A Study on Neutrinoless Double Beta Decay and Neutron-Antineutron Oscillation

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Based on:

1. **Shao-Long Chen and Yu-Qi Xiao, *Neutrinoless Double Beta Decay in the Colored Zee-Babu Model*, [2205.13118].**
2. **Shao-Long Chen and Yu-Qi Xiao, *The Decomposition of Neutron-Antineutron Oscillation Operators*, [2211.02813].**

OUTLINE

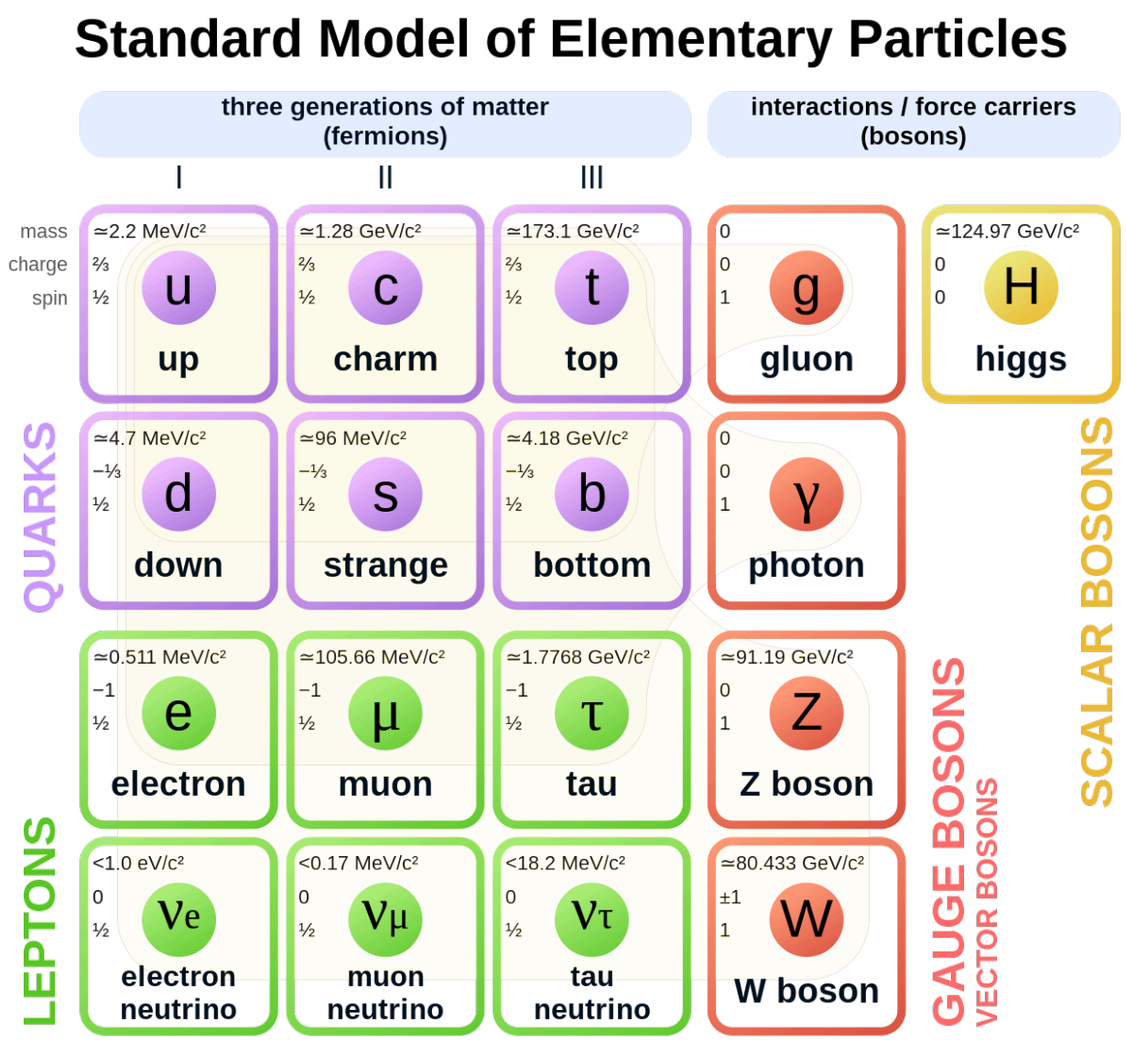
- **Brief Introduction**
- **Neutrinoless Double Beta Decay** ($|\Delta L| = 2$)
- **Neutron-Antineutron Oscillation** ($|\Delta B| = 2$)
- **Summary**

PART I

- **Brief Introduction**
- **Neutrinoless Double Beta Decay ($|\Delta L| = 2$)**
- **Neutron-Antineutron Oscillation ($|\Delta B| = 2$)**
- **Summary**

Some open questions in the Standard Model:

- ★ The origin of neutrino mass?
- ★ The origin of matter-antimatter asymmetry?
- The origin of Dark Matter?



We focus on the first two questions.

What is the origin of neutrino mass?

- Dirac neutrino: Higgs mechanism
- Majorana neutrino: seesaw mechanism

Dirac
Or Majorana ?

Neutrinoless Double Beta ($0\nu\beta\beta$) Decay

Lepton number violation $\Delta L = 2$

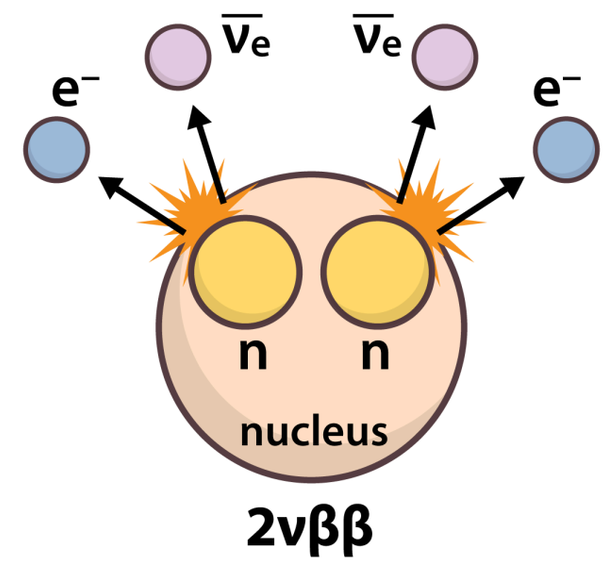
- For some nuclei, beta decay is prevented, while double beta decay is allowed.

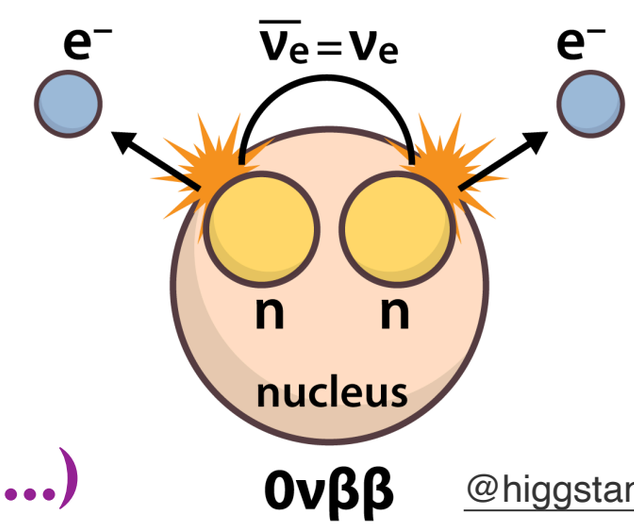
e.g. $^{76}\mathrm{Ge} \rightarrow ^{76}\mathrm{Se} + 2e^- + 2\bar{\nu}_e$.

- If neutrinos are Majorana particles, it is possible for neutrinoless double beta decay to occur.

no evidence for the decay

(KamLAND-Zen, GERDA, CUPID-1T, LEGEND-1000, ...)





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What is the origin of matter-antimatter asymmetry? (baryon asymmetry)

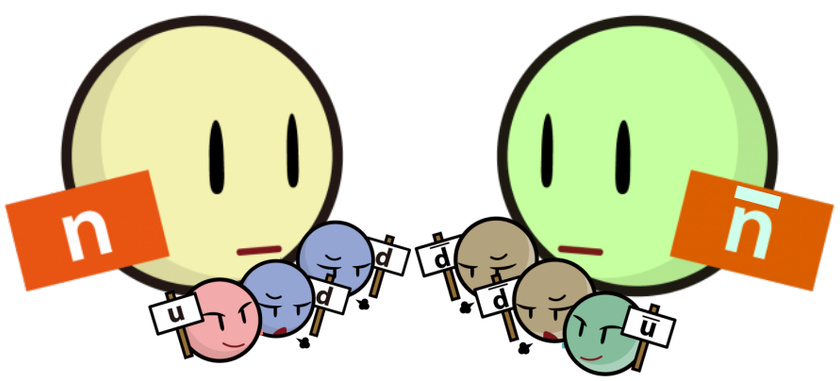
Three necessary conditions (1967, Andrei Sakharov)

- B violation (B: baryon number)
- CP violation
- Deviation from thermal equilibrium

Proton decay

Neutron-antineutron ($n - \bar{n}$) oscillation

Neutron-antineutron oscillation



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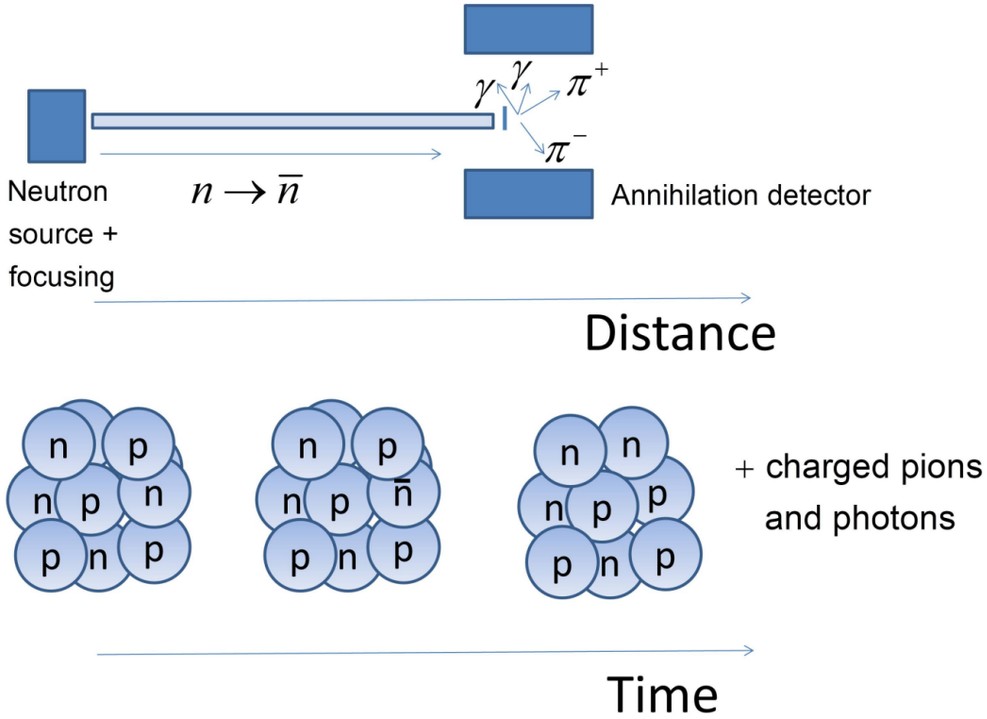
Baryon number violation
$\Delta B = -2$

Direct search: free neutrons (ILL, **NNBAR**)

Indirect search: neutrons bound inside nuclei (SK, **DUNE**, **HK**)



no evidence for the oscillation

2006.04907

PART II

- Brief Introduction
- **Neutrinoless Double Beta Decay ($|\Delta L| = 2$)**
- Neutron-Antineutron Oscillation ($|\Delta B| = 2$)
- Summary

The neutrinoless double beta decay in the colored Zee-Babu model

The Model & Constraints

- The colored Zee-Babu (cZB) model requires a leptoquark and a diquark to generate the neutrino mass.

- Three cases

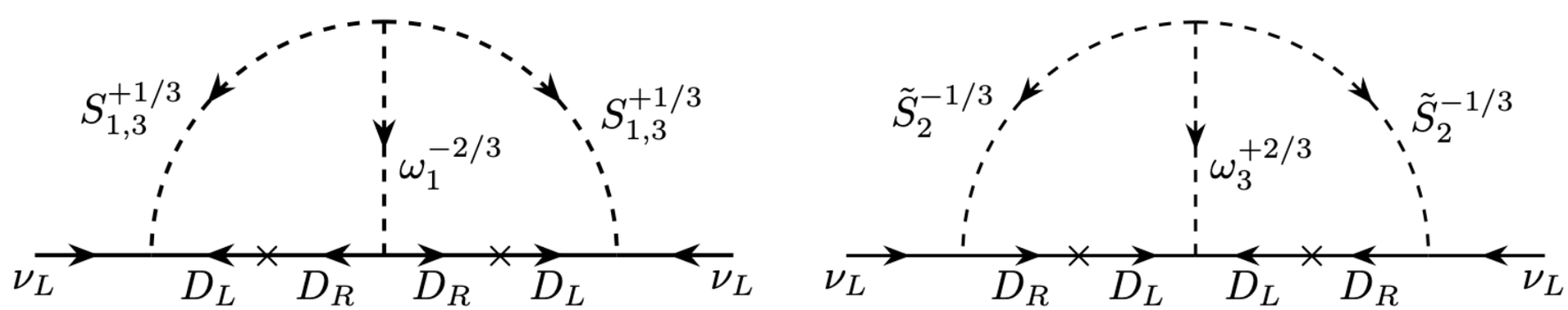
case 1: a singlet LQ $S_1 \sim (\bar{3}, 1, 1/3)$ and a singlet DQ $\omega_1 \sim (6, 1, -2/3)$,
case 2: a triplet LQ $S_3 \sim (\bar{3}, 3, 1/3)$ and a singlet DQ $\omega_1 \sim (6, 1, -2/3)$,
case 3: a doublet LQ $\tilde{S}_2 \sim (3, 2, 1/6)$ and a triplet DQ $\omega_3 \sim (6, 3, 1/3)$.

$$-\mathcal{L}_{Y1} \supset y_{1SR}^{ij}\overline{(U_R^{i\alpha})^c}E_R^j S_1^{\overline{\alpha}} + y_{1SL}^{ij}\overline{(Q_L^{i\alpha})^c}i\sigma^2 L_L^j S_1^{\overline{\alpha}} + z_{1\omega}^{ij}\overline{(D_R^{i\alpha})^c}D_R^{j\beta}\omega_1^{*\overline{\alpha}\overline{\beta}} + \text{h.c.},$$

$$-\mathcal{L}_{Y2} \supset y_{3S}^{ij}\overline{(Q_L^{i\alpha})^c}i\sigma^2(\sigma^k S_3^{k\overline{\alpha}})L_L^j + z_{1\omega}^{ij}\overline{(D_R^{i\alpha})^c}D_R^{j\beta}\omega_1^{*\overline{\alpha}\beta} + \text{h.c.},$$

$$-\mathcal{L}_{Y3} \supset y_{2S}^{ij}\overline{D_R^{i\alpha}}(\tilde{S}_2^{\alpha})^T i\sigma^2 L_L^j + z_{3\omega}^{ij}\overline{(Q_L^{i\alpha})^c}i\sigma^2(\sigma^k\omega_3^{*k\overline{\alpha}\overline{\beta}})^T Q_L^{j\beta} + \text{h.c.}$$

Neutrino mass



$$M_{\nu_a}^{kn} = 24\mu_a[y_{bS(L)}^T]^{kl} m_{D^l} z_{c\omega}^{lm} m_{D^m} y_{bS(L)}^{mn}\mathcal{I}_{lm}$$

Texture setup

$$y_{1SL} = V^T \begin{pmatrix} \# & 0 & 0 \\ 0 & 0 & \# \\ \# & \# & \# \end{pmatrix}, \quad y_{1SR} = \begin{pmatrix} \# & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \# & 0 \end{pmatrix}, \quad z_{1\omega} = \begin{pmatrix} \# & 0 & 0 \\ 0 & 0 & \# \\ 0 & \# & \# \end{pmatrix}.$$

#: short-range $0\nu\beta\beta$, #: $(g-2)_\mu$, #: standard $0\nu\beta\beta$,

#: provide enough independent parameters

Constraints from tree-level flavor violation processes
(four-fermion interaction)

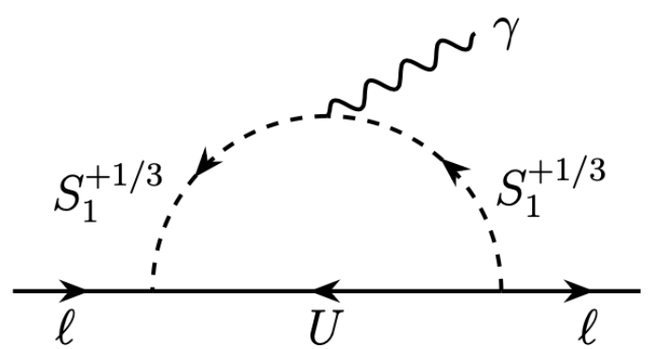
$$\begin{aligned}
\mathcal{L}_{\text{eff},1} = & \frac{1}{2M_{S_1}^2}\Big\{ (V^* y_{1SL})^{*ki} (V^* y_{1SL})^{nj} [\overline{E^i}\gamma_\mu P_L E^j][\overline{U^k}\gamma^\mu P_L U^n] \\
& + y_{1SL}^{*ki} y_{1SL}^{nj} [\overline{\nu^i}\gamma_\mu P_L \nu^j][\overline{D^k}\gamma^\mu P_L D^n] + y_{1SR}^{*ki} y_{1SR}^{nj} [\overline{E^i}\gamma_\mu P_R E^j][\overline{U^k}\gamma^\mu P_R U^n] \\
& - \Big[y_{1SL}^{*ki} (V^* y_{1SL})^{nj} [\overline{\nu^i}\gamma_\mu P_L E^j][\overline{D}^k \gamma^\mu P_L U^n] + \text{h.c.} \Big] \\
& + \Big[y_{1SR}^{*ki} y_{1SL}^{nj} [\overline{\nu^i} P_R E^j][\overline{D^k} P_R U^n] + \text{h.c.} \Big] \\
& + \Big[(V^* y_{1SL})^{*ki} y_{1SR}^{nj} [\overline{E^i} P_R E^j][\overline{U^k} P_R U^n] + \text{h.c.} \Big] \\
& - \frac{1}{4} \Big[y_{1SR}^{*ki} y_{1SL}^{nj} [\overline{\nu^i}\sigma_{\mu\nu} P_R E^j][\overline{D^k}\sigma^{\mu\nu} P_R U^n] + \text{h.c.} \Big] \\
& - \frac{1}{4} \Big[(V^* y_{1SL})^{*ki} y_{1SR}^{nj} [\overline{E^i}\sigma_{\mu\nu} P_R E^j][\overline{U^k}\sigma^{\mu\nu} P_R U^n] + \text{h.c.} \Big] \Big\}.
\end{aligned}$$

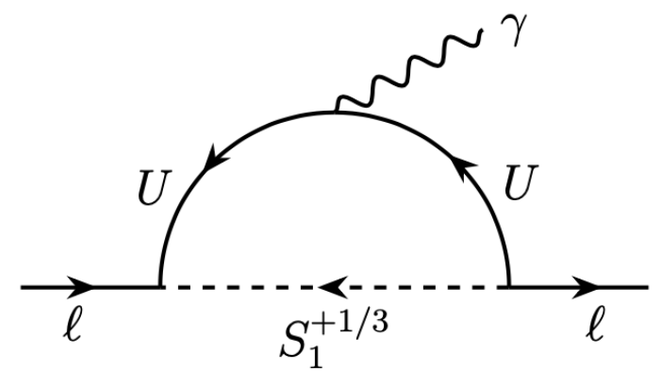
Constraints from $B_d^0 - \bar{B}_d^0$ mixing

$$|z^{11}_{1(3)\omega} z^{33}_{1(3)\omega}| < 4.6(2.3) \times 10^{-5} \times \left(\frac{M_{\omega 1(3)}}{\text{TeV}}\right)^2$$

Constraints from $(g-2)_\ell$

$$\Delta a_\mu(S_1) \simeq \frac{3m_\mu^2}{8\pi^2 M_{S_1}^2} \frac{m_t}{m_\mu} \text{Re}[y^{32}_{1SR} y'^{*32}_{1SL}] \left[\frac{1}{3} f_1\left(\frac{m_t^2}{m_{S_1}^2}\right) + \frac{2}{3} f_2\left(\frac{m_t^2}{m_{S_1}^2}\right)\right]$$





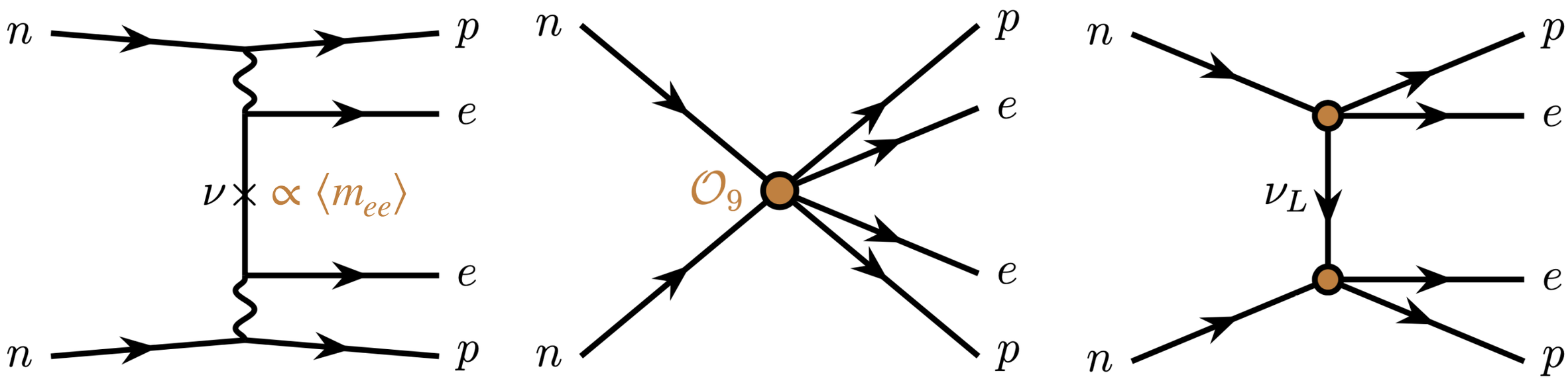
Parameter regions

case 1 : $|y'^{11}_{1SL}| < 0.12$, $|y'^{31,32}_{1SL}| < 0.3$, $|y'^{33}_{1SL}| < 0.4$, $|y'^{23}_{1SL}| < 0.005$,
$|y^{11}_{1SR}| < 1.2$, $\text{Re}[y^{32}_{1SR} y'^{*32}_{1SL}] \sim 0.0848$, $|z^{11}_{1\omega}| < 1.5$, $|z^{33}_{1\omega}| < 0.001$,

case 2 : $|y'^{11}_{3S}| < 0.12$, $|y'^{31}_{3S}| < 0.07$, $|y'^{32}_{3S}| < 0.008$, $|y'^{33}_{3S}| < 0.4$,
$|y'^{23}_{3S}| < 0.005$, $|z^{11}_{1\omega}| < 1.5$, $|z^{33}_{1\omega}| < 0.002$,

case 3 : $|y^{11}_{2S}| < 1.5$, $|y^{31}_{2S}| < 0.01$, $|y^{32}_{2S}| < 0.01$, $|y^{33}_{2S}| < 0.3$,
$|y^{23}_{2S}| < 0.3$, $|z^{11}_{3\omega}| < 0.01$, $|z^{33}_{3\omega}| < 0.15$,

The Effective Field Theory Approach



standard neutrino exchange | short-range | long-range

Focus on the standard neutrino exchange and short-range contribution

$$\mathcal{L}_{SR} = \frac{G_F^2 V_{ud}^2}{2m_p} \sum_{X,Y,Z} \Big(\epsilon_1^\chi J_X J_Y j_Z + \epsilon_2^\chi J_X^{\mu\nu} J_{Y,\mu\nu} j_Z + \epsilon_3^\chi J_X^\mu J_{Y,\mu} j_Z + \epsilon_4^\chi J_X^\mu J_{Y,\mu\nu} j^\nu + \epsilon_5^\chi J_X^\mu J_Y j_\mu \Big) + \text{h.c.}$$

$$= \frac{G_F^2 V_{ud}^2}{2m_p} \sum_{\chi,i} \epsilon_i^\chi \mathcal{O}_{i,\chi}^{0\nu\beta\beta} + \text{h.c.}$$

H. Pas, M. Hirsch, *et al.*
Hep-ph/0008182

$$J_{R/L} = \bar{u}(1 \pm \gamma_5) d\,,$$
$$J_{R,L}^\mu = \bar{u}\gamma^\mu (1 \pm \gamma_5) d\,,$$
$$J_{R/L}^{\mu\nu} = \bar{u}\sigma^{\mu\nu}(1 \pm \gamma_5) d\,,$$
$$j_{R/L} = \bar{e}(1 \mp \gamma_5) e^c,$$
$$j^\mu = \bar{e}\gamma^\mu \gamma_5 e^c\,.$$

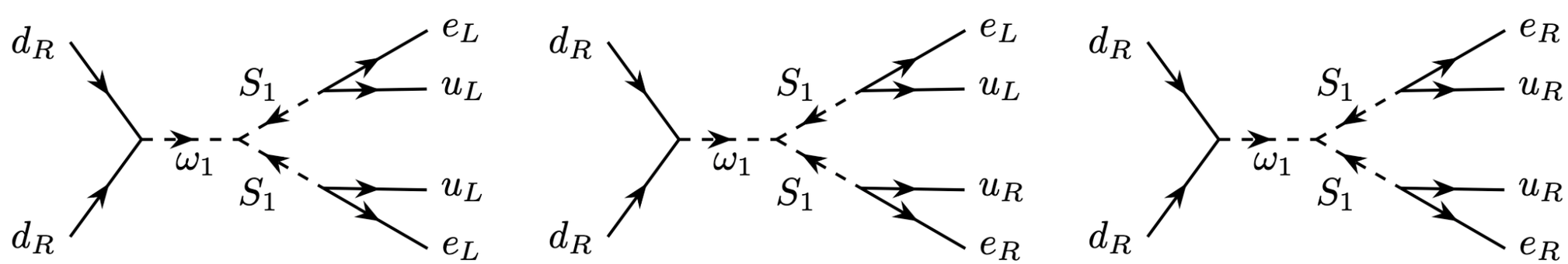
- **Inverse half-life**
  (involving the short-range mechanism and standard neutrino exchange)

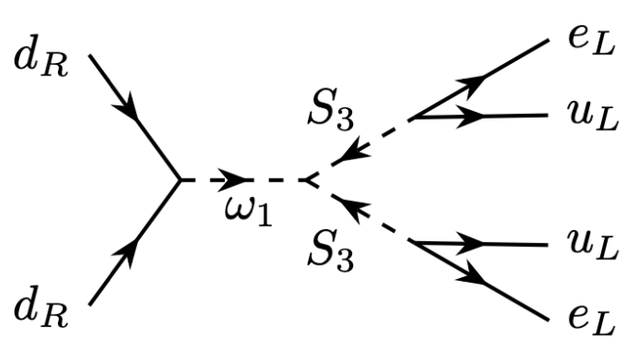
F. F. Deppisch, *et al.* 2009.10119

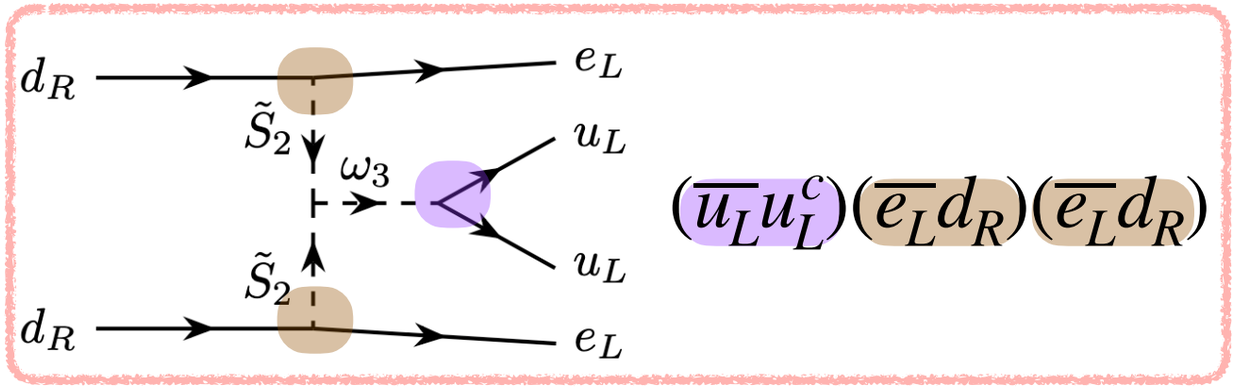
$$\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} = G_{11+}^{(0)}\left|\sum_{i=1}^{3}\epsilon_i^{XYL}\mathcal{M}_i^{XY} + \epsilon_\nu\mathcal{M}_\nu\right|^2 + G_{11+}^{(0)}\left|\sum_{i=1}^{3}\epsilon_i^{XYR}\mathcal{M}_i^{XY}\right|^2 + G_{66}^{(0)}\left|\sum_{i=4}^{5}\epsilon_i^{XY}\mathcal{M}_i^{XY}\right|^2$$
$$+ G_{16}^{(0)} \times 2\mathrm{Re}\left[\left(\sum_{i=1}^{3}\epsilon_i^{XYL}\mathcal{M}_i^{XY} - \sum_{i=1}^{3}\epsilon_i^{XYR}\mathcal{M}_i^{XY} + \epsilon_\nu\mathcal{M}_\nu\right)\left(\sum_{i=4}^{5}\epsilon_i^{XY}\mathcal{M}_i^{XY}\right)^*\right]$$
$$+ G_{11-}^{(0)} \times 2\mathrm{Re}\left[\left(\sum_{i=1}^{3}\epsilon_i^{XYL}\mathcal{M}_i^{XY} + \epsilon_\nu\mathcal{M}_\nu\right)\left(\sum_{i=1}^{3}\epsilon_i^{XYR}\mathcal{M}_i^{XY}\right)^*\right],$$

nuclear matrix elements
phase space factors

- **Feynman diagrams**







- **Operators**

$$\text{case 1}: \quad (\overline{u_L e_L})(\overline{u_L e_L})(d_R d_R) \to \frac{1}{48}\mathcal{O}_1^{RRL} - \frac{1}{192}\mathcal{O}_2^{RRL},$$
$$(\overline{u_L e_L})(\overline{u_R e_R})(d_R d_R) \to \frac{1}{96i}\mathcal{O}_4^{RR} - \frac{1}{48}\mathcal{O}_5^{RR},$$
$$(\overline{u_R e_R})(\overline{u_R e_R})(d_R d_R) \to -\frac{1}{48}\mathcal{O}_3^{RRR},$$
$$\text{case 2}: \quad (\overline{u_L e_L})(\overline{u_L e_L})(d_R d_R) \to \frac{1}{48}\mathcal{O}_1^{RRL} - \frac{1}{192}\mathcal{O}_2^{RRL},$$
$$\text{case 3}: \quad (\overline{u_L u_L})(d_R \overline{e_L})(d_R \overline{e_L}) \to \frac{1}{48}\mathcal{O}_1^{RRL} - \frac{1}{192}\mathcal{O}_2^{RRL}.$$

- **Coefficients**

$$\text{case 1}: \quad \epsilon_1^{RRL} = +\frac{1}{48}\frac{2m_p}{G_F^2 V_{ud}^2}\frac{4(y_{1SL}'^{*11})^2 z_{1\omega}^{11}\mu_1}{M_{S_1}^4 M_{\omega_1}^2}, \quad \epsilon_2^{RRL} = -\frac{1}{4}\epsilon_1^{RRL},$$
$$\epsilon_3^{RRR} = -\frac{1}{48}\frac{2m_p}{G_F^2 V_{ud}^2}\frac{4(y_{1SR}^{*11})^2 z_{1\omega}^{11}\mu_1}{M_{S_1}^4 M_{\omega_1}^2},$$
$$\epsilon_4^{RR} = +\frac{1}{96i}\frac{2m_p}{G_F^2 V_{ud}^2}\frac{4 y_{1SR}^{*11} y_{1SL}'^{*11} z_{1\omega}^{11}\mu_1}{M_{S_1}^4 M_{\omega_1}^2}, \quad \epsilon_5^{RR} = -2i\epsilon_4^{RR},$$
$$\text{case 2}: \quad \epsilon_1^{RRL} = +\frac{1}{48}\frac{2m_p}{G_F^2 V_{ud}^2}\frac{4(y_{3S}'^{*11})^2 z_{1\omega}^{11}\mu_2}{M_{S_3}^4 M_{\omega_1}^2}, \quad \epsilon_2^{RRL} = -\frac{1}{4}\epsilon_1^{RRL},$$
$$\text{case 3}: \quad \epsilon_1^{RRL} = -\frac{1}{48}\frac{2m_p}{G_F^2 V_{ud}^2}\frac{4(y_{2S}^{*11})^2 (V^* z_{3\omega} V^\dagger)^{11}\mu_3}{M_{S_2}^4 M_{\omega_3}^2}, \quad \epsilon_2^{RRL} = -\frac{1}{4}\epsilon_1^{RRL}.$$

Numerical Results

- **RGE μ-evolution matrix elements**

M. Gonz´alez, M. Hirsch

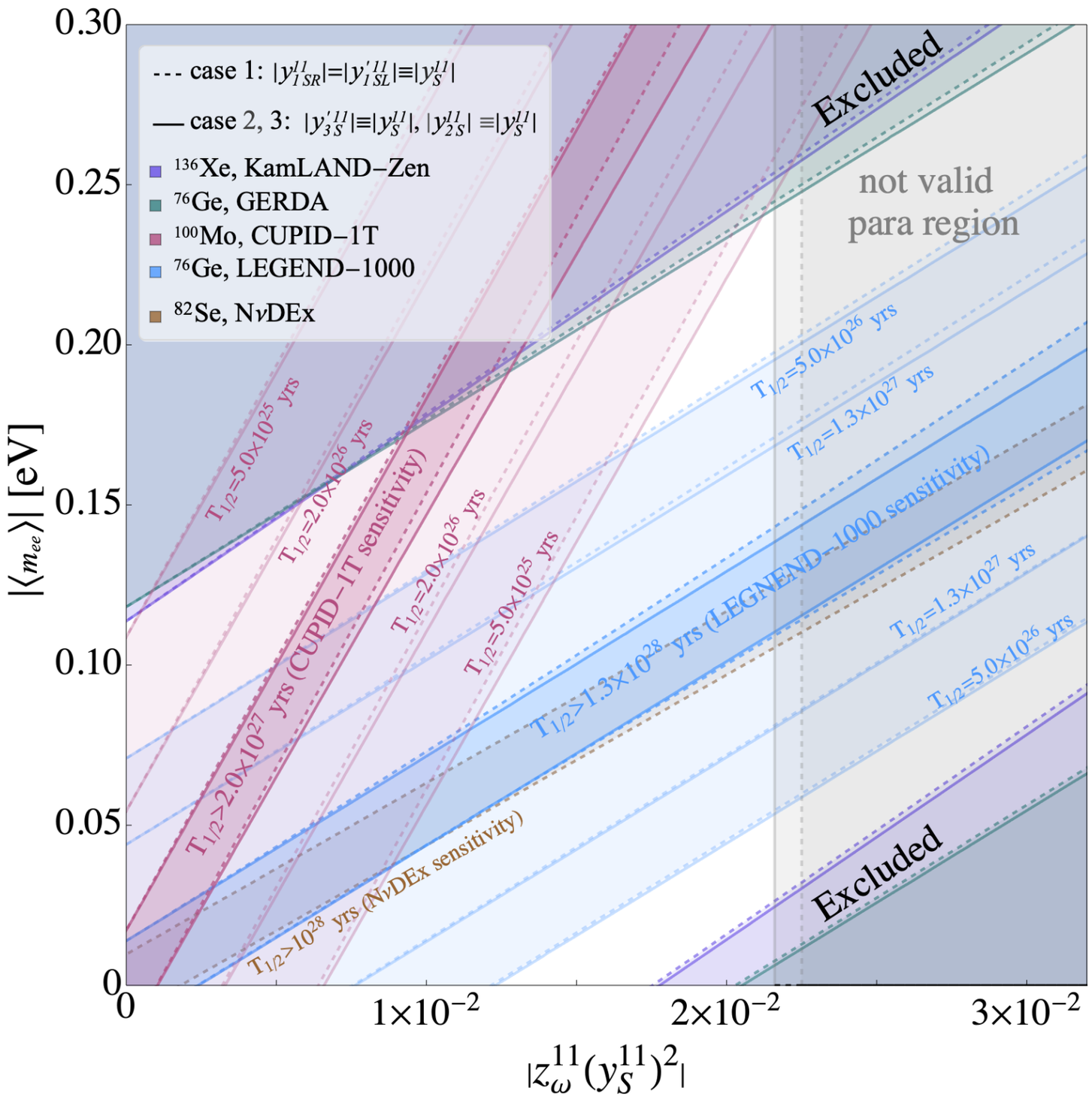
1511.03945

$$\hat{U}_{(12)}^{XX} = \begin{pmatrix} 2.39 & 0.02 \\ -3.83 & 0.35 \end{pmatrix},$$
$$\hat{U}_{(3)}^{XX} = 0.70,$$
$$\hat{U}_{(45)}^{XX} = \begin{pmatrix} 0.35 & -0.96i \\ -0.06i & 2.39 \end{pmatrix},$$

$$\mathcal{M}_1^{XX} \to \beta_1^{XX} = 2.39\mathcal{M}_1^{XX} - 3.83\mathcal{M}_2^{XX},$$
$$\mathcal{M}_2^{XX} \to \beta_2^{XX} = 0.02\mathcal{M}_1^{XX} + 0.35\mathcal{M}_2^{XX},$$
$$\mathcal{M}_3^{XX} \to \beta_3^{XX} = 0.70\mathcal{M}_3^{XX},$$
$$\mathcal{M}_4^{XX} \to \beta_4^{XX} = 0.35\mathcal{M}_4^{XX} - 0.06i\mathcal{M}_5^{XX},$$
$$\mathcal{M}_5^{XX} \to \beta_5^{XX} = -0.96i\mathcal{M}_4^{XX} + 2.39\mathcal{M}_5^{XX}.$$

Neutrinoless Double Beta Decay

$$(y_S^{*11})^2 z_\omega^{11} \simeq 300 \times \frac{\mathcal{M}_\nu}{\beta_1 - \beta_2/4} \frac{\langle m_{ee}\rangle}{\text{eV}} \times \left(\frac{1.5\ \text{TeV}}{\mu}\right)\left(\frac{M_S}{1.5\ \text{TeV}}\right)^4 \left(\frac{M_\omega}{8\ \text{TeV}}\right)^2$$



The neutrinoless double beta decay **could be hidden**.

$$\frac{\mathcal{M}_\nu}{\beta_1 - \beta_2/4} \begin{cases} \text{similar ratio value in } {}^{76}\text{Ge and } {}^{136}\text{Xe} \\ \text{different in } {}^{100}\text{Mo} \end{cases}$$

If there is no signal, the **survival region** will be reduced to the **overlap** area.

PART III

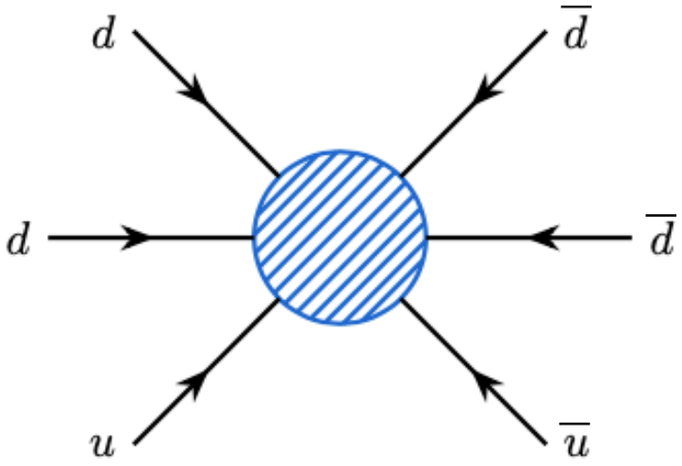
- Brief Introduction
- Neutrinoless Double Beta Decay ($|\Delta L| = 2$)
- Neutron-Antineutron Oscillation ($|\Delta B| = 2$)
- Summary

Systematic decomposition of $d = 9$ $n - \bar{n}$ oscillation operators

Operators & Transition rate

- The basis for the operators

$$\mathcal{O}^1_{\chi_1\chi_2\chi_3} = [\overline{u_i^c} P_{\chi_1} u_j][\overline{d_k^c} P_{\chi_2} d_l][\overline{d_m^c} P_{\chi_3} d_n] T^{SSS},$$
$$\mathcal{O}^2_{\chi_1\chi_2\chi_3} = [\overline{u_i^c} P_{\chi_1} d_j][\overline{u_k^c} P_{\chi_2} d_l][\overline{d_m^c} P_{\chi_3} d_n] T^{SSS},$$
$$\mathcal{O}^3_{\chi_1\chi_2\chi_3} = [\overline{u_i^c} P_{\chi_1} d_j][\overline{u_k^c} P_{\chi_2} d_l][\overline{d_m^c} P_{\chi_3} d_n] T^{AAS}.$$



and the independent effective operators are defined as

$$\mathcal{O}_1 = -4\mathcal{O}^3_{RRR}\,, \quad \mathcal{O}_2 = -4\mathcal{O}^3_{LRR}\,, \quad \mathcal{O}_3 = -4\mathcal{O}^3_{LLR}\,,$$
$$\mathcal{O}_4 = -\frac{4}{5}\mathcal{O}^1_{RRR} - \frac{16}{5}\mathcal{O}^2_{RRR}\,, \quad \mathcal{O}_5 = \mathcal{O}^1_{RLL}\,,$$
$$\mathcal{O}_6 = -4\mathcal{O}^2_{RLL}\,, \quad \mathcal{O}_7 = -\frac{4}{3}\mathcal{O}^1_{LLR} - \frac{8}{3}\mathcal{O}^2_{LLR}\,.$$

M. I. Buchoff and M. Wagman
1506.00647

- Effective Lagrangian and the transition rate

$$\mathcal{L}_{\text{eff}} = \sum_{i=1,2,3,5} \left(\mathcal{C}_i(\mu)\mathcal{O}_i(\mu) + \mathcal{C}_i^P(\mu)\mathcal{O}_i^P(\mu)\right),$$

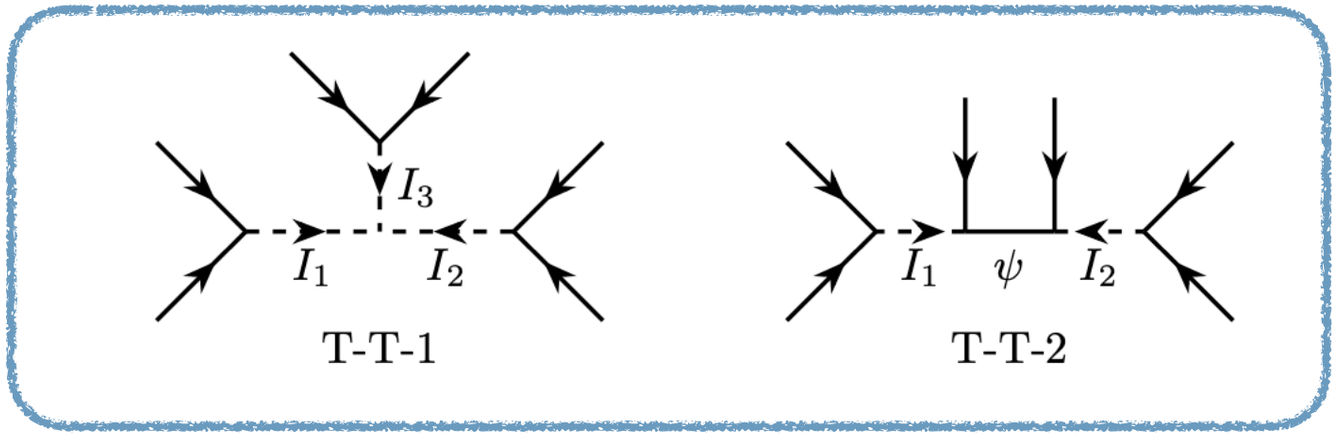
$$\tau^{-1}_{n-\bar{n}} = \langle\bar{n}|\mathcal{L}_{\text{eff}}\,|n\rangle = \left|\sum_{i=1,2,3,5} \left(\mathcal{C}_i(\mu)\mathcal{M}_i(\mu) + \mathcal{C}_i^P(\mu)\mathcal{M}_i^P(\mu)\right)\right|.$$

Wilson coefficients at the NP scale

Transition matrix elements at the NP scale

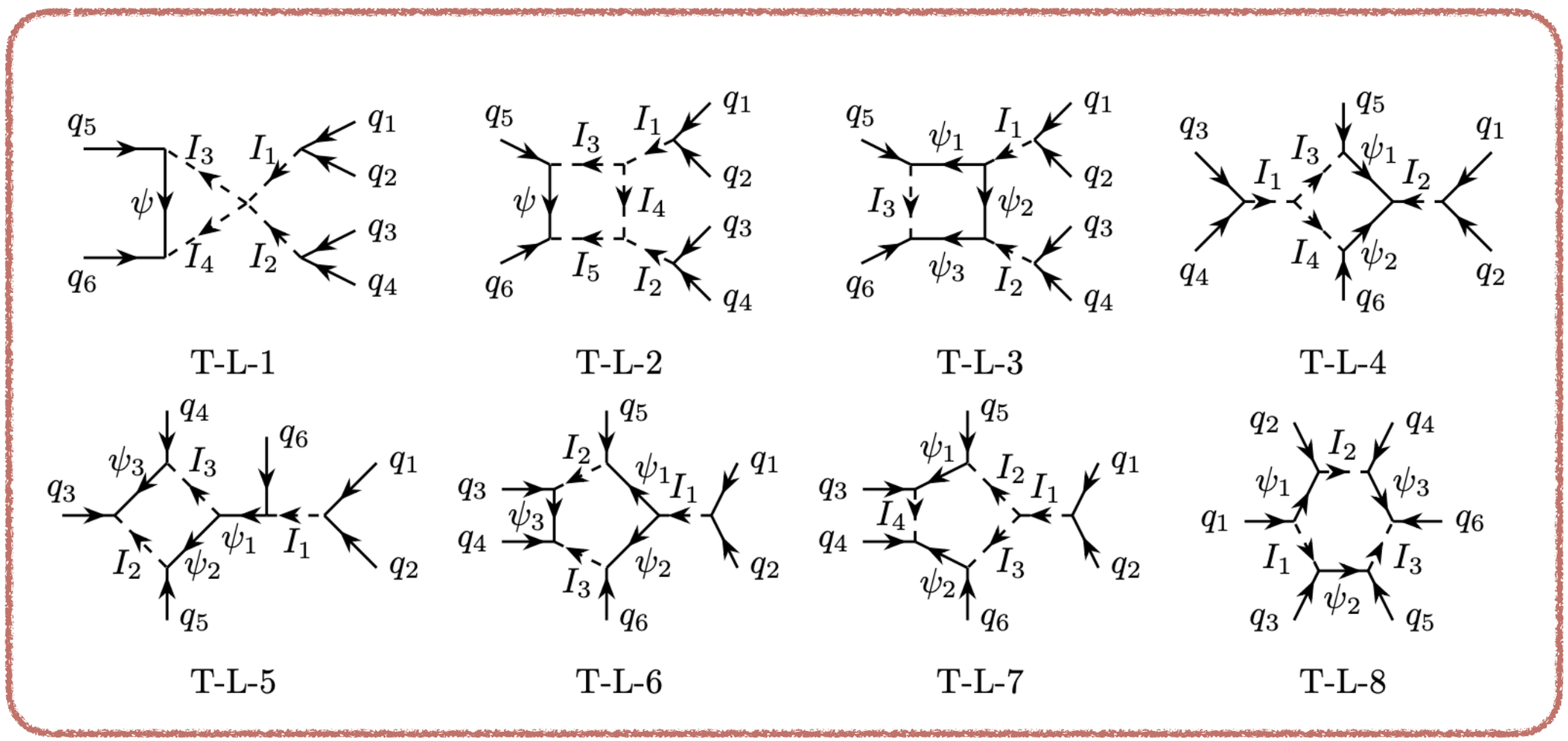
Generation of the topologies and diagrams

Tree level



consider the internal particles to be scalars or fermions

One-loop level



Neutron-Antineutron oscillation

Classification of the one-loop diagrams

- class 1 (2+2+1+1): T-L-1, T-L-2, T-L-3, T-L-4;
- class 2 (2+1+1+1+1): T-L-5, T-L-6, T-L-7;
- class 3 (1+1+1+1+1+1): T-L-8.

Assignment of chiral quarks

- Three possible cases
  (total hypercharge of the external legs is zero)

$$
\begin{aligned}
&\text{(i) } 4d_R\,,\ 2u_R\,, && d_R d_R u_R \leftrightarrow \overline{d_R}\,\overline{d_R}\,\overline{u_R}\,, \\
&\text{(ii) } 2d_R\,,\ 2d_L\,,\ 2u_L\,, && d_R d_L u_L \leftrightarrow \overline{d_R}\,\overline{d_L}\,\overline{u_L}\,, \quad d_L d_L u_L \leftrightarrow \overline{d_R}\,\overline{d_R}\,\overline{u_L}\,, \\
&\text{(iii) } 3d_R\,,\ 1d_L\,,\ 1u_R\,,\ 1u_L\,, && d_L d_R u_L \leftrightarrow \overline{d_R}\,\overline{d_R}\,\overline{u_R}\,, \quad d_R d_L u_R \leftrightarrow \overline{d_R}\,\overline{d_R}\,\overline{u_L}\,.
\end{aligned}
$$

- Find out the combination of the external legs

For case (iii)

$$
\begin{aligned}
\text{T-T-1} &: (d_R d_R)(d_R u_R)(d_L u_L)\,, \\
\text{T-T-2, class 1} &: (d_R d_R)(d_R u_R)(d_L)(u_L)\,, \quad (d_R d_R)(d_R)(u_R)(d_L u_L)\,, \\
&\quad (d_R)(d_R)(d_R u_R)(d_L u_L)\,, \\
\text{class 2} &: (d_R)(d_R)(d_R)(u_R)(d_L u_L)\,, \quad (d_R)(d_R)(d_R u_R)(d_L)(u_L)\,, \\
&\quad (d_R d_R)(d_R)(u_R)(d_L)(u_L)\,, \\
\text{class 3} &: (d_R)(d_R)(d_R)(u_R)(d_L)(u_L)\,.
\end{aligned}
$$

label the quark current as $(\overline{q^c} q^{(\prime)}) = (q q^{(\prime)})$

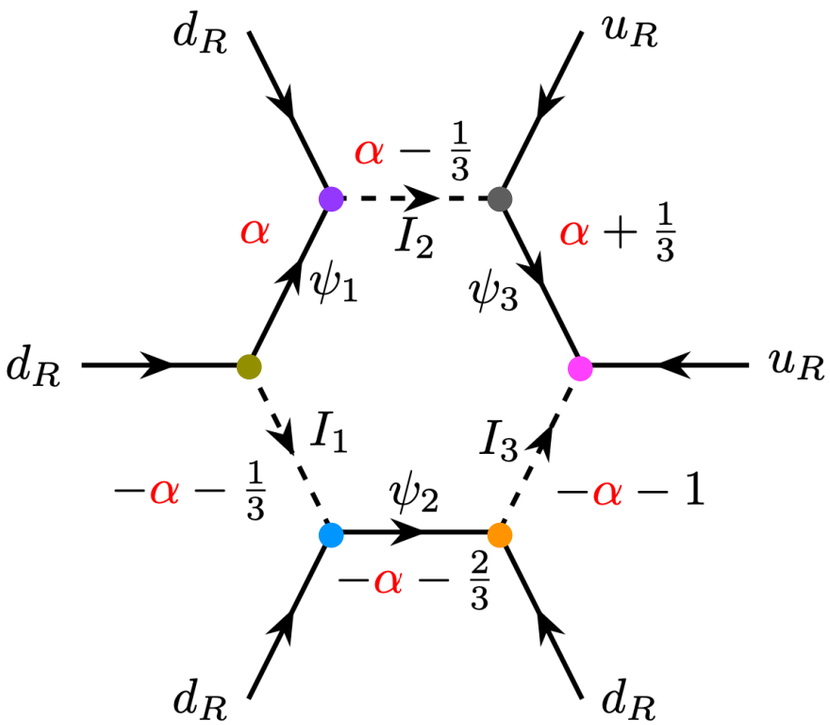
Decomposition of the operators

- $SU(3)_C$ $\quad 3 \otimes 3 = \overline{3} \oplus 6\,,\ 3 \otimes \overline{3} = 1 \oplus 8\,,\ 3 \otimes 6 \supset 8\,,\ 3 \otimes \overline{6} \supset \overline{3}\,,\ 3 \otimes 8 \supset 3 \oplus \overline{6}\,,$
  $6 \otimes 6 \supset \overline{6}\,,\ 6 \otimes \overline{6} \supset 1 \oplus 8\,,\ 6 \otimes 8 \supset \overline{3} \oplus 6\,,\ 8 \otimes 8 \supset 1 \oplus 8 \oplus 8\,.$

- $SU(2)_L$ $\quad 2 \otimes 2 = 1 \oplus 3\,,\ 2 \otimes 3 \supset 2\,,\ 3 \otimes 3 \supset 1 \oplus 3\,.$

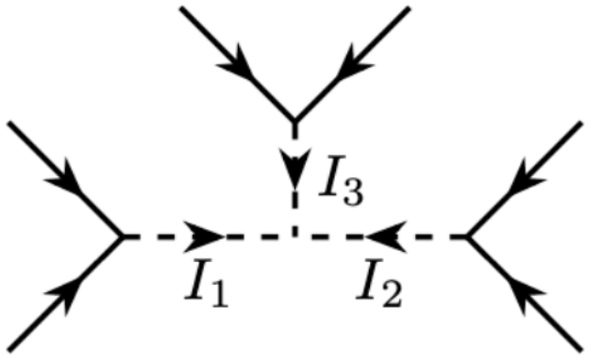
- $U(1)_Y$ $\quad$ The hypercharge Y is conserved at every vertex.

$$Y(d_R) - Y(\psi_1) - Y(I_1) = 0$$
$$Y(\psi_1) + Y(d_R) - Y(I_2) = 0$$
$$Y(I_1) + Y(d_R) - Y(\psi_2) = 0$$
$$Y(\psi_2) + Y(d_R) - Y(I_3) = 0$$
$$Y(I_2) + Y(u_R) - Y(\psi_3) = 0$$
$$Y(\psi_3) + Y(u_R) + Y(I_3) = 0$$



Lists of the decomposition

- **T-T-1**

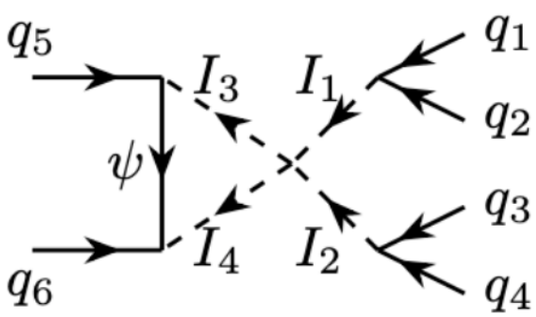


T-T-1

Operator	S_1	S_2	S_3
$(d_R d_R)(d_R d_R)(u_R u_R)$	$(6,1,-\frac{2}{3})$	$(6,1,-\frac{2}{3})$	$(6,1,\frac{4}{3})$
$(d_R u_R)(d_R u_R)(d_R d_R)$	$(\bar{3},1,\frac{1}{3})$ $(6,1,\frac{1}{3})$	$(\bar{3},1,\frac{1}{3})$ $(6,1,\frac{1}{3})$	$(6,1,-\frac{2}{3})$ $(6,1,-\frac{2}{3})$
$(d_R d_R)(d_L d_L)(u_L u_L)$	$(6,1,-\frac{2}{3})$	$(6,3,\frac{1}{3})$	$(6,3,\frac{1}{3})$
$(d_L u_L)(d_L u_L)(d_R d_R)$	$(\bar{3},1,\frac{1}{3})$ $(6,1,\frac{1}{3})$ $(\bar{3},3,\frac{1}{3})$ $(6,3,\frac{1}{3})$	$(\bar{3},1,\frac{1}{3})$ $(6,1,\frac{1}{3})$ $(\bar{3},3,\frac{1}{3})$ $(6,3,\frac{1}{3})$	$(6,1,-\frac{2}{3})$ $(6,1,-\frac{2}{3})$ $(6,1,-\frac{2}{3})$ $(6,1,-\frac{2}{3})$
$(d_L u_L)(d_R u_R)(d_R d_R)$	$(\bar{3},1,\frac{1}{3})$ $(6,1,\frac{1}{3})$	$(\bar{3},1,\frac{1}{3})$ $(6,1,\frac{1}{3})$	$(6,1,-\frac{2}{3})$ $(6,1,-\frac{2}{3})$

Neutron-Antineutron oscillation

- **T-L-1**



T-L-1

$SU(3)_C$				
I_1	I_2	I_3	I_4	ψ
$\bar{3}$	$\bar{3}$	3	8	$\bar{3}$
		3	8	6
		$\bar{3}$	$\bar{3}$	1
		$\bar{3}$	$\bar{3}$	8
		$\bar{3}$	6	8
		6	$\bar{3}$	8
		6	6	8
		$\bar{6}$	8	$\bar{3}$
		8	3	3
		8	3	$\bar{6}$
		8	$\bar{6}$	3
6	6	3	8	$\bar{3}$
		3	8	6
		$\bar{3}$	6	8
		6	$\bar{3}$	8
		6	6	8
		$\bar{6}$	8	$\bar{3}$
		8	3	3
		8	3	$\bar{6}$
		8	$\bar{6}$	3

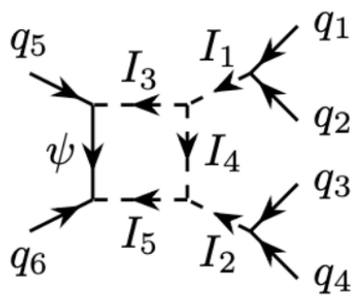
$SU(3)_C$				
I_1	I_2	I_3	I_4	ψ
$\bar{3}$	6	3	8	$\bar{3}$
		3	8	6
		$\bar{3}$	$\bar{3}$	1
		$\bar{3}$	$\bar{3}$	8
		$\bar{3}$	6	8
		6	$\bar{3}$	8
		6	6	8
		$\bar{6}$	8	$\bar{3}$
		8	3	3
		8	3	$\bar{6}$
		8	$\bar{6}$	3
6	$\bar{3}$	3	8	$\bar{3}$
		3	8	6
		$\bar{3}$	$\bar{3}$	1
		$\bar{3}$	$\bar{3}$	8
		$\bar{3}$	6	8
		6	$\bar{3}$	8
		6	6	8
		$\bar{6}$	8	$\bar{3}$
		8	3	3
		8	3	$\bar{6}$
		8	$\bar{6}$	3

$SU(2)_L$										
q_1	q_2	q_3	q_4	q_5	q_6	I_1	I_2	I_3	I_4	ψ
1	1	1	1	1	1	1	1	1	1	1
						1	1	2	2	2
						1	1	3	3	3
1	1	1	1	2	2	1	1	1	1	2
						1	1	2	2	1
						1	1	2	2	3
						1	1	3	3	2
1	1	2	2	1	1	1	1	1	1	1
						1	1	2	2	2
						1	1	3	3	3
						1	3	2	2	2
						1	3	3	3	3
1	1	2	2	2	2	1	1	1	1	2
						1	1	2	2	1
						1	1	2	2	3
						1	1	3	3	2
						1	3	1	3	2
						1	3	3	1	2
						1	3	2	2	1
						1	3	2	2	3
						1	3	3	3	2
2	2	2	2	1	1	1	1	1	1	1
						1	1	2	2	2
						1	1	3	3	3
						1	3	2	2	2
						1	3	3	3	3
						3	3	1	1	1
						3	3	2	2	2
						3	3	3	3	3
						3	3	1	3	3
						3	3	3	1	3

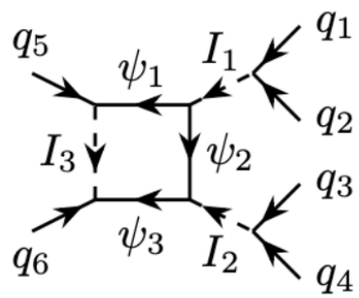
Particles						$U(1)_Y$				
q_1	q_2	q_3	q_4	q_5	q_6	I_1	I_2	I_3	I_4	ψ
d_R	d_R	d_R	d_R	u_R	u_R	$-\frac{2}{3}$	$-\frac{2}{3}$	α	$-\alpha-\frac{4}{3}$	$\alpha+\frac{2}{3}$
d_R	d_R	u_R	u_R	d_R	d_R	$-\frac{2}{3}$	$+\frac{4}{3}$	α	$-\alpha+\frac{2}{3}$	$\alpha-\frac{1}{3}$
d_R	d_R	d_R	u_R	d_R	u_R	$-\frac{2}{3}$	$+\frac{1}{3}$	α	$-\alpha-\frac{1}{3}$	$\alpha-\frac{1}{3}$
d_R	d_R	d_L	u_L	d_R	u_R					
d_R	d_R	d_R	u_R	u_R	d_R	$-\frac{2}{3}$	$+\frac{1}{3}$	α	$-\alpha-\frac{1}{3}$	$\alpha+\frac{2}{3}$
d_R	d_R	d_L	u_L	u_R	d_R					
d_R	u_R	d_R	u_R	d_R	d_R	$+\frac{1}{3}$	$+\frac{1}{3}$	α	$-\alpha+\frac{2}{3}$	$\alpha-\frac{1}{3}$
d_R	u_R	d_L	u_L	d_R	d_R					
d_L	d_L	u_L	u_L	d_R	d_R					
d_L	u_L	d_L	u_L	d_R	d_R					
d_R	d_R	d_R	u_R	d_L	u_L	$-\frac{2}{3}$	$+\frac{1}{3}$	α	$-\alpha-\frac{1}{3}$	$\alpha+\frac{1}{6}$
d_R	d_R	d_R	u_R	u_L	d_L					
d_R	d_R	d_L	d_L	u_L	u_L					
d_R	d_R	u_L	u_L	d_L	d_L					
d_R	d_R	u_L	d_L	u_L	d_L					
d_R	d_R	u_L	d_L	d_L	u_L					

Neutron-Antineutron oscillation

- **T-L-2,3**

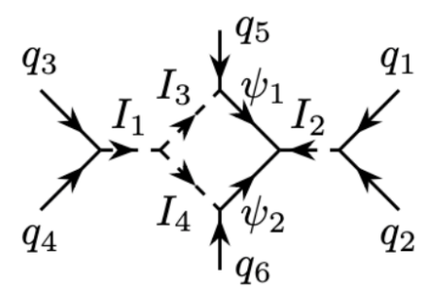


T-L-2



T-L-3

- **T-L-4**



T-L-4

$SU(3)_C$					
I_1	I_2	ψ_1	ψ_2	ψ_3	I_3
I_1	I_2	I_3	I_4	I_5	ψ
$\bar{3}$	$\bar{3}$	3	3	1	$\bar{3}$
		3	3	8	$\bar{3}$
		3	3	8	6
		3	$\bar{6}$	1	$\bar{3}$
		3	$\bar{6}$	8	$\bar{3}$
		$\bar{3}$	8	3	3
		$\bar{3}$	8	3	$\bar{6}$
		$\bar{3}$	8	$\bar{6}$	3
		6	8	3	3
		6	8	3	$\bar{6}$
		$\bar{6}$	3	8	$\bar{3}$
		$\bar{6}$	3	8	6
		8	$\bar{3}$	$\bar{3}$	1
		8	$\bar{3}$	$\bar{3}$	8
		8	$\bar{3}$	6	8
		8	6	$\bar{3}$	8
		8	6	6	8
6	6	3	3	8	$\bar{3}$
		3	3	8	6
		$\bar{3}$	8	3	3
		$\bar{3}$	8	3	$\bar{6}$
		6	8	$\bar{6}$	3
		$\bar{6}$	$\bar{6}$	1	$\bar{3}$
		$\bar{6}$	$\bar{6}$	8	$\bar{3}$
		8	$\bar{3}$	$\bar{3}$	1
		8	$\bar{3}$	$\bar{3}$	8
		8	$\bar{3}$	6	8
		8	6	$\bar{3}$	8
		8	6	6	8

$SU(3)_C$					
I_1	I_2	ψ_1	ψ_2	ψ_3	I_3
I_1	I_2	I_3	I_4	I_5	ψ
$\bar{3}$	6	3	3	8	$\bar{3}$
		3	3	8	6
		3	$\bar{6}$	8	$\bar{3}$
		$\bar{3}$	8	3	3
		$\bar{3}$	8	3	$\bar{6}$
		6	8	$\bar{6}$	3
		$\bar{6}$	3	1	$\bar{3}$
		$\bar{6}$	3	8	$\bar{3}$
		$\bar{6}$	3	8	6
		8	$\bar{3}$	$\bar{3}$	1
		8	$\bar{3}$	$\bar{3}$	8
		8	$\bar{3}$	6	8
		8	6	$\bar{3}$	8
		8	6	6	8
6	$\bar{3}$	3	3	1	$\bar{3}$
		3	3	8	$\bar{3}$
		3	3	8	6
		$\bar{3}$	8	3	3
		$\bar{3}$	8	3	$\bar{6}$
		$\bar{3}$	8	$\bar{6}$	3
		6	8	3	3
		6	8	3	$\bar{6}$
		$\bar{6}$	$\bar{6}$	8	$\bar{3}$
		8	$\bar{3}$	$\bar{3}$	1
		8	$\bar{3}$	$\bar{3}$	8
		8	$\bar{3}$	6	8
		8	6	$\bar{3}$	8
		8	6	6	8

$SU(2)_L$											
q_1	q_2	q_3	q_4	q_5	q_6	I_1	I_2	I_3	I_4	I_5	ψ
q_1	q_2	q_3	q_4	q_5	q_6	I_1	I_2	ψ_1	ψ_2	ψ_3	I_3
1	1	1	1	1	1	1	1	1	1	1	1
						1	1	2	2	2	2
						1	1	3	3	3	3
1	1	1	1	2	2	1	1	1	1	1	2
						1	1	2	2	2	1
						1	1	2	2	2	3
						1	1	3	3	3	2
1	1	2	2	1	1	1	1	1	1	1	1
						1	1	2	2	2	2
						1	1	3	3	3	3
						1	3	2	2	2	2
						1	3	3	3	3	3
2	2	1	1	1	1	1	1	1	1	1	1
						1	1	2	2	2	2
						1	1	3	3	3	3
						3	1	2	2	2	2
						3	1	3	3	3	3
1	1	2	2	2	2	1	1	1	1	1	2
						1	1	2	2	2	1
						1	1	2	2	2	3
						1	1	3	3	3	2
						1	3	1	1	3	2
						1	3	2	2	2	3
						1	3	2	2	2	1
						1	3	3	3	1	2
						1	3	3	3	3	2
2	2	2	2	1	1	1	1	1	1	1	1
						1	1	2	2	2	2
						1	1	3	3	3	3
						1	3	2	2	2	2
						1	3	3	3	3	3
						3	1	2	2	2	2
						3	1	3	3	3	3
						3	3	2	2	2	2
						3	3	3	3	3	3
						3	3	1	3	3	3
						3	3	3	1	1	1
2	2	1	1	2	2	1	1	1	1	1	2
						1	1	2	2	2	1
						1	1	2	2	2	3
						1	1	3	3	3	2
						3	1	2	2	2	1
						3	1	1	3	1	2
						3	1	3	1	3	2
						3	1	3	3	3	2
						3	1	2	2	2	3

Particles						$U(1)_Y$					
q_1	q_2	q_3	q_4	q_5	q_6	I_1	I_2	I_3	I_4	I_5	ψ
q_1	q_2	q_3	q_4	q_5	q_6	I_1	I_2	ψ_1	ψ_2	ψ_3	I_3
d_R	d_R	d_R	d_R	u_R	u_R	$-\frac{2}{3}$	$-\frac{2}{3}$	α	$-\alpha-\frac{2}{3}$	$\alpha-\frac{2}{3}$	$-\alpha$
d_R	d_R	u_R	u_R	d_R	d_R	$-\frac{2}{3}$	$+\frac{4}{3}$	α	$-\alpha-\frac{2}{3}$	$\alpha+\frac{4}{3}$	$-\alpha-1$
u_R	u_R	d_R	d_R	d_R	d_R	$+\frac{4}{3}$	$-\frac{2}{3}$	α	$-\alpha+\frac{4}{3}$	$\alpha-\frac{2}{3}$	$-\alpha+1$
d_R	d_R	d_R	u_R	d_R	u_R	$-\frac{2}{3}$	$+\frac{1}{3}$	α	$-\alpha-\frac{2}{3}$	$\alpha+\frac{1}{3}$	$-\alpha-1$
d_R	d_R	d_L	u_L	d_R	u_R						
d_R	d_R	d_R	u_R	u_R	d_R	$-\frac{2}{3}$	$+\frac{1}{3}$	α	$-\alpha-\frac{2}{3}$	$\alpha+\frac{1}{3}$	$-\alpha$
d_R	d_R	d_L	u_L	u_R	d_R						
d_R	u_R	d_R	d_R	d_R	u_R	$+\frac{1}{3}$	$-\frac{2}{3}$	α	$-\alpha+\frac{1}{3}$	$\alpha-\frac{2}{3}$	$-\alpha$
d_L	u_L	d_R	d_R	d_R	u_R						
d_R	u_R	d_R	d_R	u_R	d_R	$+\frac{1}{3}$	$-\frac{2}{3}$	α	$-\alpha+\frac{1}{3}$	$\alpha-\frac{2}{3}$	$-\alpha+1$
d_L	u_L	d_R	d_R	u_R	d_R						
d_R	u_R	d_R	u_R	d_R	d_R	$+\frac{1}{3}$	$+\frac{1}{3}$	α	$-\alpha+\frac{1}{3}$	$\alpha+\frac{1}{3}$	$-\alpha$
d_R	u_R	d_L	u_L	d_R	d_R						
u_L	u_L	d_L	d_L	d_R	d_R						
d_L	d_L	u_L	u_L	d_R	d_R						
d_L	u_L	d_R	u_R	d_R	d_R						
d_L	u_L	d_L	u_L	d_R	d_R						
d_R	d_R	d_R	u_R	d_L	u_L	$-\frac{2}{3}$	$+\frac{1}{3}$	α	$-\alpha-\frac{2}{3}$	$\alpha+\frac{1}{3}$	$-\alpha-\frac{1}{2}$
d_R	d_R	d_R	u_R	u_L	d_L						
d_R	d_R	d_L	d_L	u_L	u_L						
d_R	d_R	u_L	u_L	d_L	d_L						
d_R	d_R	u_L	d_L	u_L	d_L						
d_R	d_R	u_L	d_L	d_L	u_L						
d_L	d_L	d_R	d_R	u_L	u_L	$+\frac{1}{3}$	$-\frac{2}{3}$	α	$-\alpha+\frac{1}{3}$	$\alpha-\frac{2}{3}$	$-\alpha+\frac{1}{2}$
u_L	u_L	d_R	d_R	d_L	d_L						
d_R	u_R	d_R	d_R	d_L	u_L						
d_R	d_R	d_R	d_R	u_L	d_L						
u_L	d_L	d_R	d_R	d_L	u_L						
u_L	d_L	d_R	d_R	u_L	d_L						

TABLE V. The possible numbers of the fields in Topology-L-2 and Topology-L-3.

$SU(3)_C$					
I_1	I_2	I_3	I_4	ψ_1	ψ_2
$\bar{3}$	$\bar{3}$	3	3	$\bar{3}$	$\bar{3}$
		3	3	$\bar{3}$	6
		3	3	6	$\bar{3}$
		3	$\bar{6}$	$\bar{3}$	$\bar{3}$
		3	$\bar{6}$	6	$\bar{3}$
		$\bar{3}$	8	8	3
		$\bar{3}$	8	8	$\bar{6}$
		6	8	8	3
		6	8	8	$\bar{6}$
		$\bar{6}$	3	$\bar{3}$	$\bar{3}$
		$\bar{6}$	3	$\bar{3}$	6
		8	$\bar{3}$	3	8
		8	$\bar{3}$	6	8
		8	6	3	8
		8	6	6	8
6	6	3	3	$\bar{3}$	$\bar{3}$
		3	3	6	6
		$\bar{3}$	8	8	3
		$\bar{3}$	8	8	$\bar{6}$
		6	8	8	3
		6	8	8	$\bar{6}$
		$\bar{6}$	$\bar{6}$	$\bar{3}$	$\bar{3}$
		8	$\bar{3}$	3	8
		8	$\bar{3}$	6	8
		8	6	3	8
		8	6	6	8

$SU(3)_C$					
I_1	I_2	I_3	I_4	ψ_1	ψ_2
$\bar{3}$	6	3	3	$\bar{3}$	$\bar{3}$
		3	3	6	6
		3	$\bar{6}$	$\bar{3}$	$\bar{3}$
		$\bar{3}$	8	8	3
		$\bar{3}$	8	8	$\bar{6}$
		6	8	8	3
		6	8	8	$\bar{6}$
		$\bar{6}$	3	$\bar{3}$	$\bar{3}$
		8	$\bar{3}$	3	8
		8	$\bar{3}$	$\bar{6}$	8
		8	6	3	8
		8	6	$\bar{6}$	8
6	$\bar{3}$	3	3	$\bar{3}$	$\bar{3}$
		3	3	$\bar{3}$	6
		3	3	6	$\bar{3}$
		$\bar{3}$	8	8	3
		$\bar{3}$	8	8	$\bar{6}$
		6	8	8	3
		6	8	8	$\bar{6}$
		$\bar{6}$	$\bar{6}$	$\bar{3}$	$\bar{3}$
		8	$\bar{3}$	3	8
		8	$\bar{3}$	$\bar{6}$	8
		8	6	3	8
		8	6	$\bar{6}$	8

$SU(2)_L$											
q_1	q_2	q_3	q_4	q_5	q_6	I_1	I_2	I_3	I_4	ψ_1	ψ_2
1	1	1	1	1	1	1	1	1	1	1	1
						1	1	2	2	2	2
						1	1	3	3	3	3
1	1	1	1	2	2	1	1	1	1	2	2
						1	1	2	2	1	1
						1	1	2	2	2	2
						1	1	3	3	3	3
1	1	2	2	1	1	1	1	1	1	1	1
						1	1	2	2	2	2
						1	1	3	3	3	3
						1	3	2	2	2	2
						1	3	3	3	3	3
2	2	1	1	1	1	1	1	1	1	1	1
						1	1	2	2	2	2
						1	1	3	3	3	3
						3	1	2	2	2	2
						3	1	3	3	3	3
1	1	2	2	2	2	1	1	1	1	1	2
						1	1	2	2	2	1
						1	1	2	2	2	3
						1	1	3	3	3	2
						1	3	1	1	3	2
						1	3	2	2	2	3
						1	3	2	2	2	1
						1	3	3	3	1	2
						1	3	3	3	3	2
2	2	2	2	1	1	1	1	1	1	1	1
						1	1	2	2	2	2
						1	1	3	3	3	3
						1	3	2	2	2	2
						1	3	3	3	3	3
						3	1	2	2	2	2
						3	1	3	3	3	3
						3	3	2	2	2	2
						3	3	3	3	3	3
						3	3	1	3	3	3
						3	3	3	1	1	1
2	2	1	1	2	2	1	1	1	1	2	2
						1	1	2	2	1	1
						1	1	2	2	3	3
						1	1	3	3	2	2
						3	1	2	2	1	1
						3	1	1	3	2	2
						3	1	3	1	2	2
						3	1	3	3	2	2
						3	1	2	2	3	3

Particles						$U(1)_Y$					
q_1	q_2	q_3	q_4	q_5	q_6	I_1	I_2	I_3	I_4	ψ_1	ψ_2
d_R	d_R	d_R	d_R	u_R	u_R	$-\frac{2}{3}$	$-\frac{2}{3}$	α	$-\alpha-\frac{2}{3}$	$\alpha+\frac{2}{3}$	$-\alpha$
d_R	d_R	u_R	u_R	d_R	d_R	$-\frac{2}{3}$	$+\frac{4}{3}$	α	$-\alpha-\frac{2}{3}$	$\alpha-\frac{1}{3}$	$-\alpha-1$
u_R	u_R	d_R	d_R	d_R	d_R	$+\frac{4}{3}$	$-\frac{2}{3}$	α	$-\alpha+\frac{4}{3}$	$\alpha-\frac{1}{3}$	$-\alpha+1$
d_R	d_R	d_R	u_R	d_R	u_R	$-\frac{2}{3}$	$+\frac{1}{3}$	α	$-\alpha-\frac{2}{3}$	$\alpha-\frac{1}{3}$	$-\alpha$
d_R	d_R	d_L	u_L	d_R	u_R						
d_R	d_R	d_R	u_R	u_R	d_R	$-\frac{2}{3}$	$+\frac{1}{3}$	α	$-\alpha-\frac{2}{3}$	$\alpha+\frac{2}{3}$	$-\alpha-1$
d_R	d_R	d_L	u_L	u_R	d_R						
d_R	u_R	d_R	d_R	d_R	u_R	$+\frac{1}{3}$	$-\frac{2}{3}$	α	$-\alpha+\frac{1}{3}$	$\alpha-\frac{1}{3}$	$-\alpha+1$
d_L	u_L	d_R	d_R	d_R	u_R						
d_R	u_R	d_R	d_R	u_R	d_R	$+\frac{1}{3}$	$-\frac{2}{3}$	α	$-\alpha+\frac{1}{3}$	$\alpha+\frac{2}{3}$	$-\alpha$
d_L	u_L	d_R	d_R	u_R	d_R						
d_R	u_R	d_R	u_R	d_R	d_R	$+\frac{1}{3}$	$+\frac{1}{3}$	α	$-\alpha+\frac{1}{3}$	$\alpha-\frac{1}{3}$	$-\alpha$
d_R	u_R	d_L	u_L	d_R	d_R						
u_L	u_L	d_L	d_L	d_R	d_R						
d_L	d_L	u_L	u_L	d_R	d_R						
d_L	u_L	d_R	u_R	d_R	d_R						
d_L	u_L	d_L	u_L	d_R	d_R						
d_R	d_R	d_R	u_R	d_L	u_L	$-\frac{2}{3}$	$+\frac{1}{3}$	α	$-\alpha-\frac{2}{3}$	$\alpha+\frac{1}{6}$	$-\alpha-\frac{1}{2}$
d_R	d_R	d_R	u_R	u_L	d_L						
d_R	d_R	d_L	d_L	u_L	u_L						
d_R	d_R	u_L	u_L	d_L	d_L						
d_R	d_R	u_L	d_L	u_L	d_L						
d_R	d_R	u_L	d_L	d_L	u_L						
d_L	d_L	d_R	d_R	u_L	u_L	$+\frac{1}{3}$	$-\frac{2}{3}$	α	$-\alpha+\frac{1}{3}$	$\alpha+\frac{1}{6}$	$-\alpha+\frac{1}{2}$
u_L	u_L	d_R	d_R	d_L	d_L						
d_R	u_R	d_R	d_R	d_L	u_L						
d_R	d_R	d_R	d_R	u_L	d_L						
u_L	d_L	d_R	d_R	d_L	u_L						
u_L	d_L	d_R	d_R	u_L	d_L						

TABLE VI. The possible quantum numbers of the fields in Topology-L-4.

……

The completed lists can be found in our manuscript.

An example toy model

(Introduce $\mathbb{Z}_2$ symmetry)

Particles

SM particles	Quantum Number	New Particles	Quantum Number
$\Phi = (\phi^+, \phi^0)^T$	$(1, 2, 1/2, +1)$	S_1	$(3, 1, -1/3, -1)$
$Q_L = (U_L, D_L)^T$	$(3, 2, 1/6, +1)$	$S_2 = (S_2^{+2/3}, S_2^{-1/3})^T$	$(3, 2, 1/6, -1)$
$L_L = (\nu_L, E_L)^T$	$(1, 2, -1/2, +1)$	$\eta = (\eta^+, \eta^0)^T$	$(1, 2, 1/2, -1)$
U_R	$(3, 1, +2/3, +1)$	N_i	$(1, 1, 0, -1)$
D_R	$(3, 1, -1/3, +1)$	$\psi = (\psi^{+2/3}, \psi^{-1/3})^T$	$(3, 2, 1/6, -1)$
E_R	$(1, 1, -1, +1)$		

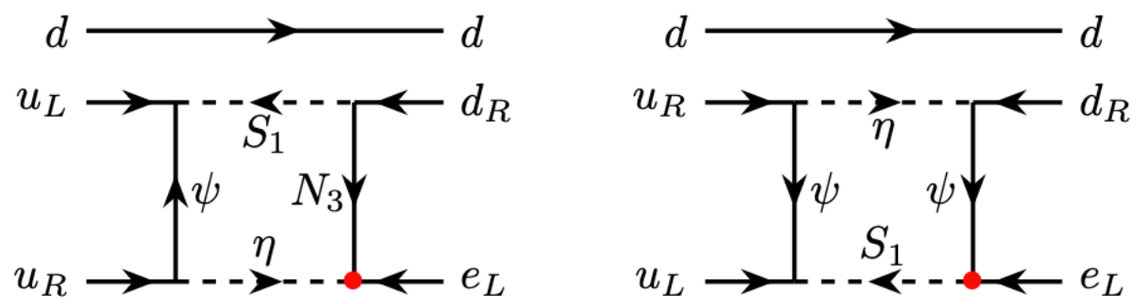
Lagrangian

$$
\begin{aligned}
-\mathcal{L}_Y \supset\ & y_N^{ij}\overline{(L_L^i)^c} i\sigma_2 \eta N_j^c + y_\psi^i \overline{(L_L^{i\alpha})^c} i\sigma_2 \psi^\alpha S_1^{*\bar\alpha} + y_1^i \overline{\psi^\alpha} i\sigma_2 \eta^* U_R^{i\alpha} \\
& + y_2^{ij}\overline{(D_R^{i\alpha})^c} N_j S_1^{*\bar\alpha} + y_3^i \overline{(Q_L^{i\alpha})^c} i\sigma_2 \psi^\beta S_1^\gamma \epsilon^{\alpha\beta\gamma} + y_4^{ij}\overline{(Q_L^{i\alpha})^c} S_2^{*\bar\alpha} N_j^c \\
& + y_5^i \overline{(\psi^\alpha)^c} i\sigma_2 S_2^\beta D_R^{i\gamma} \epsilon^{\alpha\beta\gamma} + y_6^i \overline{\psi^\alpha} \eta D_R^{i\alpha} + \text{h.c.}\,.
\end{aligned}
$$

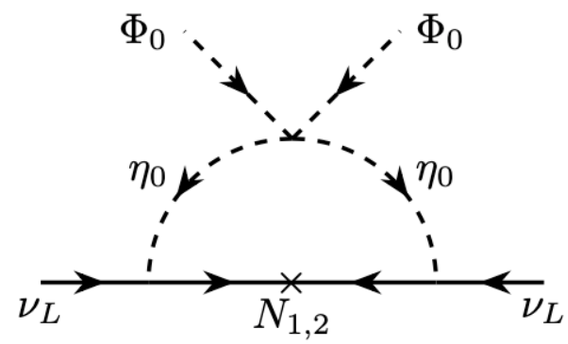
forbid proton decay

Right-handed neutrino $N_{1,2}$ couples with leptons, N_3 couples with quarks, and $y_\psi = 0$.

Neutron-Antineutron oscillation

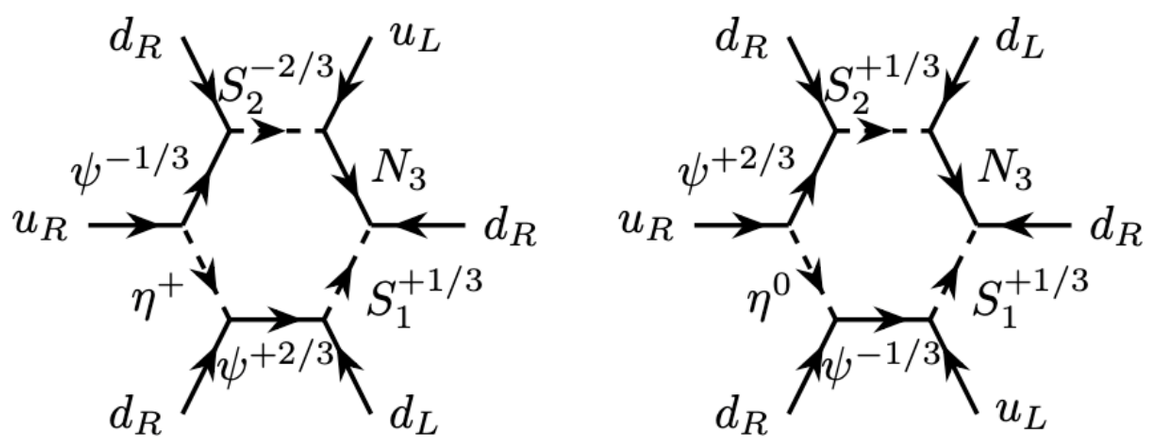


proton decay



$$m_\nu^{ij} = \frac{\lambda_5 v^2}{16\pi^2} \sum_{k=1,2} \frac{y_N^{ik} y_N^{jk}}{m_N}$$

neutrino mass (Scotogenic)



$$\mathcal{O}_2 = -4\mathcal{O}^3_{RLR}$$

$$\mathcal{C}_2 = -\tfrac{1}{4}\mathcal{C}^3_{RLR}$$

$n - \bar{n}$ oscillation

Wilson Coefficient

$$\mathcal{C}_2 = \frac{m_\psi}{64\pi^2 m_{N_3}^6}\left[\mathcal{Y}^+ \times F_6(a,b,c,d,f,g^+) - \mathcal{Y}^0 \times \left(F_6(a,b,c,d,f,g_R^0) - F_6(a,b,c,d,f,g_I^0)\right)\right]$$

Loop integral

$$F_6(a,b,c,d,f,g) = \frac{1}{4(i\pi^2)}\int d^4k \frac{k^2}{(k^2-a)(k^2-b)(k^2-c)(k^2-d)(k^2-f)(k^2-g)},$$

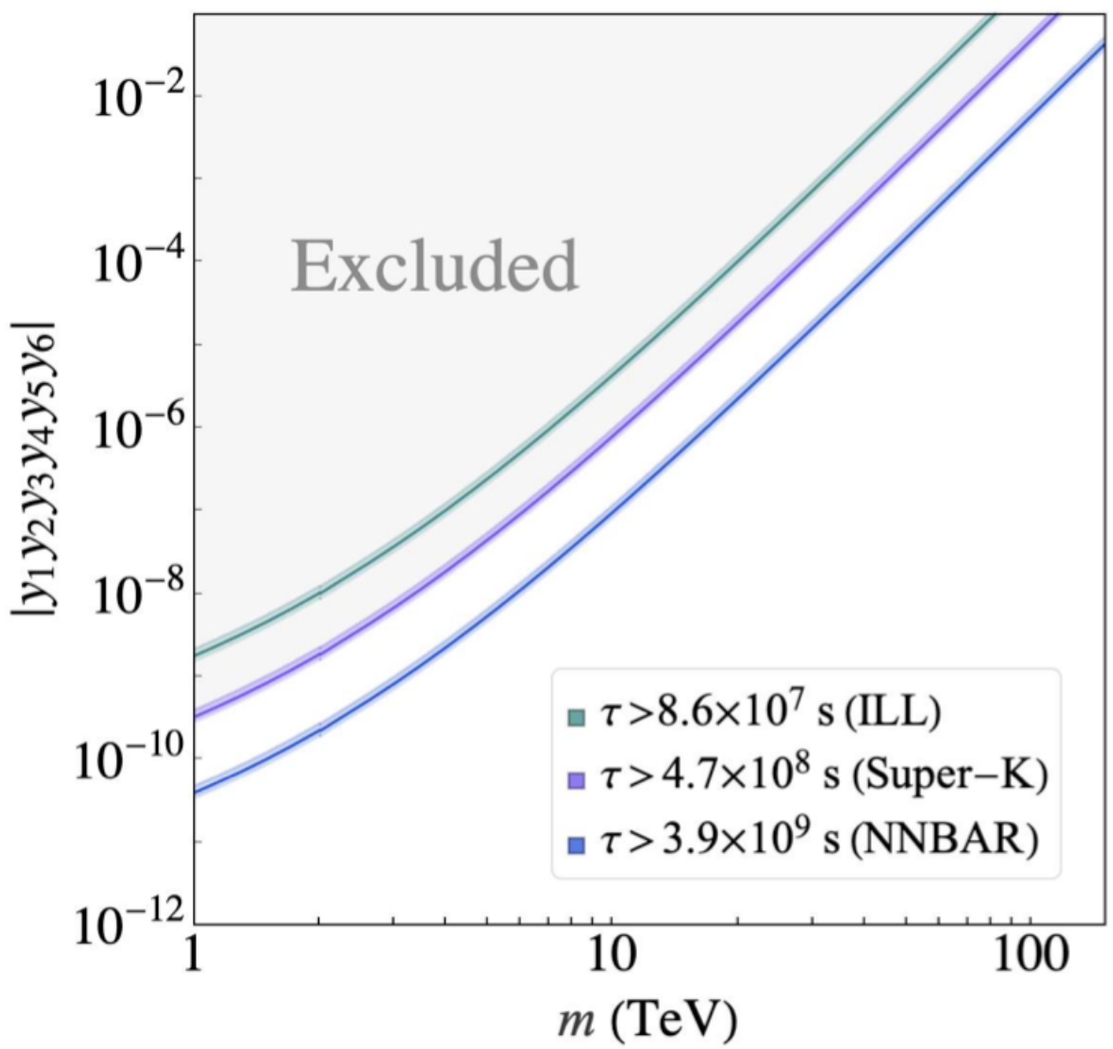
$$a \equiv \frac{m_{N_3}^2}{m_{N_3}^2},\ b = c \equiv \frac{m_\psi^2}{m_{N_3}^2},\ d = f \equiv \frac{m_{S_i}^2}{m_{N_3}^2},\ g^+ \equiv \frac{m_{\eta^+}^2}{m_{N_3}^2},\ g_R^0 \equiv \frac{m_{\eta_R^0}^2}{m_{N_3}^2},\ g_I^0 \equiv \frac{m_{\eta_I^0}^2}{m_{N_3}^2}.$$

Neutron-Antineutron oscillation

Assume that $m_N = m_{\eta^+} = m_{\eta_R^0} \simeq m_{\eta_I^0} = 2\ \text{TeV}\,,\ m_\psi = m_{S_i}\,,$

$$a = 1\,,\ b = c = d = f,\ g^+ = g_R^0 = 1\,,\ g_I^0 \simeq 1\,.$$

The loop integral $$F_6(1,b,b,b,b,1) = \frac{-b^3 - 9b^2 + 9b + 6(b+1)b\ln b + 1}{12(b-1)^5 b}\,.$$



The Super-K experiment excludes the parameter region above the lines (the region in gray).

$$|y_1^1 y_2^{13} y_3^1 y_4'^{13} y_5^1 y_6^1| \equiv |y_1 y_2 y_3 y_4 y_5 y_6|$$

$$m \equiv m_\psi = m_{S_i}$$

PART IV

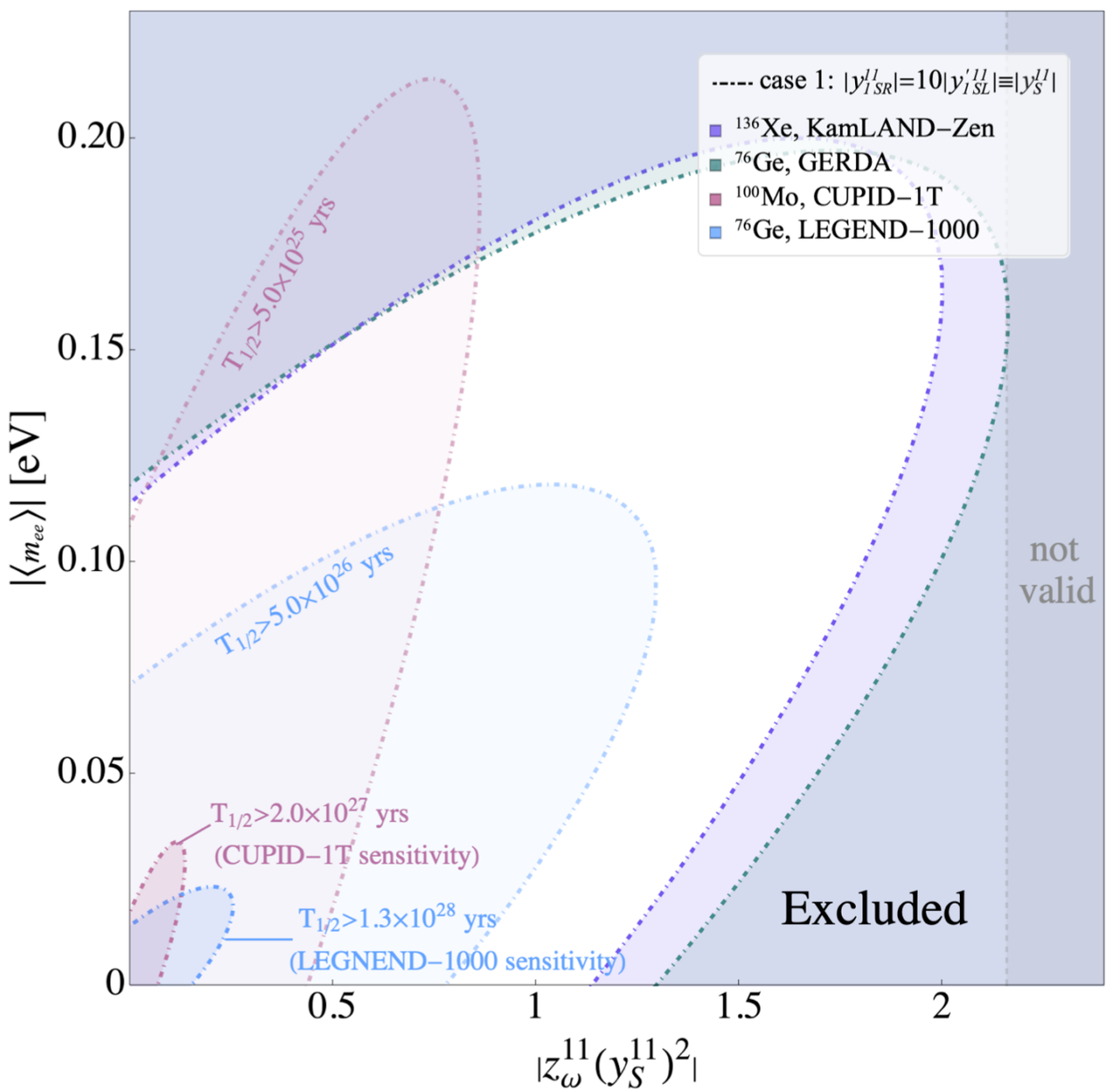
- **Brief Introduction**
- **Neutrinoless Double Beta Decay ($|\Delta L| = 2$)**
- **Neutron-Antineutron Oscillation ($|\Delta B| = 2$)**
- **Summary**

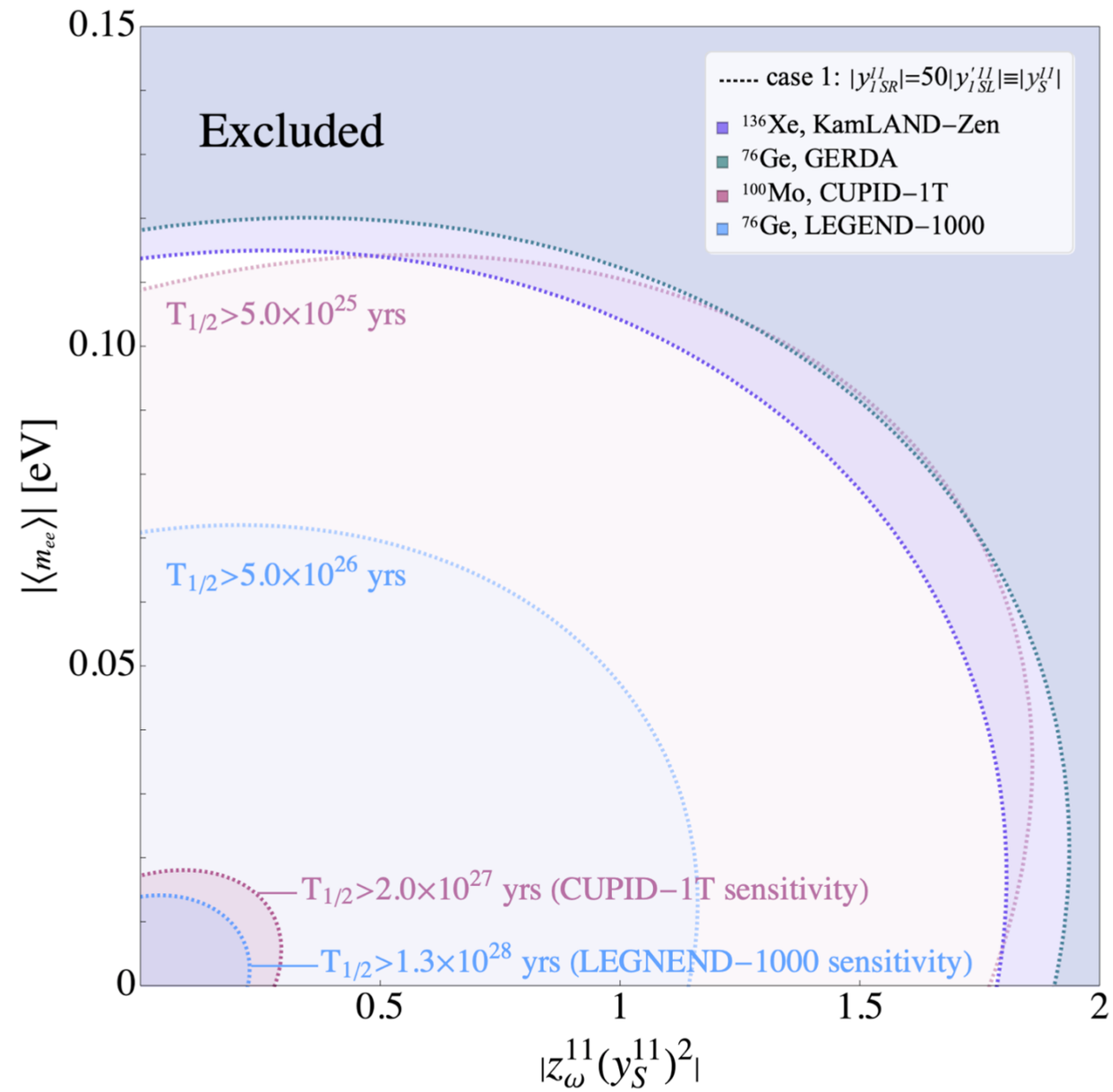
SUMMARY

- **The neutrinoless double beta decay in the colored Zee-Babu model**
  - Consider three cases of the colored Zee-Babu model
  - Focus on the interplay of standard neutrino exchange and short-range contribution of neutrinoless double beta decay
  - Find that neutrinoless double beta decay can be hidden under certain condition
  - The condition can be examined comprehensively by future complementary searches with different isotopes.
- **Systematic decomposition of $d = 9$ $n - \bar{n}$ oscillation operators**
  - Discuss the topologies' generation and the assignment of the chiral quarks.
  - Provide the completed lists of the decompositions
  - Show a toy example

Thank you for your attention

BACKUP


case 1: $|y^{11}_{1SR}|=10|y'^{11}_{1SL}|\equiv|y^{11}_{S}|$
^{136}Xe, KamLAND−Zen
^{76}Ge, GERDA
^{100}Mo, CUPID−1T
^{76}Ge, LEGEND−1000
$T_{1/2}>5.0\times10^{25}$ yrs
$T_{1/2}>5.0\times10^{26}$ yrs
$T_{1/2}>2.0\times10^{27}$ yrs
(CUPID−1T sensitivity)
$T_{1/2}>1.3\times10^{28}$ yrs
(LEGNEND−1000 sensitivity)
not valid
Excluded
$|\langle m_{ee}\rangle|$ [eV]
$|z^{11}_{\omega}(y^{11}_{S})^2|$


case 1: $|y^{11}_{1SR}|=50|y'^{11}_{1SL}|\equiv|y^{11}_{S}|$
^{136}Xe, KamLAND−Zen
^{76}Ge, GERDA
^{100}Mo, CUPID−1T
^{76}Ge, LEGEND−1000
Excluded
$T_{1/2}>5.0\times10^{25}$ yrs
$T_{1/2}>5.0\times10^{26}$ yrs
$T_{1/2}>2.0\times10^{27}$ yrs (CUPID−1T sensitivity)
$T_{1/2}>1.3\times10^{28}$ yrs (LEGNEND−1000 sensitivity)
$|\langle m_{ee}\rangle|$ [eV]
$|z^{11}_{\omega}(y^{11}_{S})^2|$