江西师范大学
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Studying D meson decays with the SU(3) flavor symmetry/breaking

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$D \to P/V/S\ell^+\nu_\ell$: Ru-Min Wang, Yue-Xin Liu, Chong Hua, Jin-Huan Sheng, Yuan-Guo Xu

$D \to P_1P_2\ell^+\nu_\ell$: Ru-Min Wang, Yi-Jie Zhang, Yi Qiao, Xiao-Dong Cheng, Yuan-Guo Xu

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Outline

1. Motivation

2. Three body semileptonic charm decays

   $D \to P/V/S\ell^+\nu_\ell$

3. Four body semileptonic charm decays

   $D \to P_1 P_2 \ell^+ \nu_\ell$

4. Conclusion

1. Motivation

- ✓ Many exclusive $c \to s/d\,\ell^+\nu_\ell$ decays have been well measured.

  many $D \to P/V\,\ell^+\nu_\ell$, one $D_s^+ \to f_0(980)e^+\nu_e$, some $D \to P_1P_2\ell^+\nu_\ell$

- ✓ A large number of charmed hadrons are produced at the LHC, BESIII and Belle-II.

- ✓ Theoretical calculations of the form factors

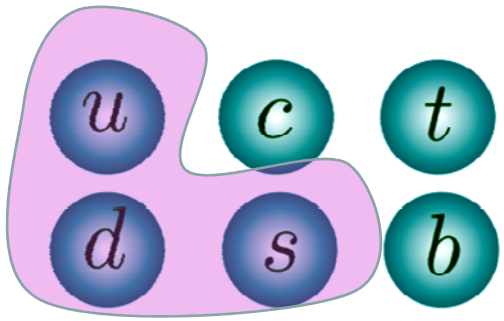
   $D \to P/V\,\ell^+\nu_\ell$

   $D \to S\ell^+\nu_\ell$, $D \to P_1P_2\ell^+\nu_\ell$ $(D \to \pi\pi\ell\nu, \pi K\ell\nu, \eta\pi\ell\nu)$

- ✓ Symmetries (for an example, SU(3) flavor symmetry) provide very important information for particle physics.

SU(3) flavor symmetry approach

- ✓ Irreducible representation approach
- ✓ Topological diagram approach



- ✓ **Advantage**： Independent of the detailed dynamics (Don't need calculate the form factors).
- ✓ **Disadvantage**： it can not determine the sizes of the amplitudes by itself.

Meson decays with the SU(3) flavor symmetry

decays \ baryons	charmed mesons	bottom mesons
$B/D \to M_1 M_2$	① Hai-Yang Cheng et. al., PRD 86, 014014 (2012); PRD 93, 114010 (2016) ② Xiao-Gang He, Wei Wang et al., EPJC 80, 359 (2020); CPC 42, 103108 (2018). ③ D.Pirtskhalava, PLB 712, 81 (2012). ④	① Si-Hong Zhou, et al, PRD 92, 094016 (2015) ; EPJC 77 (2017) 2, 125; PRD 92, 094016 (2015). ② Hai-Yang Cheng et. al., JHEP 09, 024 (2011) ; PRD 91, 014011 (2015) . ③ Xiao-Gang He, Wei Wang et al., EPJC 80, 359 (2020); CPC 42, 103108 (2018).; PRD 93, 114002 (2016); JHEP 08, 065 (2013); PRD 69, 074002 (2004); PRD 64,034002(2001); EPJC 9, 443 (1999); PRL 75, 1703 (1995). ④ .....
$B/D \to M_1 M_2 M_3$		① Si-Hong Zhou, , PRD 104, 116012 (2021). ② .....
$B/D \to M \ell \nu$ $B/D \to M_1 M_2 \ell \nu$		
$B/D \to M_1 M_2 \ell^+ \ell^-$		

Outline

1. Motivation

2. Three body semileptonic charm decays

   $D \to P/V/S\ell^+\nu_\ell$

3. Four body semileptonic charm decays

   $D \to P_1 P_2 \ell^+ \nu_\ell$

4. Conclusion

2. Three body semileptonic decays

2.1 $D \to P\ell^+\nu$ decays

2.2 $D \to \mathrm{V}\ell^+\nu$ decays

2.3 $D \to S\ell^+\nu$ decays

Outline

1. Motivation

2. Three body semileptonic charm decays

   $D \to P/V/S\ell^+\nu_\ell$

3. Four body semileptonic charm decays

   $D \to P_1 P_2 \ell^+ \nu_\ell$

4. Conclusion

3. Four body semileptonic decays

3.1 Non-resonant $D \to P_1 P_2 \ell^+ \nu_\ell$ Decays

3.2 Decays $D \to P_1 P_2 \ell^+ \nu_\ell$ with light scalar resonances

3.3 Decays $D \to P_1 P_2 \ell^+ \nu_\ell$ with vector resonances

3.4 Total branching ratios of decays $D \to P_1 P_2 \ell^+ \nu_\ell$

Mesons M^i_j $i=1,2,3$ *for* u,d,s

$$D_i = \left(D^0(c\bar{u}),\ D^+(c\bar{d}),\ D_s^+(c\bar{s})\right)$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \overline{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} \end{pmatrix},$$

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} + \frac{\omega_1}{\sqrt{3}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} + \frac{\omega_1}{\sqrt{3}} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\frac{2\omega_8}{\sqrt{6}} + \frac{\omega_1}{\sqrt{3}} \end{pmatrix},$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} cos\theta_P & -sin\theta_P \\ sin\theta_P & cos\theta_P \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix},$$

$$\begin{pmatrix} \phi \\ \omega \end{pmatrix} = \begin{pmatrix} cos\theta_V & -sin\theta_V \\ sin\theta_V & cos\theta_V \end{pmatrix} \begin{pmatrix} \omega_8 \\ \omega_1 \end{pmatrix},$$

Scalar with two quark picture:

$$S = \begin{pmatrix} \frac{a_0^0}{\sqrt{2}} + \frac{\sigma}{\sqrt{2}} & a_0^+ & K_0^+ \\ a_0^- & -\frac{a_0^0}{\sqrt{2}} + \frac{\sigma}{\sqrt{2}} & K_0^0 \\ K_0^- & \overline{K}_0^0 & f_0 \end{pmatrix}$$

$$\begin{pmatrix} f_0(980) \\ f_0(500) \end{pmatrix} = \begin{pmatrix} \cos\theta_S & \sin\theta_S \\ -\sin\theta_S & \cos\theta_S \end{pmatrix} \begin{pmatrix} f_0 \\ \sigma \end{pmatrix}$$

Scalar with four quark picture:

$\sigma = u\bar{u}d\bar{d}$, $f_0 = (u\bar{u} + d\bar{d})s\bar{s}/\sqrt{2}$,

$a_0^0 = (u\bar{u} - d\bar{d})s\bar{s}/\sqrt{2}$, $a_0^+ = u\bar{d}s\bar{s}$, $a_0^- = d\bar{u}s\bar{s}$,

$K_0^+ = u\bar{s}d\bar{d}$, $K_0^0 = d\bar{s}u\bar{u}$, $\bar{K}_0^0 = s\bar{d}u\bar{u}$, $K_0^+ = s\bar{u}d\bar{d}$,

S^{im}_{jm}

$$\begin{pmatrix} f_0(980) \\ f_0(500) \end{pmatrix} = \begin{pmatrix} \cos\phi_S & \sin\phi_S \\ -\sin\phi_S & \cos\phi_S \end{pmatrix} \begin{pmatrix} f_0 \\ \sigma \end{pmatrix}$$

5.1 Non-resonant $D \to P_1 P_2 \ell^+ \nu_\ell$ Decays

SU(3) flavor symmetry amplitudes:

$$A(D \to P_1 P_2 \ell^+ \nu_\ell)_N = c_{10} D_i P^i_j P^j_k H^k + c_{20} D_i P^i_j H^j P^k_k + c_{30} D_i H^i P^j_k P^k_j + c_{40} D_i H^i P^k_k P^j_j,$$

SU(3) flavor breaking amplitudes:

$$\begin{aligned}
\Delta A(D \to P_1 P_2 \ell^+ \nu_\ell)_N &= c_{11} D_i W^i_a P^a_j P^j_k H^k + c_{12} D_i P^i_j W^j_a P^a_k H^k + c_{13} D_i P^i_j P^j_k W^k_a H^a \\
&+ c_{21} D_i W^i_a P^a_j H^j P^k_k + c_{22} D_i P^i_j W^j_a H^a P^k_k \\
&+ c_{31} D_i W^i_a H^a P^j_k P^k_j + c_{32} D_i H^i P^j_k W^k_a P^a_j \\
&+ c_{41} D_i W^i_a H^a P^k_k P^j_j,
\end{aligned}$$

Branching ratios:

$$\frac{d\mathcal{B}(D \to P_1 P_2 \ell^+ \nu)_N}{dq^2\, dk^2} = \frac{1}{2} \tau_D |\mathcal{N}|^2 \beta_\ell (3 - \beta_\ell) |F_A|^2,$$

$$\begin{aligned}
|\mathcal{N}|^2 &= G_F^2 |V_{cq}|^2 \frac{\beta_\ell q^2 \sqrt{\lambda(m_D^2, q^2, k^2)}}{3 \cdot 2^{10} \pi^5 m_D^3} \quad \text{with} \quad \beta_\ell = 1 - \frac{m_\ell^2}{q^2}, \\
|F_A|^2 &= |F_0|^2 + \frac{2}{3}(|F_\parallel|^2 + |F_\perp|^2) + \frac{3 m_\ell^2}{q^2 (3 - \beta_\ell)} |F_t|^2,
\end{aligned}$$

Decay Amplitudes:

TABLE I: The amplitudes including the SU(3) flavor symmetry and the SU(3) flavor breaking contributions for the $D \to P_1 P_2 \ell^+ \nu_\ell$ decays. $C_1 \equiv c_{10} + c_{11} + c_{12} - 2c_{13}$, $C_2 \equiv c_{20} + c_{21} - 2c_{22}$, $C_3 \equiv c_{30} - 2c_{31}$, $C_4 \equiv c_{40} - 2c_{41}$, and $[C^{(\prime,\prime\prime)}]_R$ denotes the contributions come from the decays with R resonances.

Decay modes	Non-resonant decay amplitudes	Scalar resonant ones	Vector resonant ones
$c \to s\ell^+\nu_\ell$:			
$D^0 \to \pi^- \overline{K}^0 \ell^+ \nu_\ell$	C_1	$[C_1^\prime]_{K_0^-}$	$[C_1^{\prime\prime}]_{K^{*-}}$
$D^0 \to \pi^0 K^- \ell^+ \nu_\ell$	$\frac{1}{\sqrt{2}} C_1$	$[\frac{1}{\sqrt{2}} C_1^\prime]_{K_0^-}$	$[\frac{1}{\sqrt{2}} C_1^{\prime\prime}]_{K^{*-}}$
$D^0 \to \eta_8 K^- \ell^+ \nu_\ell$	$-\frac{1}{\sqrt{6}} C_1 + \sqrt{6} c_{12}$	...	...
$D^0 \to \eta_1 K^- \ell^+ \nu_\ell$	$\frac{2}{\sqrt{3}} (C_1 + \frac{3}{2} C_2) - \sqrt{3} c_{12}$	...	...
$D^+ \to \pi^+ K^- \ell^+ \nu_\ell$	C_1	$[C_1^\prime]_{\overline{K}_0^0}$	$[C_1^{\prime\prime}]_{K^{*0}}$
$D^+ \to \pi^0 \overline{K}^0 \ell^+ \nu_\ell$	$-\frac{1}{\sqrt{2}} C_1$	$[\frac{1}{\sqrt{2}} C_1^\prime]_{\overline{K}_0^0}$	$[\frac{1}{\sqrt{2}} C_1^{\prime\prime}]_{K^{*0}}$
$D^+ \to \eta_8 \overline{K}^0 \ell^+ \nu_\ell$	$-\frac{1}{\sqrt{6}} C_1 + \sqrt{6} c_{12}$	...	...
$D^+ \to \eta_1 \overline{K}^0 \ell^+ \nu_\ell$	$\frac{2}{\sqrt{3}} (C_1 + \frac{3}{2} C_2) - \sqrt{3} c_{12}$	...	...
$D_s^+ \to K^+ K^- \ell^+ \nu_\ell$	$C_1 + 2C_3 - 3c_{11}$	$[\cos^2\theta_S\, C_1^\prime]_{f_0(980)}$	$[C_1^{\prime\prime}]_\phi$
$D_s^+ \to K^0 \overline{K}^0 \ell^+ \nu_\ell$	$C_1 + 2C_3 - 3c_{11}$	$[\cos^2\theta_S\, C_1^\prime]_{f_0(980)}$	$[C_1^{\prime\prime}]_\phi$
$D_s^+ \to \pi^0 \pi^0 \ell^+ \nu_\ell$	$\sqrt{2} C_3 + \sqrt{2} c_{32}$	$[\sin\theta_S \cos\theta_S C_1^\prime]_{f_0(980)}$ $[-\sin\theta_S \cos\theta_S C_1^\prime]_{f_0(500)}$	...
$D_s^+ \to \pi^+ \pi^- \ell^+ \nu_\ell$	$2C_3$	$[\sqrt{2}\sin\theta_S \cos\theta_S C_1^\prime]_{f_0(980)}$ $[-\sqrt{2}\sin\theta_S \cos\theta_S C_1^\prime]_{f_0(500)}$	...
$D_s^+ \to \eta_8 \eta_8 \ell^+ \nu_\ell$	$\frac{2\sqrt{2}}{3} (C_1 + \frac{3}{2} C_3) - \sqrt{2}(2c_{11} + 2c_{12} + c_{32})$	...	...
$D_s^+ \to \eta_1 \eta_1 \ell^+ \nu_\ell$	$\frac{\sqrt{2}}{3} (C_1 + 3C_2 + 3C_3 + 9C_4) - \sqrt{2}(c_{11} + c_{12} + 3c_{21})$	...	...
$D_s^+ \to \eta_8 \eta_1 \ell^+ \nu_\ell$	$-\frac{2\sqrt{2}}{3} (C_1 + \frac{3}{2} C_2) + 2\sqrt{2}(c_{11} + c_{12} + \frac{3}{2} c_{21} + c_{32})$	...	...
$c \to d\ell^+\nu_\ell$:			
$D^0 \to K^- K^0 \ell^+ \nu_\ell$	$C_1 - 3(c_{12} - c_{13})$	$[C_1^\prime]_{a_0(980)}$	...
$D^0 \to \pi^0 \pi^- \ell^+ \nu_\ell$	...	...	$[\frac{1}{\sqrt{2}} C_1^{\prime\prime}]_{\rho^-}$
$D^0 \to \eta_8 \pi^- \ell^+ \nu_\ell$	$\sqrt{\frac{2}{3}} C_1 + \sqrt{6} c_{13}$	$[\sqrt{\frac{2}{3}} C_1^\prime]_{a_0(980)}$	$[\frac{1}{\sqrt{6}} C_1^{\prime\prime}]_{\rho^-}$
$D^0 \to \eta_1 \pi^- \ell^+ \nu_\ell$	$\frac{2}{\sqrt{3}} (C_1 + \frac{3}{2} C_2) + \sqrt{3}(2c_{13} + 3c_{22})$	$[\frac{2}{\sqrt{3}} C_1^\prime]_{a_0(980)}$	$[\frac{1}{\sqrt{3}} C_1^{\prime\prime}]_{\rho^-}$
$D^+ \to \overline{K}^0 K^0 \ell^+ \nu_\ell$	$C_1 + 2C_3 - 3(c_{12} - c_{13} - 2c_{31})$	$[\frac{1}{2} C_1^\prime]_{a_0(980)^0}$ $[\frac{1}{\sqrt{2}} \sin\theta_S \cos\theta_S C_1^\prime]_{f_0(980)}$	...
$D^+ \to K^+ K^- \ell^+ \nu_\ell$	$2C_3 + 6c_{31}$	$[-\frac{1}{2} C_1^\prime]_{a_0(980)^0}$ $[\frac{1}{\sqrt{2}} \sin\theta_S \cos\theta_S C_1^\prime]_{f_0(980)}$	...
$D^+ \to \pi^+ \pi^- \ell^+ \nu_\ell$	$C_1 + 2C_3 + 3c_{13} + 6c_{31}$	$[\sin^2\theta_S C_1^\prime]_{f_0(980)}$ $[\cos^2\theta_S C_1^\prime]_{f_0(500)}$	$[\frac{1}{2} C_1^{\prime\prime}]_{\rho^0,\omega}$
$D^+ \to \pi^0 \pi^0 \ell^+ \nu_\ell$	$\frac{1}{\sqrt{2}} (C_1 + 2C_3) + \frac{1}{\sqrt{2}} (3c_{13} + 6c_{31} + 2c_{32})$	$[\frac{1}{\sqrt{2}} \sin^2\theta_S C_1^\prime]_{f_0(980)}$ $[\frac{1}{\sqrt{2}} \cos^2\theta_S C_1^\prime]_{f_0(500)}$	...
$D^+ \to \eta_8 \pi^0 \ell^+ \nu_\ell$	$-\frac{1}{\sqrt{3}} (C_1 + C_2) - \sqrt{3}(c_{13} + c_{22})$	$[-\frac{1}{\sqrt{6}} C_1^\prime]_{a_0(980)}$	...
$D^+ \to \eta_1 \pi^0 \ell^+ \nu_\ell$	$-\sqrt{\frac{2}{3}} (C_1 + C_2) - \frac{1}{\sqrt{6}} (6c_{13} + 9c_{22})$	$[-\frac{1}{\sqrt{3}} C_1^\prime]_{a_0(980)}$	...
$D^+ \to \eta_8 \eta_8 \ell^+ \nu_\ell$	$\frac{\sqrt{2}}{9} (C_1 + 6C_3) + \frac{1}{\sqrt{2}} (c_{13} + 6c_{31} - 2c_{32})$	...	...
$D^+ \to \eta_1 \eta_1 \ell^+ \nu_\ell$	$\frac{\sqrt{2}}{3} (C_1 + 3C_2 + 3C_3 + 9C_4) + \sqrt{2}(c_{13} + 3c_{22} + 3c_{31} + 9c_{41})$	...	...
$D^+ \to \eta_8 \eta_1 \ell^+ \nu_\ell$	$\frac{\sqrt{2}}{3} (C_1 + \frac{3}{2} C_2) + \sqrt{2}(c_{13} + \frac{3}{2} c_{22} + 2c_{32})$	...	...
$D_s^+ \to K^+ \pi^- \ell^+ \nu_\ell$	$C_1 - 3c_{11} + 3c_{13}$	$[C_1^\prime]_{K_0^0}$	$[C_1^{\prime\prime}]_{K^{*0}}$
$D_s^+ \to K^0 \pi^0 \ell^+ \nu_\ell$	$-\frac{1}{\sqrt{2}} C_1 - \frac{1}{\sqrt{2}} (-3c_{11} + 3c_{13})$	$[-\frac{1}{\sqrt{2}} C_1^\prime]_{K_0^0}$	$[\frac{1}{\sqrt{2}} C_1^{\prime\prime}]_{K^{*0}}$
$D_s^+ \to \eta_8 K^0 \ell^+ \nu_\ell$	$-\frac{1}{\sqrt{6}} C_1 + \frac{1}{\sqrt{6}} (3c_{11} + 6c_{12} - 3c_{13})$	...	...
$D_s^+ \to \eta_1 K^0 \ell^+ \nu_\ell$	$\frac{2}{\sqrt{3}} (C_1 + \frac{3}{2} C_2) - \sqrt{3}(2c_{11} + c_{12} - 2c_{13} + 3c_{21} - 3c_{22})$	...	...

$$
\begin{aligned}
A(D^0 \to \pi^- \overline{K}^0 \ell^+ \nu_\ell)_N &= A(D^+ \to \pi^+ K^- \ell^+ \nu_\ell)_N = \sqrt{2} A(D^0 \to \pi^0 K^- \ell^+ \nu_\ell)_N = -\sqrt{2} A(D^+ \to \pi^0 \overline{K}^0 \ell^+ \nu_\ell)_N, \\
A(D^0 \to \eta_8 K^- \ell^+ \nu_\ell)_N &= A(D^+ \to \eta_8 \overline{K}^0 \ell^+ \nu_\ell)_N, \\
A(D^0 \to \eta_1 K^- \ell^+ \nu_\ell)_N &= A(D^+ \to \eta_1 \overline{K}^0 \ell^+ \nu_\ell)_N, \\
A(D_s^+ \to K^+ K^- \ell^+ \nu_\ell)_N &= A(D_s^+ \to K^0 \overline{K}^0 \ell^+ \nu_\ell)_N, \\
A(D^0 \to K^- K^0 \ell^+ \nu_\ell)_N &= A(D^+ \to \overline{K}^0 K^0 \ell^+ \nu_\ell)_N - A(D^+ \to K^+ K^- \ell^+ \nu_\ell)_N, \\
A(D_s^+ \to K^+ \pi^- \ell^+ \nu_\ell)_N &= -\sqrt{2} A(D_s^+ \to K^0 \pi^0 \ell^+ \nu_\ell)_N. \qquad (1
\end{aligned}
$$

If ignoring both the OZI suppressed SU(3) flavor symmetry contributions and the SU(3) flavor breaking contributions, almost all amplitudes of the non-resonant decays can be related by the coefficient c_{10}.

TABLE II: The experimental data and the SU(3) flavor symmetry predictions of the non-resonant branching ratios and the total branching ratios of the $D \to P_1 P_2 \ell^+ \nu_\ell$ decays with the $c \to s\ell^+\nu_\ell$ transitions with the 2σ errors. The experimental data are taken from PDG if not special specified. 'N' denotes the non-resonant contributions, and 'T' denotes the total contributions including the non-resonance, the light scalar meson resonances and the vector meson resonances. [b]Scale factor $S = 3.2$ is considered. The same below.

$c \to s\ell\nu$

Branching ratios	Exp. data with N	Ones with N
$\mathcal{B}(D^0 \to \pi^- \overline{K}^0 e^+ \nu_e)(\times 10^{-2})$		0.076 ± 0.041
$\mathcal{B}(D^0 \to \pi^0 K^- e^+ \nu_e)(\times 10^{-2})$		0.039 ± 0.021
$\mathcal{B}(D^0 \to \eta K^- e^+ \nu_e)(\times 10^{-6})$		3.04 ± 3.04
$\mathcal{B}(D^0 \to \eta' K^- e^+ \nu_e)(\times 10^{-6})$		3.26 ± 1.78
$\mathcal{B}(D^+ \to \pi^+ K^- e^+ \nu_e)(\times 10^{-2})$	< 0.7	0.20 ± 0.10
$\mathcal{B}(D^+ \to \pi^0 \overline{K}^0 e^+ \nu_e)(\times 10^{-2})$		0.099 ± 0.052
$\mathcal{B}(D^+ \to \eta \overline{K}^0 e^+ \nu_e)(\times 10^{-5})$		0.77 ± 0.77
$\mathcal{B}(D^+ \to \eta' \overline{K}^0 e^+ \nu_e)(\times 10^{-5})$		0.83 ± 0.45
$\mathcal{B}(D_s^+ \to K^+ K^- e^+ \nu_e)(\times 10^{-2})$		0.031 ± 0.017
$\mathcal{B}(D_s^+ \to K^0 \overline{K}^0 e^+ \nu_e)(\times 10^{-3})$		0.30 ± 0.16
$\mathcal{B}(D_s^+ \to \pi^+ \pi^- e^+ \nu_e)(\times 10^{-3})$		$\cdots$
$\mathcal{B}(D_s^+ \to \pi^0 \pi^0 e^+ \nu_e)(\times 10^{-4})$		$\cdots$
$\mathcal{B}(D_s^+ \to \eta\eta e^+ \nu_e)(\times 10^{-4})$		0.49 ± 0.42
$\mathcal{B}(D_s^+ \to \eta\eta' e^+ \nu_e)(\times 10^{-6})$		4.44 ± 2.68
$\mathcal{B}(D^0 \to \pi^- \overline{K}^0 \mu^+ \nu_\mu)(\times 10^{-2})$		0.073 ± 0.039
$\mathcal{B}(D^0 \to \pi^0 K^- \mu^+ \nu_\mu)(\times 10^{-2})$		0.038 ± 0.020
$\mathcal{B}(D^0 \to \eta K^- \mu^+ \nu_\mu)(\times 10^{-6})$		2.77 ± 2.77
$\mathcal{B}(D^0 \to \eta' K^- \mu^+ \nu_\mu)(\times 10^{-6})$		2.25 ± 1.22
$\mathcal{B}(D^+ \to \pi^+ K^- \mu^+ \nu_\mu)(\times 10^{-2})$	0.19 ± 0.10	0.19 ± 0.10
$\mathcal{B}(D^+ \to \pi^0 \overline{K}^0 \mu^+ \nu_\mu)(\times 10^{-2})$		0.095 ± 0.050
$\mathcal{B}(D^+ \to \eta \overline{K}^0 \mu^+ \nu_\mu)(\times 10^{-5})$		0.70 ± 0.70
$\mathcal{B}(D^+ \to \eta' \overline{K}^0 \mu^+ \nu_\mu)(\times 10^{-5})$		0.57 ± 0.31
$\mathcal{B}(D_s^+ \to K^+ K^- \mu^+ \nu_\mu)(\times 10^{-2})$		0.029 ± 0.016
$\mathcal{B}(D_s^+ \to K^0 \overline{K}^0 \mu^+ \nu_\mu)(\times 10^{-3})$		0.28 ± 0.15
$\mathcal{B}(D_s^+ \to \pi^+ \pi^- \mu^+ \nu_\mu)(\times 10^{-3})$		$\cdots$
$\mathcal{B}(D_s^+ \to \pi^0 \pi^0 \mu^+ \nu_\mu)(\times 10^{-4})$		$\cdots$
$\mathcal{B}(D_s^+ \to \eta\eta \mu^+ \nu_\mu)(\times 10^{-4})$		0.45 ± 0.39
$\mathcal{B}(D_s^+ \to \eta\eta' \mu^+ \nu_\mu)(\times 10^{-6})$		3.30 ± 1.99

Many br are on the orders of $O(10^{-3} - 10^{-4})$

TABLE III: The experimental data and the SU(3) flavor symmetry predictions of the non-resonant branching ratios and the total branching ratios of $D \to P_1 P_2 \ell^+ \nu_\ell$ decays with the $c \to d\ell^+\nu_\ell$ transitions with the 2σ errors.

$c \to d\ell\nu$ Suppressed by V_{cd}. $O(10^{-4} - 10^{-7})$

Branching ratios	Exp. data with N	Ones with N
$\mathcal{B}(D^0 \to K^- K^0 e^+ \nu_e)(\times 10^{-5})$		0.74 ± 0.41
$\mathcal{B}(D^0 \to \pi^0 \pi^- e^+ \nu_e)(\times 10^{-3})$		0
$\mathcal{B}(D^0 \to \eta \pi^- e^+ \nu_e)(\times 10^{-5})$		4.19 ± 2.63
$\mathcal{B}(D^0 \to \eta' \pi^- e^+ \nu_e)(\times 10^{-5})$		0.33 ± 0.22
$\mathcal{B}(D^+ \to \overline{K}^0 K^0 e^+ \nu_e)(\times 10^{-5})$		1.88 ± 1.01
$\mathcal{B}(D^+ \to K^+ K^- e^+ \nu_e)(\times 10^{-5})$		$\cdots$
$\mathcal{B}(D^+ \to \pi^+ \pi^- e^+ \nu_e)(\times 10^{-3})$		0.30 ± 0.16
$\mathcal{B}(D^+ \to \pi^0 \pi^0 e^+ \nu_e)(\times 10^{-4})$		1.54 ± 0.83
$\mathcal{B}(D^+ \to \eta \pi^0 e^+ \nu_e)(\times 10^{-5})$		5.56 ± 3.47
$\mathcal{B}(D^+ \to \eta' \pi^0 e^+ \nu_e)(\times 10^{-6})$		4.48 ± 2.96
$\mathcal{B}(D^+ \to \eta \eta e^+ \nu_e)(\times 10^{-6})$		2.74 ± 1.99
$\mathcal{B}(D^+ \to \eta \eta' e^+ \nu_e)(\times 10^{-8})$		3.20 ± 1.92
$\mathcal{B}(D_s^+ \to K^+ \pi^- e^+ \nu_e)(\times 10^{-3})$		0.078 ± 0.043
$\mathcal{B}(D_s^+ \to K^0 \pi^0 e^+ \nu_e)(\times 10^{-4})$		0.39 ± 0.22
$\mathcal{B}(D_s^+ \to \eta K^0 e^+ \nu_e)(\times 10^{-5})$		1.51 ± 0.95
$\mathcal{B}(D_s^+ \to \eta' K^0 e^+ \nu_e)(\times 10^{-7})$		4.32 ± 2.88
$\mathcal{B}(D^0 \to K^- K^0 \mu^+ \nu_\mu)(\times 10^{-5})$		0.68 ± 0.37
$\mathcal{B}(D^0 \to \pi^0 \pi^- \mu^+ \nu_\mu)(\times 10^{-3})$		0
$\mathcal{B}(D^0 \to \eta \pi^- \mu^+ \nu_\mu)(\times 10^{-5})$		4.00 ± 2.51
$\mathcal{B}(D^0 \to \eta' \pi^- \mu^+ \nu_\mu)(\times 10^{-5})$		0.30 ± 0.20
$\mathcal{B}(D^+ \to \overline{K}^0 K^0 \mu^+ \nu_\mu)(\times 10^{-5})$		1.73 ± 0.93
$\mathcal{B}(D^+ \to K^+ K^- \mu^+ \nu_\mu)(\times 10^{-5})$		$\cdots$
$\mathcal{B}(D^+ \to \pi^+ \pi^- \mu^+ \nu_\mu)(\times 10^{-3})$		0.29 ± 0.16
$\mathcal{B}(D^+ \to \pi^0 \pi^0 \mu^+ \nu_\mu)(\times 10^{-4})$		1.50 ± 0.81
$\mathcal{B}(D^+ \to \eta \pi^0 \mu^+ \nu_\mu)(\times 10^{-5})$		5.31 ± 3.31
$\mathcal{B}(D^+ \to \eta' \pi^0 \mu^+ \nu_\mu)(\times 10^{-6})$		4.03 ± 2.66
$\mathcal{B}(D^+ \to \eta \eta \mu^+ \nu_\mu)(\times 10^{-6})$		2.47 ± 1.78
$\mathcal{B}(D^+ \to \eta \eta' \mu^+ \nu_\mu)(\times 10^{-8})$		1.98 ± 1.18
$\mathcal{B}(D_s^+ \to K^+ \pi^- \mu^+ \nu_\mu)(\times 10^{-3})$		0.075 ± 0.042
$\mathcal{B}(D_s^+ \to K^0 \pi^0 \mu^+ \nu_\mu)(\times 10^{-4})$		0.38 ± 0.21
$\mathcal{B}(D_s^+ \to \eta K^0 \mu^+ \nu_\mu)(\times 10^{-5})$		1.40 ± 0.88
$\mathcal{B}(D_s^+ \to \eta' K^0 \mu^+ \nu_\mu)(\times 10^{-7})$		3.39 ± 2.26

Many of them might be observed by the BESIII and BelleII experiments in the near future。

5.2 Decays $D \to P_1 P_2 \ell^+ \nu_\ell$ with light scalar resonances

$$\mathcal{B}(D \to R\ell^+\nu_\ell, R \to P_1 P_2) = \mathcal{B}(D \to R\ell^+\nu_\ell) \times \mathcal{B}(R \to P_1 P_2),$$

TABLE IV: The strong coupling constants of the $S \to P_1 P_2$ decays by the SU(3) flavor symmetry.

strong couplings	ones for two quark state	ones for four quark state
$g_{K_0^- \to \pi^0 K^-}$	$\frac{1}{\sqrt{2}}\, g_2$	$-\frac{1}{\sqrt{2}} g_4$
$g_{K_0^- \to \pi^- \overline{K}^0}$	g_2	g_4
$g_{\overline{K}_0^0 \to \pi^+ K^-}$	g_2	g_4
$g_{\overline{K}_0^0 \to \pi^0 \overline{K}^0}$	$-\frac{1}{\sqrt{2}}\, g_2$	$\frac{1}{\sqrt{2}} g_4$
$g_{a_0(980)^- \to \eta\pi^-}$	$2\, g_2 \left(\frac{1}{\sqrt{6}} cos\theta_P - \frac{1}{\sqrt{3}} sin\theta_P\right)$	$2\, g_4' \left(\frac{1}{\sqrt{6}} cos\theta_P - \frac{1}{\sqrt{3}} sin\theta_P\right)$
$g_{a_0(980)^- \to \eta'\pi^-}$	$2\, g_2 \left(\frac{1}{\sqrt{6}} sin\theta_P + \frac{1}{\sqrt{3}} cos\theta_P\right)$	$2\, g_4' \left(\frac{1}{\sqrt{6}} sin\theta_P + \frac{1}{\sqrt{3}} cos\theta_P\right)$
$g_{a_0(980)^- \to K^0 K^-}$	g_2	g_4
$g_{a_0(980)^0 \to \eta\pi^0}$	$g_2 \left(\frac{1}{\sqrt{3}} cos\theta_P - \sqrt{\frac{2}{3}} sin\theta_P\right)$	$g_4' \left(\frac{1}{\sqrt{6}} cos\theta_P - \frac{1}{\sqrt{3}} sin\theta_P\right)$
$g_{a_0(980)^0 \to \eta'\pi^0}$	$g_2 \left(\frac{1}{\sqrt{3}} sin\theta_P + \sqrt{\frac{2}{3}} cos\theta_P\right)$	$g_4' \left(\frac{1}{\sqrt{6}} sin\theta_P + \frac{1}{\sqrt{3}} cos\theta_P\right)$
$g_{a_0(980)^0 \to K^+ K^-}$	$\frac{1}{\sqrt{2}}\, g_2$	$\frac{1}{\sqrt{2}}\, g_4$
$g_{a_0(980)^0 \to K^0 \overline{K}^0}$	$-\frac{1}{\sqrt{2}}\, g_2$	$-\frac{1}{\sqrt{2}}\, g_4$
$g_{f_0(980) \to \pi^+\pi^-}$	$\sqrt{2}\, g_2\, sin\theta_S$	$\sqrt{2}\, g_4'\, cos\phi_S + g_4 sin\phi_S$
$g_{f_0(980) \to \pi^0\pi^0}$	$g_2\, sin\theta_S$	$g_4'\, cos\phi_S - \frac{1}{\sqrt{2}} g_4 sin\phi_S$
$g_{f_0(980) \to K^+ K^-}$	$g_2\, cos\theta_S$	$\frac{1}{\sqrt{2}} g_4 cos\phi_S$
$g_{f_0(980) \to K^0 \overline{K}^0}$	$g_2\, cos\theta_S$	$\frac{1}{\sqrt{2}} g_4 cos\phi_S$
$g_{f_0(500) \to \pi^+\pi^-}$	$\sqrt{2}\, g_2\, cos\theta_S$	$-\sqrt{2}\, g_4'\, sin\phi_S + g_4 cos\phi_S$
$g_{f_0(500) \to \pi^0\pi^0}$	$g_2\, cos\theta_S$	$-g_4'\, sin\phi_S - \frac{1}{\sqrt{2}} g_4 cos\phi_S$

$$\Gamma(S \to P_1 P_2) = \frac{p_c}{8\pi m_S^2} g^2_{S \to P_1 P_2},$$

$$g^{2q}_{S \to P_1 P_2} = g_2 S^i_j P^k_i P^j_k,$$

$$g^{4q}_{S \to P_1 P_2} = g_4 S^{im}_{jn} P^j_i P^n_m + g_4' S^{im}_{jm} P^n_i P^j_n,$$

$[25^\circ, 40^\circ] \Rightarrow [25^\circ, 35^\circ]$

$[140^\circ, 165^\circ] \Rightarrow [144^\circ, 151^\circ]$

$|\phi_S| \leq 30^\circ \Rightarrow 22^\circ \leq |\phi_S| \leq 30^\circ$

TABLE VI: The experimental data and the SU(3) flavor symmetry predictions of the $D \to S\ell^+\nu_\ell$, $S \to P_1P_2$ decays wit

†denotes the results with $\frac{S'_1}{S_1} > 0$, ♯ denotes ones with $\frac{S'_1}{S_1} < 0$.

$c \to s\ell\nu$

Branching ratios	Exp. Data	Ones in the 2-quark picture with			Ones in the 4-quark picture
		$\theta_S = [25^\circ, 35^\circ]$	$\theta_S = [144^\circ, 158^\circ]$	$22^\circ \leq \lvert\theta_S\rvert \leq 30^\circ$	
$\mathcal{B}(D^0 \to K_0^- e^+\nu_e,\ K_0^- \to \pi^-\overline{K}^0)(\times 10^{-4})$		19.99 ± 7.34	19.86 ± 7.26	19.74 ± 6.97	8.37 ± 3.01
$\mathcal{B}(D^0 \to K_0^- e^+\nu_e,\ K_0^- \to \pi^0 K^-)(\times 10^{-4})$		10.18 ± 3.77	10.12 ± 3.73	10.05 ± 3.57	4.19 ± 1.50
$\mathcal{B}(D^+ \to \overline{K}_0^0 e^+\nu_e,\ \overline{K}_0^0 \to \pi^+ K^-)(\times 10^{-3})$		5.17 ± 1.92	5.19 ± 1.85	5.12 ± 1.86	2.24 ± 0.83
$\mathcal{B}(D^+ \to \overline{K}_0^0 e^+\nu_e,\ \overline{K}_0^0 \to \pi^0\overline{K}^0)(\times 10^{-3})$		2.57 ± 0.96	2.59 ± 0.92	2.55 ± 0.92	1.12 ± 0.42
$\mathcal{B}(D_s^+ \to f_0(980) e^+\nu_e,\ f_0(980) \to \pi^+\pi^-)(\times 10^{-3})$	1.30 ± 0.63 [78]	1.19 ± 0.18	1.17 ± 0.17	1.18 ± 0.17	$1.22 \pm 0.55^\dagger,\ 1.44 \pm 0.49^\sharp$
$\mathcal{B}(D_s^+ \to f_0(980) e^+\nu_e,\ f_0(980) \to \pi^0\pi^0)(\times 10^{-4})$	7.9 ± 2.9 [4]	5.95 ± 0.92	5.89 ± 0.85	5.90 ± 0.86	$7.91 \pm 2.85^\dagger,\ 7.13 \pm 2.10^\sharp$
$\mathcal{B}(D_s^+ \to f_0(980) e^+\nu_e,\ f_0(980) \to K^+K^-)(\times 10^{-4})$		5.11 ± 2.34	5.53 ± 2.78	6.28 ± 2.07	$3.33 \pm 1.53^\dagger,\ 3.07 \pm 1.34^\sharp$
$\mathcal{B}(D_s^+ \to f_0(980) e^+\nu_e,\ f_0(980) \to K^0\overline{K}^0)(\times 10^{-4})$		4.62 ± 2.12	5.01 ± 2.52	5.68 ± 1.87	$3.01 \pm 1.39^\dagger,\ 2.78 \pm 1.22^\sharp$
$\mathcal{B}(D_s^+ \to f_0(500) e^+\nu_e,\ f_0(500) \to \pi^+\pi^-)(\times 10^{-4})$		9.91 ± 2.83	9.67 ± 3.07	9.44 ± 3.30	$2.49 \pm 2.49^\dagger,\ 0.90 \pm 0.90^\sharp$
$\mathcal{B}(D_s^+ \to f_0(500) e^+\nu_e,\ f_0(500) \to \pi^0\pi^0)(\times 10^{-5})$	< 64 [4]	49.77 ± 14.23	48.57 ± 15.43	47.44 ± 16.56	$6.66 \pm 6.66^\dagger,\ 0.78 \pm 0.78^\sharp$
$\mathcal{B}(D^0 \to K_0^- \mu^+\nu_\mu,\ K_0^- \to \pi^- K^0)(\times 10^{-4})$		17.27 ± 6.48	17.16 ± 6.41	17.04 ± 6.14	7.19 ± 2.63
$\mathcal{B}(D^0 \to K_0^- \mu^+\nu_\mu,\ K_0^- \to \pi^0 K^-)(\times 10^{-4})$		8.63 ± 3.24	8.58 ± 3.20	8.52 ± 3.07	3.59 ± 1.32
$\mathcal{B}(D^+ \to \overline{K}_0^0 \mu^+\nu_\mu,\ \overline{K}_0^0 \to \pi^+ K^-)(\times 10^{-3})$		4.43 ± 1.68	4.46 ± 1.62	4.40 ± 1.62	1.92 ± 0.73
$\mathcal{B}(D^+ \to \overline{K}_0^0 \mu^+\nu_\mu,\ \overline{K}_0^0 \to \pi^0 K^0)(\times 10^{-3})$		2.22 ± 0.84	2.23 ± 0.81	2.20 ± 0.81	0.96 ± 0.36
$\mathcal{B}(D_s^+ \to f_0(980) \mu^+\nu_\mu,\ f_0(980) \to \pi^+\pi^-)(\times 10^{-3})$		1.01 ± 0.16	1.00 ± 0.15	1.00 ± 0.16	$1.02 \pm 0.46^\dagger,\ 1.23 \pm 0.42^\sharp$
$\mathcal{B}(D_s^+ \to f_0(980) \mu^+\nu_\mu,\ f_0(980) \to \pi^0\pi^0)(\times 10^{-4})$		5.05 ± 0.83	4.99 ± 0.77	5.00 ± 0.78	$6.72 \pm 2.48^\dagger,\ 6.04 \pm 1.82^\sharp$
$\mathcal{B}(D_s^+ \to f_0(980) \mu^+\nu_\mu,\ f_0(980) \to K^+K^-)(\times 10^{-4})$		4.31 ± 1.94	4.70 ± 2.34	5.34 ± 1.75	$2.79 \pm 1.28^\dagger,\ 2.59 \pm 1.14^\sharp$
$\mathcal{B}(D_s^+ \to f_0(980) \mu^+\nu_\mu,\ f_0(980) \to K^0\overline{K}^0)(\times 10^{-4})$		3.90 ± 1.76	4.25 ± 2.12	4.83 ± 1.58	$2.52 \pm 1.16^\dagger,\ 2.34 \pm 1.03^\sharp$
$\mathcal{B}(D_s^+ \to f_0(500) \mu^+\nu_\mu,\ f_0(500) \to \pi^+\pi^-)(\times 10^{-4})$		8.88 ± 2.62	8.70 ± 2.86	8.49 ± 3.05	$2.30 \pm 2.30^\dagger,\ 0.83 \pm 0.83^\sharp$
$\mathcal{B}(D_s^+ \to f_0(500) \mu^+\nu_\mu,\ f_0(500) \to \pi^0\pi^0)(\times 10^{-5})$		44.67 ± 13.23	43.85 ± 14.53	42.77 ± 15.49	$6.16 \pm 6.16^\dagger,\ 7.23 \pm 7.23^\sharp$

$O(10^{-3} - 10^{-4})$

$c \to d \ell \nu$

TABLE VII: The experimental data and the SU(3) flavor symmetry predictions of the $D \to S\ell^+\nu_\ell \to P_1P_2\ell^+\nu_\ell$ decays with the $c \to d\ell^+\nu_\ell$ transitions within 2σ errors. † denotes the results with $\frac{S_1'}{S_1} > 0$, $^\sharp$ denotes ones with $\frac{S_1'}{S_1} < 0$, and ^{a} denotes the experimental lower limits have not used to constain the predictions.

Branching ratios	Exp. Data	Ones in the 2-quark picutre with			Ones in the 4-quark picutre
		$\theta_S = [25°, 35°]$	$\theta_S = [144°, 158°]$	$22° \le \lvert\theta_S\rvert \le 30°$	
$\mathcal{B}(D^0 \to a_0(980)^- e^+\nu_e,\ a_0(980)^- \to \eta\pi^-)(\times 10^{-5})$	$13.3^{+6.8}_{-6.0^a}$	5.99 ± 2.69	5.86 ± 2.48	6.05 ± 2.57 2σ	3.81 ± 0.98 3σ
$\mathcal{B}(D^0 \to a_0(980)^- e^+\nu_e,\ a_0(980)^- \to \eta'\pi^-)(\times 10^{-6})$		2.88 ± 1.71	2.97 ± 1.77	2.97 ± 1.73	1.88 ± 0.98
$\mathcal{B}(D^0 \to a_0(980)^- e^+\nu_e,\ a_0(980)^- \to K^0K^-)(\times 10^{-6})$		29.99 ± 13.81	30.73 ± 13.81	30.57 ± 13.70	4.22 ± 1.93
$\mathcal{B}(D^+ \to a_0(980)^0 e^+\nu_e,\ a_0(980)^0 \to \eta\pi^0)(\times 10^{-5})$	17^{+16}_{-14}	7.35 ± 3.28	7.25 ± 3.13	7.32 ± 3.17	4.00 ± 1.00
$\mathcal{B}(D^+ \to a_0(980)^0 e^+\nu_e,\ a_0(980)^0 \to \eta'\pi^0)(\times 10^{-6})$		5.53 ± 3.26	5.69 ± 3.32	5.65 ± 3.20	3.08 ± 1.56
$\mathcal{B}(D^+ \to a_0(980)^0 e^+\nu_e,\ a_0(980)^0 \to K^+K^-)(\times 10^{-5})$		2.28 ± 1.06	2.30 ± 1.00	2.29 ± 0.99	0.88 ± 0.36
$\mathcal{B}(D^+ \to a_0(980)^0 e^+\nu_e,\ a_0(980)^0 \to K^0\overline{K}^0)(\times 10^{-5})$		1.99 ± 0.92	2.01 ± 0.88	2.00 ± 0.86	0.77 ± 0.31
$\mathcal{B}(D^+ \to f_0(980) e^+\nu_e,\ f_0(980) \to \pi^+\pi^-)(\times 10^{-5})$	< 2.8 [5]	1.15 ± 0.50	1.10 ± 0.58	0.96 ± 0.43	$1.65 \pm 1.15^\dagger$, $2.14 \pm 0.65^\sharp$
$\mathcal{B}(D^+ \to f_0(980) e^+\nu_e,\ f_0(980) \to \pi^0\pi^0)(\times 10^{-6})$		5.75 ± 2.53	5.51 ± 2.92	4.80 ± 2.18	$10.53 \pm 3.67^\dagger$, $10.10 \pm 5.37^\sharp$
$\mathcal{B}(D^+ \to f_0(980) e^+\nu_e,\ f_0(980) \to K^+K^-)(\times 10^{-6})$		5.07 ± 0.88	5.06 ± 0.85	5.01 ± 0.80	$4.35 \pm 2.78^\dagger$, $4.60 \pm 2.76^\sharp$
$\mathcal{B}(D^+ \to f_0(980) e^+\nu_e,\ f_0(980) \to K^0\overline{K}^0)(\times 10^{-6})$		5.07 ± 0.88	5.06 ± 0.85	5.01 ± 0.80	$4.35 \pm 2.78^\dagger$, $4.60 \pm 2.76^\sharp$
$\mathcal{B}(D^+ \to f_0(500) e^+\nu_e,\ f_0(500) \to \pi^+\pi^-)(\times 10^{-4})$	6.3 ± 1.0^a	1.44 ± 0.64	1.72 ± 0.92	1.79 ± 0.85	$3.64 \pm 2.51^\dagger$, $2.95 \pm 1.87^\sharp$
$\mathcal{B}(D^+ \to f_0(500) e^+\nu_e,\ f_0(500) \to \pi^0\pi^0)(\times 10^{-4})$		0.72 ± 0.32	0.87 ± 0.46	0.91 ± 0.43	$1.45 \pm 1.02^\dagger$, $2.08 \pm 1.57^\sharp$
$\mathcal{B}(D_s^+ \to K_0^0 e^+\nu_e,\ K_0^0 \to \pi^-K^+)(\times 10^{-5})$		22.34 ± 8.09	22.13 ± 7.97	22.34 ± 7.64	9.54 ± 3.38
$\mathcal{B}(D_s^+ \to K_0^0 e^+\nu_e,\ K_0^0 \to \pi^0K^0)(\times 10^{-5})$		11.17 ± 4.04	11.07 ± 3.99	11.17 ± 3.82	4.77 ± 1.69
$\mathcal{B}(D^0 \to a_0(980)^- \mu^+\nu_\mu,\ a_0(980)^- \to \eta\pi^-)(\times 10^{-5})$		4.95 ± 2.27	4.84 ± 2.10	5.00 ± 2.18	3.14 ± 0.84
$\mathcal{B}(D^0 \to a_0(980)^- \mu^+\nu_\mu,\ a_0(980)^- \to \eta'\pi^-)(\times 10^{-6})$		2.39 ± 1.44	2.46 ± 1.48	2.45 ± 1.45	1.56 ± 0.82
$\mathcal{B}(D^0 \to a_0(980)^- \mu^+\nu_\mu,\ a_0(980)^- \to K^0K^-)(\times 10^{-6})$		24.78 ± 11.68	25.37 ± 11.62	25.20 ± 11.53	3.51 ± 1.62
$\mathcal{B}(D^+ \to a_0(980)^0 \mu^+\nu_\mu,\ a_0(980)^0 \to \eta\pi^0)(\times 10^{-5})$		6.09 ± 2.78	6.00 ± 2.65	6.06 ± 2.69	3.30 ± 0.86
$\mathcal{B}(D^+ \to a_0(980)^0 \mu^+\nu_\mu,\ a_0(980)^0 \to \eta'\pi^0)(\times 10^{-6})$		4.58 ± 2.74	4.72 ± 2.79	4.67 ± 2.69	2.55 ± 1.31
$\mathcal{B}(D^+ \to a_0(980)^0 \mu^+\nu_\mu,\ a_0(980)^0 \to K^+K^-)(\times 10^{-5})$		1.89 ± 0.89	1.91 ± 0.85	1.89 ± 0.83	0.73 ± 0.30
$\mathcal{B}(D^+ \to a_0(980)^0 \mu^+\nu_\mu,\ a_0(980)^0 \to K^0\overline{K}^0)(\times 10^{-5})$		1.65 ± 0.78	1.66 ± 0.74	1.65 ± 0.73	0.64 ± 0.27
$\mathcal{B}(D^+ \to f_0(980) \mu^+\nu_\mu,\ f_0(980) \to \pi^+\pi^-)(\times 10^{-5})$		0.94 ± 0.43	0.91 ± 0.48	0.79 ± 0.36	$1.37 \pm 0.96^\dagger$, $1.76 \pm 0.55^\sharp$
$\mathcal{B}(D^+ \to f_0(980) \mu^+\nu_\mu,\ f_0(980) \to \pi^0\pi^0)(\times 10^{-6})$		4.74 ± 2.14	4.58 ± 2.43	3.97 ± 1.82	$8.67 \pm 3.13^\dagger$, $8.32 \pm 4.47^\sharp$
$\mathcal{B}(D^+ \to f_0(980) \mu^+\nu_\mu,\ f_0(980) \to K^+K^-)(\times 10^{-6})$		4.21 ± 0.73	4.19 ± 0.71	4.15 ± 0.67	$3.55 \pm 2.29^\dagger$, $3.76 \pm 2.26^\sharp$
$\mathcal{B}(D^+ \to f_0(980) \mu^+\nu_\mu,\ f_0(980) \to K^0\overline{K}^0)(\times 10^{-6})$		4.21 ± 0.73	4.19 ± 0.71	4.15 ± 0.67	$3.55 \pm 2.29^\dagger$, $3.76 \pm 2.26^\sharp$
$\mathcal{B}(D^+ \to f_0(500) \mu^+\nu_\mu,\ f_0(980) \to \pi^+\pi^-)(\times 10^{-4})$		1.28 ± 0.59	1.54 ± 0.84	1.61 ± 0.79	$3.30 \pm 2.39^\dagger$, $2.68 \pm 1.74^\sharp$
$\mathcal{B}(D^+ \to f_0(500) \mu^+\nu_\mu,\ f_0(980) \to \pi^0\pi^0)(\times 10^{-4})$		0.64 ± 0.30	0.78 ± 0.43	0.81 ± 0.40	$1.32 \pm 0.95^\dagger$, $1.89 \pm 1.46^\sharp$
$\mathcal{B}(D_s^+ \to K_0^0 \mu^+\nu_\mu,\ K_0^0 \to \pi^-K^+)(\times 10^{-5})$		19.61 ± 7.20	19.43 ± 7.10	19.60 ± 6.80	8.38 ± 3.01
$\mathcal{B}(D_s^+ \to K_0^0 \mu^+\nu_\mu,\ K_0^0 \to \pi^0K^0)(\times 10^{-5})$		9.80 ± 3.60	9.71 ± 3.55	9.80 ± 3.40	4.19 ± 1.50

Smaller than data

$O(10^{-4} - 10^{-6})$

5.3 Decays $D \to P_1 P_2 \ell^+ \nu_\ell$ with vector resonances

$$\mathcal{B}(D \to R\ell^+\nu_\ell, R \to P_1P_2) = \mathcal{B}(D \to R\ell^+\nu_\ell) \times \mathcal{B}(R \to P_1P_2),$$

$\mathcal{B}(V \to P_1P_2)$ can be written as [79]

$$\mathcal{B}(V \to P_1P_2) = \frac{\tau_V p_c'^3}{6\pi m_V^2} g^2_{V\to P_1P_2}, \tag{20}$$

with $p_c' \equiv \frac{\sqrt{\lambda(m_V^2, m_{P_1}^2, m_{P_2}^2)}}{2m_V}$, and $g_{V\to P_1P_2}$ are the strong coupling constants. Many involved $\mathcal{B}(V \to P_1P_2)$ are measured [11]

$$\begin{aligned}
&\mathcal{B}(K^{*+} \to \pi K) = (99.902 \pm 0.018)\%, && \mathcal{B}(K^{*0} \to \pi K) = (99.754 \pm 0.042)\%,\\
&\mathcal{B}(\rho^+ \to \pi^0\pi^+) = 100\%, && \mathcal{B}(\rho^0 \to \pi^+\pi^-) = 100\%,\\
&\mathcal{B}(\phi \to K^+K^-) = (49.1 \pm 1.0)\% \text{ with } S = 1.3, && \mathcal{B}(\omega \to \pi^+\pi^-) = (1.53^{+0.22}_{-0.26})\% \text{ with } S = 1.2.
\end{aligned} \tag{21}$$

Using the following relations

$$\boxed{\begin{aligned}
&\sqrt{2} g_{K^{*-}\to\pi^0K^-} = g_{K^{*-}\to\pi^-K^0}, && \sqrt{2} g_{K^{*0}\to\pi^0K^0} = g_{K^{*0}\to\pi^-K^+},\\
&g_{\rho^-\to\pi^0\pi^-} = \sqrt{3} g_{\rho^-\to\eta_8\pi^-} = \sqrt{3/2} g_{\rho^-\to\eta_1\pi^-}, && g_{\phi\to K^+K^-} = g_{\phi\to K^0\overline{K}^0},
\end{aligned}} \tag{22}$$

other $\mathcal{B}(V \to P_1P_2)$ can be obtained

$$\begin{aligned}
&\mathcal{B}(K^{*0} \to \pi^0K^0) = (33.02 \pm 0.02)\%, && \mathcal{B}(K^{*0} \to \pi^-K^+) = (66.74 \pm 0.04)\%,\\
&\mathcal{B}(K^{*+} \to \pi^0K^+) = (33.62 \pm 0.01)\%, && \mathcal{B}(K^{*+} \to \pi^-K^0) = (66.28 \pm 0.01)\%,\\
&\mathcal{B}(\rho^+ \to \eta\pi^+) = (4.38 \pm 0.66)\%, && \mathcal{B}(\phi \to K^0K^0) = (32.42 \pm 1.04)\%.
\end{aligned} \tag{23}$$

TABLE VIII: The experimental data and the SU(3) flavor symmetry predictions of $D \to V\ell^+\nu_\ell \to P_1P_2\ell^+\nu_\ell$ decays within 2σ errors.

Branching ratios	Exp. Data	Our predictions	Previous ones
$c \to se^+\nu_e$:			
$\mathcal{B}(D^0 \to K^{*-}e^+\nu_e,\ K^{*-} \to \pi^-\overline{K}^0)(\times 10^{-2})$		1.42 ± 0.07	
$\mathcal{B}(D^0 \to K^{*-}e^+\nu_e,\ K^{*-} \to \pi^0K^-)(\times 10^{-3})$		7.18 ± 0.37	7.17 [20]
$\mathcal{B}(D^+ \to \overline{K}^{*0}e^+\nu_e,\ \overline{K}^{*0} \to \pi^+K^-)(\times 10^{-2})$	3.77 ± 0.34	3.64 ± 0.11	3.51 [20]
$\mathcal{B}(D^+ \to \overline{K}^{*0}e^+\nu_e,\ \overline{K}^{*0} \to \pi^0\overline{K}^0)(\times 10^{-2})$		1.80 ± 0.06	
$\mathcal{B}(D_s^+ \to \phi e^+\nu_e,\ \phi \to K^+K^-)(\times 10^{-2})$		1.20 ± 0.10	
$\mathcal{B}(D_s^+ \to \phi e^+\nu_e,\ \phi \to K^0\overline{K}^0)(\times 10^{-3})$		7.94 ± 0.65	
$c \to s\mu^+\nu_\mu$:			
$\mathcal{B}(D^0 \to K^{*-}\mu^+\nu_\mu,\ K^{*-} \to \pi^-\overline{K}^0)(\times 10^{-2})$		1.33 ± 0.07	
$\mathcal{B}(D^0 \to K^{*-}\mu^+\nu_\mu,\ K^{*-} \to \pi^0K^-)(\times 10^{-3})$		6.76 ± 0.35	7.17 [20]
$\mathcal{B}(D^+ \to \overline{K}^{*0}\mu^+\nu_\mu,\ \overline{K}^{*0} \to \pi^+K^-)(\times 10^{-2})$	3.52 ± 0.20	3.43 ± 0.11	3.51 [20]
$\mathcal{B}(D^+ \to \overline{K}^{*0}\mu^+\nu_\mu,\ \overline{K}^{*0} \to \pi^0\overline{K}^0)(\times 10^{-2})$		1.70 ± 0.05	
$\mathcal{B}(D_s^+ \to \phi\mu^+\nu_\mu,\ \phi \to K^+K^-)(\times 10^{-2})$		1.13 ± 0.09	
$\mathcal{B}(D_s^+ \to \phi\mu^+\nu_\mu,\ \phi \to K^0\overline{K}^0)(\times 10^{-3})$		7.46 ± 0.62	
$c \to de^+\nu_e$:			
$\mathcal{B}(D^0 \to \rho^-e^+\nu_e,\ \rho^- \to \pi^0\pi^-)(\times 10^{-3})$		1.85 ± 0.11	1.63 [20]
$\mathcal{B}(D^0 \to \rho^-e^+\nu_e,\ \rho^- \to \eta\pi^-)(\times 10^{-5})$		8.23 ± 1.59	
$\mathcal{B}(D^+ \to \rho^0e^+\nu_e,\ \rho^0 \to \pi^+\pi^-)(\times 10^{-3})$		2.40 ± 0.12	1.57 ± 0.07 [13], 2.10 [20]
$\mathcal{B}(D^+ \to \omega e^+\nu_e,\ \omega \to \pi^+\pi^-)(\times 10^{-5})$		3.55 ± 0.82	
$\mathcal{B}(D_s^+ \to K^{*0}e^+\nu_e,\ K^{*0} \to \pi^-K^+)(\times 10^{-3})$		1.49 ± 0.10	
$\mathcal{B}(D_s^+ \to K^{*0}e^+\nu_e,\ K^{*0} \to \pi^0K^0)(\times 10^{-4})$		7.39 ± 0.51	
$c \to d\mu^+\nu_\mu$:			
$\mathcal{B}(D^0 \to \rho^-\mu^+\nu_\mu,\ \rho^- \to \pi^0\pi^-)(\times 10^{-3})$		1.76 ± 0.10	
$\mathcal{B}(D^0 \to \rho^-\mu^+\nu_\mu,\ \rho^- \to \eta\pi^-)(\times 10^{-5})$		7.83 ± 1.51	
$\mathcal{B}(D^+ \to \rho^0\mu^+\nu_\mu,\ \rho^0 \to \pi^+\pi^-)(\times 10^{-3})$		2.29 ± 0.11	1.57 ± 0.07 [13]
$\mathcal{B}(D^+ \to \omega\mu^+\nu_\mu,\ \omega \to \pi^+\pi^-)(\times 10^{-5})$		3.38 ± 0.78	
$\mathcal{B}(D_s^+ \to K^{*0}\mu^+\nu_\mu,\ K^{*0} \to \pi^-K^+)(\times 10^{-3})$		1.42 ± 0.10	
$\mathcal{B}(D_s^+ \to K^{*0}\mu^+\nu_\mu,\ K^{*0} \to \pi^0K^0)(\times 10^{-4})$		7.03 ± 0.48	

$O(10^{-2} - 10^{-3})$

$O(10^{-3} - 10^{-5})$

5.4 Total branching ratios of decays $D \to P_1 P_2 \ell^+ \nu_\ell$

$c \to s\ell\nu$

Branching ratios	Exp. data with T	Ones with T
$\mathcal{B}(D^0 \to \pi^- \overline{K}^0 e^+ \nu_e)(\times 10^{-2})$	1.44 ± 0.08	1.57 ± 0.14
$\mathcal{B}(D^0 \to \pi^0 K^- e^+ \nu_e)(\times 10^{-2})$	$1.6^{+2.6}_{-1.0}$	0.80 ± 0.07
$\mathcal{B}(D^0 \to \eta K^- e^+ \nu_e)(\times 10^{-6})$		3.04 ± 3.04
$\mathcal{B}(D^0 \to \eta' K^- e^+ \nu_e)(\times 10^{-6})$		3.26 ± 1.78
$\mathcal{B}(D^+ \to \pi^+ K^- e^+ \nu_e)(\times 10^{-2})$	4.02 ± 1.15^b	4.06 ± 0.30
$\mathcal{B}(D^+ \to \pi^0 \overline{K}^0 e^+ \nu_e)(\times 10^{-2})$		2.01 ± 0.15
$\mathcal{B}(D^+ \to \eta \overline{K}^0 e^+ \nu_e)(\times 10^{-5})$		0.77 ± 0.77
$\mathcal{B}(D^+ \to \eta' \overline{K}^0 e^+ \nu_e)(\times 10^{-5})$		0.83 ± 0.45
$\mathcal{B}(D_s^+ \to K^+ K^- e^+ \nu_e)(\times 10^{-2})$		1.27 ± 0.13
$\mathcal{B}(D_s^+ \to K^0 \overline{K}^0 e^+ \nu_e)(\times 10^{-3})$		8.56 ± 0.95
$\mathcal{B}(D_s^+ \to \pi^+ \pi^- e^+ \nu_e)(\times 10^{-3})$		1.47 ± 0.79
$\mathcal{B}(D_s^+ \to \pi^0 \pi^0 e^+ \nu_e)(\times 10^{-4})$		8.58 ± 3.50
$\mathcal{B}(D_s^+ \to \eta \eta e^+ \nu_e)(\times 10^{-4})$		0.49 ± 0.42
$\mathcal{B}(D_s^+ \to \eta \eta' e^+ \nu_e)(\times 10^{-6})$		4.44 ± 2.68
$\mathcal{B}(D^0 \to \pi^- \overline{K}^0 \mu^+ \nu_\mu)(\times 10^{-2})$		1.48 ± 0.13
$\mathcal{B}(D^0 \to \pi^0 K^- \mu^+ \nu_\mu)(\times 10^{-2})$		0.75 ± 0.07
$\mathcal{B}(D^0 \to \eta K^- \mu^+ \nu_\mu)(\times 10^{-6})$		2.77 ± 2.77
$\mathcal{B}(D^0 \to \eta' K^- \mu^+ \nu_\mu)(\times 10^{-6})$		2.25 ± 1.22
$\mathcal{B}(D^+ \to \pi^+ K^- \mu^+ \nu_\mu)(\times 10^{-2})$	3.65 ± 0.68	3.81 ± 0.27
$\mathcal{B}(D^+ \to \pi^0 \overline{K}^0 \mu^+ \nu_\mu)(\times 10^{-2})$		1.89 ± 0.14
$\mathcal{B}(D^+ \to \eta \overline{K}^0 \mu^+ \nu_\mu)(\times 10^{-5})$		0.70 ± 0.70
$\mathcal{B}(D^+ \to \eta' \overline{K}^0 \mu^+ \nu_\mu)(\times 10^{-5})$		0.57 ± 0.31
$\mathcal{B}(D_s^+ \to K^+ K^- \mu^+ \nu_\mu)(\times 10^{-2})$		1.19 ± 0.12
$\mathcal{B}(D_s^+ \to K^0 \overline{K}^0 \mu^+ \nu_\mu)(\times 10^{-3})$		7.99 ± 0.87
$\mathcal{B}(D_s^+ \to \pi^+ \pi^- \mu^+ \nu_\mu)(\times 10^{-3})$		1.25 ± 0.69
$\mathcal{B}(D_s^+ \to \pi^0 \pi^0 \mu^+ \nu_\mu)(\times 10^{-4})$		7.34 ± 3.08
$\mathcal{B}(D_s^+ \to \eta \eta \mu^+ \nu_\mu)(\times 10^{-4})$		0.45 ± 0.39
$\mathcal{B}(D_s^+ \to \eta \eta' \mu^+ \nu_\mu)(\times 10^{-6})$		3.30 ± 1.99

$c \to d\ell\nu$

Branching ratios	Exp. data with T	Ones with T
$\mathcal{B}(D^0 \to K^- K^0 e^+ \nu_e)(\times 10^{-5})$		1.16 ± 0.60
$\mathcal{B}(D^0 \to \pi^0 \pi^- e^+ \nu_e)(\times 10^{-3})$	1.45 ± 0.14	1.85 ± 0.11
$\mathcal{B}(D^0 \to \eta \pi^- e^+ \nu_e)(\times 10^{-5})$		16.22 ± 5.13
$\mathcal{B}(D^0 \to \eta' \pi^- e^+ \nu_e)(\times 10^{-5})$		0.52 ± 0.32
$\mathcal{B}(D^+ \to \overline{K}^0 K^0 e^+ \nu_e)(\times 10^{-5})$		3.07 ± 1.57
$\mathcal{B}(D^+ \to K^+ K^- e^+ \nu_e)(\times 10^{-5})$		1.31 ± 0.64
$\mathcal{B}(D^+ \to \pi^+ \pi^- e^+ \nu_e)(\times 10^{-3})$	2.45 ± 0.20	3.12 ± 0.53
$\mathcal{B}(D^+ \to \pi^0 \pi^0 e^+ \nu_e)(\times 10^{-4})$		3.09 ± 1.88
$\mathcal{B}(D^+ \to \eta \pi^0 e^+ \nu_e)(\times 10^{-5})$		9.56 ± 4.45
$\mathcal{B}(D^+ \to \eta' \pi^0 e^+ \nu_e)(\times 10^{-6})$		7.56 ± 4.50
$\mathcal{B}(D^+ \to \eta \eta e^+ \nu_e)(\times 10^{-6})$		2.74 ± 1.99
$\mathcal{B}(D^+ \to \eta \eta' e^+ \nu_e)(\times 10^{-8})$		3.20 ± 1.92
$\mathcal{B}(D_s^+ \to K^+ \pi^- e^+ \nu_e)(\times 10^{-3})$		1.67 ± 0.18
$\mathcal{B}(D_s^+ \to K^0 \pi^0 e^+ \nu_e)(\times 10^{-4})$		8.26 ± 0.87
$\mathcal{B}(D_s^+ \to \eta K^0 e^+ \nu_e)(\times 10^{-5})$		1.51 ± 0.95
$\mathcal{B}(D_s^+ \to \eta' K^0 e^+ \nu_e)(\times 10^{-7})$		4.32 ± 2.88
$\mathcal{B}(D^0 \to K^- K^0 \mu^+ \nu_\mu)(\times 10^{-5})$		1.03 ± 0.53
$\mathcal{B}(D^0 \to \pi^0 \pi^- \mu^+ \nu_\mu)(\times 10^{-3})$		1.76 ± 0.10
$\mathcal{B}(D^0 \to \eta \pi^- \mu^+ \nu_\mu)(\times 10^{-5})$		15.00 ± 4.78
$\mathcal{B}(D^0 \to \eta' \pi^- \mu^+ \nu_\mu)(\times 10^{-5})$		0.45 ± 0.28
$\mathcal{B}(D^+ \to \overline{K}^0 K^0 \mu^+ \nu_\mu)(\times 10^{-5})$		2.71 ± 1.41
$\mathcal{B}(D^+ \to K^+ K^- \mu^+ \nu_\mu)(\times 10^{-5})$		1.08 ± 0.53
$\mathcal{B}(D^+ \to \pi^+ \pi^- \mu^+ \nu_\mu)(\times 10^{-3})$		2.97 ± 0.50
$\mathcal{B}(D^+ \to \pi^0 \pi^0 \mu^+ \nu_\mu)(\times 10^{-4})$		2.90 ± 1.76
$\mathcal{B}(D^+ \to \eta \pi^0 \mu^+ \nu_\mu)(\times 10^{-5})$		8.60 ± 4.16
$\mathcal{B}(D^+ \to \eta' \pi^0 \mu^+ \nu_\mu)(\times 10^{-6})$		6.58 ± 3.95
$\mathcal{B}(D^+ \to \eta \eta \mu^+ \nu_\mu)(\times 10^{-6})$		2.47 ± 1.78
$\mathcal{B}(D^+ \to \eta \eta' \mu^+ \nu_\mu)(\times 10^{-8})$		1.98 ± 1.18
$\mathcal{B}(D_s^+ \to K^+ \pi^- \mu^+ \nu_\mu)(\times 10^{-3})$		1.58 ± 0.17
$\mathcal{B}(D_s^+ \to K^0 \pi^0 \mu^+ \nu_\mu)(\times 10^{-4})$		7.83 ± 0.82
$\mathcal{B}(D_s^+ \to \eta K^0 \mu^+ \nu_\mu)(\times 10^{-5})$		1.40 ± 0.88
$\mathcal{B}(D_s^+ \to \eta' K^0 \mu^+ \nu_\mu)(\times 10^{-7})$		3.39 ± 2.26

- **Vector resonant** contributions are dominant

$$D^0 \to \pi^-\overline{K}^0\ell^+\nu_\ell, \pi^0K^-\ell^+\nu_\ell, \pi^0\pi^-\ell^+\nu_\ell,\ D^+ \to \pi^+K^-\ell^+\nu_\ell, \pi^0\overline{K}^0\ell^+\nu_\ell, \pi^+\pi^-\ell^+\nu_\ell,$$

$$D_s^+ \to K^+K^-\ell^+\nu_\ell, K^0\overline{K}^0\ell^+\nu_\ell, K^+\pi^-\ell^+\nu_\ell, K^0\pi^0\ell^+\nu_\ell$$

- All **three kinds** of contributions are important

$$D^0 \to \eta\pi^-\ell^+\nu_\ell$$

- Both the **non-resonant** and the **scalar resonant** contributions are important

$$D^0 \to K^-K^0\ell^+\nu_\ell, \eta'\pi^-\ell^+\nu_\ell$$

$$D^+ \to \overline{K}^0K^0\ell^+\nu_\ell, \pi^0\pi^0\ell^+\nu_\ell, \eta\pi^0\ell^+\nu_\ell, \eta'\pi^0\ell^+\nu_\ell$$

Outline

1. Motivation

2. Three body semileptonic charm decays

   $D \to P/V/S\ell^+\nu_\ell$

3. Four body semileptonic charm decays

   $D \to P_1 P_2 \ell^+ \nu_\ell$

4. Conclusion

4. Conclusion

- ✓ SU(3) Flavor Symmetry approach works well in the $D \to P/V\ell^+\nu_\ell$ and $D \to P_1P_2\ell^+\nu_\ell$.

- ✓ Predicted the not-yet-measured observables, and some of them are obtained for the first time.

- ✓ Our predictions of $D \to S\ell^+\nu_\ell, D \to P_1P_2\ell^+\nu_\ell$ could be tested at BESIII, LHCb and Belle-II.

Thank you!

2. Three body semileptonic decays

2.1 Effective Hamiltonian

2.2 $D \to P\ell^+\nu$ decays

2.3 $D \to \mathrm{V}\ell^+\nu$ decays

2.4 $D \to S\ell^+\nu$ decays

2.1 Effective Hamiltonian

$$\mathcal{H}_{eff}(c \to q\ell^+\nu_\ell) = \frac{G_F}{\sqrt{2}} V_{cq}^* \bar{q}\gamma^\mu(1-\gamma_5)c\ \bar{\nu}_\ell\gamma_\mu(1-\gamma_5)\ell,$$

$$\mathcal{M}(D \to M\ell^+\nu_\ell) = \frac{G_F}{\sqrt{2}} V_{cb} \sum_{mm'} g_{mm'} L_m^{\lambda_\ell\lambda_\nu} H_{m'}^{\lambda_M},$$

$$L_m^{\lambda_\ell\lambda_\nu} = \epsilon_\alpha(m)\bar{\nu}_\ell\gamma^\alpha(1-\gamma_5)\ell,$$

$$H_{m'}^{\lambda_M} = \begin{cases} \epsilon_\beta^*(m')\langle P/S(p_{P/S})|\bar{q}\gamma^\beta(1-\gamma_5)c|D(p_D)\rangle \\ \epsilon_\beta^*(m')\langle V(p_V,\epsilon^*)|\bar{q}\gamma^\beta(1-\gamma_5)c|D(p_D)\rangle \end{cases},$$

form factors

SU(3) flavor symmetry

Observables

differential branching ratios:

$$\frac{d\mathcal{B}(D_{(s)} \to M\ell^+\nu_\ell)}{dq^2} = \frac{\tau_D G_F^2 |V_{cq}|^2 \lambda^{1/2}(q^2 - m_\ell^2)^2}{24(2\pi)^3 M_{D_{(s)}}^3 q^2}\mathcal{H}_{\text{total}},$$

$$\mathcal{H}_{\text{total}} \equiv (\mathcal{H}_U + \mathcal{H}_L)\left(1 + \frac{m_\ell^2}{2q^2}\right) + \frac{3m_\ell^2}{2q^2}\mathcal{H}_S.$$

lepton flavor universality :

$$R^{\mu/e} = \frac{\int_{q_{min}}^{q_{max}} d\mathcal{B}(D_{(s)} \to M\mu^+\nu_\mu)/dq^2}{\int_{q_{min}}^{q_{max}} d\mathcal{B}(D_{(s)} \to Me^+\nu_e)/dq^2}$$

forward-backward asymmetries:

$$A_{FB}^\ell(q^2) = \frac{3}{4}\frac{\mathcal{H}_P - \frac{2m_\ell^2}{q^2}\mathcal{H}_{SL}}{\mathcal{H}_{\text{total}}}$$

longitudinal polarizations of l^+:

$$P_L^\ell(q^2) = \frac{(\mathcal{H}_U + \mathcal{H}_L)\left(1 - \frac{m_\ell^2}{2q^2}\right) - \frac{3m_\ell^2}{2q^2}\mathcal{H}_S}{\mathcal{H}_{\text{total}}}$$

longitudinal polarization fractions of V:

$$F_L(q^2) = \frac{\mathcal{H}_L\left(1 + \frac{m_\ell^2}{2q^2}\right) + \frac{3m_\ell^2}{2q^2}\mathcal{H}_S}{\mathcal{H}_{\text{total}}}$$

lepton-side convexity parameters:

$$C_F^\ell(q^2) = \frac{3}{4}\left(1 - \frac{m_\ell^2}{q^2}\right)\frac{\mathcal{H}_U - 2\mathcal{H}_L}{\mathcal{H}_{\text{total}}}$$

Mesons $M^i_j \quad i=1,2,3$ *for* u,d,s

$$D_i = \left(D^0(c\bar{u}),\ D^+(c\bar{d}),\ D_s^+(c\bar{s})\right)$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \overline{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} \end{pmatrix},$$

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} + \frac{\omega_1}{\sqrt{3}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} + \frac{\omega_1}{\sqrt{3}} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\frac{2\omega_8}{\sqrt{6}} + \frac{\omega_1}{\sqrt{3}} \end{pmatrix},$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} cos\theta_P & -sin\theta_P \\ sin\theta_P & cos\theta_P \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix},$$

$$\begin{pmatrix} \phi \\ \omega \end{pmatrix} = \begin{pmatrix} cos\theta_V & -sin\theta_V \\ sin\theta_V & cos\theta_V \end{pmatrix} \begin{pmatrix} \omega_8 \\ \omega_1 \end{pmatrix},$$

Scalar with two quark picture:

$$S = \begin{pmatrix} \frac{a_0^0}{\sqrt{2}} + \frac{\sigma}{\sqrt{2}} & a_0^+ & K_0^+ \\ a_0^- & -\frac{a_0^0}{\sqrt{2}} + \frac{\sigma}{\sqrt{2}} & K_0^0 \\ K_0^- & \overline{K}_0^0 & f_0 \end{pmatrix}$$

$$\begin{pmatrix} f_0(980) \\ f_0(500) \end{pmatrix} = \begin{pmatrix} \cos\theta_S & \sin\theta_S \\ -\sin\theta_S & \cos\theta_S \end{pmatrix} \begin{pmatrix} f_0 \\ \sigma \end{pmatrix}$$

Scalar with four quark picture:

$\sigma = u\bar{u}d\bar{d}$, $\quad f_0 = (u\bar{u} + d\bar{d})s\bar{s}/\sqrt{2}$,

$a_0^0 = (u\bar{u} - d\bar{d})s\bar{s}/\sqrt{2}$, $\quad a_0^+ = u\bar{d}s\bar{s}$, $\quad a_0^- = d\bar{u}s\bar{s}$,

$K_0^+ = u\bar{s}d\bar{d}$, $\quad K_0^0 = d\bar{s}u\bar{u}$, $\quad \bar{K}_0^0 = s\bar{d}u\bar{u}$, $\quad K_0^+ = s\bar{u}d\bar{d}$,

S^{im}_{jm}

$$\begin{pmatrix} f_0(980) \\ f_0(500) \end{pmatrix} = \begin{pmatrix} \cos\phi_S & \sin\phi_S \\ -\sin\phi_S & \cos\phi_S \end{pmatrix} \begin{pmatrix} f_0 \\ \sigma \end{pmatrix}$$

Hadronic helicity amplitude：

$$\begin{cases} H(D \to M\ell^+\nu_\ell)^{2q} = c_0^M D_i M_j^i H^j, \\ \Delta H(D \to M\ell^+\nu_\ell)^{2q} = c_1^M D_a W_i^a M_j^i H^j + c_2^M D_i M_a^i W_j^a H^j, \end{cases}$$

$$\begin{cases} H(D \to S\ell^+\nu_\ell)^{4q} = c_0'^S D_i S_{jm}^{im} H^j \\ \Delta H(D \to S\ell^+\nu_\ell)^{4q} = c_1'^S D_a W_i^a S_{jm}^{im} H^j + c_2'^S D_i S_{am}^{im} W_j^a H^j + c_1'^S D_i S_{ja}^{im} W_m^a H^j. \end{cases}$$

c_i, c_i' : nonperturbative coefficients

H^i : CKM matrix elements

$$W = (W_j^i) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix},$$

2.2 $D \to P\ell^+\nu_\ell$ decays

Amplitudes:

TABLE I: The hadronic helicity amplitudes for the $D \to P\ell^+\nu$ decays including both the SU(3) flavor symmetry and the SU(3) flavor breaking contributions. $A_1 \equiv c_0^P + c_1^P - 2c_2^P$, $A_2 \equiv c_0^P - 2c_1^P - 2c_2^P$, $A_3 \equiv c_0^P + c_1^P + c_2^P$, $A_4 \equiv c_0^P - 2c_1^P + c_2^P$. $A_1 = A_2 = A_3 = A_4 = c_0^P$ if neglecting the SU(3) flavor breaking c_1^P and c_2^P terms.

Hadronic helicity amplitudes	SU(3) flavor amplitudes
$H(D^0 \to K^-\ell^+\nu_\ell)$	$A_1 V_{cs}^*$
$H(D^+ \to \overline{K}^0\ell^+\nu_\ell)$	$A_1 V_{cs}^*$
$H(D_s^+ \to \eta\ell^+\nu_\ell)$	$\left(-cos\theta_P\sqrt{2/3} - sin\theta_P/\sqrt{3}\right)A_2 V_{cs}^*$
$H(D_s^+ \to \eta'\ell^+\nu_\ell)$	$\left(-sin\theta_P\sqrt{2/3} + cos\theta_P/\sqrt{3}\right)A_2 V_{cs}^*$
$H(D_s^+ \to \pi^0\ell^+\nu_\ell)$	$-\delta\left(-cos\theta_P\sqrt{2/3} - sin\theta_P/\sqrt{3}\right)A_2 V_{cs}^*$
$H(D^0 \to \pi^-\ell^+\nu_\ell)$	$A_3 V_{cd}^*$
$H(D^+ \to \pi^0\ell^+\nu_\ell)$	$-\frac{1}{\sqrt{2}}A_3 V_{cd}^*$
$H(D^+ \to \eta\ell^+\nu_\ell)$	$\left(cos\theta_P/\sqrt{6} - sin\theta_P/\sqrt{3}\right)A_3 V_{cd}^*$
$H(D^+ \to \eta'\ell^+\nu_\ell)$	$\left(sin\theta_P/\sqrt{6} + cos\theta_P/\sqrt{3}\right)A_3 V_{cd}^*$
$H(D_s^+ \to K^0\ell^+\nu_\ell)$	$A_4 V_{cd}^*$

Branching ratios:

TABLE II: Branching ratios of the $D \to P\ell^+\nu$ d[illegible] Denotes that the corresponding experi[illegible] ata from PDG [1] are not used to constrain A_i in this case.

Branching ratios	Exp. data	Ones in C_1	Ones in C_2	Ones in C_3	Ones in C_4	Previous ones
$\mathcal{B}(D^+ \to \overline{K}^0 e^+\nu_e)(\times 10^{-2})$	8.72 ± 0.18	8.84 ± 0.06	8.83 ± 0.07	8.84 ± 0.06	8.83 ± 0.07	
$\mathcal{B}(D^+ \to \pi^0 e^+\nu_e)(\times 10^{-3})$	3.72 ± 0.34	3.75 ± 0.05	$5.40 \pm 1.33^\dagger$	$5.04 \pm 0.12^\dagger$	3.70 ± 0.11	
$\mathcal{B}(D^+ \to \eta e^+\nu_e)(\times 10^{-3})$	1.11 ± 0.14	1.15 ± 0.05	1.20 ± 0.05	1.20 ± 0.05	0.92 ± 0.08	
$\mathcal{B}(D^+ \to \eta' e^+\nu_e)(\times 10^{-4})$	2.0 ± 0.8	2.59 ± 0.14	2.22 ± 0.34	2.09 ± 0.14	1.50 ± 0.20	
$\mathcal{B}(D^0 \to K^- e^+\nu_e)(\times 10^{-2})$	3.549 ± 0.052	3.52 ± 0.02	3.52 ± 0.03	3.52 ± 0.03	3.52 ± 0.02	
$\mathcal{B}(D^0 \to \pi^- e^+\nu_e)(\times 10^{-3})$	$2.91 + 0.08$	$2.95 + 0.03$	$4.23 + 1.03^\dagger$	$3.97 + 0.09^\dagger$	$2.89 + 0.06$	
$\mathcal{B}(D_s^+ \to \eta e^+\nu_e)(\times 10^{-2})$	2.32 ± 0.16	2.37 ± 0.11	2.34 ± 0.14	2.36 ± 0.12	2.32 ± 0.16	
$\mathcal{B}(D_s^+ \to \eta' e^+\nu_e)(\times 10^{-3})$	8.0 ± 1.4	9.05 ± 0.04	8.25 ± 1.13	8.04 ± 0.43	8.02 ± 1.38	
$\mathcal{B}(D_s^+ \to K^0 e^+\nu_e)(\times 10^{-3})$	3.4 ± 0.8	3.10 ± 0.08	3.56 ± 0.39	3.54 ± 0.12	3.40 ± 0.80	
$\mathcal{B}(D_s^+ \to \pi^0 e^+\nu_e)(\times 10^{-5})$	$\cdots$	1.51 ± 0.07	2.10 ± 0.56	1.96 ± 0.10	1.92 ± 0.13	2.65 ± 0.38 [76]
$\mathcal{B}(D^+ \to \overline{K}^0 \mu^+\nu_\mu)(\times 10^{-2})$	8.76 ± 0.38	8.56 ± 0.06	8.69 ± 0.15	8.61 ± 0.06	8.61 ± 0.06	
$\mathcal{B}(D^+ \to \pi^0 \mu^+\nu_\mu)(\times 10^{-3})$	3.50 ± 0.30	3.67 ± 0.05	$5.32 \pm 1.31^\dagger$	$4.96 \pm 0.12^\dagger$	3.64 ± 0.10	
$\mathcal{B}(D^+ \to \eta \mu^+\nu_\mu)(\times 10^{-3})$	1.04 ± 0.22	1.11 ± 0.05	1.18 ± 0.07	1.17 ± 0.05	0.90 ± 0.08	1.21 [7] 0.75 ± 0.15 [79]
$\mathcal{B}(D^+ \to \eta' \mu^+\nu_\mu)(\times 10^{-4})$	$\cdots$	2.42 ± 0.13	2.10 ± 0.33	1.96 ± 0.13	1.41 ± 0.19	2.11 [7] 1.06 ± 0.20 [79]
$\mathcal{B}(D^0 \to K^- \mu^+\nu_\mu)(\times 10^{-2})$	3.41 ± 0.08	3.41 ± 0.02	3.44 ± 0.05	3.43 ± 0.02	3.43 ± 0.02	
$\mathcal{B}(D^0 \to \pi^- \mu^+\nu_\mu)(\times 10^{-3})$	$2.67 + 0.24$	$2.89 + 0.02$	$4.17 + 1.01^\dagger$	$3.90 + 0.09^\dagger$	$2.85 + 0.06$	
$\mathcal{B}(D_s^+ \to \eta \mu^+\nu_\mu)(\times 10^{-2})$	2.4 ± 1.0	2.30 ± 0.10	2.30 ± 0.17	2.31 ± 0.12	2.26 ± 0.16	
$\mathcal{B}(D_s^+ \to \eta' \mu^+\nu_\mu)(\times 10^{-2})$	1.1 ± 1.0	0.86 ± 0.03	0.79 ± 0.11	0.77 ± 0.04	0.76 ± 0.13	
$\mathcal{B}(D_s^+ \to K^0 \mu^+\nu_\mu)(\times 10^{-3})$	$\cdots$	3.01 ± 0.08	3.51 ± 0.38	3.46 ± 0.11	3.33 ± 0.78	3.9 [7] 3.85 ± 0.76 [79]
$\mathcal{B}(D_s^+ \to \pi^0 \mu^+\nu_\mu)(\times 10^{-5})$	$\cdots$	1.48 ± 0.07	2.09 ± 0.53	1.93 ± 0.10	1.89 ± 0.13	
$\mathcal{B}(D_s^+ \to \pi^0 \tau^+\nu_\tau)(\times 10^{-10})$	$\cdots$	3.45 ± 0.21	160.34 ± 149.53	4.20 ± 0.26	4.08 ± 0.34	$(27 \sim 36)$ [76]

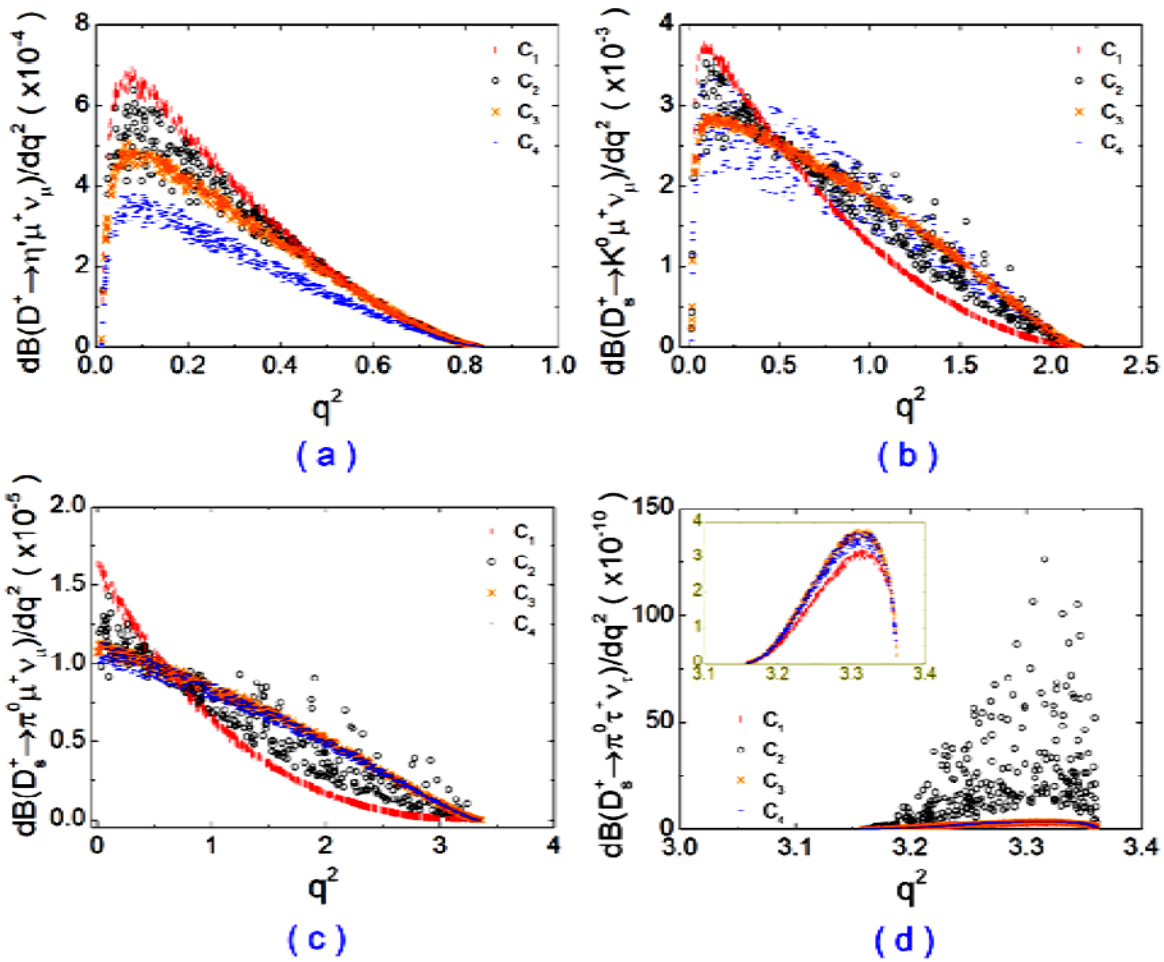


FIG. 1: The q^2 dependence of the differential branching ratios for some $D \to P\ell^+\nu_\ell$ with present experimental bounds.

2.3 $D \to V \ell^+ \nu_\ell$ decays

Amplitudes:

TABLE IV: The hadronic helicity amplitudes for $D \to V\ell^+\nu$ decays including both the SU(3) flavor symmetry and the SU(3) flavor breaking contributions. $B_1 = c_0^V + c_1^V - 2c_2^V$, $B_2 = c_0^V - 2c_1^V - 2c_2^V$, $B_3 = c_0^V + c_1^V + c_2^V$, $B_4 = c_0^V - 2c_1^V + c_2^V$. If neglecting the SU(3) flavor breaking c_1^V and c_2^V terms, $B_1 = B_2 = B_3 = B_4 = c_0^V$.

Hadronic helicity amplitudes	SU(3) IRA amplitudes
$H(D^0 \to K^{*-}\ell^+\nu_\ell)$	$B_1 V_{cs}^*$
$H(D^+ \to \overline{K}^{*0}\ell^+\nu_\ell)$	$B_1 V_{cs}^*$
$H(D_s^+ \to \phi\ell^+\nu_\ell)$	$\left(-cos\theta_V\sqrt{2/3} - sin\theta_V/\sqrt{3}\right) B_2 V_{cs}^*$
$H(D_s^+ \to \omega\ell^+\nu_\ell)$	$\left(-sin\theta_V\sqrt{2/3} + cos\theta_V/\sqrt{3}\right) B_2 V_{cs}^*$
$H(D^0 \to \rho^-\ell^+\nu_\ell)$	$B_3 V_{cd}^*$
$H(D^+ \to \rho^0\ell^+\nu_\ell)$	$-\frac{1}{\sqrt{2}} B_3 V_{cd}^*$
$H(D^+ \to \phi\ell^+\nu_\ell)$	$\left(cos\theta_V/\sqrt{6} - sin\theta_V/\sqrt{3}\right) B_3 V_{cd}^*$
$H(D^+ \to \omega\ell^+\nu_\ell)$	$\left(sin\theta_V/\sqrt{6} + cos\theta_V/\sqrt{3}\right) B_3 V_{cd}^*$
$H(D_s^+ \to K^{*0}\ell^+\nu_\ell)$	$B_4 V_{cd}^*$

Branching ratios:

TABLE V: Branching ratios of the $D \to V\ell^+\nu$ within 2σ errors. †The experimental data of $\mathcal{B}(D^+ \to \omega e^+\nu_e)$ and $\mathcal{B}(D^0 \to \rho^-\mu^+\nu_\mu)$ from PDG [1] are not used in the $C_{1,2,3}$ cases.

Branching ratios	Exp. data	Ones in C_1	Ones in C_2	Ones in C_3	Ones in C_4
$\mathcal{B}(D^+ \to \overline{K}^{*0}e^+\nu_e)(\times 10^{-2})$	5.40 ± 0.20	5.44 ± 0.15	5.42 ± 0.18	5.36 ± 0.08	5.44 ± 0.16
$\mathcal{B}(D^+ \to \rho^0 e^+\nu_e)(\times 10^{-3})$	$2.18^{+0.34}_{-0.50}$	2.31 ± 0.07	2.39 ± 0.13	2.33 ± 0.05	1.83 ± 0.15
$\mathcal{B}(D^+ \to \omega e^+\nu_e)(\times 10^{-3})$	1.69 ± 0.22	$2.24 \pm 0.07^\dagger$	$2.33 \pm 0.12^\dagger$	$2.26 \pm 0.04^\dagger$	1.77 ± 0.14
$\mathcal{B}(D^+ \to \phi e^+\nu_e)(\times 10^{-7})$	< 130	3.13 ± 0.12	3.11 ± 0.19	3.07 ± 0.07	2.38 ± 0.23
$\mathcal{B}(D^0 \to K^{*-}e^+\nu_e)(\times 10^{-2})$	2.15 ± 0.32	2.12 ± 0.09	2.13 ± 0.10	2.08 ± 0.06	2.13 ± 0.10
$\mathcal{B}(D^0 \to \rho^- e^+\nu_e)(\times 10^{-3})$	1.50 ± 0.24	1.79 ± 0.08	1.86 ± 0.11	1.80 ± 0.06	1.41 ± 0.13
$\mathcal{B}(D_s^+ \to \phi e^+\nu_e)(\times 10^{-2})$	2.39 ± 0.32	2.46 ± 0.12	2.43 ± 0.14	2.40 ± 0.10	2.39 ± 0.32
$\mathcal{B}(D_s^+ \to \omega e^+\nu_e)(\times 10^{-5})$	< 200	2.45 ± 0.13	2.56 ± 0.20	2.47 ± 0.10	2.49 ± 0.38
$\mathcal{B}(D_s^+ \to K^{*0}e^+\nu_e)(\times 10^{-3})$	2.15 ± 0.56	2.17 ± 0.10	2.25 ± 0.13	2.17 ± 0.08	2.15 ± 0.56
$\mathcal{B}(D^+ \to \overline{K}^{*0}\mu^+\nu_\mu)(\times 10^{-2})$	5.27 ± 0.30	5.12 ± 0.15	5.13 ± 0.16	5.05 ± 0.08	5.12 ± 0.15
$\mathcal{B}(D^+ \to \rho^0\mu^+\nu_\mu)(\times 10^{-3})$	2.4 ± 0.8	2.19 ± 0.07	2.29 ± 0.13	2.22 ± 0.04	1.74 ± 0.14
$\mathcal{B}(D^+ \to \omega\mu^+\nu_\mu)(\times 10^{-3})$	1.77 ± 0.42	2.13 ± 0.06	2.23 ± 0.12	2.15 ± 0.04	1.68 ± 0.13
$\mathcal{B}(D^+ \to \phi\mu^+\nu_\mu)(\times 10^{-7})$	$\cdots$	2.89 ± 0.11	2.89 ± 0.17	2.84 ± 0.07	2.20 ± 0.21
$\mathcal{B}(D^0 \to K^{*-}\mu^+\nu_\mu)(\times 10^{-2})$	1.89 ± 0.48	1.99 ± 0.09	2.01 ± 0.09	1.96 ± 0.06	2.01 ± 0.10
$\mathcal{B}(D^0 \to \rho^-\mu^+\nu_\mu)(\times 10^{-3})$	1.35 ± 0.26	$1.70 \pm 0.07^\dagger$	$1.78 \pm 0.11^\dagger$	$1.72 \pm 0.06^\dagger$	1.34 ± 0.13
$\mathcal{B}(D_s^+ \to \phi\mu^+\nu_\mu)(\times 10^{-2})$	1.9 ± 1.0	2.30 ± 0.12	2.29 ± 0.12	2.25 ± 0.09	2.24 ± 0.30
$\mathcal{B}(D_s^+ \to \omega\mu^+\nu_\mu)(\times 10^{-5})$	$\cdots$	2.34 ± 0.12	2.47 ± 0.19	2.37 ± 0.09	2.38 ± 0.36
$\mathcal{B}(D_s^+ \to K^{*0}\mu^+\nu_\mu)(\times 10^{-3})$	$\cdots$	2.06 ± 0.10	2.15 ± 0.13	2.07 ± 0.08	2.05 ± 0.53

(2.23 ± 0.32)

2.4 $D \to S\ell^+\nu_\ell$ decays

Amplitudes:

TABLE VIII: The hadronic helicity amplitudes for $D \to S\ell^+\nu$ decays including both the SU(3) flavor symmetry and the SU(3) flavor breaking contributions. In the two quark picture of the scalar mesons, $E_1 \equiv c_0^S + c_1^S - 2c_2^S$, $E_2 \equiv c_0^S - 2c_1^S - 2c_2^S$, $E_3 \equiv c_0^S + c_1^S + c_2^S$, $E_4 \equiv c_0^S - 2c_1^S + c_2^S$. $E_1 = E_2 = E_3 = E_4 = c_0^S$ if neglecting the SU(3) flavor breaking c_1^S and c_2^S terms. In the four quark picture of the scalar mesons, $E_1' \equiv c_0'^S + c_1'^S - 2c_2'^S + c_3'^S$, $E_2' \equiv c_0'^S - 2c_1'^S - 2c_2'^S + c_3'^S$, $E_3' \equiv c_0'^S + c_1'^S + c_2'^S - 2c_3'^S$, $E_4' \equiv c_0'^S + c_1'^S + c_2'^S + c_3'^S$, $E_5' \equiv c_0'^S - 2c_1'^S + c_2'^S + c_3'^S$, $E_1' = E_2' = E_3' = E_4' = E_5' = c_0'^S$ if neglecting the SU(3) flavor breaking $c_1'^S$, $c_2'^S$ and $c_3'^S$ terms.

Hadronic helicity amplitudes	ones for two-quark scenario	ones for four-quark scenario
$H(D^0 \to K_0^- \ell^+\nu_\ell)$	$E_1 V_{cs}^*$	$E_1' V_{cs}^*$
$H(D^+ \to \overline{K}_0^0 \ell^+\nu_\ell)$	$E_1 V_{cs}^*$	$E_1' V_{cs}^*$
$H(D_s^+ \to f_0 \ell^+\nu_\ell)$	$E_2 V_{cs}^*$	$\sqrt{2} E_2' V_{cs}^*$
$H(D_s^+ \to f_0(980) \ell^+\nu_\ell)$	$cos\theta_S\ E_2 V_{cs}^*$	$\sqrt{2} cos\phi_S\ E_2' V_{cs}^*$
$H(D_s^+ \to f_0(500) \ell^+\nu_\ell)$	$-sin\theta_S\ E_2 V_{cs}^*$	$-\sqrt{2} sin\phi_S\ E_2' V_{cs}^*$
$H(D^0 \to a_0^- \ell^+\nu_\ell)$	$E_3 V_{cd}^*$	$E_3' V_{cd}^*$
$H(D^+ \to a_0^0 \ell^+\nu_\ell)$	$-\frac{1}{\sqrt{2}} E_3 V_{cd}^*$	$-\frac{1}{\sqrt{2}} E_3' V_{cd}^*$
$H(D^+ \to f_0 \ell^+\nu_\ell)$	0	$\frac{1}{\sqrt{2}} E_3' V_{cd}^*$
$H(D^+ \to \sigma \ell^+\nu_\ell)$	$\frac{1}{\sqrt{2}} E_3 V_{cd}^*$	$E_4' V_{cd}^*$
$H(D^+ \to f_0(980) \ell^+\nu_\ell)$	$\frac{1}{\sqrt{2}} sin\theta_S\ E_3 V_{cd}^*$	$(\frac{1}{\sqrt{2}} E_3' cos\phi_S + E_4' sin\phi_S) V_{cd}^*$
$H(D^+ \to f_0(500) \ell^+\nu_\ell)$	$\frac{1}{\sqrt{2}} cos\theta_S\ E_3 V_{cd}^*$	$(-\frac{1}{\sqrt{2}} E_3' sin\phi_S + E_4' cos\phi_S) V_{cd}^*$
$H(D_s^+ \to K_0^0 \ell^+\nu_\ell)$	$E_4 V_{cd}^*$	$E_5' V_{cd}^*$

Experimental data:

$$\mathcal{B}(D_s^+ \to f_0(980)e^+\nu_e) = (2.3 \pm 0.8) \times 10^{-3}$$

$$\mathcal{B}(D_s^+ \to f_0(980)e^+\nu_e,\ f_0(980) \to \pi^+\pi^-) = (1.30 \pm 0.63) \times 10^{-3} \quad [81],$$

$$\mathcal{B}(D_s^+ \to f_0(980)e^+\nu_e,\ f_0(980) \to \pi^0\pi^0) = (7.9 \pm 2.9) \times 10^{-4} \quad [82],$$

$$\mathcal{B}(D^0 \to a_0(980)^- e^+\nu_e,\ a_0(980)^- \to \eta\pi^-) = (1.33^{+0.68}_{-0.60}) \times 10^{-4} \quad [1],$$

$$\mathcal{B}(D^+ \to a_0(980)^0 e^+\nu_e,\ a_0(980)^0 \to \eta\pi^0) = (1.7^{+1.6}_{-1.4}) \times 10^{-4} \quad [1],$$

$$\mathcal{B}(D^+ \to f_0(500)e^+\nu_e,\ f_0(500) \to \pi^+\pi^-) = (6.3 \pm 1.0) \times 10^{-4} \quad [1],$$

$$\mathcal{B}(D^+ \to f_0(980)e^+\nu_e,\ f_0(980) \to \pi^+\pi^-) < 2.8 \times 10^{-5} \quad [83],$$

$$\mathcal{B}(D_s^+ \to f_0(500)e^+\nu_e,\ f_0(500) \to \pi^0\pi^0) < 6.4 \times 10^{-4} \quad [82].$$

Branching ratios:

TABLE IX: Branching ratios of $D \to S\ell^+\nu$ decays within 2σ errors. As given in Ref. [80], g_4' and g_4 are strong coupling constants obtained by the SU(3) flavor symmetry in $S \to P_1 P_2$ decays. [a]denotes the results with $\frac{g_4'}{g_4} > 0$, and [b]denotes ones with $\frac{g_4'}{g_4} < 0$. †denotes the results with two quark picture, and ‡denotes the results with four quark picture.

②

Branching ratios	ones for 2q state in S_1			ones for 2q state in S_2			ones for 4q	ones for 4q	Previous ones
	$[25°, 40°]$	$[140°, 165°]$	$[-30°, 30°]$	$[25°, 35°]$	$[144°, 158°]$	$22° \le \lvert\theta_S\rvert \le 30°$	state in S_1	state in S_2	③
$\mathcal{B}(D^0 \to K_0^- e^+ \nu_e)(\times 10^{-3})$	3.38 ± 2.12	3.18 ± 2.05	2.57 ± 1.58	3.02 ± 1.11	3.00 ± 1.10	2.98 ± 1.05	1.11 ± 0.63	1.25 ± 0.45	$0.103 \pm 0.115^\dagger$ [67]
$\mathcal{B}(D^+ \to \overline{K}_0^0 e^+ \nu_e)(\times 10^{-3})$	8.66 ± 5.55	7.99 ± 5.02	7.02 ± 4.48	7.74 ± 2.88	7.78 ± 2.77	7.68 ± 2.78	2.85 ± 1.65	3.36 ± 1.25	$38.8 \pm 5.6^\dagger$ [67]
DATA $\mathcal{B}(D_s^+ \to f_0(980) e^+ \nu_e)(\times 10^{-3})$	2.30 ± 0.80	2.30 ± 0.80	2.30 ± 0.80	2.58 ± 0.52	2.57 ± 0.53	2.71 ± 0.39	2.30 ± 0.80	2.49 ± 0.61^a 2.54 ± 0.56^b	$2.1 \pm 0.2^\dagger$ [78], $2^{+0.5}_{-0.4}{}^\dagger$ [84]
$\mathcal{B}(D_s^+ \to f_0(500) e^+ \nu_e)(\times 10^{-3})$	6.73 ± 6.11	5.98 ± 5.75	3.25 ± 3.25	1.49 ± 0.43	1.45 ± 0.46	1.42 ± 0.50	0.37 ± 0.37	0.31 ± 0.31^a 0.17 ± 0.17^b	
$\mathcal{B}(D^0 \to K_0^- \mu^+ \nu_\mu)(\times 10^{-3})$	2.90 ± 1.84	2.73 ± 1.77	2.20 ± 1.36	2.59 ± 0.97	2.57 ± 0.96	2.56 ± 0.92	0.95 ± 0.54	1.09 ± 0.39	$0.103 \pm 0.115^\dagger$ [67]
$\mathcal{B}(D^+ \to \overline{K}_0^0 \mu^+ \nu_\mu)(\times 10^{-3})$	7.46 ± 4.81	6.87 ± 4.33	6.04 ± 3.88	6.65 ± 2.52	6.69 ± 2.43	6.59 ± 2.43	2.45 ± 1.43	2.89 ± 1.09	$38.8 \pm 5.6^\dagger$ [67]
$\mathcal{B}(D_s^+ \to f_0(980) \mu^+ \nu_\mu)(\times 10^{-3})$	1.95 ± 0.70	1.95 ± 0.70	1.95 ± 0.69	2.20 ± 0.45	2.20 ± 0.45	2.32 ± 0.33	1.95 ± 0.70	2.12 ± 0.54^a 2.16 ± 0.49^b	$2.1 \pm 0.2^\dagger$ [78]
④ $\mathcal{B}(D_s^+ \to f_0(500) \mu^+ \nu_\mu)(\times 10^{-3})$	6.21 ± 5.66	5.53 ± 5.32	3.01 ± 3.01	1.33 ± 0.39	1.31 ± 0.43	1.28 ± 0.46	0.34 ± 0.34	0.29 ± 0.29^a 0.16 ± 0.16^b	
$\mathcal{B}(D^0 \to a_0^- e^+ \nu_e)(\times 10^{-5})$	9.99 ± 6.54	9.56 ± 6.50	8.34 ± 5.67	9.22 ± 3.98	9.09 ± 3.65	9.17 ± 3.58	3.42 ± 2.06	4.32 ± 1.17	$16.8 \pm 1.5^\dagger$ [78], $40.8^{+13.7}_{-12.2}{}^\dagger$ [86], $24.4 \pm 3.0^\dagger$ [67]
$\mathcal{B}(D^+ \to a_0^0 e^+ \nu_e)(\times 10^{-5})$	13.09 ± 8.62	12.62 ± 8.67	10.89 ± 7.35	12.09 ± 5.19	11.81 ± 4.71	11.97 ± 4.66	4.49 ± 2.71	5.68 ± 1.52	$21.8 \pm 3.8^\dagger$ [78], $54.0^{+17.8}_{-15.9}{}^\dagger$ [86] $6 \sim 8^\dagger$ [85], $5 \sim 5.4^\ddagger$ [85]
$\mathcal{B}(D^+ \to f_0(980) e^+ \nu_e)(\times 10^{-5})$	3.92 ± 2.92	3.48 ± 3.13	1.59 ± 1.59	2.62 ± 0.82	2.52 ± 0.94	2.40 ± 0.80	3.14 ± 1.98	3.35 ± 1.80^a 3.69 ± 1.35^b	$7.78 \pm 0.68^\dagger$ [78], $5.7 \pm 1.3^\dagger$ [87] $0.4 \sim 3.5^\dagger$ [85], $1.9 \sim 6.3^\ddagger$ [85]
$\mathcal{B}(D^+ \to f_0(500) e^+ \nu_e)(\times 10^{-4})$	4.05 ± 3.20	4.08 ± 3.10	4.21 ± 3.28	2.16 ± 0.96	2.59 ± 1.38	2.70 ± 1.28	4.97 ± 4.13	4.97 ± 3.34^a 4.95 ± 3.36^b	$0.4 \sim 0.6^\dagger$ [85], $0.88 \sim 1.4^\ddagger$ [85]
$\mathcal{B}(D_s^+ \to K_0^0 e^+ \nu_e)(\times 10^{-4})$	3.73 ± 2.37	3.41 ± 2.13	2.99 ± 1.88	3.35 ± 1.21	3.32 ± 1.20	3.35 ± 1.15	1.25 ± 0.71	1.43 ± 0.51	$26.5 \pm 2.8^\dagger$ [67]
$\mathcal{B}(D^0 \to a_0^- \mu^+ \nu_\mu)(\times 10^{-5})$	8.25 ± 5.45	7.89 ± 5.42	6.91 ± 4.75	7.61 ± 3.37	7.51 ± 3.10	7.57 ± 3.04	2.83 ± 1.72	3.57 ± 0.99	$16.3 \pm 1.4^\dagger$ [78], $24.4 \pm 3.0^\dagger$ [67]
$\mathcal{B}(D^+ \to a_0^0 \mu^+ \nu_\mu)(\times 10^{-5})$	10.83 ± 7.19	10.44 ± 7.23	9.04 ± 6.16	10.00 ± 4.41	9.76 ± 4.00	9.89 ± 3.97	3.73 ± 2.28	4.69 ± 1.30	$21.2 \pm 3.7^\dagger$ [78]
$\mathcal{B}(D^+ \to f_0(980) \mu^+ \nu_\mu)(\times 10^{-5})$	3.23 ± 2.41	2.88 ± 2.60	1.32 ± 1.32	2.15 ± 0.70	2.09 ± 0.78	1.99 ± 0.66	2.56 ± 1.62	2.74 ± 1.48^a 3.20 ± 1.14^b	$7.87 \pm 0.67^\dagger$ [78]
$\mathcal{B}(D^+ \to f_0(500) \mu^+ \nu_\mu)(\times 10^{-4})$	3.69 ± 2.96	3.71 ± 2.86	3.84 ± 3.04	1.92 ± 0.88	2.32 ± 1.27	2.42 ± 1.19	4.54 ± 3.81	4.52 ± 3.10^a 4.49 ± 3.12^b	
$\mathcal{B}(D_s^+ \to K_0^0 \mu^+ \nu_\mu)(\times 10^{-4})$	3.28 ± 2.10	3.00 ± 1.88	2.62 ± 1.66	2.94 ± 1.08	2.91 ± 1.06	2.94 ± 1.02	1.10 ± 0.63	1.26 ± 0.45	$26.5 \pm 2.8^\dagger$ [67]