Two-body decays $\Lambda_b \to p\pi^-, pK^-$ in PQCD

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In collaboration with Ya Li, Shen-Yue Long, Hsiang-nan Li, Zhen-Jun Xiao and Fu-Sheng Yu

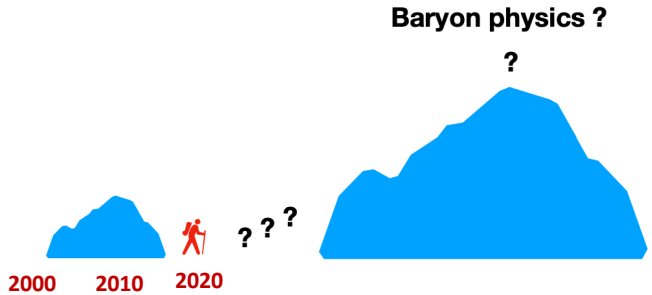
Outline

- Motivation
- Framework
- Results
- Summary

Motivation

Heavy baryon physics

- Weak decays of heavy hadrons related tightly to SM, Researches on meson achieved fruitful results (QCDF, PQCD, SCET,……).



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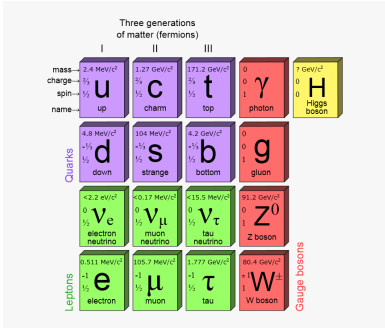
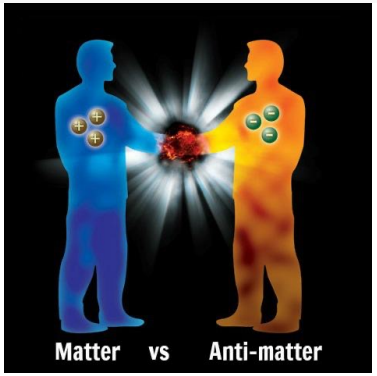




- CPV also plays an important role in evaluation of Universe.
- Sakharov three requirements: [Sakharov ,1967]
  - baryon number violation; C and CP violation; Out of thermal equilibrium
- As the most important issue in flavor physics, CP violation only confirmed in K, B, D meson.

$$a_{CP}^{T-odd}(\Lambda_b \to p\pi^+\pi^-\pi^-) = (-0.7 \pm 0.7 \pm 0.2)\%$$

$$A_{CP}(\Lambda_b \to p\pi^-) = (-3.5 \pm 1.7 \pm 2.0)\%, \qquad A_{CP}(\Lambda_b \to pK^-) = (-2.0 \pm 1.3 \pm 1.0)\%$$ [LHCb,2018]

- Baryon physics has lots of opportunities not only theoretically, but experimentally.

	2012	2018	2023	2029	2035
LHCb	Run-I	Run-II	Run-III	Run-IV	Run-V
luminosity	$3fb^{-1}$	$9fb^{-1}$	$23fb^{-1}$	$50fb^{-1}$	$300fb^{-1}$

Motivation

PQCD to predict CPV in B meson

- CPV can be predicted well in PQCD in B meson decays.

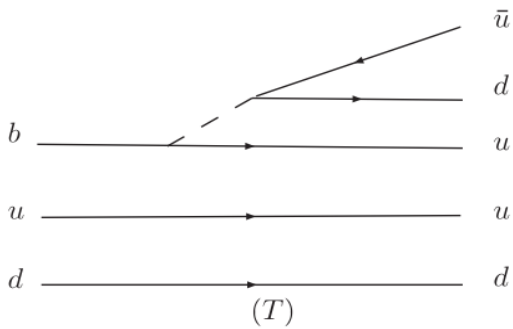
  [Keum, Li, Sanda, 2000] **[Lu, Ukai, Yang, 2000]**

- W-exchange/annihilation diagrams are not suppressed, but have large contributions to CP asymmetry.
- More W-exchange diagrams included, PQCD is hopeful to predict correct CPV of b-baryon decays.
- The only prediction of b-baryon CPV by PQCD is given by [Lu, Wang, Zou, Ali, Kramer, 2009]

$$\langle P(p',s')|\bar{u}\gamma_\mu(1-\gamma_5)b|\Lambda_b(p,s)\rangle = \overline{P}(p',s')(f_1\gamma_\mu - if_2\sigma_{\mu\nu}q^\nu + f_3 q_\mu)\Lambda_b(p,s) - \overline{P}(p',s')(g_1\gamma_\mu - ig_2\sigma_{\mu\nu}q^\nu + g_3 q_\mu)\gamma_5\Lambda_b(p,s).$$

Form factor f_1	Light-cone sum rule(2011)	Lattice(2015)	PQCD(2009)
	0.13	0.22	$2.2^{+0.08}_{-0.5}\times10^{-3}$

$$\mathcal{M} = \bar{p}(p')[f_1 + f_2\gamma_5]\Lambda_b(p),$$



	Factorizable	Nonfactorizable
$f_1(\Lambda_b \to p\pi)$	$2.43\times10^{-10} - i4.39\times10^{-10}$	$-2.43\times10^{-9} - i2.05\times10^{-9}$
$f_2(\Lambda_b \to p\pi)$	$2.64\times10^{-10} - i6.54\times10^{-10}$	$-1.75\times10^{-9} - i1.20\times10^{-9}$
$f_1(\Lambda_b \to pK)$	$-3.17\times10^{-10} - i1.22\times10^{-10}$	$-0.88\times10^{-9} + i0.54\times10^{-10}$
$f_2(\Lambda_b \to pK)$	$1.74\times10^{-10} - i1.96\times10^{-10}$	$-1.06\times10^{-9} + i1.67\times10^{-9}$

Sildes from my report on HFCPV2021

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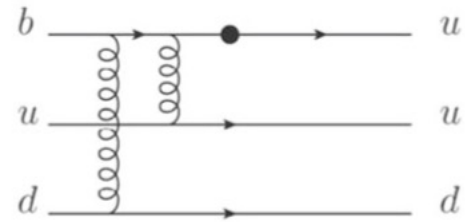
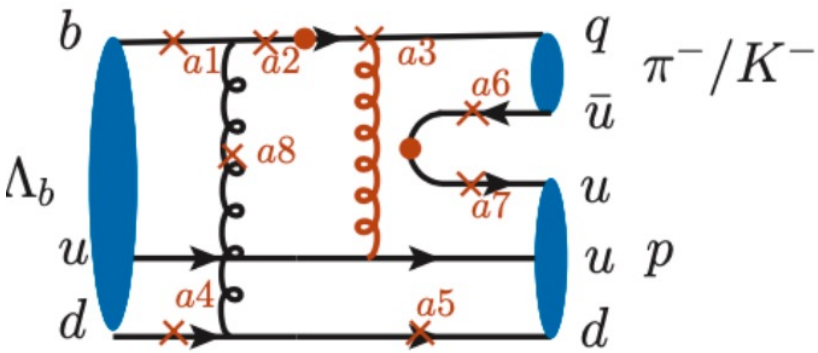


One of diagrams for $\Lambda_b \to p$

Table 4 Form factor $f_1(0)$ from various twist combinations of the Λ_b baryon and proton LCDAs. The first (second) theoretical errors of the total results come from the variations of the relevant parameters in the Λ_b baryon (proton) LCDAs

	Twist-3	Twist-4	Twist-5	Twist-6	Total
Exponential					
Twist-2	0.0007	−0.00007	−0.0005	− 0.000003	0.0001
Twist-3^{+-}	− 0.0001	0.002	0.0004	− 0.000004	0.002
Twist-3^{-+}	− 0.0002	0.0060	0.000004	0.00007	0.006
Twist-4	0.01	0.00009	0.25	0.0000007	0.26
Total	0.01	0.008	0.25	0.00007	0.27±0.09±0.07



$$= \int_0^1 d[x]d[x']dx_q \int d[\boldsymbol{b}]d[\boldsymbol{b}']d\boldsymbol{b}_q \tilde{\phi}_{\Lambda_b}([x],[\boldsymbol{b}],\mu^2)$$
$$* \widetilde{T_H}\left([x],[x'],[\boldsymbol{b}],[\boldsymbol{b}'],\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)$$
$$* \tilde{\phi}_p([x'],[\boldsymbol{b}'],\mu^2)\tilde{\phi}_M(x_q,\boldsymbol{b}_q,\mu^2) * S_t([x],[x'])$$

One of diagrams for $\Lambda_b \to p\pi, pK$

Results

The magnitudes are in unit of 10^{-9} and strong phases are in degrees, without CKM matrix elements.

$\Lambda_b \to p\pi$	$\lvert f_1 \rvert$	$\phi(f_1)^\circ$	$Real(f_1)$	$Imag(f_1)$	$\lvert f_2 \rvert$	$\phi(f_2)^\circ$	$Real(f_2)$	$Imag(f_2)$
T	1484	-39	1147	-942	3534	-38	2770	-2196
C_2	133	-16	128	-38	786	132	-531	580
E_2	196	32	166	104	24	69	8.6	23
B	101	-126	-60	-81	59	-141	-46	-37
$P_{QCD}^{C_1}$	149	156	-137	59	92	77	20	90
$P_{EW}^{C_1}$	7.1	-67	2.9	-6.6	20	-80	3.5	-20
$P_{QCD}^{C_2}$	81	175	-81	7.1	51	-2	51	-2.1
$P_{EW}^{C_2}$	1.4	114	-0.6	1.3	7.9	97	-0.9	7.9
$P_{QCD}^{E_1^u}$	17	-98	-2.3	-17	23	58	12	19
$P_{EW}^{E_1^u}$	1.2	-40	0.9	-0.8	3.1	-84	0.3	-3.1
$P_{QCD}^{E_1^d}$	14	-111	-5.1	-13	29	41	22	19
$P_{EW}^{E_1^d}$	0.6	-69	0.2	-0.6	0.8	27	0.7	0.4
$P_{QCD}^{E_2}$	14	-111	-5.1	-13	29	41	22	19
$P_{EW}^{E_2}$	0.6	-69	0.2	-0.6	0.8	27	0.7	0.4
P_{QCD}^{B}	6.1	-58	3.2	-5.2	1.9	-116	-0.9	-1.7
P_{EW}^{B}	0.9	-102	-0.2	-0.9	0.2	-72	0.1	-0.2

hierarchies among f_1 (with CKM) (Tree) $V_{ub}V_{ud}^* \sim \lambda^3$ (Penguin) $V_{tb}V_{td}^* \sim \lambda^3$

$$|T| \gg |P_{QCD}^{C_1}| > |E_2| \gtrsim |P_{QCD}^{C_2}| > |C_2| > |B| > |P_{QCD}^{E_1^u}| \approx |P_{QCD}^{E_1^d}| > |P_{EW}^{C_1}| > |P_{QCD}^{B}| > |P_{EW}^{C_2}| \approx |P_{EW}^{E_1^u}| \approx |P_{EW}^{B}| > |P_{EW}^{E_1^d}|$$

hierarchies among f_2 (with CKM)

$$|T| \gg |C_2| \gg |P_{QCD}^{C_1}| > |E_2| \approx |P_{QCD}^{C_2}| > |P_{QCD}^{E_1^d}| > |B| \approx |P_{EW}^{C_1}| > |P_{QCD}^{E_1^u}| > |P_{EW}^{C_2}| > |P_{EW}^{E_1^u}| > |P_{QCD}^{B}| > |P_{EW}^{E_1^d}| > |P_{EW}^{B}|$$

The magnitudes are in unit of 10^{-9} and strong phases are in degrees, without CKM matrix elements.

$\Lambda_b \to pK$	$\lvert f_1 \rvert$	$\phi(f_1)^\circ$	$Real(f_1)$	$Imag(f_1)$	$\lvert f_2 \rvert$	$\phi(f_2)^\circ$	$Real(f_2)$	$Imag(f_2)$
T	1912	-34	1580	-1078	3821	-43	2772	-2631
E_2	237	38	187	145	36	74	10	35
$P_{QCD}^{C_1}$	159	157	-154	65	112	77	25	109
$P_{EW}^{C_1}$	11	-62	5.0	-9.3	25	-78	5.0	-24
$P_{QCD}^{E_1^u}$	25	-97	-3.0	-25	26	58	14	22
$P_{EW}^{E_1^u}$	1.5	-23	1.4	-0.6	3.6	-84	0.4	-3.6
$P_{QCD}^{E_1^d}$	34	-112	-13	-31	34	69	12	32
$P_{EW}^{E_1^d}$	0.6	-59	0.3	-0.5	0.8	67	0.3	0.7

hierarchies among f_1 (with CKM) (Tree) $V_{ub}V_{us}^* \sim \lambda^4$ (Penguin) $V_{tb}V_{ts}^* \sim \lambda^2$

$$|P_{QCD}^{C_1}| \gg |T| \gtrsim |P_{QCD}^{E_1^d}| > |P_{QCD}^{E_1^u}| > |P_{EW}^{C_1}| > |E_2| > |P_{EW}^{E_1^u}| > |P_{EW}^{E_1^d}|$$

hierarchies among f_2 (with CKM)

$$|P_{QCD}^{C_1}| > |T| > |P_{QCD}^{E_1^d}| > |P_{QCD}^{E_1^u}| \approx |P_{EW}^{C_1}| > |P_{EW}^{E_1^u}| > |E_2| > |P_{EW}^{E_1^d}|$$

- The same as hierarchies between topological diagrams of B meson decays *[Cheng, Oh, 2011]*

$$f_1(\Lambda_b \to p\pi) = (-2.86^{+1.17+1.03}_{-0.96-1.12} - 6.42^{+2.37+1.99}_{-2.95-2.76} i) \times 10^{-9},$$
$$f_2(\Lambda_b \to p\pi) = (-1.35^{+0.45+0.51}_{-0.58-0.67} - 7.96^{+2.64+2.42}_{-3.04-3.44} i) \times 10^{-9},$$
$$f_1(\Lambda_b \to pK) = (6.36^{+1.71+2.10}_{-1.46-2.29} - 1.67^{+0.43+0.65}_{-0.52-0.47} i) \times 10^{-9},$$
$$f_2(\Lambda_b \to pK) = (-3.40^{+0.88+1.36}_{-0.98-1.54} - 8.47^{+2.28+3.23}_{-3.13-3.66} i) \times 10^{-9}.$$

$$R_{\pi K} \equiv \frac{\mathcal{B}r(\Lambda_b \to p\pi)}{\mathcal{B}r(\Lambda_b \to pK)} \qquad A_{CP}^{dir}(\Lambda_b \to pM) \equiv \frac{\mathcal{B}r(\Lambda_b \to pM) - \mathcal{B}r(\bar{\Lambda}_b \to \bar{p}\bar{M})}{\mathcal{B}r(\Lambda_b \to pM) + \mathcal{B}r(\bar{\Lambda}_b \to \bar{p}\bar{M})}$$

	PQCD(this work)	Experiment
$\mathcal{B}r(\Lambda_b \to p\pi)$	$(27.3^{+26.2}_{-18.5}) \times 10^{-6}$	$(4.5 \pm 0.8) \times 10^{-6}$
$\mathcal{B}r(\Lambda_b \to pK)$	$(29.4^{+34.3}_{-19.6}) \times 10^{-6}$	$(5.4 \pm 1.0) \times 10^{-6}$
$R_{\pi K}(\Lambda_b)$	$0.93^{+0.02}_{-0.20}$	0.82 ± 0.12
$A_{CP}^{dir}(\Lambda_b \to p\pi)$	$(-3.63^{+3.41}_{-0.07})\%$	$(-2.5 \pm 2.9)\%$
$A_{CP}^{dir}(\Lambda_b \to pK)$	$(8.26^{+0.32}_{-7.78})\%$	$(-2.5 \pm 2.2)\%$

- SU(3) breaking effects

$$\left|\frac{T}{T'}\right|_{f_1} = 0.78, \quad \left|\frac{T}{T'}\right|_{f_2} = 0.92, \quad \left|\frac{P_{QCD}^{C_1}}{P_{QCD}^{C_1\prime}}\right|_{f_1} = 0.84, \quad \left|\frac{P_{QCD}^{C_1}}{P_{QCD}^{C_1\prime}}\right|_{f_2} = 0.82, \quad \left|\frac{P_{EW}^{C_1}}{P_{EW}^{C_1\prime}}\right|_{f_1} = 0.85, \quad \left|\frac{P_{EW}^{C_1}}{P_{EW}^{C_1\prime}}\right|_{f_2} = 0.80,$$

$$\left|\frac{E_2}{E_2'}\right|_{f_1} = 0.82, \quad \left|\frac{E_2}{E_2'}\right|_{f_2} = 0.87, \quad \left|\frac{P_{QCD}^{E_1^u}}{P_{QCD}^{E_1^u\prime}}\right|_{f_1} = 0.78, \quad \left|\frac{P_{QCD}^{E_1^u}}{P_{QCD}^{E_1^u\prime}}\right|_{f_2} = 0.88, \quad \left|\frac{P_{EW}^{E_1^u}}{P_{EW}^{E_1^u\prime}}\right|_{f_1} = 0.80, \quad \left|\frac{P_{EW}^{E_1^u}}{P_{EW}^{E_1^u\prime}}\right|_{f_2} = 0.86,$$

$$\left|\frac{P_{QCD}^{E_1^d}}{P_{QCD}^{E_1^d\prime}}\right|_{f_1} = 0.82, \quad \left|\frac{P_{QCD}^{E_1^d}}{P_{QCD}^{E_1^d\prime}}\right|_{f_2} = 0.85, \quad \left|\frac{P_{EW}^{E_1^d}}{P_{EW}^{E_1^d\prime}}\right|_{f_1} = 0.81, \quad \left|\frac{P_{EW}^{E_1^d}}{P_{EW}^{E_1^d\prime}}\right|_{f_2} = 0.85,$$

- The SU(3) breaking effects are consistent with the B meson cases. ***[Cheng, Oh, 2011]***

Table 5: The parameters $f_1(f_2)$ for the topology T_f from various twist combinations of the Λ_b baryon and proton LCDAs. Only the central values are listed for simplicity.

	twist-3	twist-4	twist-5	twist-6
$\Lambda_b \to p\pi$				
twist-2	6.9(9.6)	-0.51(0.72)	-4.6(-0.64)	-0.02(0.03)
twist-3^{+-}	-0.48(0.68)	22.3(31.2)	0.14(-0.21)	-0.25(-0.37)
twist-3^{-+}	-0.80(1.1)	10.1(14.3)	-0.24(0.33)	0.59(0.84)
twist-4	72.1(99.7)	37.5(-52.4)	1254(1770)	0.003(-0.005)
$\Lambda_b \to pK$				
twist-2	8.77(12.4)	-0.67(0.95)	-5.72(-8.11)	-0.03(0.05)
twist-3^{+-}	-0.57(0.82)	28.4(41.2)	0.20(-0.29)	-0.34(-0.49)
twist-3^{-+}	-0.99(1.36)	14.3(19.2)	-0.28(0.42)	0.75(1.10)
twist-4	91.1(123)	45.8(-65.8)	1570(2214)	0.04(-0.05)

Table 6: The same as Table 5 but for the topology T_{nf}.

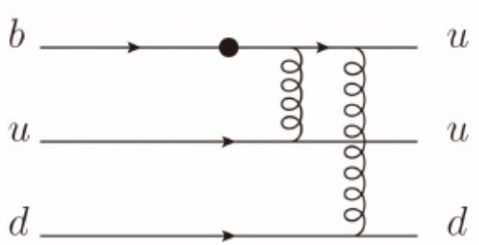
	twist-3	twist-4	twist-5	twist-6
$\Lambda_b \to p\pi$				
twist-2	-508-1093i(-1753-1546i)	-73+45i(-300-46i)	0.34-3.2i(-29-6.4i)	0.88-0.80i(-0.66+1.6i)
twist-3^{+-}	-112-71i(-531+87i)	-54+29i(-323+45i)	33+23i(182-31i)	0.014+0i(-0.03-0.005i)
twist-3^{-+}	632+419i(3150-571i)	-32-104i(923-149i)	29+21i(80-29i)	-0.031-0.004i(-0.02-0.001i)
twist-4	108-70i(-253-87i)	-277-135i(-227+136i)	-10-0.71i(-1.3+1.5i)	-0.31-0.31i(-1.5-0.46i)
$\Lambda_b \to pK$				
twist-2	-547-1201i(-2074-1849i)	-84+48i(-364-49i)	0.39-4.1i(-31-7.0i)	-1.0-0.9i(-0.7+1.9i)
twist-3^{+-}	-126-76i(-649+97i)	-57+42.4i(-349+52)	33+27i(217-39i)	0.015+0.002i(-0.035-0.005i)
twist-3^{-+}	700+427i(316-64)	-27.8-114i(1030-156i)	34+25i(97-36.4i)	-0.04-0.01i(-0.02-0.002i)
twist-4	115-71i(-290-118i)	-322-129i(-259+166i)	-12-0.94i(-1.4+1.66i)	-0.35-0.52i(-1.63-0.61i)

Table 8: The same as Table.5 but for the topology $P_{QCD}^{C_1}$.

	twist-3	twist-4	twist-5	twist-6
$\Lambda_b \to p\pi$				
twist-2	-2.3+47i(3.3+60i)	-1.5-0.47i(3.3+1.6i)	0.36+0.002i(0.12+0.54i)	0.03-0.02i(-0.05-0.13i)
twist-3^{+-}	-1.2+1.1i(-3.3-2.2i)	-0.68+6.2i(4.3-15i)	-0.04+0.08i(2.6+2.3i)	0.02-0.01i(-0.01-0.01i)
twist-3^{-+}	0.89-14i(0.41+21i)	-0.025+4.7i(3.6+2.2i)	-0.15+0.26i(-0.22+2.42i)	-0.05+0.005i(0.015+0.01i)
twist-4	-2.4+1.1i(2.3+13i)	-1.6+5.1i(-8.0+3.3i)	-114+0.1i(9.1-0.51i)	-0.11+0.05i(0.31+0.07i)
$\Lambda_b \to pK$				
twist-2	-2.9+59.5i(4.5+73i)	-1.84-0.41i(4.2+2.1i)	0.68-0.01i(0.13+0.74i)	0.03-0.02i(-0.67-0.21i)
twist-3^{+-}	-1.46+1.36i(-3.9-2.5i)	-0.8+8.0i(6.0-21.2i)	-0.06+0.1i(3.3+2.9i)	0.03-0.01i(-0.01-0.01i)
twist-3^{-+}	1.1-17.0i(0.48+27)	-0.03+5.81i(4.5+2.5i)	-0.16+0.36i(-0.3+3.3i)	-0.07+0.007i(0.02+0.01i)
twist-4	-3.0+1.24i(2.7+18.1i)	-2.24+5.79i(-11+4.6i)	-142+0.15i(11-0.69i)	-0.16+0.07i(0.41+0.01i)

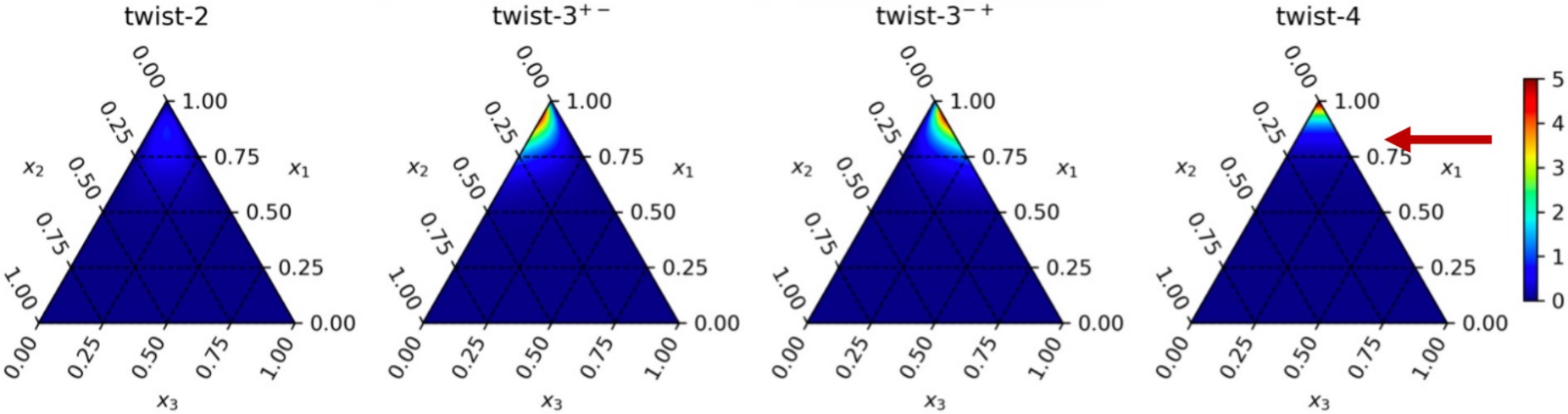
Summary

- Baryon physics have lots of opportunities.
- PQCD approach applies well to b-baryon decays.
- Contributions from high twist LCDAs are important.
- Our results suffer from large uncertainties due to non-perturbative inputs of baryons LCDAs
- More researches of high-twist LCDAs of baryons are urgently needed



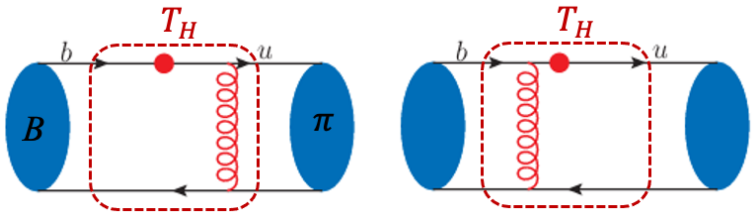
$$= \int [dx][db] \cdot \phi_{\Lambda_b} \cdot T_H \cdot \phi_p$$

	twist-3	twist-4	twist-5	twist-6
twist-2	~ 0	$r \cdot 2\sqrt{2}(1-x_1)x_3$	$r^2 \cdot 2\sqrt{2}x_3$	$r^3 \cdot 4\sqrt{2}(1-x_1)(1-x_2')$
twist-3^{+-}	$x_3(1-x_1)$	$r \cdot x_3$	$r^2 \cdot (1-x_1)(1-x_2')$	~ 0
twist-3^{-+}	~ 0	$r \cdot x_3$	$r^2 \cdot (1-x_1)(1-x_2')$	$r^3 \cdot (1-x_2')$
twist-4	$4\sqrt{2}x_3$	$r \cdot 2\sqrt{2}(1-x_1)(1-x_2')$	$r^2 \cdot 2\sqrt{2}(1-x_2')$	~ 0



A brief review of PQCD

$\langle \pi | H_{eff} | B \rangle$



$$M_W(80GeV) \gg M_b(5GeV) \gg \sqrt{M_b \Lambda_{QCD}}(1.5GeV) \gg \Lambda_{QCD}(0.5GeV)$$

$$M_{B\to\pi} = M_1(M_W, \mu_1) * M_2(\mu_1, M_b, \mu_2) * M_3(\mu_2, \Lambda_{QCD})$$

Under collinear factorization:

$$M(Q^2) = \int_0^1 dx_1 dx_2\, \phi_B(x_2, \mu^2) * T_H\left(x_1, x_2, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) * \phi_\pi(x_1, \mu^2)$$

1、propagators $\sim \frac{1}{x_1 x_2 Q^2} \to \infty\ when\ x \to 0$

2、$\alpha_s(x_1 x_2 Q^2) \to \infty\ when\ x \to 0$

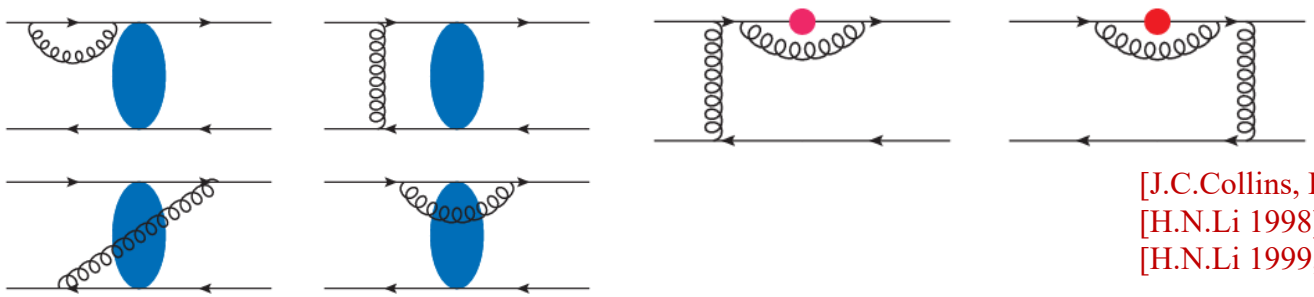
k_T factorization: retain transverse momentum of parton, propagator $\sim \frac{1}{x_1 x_2 Q^2 + \boldsymbol{k}_T^2}$

$$M(Q^2) = \int_0^1 dx_1 dx_2 \int d\boldsymbol{k}_{1T} d\boldsymbol{k}_{2T} \phi_B(x_2, \boldsymbol{k}_{2T}, \mu^2) * T_H\left(x_1, x_2, \boldsymbol{k}_{2T}, \boldsymbol{k}_{1T}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) * \phi_\pi(x_1, \boldsymbol{k}_{1T}, \mu^2)$$

- Fourier tranfer over k_T

$$M(Q^2)$$
$$= \int_0^1 dx_1 dx_2 \int d\boldsymbol{k}_{1T} d\boldsymbol{k}_{2T} \int \frac{d\boldsymbol{b}_1}{(2\pi)^2}\frac{d\boldsymbol{b}_1}{(2\pi)^2} \exp[-i(k_{1T}b_1 + k_{2T}b_2)] \phi_B(x_2, \boldsymbol{k}_{2T}, \mu^2) * T_H\left(x_1, x_2, \boldsymbol{k}_{2T}, \boldsymbol{k}_{1T}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) * \phi_\pi(x_1, \boldsymbol{k}_{1T}, \mu^2)$$
$$= \int_0^1 dx_1 dx_2 \int \frac{d\boldsymbol{b}_1}{(2\pi)^2}\frac{d\boldsymbol{b}_1}{(2\pi)^2} \int d\boldsymbol{k}_{1T} d\boldsymbol{k}_{2T} \exp[-i(k_{1T}b_1 + k_{2T}b_2)] \phi_B(x_2, \boldsymbol{k}_{2T}, \mu^2) * T_H\left(x_1, x_2, \boldsymbol{k}_{2T}, \boldsymbol{k}_{1T}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) * \phi_\pi(x_1, \boldsymbol{k}_{1T}, \mu^2)$$
$$= \int_0^1 dx_1 dx_2 \int d\boldsymbol{b}_1 d\boldsymbol{b}_2 \tilde{\phi}_B(x_2, \boldsymbol{b}_2, \mu^2) * \widetilde{T_H}\left(x_1, x_2, \boldsymbol{b}_2, \boldsymbol{b}_1, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) * \tilde{\phi}_\pi(x_1, \boldsymbol{b}_1, \mu^2)$$

- Resum double-log to get Sudakov factors $\tilde{\phi}_\pi(x_1, \boldsymbol{b}_1, \mu^2 = \phi_\pi(x_1, b_1)\exp[-S(\mu, b_1)]$



[J.C.Collins, D.E.Soper 1981]
[H.N.Li 1998]
[H.N.Li 1999, 2002]

$$M(Q^2) = \int_0^1 dx_1 dx_2 \int d\boldsymbol{b}_1 d\boldsymbol{b}_2 \tilde{\phi}_B(x_2, \boldsymbol{b}_2, \mu^2) * \widetilde{T_H}\left(x_1, x_2, \boldsymbol{b}_2, \boldsymbol{b}_1, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) * \tilde{\phi}_\pi(x_1, \boldsymbol{b}_1, \mu^2) * S_t(x_1, x_2)$$

- PQCD in b-baryon two-body decays:

$$M = \int_0^1 d[x] d[x'] dx_q \int d[\boldsymbol{b}] d[\boldsymbol{b}'] d\boldsymbol{b}_q \tilde{\phi}_{\Lambda_b}([x], [\boldsymbol{b}], \mu^2)$$
$$* \widetilde{T_H}\left([x], [x'], [\boldsymbol{b}], [\boldsymbol{b}'], \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) * \tilde{\phi}_p([x'], [\boldsymbol{b}'], \mu^2) \tilde{\phi}_M(x_q, \boldsymbol{b}_q, \mu^2) * S_t([x], [x'])$$

LCDAs of baryons

Λ_b LCDA

[P.Ball, V.M.Braun, E.Gardi (2008)]
[G.Bell, T.Feldmann, Yu-Ming Wang, M.W.Y.Yip (2013)]
[Yu-Ming Wang, Yue-Long Shen (2016)]

- introduce the general light-cone hadronic matrix element of Λ_b baryon

$$\Phi^{\alpha\beta\delta}_{\Lambda_b}(t_1,t_2) \equiv \epsilon_{ijk}\langle 0|[u_i^T(t_1\bar{n})]_\alpha[0,t_1\bar{n}][d_j(t_2\bar{n})]_\beta[0,t_2\bar{n}][b_k(0)]_\delta|\Lambda_b(v)\rangle$$
$$= \frac{1}{4}\Big\{ f^{(1)}_{\Lambda_b}(\mu)[\tilde{M}_1(v,t_1,t_2)\gamma_5 C^T]_{\beta\alpha} + f^{(2)}_{\Lambda_b}(\mu)[\tilde{M}_2(v,t_1,t_2)\gamma_5 C^T]_{\beta\alpha}\Big\}[\Lambda_b(v)]_\delta \quad (23)$$

- performing the Fourier transformation and including the NLO terms off the light-cone leads to the momentum space light-cone projector

$$M_2(\omega_1,\omega_2) = \frac{\not{n}}{\sqrt{2}}\psi_2(\omega_1,\omega_2) + \frac{\not{\bar{n}}}{\sqrt{2}}\psi_4(\omega_1,\omega_2)$$
$$M_1(\omega_1,\omega_2) = \frac{\not{\bar{n}}\not{n}}{4}\psi_3^{+-}(\omega_1,\omega_2) + \frac{\not{n}\not{\bar{n}}}{4}\psi_3^{-+}(\omega_1,\omega_2)$$

- ***G.Bell, T.Feldmann, Yu-Ming Wang, M.W.Y.Yip (2013)***

$$\psi_2(\omega_1,\omega_2) = \frac{\omega_1\omega_2}{\omega_0^4}e^{-(\omega_1+\omega_2)/\omega_0},$$
$$\psi_3^{+-}(\omega_1,\omega_2) = \frac{2\omega_1}{\omega_0^3}e^{-(\omega_1+\omega_2)/\omega_0},$$
$$\psi_3^{-+}(\omega_1,\omega_2) = \frac{2\omega_2}{\omega_0^3}e^{-(\omega_1+\omega_2)/\omega_0},$$
$$\psi_4(\omega_1,\omega_2) = \frac{1}{\omega_0^2}e^{-(\omega_1+\omega_2)/\omega_0}.$$

Proton LCDA

[V.M.Braun, R.J.Fries, N.Mahnke, E.Stein (2001)]

$$
\begin{aligned}
\bar{\Phi}^{\alpha\beta\gamma}_{proton} \equiv & \langle \mathcal{P}(p')|\bar{u}^i_\alpha(0)\bar{u}^j_\beta(z_1)\bar{d}^k_\gamma(z_2)|0\rangle \\
= & \frac{1}{4}\{S_1 m_p C_{\beta\alpha}(\bar{N}^+\gamma_5)_\gamma + S_2 m_p C_{\beta\alpha}(\bar{N}^-\gamma_5)_\gamma + P_1 m_p (C\gamma_5)_{\beta\alpha}\bar{N}^+_\gamma + P_2 m_p (C\gamma_5)_{\beta\alpha}\bar{N}^-_\gamma + V_1 (C\not{P})_{\beta\alpha}(\bar{N}^+\gamma_5)_\gamma \\
& + V_2 (C\not{P})_{\beta\alpha}(\bar{N}^-\gamma_5)_\gamma + V_3 \frac{m_p}{2}(C\gamma_\perp)_{\beta\alpha}(\bar{N}^+\gamma_5\gamma^\perp)_\gamma + V_4 \frac{m_p}{2}(C\gamma_\perp)_{\beta\alpha}(\bar{N}^-\gamma_5\gamma^\perp)_\gamma + V_5 \frac{m_p^2}{2Pz}(C\not{z})_{\beta\alpha}(\bar{N}^+\gamma_5)_\gamma \\
& + V_6 \frac{m_p^2}{2Pz}(C\not{z})_{\beta\alpha}(\bar{N}^-\gamma_5)_\gamma + A_1 (C\gamma_5\not{P})_{\beta\alpha}(\bar{N}^+)_\gamma + A_2 (C\gamma_5\not{P})_{\beta\alpha}(\bar{N}^-)_\gamma + A_3 \frac{m_p}{2}(C\gamma_5\gamma_\perp)_{\beta\alpha}(\bar{N}^+\gamma^\perp)_\gamma \\
& + A_4 \frac{m_p}{2}(C\gamma_5\gamma_\perp)_{\beta\alpha}(\bar{N}^-\gamma^\perp)_\gamma + A_5 \frac{m_p^2}{2Pz}(C\gamma_5\not{z})_{\beta\alpha}(\bar{N}^+)_\gamma + A_6 \frac{m_p^2}{2Pz}(C\gamma_5\not{z})_{\beta\alpha}(\bar{N}^-)_\gamma - T_1 (iC\sigma_{\perp P})_{\beta\alpha}(\bar{N}^+\gamma_5\gamma^\perp)_\gamma \\
& - T_2 (iC\sigma_{\perp P})_{\beta\alpha}(\bar{N}^-\gamma_5\gamma^\perp)_\gamma - T_3 \frac{m_p}{Pz}(iC\sigma_{Pz})_{\beta\alpha}(\bar{N}^+\gamma_5)_\gamma - T_4 \frac{m_p}{Pz}(iC\sigma_{zP})_{\beta\alpha}(\bar{N}^-\gamma_5)_\gamma - T_5 \frac{m_p^2}{2Pz}(iC\sigma_{\perp z})_{\beta\alpha}(\bar{N}^+\gamma_5\gamma^\perp)_\gamma \\
& - T_6 \frac{m_p^2}{2Pz}(iC\sigma_{\perp z})_{\beta\alpha}(\bar{N}^-\gamma_5\gamma^\perp)_\gamma + T_7 \frac{m_p}{2}(C\sigma_{\perp\perp'})_{\beta\alpha}(\bar{N}^+\gamma_5\sigma^{\perp\perp'})_\gamma + T_8 \frac{m_p}{2}(C\sigma_{\perp\perp'})_{\beta\alpha}(\bar{N}^-\gamma_5\sigma^{\perp\perp'})_\gamma\}
\end{aligned}
$$

TABLE I: Twist classification of proton distribution amplitudes.

	twist-3	twist-4	twist-5	twist-6
Vector	V_1	V_2, V_3	V_4, V_5	V_6
Pseudo-Vector	A_1	A_2, A_3	A_4, A_5	A_6
Tensor	T_1	T_2, T_3, T_7	T_4, T_5, T_8	T_6
Scalar		S_1	S_2	
Pesudo-Scalar		P_1	P_2	

Light meson LCDA

- ***Ball, 2005, 2006***

$$\Phi_\pi(p,x,\zeta) \equiv \frac{i}{\sqrt{2N_C}}\gamma_5\left[\not{p}\phi_\pi^A(x) + m_0^\pi\phi_\pi^P(x) + \zeta m_0^\pi(\not{p}\not{n}-1)\phi_\pi^T(x)\right].$$

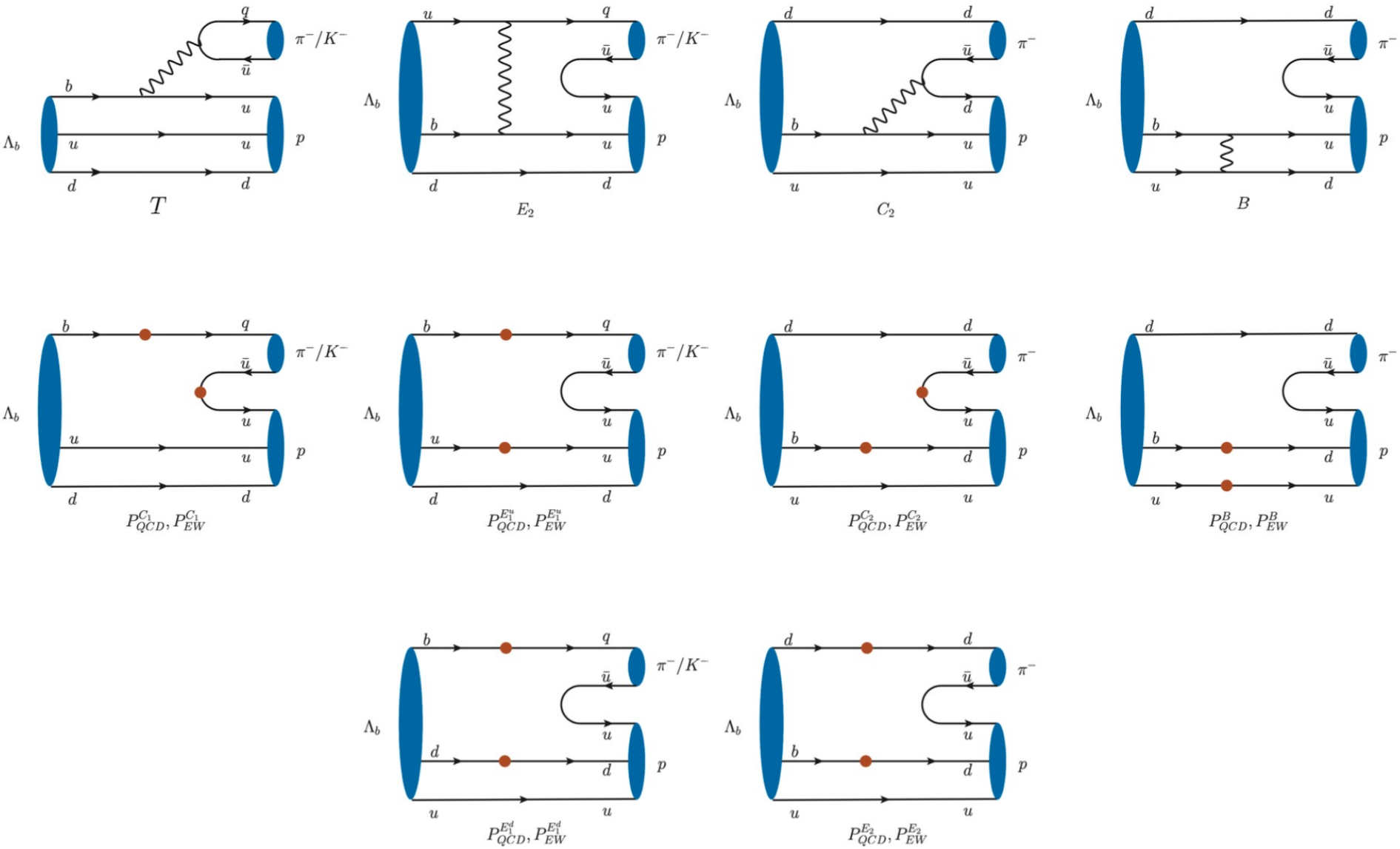
$$\begin{aligned}
\phi^A_{\pi(K)}(x) &= \frac{f_{\pi(K)}}{2\sqrt{2N_c}}6x(1-x)\left[1+a_1^{\pi(K)}C_1^{3/2}(2x-1)+a_2^{\pi(K)}C_2^{3/2}(2x-1)\right.\\
&\quad\left.+a_4^{\pi(K)}C_4^{3/2}(2x-1)\right],\\
\phi^P_{\pi(K)}(x) &= \frac{f_{\pi(K)}}{2\sqrt{2N_c}}\left[1+\left(30\eta_3-\frac{5}{2}\rho^2_{\pi(K)}\right)C_2^{1/2}(2x-1)\right.\\
&\quad\left.-3\left\{\eta_3\omega_3+\frac{9}{20}\rho^2_{\pi(K)}(1+6a_2^{\pi(K)})\right\}C_4^{1/2}(2x-1)\right],\\
\phi^T_{\pi(K)}(x) &= \frac{f_{\pi(K)}}{2\sqrt{2N_c}}(1-2x)\left[1\right.\\
&\quad\left.+6\left(5\eta_3-\frac{1}{2}\eta_3\omega_3-\frac{7}{20}\rho^2_{\pi(K)}-\frac{3}{5}\rho^2_{\pi(K)}a_2^{\pi(K)}\right)(1-10x+10x^2)\right]
\end{aligned}$$

$$\begin{aligned}
C_1^{3/2}(t) &= 3\,t\,,\\
C_2^{1/2}(t) &= \frac{1}{2}\left(3\,t^2-1\right),\quad C_2^{3/2}(t) = \frac{3}{2}\left(5\,t^2-1\right),\\
C_4^{1/2}(t) &= \frac{1}{8}\left(3-30\,t^2+35\,t^4\right),\quad C_4^{3/2}(t) = \frac{15}{8}\left(1-14\,t^2+21\,t^4\right).
\end{aligned}$$

$$\begin{aligned}
a_1^\pi &= 0,\quad a_2^{\pi,K} = 0.25\pm0.15,\quad a_4^\pi = -0.015,\quad a_1^K = 0.06,\\
\rho_\pi &= m_\pi/m_0^\pi,\quad \rho_K = m_K/m_0^K,\quad \eta_3^{\pi,K,\eta} = 0.015,\quad \omega_3^{\pi,K,\eta} = -3,
\end{aligned}$$

$$m_0^\pi = 1.4\pm0.1\ \text{GeV}\ ,\quad m_0^K = 1.6\pm0.1\ \text{GeV}\qquad \rho_{\pi(K)} = m_{\pi(K)}/m_0^{\pi(K)}$$

Topologies of $\Lambda_b \to p\pi, pK$



Feynman diagrams E_2 topology