Inclusive weak-annihilation decays and lifetimes of beauty-charmed baryons

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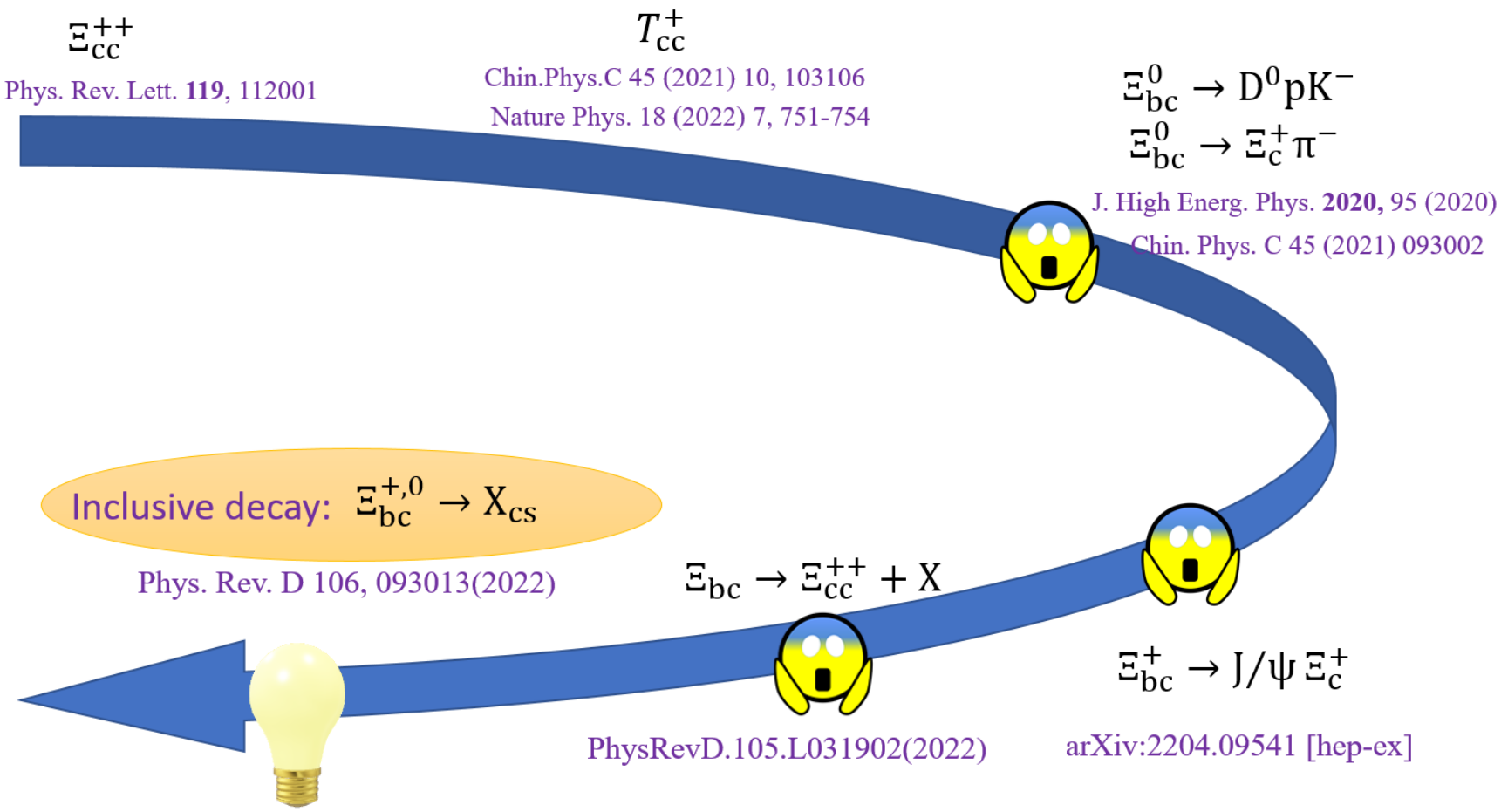
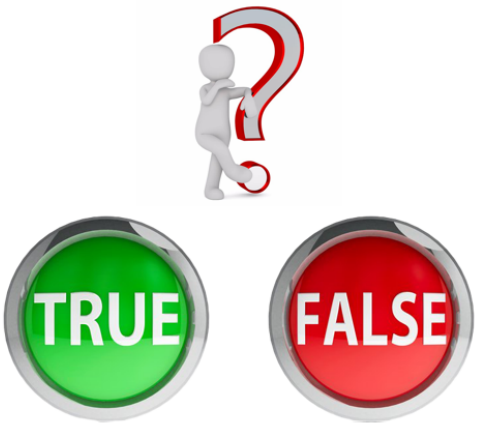
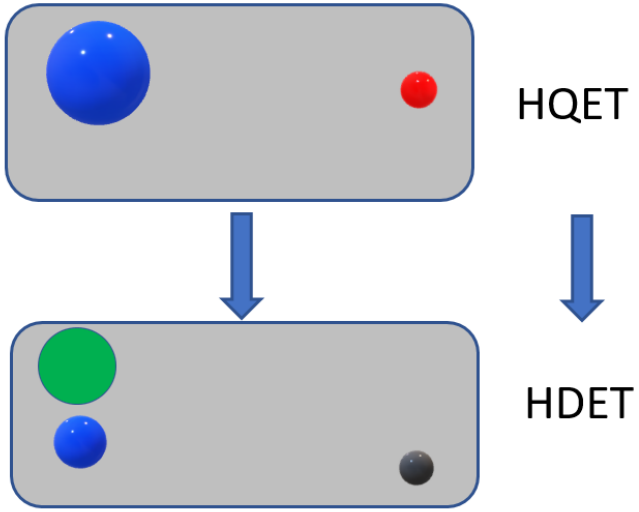
Introduction



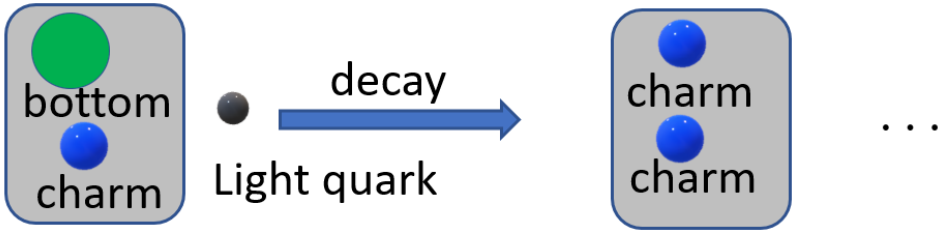
Introduction

- Our goal 1: test the heavy diquark effective theory

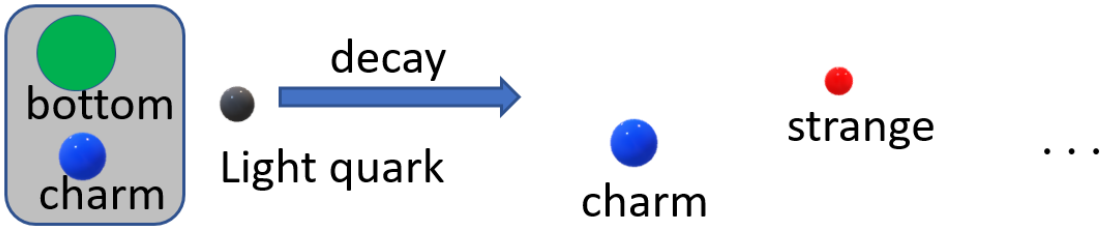




$\Xi_{bc} \to \Xi_{cc}^{++} + X$

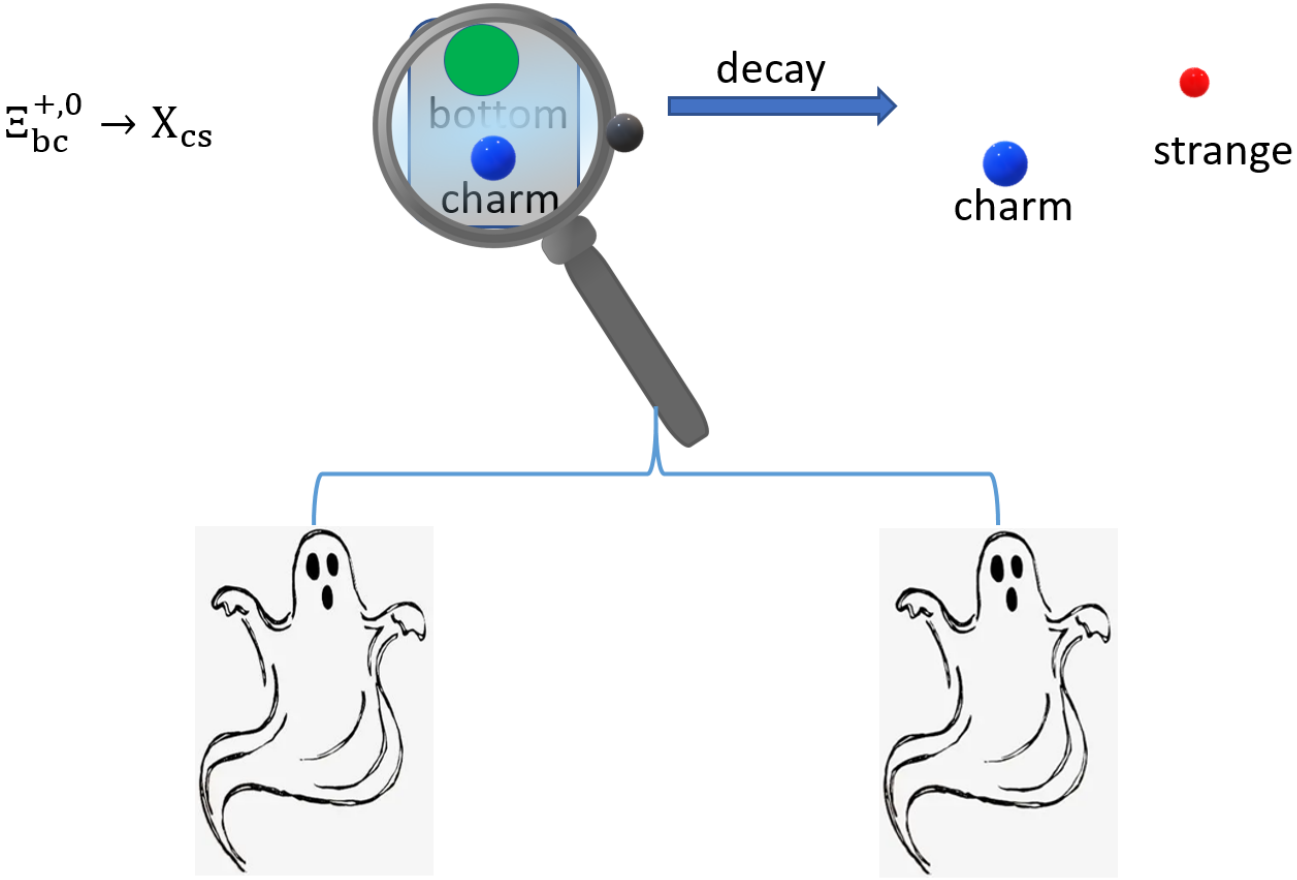


$\Xi_{bc}^{+,0} \to X_{cs}$



Introduction

- Our goal 2: determine the diquark constituent



Scalar state

Eur. Phys. J. C 16, 461–469 (2000), V.V. Kiselev

Axial-vector state

PhysRevD.99.073006(2019) , Hai-Yang Cheng

$$\left|\Xi_{bc}^{+,0}\right\rangle = \cos\theta\,|\Xi_{\mathcal{X}}\rangle + \sin\theta e^{i\phi}\,|\Xi_{\mathcal{S}}\rangle\,,$$

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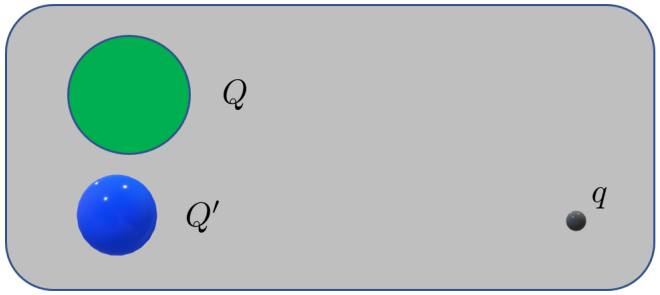
Heavy Diquark Effective Theory



$$
\begin{aligned}
r_{QQ'} &\sim 1/(m_Q v) \\
r_{Qq} &\sim 1/(\Lambda_{QCD}) \\
r_{QQ'}/r_{Qq} &\sim \Lambda_{QCD}/(m_Q v) \ll 1
\end{aligned}
$$

Diquark Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathcal{S}} &= i\mathcal{S}_v^\dagger v\cdot D\mathcal{S}_v - \frac{1}{2m_{\mathcal{S}}}\mathcal{S}_v^\dagger D^2\mathcal{S}_v + \mathcal{O}(\frac{1}{m_{\mathcal{S}}^2}) \\
\mathcal{L}_{\mathcal{X}} &= -i\mathcal{X}_{v\mu}^\dagger v\cdot D\mathcal{X}_v^\mu + \frac{1}{2m_{\mathcal{X}}}\mathcal{X}_{v\mu}^\dagger D^2\mathcal{X}_v^\mu + \frac{ig}{2m_{\mathcal{X}}}\mathcal{X}_{v\mu}^\dagger \bar{G}^{\mu\nu}\mathcal{X}_{v\nu} + \mathcal{O}(\frac{1}{m_{\mathcal{X}}^2}) \quad (1)
\end{aligned}
$$

arXiv:2002.02785,Yu-Ji Shi, et al

Redefined diquark fields

$$
\begin{aligned}
\mathcal{S}(x) &= \exp[-im_{\mathcal{S}} v\cdot x]\mathcal{S}_v(x)/\sqrt{2m_{\mathcal{S}}} \\
\mathcal{X}^\mu(x) &= \exp[-im_{\mathcal{X}} v\cdot x](\mathcal{X}_v^\mu(x) + Y_v^\mu(x))/\sqrt{2m_{\mathcal{X}}} \quad (2)
\end{aligned}
$$

Heavy Diquark Effective Theory

The effective Hamiltonian involved in the $bc \to cs$ processes is

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} V_{cs}^* V_{cb} \sum_{i=1}^{6} C_i O_i$$
$$O_1 = \bar{c}_\lambda \gamma^\mu P_L b_\rho \bar{s}_\rho \gamma_\mu P_L c_\lambda \ , \ O_2 = \bar{c}\gamma^\mu P_L b \ \bar{s}\gamma_\mu P_L c \ ,$$
$$O_3 = \bar{s}\gamma^\mu P_L b \ \bar{c}\gamma_\mu P_L c \ , \ O_4 = \bar{s}_\lambda \gamma^\mu P_L b_\rho \bar{c}_\rho \gamma_\mu P_L c_\lambda \ ,$$
$$O_5 = \bar{s}\gamma^\mu P_L b \ \bar{c}\gamma_\mu P_R c \ , \ O_6 = \bar{s}_\lambda \gamma^\mu P_L b_\rho \bar{c}_\rho \gamma_\mu P_R c_\lambda \ . \quad (3)$$

arXiv:hep-ph/9512380

The scalar and axial-vector diquark states are

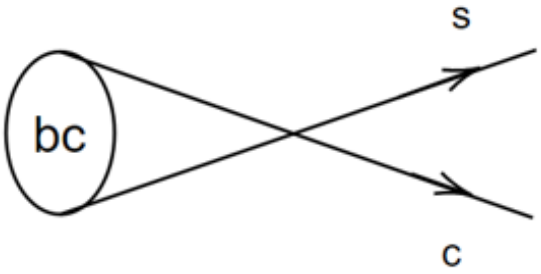
$$|\mathcal{S}^i_{bc}(v)\rangle = \frac{\sqrt{E_{\mathcal{S}}}}{4\sqrt{2m_b m_c}} \int \frac{d^3\mathrm{k}}{(2\pi)^3} \phi^*(\mathrm{k}) \epsilon_{ijk} \left[C\gamma_5(1+\not{v})\right]^\dagger_{\beta\gamma} Q^\dagger_{j\beta}(v,\mathrm{k}) Q'^\dagger_{k\gamma}(v,-\mathrm{k})|0\rangle$$
$$|\mathcal{X}^i_{bc}(v,\epsilon)\rangle = \frac{-\sqrt{E_{\mathcal{X}}}}{4\sqrt{2m_b m_c}} \int \frac{d^3\mathrm{k}}{(2\pi)^3} \phi^*(\mathrm{k}) \epsilon_{ijk} \left[C\not{\epsilon}(1+\not{v})\right]^\dagger_{\beta\gamma} Q^\dagger_{j\beta}(v,\mathrm{k}) Q'^\dagger_{k\gamma}(v,-\mathrm{k})|0\rangle \quad (4)$$

arXiv:2002.02785

where the heavy quark operators are defined by

$$Q(v,\mathrm{k}) = \sum_s u^s(p) a^s_{\mathrm{p}}, \ \text{with } \mathrm{p} \equiv m_Q \mathrm{v} + \mathrm{k} \ .$$

Heavy Diquark Effective Theory



the scalar diquark-quark interaction terms

$$
\begin{aligned}
\mathcal{H}_{\mathcal{S}} &\ni A_{12}\epsilon_{ijk}\,\mathcal{S}^i\,\bar{c}_j P_R C\bar{s}_k^T + A_{34}\epsilon_{ijk}\,\mathcal{S}^i\,\bar{c}_j P_R C\bar{s}_k^T \\
&\quad + A_{56}/m_{bc}\,\epsilon_{ijk}\,(iD_\mu\mathcal{S}^i)\,\bar{c}_j\gamma^\mu P_R C\bar{s}_k^T\ , \\
A_{12} &= 4\sqrt{m_{bc}}G_F V_{cs}^* V_{cb}(C_2 - C_1)\psi_{bc}(0)\ , \\
A_{34} &= 4\sqrt{m_{bc}}G_F V_{cs}^* V_{cb}(C_4 - C_3)\psi_{bc}(0)\ , \\
A_{56} &= 2\sqrt{m_{bc}}G_F V_{cs}^* V_{cb}(C_5 - C_6)\psi_{bc}(0)\ ,
\end{aligned}
\tag{5}
$$

the axial-vector diquark-quark interaction term

$$
\mathcal{H}_{\mathcal{X}} \ni B_{5,6}\epsilon_{ijk}\bar{c}_j\gamma^\mu P_R C\bar{s}_k^T\mathcal{X}^i_{v,\mu}\ , B_{56} = -A_{56}. \tag{6}
$$

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Operator Product Expansion

Optical theorem

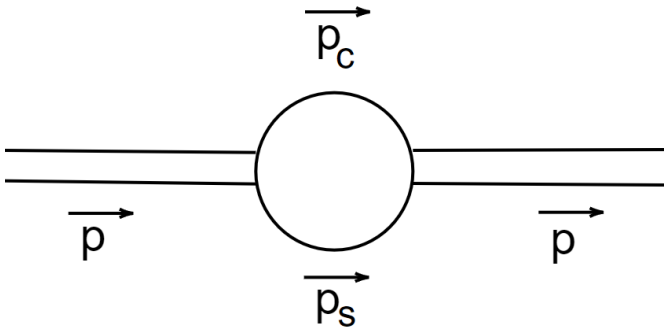

Figure: The leading order diagram for the diquark-to-diquark amplitudes

$$\Gamma(\Xi_{bc}^{+,0} \to X_{cs}) = \frac{\mathrm{Im}\langle \Xi_{bc}^{+,0} | \mathcal{T} | \Xi_{bc}^{+,0} \rangle}{m_\Xi}$$

$$\mathcal{T} = i \int d^4x \; T\{\mathcal{H}_I^\dagger(x)\mathcal{H}_I(0)\;,\}$$

$$\mathcal{H}_I = \mathcal{H}_\mathcal{S} + \mathcal{H}_\mathcal{X}. \qquad (7)$$

Operator product expansion

$$\mathcal{T} = \sum_i F_i Q_i^{(n)}\;,$$

where the $Q_i^{(n)}$ are local operators with dimension n and F_i are the corresponding coefficients.

Operator Product Expansion

$\mathcal{S}_{bc} \to cs \to \mathcal{S}_{bc} : \mathrm{Im}\ \mathcal{M}(S_{bc} \to S_{bc}) = A\frac{(p^2-m_c^2)^2}{8\pi p^2}$

$$A = |A_{12} + A_{34} + \frac{m_c^2}{m_{bc}^2}A_{56}|^2, p = m_{bc}v + k$$

Table: the order k^0 terms

	Original field	Redined field
operator	$S_{bc}^\dagger S_{bc}$	$Q_1^{s(3)} = S_v^\dagger S_v$
coefficient	$\frac{A(m_{bc}^2-m_c^2)^2}{8\pi m_{bc}^2}$	$F_1^s = \frac{A(m_{bc}^2-m_c^2)^2}{16\pi m_{bc}^3}$

Table: the order k^1 terms

	Original field	Redined field
operator	$S_{bc}^\dagger(iD\cdot v - m_{bc})S_{bc}$	$Q_1^{s(4)} = S_v^\dagger(iD\cdot v)S_v$
coefficient	$\frac{A(m_{bc}^4-m_c^4)^2}{4\pi m_{bc}^3}$	$F_2^s = \frac{A(m_{bc}^4-m_c^4)^2}{8\pi m_{bc}^4}$

Operator Product Expansion

Table: the order k^2 terms

	Original field	Redined field
operator	$S_{bc}^\dagger(iD - m_{bc}v)^2 S_{bc}$	$Q_1^{s(5)} = S_v^\dagger(iD)^2 S_v$
coefficient	$\frac{A(m_{bc}^4 - m_c^4)}{8\pi m_{bc}^4}$	$F_3^s = \frac{A(m_{bc}^4 - m_c^4)^2}{16\pi m_{bc}^5}$

$$\text{EOM:} 2imv \cdot DS_v - D^2 S_v = 0$$

The matrix element

$$\left\langle \Xi_S | S_v^\dagger S_v | \Xi_S \right\rangle \equiv 2m_\Xi \mu_3 = 2m_\Xi (1 - \frac{\mu_\pi^2}{2m_{bc}^2}) \,,$$

$$\text{with } \left\langle \Xi_S | S_v^\dagger (iD)^2 S_v | \Xi_S \right\rangle = -2m_\Xi \mu_\pi^2 \,. \tag{8}$$

The parameter μ_π^2 takes the value 0.43 ± 0.24[arXiv:1802.09409].

Operator Product Expansion

$\mathcal{X}_{bc} \to cs \to \mathcal{X}_{bc} : \mathrm{Im}\, \mathcal{M}_{5,6}(\mathcal{X}_{bc} \to \mathcal{X}_{bc}) = B\frac{(p^2-m_c^2)^2}{8\pi p^2}$

$$B = |B_{56}|^2(2 + \frac{m_c^2}{m_{bc}^2})$$

$$Q_1^{a(3)} = \mathcal{X}_v^{\mu\dagger}\mathcal{X}_{v\mu}, F_1^a = \frac{B(m_{bc}^2 - m_c^2)^2}{16\pi m_{bc}^3}$$

Analogous to the scalar case, the hadronic matrix element of $Q_1^{a(3)}$ is

$$\left\langle \Xi_{\mathcal{X}} | \mathcal{X}_v^{\mu\dagger}\mathcal{X}_{v\mu} | \Xi_{\mathcal{X}} \right\rangle \equiv 2m_{\Xi}\mu_3 \ . \tag{9}$$

The $S_{bc} \to cs \to X_{bc}$ amplitude at the leading order vanishes because of angular momentum conservation, so there would be no interference between the scalar and axial-vector constituents in the $\Xi_{bc}^{+,0}$ baryons.

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Numerical Analysis

the leading-order inclusive decay width

$$
\begin{aligned}
\textit{scalar} \qquad & \Gamma(\Xi_S \to X_{cs}) \\
= \quad & \frac{2G_F^2}{\pi m_{bc}^2}[C_2 - C_1 + C_4 - C_3 + \frac{m_c}{2m_{bc}}(C_5 - C_6)]^2 \\
& |V_{cs}^* V_{cb}|^2 |\psi_{bc}(0)|^2 (m_{bc}^2 - m_c^2)^2 (1 - \frac{\mu_\pi^2}{2m_{bc}^2}) \\
\simeq \quad & (5.7 \pm 2.1) \times 10^{-13}\ \text{GeV} \qquad (10) \\
\textit{axial} - \textit{vector} \qquad & \Gamma_1(\Xi_{\mathcal{X}} \to X_{cs}) \\
= \quad & \frac{G_F^2}{6\pi m_{bc}^2} |V_{cs}^* V_{cb}|^2 (C_5 - C_6)^2 |\psi_{bc}(0)|^2 \\
& (m_{bc}^2 - m_c^2)^2 (2 + \frac{m_c^2}{m_{bc}^2})(1 - \frac{\mu_\pi^2}{2m_{bc}^2}) \\
\simeq \quad & (9.0 \pm 3.4) \times 10^{-17}\ \text{GeV} \qquad (11)
\end{aligned}
$$

Numerical Analysis

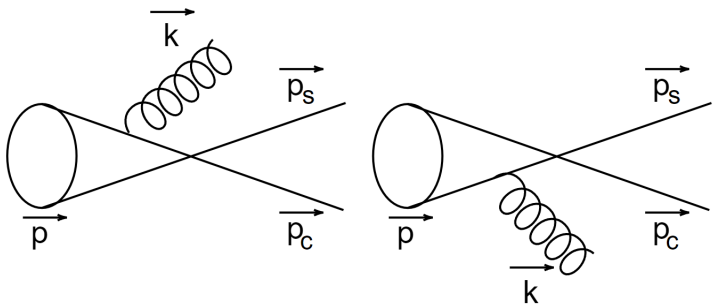

Figure: The next-to-leading order Feynman diagrams contributing to $\Xi_{\mathcal{X}} \to X_{cs}$ with a real gluon emitted from the diquark.

$$
\begin{aligned}
& \Gamma_2(\Xi_{\mathcal{X}} \to X_{cs}) \\
= \quad & \frac{\alpha_s G_F^2 \left|V_{cs}^* V_{cb}\right|^2}{36\pi^2 m_{bc}^4}[(C_1^2 + C_2^2)(x^2 - xy + y^2) + C_1 C_2 (x^2 - 4xy + y^2)]|\psi_{bc}(0)|^2 \\
& \int_0^{m_b^2} ds_{12} \left[\frac{1}{2}\left(-m_c^2 - s_{12} + m_{bc}^2\right)\sqrt{-2m_c^2\left(s_{12} + m_{bc}^2\right) + \left(m_{bc}^2 - s_{12}\right)^2 + m_c^4}\right] \\
\simeq \quad & (4.7 \pm 1.8) \times 10^{-16}\ \text{GeV} \qquad (12)
\end{aligned}
$$

where $x = m_{bc}/m_b, y = m_{bc}/m_c$.

$$\Gamma(\Xi_{\mathcal{X}} \to X_{cs}) \simeq (5.6 \pm 2.2) \times 10^{-16}\ \text{GeV}\ . \qquad (13)$$

Numerical Analysis

The inclusive $\Xi_{bc}^{+,0} \to X_{cs}$ decay width

$$\Gamma(\Xi_{bc}^{+,0} \to X_{cs}) = \sin^2\theta\, \Gamma(\Xi_{\mathcal{S}} \to X_{cs}) + \cos^2\theta\, \Gamma(\Xi_{\mathcal{X}} \to X_{cs})\,. \quad (14)$$

The total decay width

	$\Gamma_{\mathcal{S}}^{\rm an}$	$\Gamma_{\mathcal{X}}^{\rm an}$	$\Gamma_{\mathcal{SX}}^{\rm an}$	$\Gamma^{\rm other}$
Ξ_{bc}^0	1.87 ± 0.12	3.81 ± 0.30	2.20 ± 0.17	1.50 ± 0.31
Ξ_{bc}^+	0.61 ± 0.22	0.035 ± 0.008	0.020 ± 0.005	2.94 ± 0.73

Table: Various contributions to the total decay widths of $\Xi_{bc}^{+,0}$ in units of $10^{-12}\mathrm{GeV}$, including the $\{bc,bu\}$ and $\{bc,cd\}$ weak-annihilation contributions to Ξ_{bc}^+ and Ξ_{bc}^0, respectively.

$$\Gamma(\Xi_{bc}^{+,0}) = \sin^2\theta\, \Gamma_{\mathcal{S}}^{\rm an} + \cos^2\theta\, \Gamma_{\mathcal{X}}^{\rm an} + 2\sin\theta\cos\theta\cos\phi\Gamma_{\mathcal{SX}}^{\rm an} + \Gamma^{\rm other}$$

Numerical Analysis

The transition operators

$$\mathcal{T}_{bq}^{an} = \frac{G_F^2 \left|V_{cb}\right|^2}{2\pi m_{bu}^2} \left(m_{bu}^2 - m_c^2\right)^2 \left(C_1 - C_2\right)^2 \left(\bar{b}\gamma_\mu(1-\gamma_5)b\right)\left(\bar{q}\gamma^\mu(1-\gamma_5)q\right) \tag{15}$$

The hadronic matrix elements are estimated in the non-relativistic potential mode.

$$\begin{aligned}
\left\langle \Xi_{\mathcal{S}}^+ \middle| \left(\bar{Q}\gamma_\mu(1-\gamma_5)Q\right)\left(\bar{q}\gamma^\mu(1-\gamma_5)q\right) \middle| \Xi_{\mathcal{S}}^+ \right\rangle &= 2m_{bc}\left|\psi_{q,bc}(0)\right|^2, \\
\left\langle \Xi_{\mathcal{X}}^+ \middle| \left(\bar{Q}\gamma_\mu(1-\gamma_5)Q\right)\left(\bar{q}\gamma^\mu(1-\gamma_5)q\right) \middle| \Xi_{\mathcal{X}}^+ \right\rangle &= 6m_{bc}\left|\psi_{q,bc}(0)\right|^2, \\
\left\langle \Xi_{\mathcal{S}}^+ \middle| \left(\bar{Q}\gamma_\mu(1-\gamma_5)Q\right)\left(\bar{q}\gamma^\mu(1-\gamma_5)q\right) \middle| \Xi_{\mathcal{X}}^+ \right\rangle &= 2\sqrt{3}m_{bc}\left|\psi_{q,bc}(0)\right|^2 \quad (16)
\end{aligned}$$

PhysRevD.99.073006(2019), Hai-Yang Cheng

Numerical Analysis

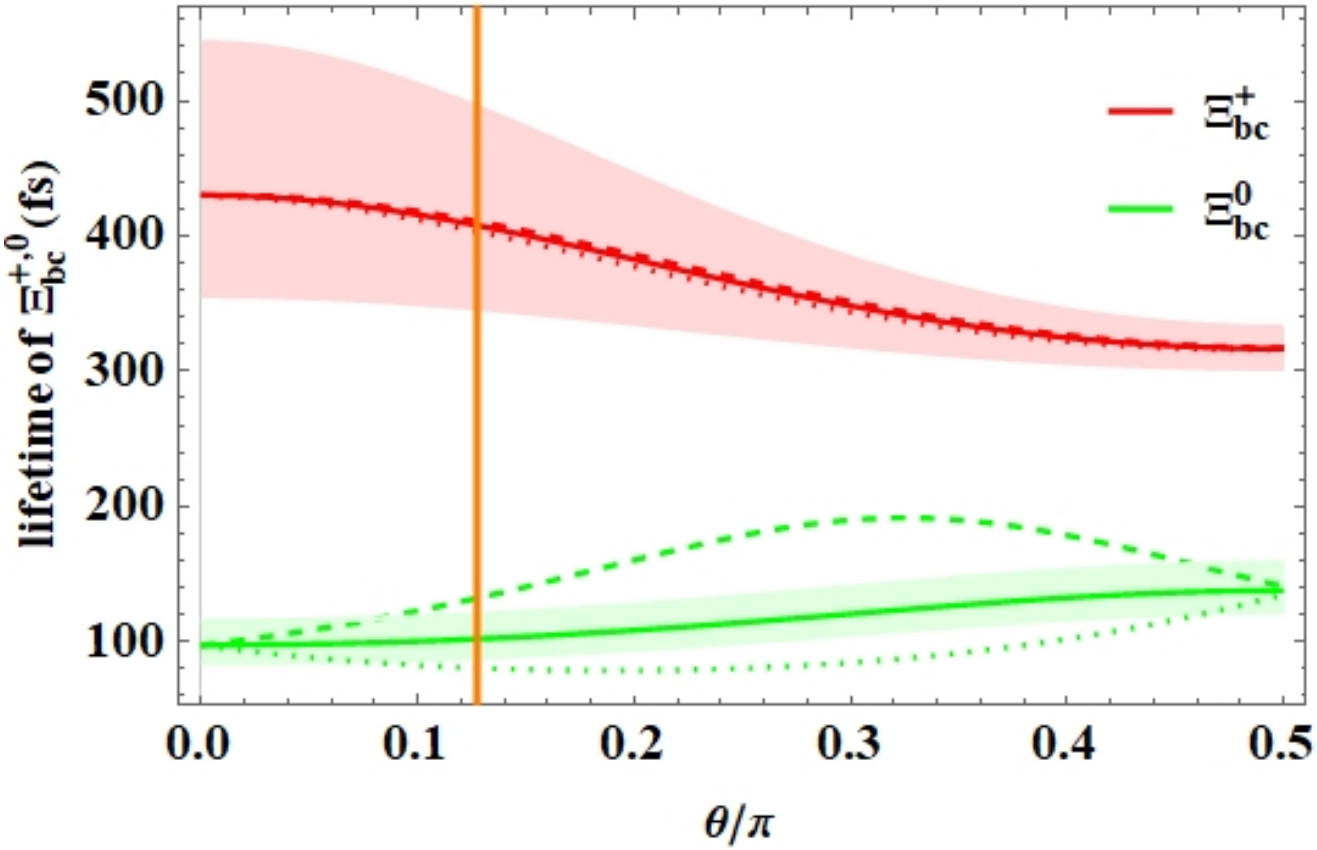

Figure: The lifetimes of Ξ_{bc}^{+0} as functions of the mixing angle θ. The solid, dotted and dashed curves correspond to the choices of the phase angle $cos\phi = 0$, $cos\phi = 1$, $cos\phi = -1$, respectively. The vertical line corresponds to the reference value $sin\theta = 0.39$ [arXiv:0803.3350]

Heavy Diquark Effective Theory

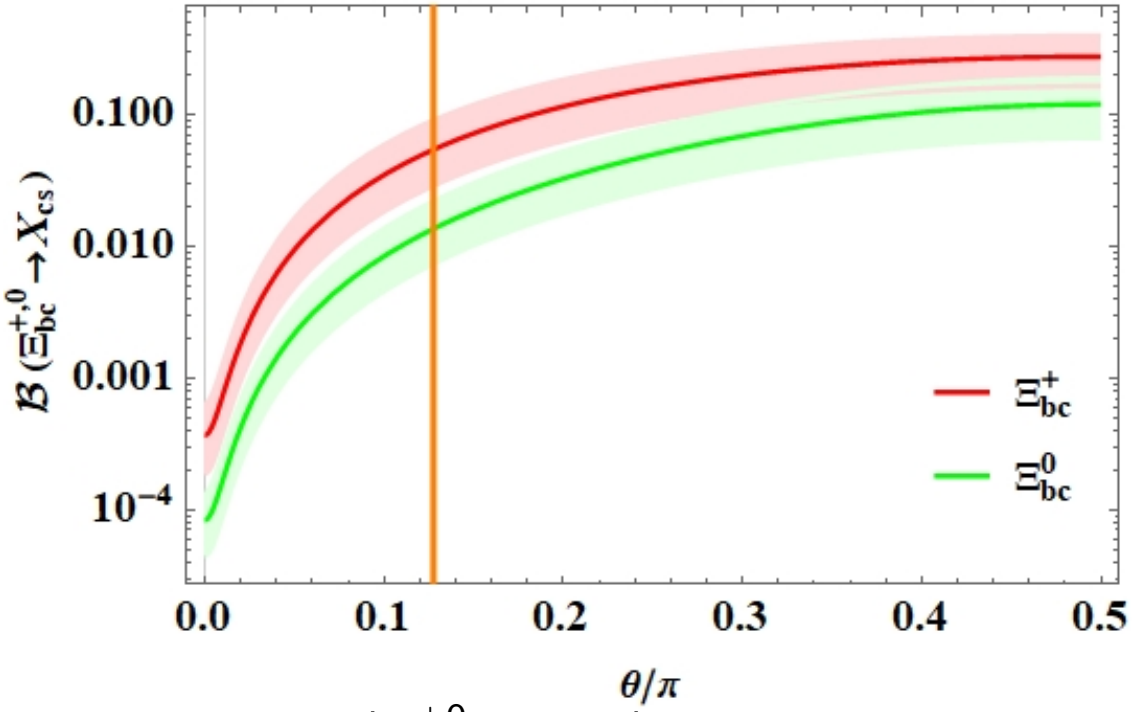

Figure: The branching ratios $B(\Xi_{bc}^{+0} \to X_{cs})$ as functions of the mixing angle θ.

$$\mathcal{B}(\Xi_{bc}^{0} \to X_{cs}) \simeq (1.4^{+0.9}_{-0.6}), \mathcal{B}(\Xi_{bc}^{+} \to X_{cs}) \simeq (5.4^{+3.8}_{-2.6}).$$

$\Xi_{bc}^{+} \to D^{(*)+}\Lambda$, $\Xi_{bc}^{+} \to \Lambda_c^{+}\bar{K}^{(*)0}$, $\Xi_{bc}^{+} \to D^{(*)+}K^{-}p$, and $\Xi_{bc}^{+} \to D^{0}\bar{K}^{(*)0}p$

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Conclusion

- We have calculated the inclusive $\Xi_{bc}^{+,0} \to X_{cs}$ decay widths making use of the local OPE technique in the framework of HDET.
- We have calculated the $\Xi_{bc}^{+,0}$ lifetime as the function of the mixing angle θ.
- We propose that the decay channels $\Xi_{bc}^{+} \to D^{(*)+}\Lambda$, $\Xi_{bc}^{+} \to \Lambda_c^{+}\bar{K}^{(*)0}$, $\Xi_{bc}^{+} \to D^{(*)+}K^{-}p$, and $\Xi_{bc}^{+} \to D^{0}\bar{K}^{(*)0}p$ should be used in Ξ_{bc} searches.

Thanks!