Two-loop QCD corrections to Bc and Bc* decay constants

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Based on:

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Outline

Introduction

➢ **Beauty-charmed meson family discovered in particle physics experiment**

- Bc(1S), CDF collaboration, 1998 [F. Abe et al. (CDF Collaboration),PRL(1998)]
- Bc(2S), ATLAS collaboration, 2014 [G. Aad et al. (ATLAS Collaboration),PRL(2014)]
- Bc*(2S), CMS and LHCb collaborations, 2019

[A. M. Sirunyan et al. (CMS Collaboration),PRL(2019)]
[R. Aaij et al. (LHCb Collaboration),PRL(2019)]

➢ **Difficulty for experiment measurement**

- composed of two different heavy flavor quarks
- the ground state Bc(1S) only weak decays
- Absolute branching ratios are hardly measured

Introduction

➢ **Theoretical investigation**

- Need study decay constants to obtain leptonic branching ratios
- Decay constants are essentially nonperturbative and universal
- But lattice QCD studies lesser due to doubly heavy flavors

➢ **Nonrelativistic QCD (NRQCD) effective theory**

scale : $\boxed{M \gg Mv \sim \Lambda_{QCD}}$

perturbative nonperturbative

- Decay constants ~ Short-distance perturbative matching coefficients × long-distance nonperturbative NRQCD matrix elements (LDMEs)
- Systematical calculation of expansion in power of α_s and v order by order

Introduction

- **Review high order calculation for Bc and Bc* decay constants with NRQCD**

- The study of Bc at leading order (LO) of α_s [C.H.Chang,Y.Q.Chen,PRD(1994)]
- NLO for Bc [E.Braaten,S.Fleming,PRD(1995)]
- NLO for Bc* [D.S.Hwang,S.Kim,PRD(1999)]
- High order relativistic corrections resummed [J.Lee,W.Sang,S.Kim,JHEP(2011)]
- Approximate NNLO for Bc [A.I.Onishchenko,O.L.Veretin,EPJC(2007)]
- Full analytical NNLO for Bc [L.B.Chen,C.F.Qiao,PLB(2015)]
- NNNLO for Bc [F.Feng,et al.,2208.04302]
- NNLO for Bc* [W.Tao,R.Zhu,Z.J.Xiao,2209.15521]
- NNNLO for Bc* [W.Sang,H.F.Zhang,M.Z.Zhou,2210.02979]

Outline

1. Introduction
2. Calculation Procedure
3. Results
4. Summary

Definition

➢ QCD definition for decay constants

$$\left\langle 0 \left| \bar{b}\gamma^{\mu}\gamma_5 c \right| B_c(P) \right\rangle = i f^{a}_{B_c} P^{\mu},$$

$$\left\langle 0 \left| \bar{b}\gamma_5 c \right| B_c(P) \right\rangle = i f^{p}_{B_c} m_{B_c},$$

$$\left\langle 0 \left| \bar{b}\gamma^{\mu} c \right| B^{*}_c(P,\varepsilon) \right\rangle = f^{v}_{B^{*}_c} m_{B^{*}_c} \varepsilon^{\mu},$$

$a \equiv (a, 0)$:timelike component of axial current

p:pseudoscalar current

v:vector current

$f^{a}_{B_c} \equiv f^{p}_{B_c}$

➢ NRQCD definition for decay constants

$$f^{p}_{B_c} = \sqrt{\frac{2}{m_{B_c}}} C_p(m_b, m_c, \mu_f) \left\langle 0 \left| \chi^{\dagger}_b \psi_c \right| B_c(\mathbf{P}) \right\rangle (\mu_f) + O(v^2)$$

mathing coefficients

NRQCD LDMEs

$$f^{v}_{B^{*}_c} = \sqrt{\frac{2}{m_{B^{*}_c}}} C_v(m_b, m_c, \mu_f) \left\langle 0 \left| \chi^{\dagger}_b \sigma \cdot \varepsilon \psi_c \right| B^{*}_c(\mathbf{P}) \right\rangle (\mu_f) + O(v^2)$$

Matching & Renormalization

- **Perturbative matching formulae**

$$Z_J Z_{2,b}^{\frac{1}{2}} Z_{2,c}^{\frac{1}{2}} \Gamma_J = C_J \tilde{Z}_J^{-1} \tilde{Z}_{2,b}^{\frac{1}{2}} \tilde{Z}_{2,c}^{\frac{1}{2}} \tilde{\Gamma}_J$$

Z_J:QCD on-shell current renormalization constants

$$Z_a = Z_v = 1, Z_p = \frac{m_b Z_{m,b} + m_c Z_{m,c}}{m_b + m_c}$$

$\tilde{Z}_J$:NRQCD $\overline{\mathrm{MS}}$ current renormalization constants

Z_2/Z_m :QCD on-shell quark field/mass renormalization constants

$\tilde{Z}_2$: NRQCD on-shell quark field renormalization constants

$$\tilde{Z}_{2,b} = \tilde{Z}_{2,c} = 1$$

$\Gamma_J/\tilde{\Gamma}_J$:unrenormalized QCD/NRQCD current vertex function

$$\tilde{\Gamma}_J=1$$

NRQCD current renormalization constants

- **NRQCD current renormalization constants and anomalous dimensions**

$$\tilde{Z}_J = 1 - \left(\frac{\alpha_s^{(n_l)}\left(\mu_f\right)}{\pi}\right)^2 \frac{\gamma_J^{(2)}(x)}{4\epsilon} + \mathcal{O}(\alpha_s^3),$$

$$\gamma_J = \frac{d\ln\tilde{Z}_J}{d\ln\mu_f} = \frac{-2\,\partial\tilde{Z}_J^{(1)}}{\partial\ln\alpha_s^{(n_l)}(\mu_f)} = \left(\frac{\alpha_s^{(n_l)}\left(\mu_f\right)}{\pi}\right)^2 \gamma_J^{(2)}(x) + \mathcal{O}(\alpha_s^3)$$

$\tilde{Z}_J^{(1)}$ denotes the coefficient of the $\frac{1}{\epsilon}$ pole in $\tilde{Z}_J$

$$\gamma_p^{(2)}(x) = -\pi^2\left(\frac{C_F C_A}{2} + \frac{(1+6x+x^2)C_F^2}{2(1+x)^2}\right),$$

$$\gamma_v^{(2)}(x) = -\pi^2\left(\frac{C_F C_A}{2} + \frac{(3+2x+3x^2)C_F^2}{6(1+x)^2}\right)$$

The renormalization coupling

➢ Matching and Calculation of α_s

- Decoupling $\alpha_s^{(n_f=n_b+n_c+n_l)}(\mu)$ to $\alpha_s^{(n_l)}(\mu)$

$$\alpha_s^{(n_f)}(\mu) = \alpha_s^{(n_l)}(\mu)\left(1 + \frac{\alpha_s^{(n_l)}(\mu)}{\pi} T_F \left(\frac{n_b}{3}\ln\frac{\mu^2}{m_b^2} + \frac{n_c}{3}\ln\frac{\mu^2}{m_c^2} + \mathcal{O}(\epsilon)\right) + \mathcal{O}(\alpha_s^2)\right)$$

- Renormalization group running equations

$$\alpha_s^{(n_l)}\left(\mu_f\right) = \left(\frac{\mu}{\mu_f}\right)^{2\epsilon} \alpha_s^{(n_l)}(\mu) + \mathcal{O}(\alpha_s^2),$$

μ:renormalization scale
μ_f:NRQCD factorization scale

$$\alpha_s^{(n_l)}(\mu) = \frac{4\pi}{\beta_0^{(n_l)}\ln\frac{\mu^2}{\Lambda_{QCD}^{(n_l)\,2}}}\left(1 - \frac{\beta_1^{(n_l)}\ln\ln\frac{\mu^2}{\Lambda_{QCD}^{(n_l)\,2}}}{\beta_0^{(n_l)2}\ln\frac{\mu^2}{\Lambda_{QCD}^{(n_l)\,2}}}\right) + \mathcal{O}\left(\frac{1}{\ln^3\frac{\mu^2}{\Lambda_{QCD}^{(n_l)\,2}}}\right)$$

Calculation steps

➢ **Higher order calculation steps**

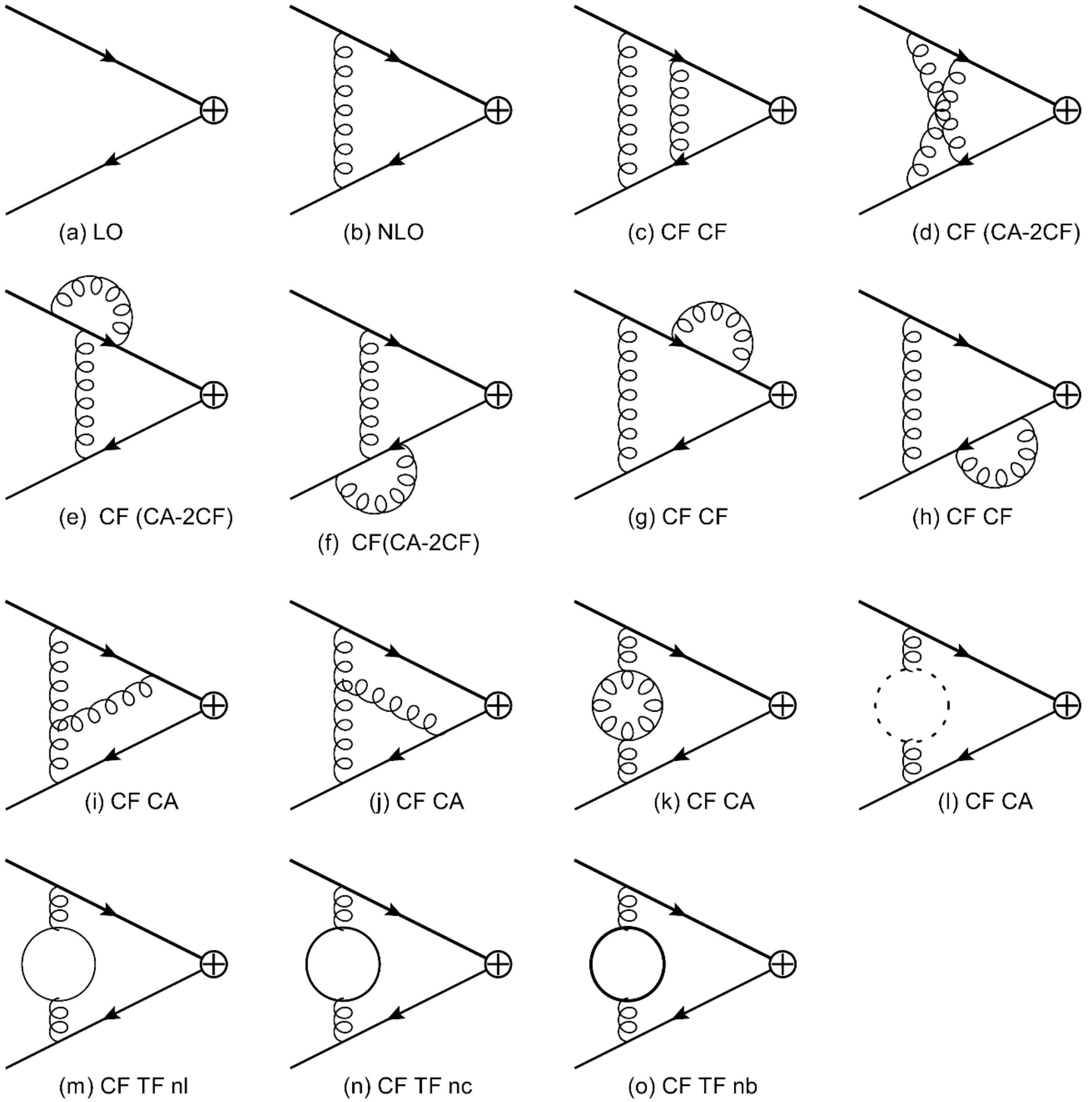
- **FeynCalc** obtains diagrams and corresponding amplitudes,
  $Apart decomposes every amplitude into several Feynman integral families
- **FIRE / Kira / FiniteFlow** reduces every Feynman integral family to master integral family
- **Kira+FIRE+Mathematica code** reduces all of master integral families to the minimal set of master integral families
- **AMFlow** with **Kira/FiniteFlow** calculates the minimal set of master integral families family by family
- Renormalization

Renormalization

- **Renormalization at NNLO**
  - two loop diagrams
  - tree diagram inserted with one α_s^2-order counter-term vertex
  - tree diagram inserted with two α_s-order counter-term vertexes (vanishing)
  - one loop diagram inserted with one α_s-order counter-term vertex

Diagrams

- ➢ **Tree,1loop,2loop**



Outline

Matching coefficients results

➢ Matching coefficient formula

$$
\begin{aligned}
C_J(\mu_f,\mu,m_b,x) &= 1+\frac{\alpha_s^{(n_f)}(\mu)}{\pi}C_J^{(1)}(x)\\
&+\left(\frac{\alpha_s^{(n_f)}(\mu)}{\pi}\right)^2\Bigg(C_J^{(1)}(x)\frac{\beta_0^{(n_f)}}{4}\ln\frac{\mu^2}{m_b^2}+\frac{\gamma_J^{(2)}(x)}{2}\ln\frac{\mu_f^2}{m_b^2}\\
&+C_F^2C_J^{FF}(x)+C_FC_AC_J^{FA}(x)+C_FT_Fn_lC_J^{FL}(x)+C_FT_FC_J^{FH}(x)\Bigg)\\
&+\mathcal{O}(\alpha_s^3).
\end{aligned}
$$

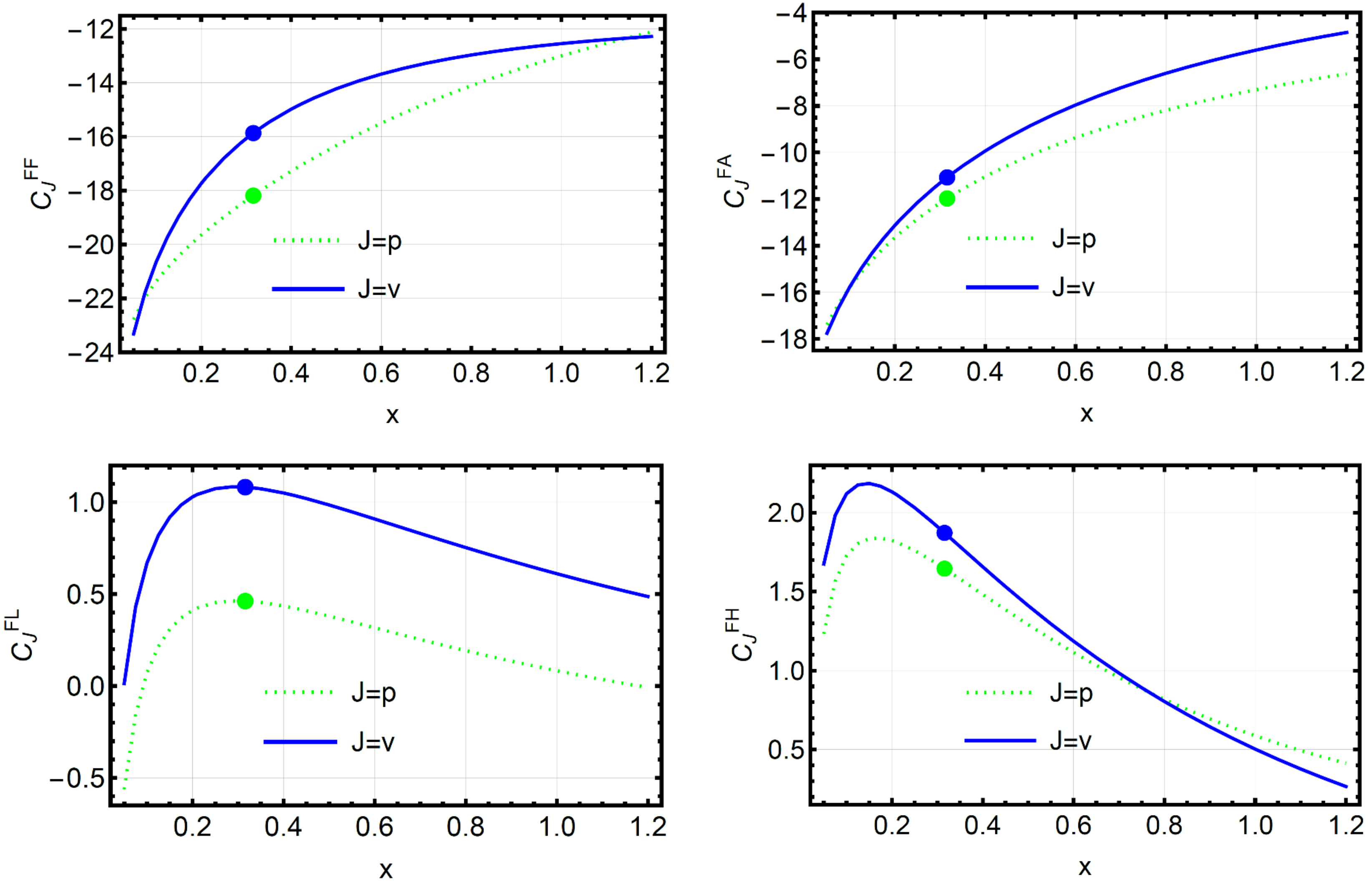
- $x=\frac{m_c}{m_b}$
- NLO matching coefficients:

$$
C_p^{(1)}(x)=\frac{3}{4}C_F\left(\frac{x-1}{x+1}\ln x-2\right)
$$

$$
C_v^{(1)}(x)=\frac{3}{4}C_F\left(\frac{x-1}{x+1}\ln x-\frac{8}{3}\right)
$$

Matching coefficients results

➢ **x dependence for** C_J^{FF}, C_J^{FA}, C_J^{FL}, and C_J^{FH}



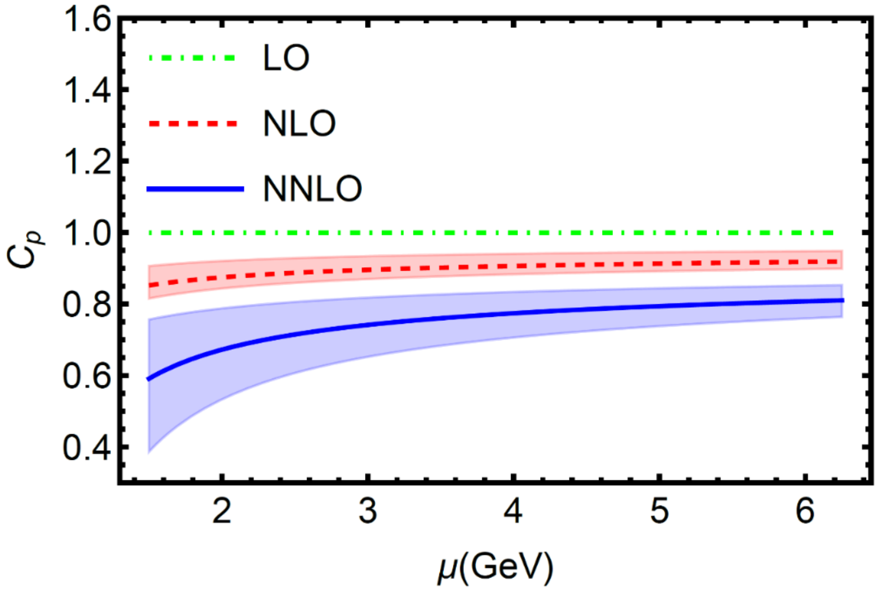
Matching coefficients results

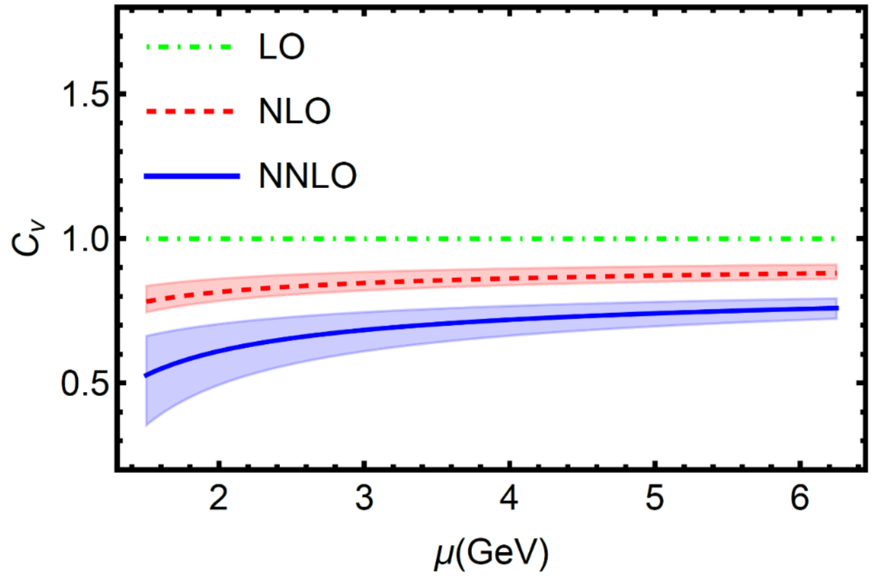
➢ numerical results for matching coefficients

- $\mu_f \in [1.5, 1.2, 1]$GeV, $\mu \in [6.25, 4.75, 3]$GeV, $m_b \in [5.25, 4.75, 4.25]$GeV, $m_c \in [2, 1.5, 1]$GeV

	LO	NLO	NNLO
C_p	1	$0.9117^{-0+0.0072+0.0061-0.0156}_{+0-0.0160-0.0064+0.0263}$	$0.7897^{-0.0310+0.0206+0.0119+0.0149}_{+0.0253-0.0482-0.0133-0.0141}$
C_v	1	$0.8697^{-0+0.0107+0.0061-0.0156}_{+0-0.0236-0.0064+0.0263}$	$0.7363^{-0.0234+0.0230+0.0106+0.0117}_{+0.0191-0.0526-0.0117-0.0121}$

- μ dependence for matching coefficients





Decay constants results

➢ Bc and Bc* decay constants

- LDMEs $\langle 0 | \chi_b^\dagger \sigma \cdot \varepsilon \psi_c | B_c^*(\mathbf{P}) \rangle \approx \langle 0 | \chi_b^\dagger \psi_c | B_c(\mathbf{P}) \rangle \approx \sqrt{2N_c}\, \psi_{B_c}(0)$,
- $|\psi_{B_c}(0)|^2 \in [0.13, 0.12, 0.10]\mathrm{GeV}^3$, $\mu_f \in [1.5, 1.2, 1]\mathrm{GeV}$, $\mu \in [6.25, 4.75, 3]\mathrm{GeV}$, $m_b \in [5.25, 4.75, 4.25]\mathrm{GeV}$, $m_c \in [2, 1.5, 1]\mathrm{GeV}$

	$\frac{f^p_{B_c}}{10^{-1}\mathrm{GeV}}$	$\frac{f^v_{B_c^*}}{10^{-1}\mathrm{GeV}}$
LO	$4.79^{+0.20+0+0+0+0}_{-0.42-0-0-0-0}$	$4.78^{+0.20+0+0+0+0}_{-0.42-0-0-0-0}$
NLO	$4.37^{+0.18+0+0.03+0.03-0.07}_{-0.38-0-0.08-0.03+0.13}$	$4.15^{+0.17+0+0.05+0.03-0.07}_{-0.36-0-0.11-0.03+0.13}$
NNLO	$3.78^{+0.15-0.15+0.10+0.06+0.07}_{-0.33+0.12-0.23-0.06-0.07}$	$3.52^{+0.14-0.11+0.11+0.05+0.06}_{-0.31+0.09-0.25-0.06-0.06}$

Decay widths & branching ratios

➢ Bc and Bc* leptonic decay widths and branching ratios

- Formulae

$$\Gamma(B_c^+ \to l^+ + \nu_l) = \frac{|V_{bc}|^2}{8\pi} G_F^2 m_{B_c} m_l^2 \left(1 - \frac{m_l^2}{m_{B_c}^2}\right)^2 f_{B_c}^{p\,2},$$

$$\Gamma(B_c^{*+} \to l^+ + \nu_l) = \frac{|V_{bc}|^2}{12\pi} G_F^2 m_{B_c^*}^3 \left(1 - \frac{m_l^2}{m_{B_c^*}^2}\right)^2 \left(1 + \frac{m_l^2}{2m_{B_c^*}^2}\right) f_{B_c^*}^{v\,2}$$

- Decay widths

	$\frac{\Gamma(B_c^+ \to e^+ + \nu_e)}{10^{-21}\,\mathrm{GeV}}$	$\frac{\Gamma(B_c^{*+} \to e^+ + \nu_e)}{10^{-13}\,\mathrm{GeV}}$
LO	$3.39^{+0.28+0+0+0+0}_{-0.56-0-0-0-0}$	$3.45^{+0.29+0+0+0+0}_{-0.57-0-0-0-0}$
NLO	$2.82^{+0.23+0+0.04+0.04-0.10}_{-0.47-0-0.10-0.04+0.16}$	$2.61^{+0.22+0+0.06+0.04-0.09}_{-0.43-0-0.14-0.04+0.16}$
NNLO	$2.11^{+0.18-0.16+0.11+0.06+0.08}_{-0.35+0.14-0.25-0.07-0.07}$	$1.87^{+0.16-0.12+0.12+0.05+0.06}_{-0.31+0.10-0.26-0.06-0.06}$

	$\frac{\Gamma(B_c \to \mu^+ + \nu_\mu)}{10^{-16}\,\mathrm{GeV}}$	$\frac{\Gamma(B_c^* \to \mu^+ + \nu_\mu)}{10^{-13}\,\mathrm{GeV}}$
LO	$1.45^{+0.12+0+0+0+0}_{-0.24-0-0-0-0}$	$3.45^{+0.29+0+0+0+0}_{-0.57-0-0-0-0}$
NLO	$1.20^{+0.10+0+0.02+0.02-0.04}_{-0.20-0-0.04-0.02+0.07}$	$2.61^{+0.22+0+0.06+0.04-0.09}_{-0.43-0-0.14-0.04+0.16}$
NNLO	$0.90^{+0.08-0.07+0.05+0.03+0.03}_{-0.15+0.06-0.11-0.03-0.03}$	$1.87^{+0.16-0.12+0.12+0.05+0.06}_{-0.31+0.10-0.26-0.06-0.06}$

Decay widths & branching ratios

- Decay widths

	$\frac{\Gamma(B_c^+ \to \tau^+ + \nu_\tau)}{10^{-14}\mathrm{GeV}}$	$\frac{\Gamma(B_c^{*+} \to \tau^+ + \nu_\tau)}{10^{-13}\mathrm{GeV}}$
LO	$3.47^{+0.29+0+0+0+0}_{-0.58-0-0-0-0}$	$3.04^{+0.25+0+0+0+0}_{-0.51-0-0-0-0}$
NLO	$2.88^{+0.24+0+0.05+0.04-0.10}_{-0.48-0-0.10-0.04+0.17}$	$2.30^{+0.19+0+0.06+0.03-0.08}_{-0.38-0-0.12-0.03+0.14}$
NNLO	$2.16^{+0.18-0.17+0.11+0.07+0.08}_{-0.36+0.14-0.26-0.07-0.08}$	$1.65^{+0.14-0.10+0.10+0.05+0.05}_{-0.27+0.09-0.23-0.05-0.05}$

- Decay branching ratios

	$e^+\nu_e$	$\mu^+\nu_\mu$	$\tau^+\nu_\tau$
B_c	$\left(1.64^{+0.44}_{-0.71}\right) \times 10^{-9}$	$\left(7.00^{+1.89}_{-3.01}\right) \times 10^{-5}$	$\left(1.68^{+0.45}_{-0.72}\right) \times 10^{-2}$
B_c^*	$\left(2.34^{+0.61}_{-1.01}\right) \times 10^{-6}$	$\left(2.34^{+0.61}_{-1.01}\right) \times 10^{-6}$	$\left(2.06^{+0.54}_{-0.89}\right) \times 10^{-6}$

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Summary

- Verify Bc decay constant from pseudoscalar current is identical with the Bc decay constant from axial-vector current
- Obtain the novel anomalous dimension for the flavor-changing heavy quark vector current
- Obtain NNLO result for the decay constant of Bc*
- The obtained branching ratio of $\mathrm{B}_c^{*+} \to \mu^+ + \nu_\mu$ isn't small, which can be a good channel to detect Bc*

Thank you!