重味强子多体衰变中的角分布不对称及相关CP破坏

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Based on 2208.13411, 2209.02348, 2209.13196

南华大学

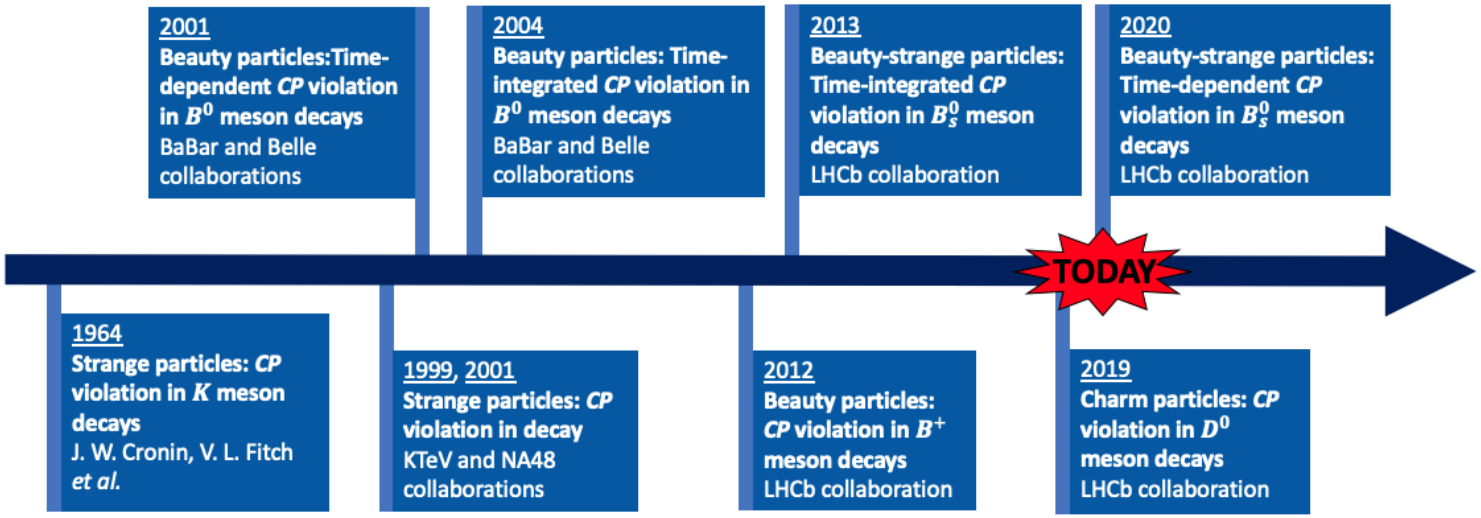
全国第十九届重味物理和CP破坏研讨会
09-11/12/2022

1. current exp. status of CPV
2. Forward-Backward Asymmetry induced CPA in three-body decays
3. Angular-correlation related CPA in four-body decays
4. Summary and Outlook

1. current exp. status of CPV

current exp. status of CPV

- B meson: $\surd\surd\cdots$
- B_s meson: $\surd$
- D meson: $\surd$ (very small CPV)
- baryon decay: $\times$ (lower statistics than meson cases)



current status of CPV in baryon decays

- no confirmation of CPV in baryon sector from exp. side.
  - 2-body and multi-body decays of heavy baryons
  - regional and integrated CPA
  - Triple Product Asymmetry induced CPA in four-body decay
  - decay asymmetry prameters ($\Lambda_c^+ \to \Lambda\pi^+$, $\Lambda \to p\pi^-$)
  - energy test method

2. Forward-Backward Asymmetry induced CPA in three-body decays

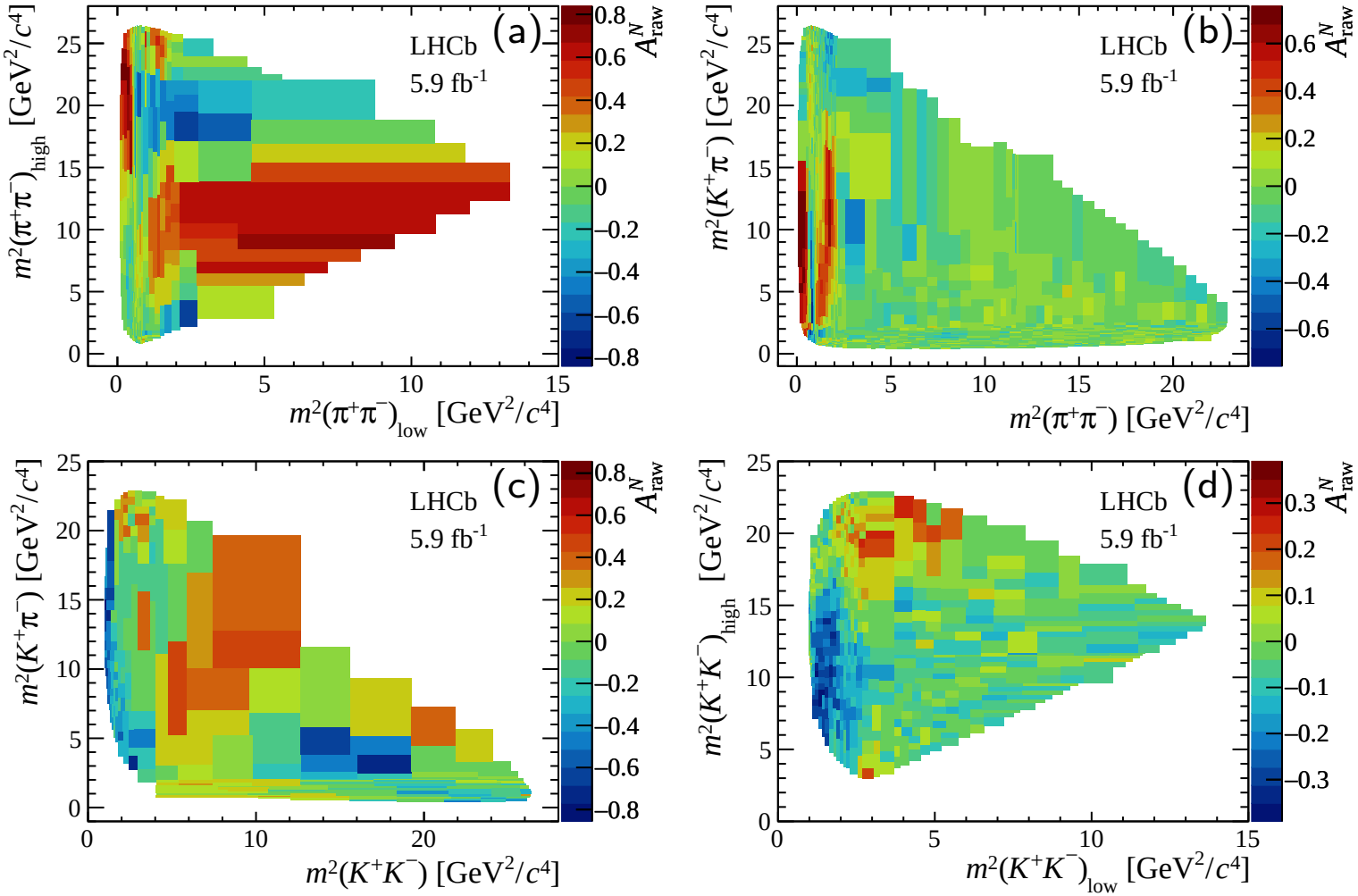
Overall CPV in 3-body decays of charged B mesons

LHCb, 2206.07622 (PRD)

$$A_{CP}(\pi\pi\pi) = +0.080 \pm 0.004 \pm 0.003 \pm 0.003$$
$$A_{CP}(K\pi\pi) = +0.011 \pm 0.002 \pm 0.003 \pm 0.003$$
$$A_{CP}(KK\pi) = -0.114 \pm 0.007 \pm 0.003 \pm 0.003$$
$$A_{CP}(KKK) = -0.037 \pm 0.002 \pm 0.002 \pm 0.003$$

Regional CPV in three-body decays of B mesons

2206.07622



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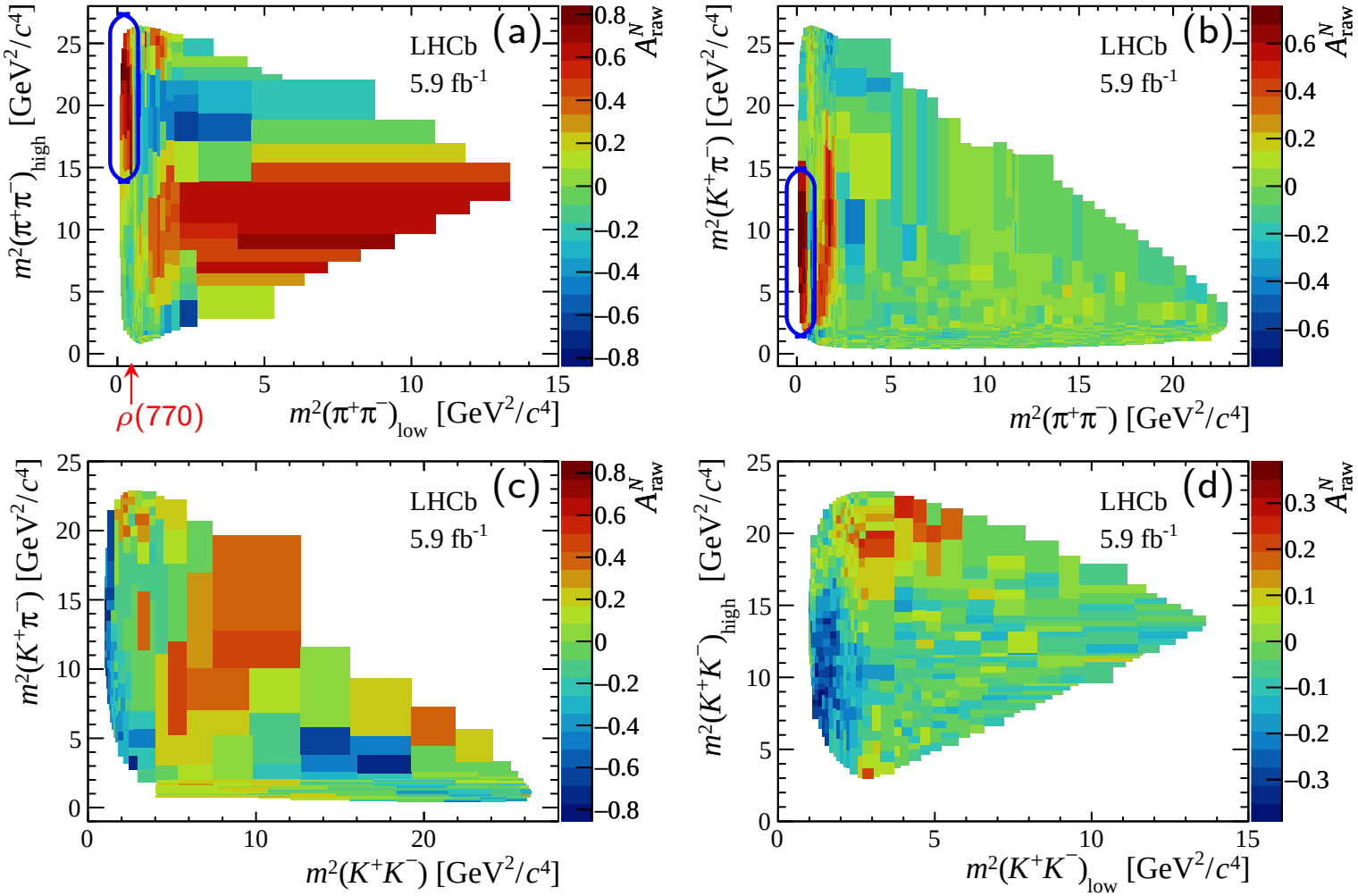
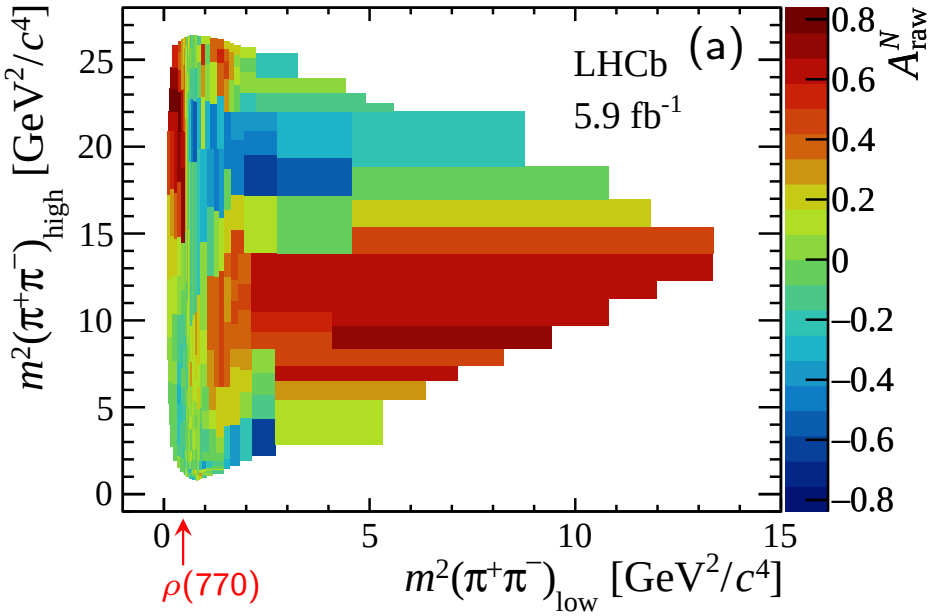
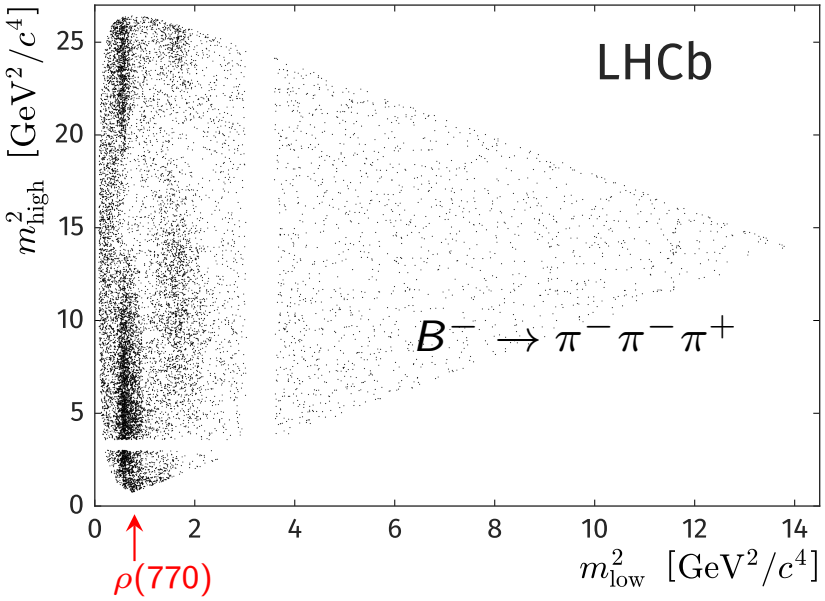


Corelated behaviour between reg. CPA and event distributions





Origins of large reg. CPA: interfer. of S- and P-waves: $f_0(500)$ and $\rho(770)^0$

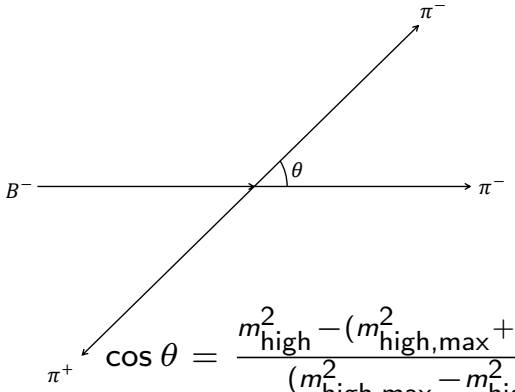
$$\mathcal{A} = a_S + e^{i\delta} a_P \cos\theta, \qquad a_{S(P)} = a^{\text{tree}}_{S(P)} + a^{\text{penguin}}_{S(P)}$$

$$|\mathcal{A}|^2 \propto |a_S|^2 + |a_P|^2 \cos^2\theta + 2\Re\left(a_S^* a_P e^{i\delta}\right)\cos\theta$$

$$A_{CP} \propto \left|a^{\text{tree}}\right| \left|a^{\text{penguin}}\right| \sin(\theta_2 - \theta_1)\sin(\delta_2 - \delta_1)$$

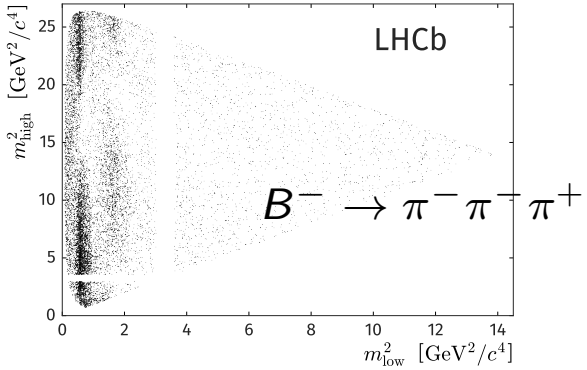
(ZHZ, X.-H. Guo, Y.-D. Yang, PRD87, 076007)

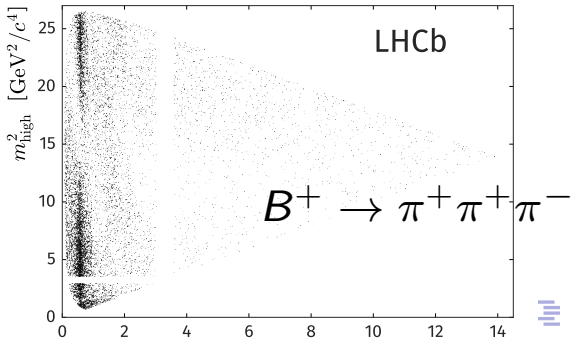
$B^\pm \to \pi^+\pi^-\pi^\pm$



$$\cos\theta = \frac{m^2_{\text{high}} - (m^2_{\text{high,max}} + m^2_{\text{high,min}})/2}{(m^2_{\text{high,max}} - m^2_{\text{high,min}})/2}$$

PRL 124 (2020) 031801 [1909.05211]





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Forward-Backward Asymmetry (FBA)

$$A^{FB}_{B^-} = \frac{\left(-\int_{-1}^{0} + \int_{0}^{+1}\right)|\mathcal{A}|^2}{\int_{-1}^{+1}|\mathcal{A}|^2} = \frac{\Re(\langle a_S^* a_P e^{i\delta}\rangle)}{|\langle a_P\rangle|^2/3 + |\langle a_S\rangle|^2}.$$

(ZHZ, PLB820, 136537)

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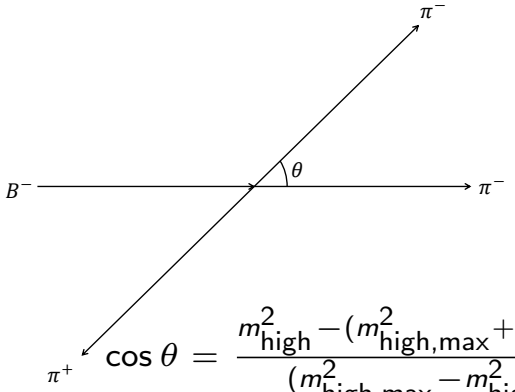

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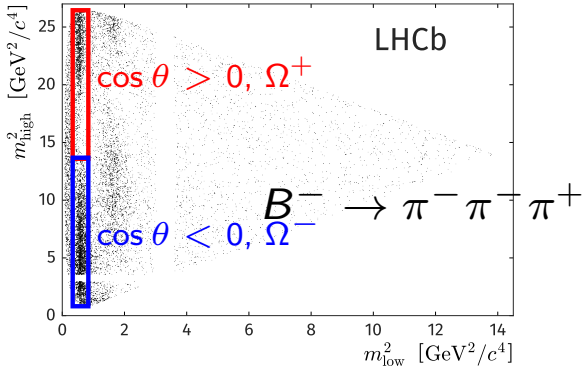

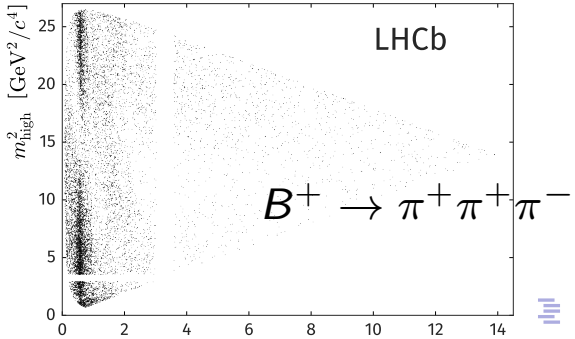

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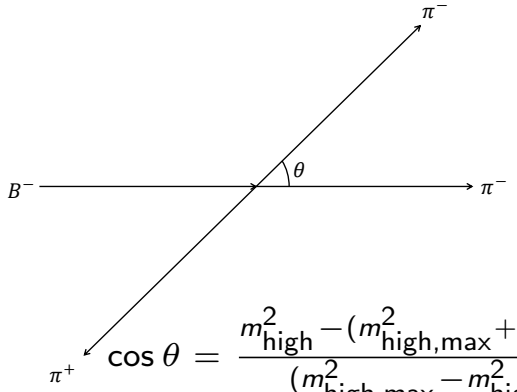
(ZHZ, PLB820, 136537)

FBA Induced *CP* Asymmetry (FB-*CP*A)

$$A^{FB}_{CP} = \frac{1}{2}(A^{FB}_{B^-} - A^{FB}_{B^+}).$$

$B^\pm \to \pi^+\pi^-\pi^\pm$



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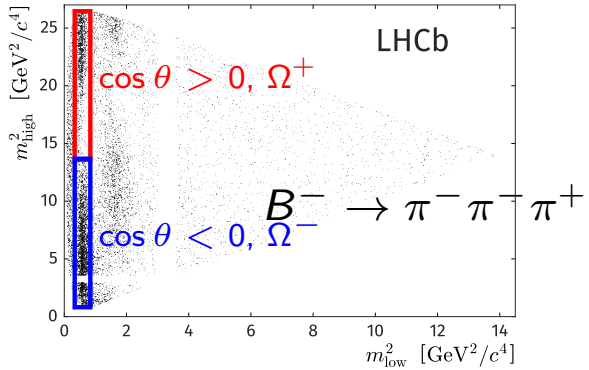
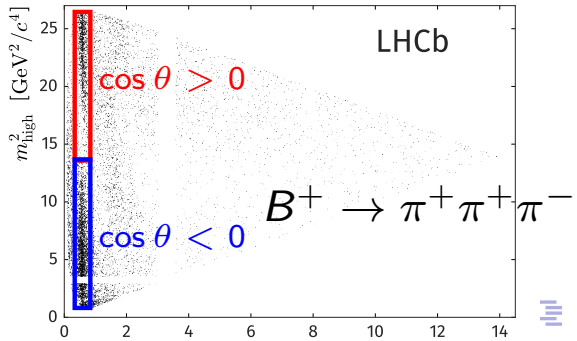
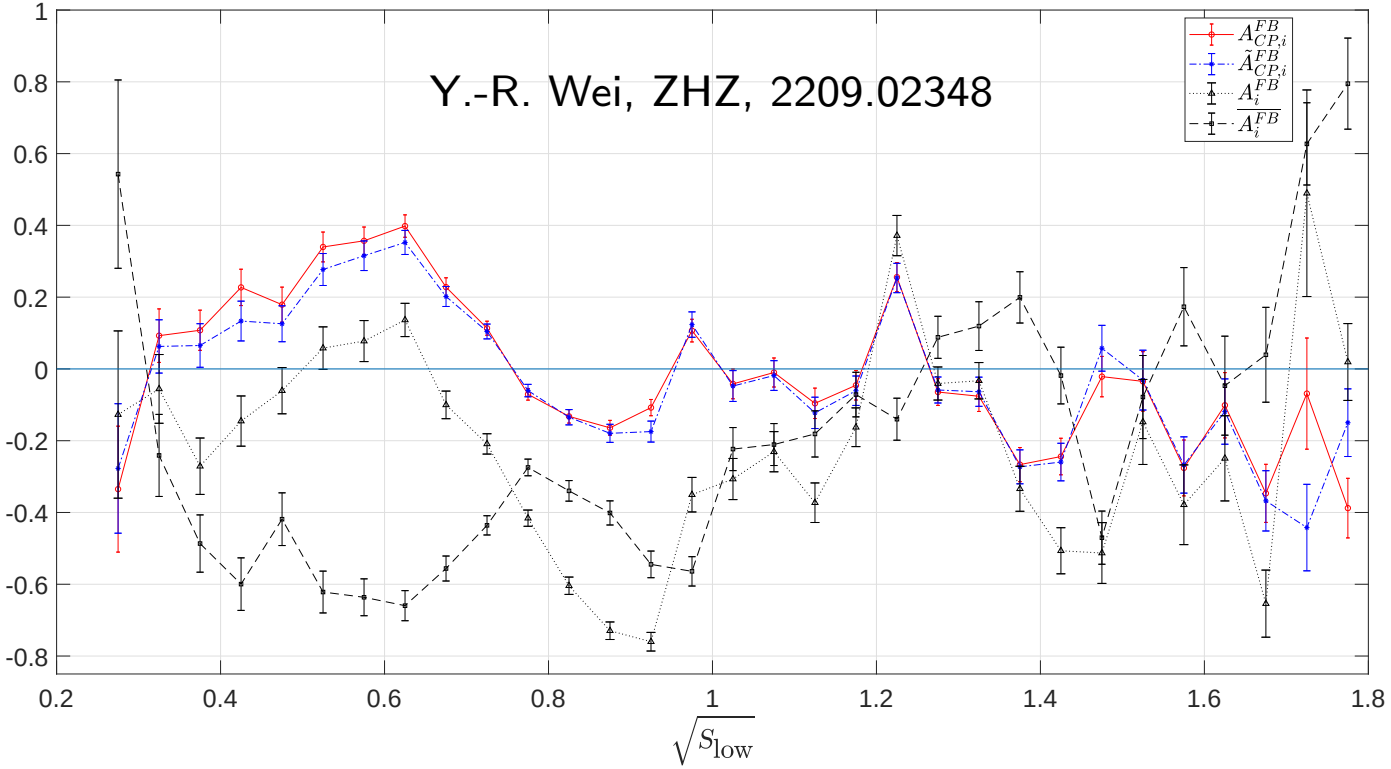




FB-CPA v.s. reg. CPA based on LHCb data



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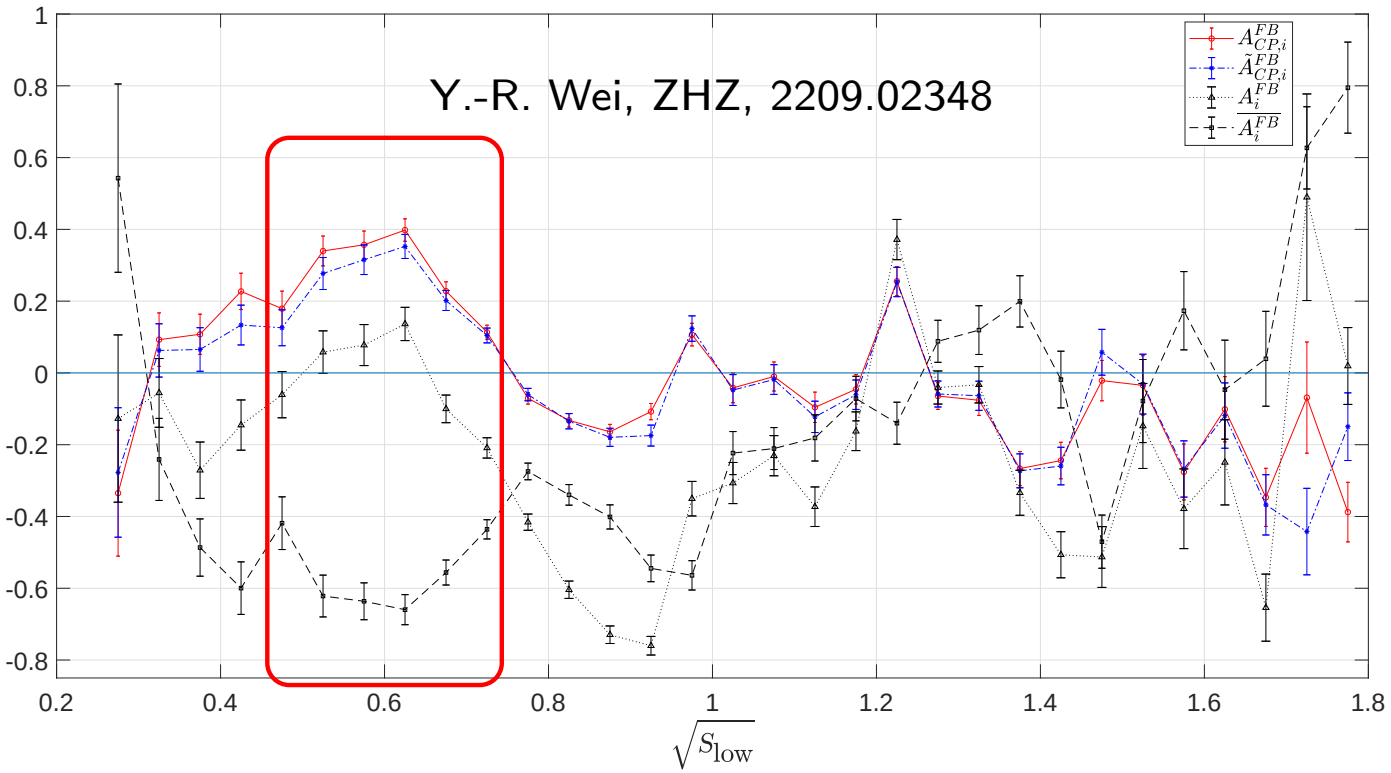
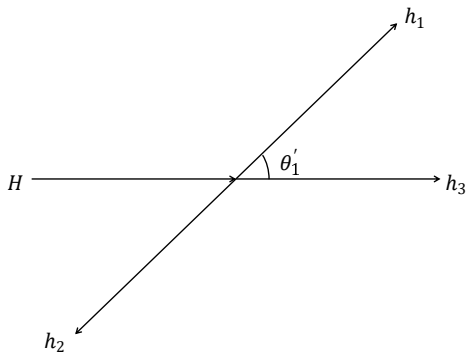


A_{CP}^{FB}	$\tilde{A}_{CP}^{FB}$	A_{CP}^{Ω}	$A_{CP}^{\Omega^+}$	$A_{CP}^{\Omega^-}$
0.224 ± 0.012	0.194 ± 0.013	0.099 ± 0.013	0.405 ± 0.020	-0.074 ± 0.017

FBA for general cases: $H \to R_i h_3$, $R_i \to h_1 h_2$, $i = 1, 2$

Decay amplitude squared

$$\overline{|\mathcal{M}|^2} = \sum_j P_j(c_{\theta_1^*}) w^{(j)}.$$



Selection Rules for the two resonances R_1 and R_2 in $w^{(j)}$

- Interference terms show up only when
  - $(-)^j P_{R_1} P_{R_2} = +1$.
  - $j = |j_{R_1} - j_{R_2}|, \cdots, j_{R_1} + j_{R_2}$.
- Non-interfer. terms show up only for
  - even j.
  - $j = 0, \cdots, 2j_{R_1/R_2}$.

- It is the Spins and the Parities of the intermediate resonances directly determines j, not the angular momentum between $h_1 h_2$.

FBA: interference of resonances with opposite parities

Well separated for opposite P_{R_1} and P_{R_2}

- Inter. terms in P_j odd j
- Non-inter. terms in P_j even j.

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FBA and Generalized FBA

$$\overline{|\mathcal{M}|^2} = \sum_j P_j(c_{\theta_1^*}) w^{(j)}$$

$$A^{FB} = \frac{\left(-\int_{-1}^{0} + \int_{0}^{+1}\right) \overline{|\mathcal{M}^J|^2} dc_{\theta_1'}}{\int_{-1}^{+1} \overline{|\mathcal{M}^J|^2} dc_{\theta_1'}}$$

$$A_j^{FB} = \frac{\left(-\int_{-1}^{x_1^{(j)}} + \int_{x_1^{(j)}}^{x_2^{(j)}} - \int_{x_2^{(j)}}^{x_3^{(j)}} \cdots + \int_{x_j^{(j)}}^{+1}\right) \overline{|\mathcal{M}^J|^2} dc_{\theta_1'}}{\int_{-1}^{+1} \overline{|\mathcal{M}^J|^2} dc_{\theta_1'}}$$

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Examples

- $\Xi_b^- \to pK^-K^-$: $\Sigma(1775) - \Sigma(1925)$ $\left(\left(\frac{5}{2}\right)^+ - \left(\frac{5}{2}\right)^-\right)$
- $\Lambda_b^0 \to pK^*(892)^0\pi^-$: $N(1440) - N(1520)$ $\left(\left(\frac{1}{2}\right)^+ - \left(\frac{3}{2}\right)^-\right)$
- $\Lambda_b^0 \to p\pi^0\pi^-$ ($\Lambda_b^0 \to p\rho^0\pi^-$): $N(1440) - N(1520)$

FB-CPA of $\Lambda_b^0 \to p\pi^0\pi^-$: interference of $N(1440)^+$ and $N(1520)^+$

$N(1440)^+$ and $N(1520)^+$: spin-parities are $\frac{1}{2}^+$ and $\frac{3}{2}^-$

$$\mathcal{M} = \mathcal{M}_{N_1} + \mathcal{M}_{N_2} e^{i\delta}$$

$$\mathcal{M}_{N_j} = \frac{1}{s_{N_j}} \sum_{\text{pol. } N_j} \mathcal{M}_{\Lambda_b \to N_j \pi^-} \mathcal{M}_{N_j \to p\pi^0}$$

According to SRs:

- $N(1440)$ in P_0
- $N(1520)$ in P_0 and P_2
- interf. in P_1.

$$\overline{|\mathcal{M}|^2} \propto \left[\frac{|\alpha_{N_1}|^2}{|s_{N_1}|^2} + \frac{|\alpha_{N_2}|^2}{|s_{N_2}|^2} \left(1 + 3\cos^2\theta\right) + 12\Re\left(\frac{\alpha_{N_1}\alpha_{N_2}^* e^{i\delta}}{s_{N_1} s_{N_2}^*} \right) \cos\theta \right],$$

3 Angular-correlation related CPA in four-body decays

Kinematics

"branching" four-body decay

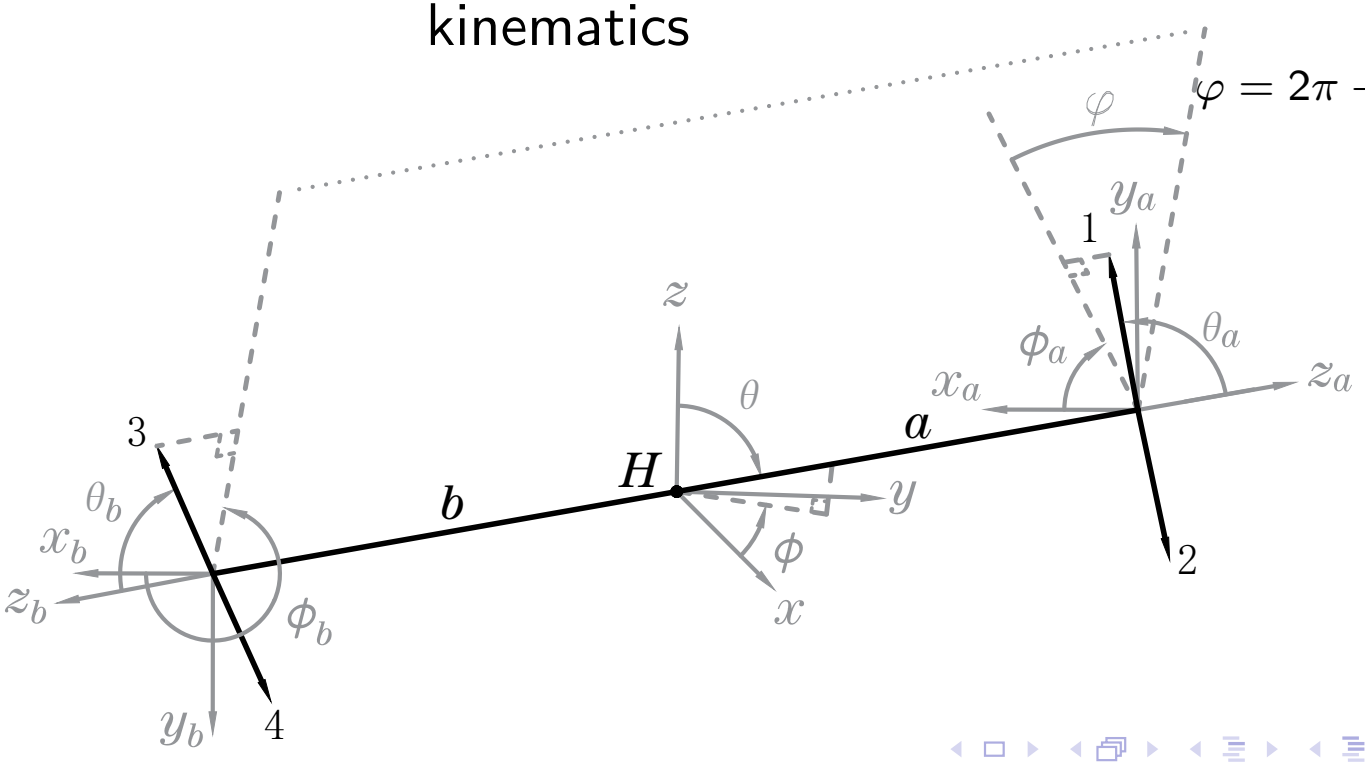
$H \to a^{(\prime)}(\to 12) b^{(\prime)}(\to 34)$

Reasons for four-body decays

- large statistics (such as Λ_b)
- more room for CPV

kinematics

$\varphi = 2\pi - (\phi_a + \phi_b)$



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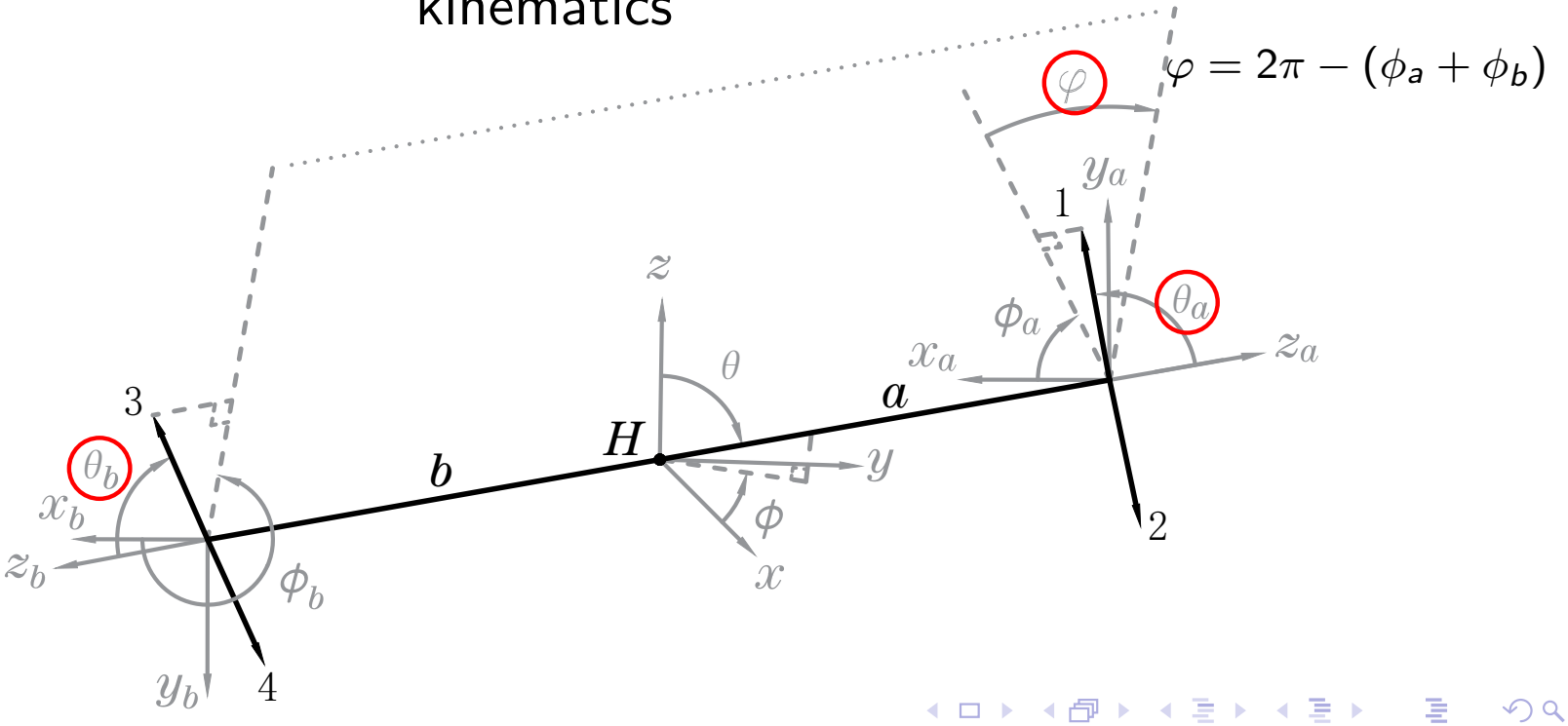


Dynamics

interferences

- $H \to a(\to 12)b(\to 34)$ and $H \to a'(\to 12)b(\to 34)$
- $H \to a(\to 12)b(\to 34)$ and $H \to a(\to 12)b'(\to 34)$
- Genuine Interference (GI) to four-body decay:
  $H \to a(\to 12)b(\to 34)$ and $H \to a'(\to 12)b'(\to 34)$

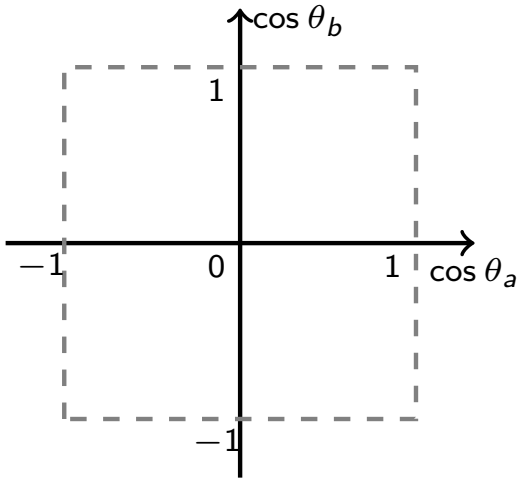
Integrate out φ

Decay amplitudes squared

$$\int \overline{|\mathcal{A}|^2} d\varphi \propto \sum_{jl} \Gamma_{jl}(s_{12}, s_{34}) P_j(c_{\theta_a}) P_l(c_{\theta_b})$$

$$\Gamma_{jl} = \sum_{a,a',b,b'} \frac{\mathcal{W}_{jl}^{(ab,a'b')} \mathcal{G}_j^{(aa')} \mathcal{G}_l^{(bb')}}{\mathcal{I}_a \mathcal{I}_{a'}^* \mathcal{I}_b \mathcal{I}_{b'}^*},$$

2 Dim Phase Space



$$\mathcal{W}_{jl}^{(ab,a'b')} = \sum_{\sigma\rho} (-)^{\sigma - s_a + \rho - s_b} \langle s_a - \sigma s_{a'} \sigma | j0 \rangle \langle s_b - \rho s_{b'} \rho | l0 \rangle \mathcal{F}_{\sigma\rho}^{H\to ab} \mathcal{F}_{\sigma\rho}^{H\to a'b'*},$$

$$\mathcal{G}_j^{(aa')} = \sum_{\lambda_1\lambda_2} (-)^{s_a - \lambda_{12}} \langle s_a - \lambda_{12} s_{a'} \lambda_{12} | j0 \rangle \mathcal{F}_{\lambda_1\lambda_2}^{a\to 12} \mathcal{F}_{\lambda_1\lambda_2}^{a'\to 12*},$$

$$\mathcal{G}_l^{(bb')} = \sum_{\lambda_3\lambda_4} (-)^{s_b - \lambda_{34}} \langle s_b - \lambda_{34} s_{b'} \lambda_{34} | l0 \rangle \mathcal{F}_{\lambda_3\lambda_4}^{b\to 34} \mathcal{F}_{\lambda_3\lambda_4}^{b'\to 34*},$$

Selection Rules

$$\mathcal{G}_j^{(aa')} = \sum_{\lambda_1\lambda_2} (-)^{s_a-\lambda_{12}} \langle s_a - \lambda_{12} s_{a'} \lambda_{12} | j0 \rangle \mathcal{F}^{a\to 12}_{\lambda_1\lambda_2} \mathcal{F}^{a'\to 12*}_{\lambda_1\lambda_2},$$

$$\mathcal{G}_l^{(bb')} = \sum_{\lambda_3\lambda_4} (-)^{s_b-\lambda_{34}} \langle s_b - \lambda_{34} s_{b'} \lambda_{34} | l0 \rangle \mathcal{F}^{b\to 34}_{\lambda_3\lambda_4} \mathcal{F}^{b'\to 34*}_{\lambda_3\lambda_4},$$

Interf.

$$P_a P_{a'} (-)^j = 1, \qquad P_b P_{b'} (-)^l = 1$$

$$|s_a - s_{a'}| \le j \le s_a + s_{a'}, \qquad |s_b - s_{b'}| \le l \le s_b + s_{b'}$$

Non-interf.

$$j \text{ even}, \qquad l \text{ even}$$

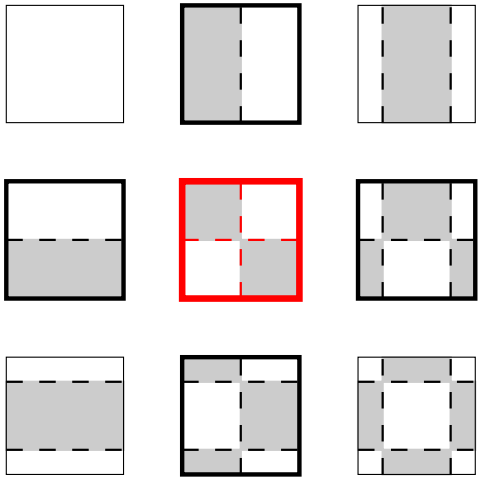
$$0 \le j \le 2s_{a^{(\prime)}}, \qquad 0 \le l \le 2s_{b^{(\prime)}}$$

If a and a' as well as b and b' have opposite parities, interfer. and non-interfer. terms will be well separated.

Applications to $\Lambda_b^0 \to p\pi^-\pi^+\pi^-$: $N(1440) - N(1520)$ and $f_0(500) - \rho(770)$

$\Lambda_b^0 \to N(\to p\pi^-)f/\rho(\to pi^+\pi^-)$: c_{θ_a} and c_{θ_b} are correlated.

$$(\Gamma_{jl}) \sim \begin{pmatrix} \text{Non-int} & \begin{matrix}(N_{1440}N_{1520})|f|^2, \\ (N_{1440}N_{1520})|\rho|^2\end{matrix} & \text{Non-int} \\ \begin{matrix}(f\rho)|N_{1440}|^2, \\ (f\rho)|N_{1520}|^2\end{matrix} & (N_{1440}N_{1520}f\rho)_{GI} & (f\rho)|N_{1520}|^2 \\ \text{Non-int} & (N_{1440}N_{1520})|\rho|^2 & \text{Non-int} \end{pmatrix}.$$



GI term corresponding to $\cos\theta_a \cos\theta_b$

two-fold FBA (TFFBA): $j = 1 = l$

$$\tilde{A}^{11} = \frac{(N_I - N_{II} + N_{III} - N_{IV})}{N}$$

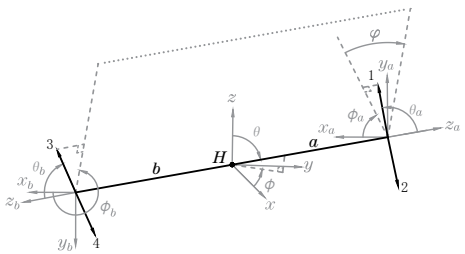
TFFBA-CPA

$$A_{CP}^{11} = \frac{1}{2}(\tilde{A}^{11} - \overline{\tilde{A}^{11}})$$

Without integrating out φ

GI terms

- $d^1_{1,0}(\theta_a)d^1_{1,0}(\theta_b)\cos\varphi \sim \sin\theta_a \sin\theta_b \sin\varphi$
- $d^1_{1,0}(\theta_a)d^1_{1,0}(\theta_b)\sin\varphi \sim \sin\theta_a \sin\theta_b \cos\varphi$



$\sin\varphi$: TPA induced CPA (Up-Down Asymmetry induced CPA)

$$A_T \equiv \frac{N(C_T > 0) - N(C_T < 0)}{N(C_T > 0) + N(C_T < 0)} = \frac{N_U - N_D}{N_U + N_D},$$

$$N_{U/D} \equiv N(\sin\varphi > 0), \quad C_T \equiv (\mathbf{p}_1 \times \mathbf{p}_2)\cdot\mathbf{p}_3$$

$$A^T_{CP} \equiv \frac{1}{2}(A_T + \overline{A}_T),$$

$\cos\varphi$: Left-Right Asymmetry induced CPA

$$A^{LR} = \frac{N(C_q > 0) - N(C_q < 0)}{N(C_q > 0) + N(C_q < 0)} = \frac{N_L - N_R}{N_L + N_R},$$

$$N_{L/R} \equiv N(\cos\varphi \gtrless 0), \quad C_q \equiv (\mathbf{p}_1 \times \mathbf{p}_2)\cdot(\mathbf{p}_3 \times \mathbf{p}_4)$$

$$A^{LR}_{CP} \equiv \frac{1}{2}(A^{LR} - \overline{A}^{LR}).$$

4 Summary and Outlook

Summary and Outlook

- interfer. of resonances with opposite parities generate FBA and FB-CPA in three-body decays of heavy hadrons
- Genuine Interfer. of resonances with opposite parities generate 2-dim-FBA and 2-dim-FB-CPA in four-body decays of heavy hadrons, (GIs also contribute to UD-CPA(TPA-CPA), LR-CPA).
- Look forward to the experimental measurements.

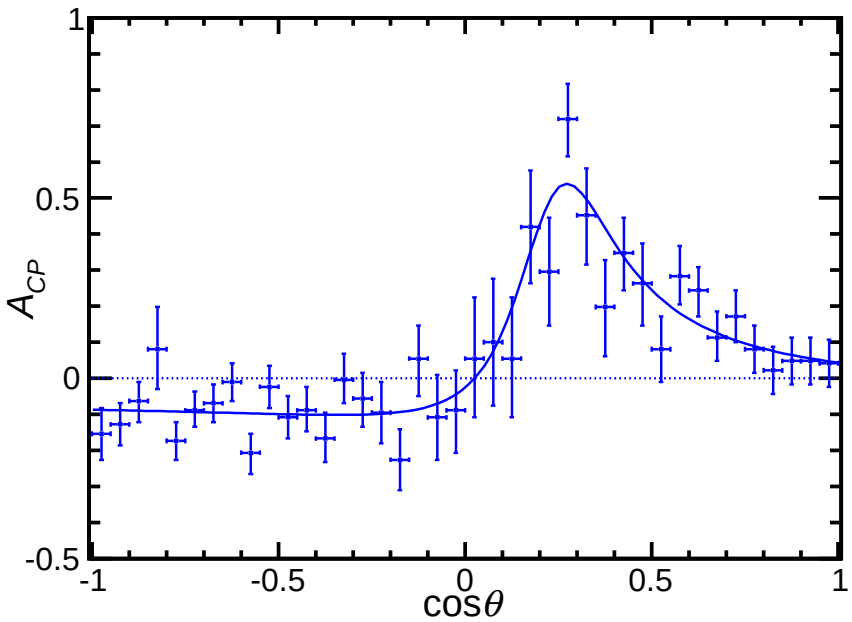
Thank you for your attentions!

Backup

Fit reg. CPA with only $f_0(500)$ and $\rho^0(770)$

$$\mathcal{A}_{B^-\to\pi^-\pi^+\pi^-} = \cos\theta\mathcal{R}_1\frac{c_\rho e^{i\delta_\rho}}{s_{\rm low}-m_\rho^2+im_\rho\Gamma_\rho}+\mathcal{R}_2\frac{c_f e^{i\delta_f}}{s_{\rm low}-m_f^2+im_f\Gamma_f},$$

$$\overline{\mathcal{A}}_{\mathcal{B}^+\to\pi^-\pi^+\pi^+} = \cos\theta\mathcal{R}_1\frac{\bar{c}_\rho e^{i\bar{\delta}_\rho}}{s_{\rm low}-m_\rho^2+im_\rho\Gamma_\rho}+\mathcal{R}_2\frac{\bar{c}_f e^{i\bar{\delta}_f}}{s_{\rm low}-m_f^2+im_f\Gamma_f}.$$



Para.	Value	Para.	Value
c_ρ	1	$\bar{c}_\rho$	1.05 ± 0.02
δ_ρ	0	$\bar{\delta}_\rho$	0
c_f	0.49 ± 0.07	$\bar{c}_f$	0.50 ± 0.06
δ_f	-0.64 ± 0.06	$\bar{\delta}_f$	-1.36 ± 0.31

Footnote for CPV in cascade 3-body decays $H \to R_i(\to h_1 h_2) h_3$

- interference within the same intermediate resonance $H \to R(\to h_1 h_2) h_3$
  - For most cases, the strong process $R \to h_1 h_2$ is governed by ONE decay amplitude, hence the strong phase difference can only be generated from the weak process $H \to R h_3$, between the tree and the penguin parts.
- interference between different resonances $H \to R_1(\to h_1 h_2) h_3$ and $H \to R_2(\to h_1 h_2) h_3$
  - strong phase can rise up in the weak process $H \to R_i h_3$, or from the phase diff between $R_1 \to h_1 h_2$ and $R_2 \to h_1 h_2$

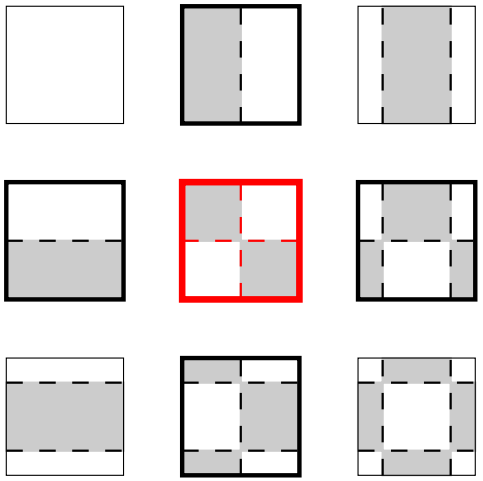
$$|\mathcal{A}|^2 \propto |a_S|^2 + |a_P|^2 \cos^2\theta + 2\Re\left(a_S a_P e^{i\delta}\right)\cos\theta$$

$$\overline{|\mathcal{M}|^2} \propto \left[\frac{|\alpha_{N_1}|^2}{|s_{N_1}|^2} + \frac{|\alpha_{N_2}|^2}{|s_{N_2}|^2}\left(1 + 3\cos^2\theta\right) + 12\Re\left(\frac{\alpha_{N_1}\alpha_{N_2}^* e^{i\delta}}{s_{N_1} s_{N_2}^*}\right)\cos\theta\right]$$

Applications to $\Lambda_b^0 \to p\pi^-\pi^+\pi^-$: $N(1440) - N(1520)$ and $f_0(500) - \rho(770)$

$\Lambda_b^0 \to N(\to p\pi^-)f/\rho(\to pi^+\pi^-)$: c_{θ_a} and c_{θ_b} are correlated.

$$(\Gamma_{jl}) \sim \begin{pmatrix} \text{Non-int} & \begin{matrix}(N_{1440}N_{1520})|f|^2, \\ (N_{1440}N_{1520})|\rho|^2\end{matrix} & \text{Non-int} \\ \begin{matrix}(f\rho)|N_{1440}|^2, \\ (f\rho)|N_{1520}|^2\end{matrix} & (N_{1440}N_{1520}f\rho)_{GI} & (f\rho)|N_{1520}|^2 \\ \text{Non-int} & (N_{1440}N_{1520})|\rho|^2 & \text{Non-int} \end{pmatrix}.$$



GI term corresponding to $\cos\theta_a \cos\theta_b$

dir-CPV-subtracted TFFBA-CPA

$$\hat{A}_{CP}^{11} = \frac{(N_I - N_{II} + N_{III} - N_{IV}) - (\bar{N}_I - \bar{N}_{II} + \bar{N}_{III} - \bar{N}_{IV})}{N + \bar{N}}$$

Without integrating out φ

Without integrating out φ, without the assumption of unpolarized H:

$$\overline{|\mathcal{A}|^2} \propto \sum \gamma^{jl}_{\sigma_{a'a},\sigma_{b'b}} d^j_{\sigma_{a'a},0}(\theta_a) d^l_{\sigma_{b'b},0}(\theta_b) e^{i\sigma_{aa'}\phi_a} e^{i\sigma_{bb'}\phi_b},$$

$$\gamma^{jl}_{\sigma_{a'a},\sigma_{b'b}} = \frac{w^{(ab,a'b')}_{jl} \mathcal{G}^{(aa')}_j \mathcal{G}^{(bb')}_l}{\mathcal{I}_a \mathcal{I}^*_{a'} \mathcal{I}_b \mathcal{I}^*_{b'}},$$

$$\begin{aligned} w^{(ab,a'b')jl}_{\sigma_{a'a},\sigma_{b'b}} =& \langle s_a - \sigma_a s_{a'} \sigma_{a'} | j \sigma_{a'a} \rangle \langle s_b - \sigma_b s_{b'} \sigma_{b'} | l \sigma_{b'b} \rangle \\ & \times (-)^{\sigma_a - s_a + \sigma_b - s_b} \mathcal{F}^{H\to ab}_{\sigma_a \sigma_b} \mathcal{F}^{H\to a'b'*}_{\sigma_{a'} \sigma_{b'}} \\ & \times P_{\sigma_{ab},\sigma_{a'b'}}(\theta), \end{aligned}$$

$$P(\theta) = \frac{1}{2}\left(1 + \begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix} P_z\right),$$

For unpolorized H, $P_z = 0$:
$\sigma_{aa'} = \sigma_{bb'}$, and
$e^{i\sigma_{aa'}\phi_a} e^{i\sigma_{bb'}\phi_b} \to e^{-i\sigma_{aa'}\varphi}$.