

# 格点规范场论模拟电弱对称性破缺

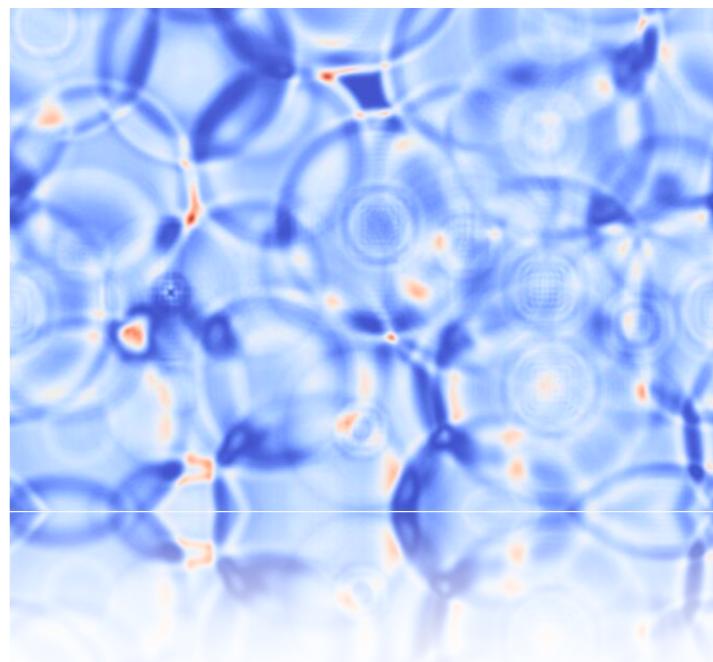
边立功  
重庆大学

2022/12/10

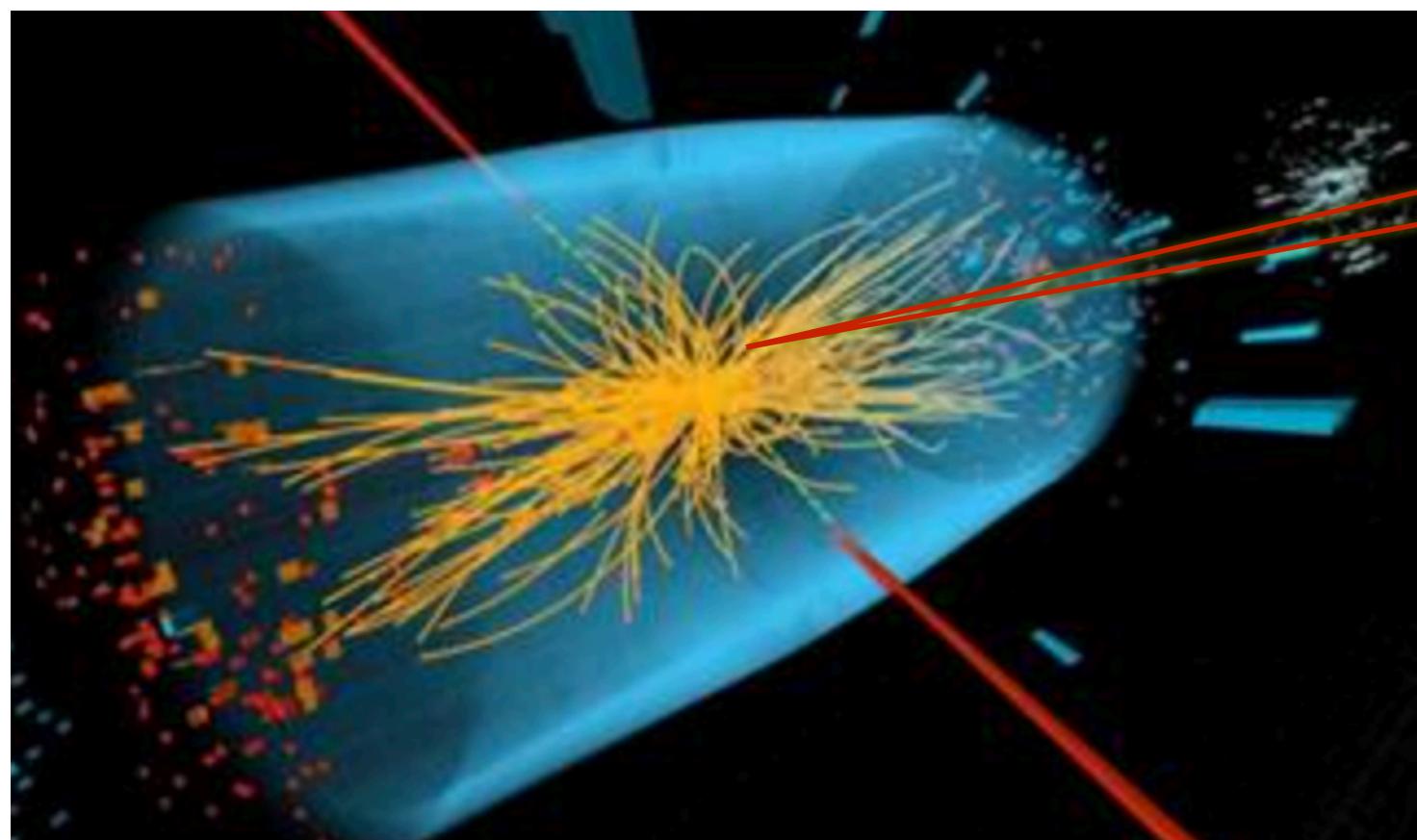
全国第十九届重味物理和CP破坏研讨会暨会议20周年庆典大会 (HFCPV2022)

# 目录

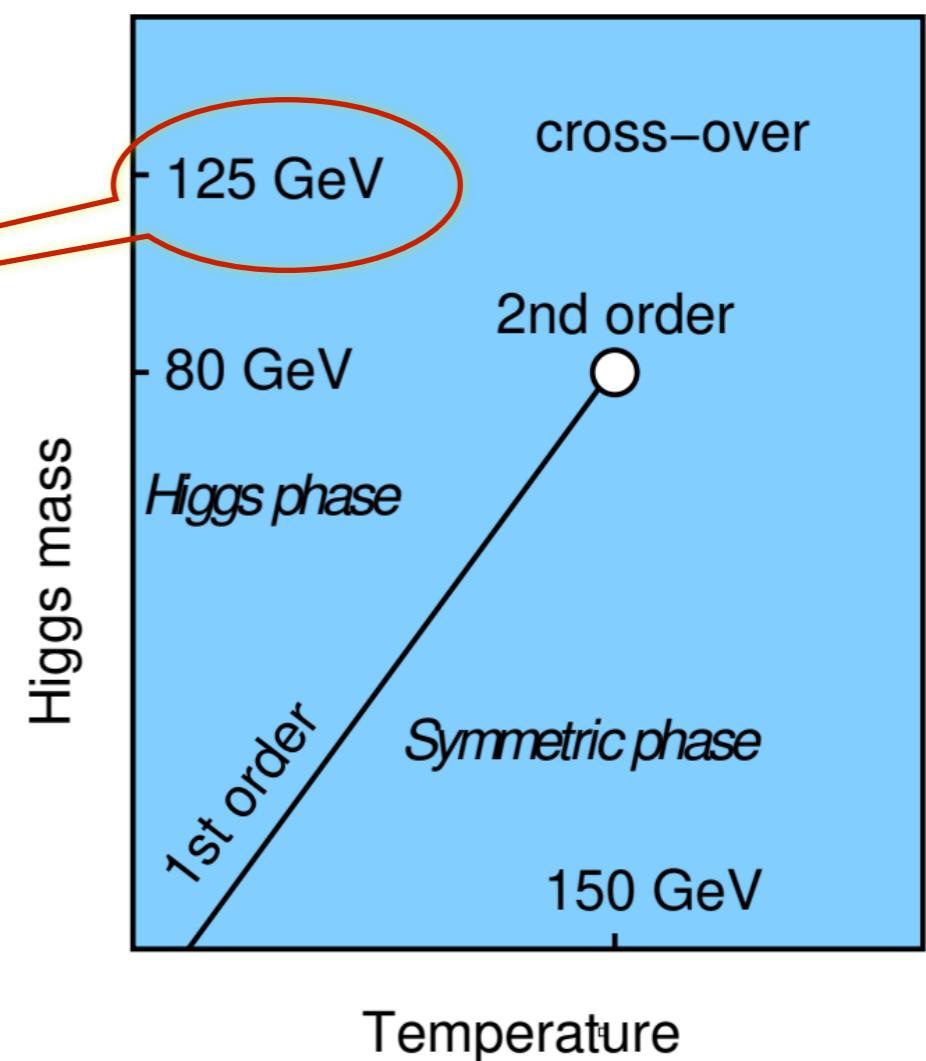
- 背景与动机
- 一步电弱相变
- 多步电弱相变
- 总结与展望



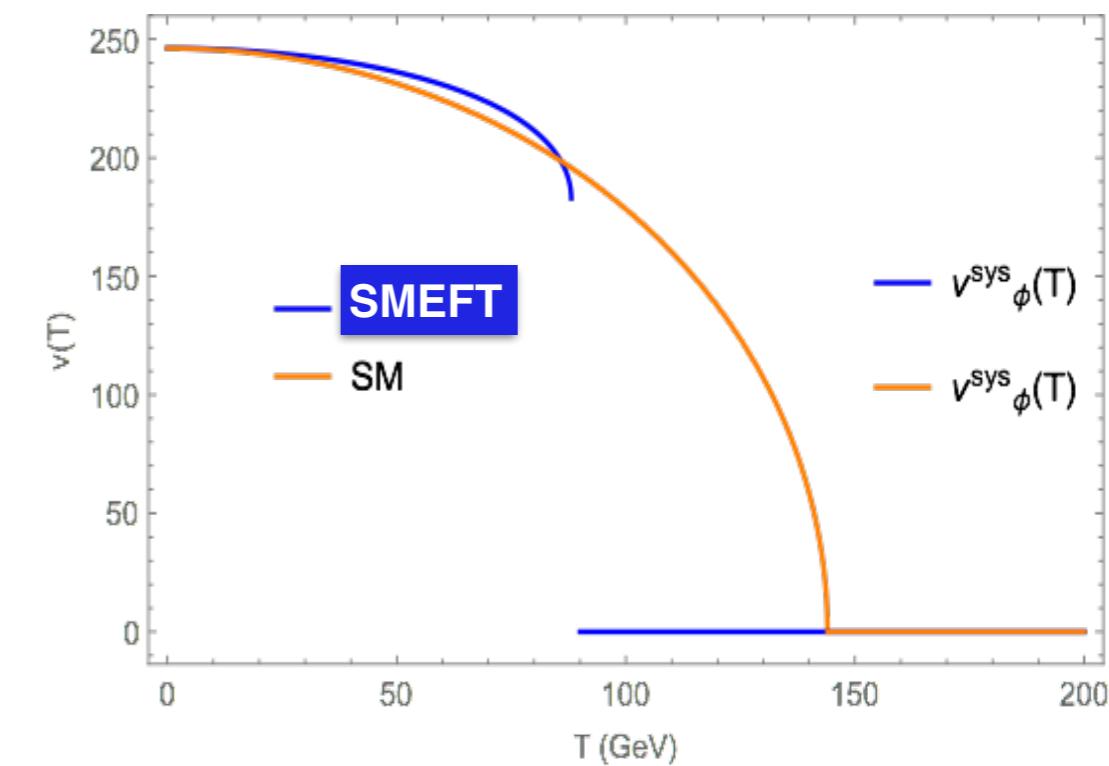
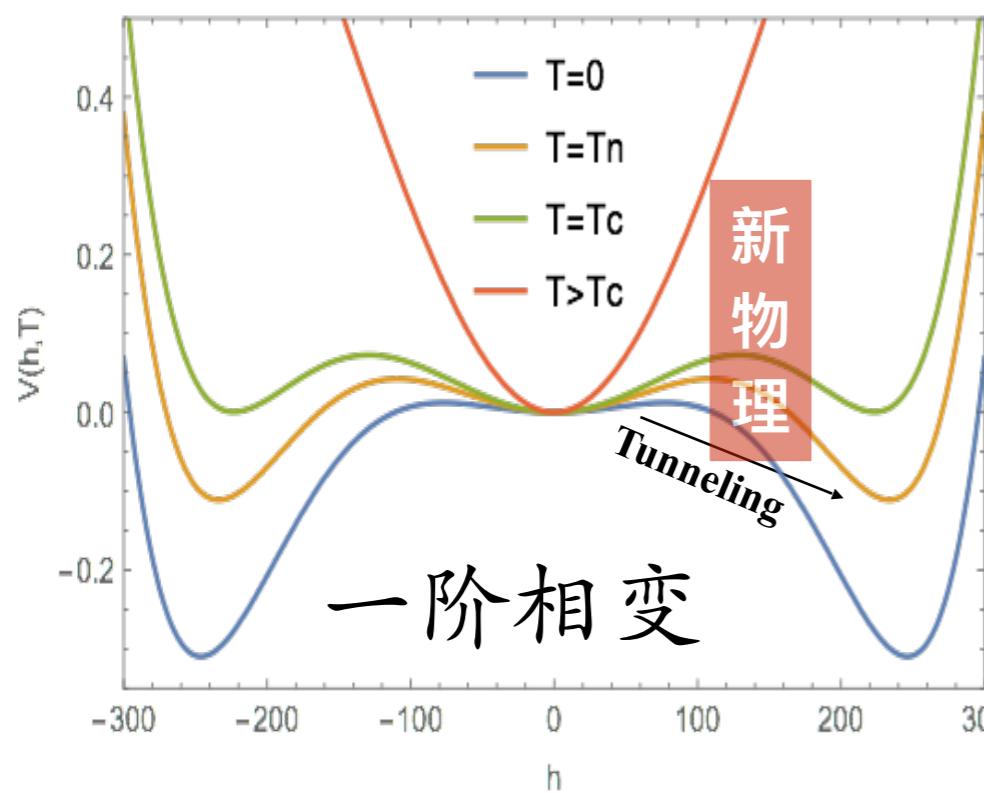
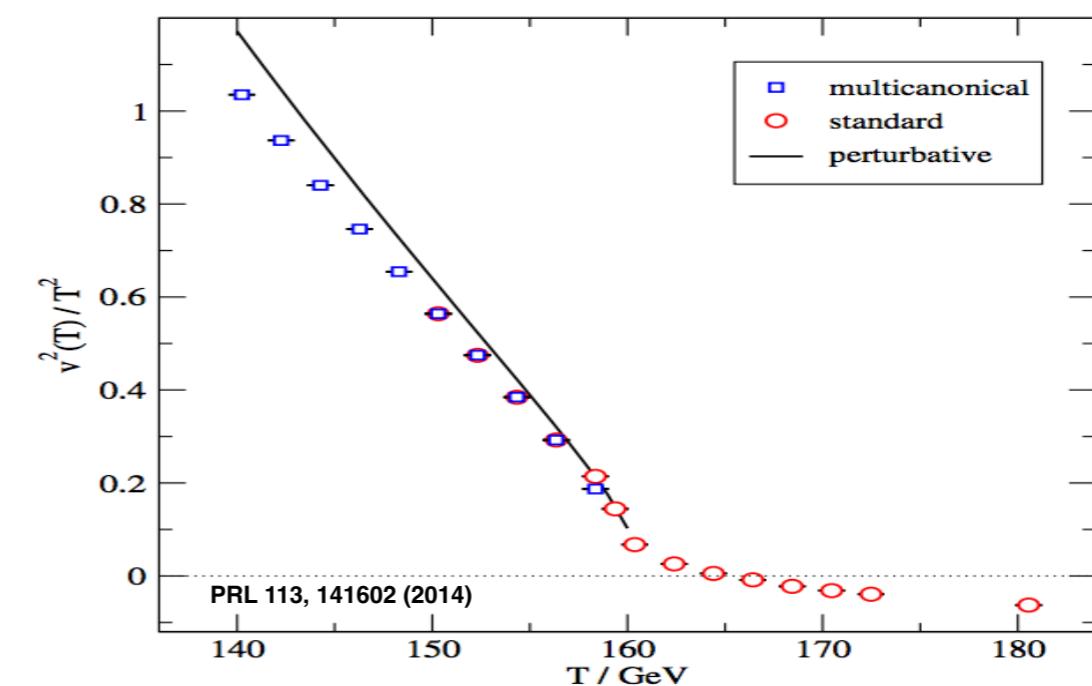
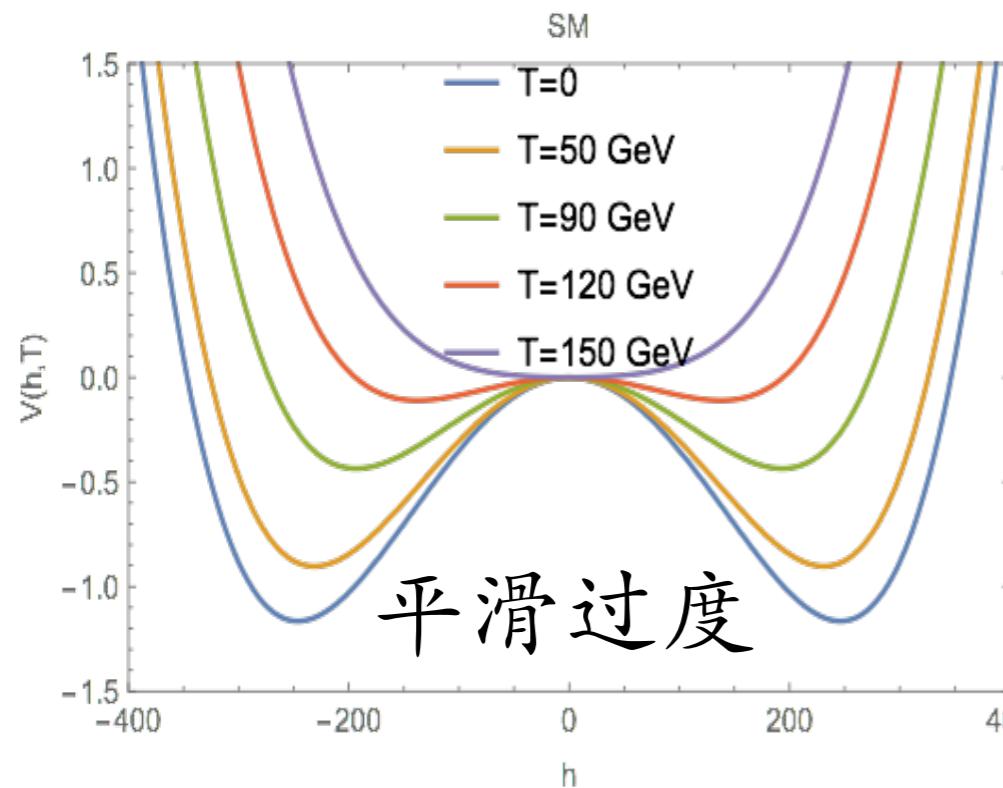
## LHC 探测实验



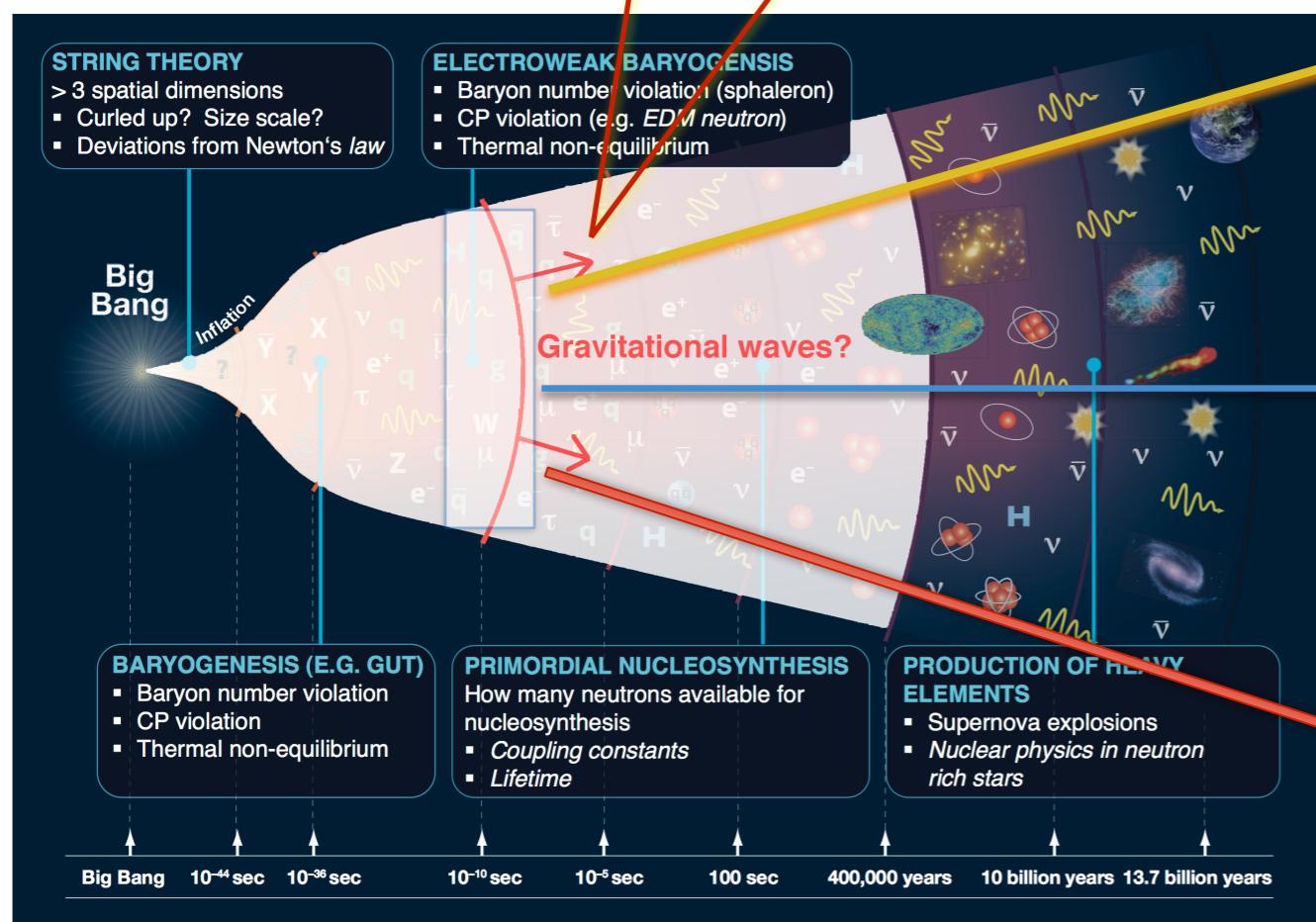
## 希格斯与电弱相变



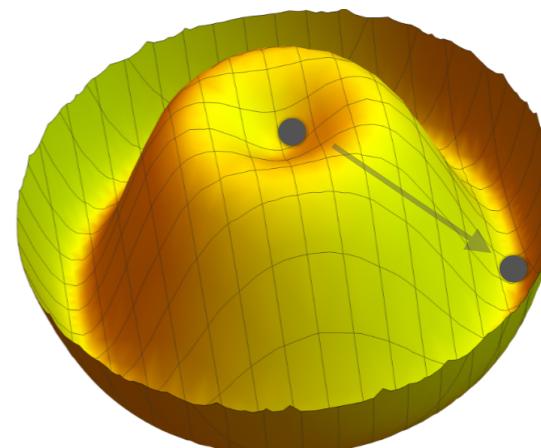
# ► 电弱相变模式



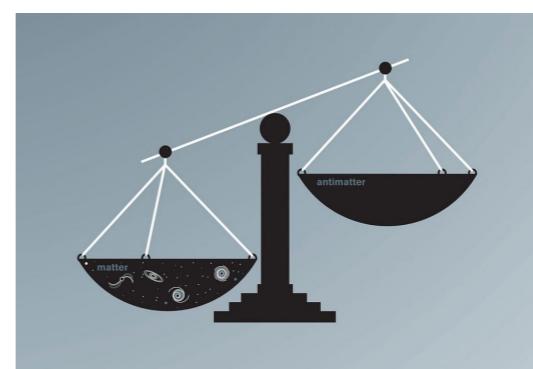
## 超出粒子物理标准模型 新物理



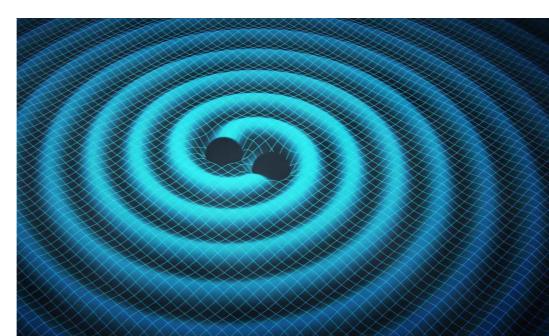
## 早期宇宙演化



电弱对称性破缺，基本粒子质量起源



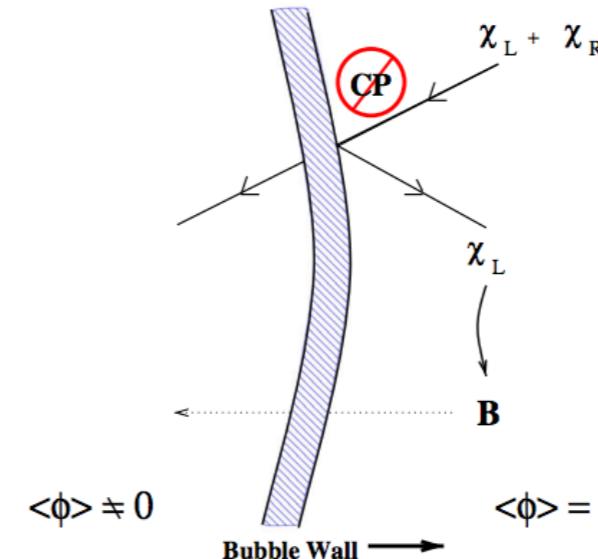
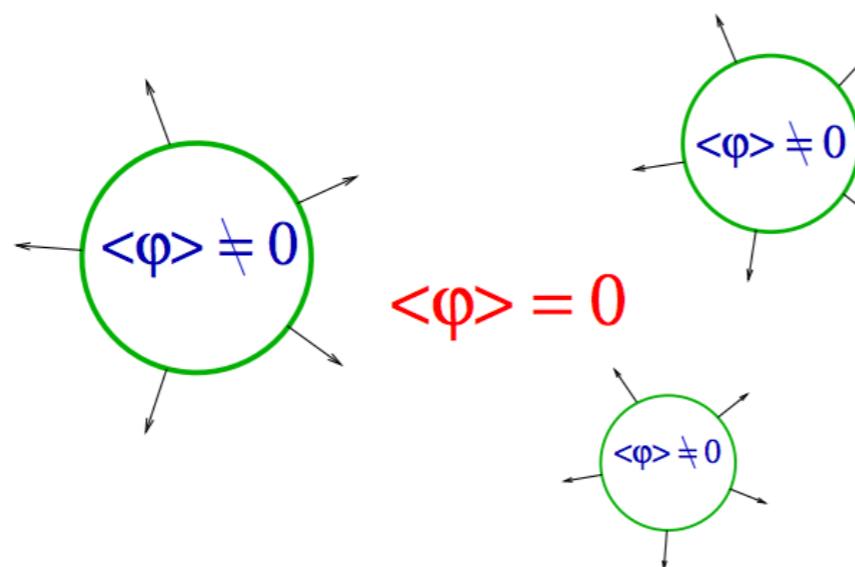
解释宇宙正反物质不对称



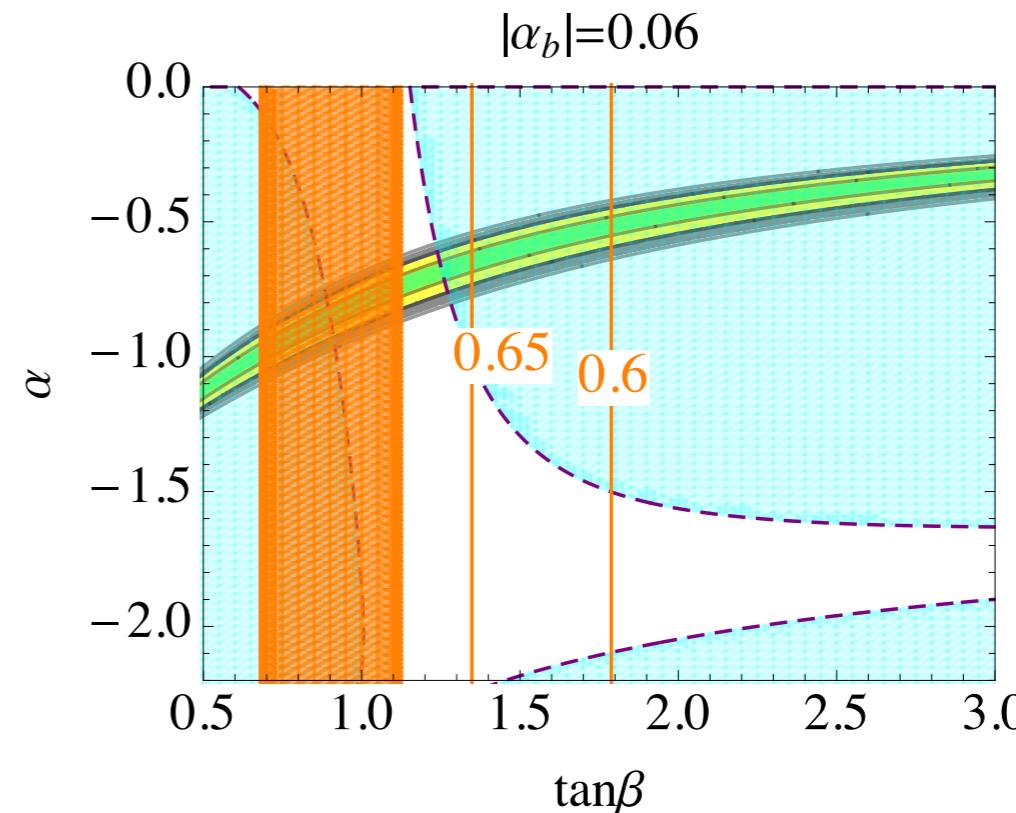
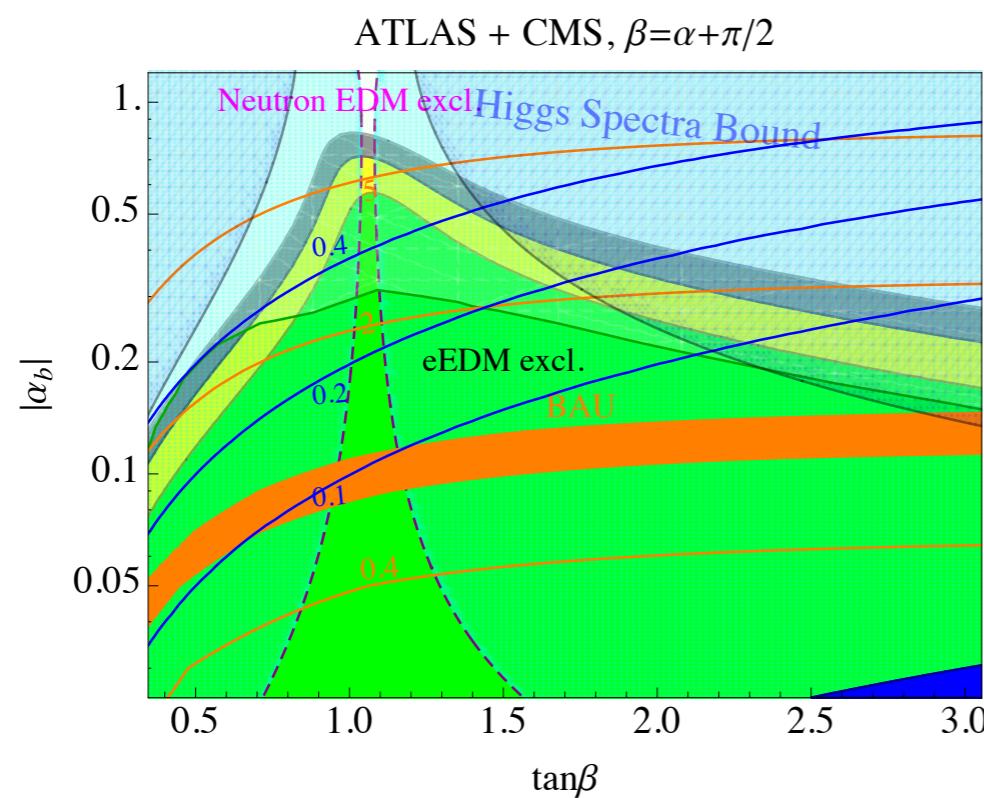
预言相变引力波，检验爱因斯坦广义相对论

# 正反物质不对称&强一阶电弱相变

## 电弱重子数产生机制



Chup etal, Rev Mod Phys.91.015001

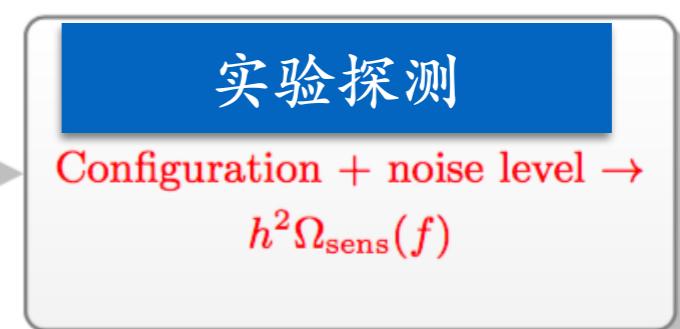
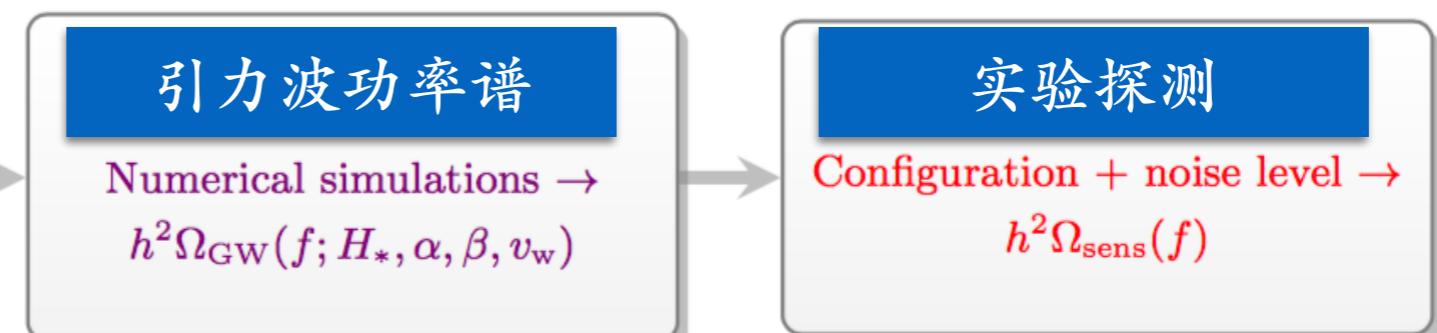
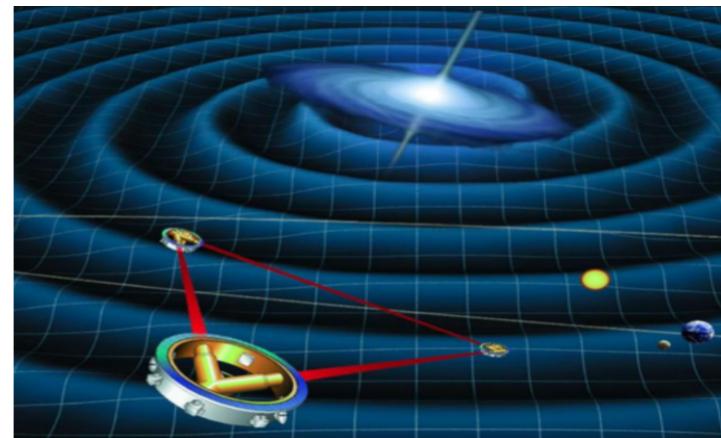
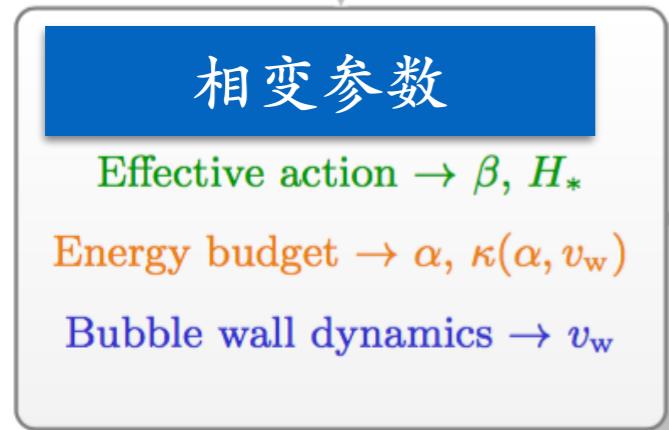


Bian, Liu\*, Shu\*, Phys.Rev.Lett. 115 (2015) 021801

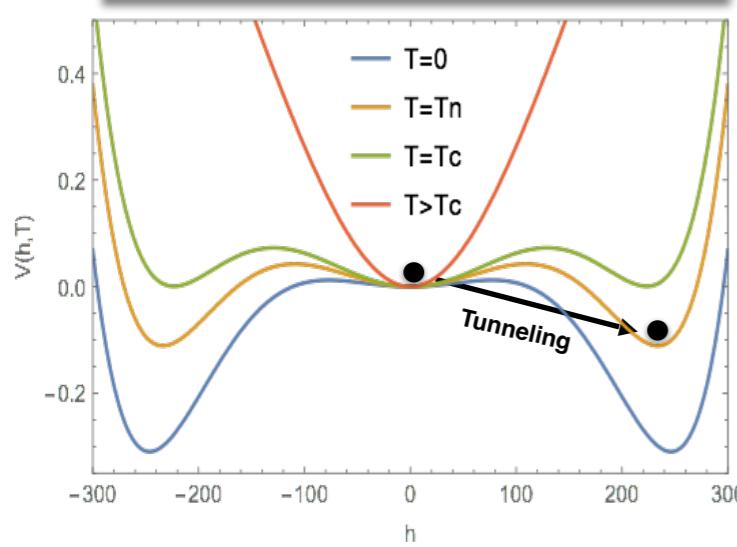
# 新物理&相变引力波

重要的引力波源，主要科学目标之一

PTA,LIGO,LISA,天琴,太极,...



有限温场论计算  
格点场论模拟建立理论和实验的桥梁



# ► 格点电弱理论

$\Phi(t, x)$  : Higgs field doublet defined on sites;

$U_i(t, x)$  and  $V_i(t, x)$  : SU(2) and U(1) link fields, defined on the link between the neighboring sites  $x$  and  $x + i$ ,  $\Phi(t, x)$ ,  $U_i(t, x)$  and  $V_i(t, x)$  are defined at time steps  $t + \Delta t, t + 2\Delta t, \dots$ ;

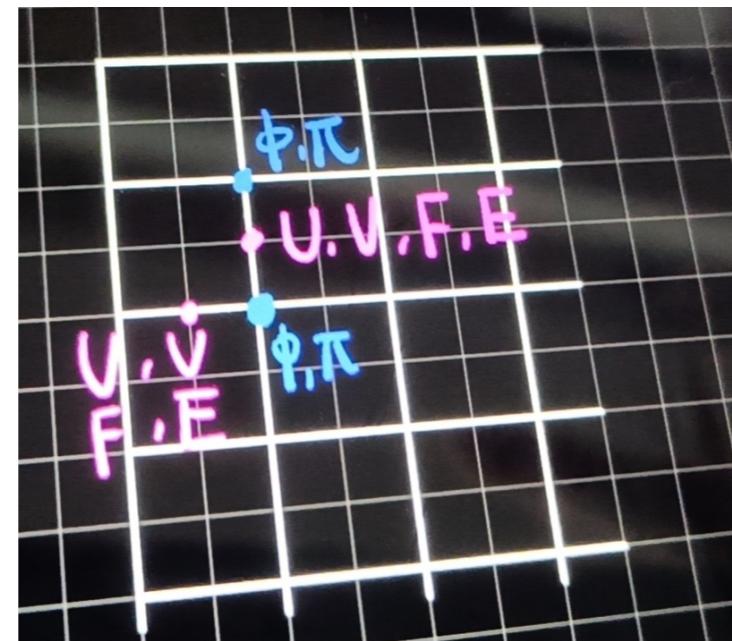
Conjugate momentum fields:  $\Pi(t + \Delta t/2, x)$ ,  $F(t + \Delta t/2, x)$  and  $E(t + \Delta t/2, x)$ , are defined at time steps  $t + \Delta t/2, t + 3\Delta t/2$ .

$$U_i(t, x) = \exp \left( -\frac{i}{2} g \Delta x \sigma^a W_i^a \right)$$

$$U_0(t, x) = \exp \left( -\frac{i}{2} g \Delta t \sigma^a W_0^a \right)$$

$$V_i(t, x) = \exp \left( -\frac{i}{2} g \Delta x B_i \right)$$

$$V_0(t, x) = \exp \left( -\frac{i}{2} g \Delta t B_0 \right).$$



$$D_i \Phi = \frac{1}{\Delta x} [U_i(t, x)V_i(t, x)\Phi(t, x + i) - \Phi(t, x)]$$

$$D_0 \Phi = \frac{1}{\Delta t} [U_0(t, x)V_0(t, x)\Phi(t + \Delta t, x) - \Phi(t, x)].$$

$$\Phi(t + \Delta t, x) = \Phi(t, x) + \Delta t \Pi(t + \Delta t/2, x)$$

$$V_i(t + \Delta t, x) = \frac{1}{2} g' \Delta x \Delta t E_i(t + \Delta t/2, x) V_i(t, x)$$

$$U_i(t + \Delta t, x) = g \Delta x \Delta t F_i(t + \Delta t/2, x) U_i(t, x),$$

Temporal gauge  
 $U_0(t, x) = I_2, V_0(t, x) = 1$

leapfrog

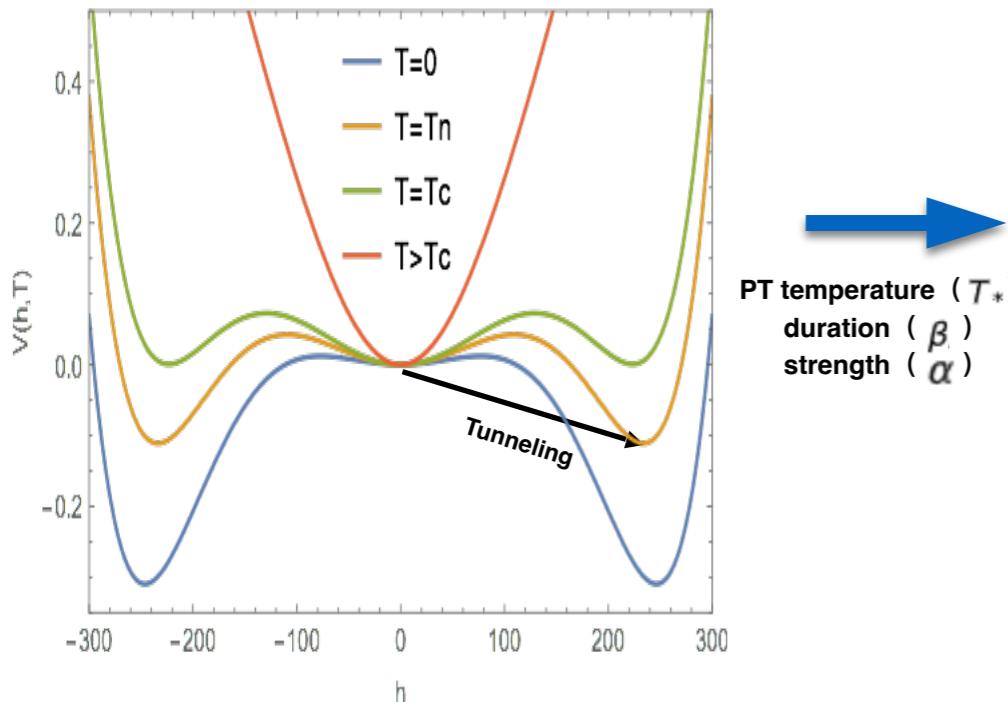
## Field basis equation of motion

$$\begin{aligned}\partial_0^2 \Phi &= D_i D_i \Phi - \frac{dV(\Phi)}{d\Phi}, \\ \partial_0^2 B_i &= -\partial_j B_{ij} + g' \operatorname{Im}[\Phi^\dagger D_i \Phi], \\ \partial_0^2 W_i^a &= -\partial_k W_{ik}^a - g \epsilon^{abc} W_k^b W_{ik}^c + g \operatorname{Im}[\Phi^\dagger \sigma^a D_i \Phi], \\ \partial_0 \partial_j B_j - g' \operatorname{Im}[\Phi^\dagger \partial_0 \Phi] &= 0, \\ \partial_0 \partial_j W_j^a + g \epsilon^{abc} W_j^b \partial_0 W_j^c - g \operatorname{Im}[\Phi^\dagger \sigma^a \partial_0 \Phi] &= 0.\end{aligned}$$

## Lattice implementation

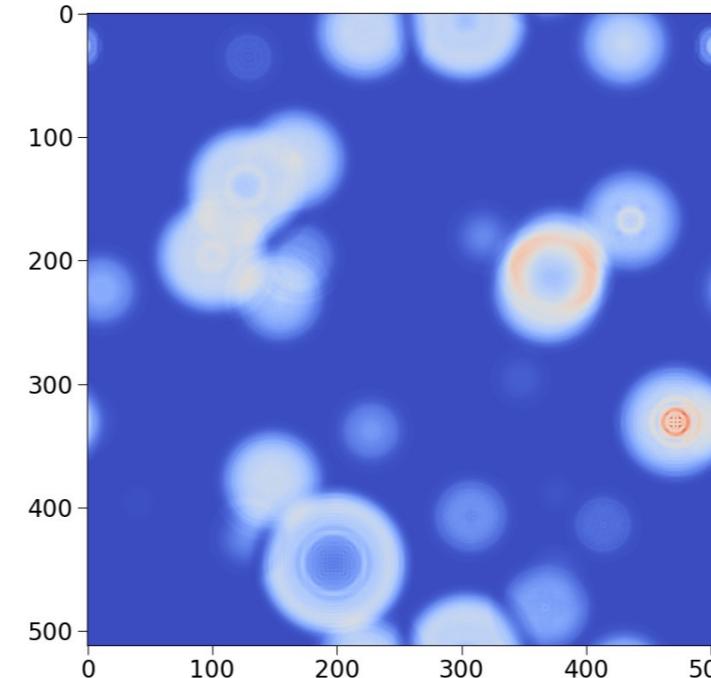
$$\begin{aligned}\Pi(t + \Delta t/2, x) &= \Pi(t - \Delta t/2, x) + \Delta t \left\{ \frac{1}{\Delta x^2} \sum_i [U_i(t, x) V_i(t, x) \Phi(t, x+i) \right. \\ &\quad \left. - 2\Phi(t, x) + U_i^\dagger(t, x-i) V_i^\dagger(t, x-i) \Phi(t, x-i)] - \frac{\partial U}{\partial \Phi^\dagger} \right\} \\ \operatorname{Im}[E_k(t + \Delta t/2, x)] &= \operatorname{Im}[E_k(t - \Delta t/2, x)] + \Delta t \left\{ \frac{g'}{\Delta x} \operatorname{Im}[\Phi^\dagger(t, x+k) U_k^\dagger(t, x) V_k^\dagger(t, x) \Phi(t, x)] \right. \\ &\quad \left. - \frac{2}{g' \Delta x^3} \sum_i \operatorname{Im}[V_k(t, x) V_i(t, x+k) V_k^\dagger(t, x+i) V_i^\dagger(t, x) \right. \\ &\quad \left. + V_i(t, x-i) V_k(t, x) V_i^\dagger(t, x+k-i) V_k^\dagger(t, x-i)] \right\} \\ \operatorname{Tr}[i\sigma^m F_k(t + \Delta t/2, x)] &= \operatorname{Tr}[i\sigma^m F_k(t - \Delta t/2, x)] + \Delta t \left\{ \frac{g}{\Delta x} \operatorname{Re}[\Phi^\dagger(t, x+k) U_k^\dagger(t, x) V_k^\dagger(t, x) i\sigma^m \Phi(t, x)] \right. \\ &\quad \left. - \frac{1}{g \Delta x^3} \sum_i \operatorname{Tr}[i\sigma^m U_k(t, x) U_i(t, x+k) U_k^\dagger(t, x+i) U_i^\dagger(t, x) \right. \\ &\quad \left. + i\sigma^m U_k(t, x) U_i^\dagger(t, x+k-i) U_k^\dagger(t, x-i) U_i(t, x-i)] \right\},\end{aligned}$$

### Finite-T Veff



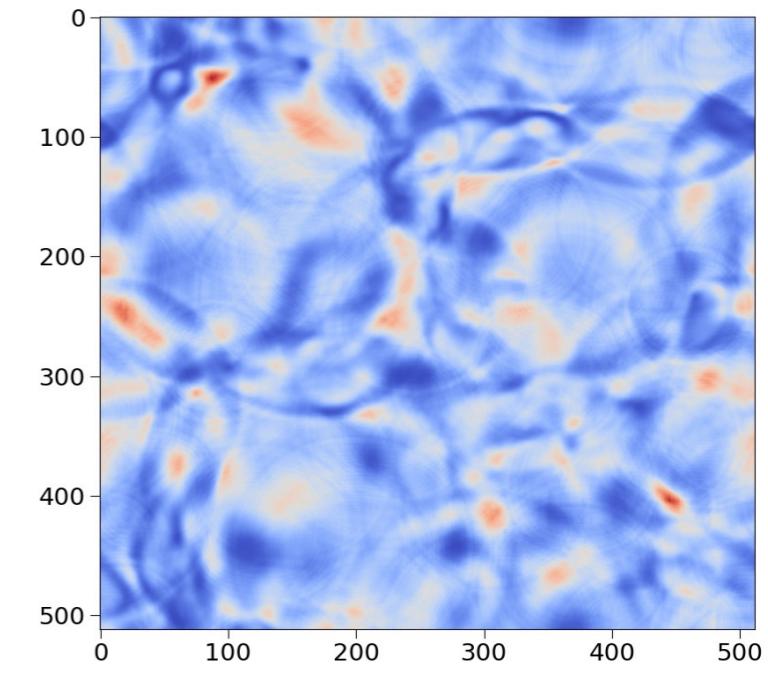
Finite-T calculation

### Nucleation



Lattice Simulation

### Expansion&Percolation



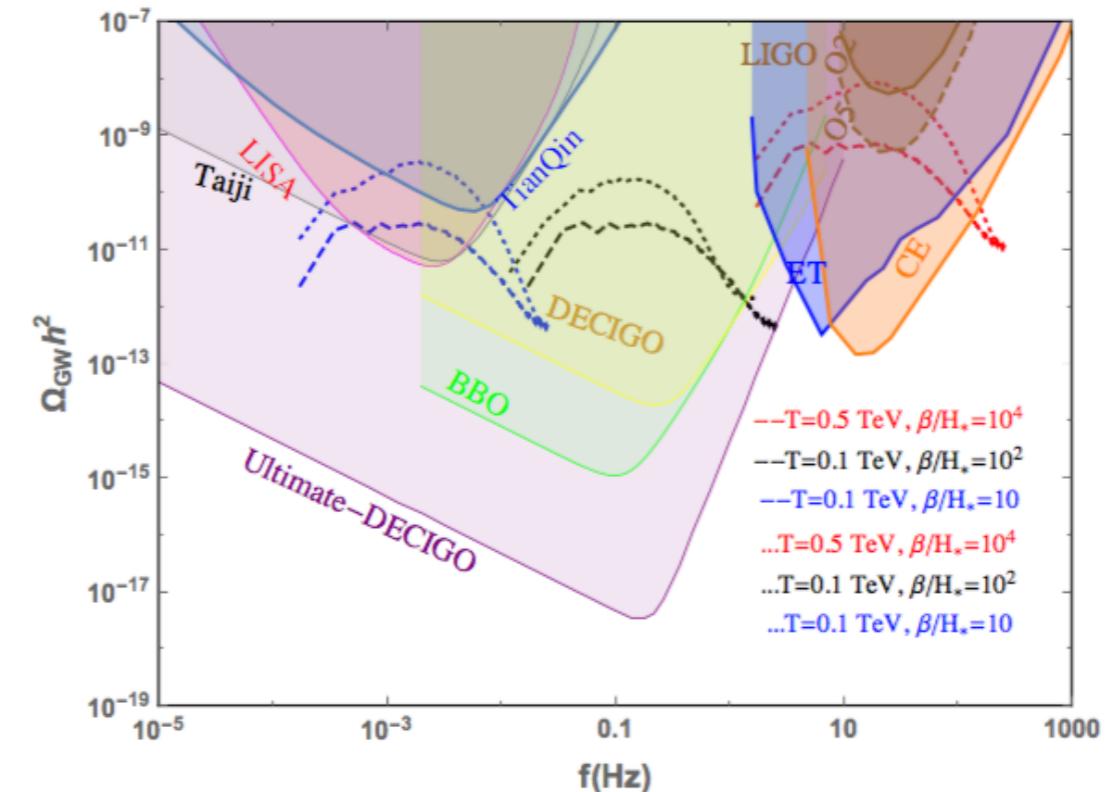
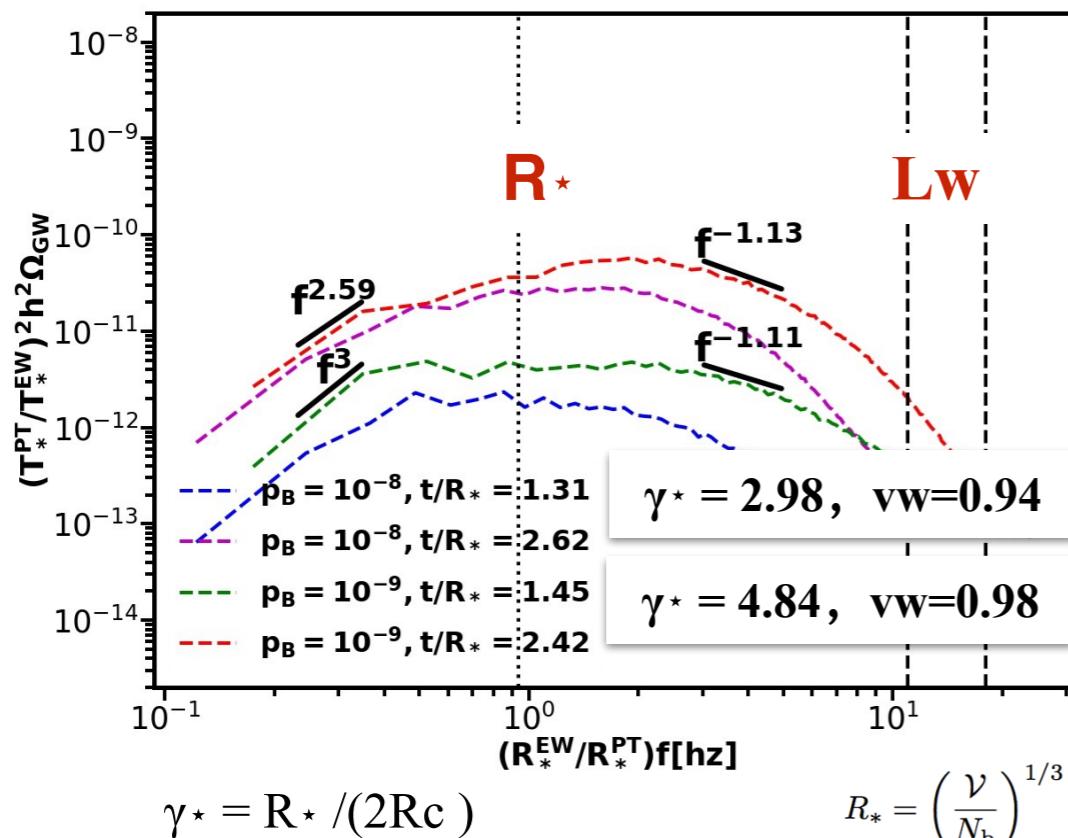
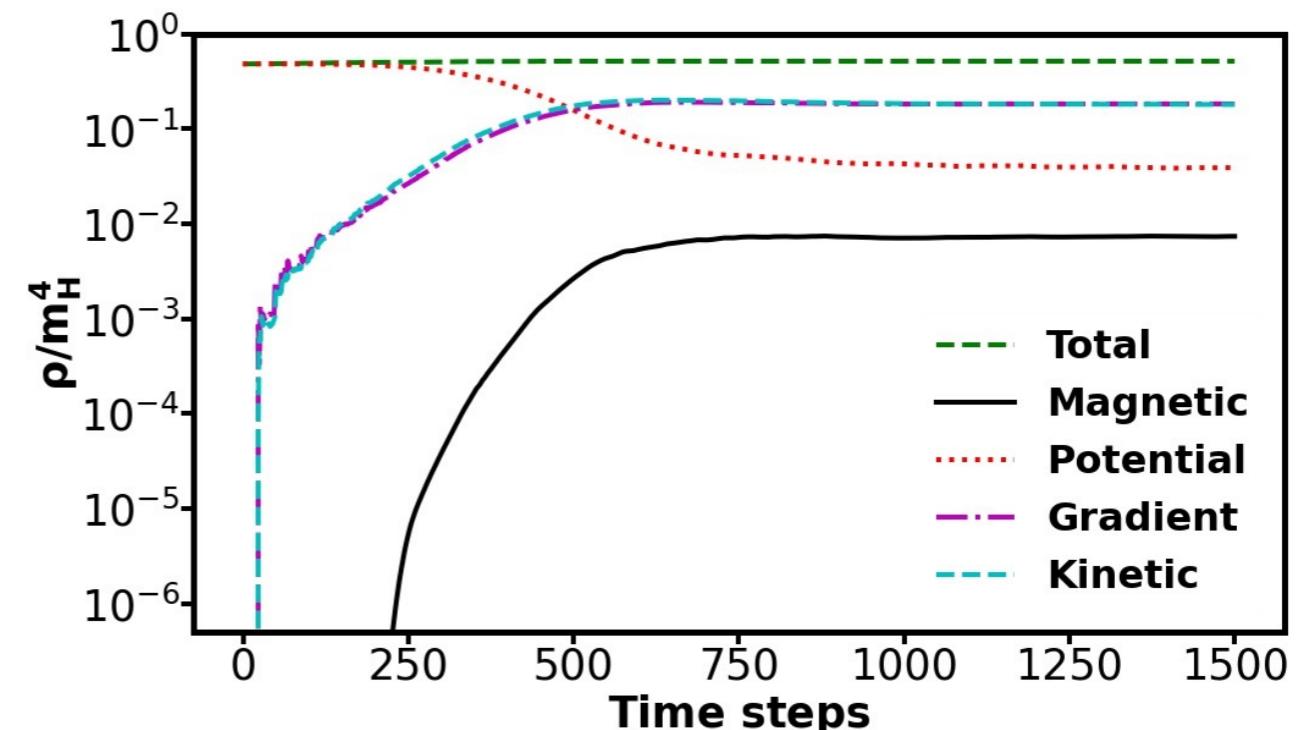
# ► 一步一阶电弱相变真空泡碰撞、合并产生引力波

$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{TT}$$

$$T_{\mu\nu} = \partial_\mu \Phi^\dagger \partial_\nu \Phi - g_{\mu\nu} \frac{1}{2} \text{Re}[(\partial_i \Phi^\dagger \partial^i \Phi)^2]$$

$$\langle \dot{h}_{ij}^{TT}(\mathbf{k}, t) \dot{h}_{ij}^{TT}(\mathbf{k}', t) \rangle = P_h(\mathbf{k}, t) (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}')$$

$$\frac{d\Omega_{\text{gw}}}{d\ln(k)} = \frac{1}{32\pi G \rho_c} \frac{k^3}{2\pi^2} P_h(\mathbf{k}, t)$$



Di, Wang, Zhou, **Bian\***, Cai\*, Liu\*, Phys.Rev.Lett. 126 (2021) 251102

# 真空泡碰撞、合并、流体演化产生引力波

## 有限温度有效势能

$$V(\phi, T) = \frac{1}{2}\gamma(T^2 - T_0^2)\phi^2 - \frac{1}{3}AT\phi^3 + \frac{1}{4}\lambda\phi^4$$

新物理

## 标量场-相对论理想流体运动方程

$$-\ddot{\phi} + \nabla^2\phi - \frac{\partial V}{\partial \phi} = \eta W(\dot{\phi} + V^i \partial_i \phi)$$

$\eta$ : 粒子和真空泡壁  
相互作用

$$\begin{aligned} \dot{E} + \partial_i(EV^i) + p[\dot{W} + \partial_i(WV^i)] - \frac{\partial V}{\partial \phi}W(\dot{\phi} + V^i \partial_i \phi) \\ = \eta W^2(\dot{\phi} + V^i \partial_i \phi)^2 \end{aligned}$$

$$\dot{Z}_i + \partial_j(Z_i V^j) + \partial_i p + \frac{\partial V}{\partial \phi} \partial_i \phi = -\eta W(\dot{\phi} + V^j \partial_j \phi) \partial_i \phi$$

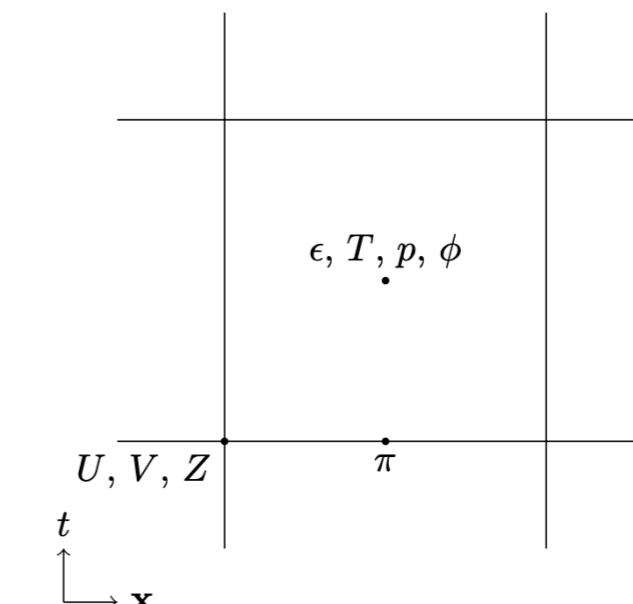
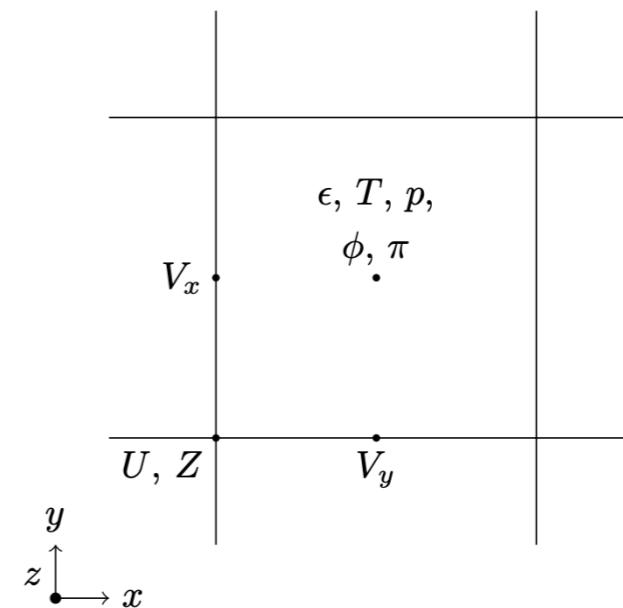
equation of state

$$\epsilon(T, \phi) = 3aT^4 + V(\phi, T) - T \frac{\partial V}{\partial T},$$

$$p(T, \phi) = aT^4 - V(\phi, T)$$

fluid momentum density  $Z_i = W(\epsilon + p)U_i$

fluid energy density  $E = W\epsilon$

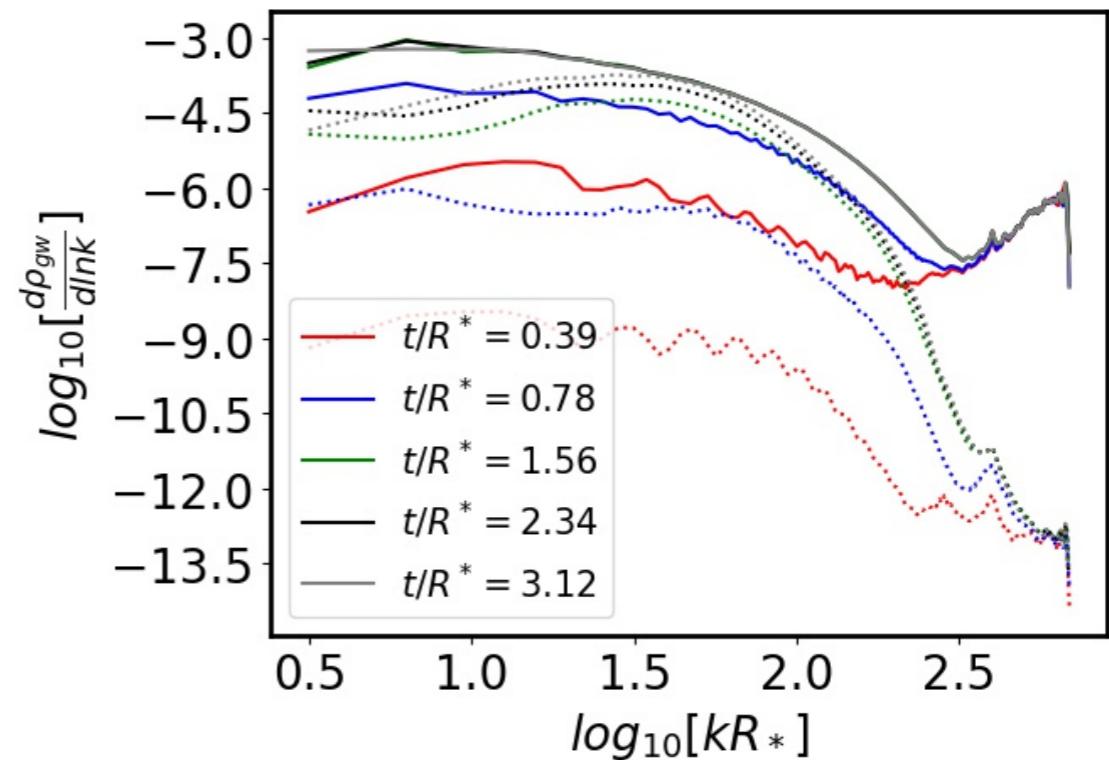
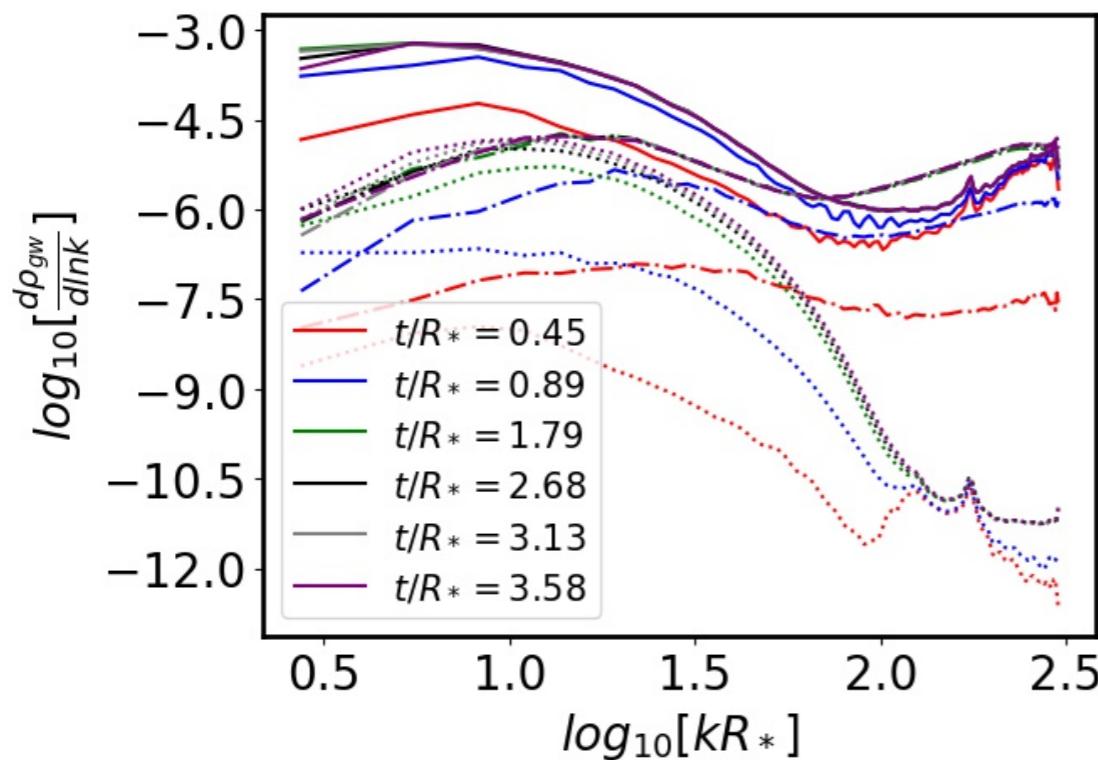
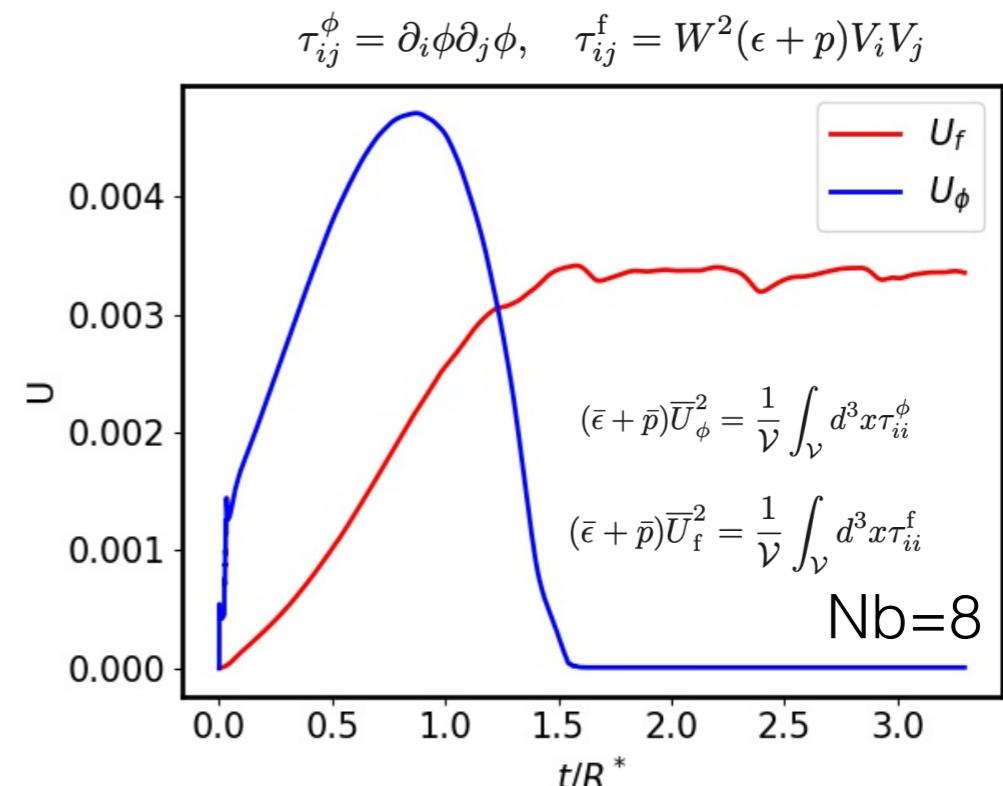
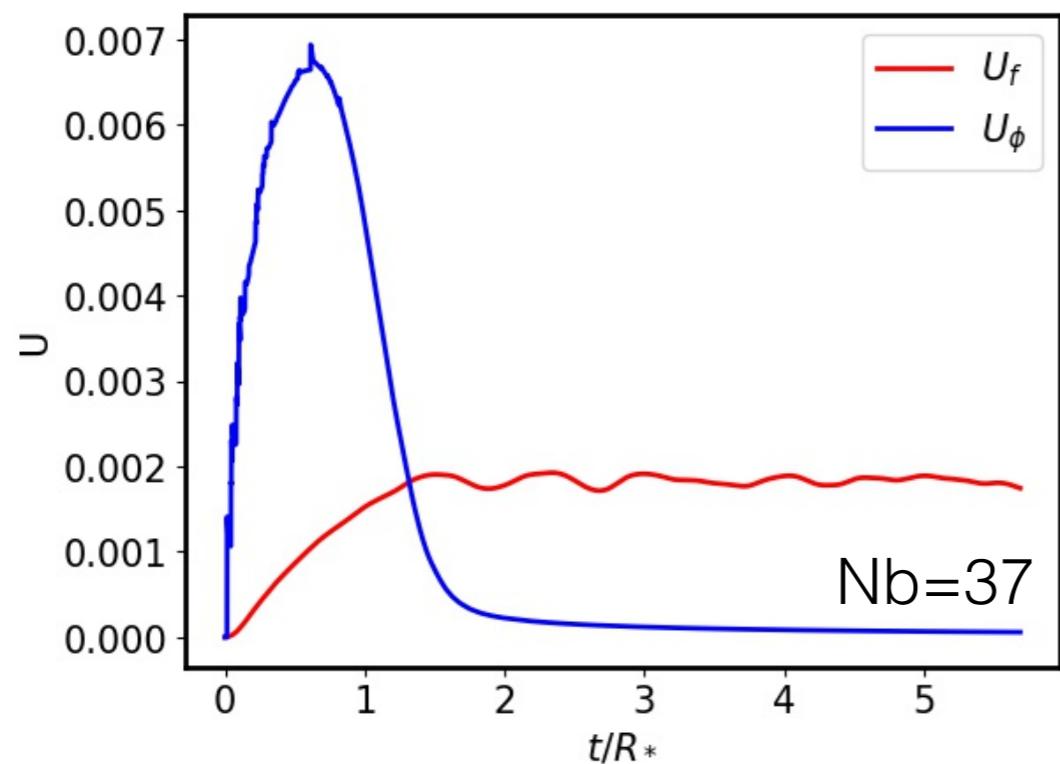


$V^i$  is the fluid 3-velocity

$U^i = W V^i$ ,  $W$ : relativistic  $\gamma$ -factor

1504.03291

# 真空泡碰撞、合并、流体演化产生引力波



To appear

# ► 对撞机&引力波甄别强一阶电弱相变引发的对称性破缺模式

## ■ Higgs&GWs

### SM+Scalar Singlet

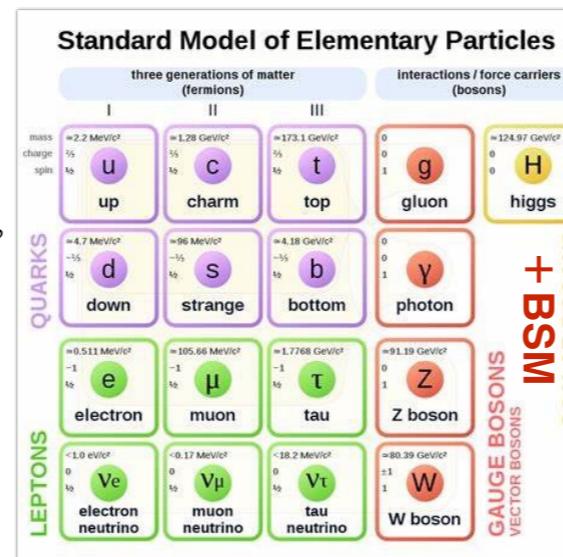
Profumo, Ramsey-Musolf, Wainwright, Winslow  
14, **Bian**, Huang, Shu 15, Cheng, **Bian** 17, **Bian**,  
Tang 18, Chen, Li, Wu, **Bian**, 19...

### SM+Scalar Doublet

Dorsch, Huber, Mimasu, No.14, Bernon,  
**Bian**, Jiang 17, **Bian**, Liu 18, Huang, Yu,  
18,...

### SM+Scalar Triplet

Zhou, Cheng, Deng, **Bian**, Wu  
18, Zhou, **Bian**, Guo, Wu 19, Ramsey-  
Musolf et al 21, Zhou,  
**Bian**, Du, 22,...



### Composite Higgs

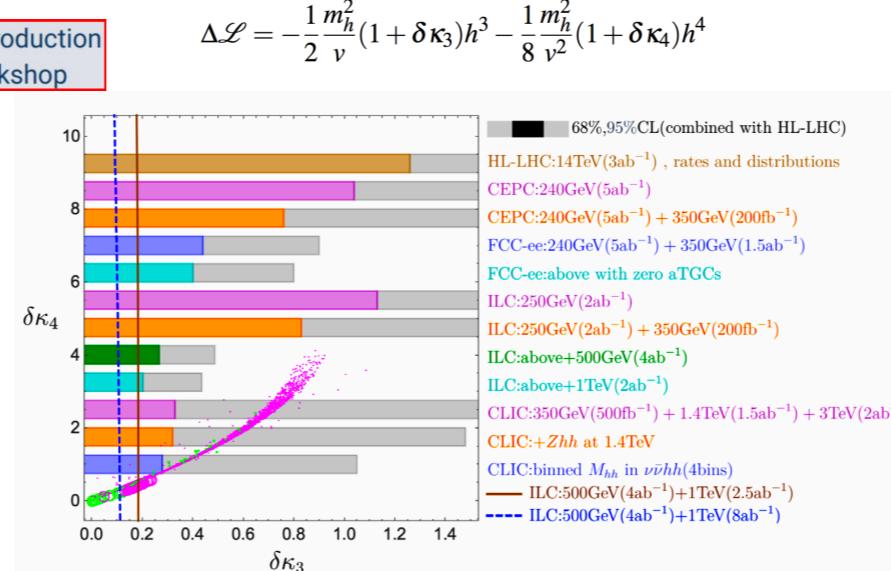
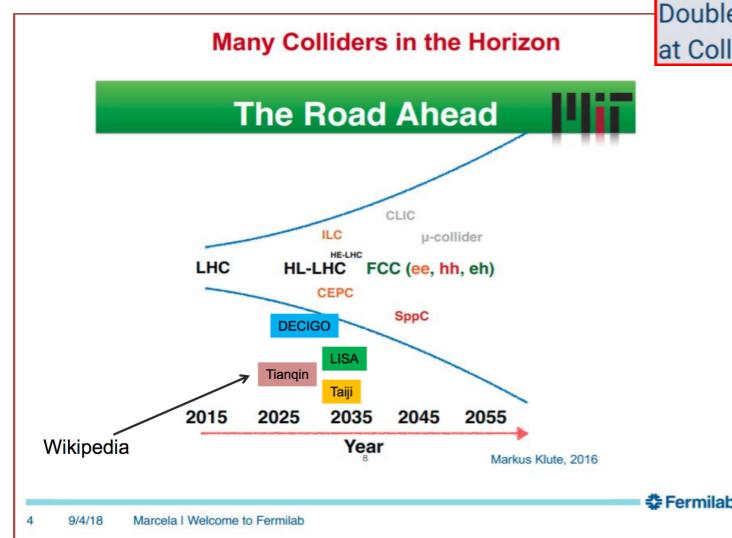
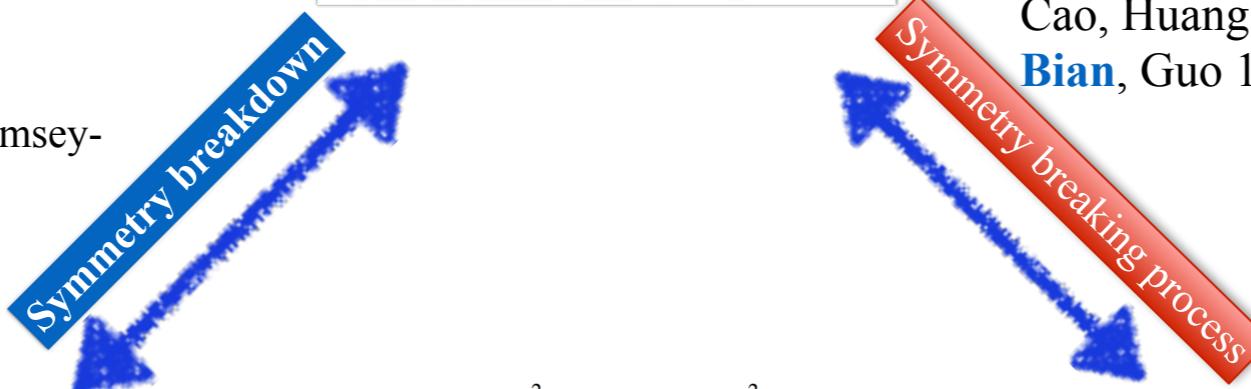
Bruggisser, Harling, Matsedonskyi, Servant, 18,  
**Bian**, Wu, Xie 19, **Bian**, Wu, Xie 20, ...

### NMSSM

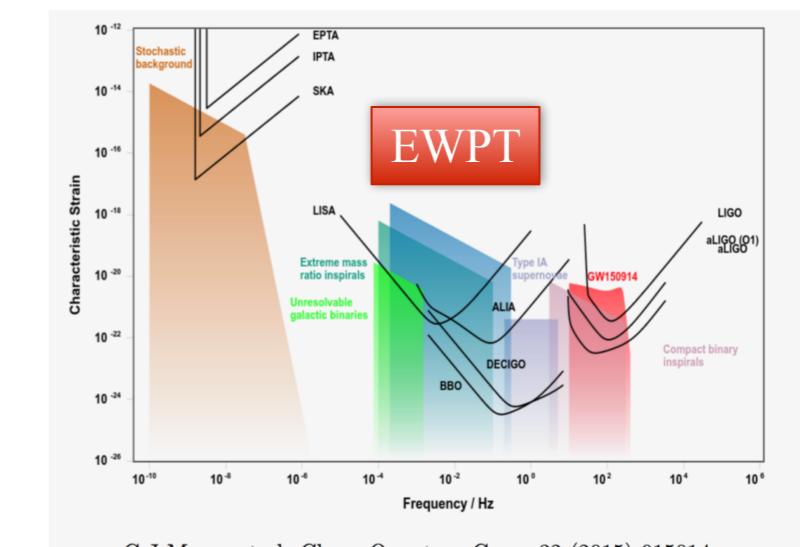
Bi, **Bian**, Huang, Shu, Yin 15, **Bian**, Guo, Shu  
17, Baum, Carena, Shah, Wagner, Wang 20, ...

### SMEFT

Cao, Huang, Xie, & Zhang 17, Zhou,  
**Bian**, Guo 19, Cai, Hashino, Wang, Yu, 22...



SNR > 10 for two-step and one-step SFOEWPT



PTA,LIGO,LISA,天琴,太极,...

# ► 脉冲星计时阵列实验PPTA 限制一阶相变

■ PPTA DR2观测数据限制早期宇宙低能标相变参数，进而限制暗区和类QCD相变

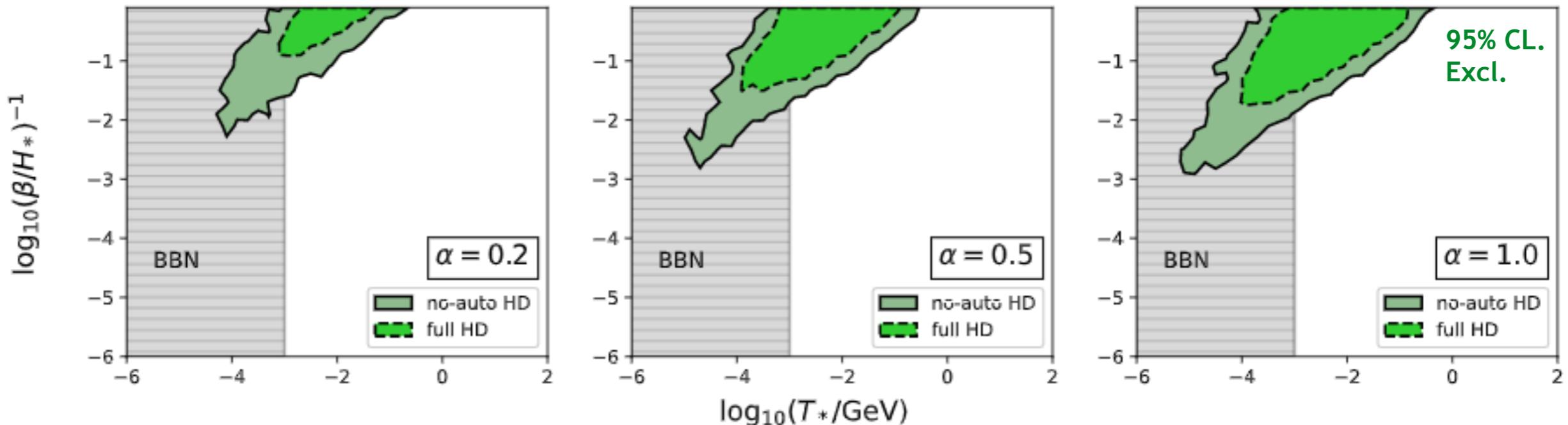
PHYSICAL REVIEW LETTERS 127, 251303 (2021)

Editors' Suggestion

Featured in Physics

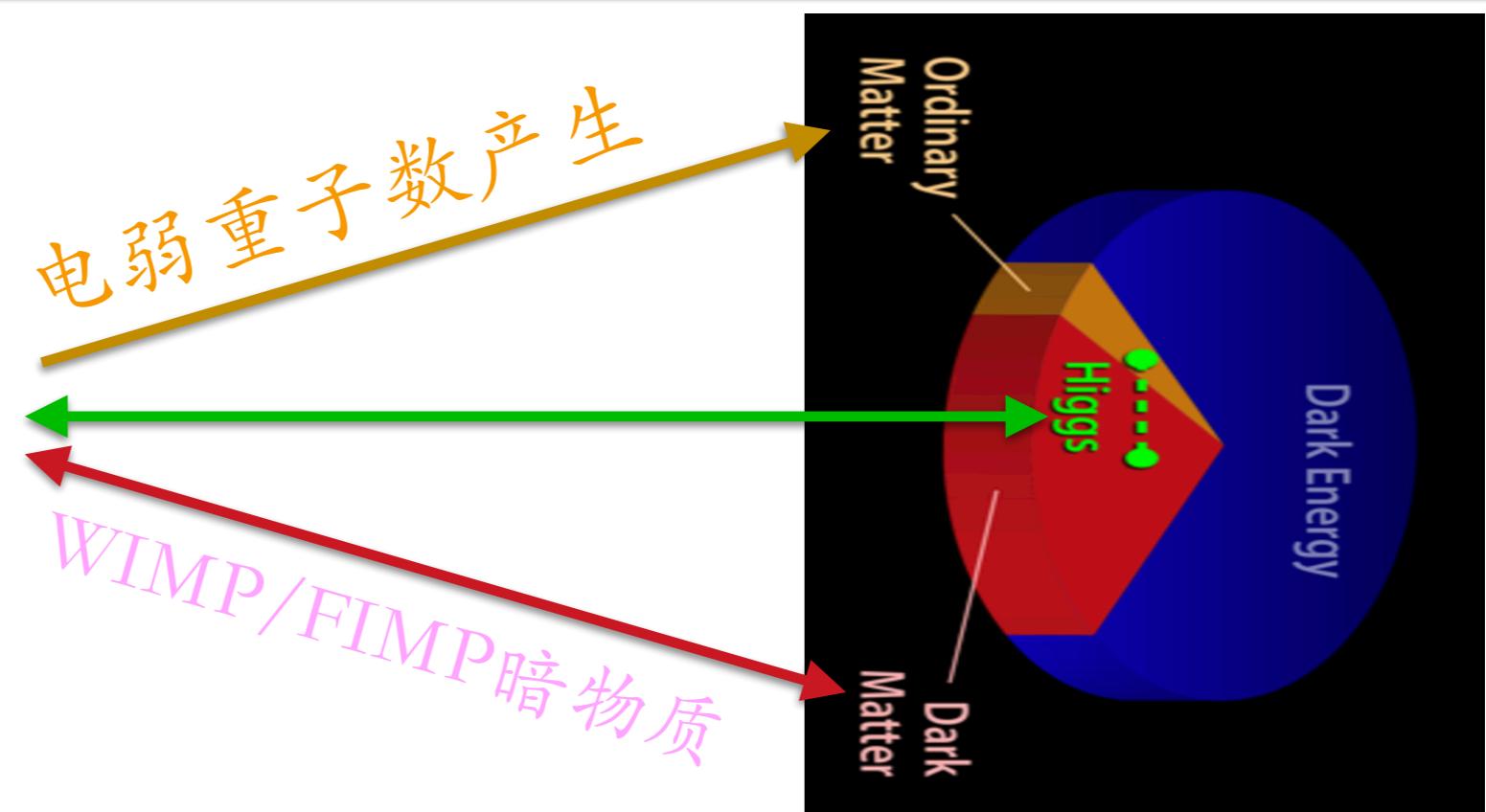
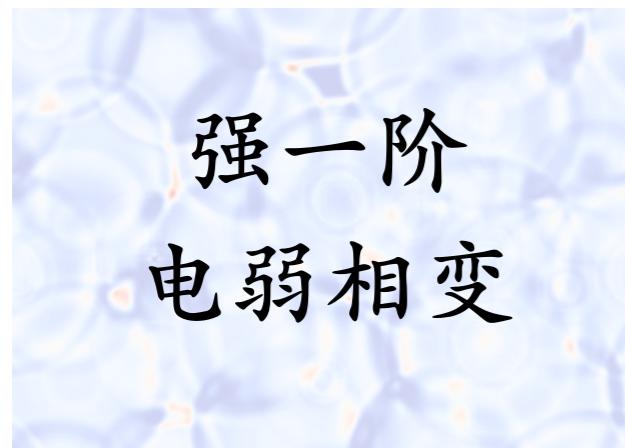
## Constraining Cosmological Phase Transitions with the Parkes Pulsar Timing Array

Xiao Xue<sup>1,2,3</sup>, Ligong Bian<sup>4,5,\*</sup>, Jing Shu,<sup>1,2,6,7,8,†</sup>, Qiang Yuan<sup>9,10,7,‡</sup>, Xingjiang Zhu<sup>11,12,13,§</sup>, N. D. Ramesh Bhat,<sup>14</sup>, Shi Dai<sup>15</sup>, Yi Feng<sup>16</sup>, Boris Goncharov<sup>11,12</sup>, George Hobbs,<sup>17</sup>, Eric Howard<sup>17,18</sup>, Richard N. Manchester<sup>17</sup>, Christopher J. Russell<sup>19</sup>, Daniel J. Reardon<sup>12,20</sup>, Ryan M. Shannon<sup>12,20</sup>, Renée Spiewak<sup>21,20</sup>, Nithyanandan Thyagarajan<sup>22</sup>, and Jingbo Wang<sup>23</sup>



## LIGO-Virgo O3 实验数据限制PeV能标一阶相变

Romero, Martinovic, Callister, Guo, et al., Phys.Rev.Lett. 126 (2021) 15, 151301



### Electroweak baryogenesis and dark matter from a singlet Higgs

#2

James M. Cline (McGill U.), Kimmo Kainulainen (Jyvaskyla U. and Helsinki Inst. of Phys. and Helsinki U.) (Oct, 2012)

Published in: JCAP 01 (2013) 012 • e-Print: [1210.4196](#) [hep-ph]

[pdf](#) [DOI](#) [cite](#)

217 citations

### 标准模型+实标量场

#### Gravitational wave, collider and dark matter signals from a scalar singlet electroweak baryogenesis

#4

Ankit Beniwal (Adelaide U. and Adelaide U., Sch. Chem. Phys.), Marek Lewicki (Adelaide U. and Warsaw U. and Adelaide U., Sch. Chem. Phys.), James D. Wells (DESY and Michigan U., MCTP), Martin White (Adelaide U., Sch. Chem. Phys. and Adelaide U.), Anthony G. Williams (Adelaide U., Sch. Chem. Phys. and Adelaide U.) (Feb 20, 2017)

Published in: JHEP 08 (2017) 108 • e-Print: [1702.06124](#) [hep-ph]

[pdf](#) [DOI](#) [cite](#)

131 citations

### Impact of a complex singlet: Electroweak baryogenesis and dark matter

#6

Minyuan Jiang (Beijing, Inst. Theor. Phys. and Beijing, KITPC and Nanjing U.), Ligong Bian (Beijing, Inst. Theor. Phys. and Beijing, KITPC), Weicong Huang (Beijing, Inst. Theor. Phys. and Beijing, KITPC), Jing Shu (Beijing, Inst. Theor. Phys. and Beijing, KITPC) (Feb 26, 2015)

Published in: Phys.Rev.D 93 (2016) 6, 065032 • e-Print: [1502.07574](#) [hep-ph]

[pdf](#)

[DOI](#)

[cite](#)

### 标准模型+复标量场

#5

#### Unified explanation for dark matter and electroweak baryogenesis with direct detection and gravitational wave signatures

#5

Mikael Chala (DESY), Germano Nardini (U. Bern, AEC), Ivan Sobolev (Lomonosov Moscow State U. and Moscow, INR) (May 27, 2016)

Published in: Phys.Rev.D 94 (2016) 5, 055006 • e-Print: [1605.08663](#) [hep-ph]

[pdf](#)

[DOI](#)

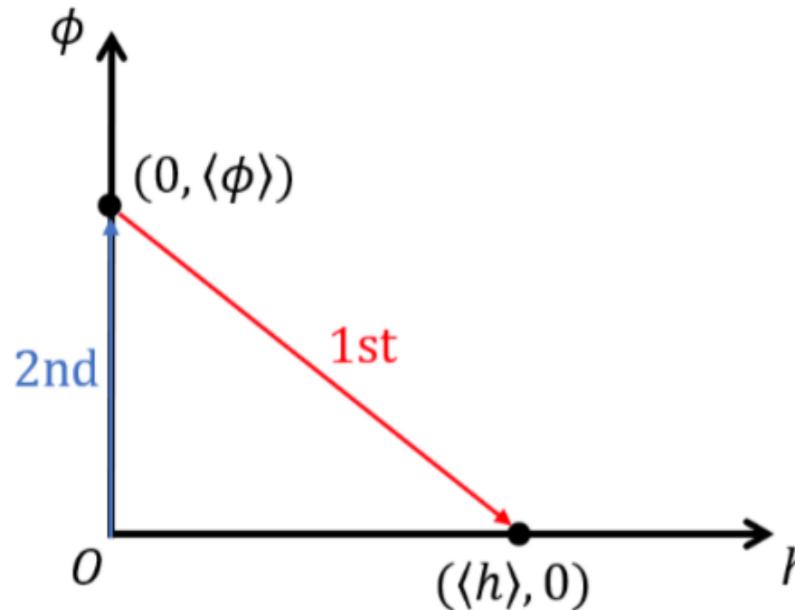
[cite](#)

### 复合希格斯

125 citations

# 两步相变

## Type-a

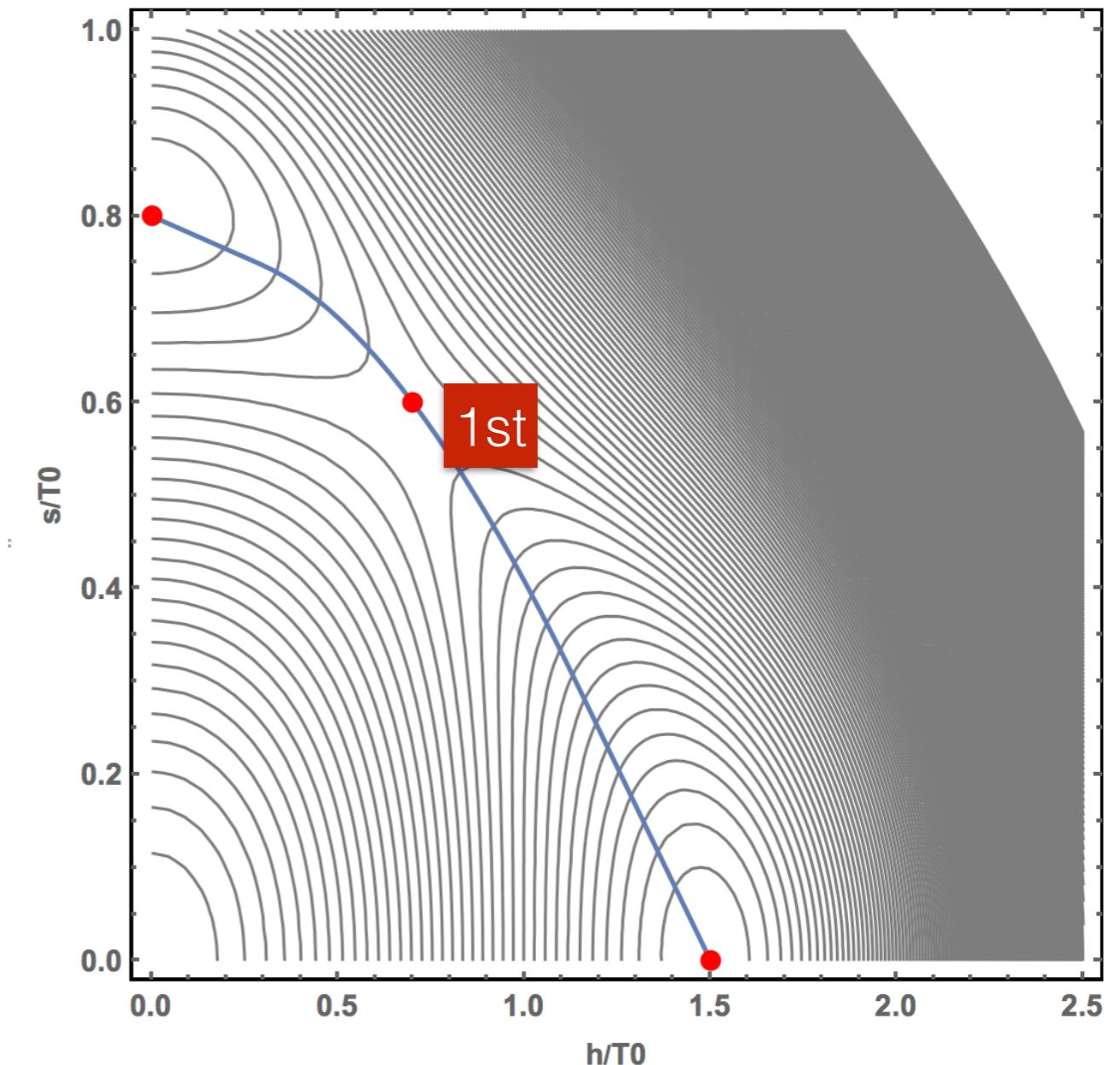


$$V_a(\phi, h, T) = \frac{1}{2}(\mu_\phi^2 + c_\phi T^2)\phi^2 + \frac{1}{2}\lambda_{h\phi}h^2\phi^2 + \frac{1}{4}\lambda_\phi\phi^4$$

$$+ \frac{1}{2}(-\mu_h^2 + c_h T^2)h^2 + \frac{1}{4}\lambda_h h^4$$

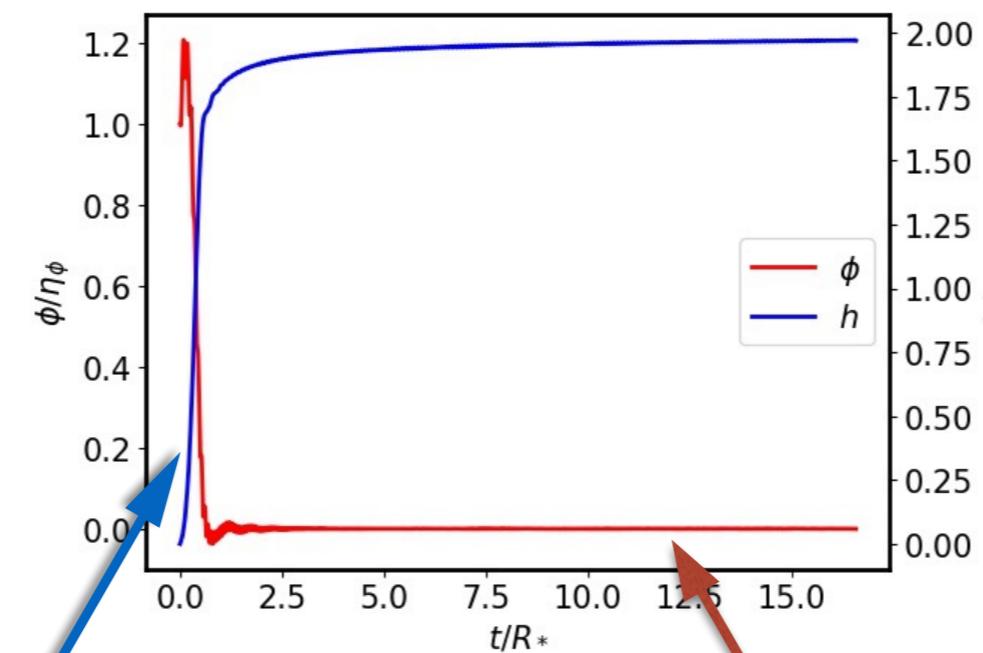
$$c_\phi = \lambda_\phi/4 + \lambda_{h\phi}/3$$

$$c_h = (2m_W^2 + m_Z^2 + 2m_t^2)/(4v^2) + \lambda_h/2 + \lambda_{h\phi}/12$$

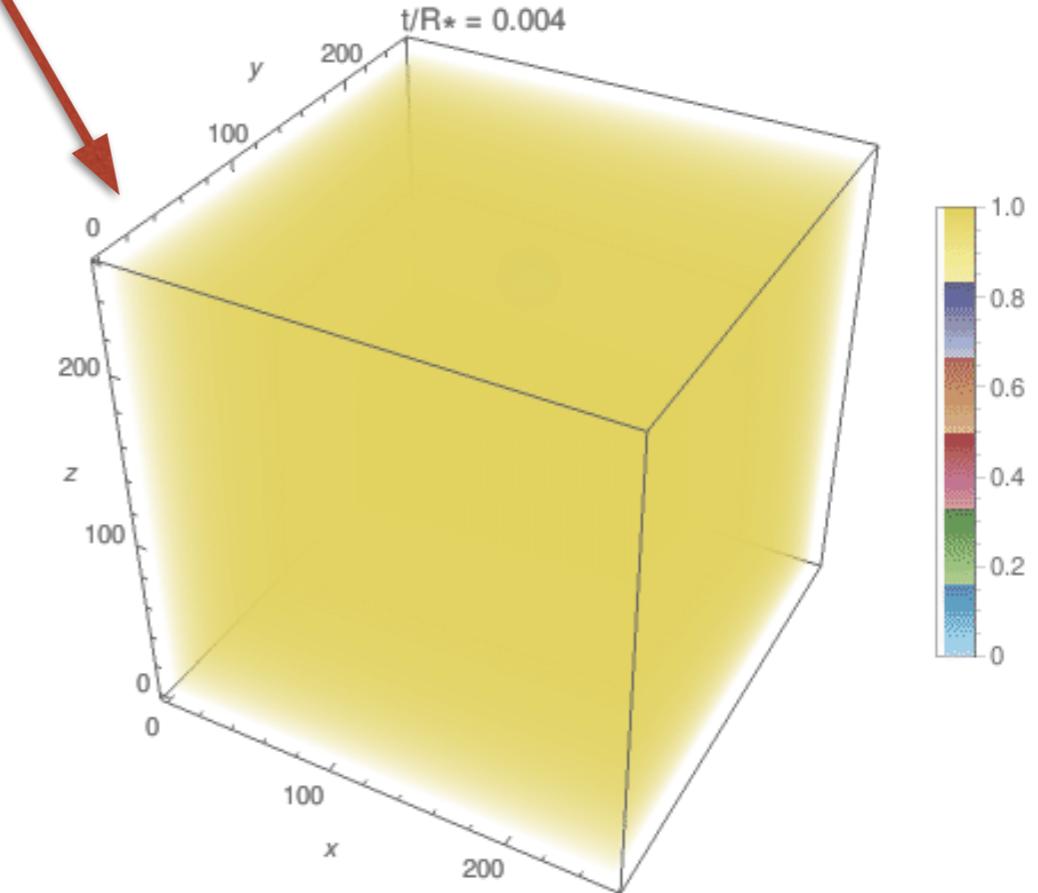
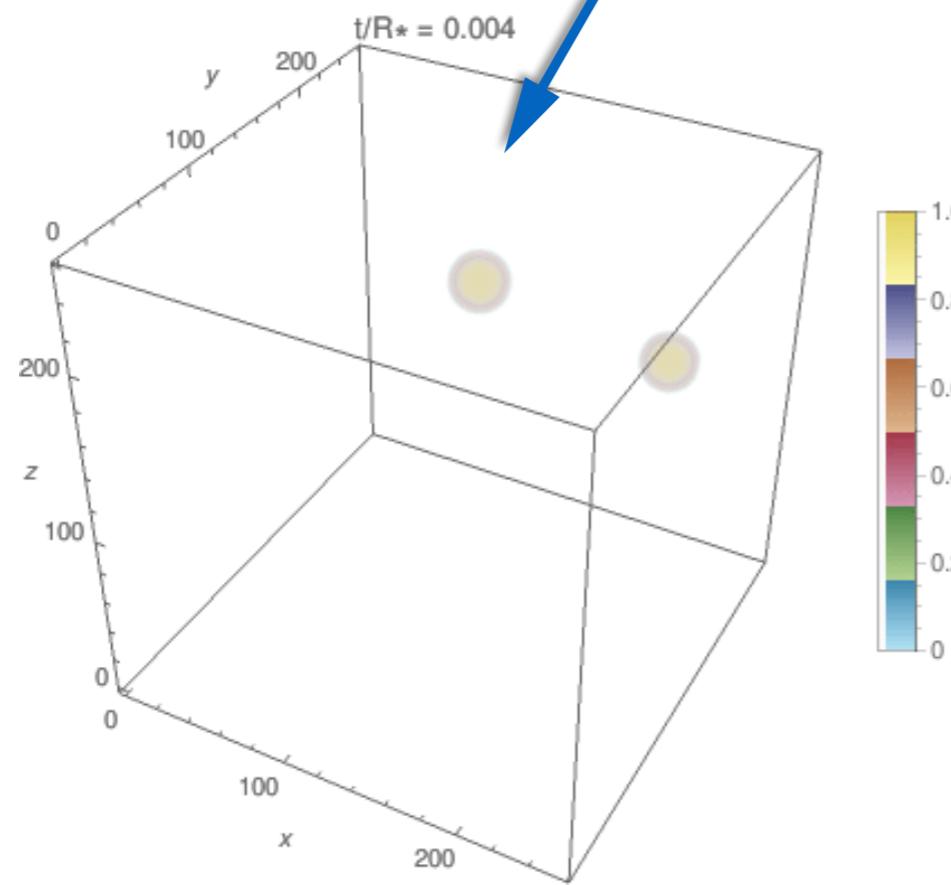


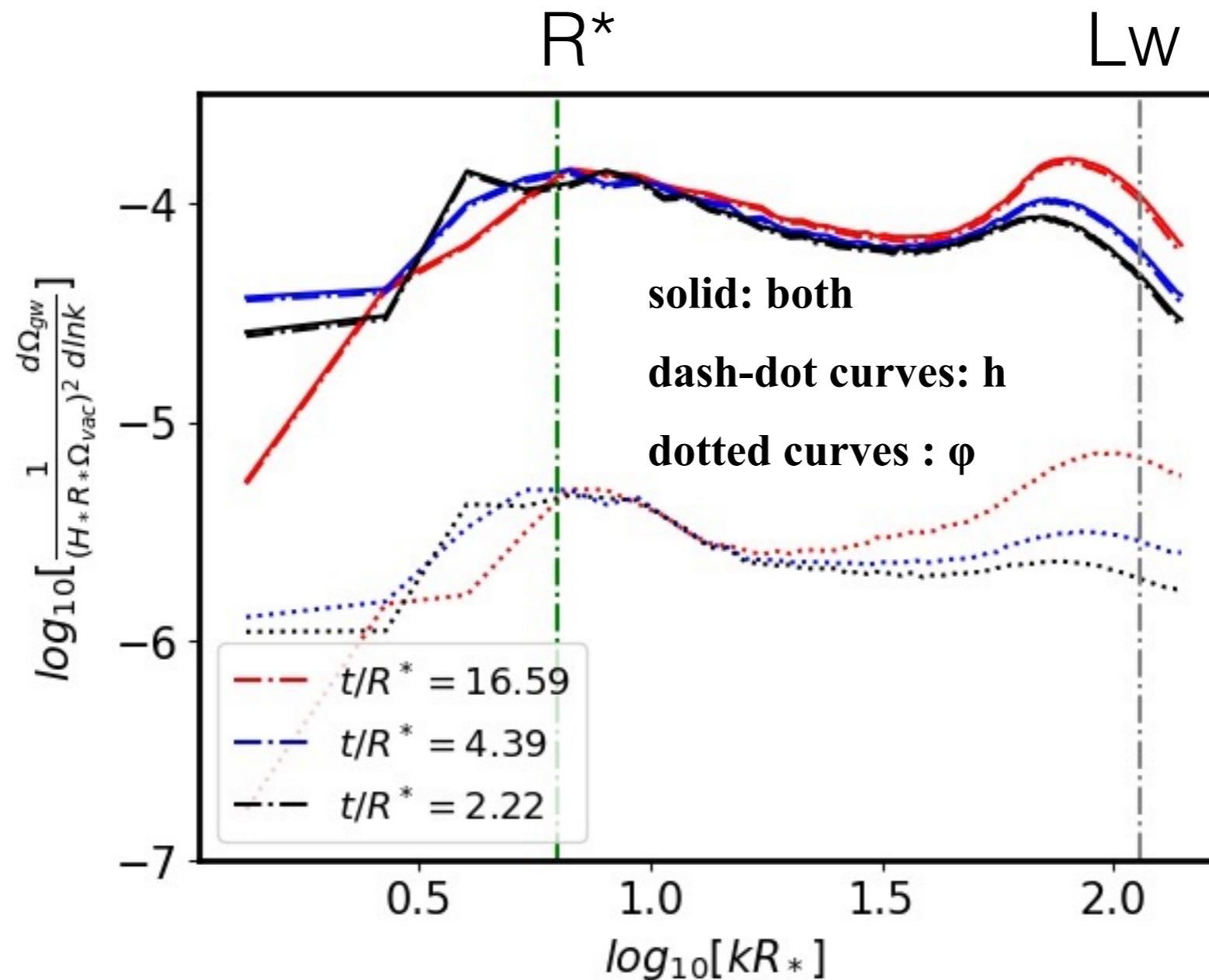
# ▶ 两步相变

Type-a



$$h(t=0, r) = \eta_h/2 \left[ 1 - \tanh \left( \frac{r - R_0}{L_w} \right) \right]$$
$$\phi(t=0, r) = \eta_\phi/2 \left[ 1 + \tanh \left( \frac{r - R_0}{L_w} \right) \right]$$



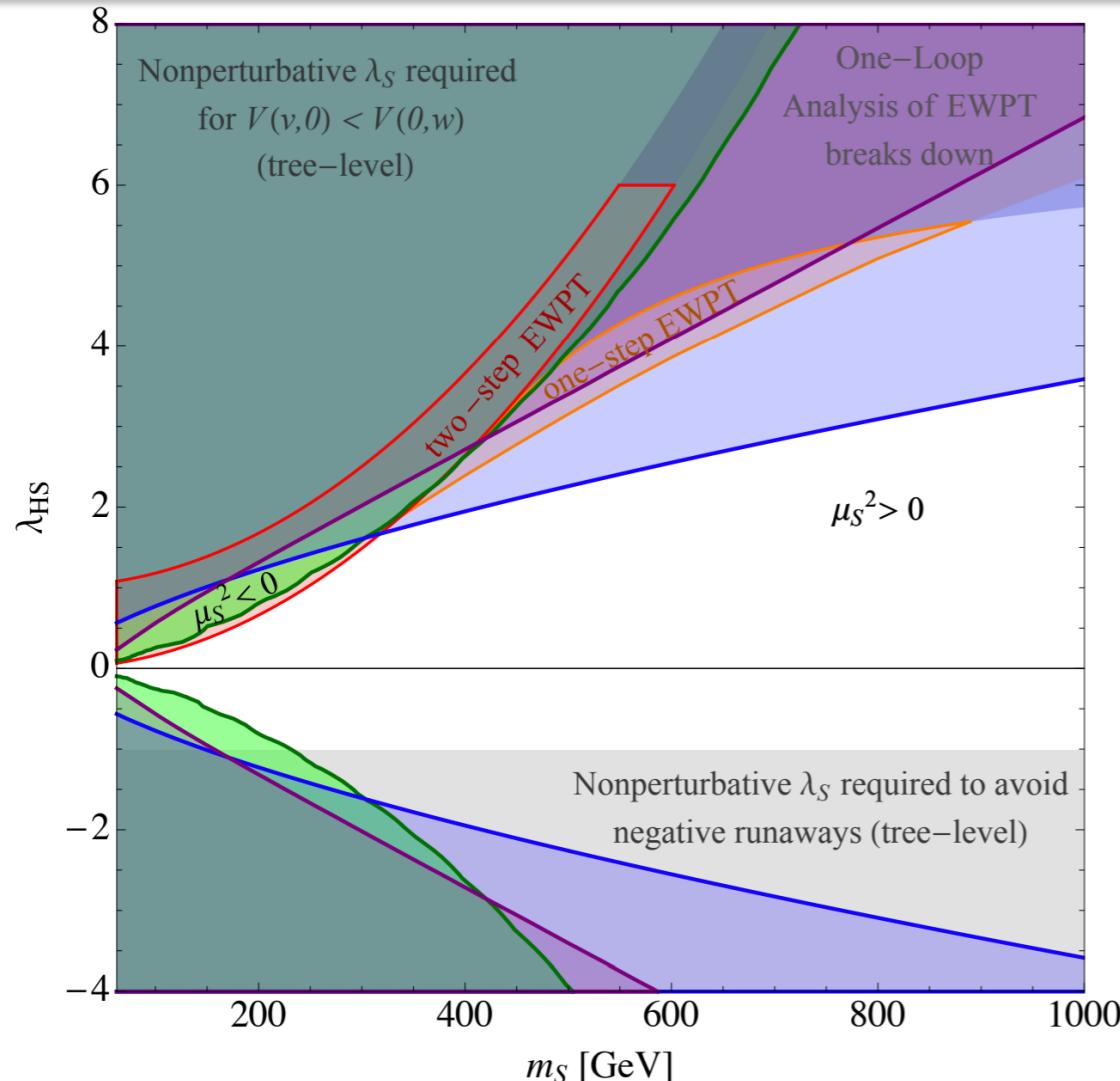
**Type-a**

# ► 对撞机探测两步一阶相变

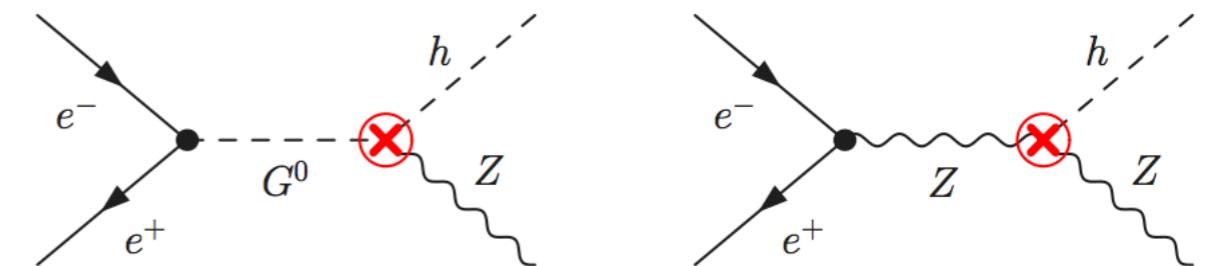
◎ Zh@ILC/CEPC

$$V_0 = -\mu^2 |H|^2 + \lambda |H|^4 + \frac{1}{2} \mu_S^2 S^2 + \lambda_{HS} |H|^2 S^2 + \frac{1}{4} \lambda_S S^4$$

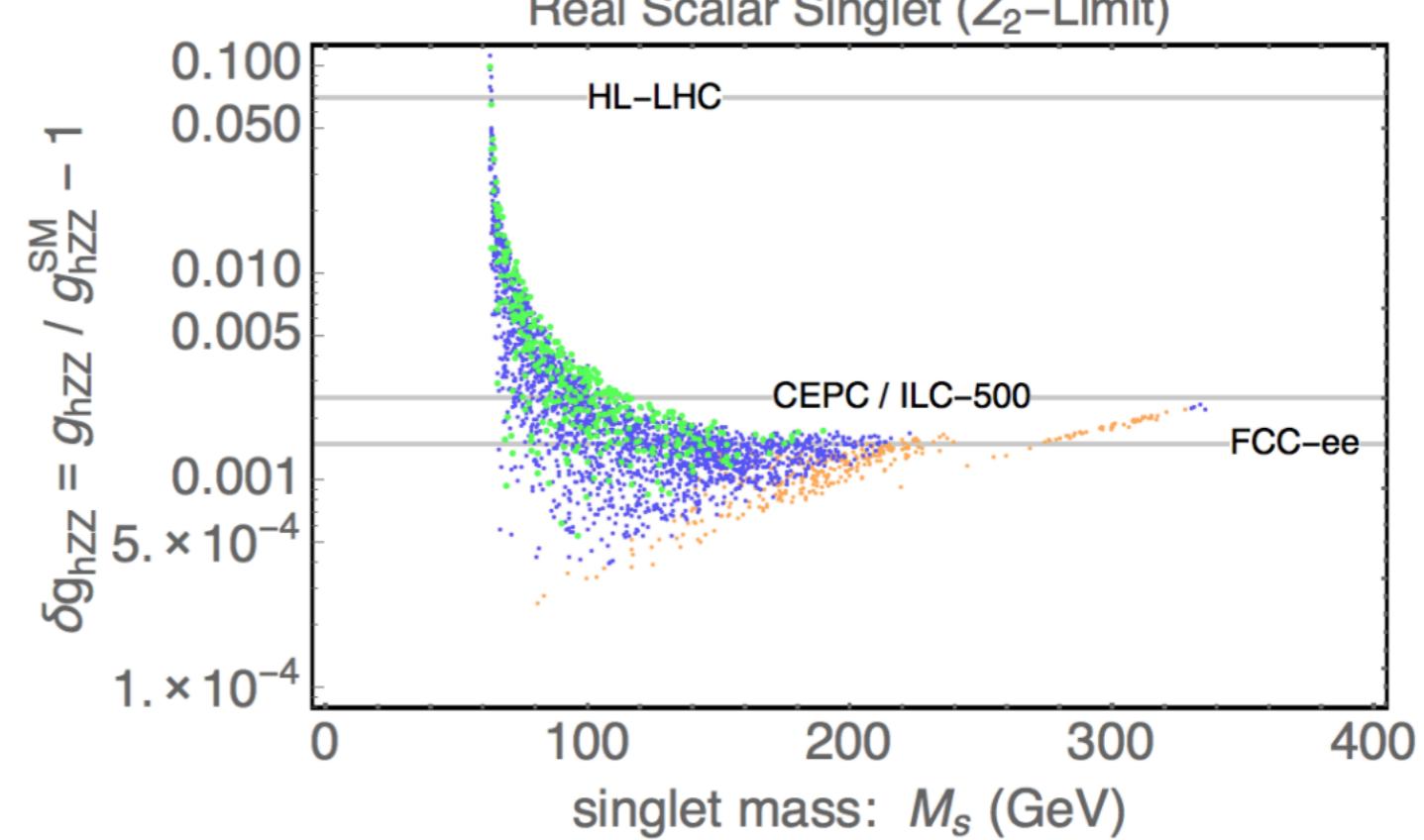
$$V_{\text{eff}}(h, T) = V_0(h) + V_0^{\text{CW}}(h) + V_T(h, T) + V_r(h, T)$$



Curtin, Meade, Yu, 1409.0005



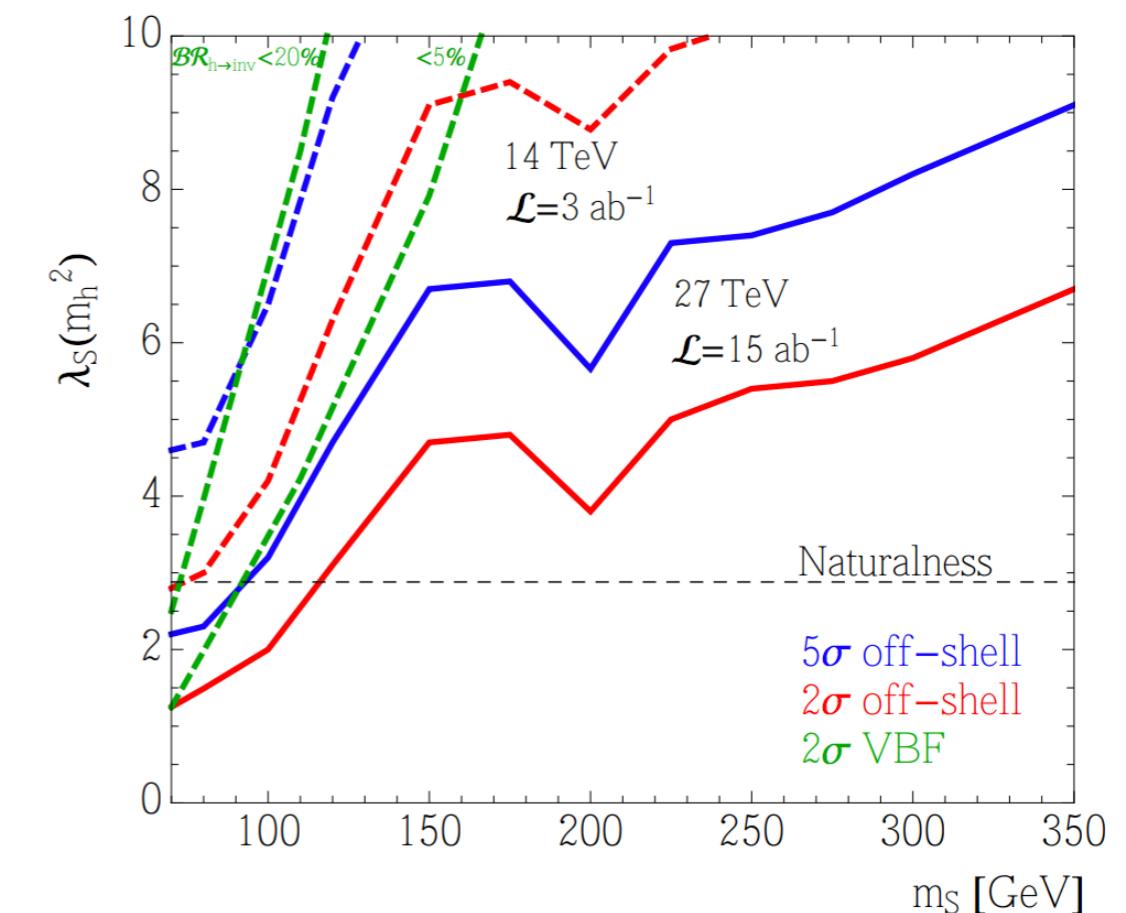
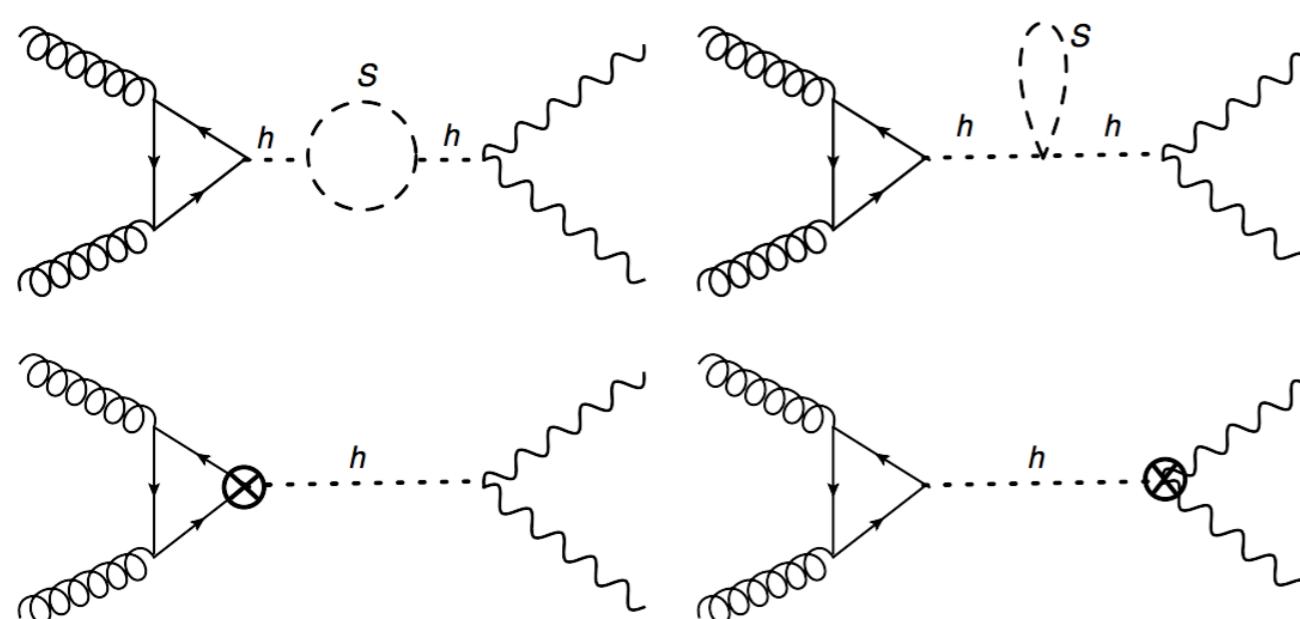
Craig, Englert, and McCullough, 1305.5251



Huang, Long, and Wang, 1608.06619

## ► 对撞机探测两步一阶相变

### ◎ Off-shell Higgs@LHC



Goncalves, Han, and Mukhopadhyay, 1710.02149

See also: Lee, Park, and Qian, 1812.02679

# Probing the Higgs Potential shape and EWPT patterns with GW production and Colliders complementarily

## ❖ Lattice simulation

- PT GW and Electroweak sphaleron simulations
- Topological defects: Magnetic monopoles, cosmic strings, domain walls

Axion string-wall ongoing project with R-G Cai and J Shu

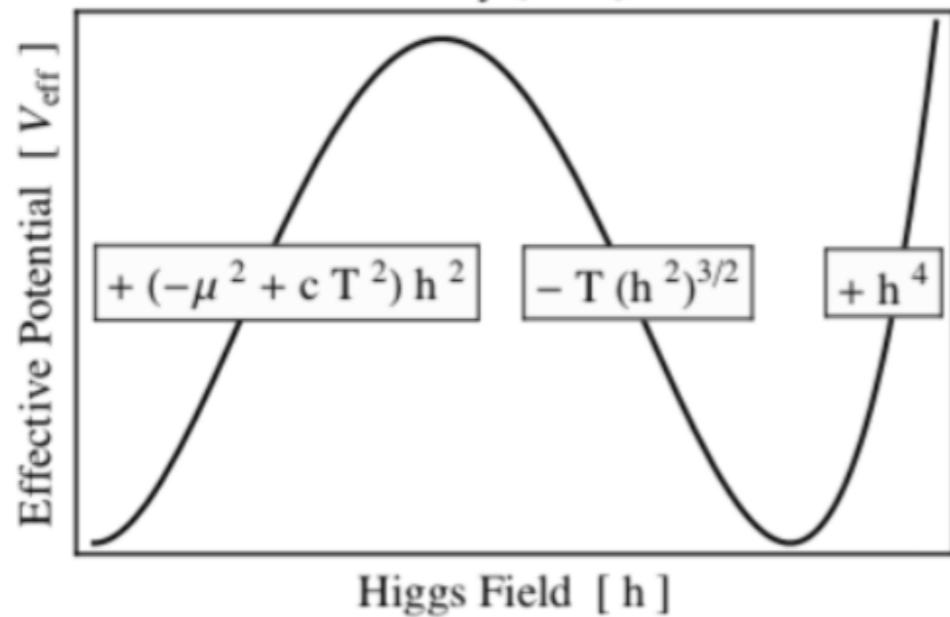
## ❖ Pheno

1. Baryon Asymmetry of the Universe and GW from FOPT
  - Sphaleron process, bubble velocity
2. DM and GW from FOPT
  - DM and high/low-scale PT, DM out-of-equilibrium & FOPT, PBH DM&FOPT

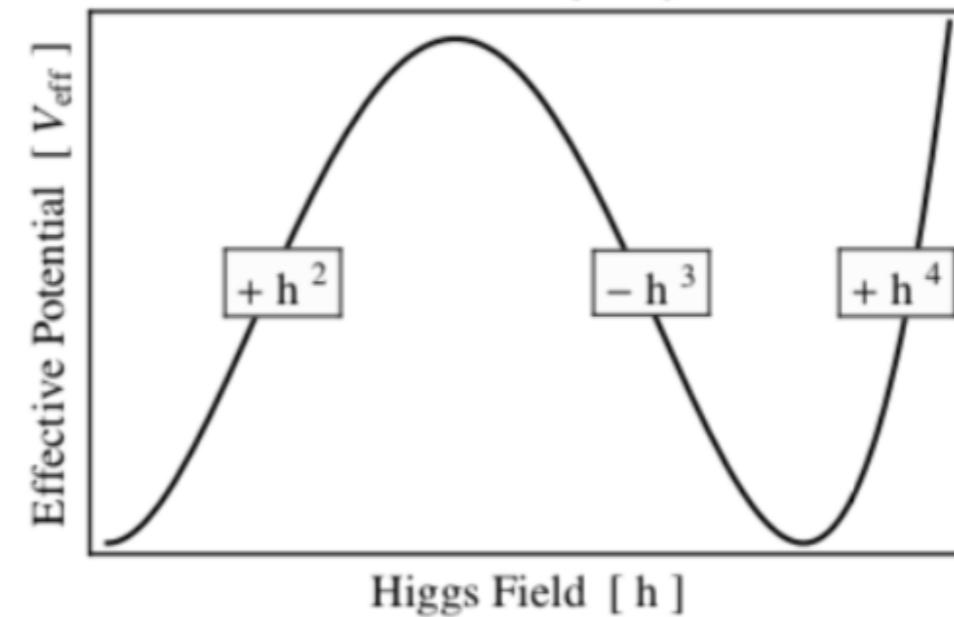
谢谢！

# 一步一阶相变模型分类

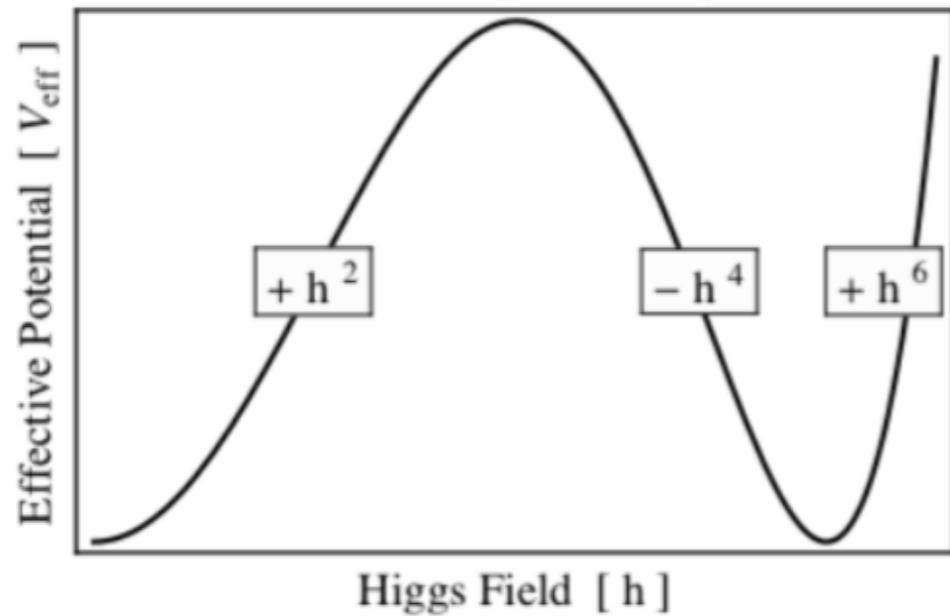
I. Thermally (BEC) Driven



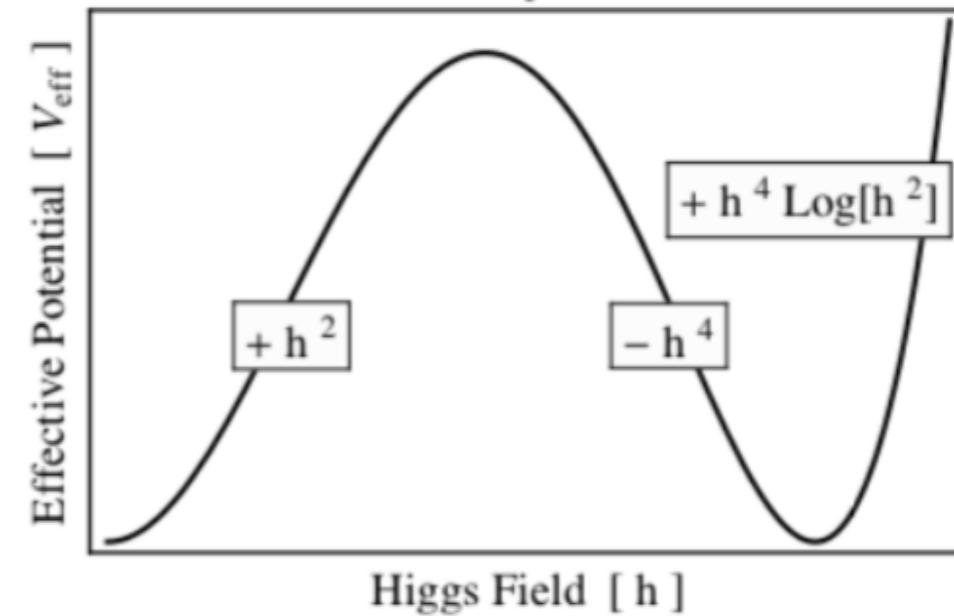
IIA. Tree-Level (Ren.) Driven



IIB. Tree-Level (Non-Ren.) Driven



III. Loop Driven



Chung, Long, Wang, Phys.Rev.D 87 (2013) 2, 023509

# 一阶相变引力波功率谱

## • Bubble collisions

$$\Omega_{\text{col}} h^2 = 1.67 \times 10^{-5} \left( \frac{H_*}{\beta} \right)^2 \left( \frac{\kappa\alpha}{1+\alpha} \right)^2 \left( \frac{100}{g_*} \right)^{1/3} \left( \frac{0.11 v_b^3}{0.42 + v_b^2} \right) \frac{3.8(f/f_{\text{env}})^{2.8}}{1 + 2.8(f/f_{\text{env}})^{3.8}}$$

peak frequency:  $f_{\text{env}} = 16.5 \times 10^{-6} \left( \frac{f_*}{H_*} \right) \left( \frac{T_*}{100 \text{GeV}} \right) \left( \frac{g_*}{100} \right)^{1/6} \text{Hz}$

## • Sound Wave

$$\Omega h_{\text{sw}}^2(f) = 2.65 \times 10^{-6} (H_* \tau_{\text{sw}}) \left( \frac{\beta}{H} \right)^{-1} v_b \left( \frac{\kappa_\nu \alpha}{1+\alpha} \right)^2 \left( \frac{g_*}{100} \right)^{-\frac{1}{3}} \left( \frac{f}{f_{\text{sw}}} \right)^3 \left( \frac{7}{4 + 3(f/f_{\text{sw}})^2} \right)^{7/2}$$

phase transition duration:  $\tau_{\text{sw}} = \min \left[ \frac{1}{H_*}, \frac{R_*}{\bar{U}_f} \right], H_* R_* = v_b (8\pi)^{1/3} (\beta/H)^{-1}$

Root-mean-square four-velocity of the plasma:

$$\bar{U}_f^2 \approx \frac{3}{4} \frac{\kappa_\nu \alpha}{1+\alpha}$$

peak frequency:

$$f_{\text{sw}} = 1.9 \times 10^{-5} \frac{\beta}{H} \frac{1}{v_b} \frac{T_*}{100} \left( \frac{g_*}{100} \right)^{\frac{1}{6}} \text{Hz}$$

## • MHD turbulence

$$\Omega h_{\text{turb}}^2(f) = 3.35 \times 10^{-4} \left( \frac{\beta}{H} \right)^{-1} \left( \frac{\epsilon \kappa_\nu \alpha}{1+\alpha} \right)^{\frac{3}{2}} \left( \frac{g_*}{100} \right)^{-\frac{1}{3}} v_b \frac{(f/f_{\text{turb}})^3 (1+f/f_{\text{turb}})^{-\frac{11}{3}}}{[1 + 8\pi f a_0 / (a_* H_*)]}$$

peak frequency:  $f_{\text{turb}} = 2.7 \times 10^{-5} \frac{\beta}{H} \frac{1}{v_b} \frac{T_*}{100} \left( \frac{g_*}{100} \right)^{\frac{1}{6}} \text{Hz}$