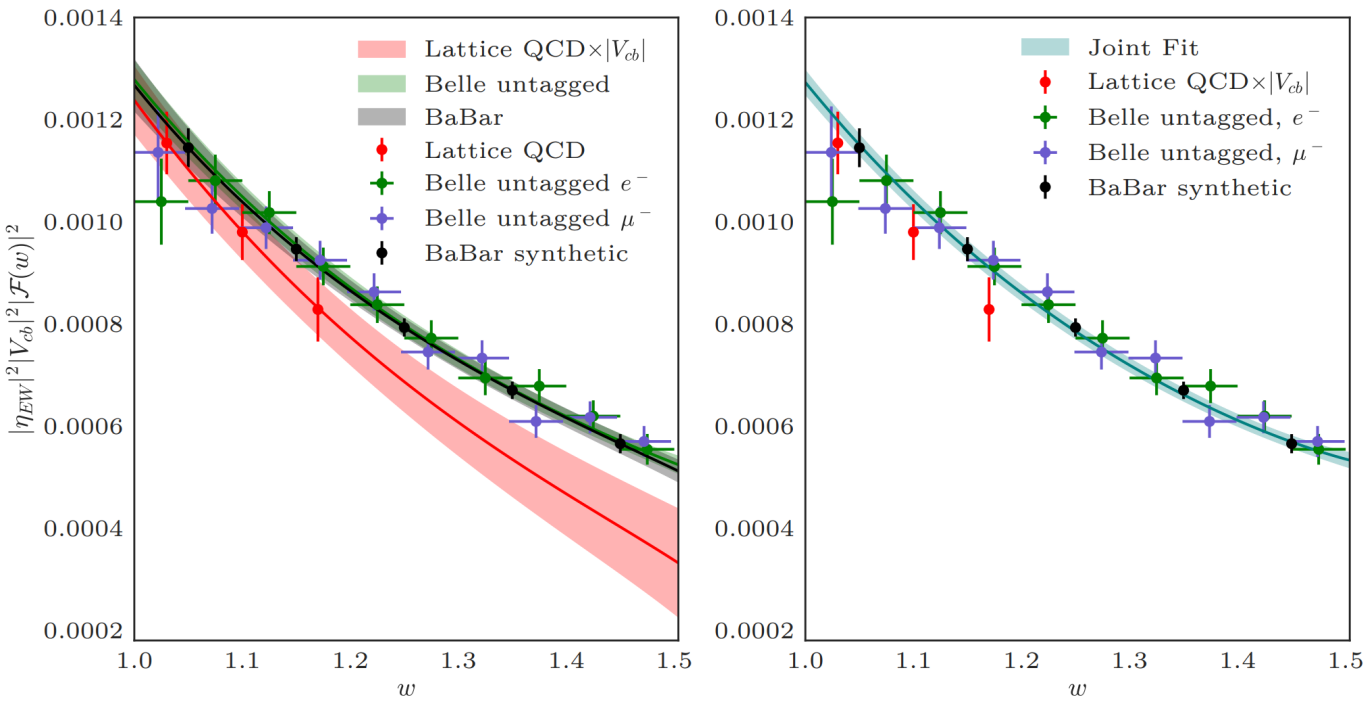


# Semileptonic $B$ decays

- $B - D^*$ form factors at nonzero recoil

FNAL/MILC, 2105.14019




Determination of $V_{cb}$

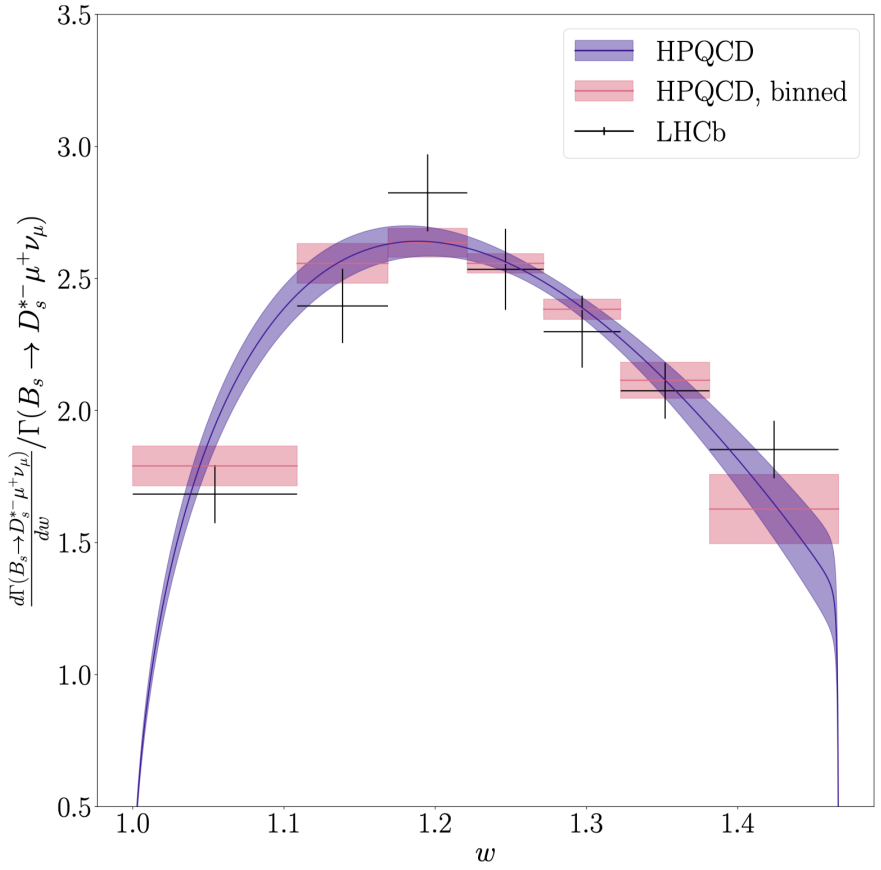
| | Lattice QCD | Lattice + BaBar | Lattice + Belle | Lattice + both |
|---|---|---|---|---|
| $a_0$ | 0.0330(12) | 0.0331(12) | 0.0325(10) | 0.0320(10) |
| $a_1$ | −0.155(55) | −0.089(40) | −0.160(44) | −0.148(31) |
| $a_2$ | −0.12(98) | −0.16(21) | −0.70(94) | −0.60(22) |
| $b_0$ | 0.01229(23) | 0.01229(22) | 0.01238(22) | 0.01246(22) |
| $b_1$ | −0.003(12) | 0.0123(69) | 0.015(10) | 0.0038(46) |
| $b_2$ | 0.07(53) | 0.36(17) | −0.30(24) | 0.02(12) |
| $c_1$ | −0.0058(25) | −0.0008(11) | 0.0010(17) | 0.00008(94) |
| $c_2$ | −0.013(91) | 0.054(46) | 0.035(57) | 0.080(36) |
| $c_3$ | | −0.12(83) | −0.34(76) | −1.11(56) |
| $d_0$ | 0.0509(15) | 0.0516(15) | 0.0521(15) | 0.0526(14) |
| $d_1$ | −0.327(67) | −0.197(50) | −0.179(49) | −0.194(43) |
| $d_2$ | −0.03(96) | 0.19(92) | −0.01(90) | −0.004(898) |
| $\chi^2/\mathrm{dof}$ | 0.64/3 | 9.28/5 | 111/81 | 126/84 |
| $\sum_i^N a_i^2$ | 0.04(24) | 0.035(71) | 0.5(1.3) | 0.39(27) |
| $\sum_i^N (b_i^2 + c_i^2)$ | 0.005(70) | 0.15(18) | 0.21(48) | 1.2(1.3) |
| $\sum_i^N d_i^2$ | 0.110(61) | 0.08(35) | 0.035(25) | 0.040(15) |
| $\lvert V_{cb}\rvert \times 10^3$ | | 39.66(91) | 38.18(82) | 38.40(74) |

$$|V_{cb}| = (38.40 \pm 0.74) \times 10^{-3}$$

# Semileptonic $B$ decays

- $B_s - D_s^*$ **form factors and** $R(D_s^*)$

HPQCD, PRD105, 094506(2022)




- Both $b$ and $c$ quarks treated using the same lattice action as the light quarks $\Rightarrow$ requires extrapolations in $m_b$ but largely eliminates the renormalization uncertainty.

LHCb, PRD101, 072004 (2020)

✓ $|V_{cb}|$ results with using LHCb measurements:

$$|V_{cb}| = 42.2(1.5)_{\text{latt}}(1.7)_{\text{exp}}(0.4)_{\text{EM}}$$

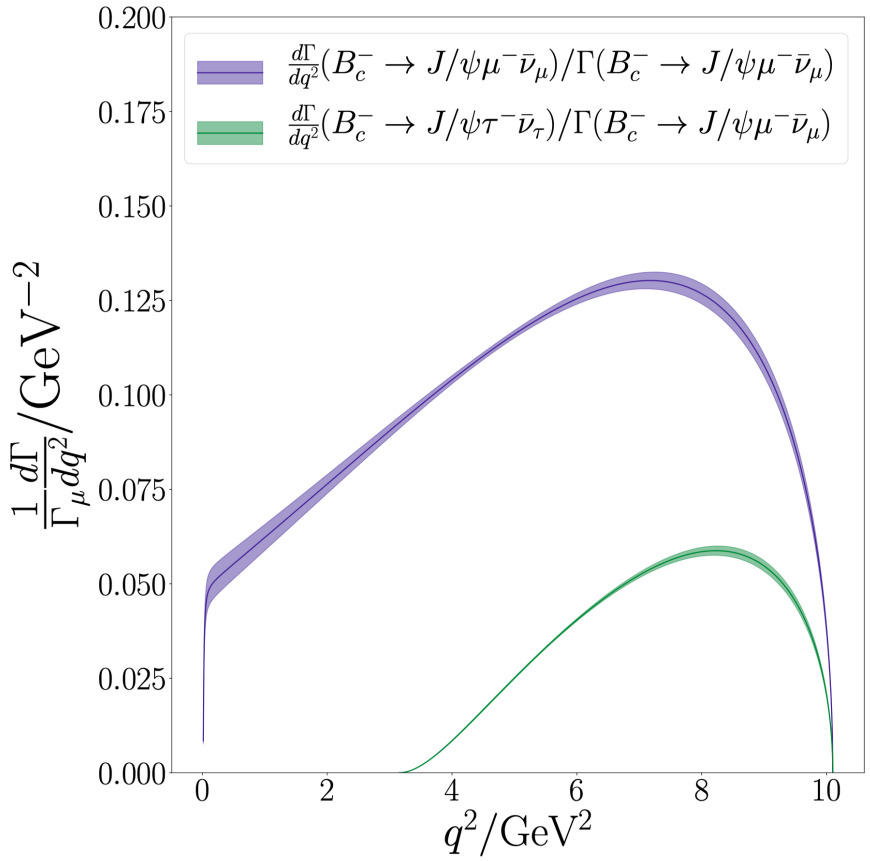
✓ Prediction for $\tau$- / $\mu$-ratio:

$$R(D_s^*)_{\text{SM}} = 0.2490(60)_{\text{latt}}(35)_{\text{EM}}$$

# Semileptonic $B$ decays

- $B_c - J/\psi$ **form factors and** $R(J/\psi)$

HPQCD, PRD102, 094518(2020); PRL125, 222003(2020)




- Same lattice methods and setups as the previous page

✓ LQCD prediction for $\tau$- / $\mu$-ratio:

$$R(J/\psi)_{\mathrm{SM}} = 0.2582(38)$$

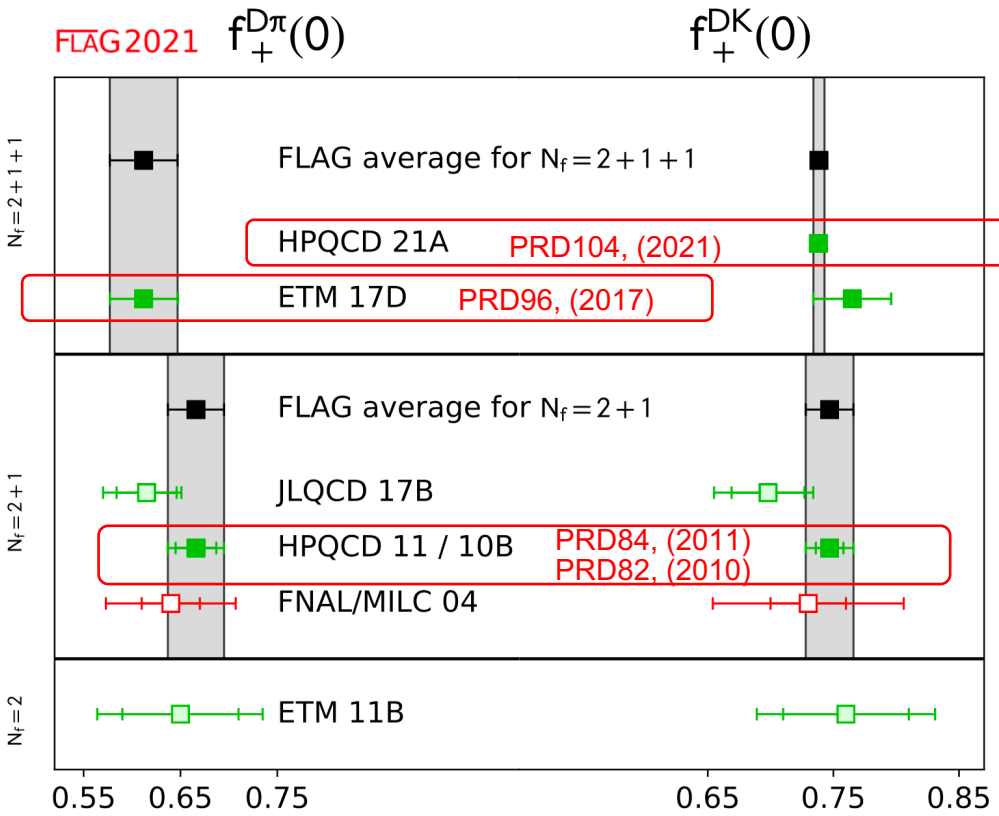
✓ For comparison, the LHCb result:

$$R(J/\psi)_{\mathrm{exp}} = 0.71(17)(18)$$

LHCb, PRL120, 121801 (2018)

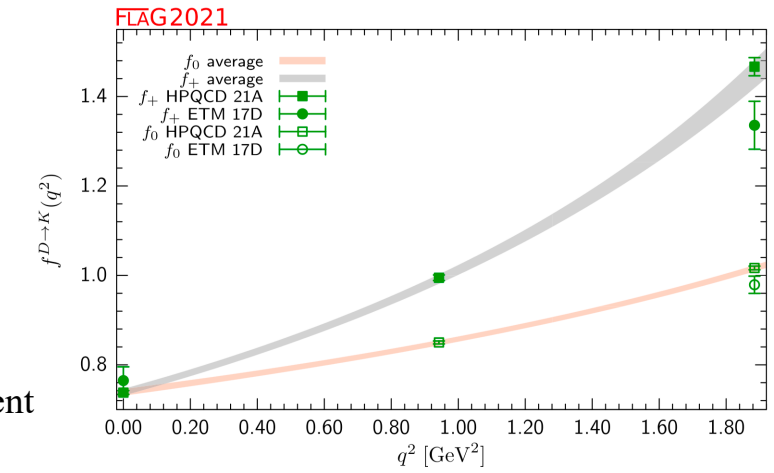
# Semileptonic $D$ decays from LQCD

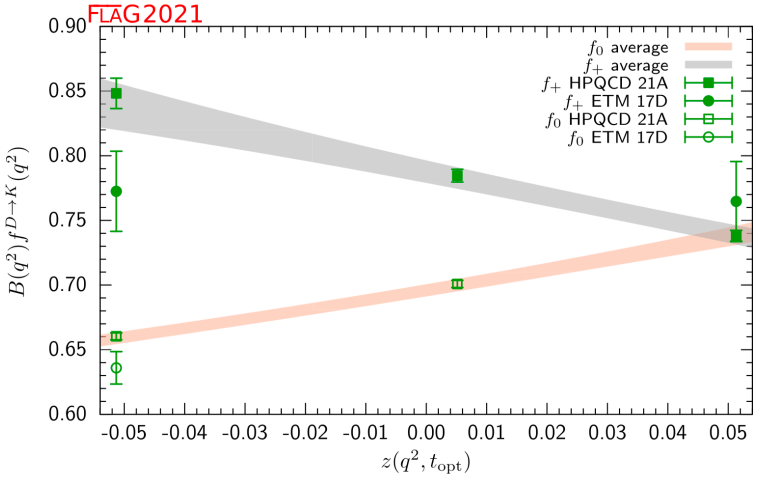
- $D-\pi/K$ form factors at zero-recoil




Significant improvement

- $D-\pi/K$ form factors at nonzero-recoil



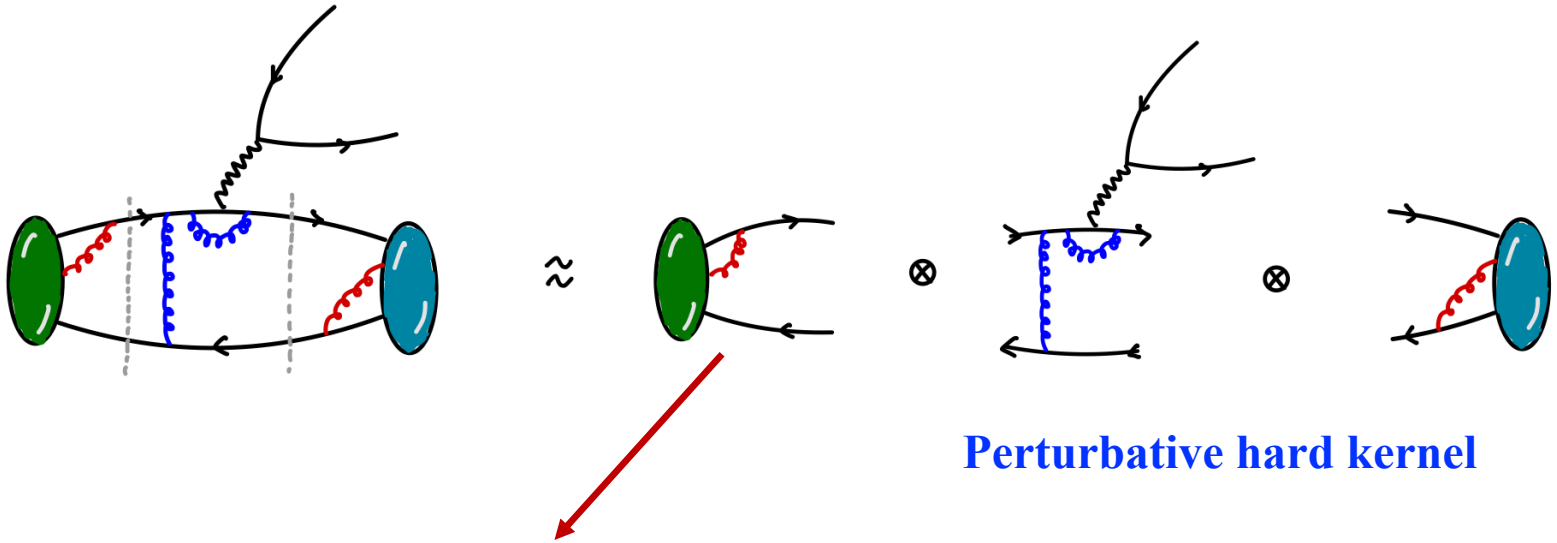



# Distribution amplitudes (DAs)

Retrieve the lost "REAL TIME"

# EFT needed nonperturbative quantities

- **QCD factorization of exclusive processes**




**Wave functions / Distribution amplitudes (nonperturbative)**

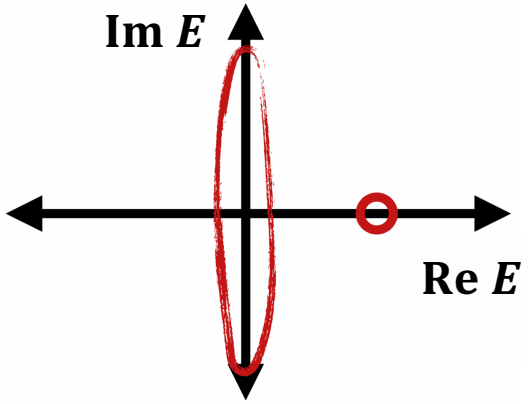
$$\int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} \langle 0 | \bar{\psi}_1(0) n \cdot \gamma\gamma_5 U(0,\xi^-) \psi_2(\xi^-) | H(p) \rangle$$

Light-cone correlations $\Rightarrow$ real time dependent!

# Retrieve the "REAL TIME" on LQCD

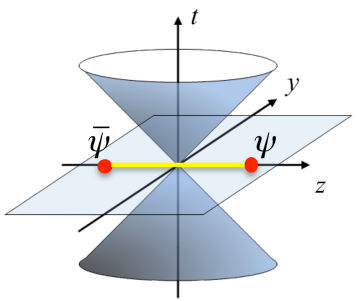
LQCD, must we give in return?

Can only calculate on Euclidean time.



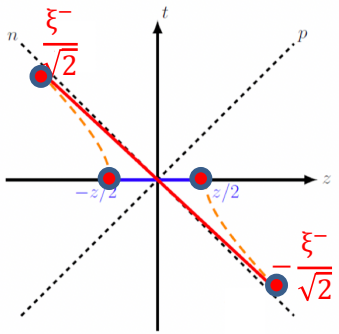

✓ **Large-momentum effective theory:**

**connecting Euclidean lattice and physical observables**

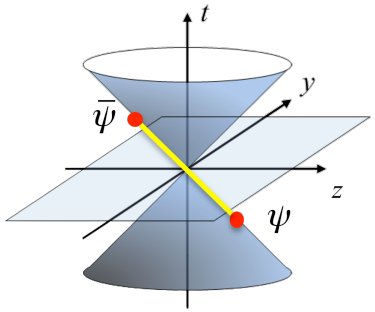



Lorentz boost

*Ji, PRL110(2013), RMP93(2021), …*

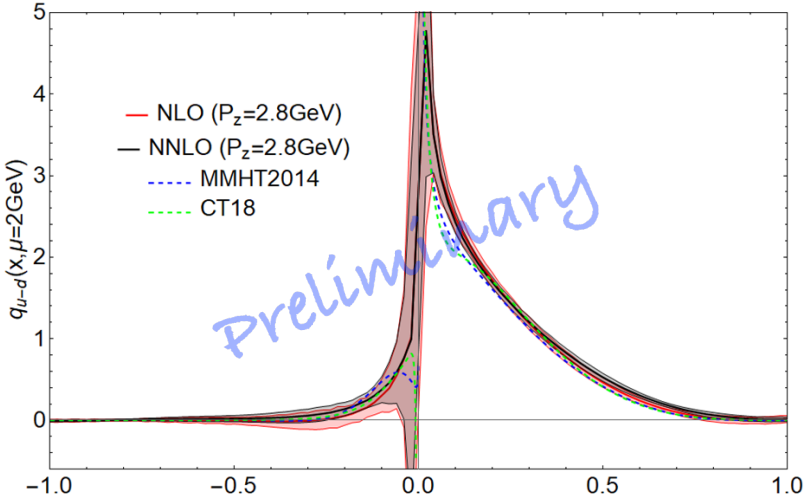



EFT

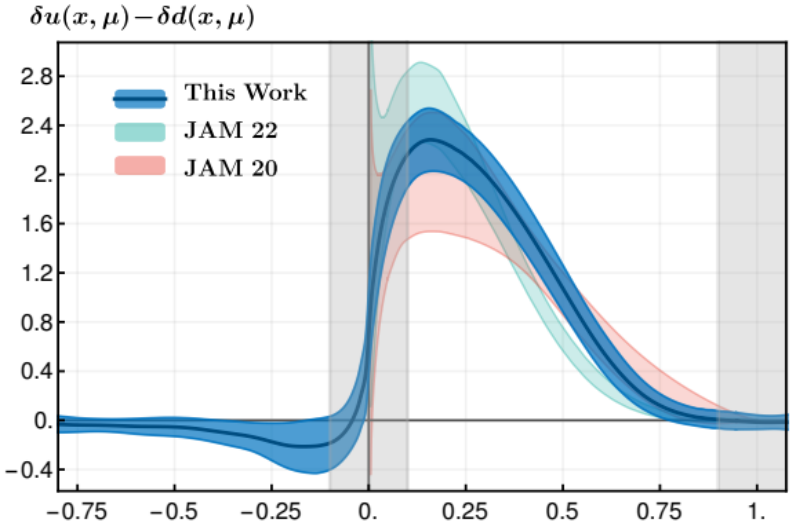


# Retrieve the "REAL TIME" on LQCD

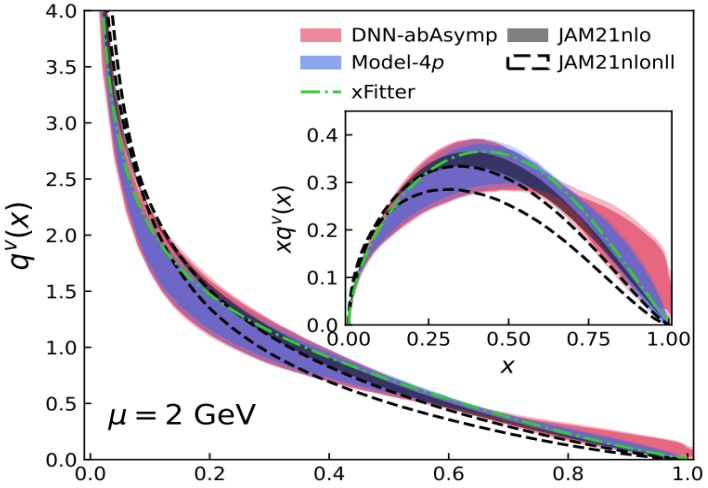
✓ **Achieved great success in the studies of PDF:**




*Proton unpolarized PDF, in preparation*




*Proton transversity PDF, 2208.08008*



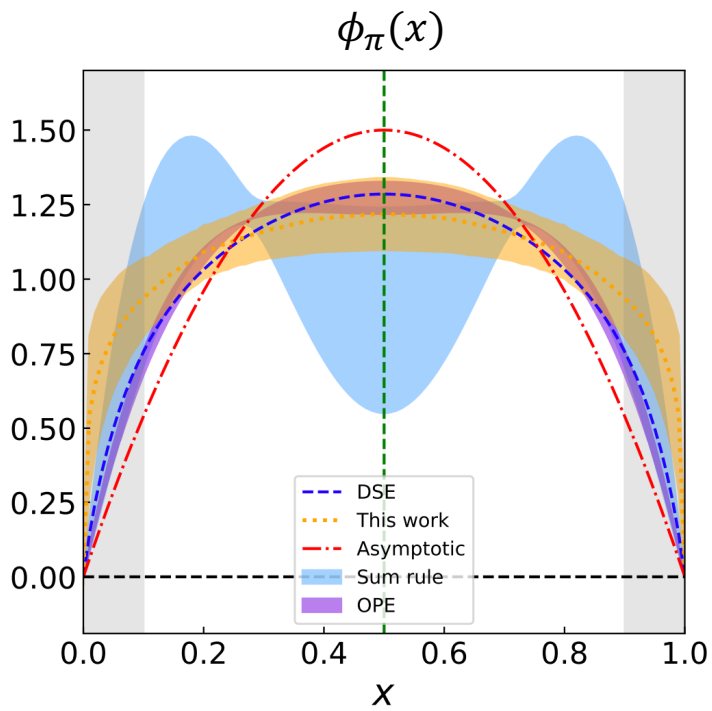

*Pion valance PDF, 2208.02297*

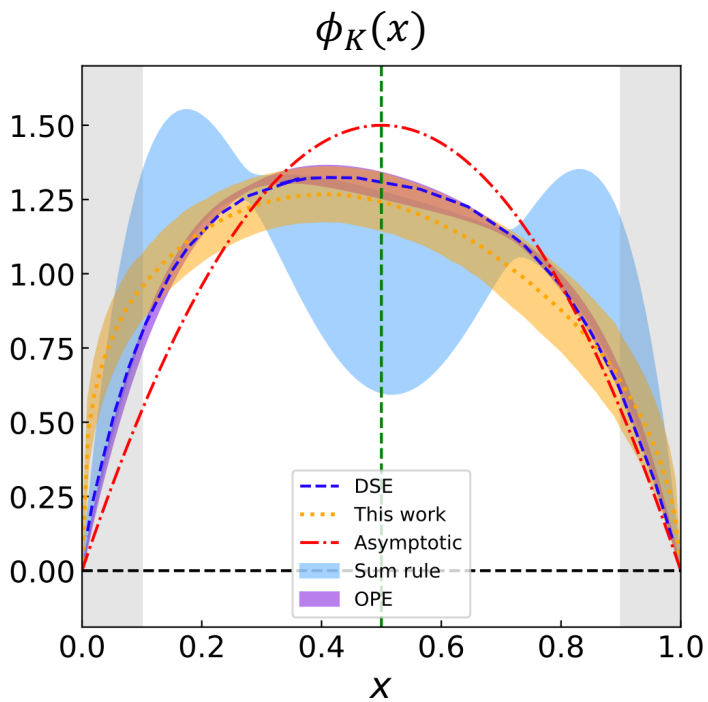
# LQCD determination of light-cone distribution amplitudes (LCDAs)

➢ **Pseudoscalar meson:**

(LPC) Hua, *et al.*, PRL129, 132001(2022)

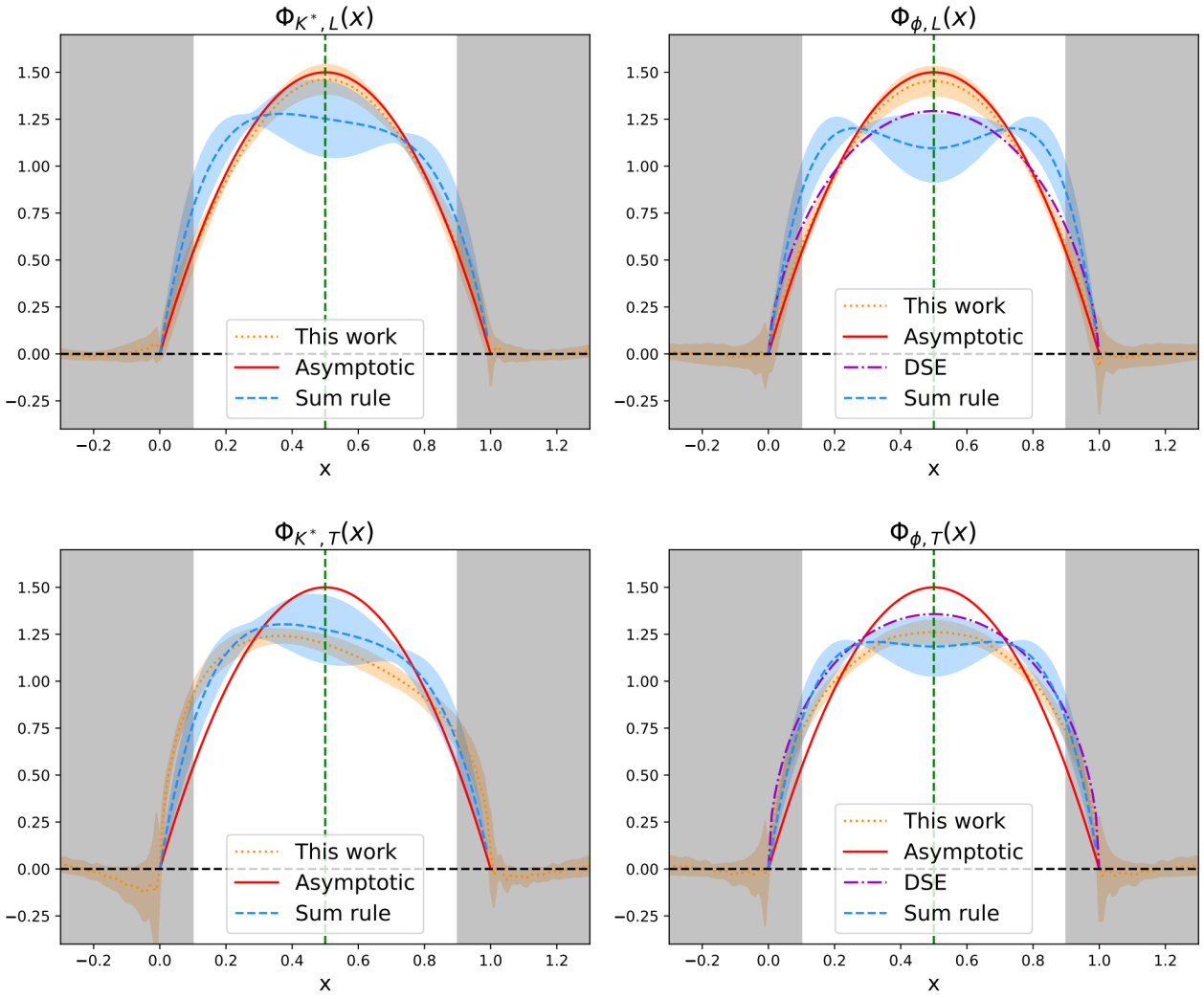
***More details see Jun Hua's talk……***




- ✓ Physical mass
- ✓ Continuum limit
- ✓ Hybrid renormalization scheme

# LQCD determination of light-cone distribution amplitudes (LCDAs)

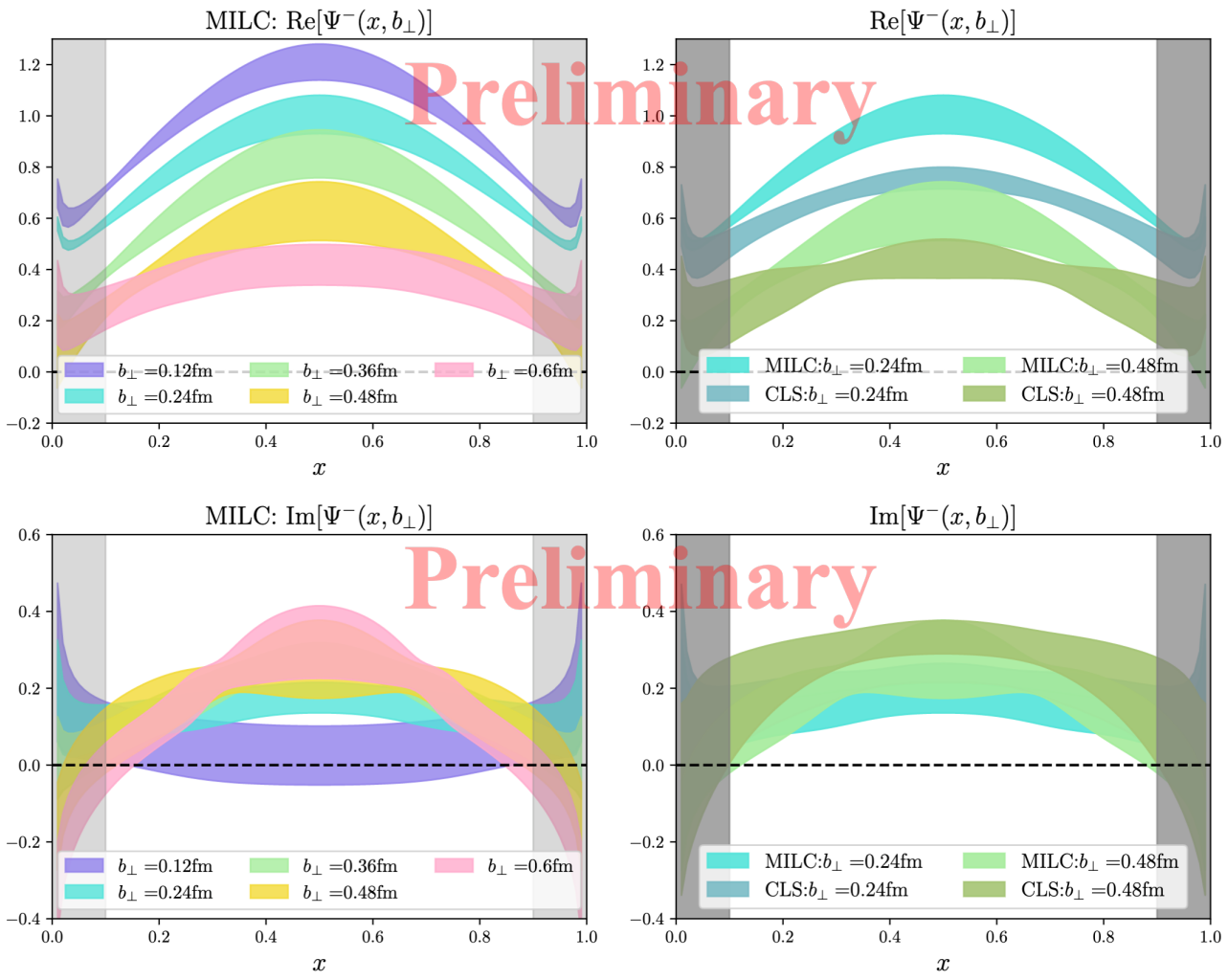



- **Vector meson:**

  (LPC) Hua, *et al.*, PRL127, 026002(2021)

  - ✓ Physical mass
  - ✓ Continuum limit
  - ✓ Hybrid renormalization scheme

# LQCD determination of TMD wave functions




***More details see Zhi-fu Deng's talk……***

- **TMD wave functions:**

(LPC) Chu, Hua, *et at*, in preparation

- ✓ MILC + CLS ensembles
- ✓ State-of-art lattice and theoretical technics
- ✓ Latest soft function, Collin-Soper kernels……

# Challenges and opportunities

QCD describes the properties of observed matter in terms of fundamental variables and their interactions.

Significant progress in lattice calculations in the past years although still many open questions and unsolved problems remain - phenomenological and theoretical.

Extensive analyses already in light meson sector. Heavy flavor physics on lattice is underway.....

# Challenges and opportunities

- **Form factor sector:**
  - More charmed and bottom baryon decays
  - Hyperon decays (super fine lattice?)
- **Other matrix elements:**
  - Lifetime (4-quark current)
  - Inclusive decays (4-point correlation function)
- **Distribution amplitudes:**
  - B LCDA
  - Baryon & heavy baryon LCDA
- **Scope of LQCD continues to increase:**
  - New methods, new technics, new measurements
  - Finer ensembles, higher precisions
  - More contributors
  - ……

Thank you for your attention!