$\Xi_c \to \Xi \ell \nu$ in QCD sum rules

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Outline

- Introduction
- Two-point correlation function: pole residue
- Three-point correlation function: form factors
- Summary and outlook

Introduction

Introduction

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BELLE:v1

$$\mathcal{B}(\Xi_c^0 \to \Xi^- e^+ \nu_e) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50)\%,$$
$$\mathcal{B}(\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu) = (1.71 \pm 0.17 \pm 0.13 \pm 0.50)\%,$$

BELLE:v4

$$\mathcal{B}(\Xi_c^0 \to \Xi^- e^+ \nu_e) = (1.31 \pm 0.04 \pm 0.07 \pm 0.38)\%,$$
$$\mathcal{B}(\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu) = (1.27 \pm 0.06 \pm 0.10 \pm 0.37)\%,$$

ALICE

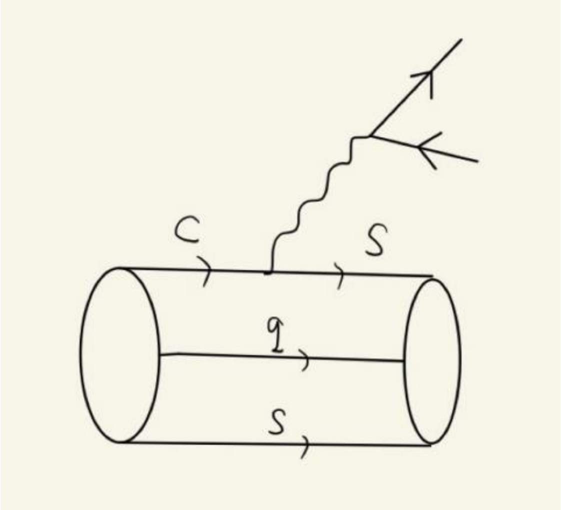
$$\mathcal{B}(\Xi_c^0 \to \Xi^- e^+ \nu_e) = (2.43 \pm 0.25 \pm 0.35 \pm 0.72)\%,$$ PoS ICHEP2020 (2021) 524

Much smaller uncertainties than the world average $(1.8 \pm 1.2)\%$ in PDG2020

Introduction

- Allow an independent determination of |Vcs|

- Form factors are inputs for the analysis of non-leptonic decays



$\Xi_c \to \Xi l \nu$

Chin.Phys.C 46 (2022) 1, 011002

LQCD:

$$\mathcal{B}(\Xi_c^0 \to \Xi^- e^+ \nu_e) = (2.38 \pm 0.30 \pm 0.32 \pm 0.07)\%,$$
$$\mathcal{B}(\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu) = (2.29 \pm 0.29 \pm 0.30 \pm 0.06)\%.$$

Two-point correlation function: pole residue

Two-point correlation function

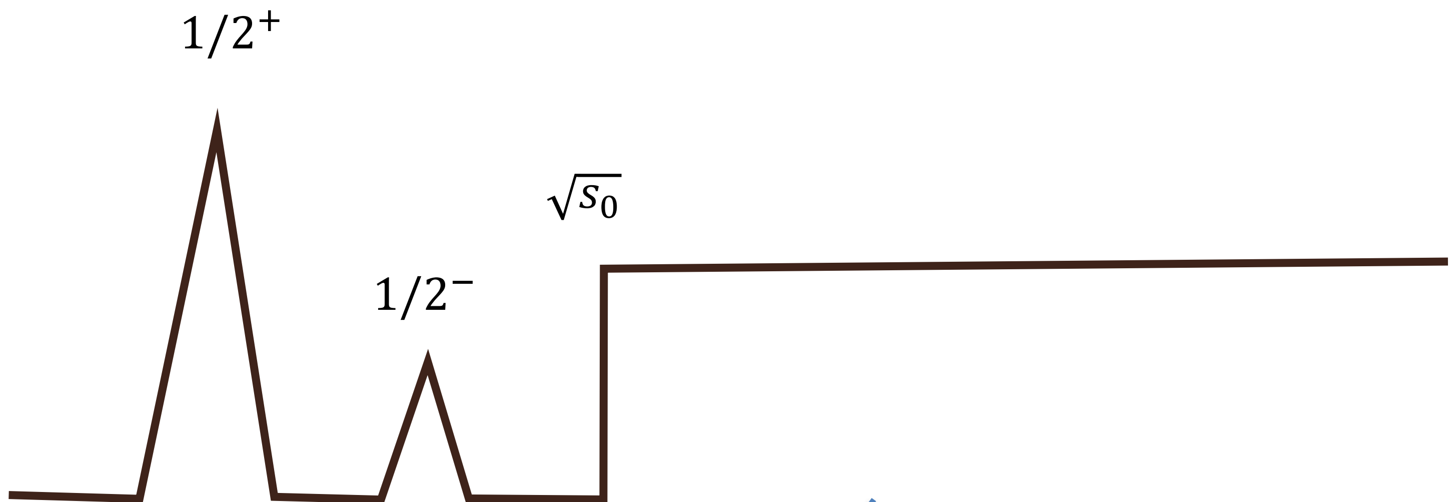
$$J_{\Xi_Q} = \epsilon_{abc}(q_a^T C\gamma_5 s_b)Q_c$$
$$J_{\Xi} = \epsilon_{abc}(s_a^T C\gamma^\mu s_b)\gamma_\mu\gamma_5 q_c$$

$$\Pi(p) = i\int d^4x e^{ip\cdot x}\langle 0|T\{J(x)\bar{J}(0)\}|0\rangle$$

$$\Pi^{\mathrm{had}}(p) = \lambda_+^2\frac{\not{p}+M_+}{M_+^2-p^2} + \lambda_-^2\frac{\not{p}-M_-}{M_-^2-p^2} + \cdots$$
$$\Pi^{\mathrm{QCD}}(p) = A(p^2)\not{p} + B(p^2)$$

$$(M_+ + M_-)\lambda_+^2\exp(-M_+^2/T_+^2) = \int_\Delta^{s_+} ds(M_-\rho^A + \rho^B)\exp(-s/T_+^2)$$

Weighted average



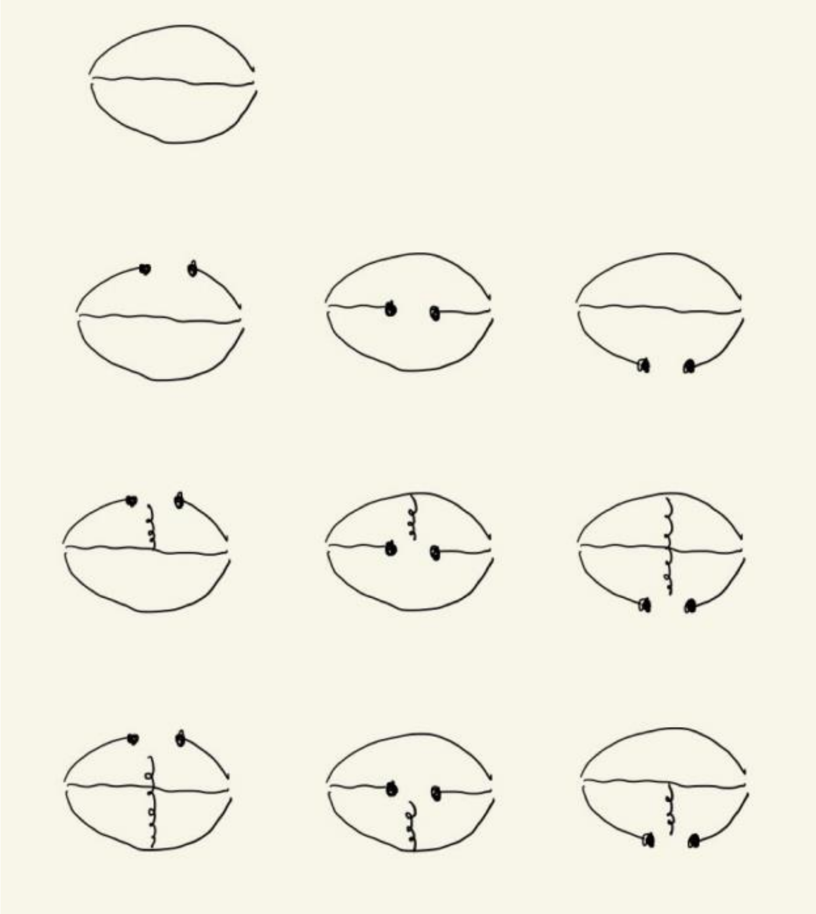
$$\Pi^{\mathrm{had}}(p) = \lambda_+^2 \frac{\not{p} + M_+}{M_+^2 - p^2} + \lambda_-^2 \frac{\not{p} - M_-}{M_-^2 - p^2} + \cdots$$

All the diagrams considered

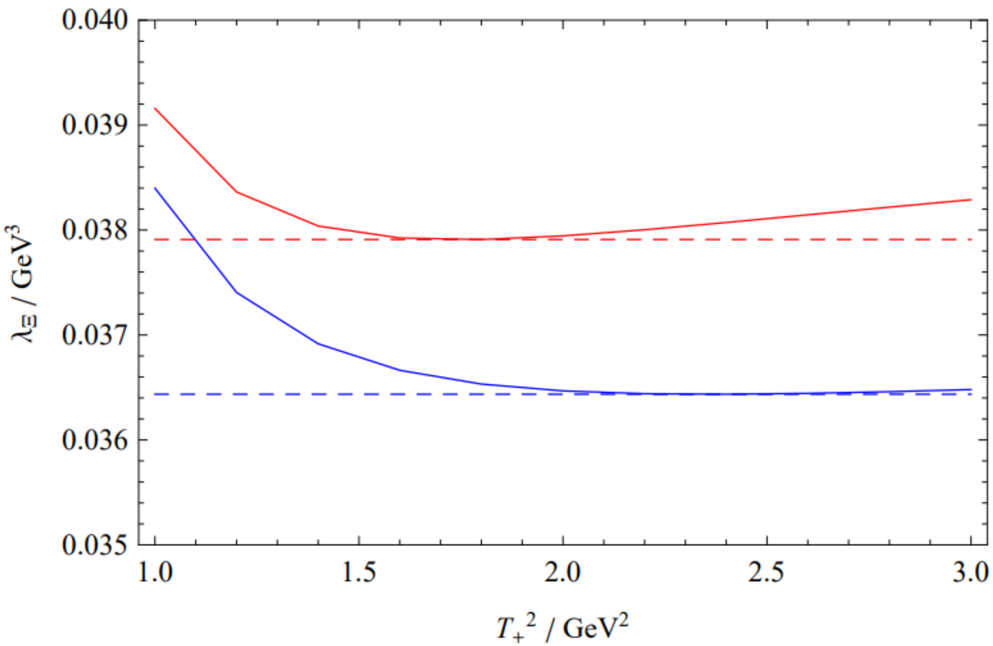
Dim-0: Perturbative

Dim-3: Quark condensate

Dim-5: Quark-gluon condensate



The results for 2PCF of Ξ



$(s_+/\mathrm{GeV}^2,\ T_+^2/\mathrm{GeV}^2)$	λ_+/GeV^3	M_+/GeV	$M_+^{\mathrm{exp}}/\mathrm{GeV}$
$(1.70^2, 2.4),\ (1.75^2, 1.8)$	0.0364 ± 0.0015	1.317 ± 0.003	1.315

optimal suboptimal

Three-point correlation function

QCD Sum Rules Analysis of Weak Decays of Doubly-Heavy Baryons

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We calculate the weak decay form factors of doubly-heavy baryons using three-point QCD sum rules. The Cutkosky rules are used to derive the double dispersion relations. We include perturbative contributions and condensation contributions up to dimension five, and point out that the perturbative contributions and condensates with lowest dimensions dominate. An estimate of part of gluon-gluon condensates show that it plays a less important role. With these form factors at hand, we present a phenomenological study of semileptonic decays. The future experimental facilities can test these predictions, and deepen our understanding of the dynamics in decays of doubly-heavy baryons.

Eur.Phys.J.C 79 (2019) 6, 501 • e-Print: 1903.03921 [hep-ph]

3 key techniques

1 -- Too many Dirac structures?

$$\Pi_\mu \sim (\not{p}_2 + M_2)(F_1 \frac{p_{1\mu}}{M_1} + F_2 \frac{p_{2\mu}}{M_2} + F_3 \gamma_\mu)(\not{p}_1 + M_1)$$

$$\not{p}_2 p_{1\mu} \not{p}_1$$

$$\not{p}_2 p_{1\mu}$$

$$\vdots$$

2*3*2 = 12 种Dirac结构

$$1/2^+ \to 1/2^+$$

$$1/2^+ \to 1/2^-$$

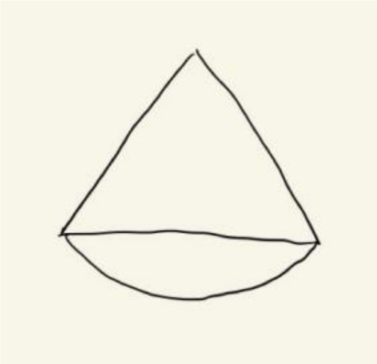
$$1/2^- \to 1/2^+$$

$$1/2^- \to 1/2^-$$

3*4 = 12 个形状因子

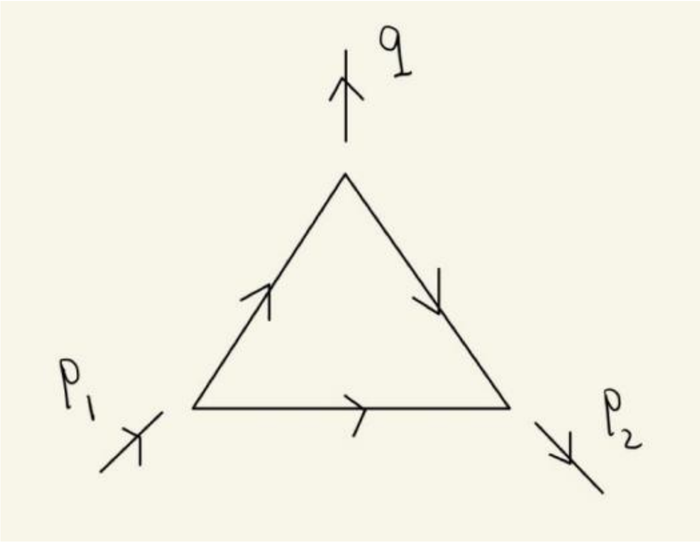
2 -- Verify the DDR

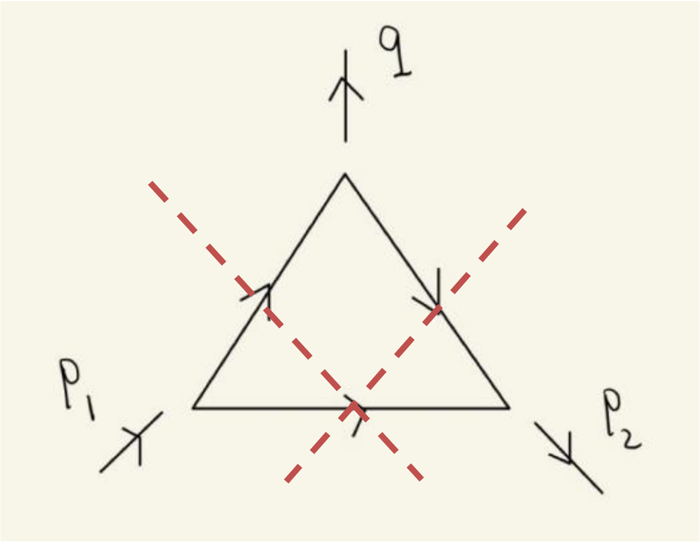
$$A_i(p_1^2, p_2^2, q^2) = \int^{\infty} ds_1 \int^{\infty} ds_2 \frac{\rho_i(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)}$$



?

2 -- Verify the DDR





Way 1:

FINITE!

Calculate it directly!

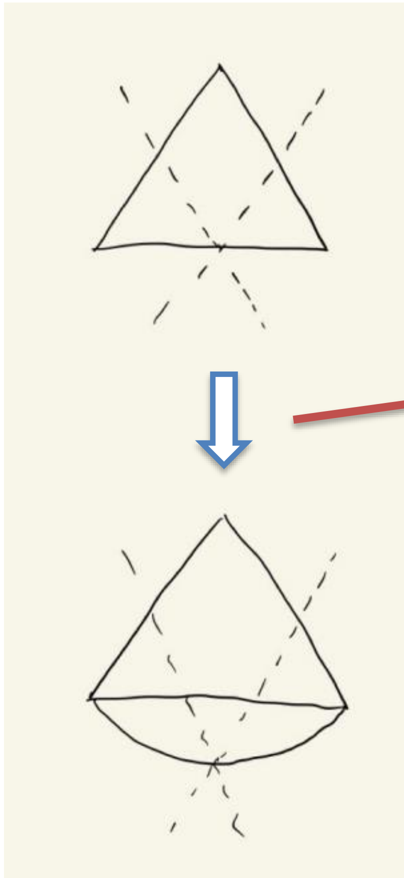
Way 2:

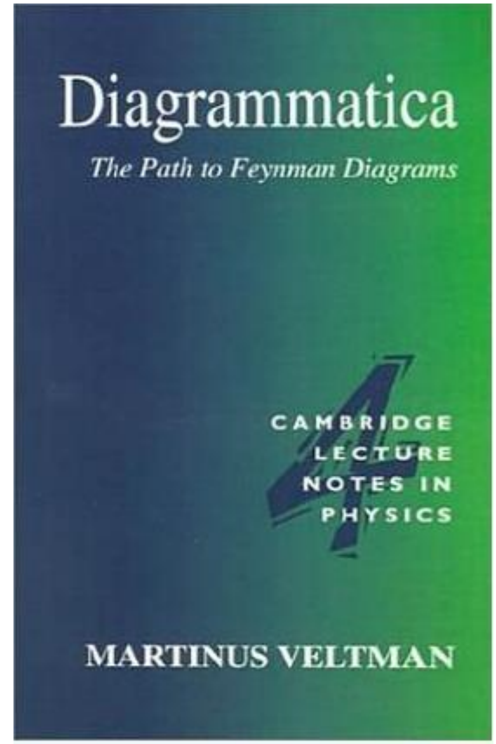
$$\Pi(p_1, p_2) = \int_{m_1^2}^{\infty} ds_1 \int_{m_1'^2}^{\infty} ds_2 \frac{\rho(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)}$$

Cutkosky cutting rule

$$\frac{1}{k^2 - m^2} \to (-2\pi i)\delta(k^2 - m^2)$$

2 -- Verify the DDR

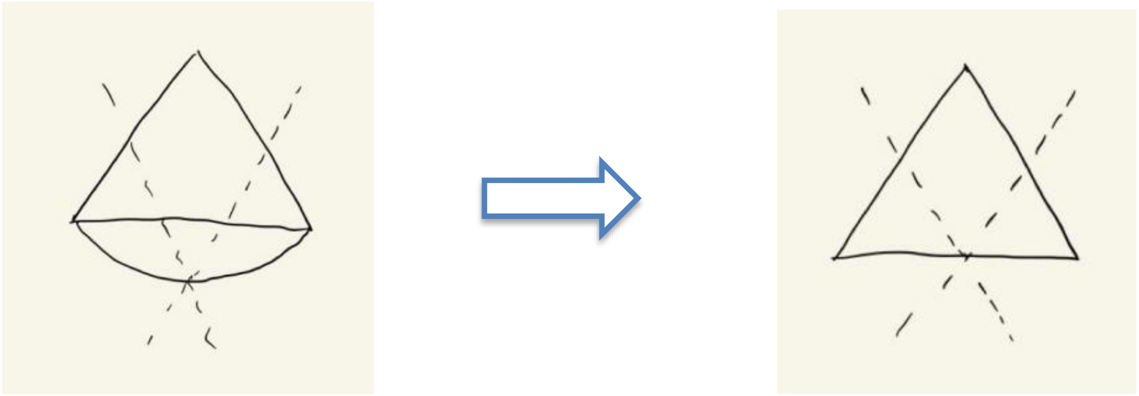




Veltman: Largest time equatioin

Can also be checked numerically

3 -- To deal with the phase space



$$\int_{\triangle}\int_{2}$$

3PCF @ hadron level

$$\Pi^V_\mu(p_1,p_2) = i^2 \int d^4x d^4y e^{-ip_1\cdot x + ip_2\cdot y}\langle 0|T\{J_\Xi(y)V_\mu(0)\bar{J}_{\Xi_c}(x)\}|0\rangle$$

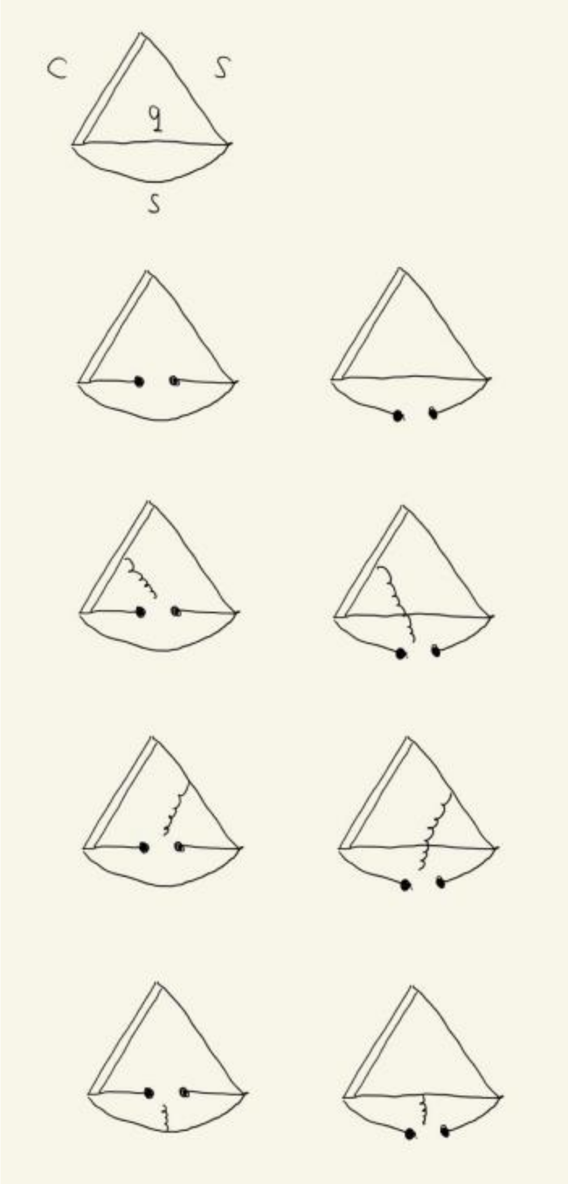
$$\begin{aligned}
\Pi^{V,\text{had}}_\mu(p_1,p_2) &= \lambda_f^+\lambda_i^+ \frac{(\not{p}_2+M_2^+)(\frac{p_{1\mu}}{M_1^+}F_1^{++}+\frac{p_{2\mu}}{M_2^+}F_2^{++}+\gamma_\mu F_3^{++})(\not{p}_1+M_1^+)}{(p_2^2-M_2^{+2})(p_1^2-M_1^{+2})} && 1/2^+\to 1/2^+\\
&+ \lambda_f^+\lambda_i^- \frac{(\not{p}_2+M_2^+)(\frac{p_{1\mu}}{M_1^-}F_1^{+-}+\frac{p_{2\mu}}{M_2^+}F_2^{+-}+\gamma_\mu F_3^{+-})(\not{p}_1-M_1^-)}{(p_2^2-M_2^{+2})(p_1^2-M_1^{-2})} && 1/2^-\to 1/2^+\\
&+ \lambda_f^-\lambda_i^+ \frac{(\not{p}_2-M_2^-)(\frac{p_{1\mu}}{M_1^+}F_1^{-+}+\frac{p_{2\mu}}{M_2^-}F_2^{-+}+\gamma_\mu F_3^{-+})(\not{p}_1+M_1^+)}{(p_2^2-M_2^{-2})(p_1^2-M_1^{+2})} && 1/2^+\to 1/2^-\\
&+ \lambda_f^-\lambda_i^- \frac{(\not{p}_2-M_2^-)(\frac{p_{1\mu}}{M_1^-}F_1^{--}+\frac{p_{2\mu}}{M_2^-}F_2^{--}+\gamma_\mu F_3^{--})(\not{p}_1-M_1^-)}{(p_2^2-M_2^{-2})(p_1^2-M_1^{-2})} && 1/2^-\to 1/2^-\\
&+ \cdots
\end{aligned}$$

3PCF @ QCD level

Dim-0: Perturbative

Dim-3: Quark condensate

Dim-5: Quark-gluon condensate



Some criteria and parameter selection

- Pole dominance

$$r \equiv \frac{\int^{s_1^0} ds_1 \int^{s_2^0} ds_2}{\int^{\infty} ds_1 \int^{\infty} ds_2} \gtrsim 0.25$$

- OPE convergence --- dim-5/Total should not be too large

$$s_1^0 = (2.85\ \mathrm{GeV})^2, \quad s_2^0 = (1.70\ \mathrm{GeV})^2$$

From 2PCF

$$T_1^2 \sim \mathcal{O}(M_1^2), \quad T_2^2 \sim \mathcal{O}(M_2^2)$$

$$T_1^2 = 3.5\ T_2^2 \text{ with } T_2^2 \in [1.4, 2.2]\ \mathrm{GeV}^2$$

Main results

TABLE III: Central values and uncertainties for the form factors F_i and G_i at $q^2 = 0$.

F	Central value	Err from $T^2_{1,2}$	Err from s^0_1	Err from s^0_2	Err from λ_i	Err from λ_f
$F_1(0)$	0.92	0.09	0.01	0.05	0.04	0.04
$F_2(0)$	0.00	0.01	0.01	0.01	0.00	0.00
$F_3(0)$	-1.43	0.10	0.02	0.08	0.07	0.06
$G_1(0)$	1.22	0.08	0.01	0.09	0.06	0.05
$G_2(0)$	-0.49	0.01	0.01	0.03	0.02	0.02
$G_3(0)$	-0.64	0.05	0.01	0.03	0.03	0.03

$s_1^0 = (2.85\ \mathrm{GeV})^2, \quad s_2^0 = (1.70\ \mathrm{GeV})^2$

10-20% uncertaity

(T_1^2, T_2^2) are taken as $(6.3, 1.8)$ GeV^2

Simplified z-expansion

$$f(q^2) = \frac{a + b\ z(q^2)}{1 - q^2/m^2_{\mathrm{pole}}},$$

F	(a, b)
F_1	$(1.21, -2.77)$
F_2	$(-0.02, 0.27)$
F_3	$(-1.58, 1.46)$
G_1	$(1.71, -4.72)$
G_2	$(-0.76, 2.55)$
G_3	$(-0.52, -1.14)$

Phenomenological applications

$$\frac{d\Gamma_L}{dq^2} = \frac{G_F^2 |V_{\rm CKM}|^2 q^2\ p\ (1-\hat{m}_l^2)^2}{384\pi^3 M_1^2} \Big((2+\hat{m}_l^2)(|H_{-\frac{1}{2},0}|^2 + |H_{\frac{1}{2},0}|^2) + 3\hat{m}_l^2 (|H_{-\frac{1}{2},t}|^2 + |H_{\frac{1}{2},t}|^2) \Big)$$

$$\frac{d\Gamma_T}{dq^2} = \frac{G_F^2 |V_{\rm CKM}|^2 q^2\ p\ (1-\hat{m}_l^2)^2 (2+\hat{m}_l^2)}{384\pi^3 M_1^2} (|H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2),$$

$$\Xi_c^+ \to \Xi^0 e^+ \nu_e, \qquad \mathcal{B} = (10.2 \pm 2.2)\%, \qquad \tau(\Xi_c^+) = (456 \pm 5)\ \text{fs}$$

$$\Xi_c^0 \to \Xi^- e^+ \nu_e, \qquad \mathcal{B} = (3.4 \pm 0.7)\%, \qquad \tau(\Xi_c^0) = (153 \pm 6)\ \text{fs}$$

Comparison with other results

TABLE V: Our predictions of the form factors at $q^2 = 0$ are compared with those from the light-front quark model (LFQM) in [9], LFQM in [7], relativistic quark model (RQM) in [8], and light-cone sum rules (LCSR) in [13]. The form factors in other works are multiplied by a minus sign to make all f_1 and g_1 have the same sign.

F	This work	LFQM [9]	LFQM [7]	RQM [8]	LCSR [13]
$f_1(0)$	-0.71 ± 0.18	-0.77 ± 0.02	-0.567	-0.590	-0.194 ± 0.050
$f_2(0)$	0.46 ± 0.06	0.96 ± 0.02	0.305	0.441	0.144 ± 0.037
$f_3(0)$	0.46 ± 0.06	- -	- -	-0.388	0.187 ± 0.049
$g_1(0)$	-0.71 ± 0.08	-0.69 ± 0.01	-0.491	-0.582	-0.311 ± 0.081
$g_2(0)$	0.14 ± 0.08	0.01 ± 0.00	0.046	-0.184	0.061 ± 0.015
$g_3(0)$	1.07 ± 0.08	- -	- -	1.144	0.126 ± 0.033

TABLE VI: Our prediction for the $\Xi_c \to \Xi e^+ \nu_e$ decay width (in units of 10^{-13} GeV) are compared with those from other works.

This work	LCSR [13]	SU(3) [12]	RQM [8]	LFQM [7]	LQCD [14]	PDG2020 [2]	ALICE [28]	Belle [1]
1.45 ± 0.31	4.26 ± 1.49	1.6 ± 0.1	1.40	0.80	1.02 ± 0.19	0.77 ± 0.52	1.04 ± 0.36	0.740 ± 0.224

Summary and outlook

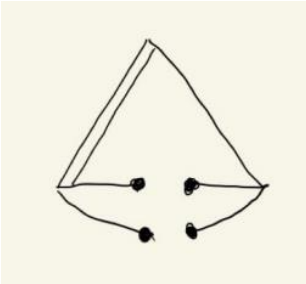
Summary

- The form factors of $\Xi_c \to \Xi$ are investigated in QCDSR.
- 2PCF is studied to obtain the pole residue of Ξ.
- Contributions from up to dim-5 operators have been considered.
- A stable Borel window can be found for the 2PCF of Ξ. For 3PCF of $\Xi_c \to \Xi$, some criteria have to be adopted to select the relatively optimal Borel parameters. About 10-20% uncertainties are introduced.

$$\Xi_c^+ \to \Xi^0 e^+ \nu_e, \qquad \mathcal{B} = (10.2 \pm 2.2)\%,$$
$$\Xi_c^0 \to \Xi^- e^+ \nu_e, \qquad \mathcal{B} = (3.4 \pm 0.7)\%,$$

Outlook

- The typical contribution ratio from dim-0,3,5 to the form factors is roughly 1 : 2 : 1. Contributions from higher dimension operators should be considered.



$$\Xi_c^0 \to \Xi^- e^+ \nu_e, \qquad \mathcal{B} = (3.4 \pm 0.7)\%, \Longrightarrow (3.4 \pm 1.7)\%.$$

- The dependence on the parameters $s_{1,2}^0$ and $T_{1,2}^2$

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Inverse problems

Thank you for your attention!