SHANDONG UNIVERSITY

Evolution of Charm-meson Ratios in an Expanding Hadron Gas

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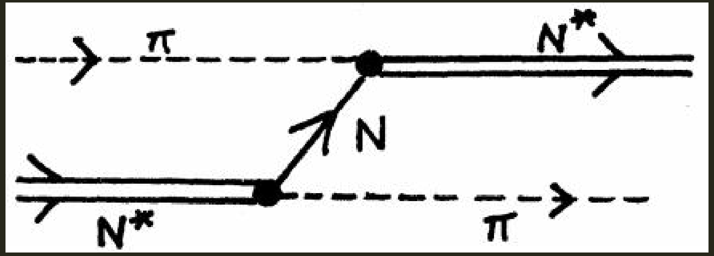
CONTENTS

t-channel singularity

- A t-channel scattering process: an unstable particle decays and one of its decay products is scattered.
- The singularity arises if the exchanged particle can be on-shell.

t-channel singularity

- First pointed out by Peierls in 1961 in π N* scattering



The exchanged N can be on-shell, which leads to a divergence in the cross section.

Peierls, PRL 6, 641-643 (1961)

t-channel singularity

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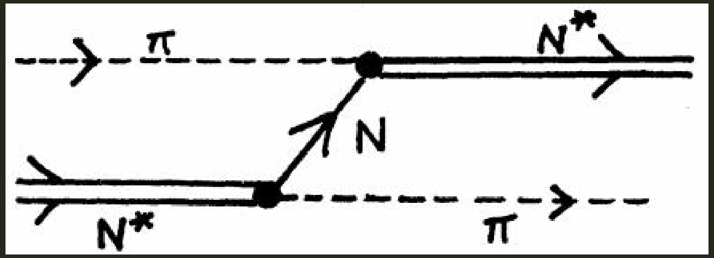


The exchanged N can be on-shell, which leads to a divergence in the cross section.

- Peierls suggested that reaction rate could be regularized by inserting width of N* into N propagator, but the cross section is unphysical.

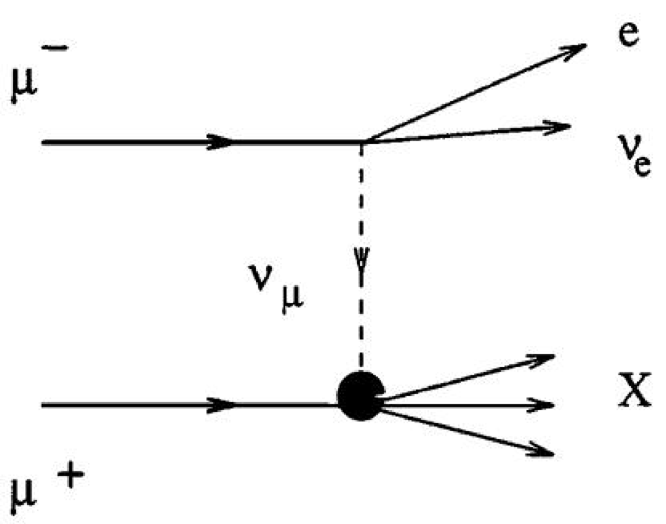
Peierls, PRL 6, 641-643 (1961)

t-channel singularity

- The t-channel singularities are unavoidable in reactions involving unstable particles.

t-channel singularity

- The t-channel singularities is unavoidable in reactions involving unstable particles.
- Other examples



Accounting for the finite sizes of the colliding beams results in the regularization of this singularity.

K. Melnikov and V.G. Serbo, PRL 76, 3263 (1996)

Elastic scattering: $W^- e^- \to e^- W^-$ mediated by ν_e, *etc.*

Inelastic scattering: $W^- \nu_e \to e^- Z$ mediated by vbar$_e$, *etc.*

t-channel singularity

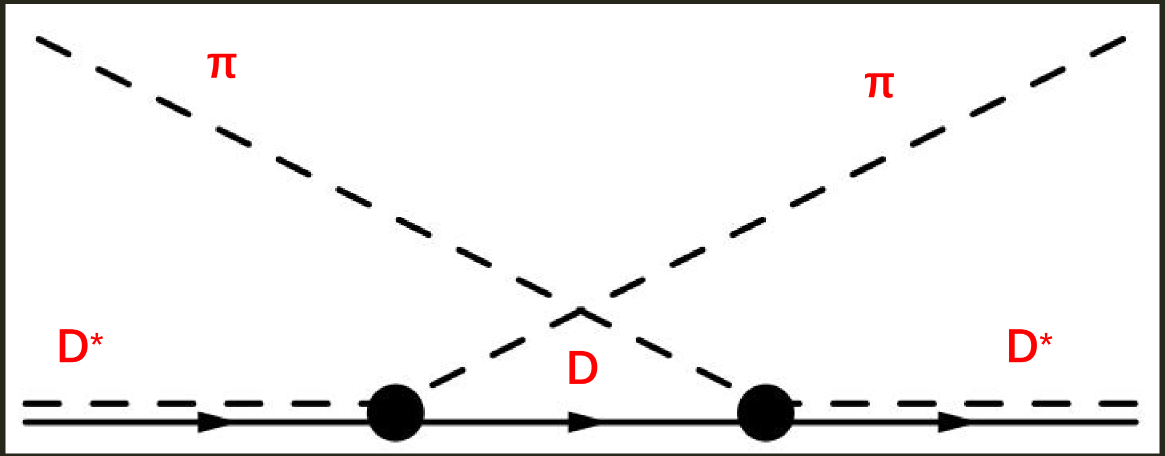
- In a thermal medium, a t-channel singularity is regularized by the thermal width of the exchanged particle.

$$\frac{1}{t-M^2} \longrightarrow \frac{1}{t-M^2-\Pi}, \quad \Pi \approx 2M\delta M - iM\Gamma$$

Grzadkowski, Iglicki, and Mr´owczy´nski ,
NPB 984, 115967 (2022)

Motivation

- $\pi D^* \to \pi D^*$ scattering has t-channel singularity because exchanged D can be on-shell.



t-channel singularity region: $2M_*^2 - M^2 + 2m^2 < s < (M_*^2 - m^2)^2/M^2,$

M*: mass of D*

M: mass of D

m: mass of π

- In hadron gas, the divergences are regularized by thermal width of D.

Motivation

- Do t-channel singularities in charm-meson reactions have any observable consequences?

One possibility is that t-channel singularities could modify the charm-meson abundances produced in a high-energy collision through the interactions with hadron gas.

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One possibility is that t-channel singularities could modify the charm-meson abundances produced in a high-energy collision through the interactions with hadron gas.

- The observed numbers N_0 and N_+ of D^0 and D^+ can be predicted in terms of the numbers $(N_a)_0$ and $(N_{*a})_0$ before D^* decays and the branching fraction B_{+0} = 68% for $D^{*+} \to D^0\pi^+$:

$$N_0 = (N_0)_0 + (N_{*0})_0 + B_{+0}\,(N_{*+})_0\,,$$
$$N_+ = (N_+)_0 + 0 + (1 - B_{+0})\,(N_{*+})_0\,,$$

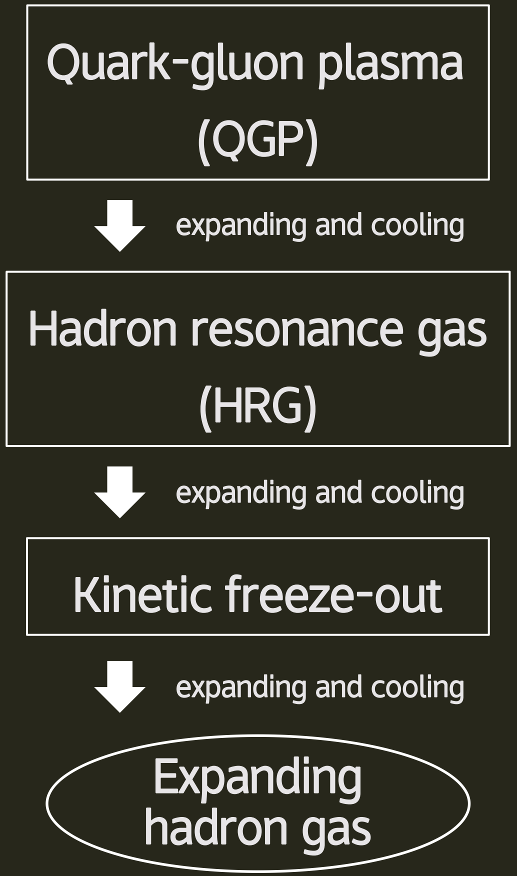
We will show that the charm-meson abundances are modified by t-channel singularities.

Charm-mesons in heavy-ion collision

- Heavy-ion collisions proceed through several stages:

Quark-gluon plasma (QGP)	The formation and thermalization of the QGP
⬇ expanding and cooling	
Hadron resonance gas (HRG)	The deconfined quarks and gluons hadronize into HRG
⬇ expanding and cooling	
Kinetic freeze-out	Particles stop interacting, momentum distributions frozen (We will show that scattering reactions with t-channel singularities can continue after kinetic freezeout)
⬇ expanding and cooling	
Expanding hadron gas	

Charm-mesons in heavy-ion collision



Charm quarks are primarily created in the hard collisions of the heavy ions.

Hadronization:

Statistical Hadronization Model (SHM)

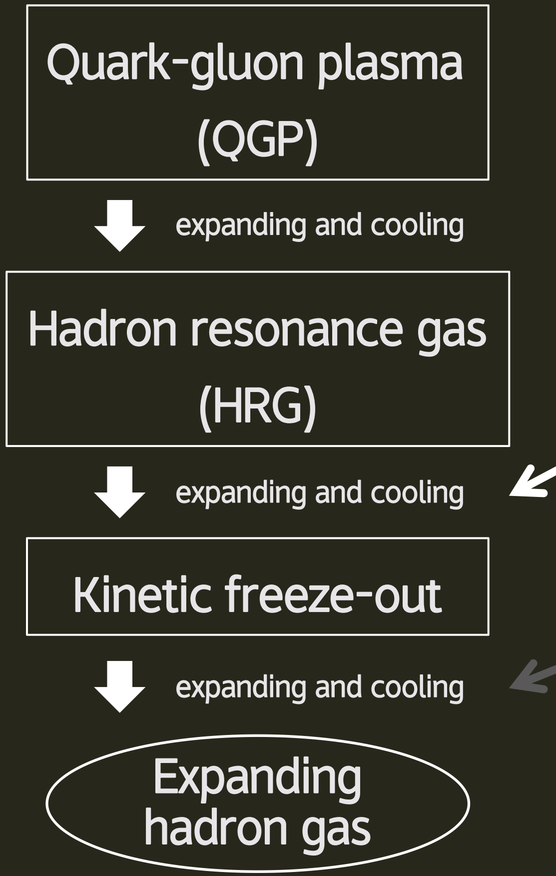
SHMc (SHM for charm)

Multiplicities before D^* decays ($c\tau$ > 2000 fm) predicted by SHMc:

$$(dN_0/dy)_0 = 2.12, \qquad (dN_+/dy)_0 = 2.03,$$
$$(dN_{*0}/dy)_0 = 2.59, \qquad (dN_{*+}/dy)_0 = 2.52.$$

A. Andronic, P. Braun-Munzinger and J. Stachel, Nucl. Phys. A 772, 167-199 (2006);
A. Andronic, *etc.*, JHEP 07, 035 (2021).

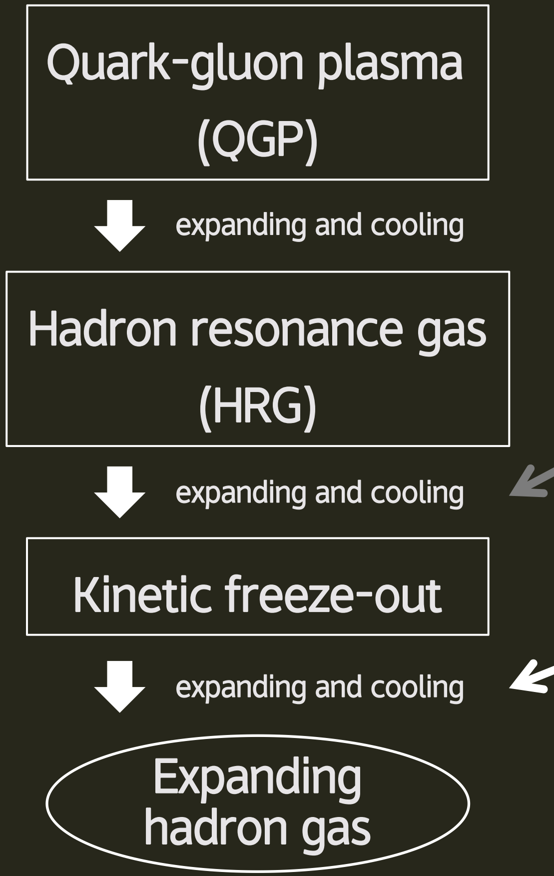
Charm-mesons in heavy-ion collision



The t-channel singularities in charm meson reactions could have significant effects

- either during the expansion and cooling of the HRG between hadronization and kinetic freezeout (requires a full treatment of the HRG)
- or during the expansion of the hadron gas after kinetic freezeout (thermal widths are determined primarily by the pion number density).

Charm-mesons in heavy-ion collision



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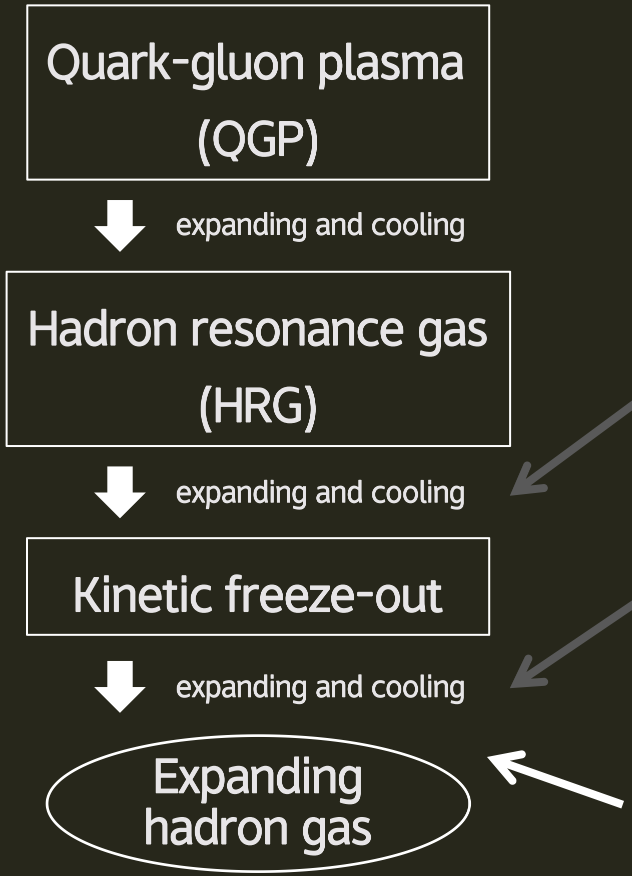
Charm-mesons in heavy-ion collision



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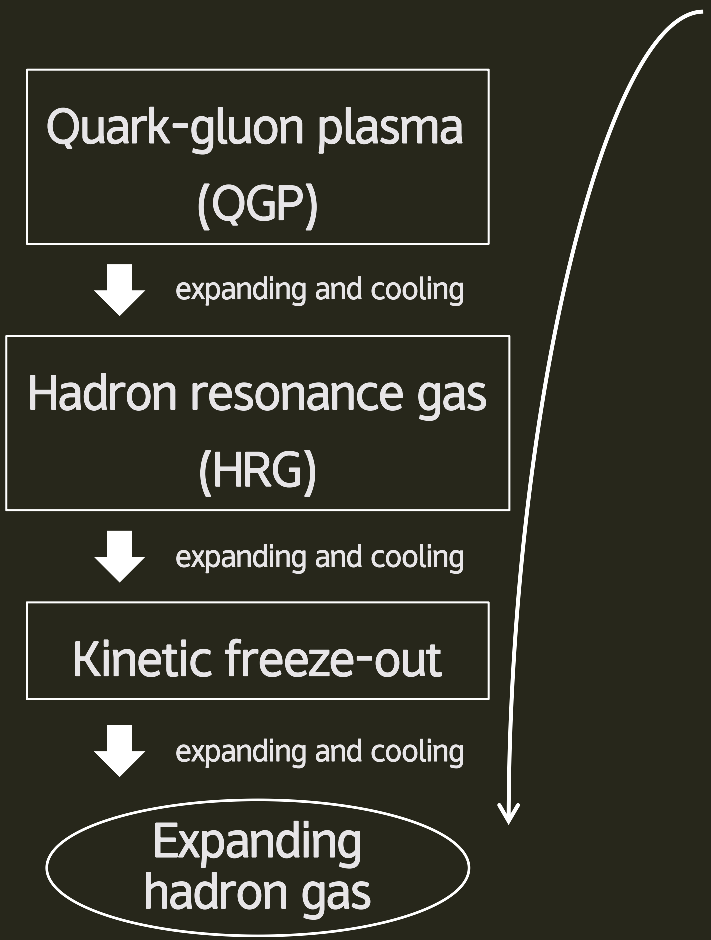
- or during the expansion of the hadron gas after kinetic freezeout (thermal widths are determined primarily by the pion number density (5 times of kions)).

We study the effects of t-channel singularities in charm meson reactions in the expansion of the hadron (pion) gas after kinetic freeze-out.

Expanding hadron gas



Expanding hadron gas is mainly PIONS !

Volume $V(\tau)$ for $\tau > \tau_F$: $V(\tau) = \pi \left[R_F + v_F(\tau - \tau_F)\right]^2 c\tau,$

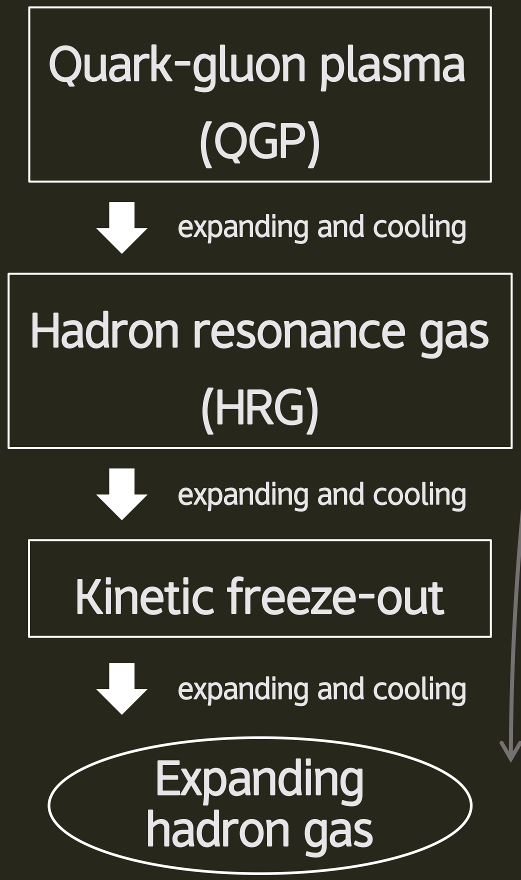
$\tau_F = 21.5$ fm/c, $R_F = 24.0$ fm, and $v_F = 1.00$ c

Number density for pions as system expanding:

$$\mathfrak{n}_\pi(\tau) = [V(\tau_F)/V(\tau)]\mathfrak{n}_\pi(\tau_F).$$

J.D. Bjorken, PRD 27, 140-151 (1983));
J. Hong, *etc.*, PRC 98, 014913 (2018);
L.M. Abreu, PRD 103, 036013 (2021).

Expanding hadron gas



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Number density for pions as system expanding:

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We can estimate the charm-meson number densities at times τ before $D*$ decays:

$$\mathfrak{n}_{D^{(*)}}(\tau) = [(dN_{D^{(*)}}/dy)/(dN_\pi/dy)]_0\, \mathfrak{n}_\pi(\tau).$$

$$dN_\pi/dy = 769 \pm 34.$$

J.D. Bjorken, PRD 27, 140-151 (1983));
J. Hong, *etc.*, PRC 98, 014913 (2018);
L.M. Abreu, PRD 103, 036013 (2021).
S. Acharya et al. [ALICE], PRC 101, 044907 (2020)

2 Thermal mass shifts, widths and reaction rates

Pion momentum distribution

- We consider a pion gas in which the momentum distribution of the pions is a Bose-Einstein distribution with temperature T.

2 Thermal mass shifts, widths and reaction rates

Pion momentum distribution

- We consider a pion gas in which the momentum distribution of the pions is a Bose-Einstein distribution with temperature T.

Pion momentum distribution for isothermally expanding pion gas:

$$\mathfrak{f}_\pi(\omega_q) = \frac{\mathfrak{n}_\pi}{\mathfrak{n}_\pi^{(\mathrm{eq})}} \frac{1}{e^{\beta\omega_q} - 1}$$

where $\beta = 1/T$.

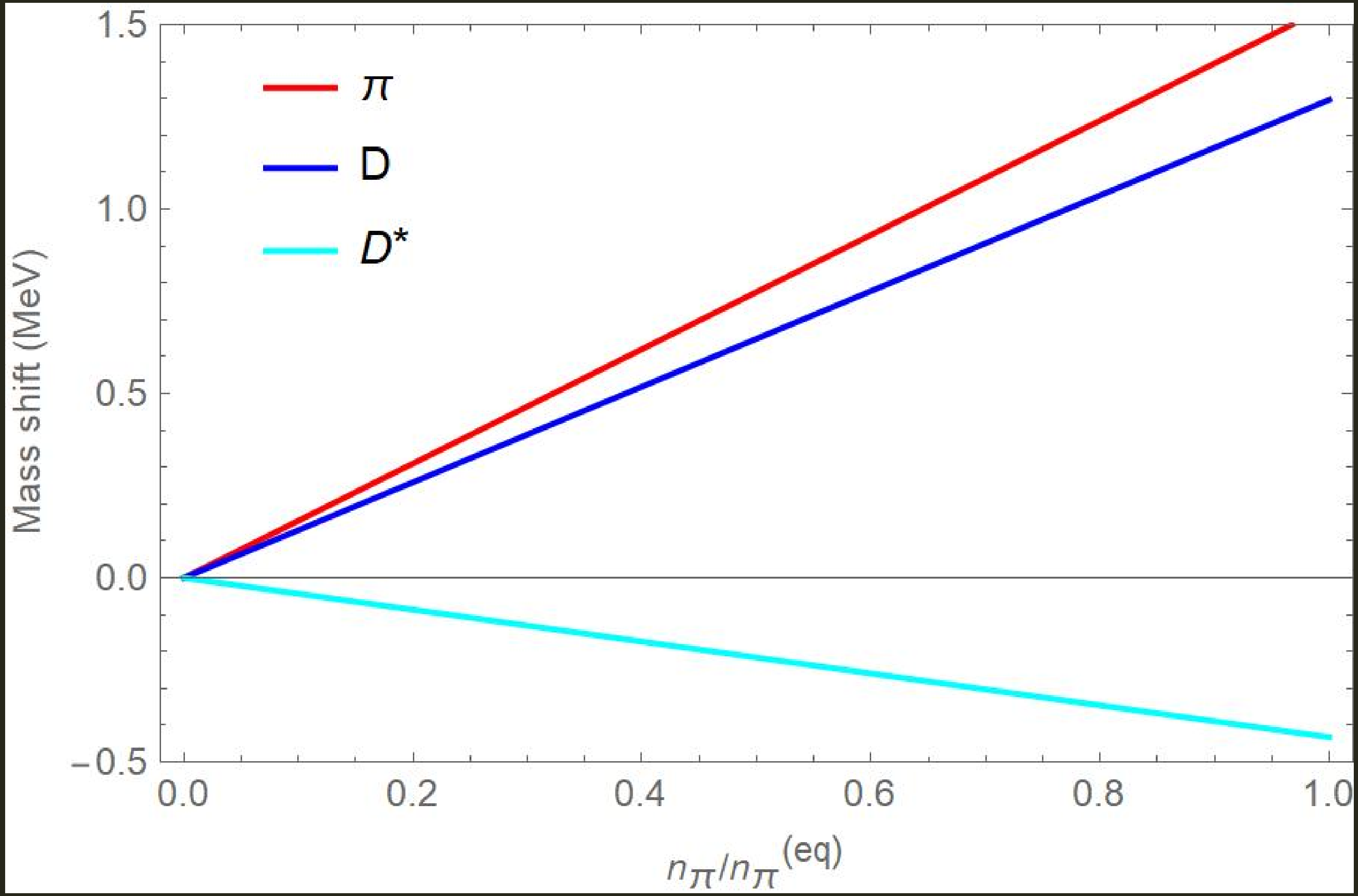
In which the number density $\mathfrak{n}_\pi^{(eq)}$ in thermal equilibrium at temperature T_F=115 MeV:

$$\mathfrak{n}_\pi^{(\mathrm{eq})} = \int \frac{d^3q}{(2\pi)^3} \frac{1}{e^{\omega_q/T_F} - 1}.$$

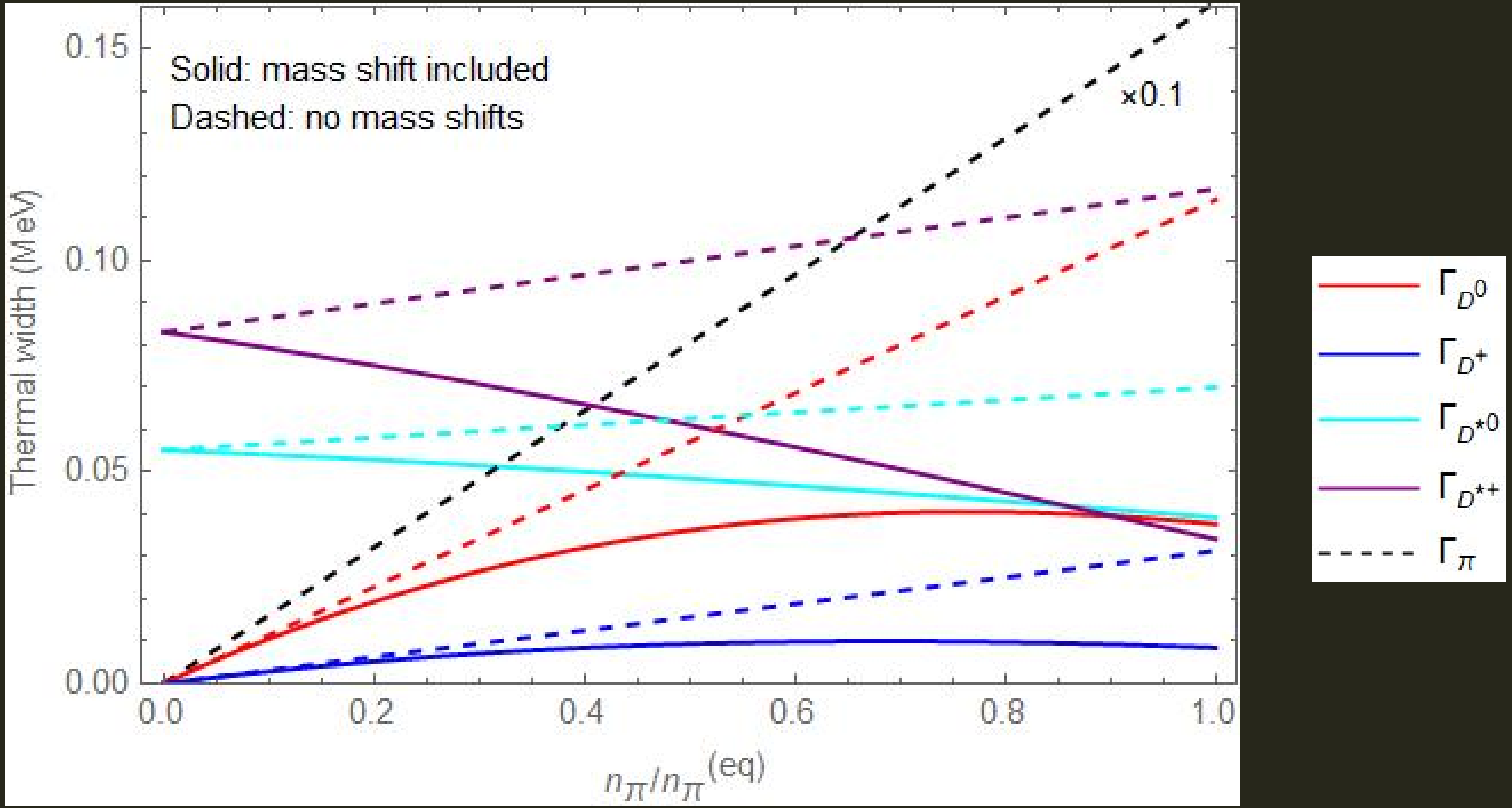
Integral of a function weighted by the pion momentum distribution:

$$\int \frac{d^3q}{(2\pi)^3} \mathfrak{f}_\pi(\omega_q)\, F(\boldsymbol{q}) = \mathfrak{n}_\pi \langle F(\boldsymbol{q}) \rangle.$$

D$^{(*)}$ mass shift and thermal width



D(*) mass shift and thermal width



2 Thermal mass shifts, widths and reaction rates

π $D^{(*)}$ reaction rates in pion gas

√ The reaction rates of π $D^{(*)}$ near the kinetic freezeout temperature would have large effects on the charm-meson abundances:

- spin transitions between D and D*,
- isospin transitions between $D^{(*)0}$ and $D^{(*)+}$.

π D(*) reaction rates in pion gas

- $D^a \pi \rightarrow D^{*b}$: increase D* density, but decrease D density.

$$\langle v\sigma_{\pi D^+ \to D^{*+}} \rangle = \left[\mathfrak{f}_\pi(\Delta)/\mathfrak{n}_\pi \right] \Gamma_{D^{*+} \to D^+ \pi},$$
$$\langle v\sigma_{\pi D^0 \to D^{*+}} \rangle = \left[\mathfrak{f}_\pi(\Delta)/\mathfrak{n}_\pi \right] \Gamma_{D^{*+} \to D^0 \pi},$$
$$\langle v\sigma_{\pi D^0 \to D^{*0}} \rangle = \left[\mathfrak{f}_\pi(\Delta)/\mathfrak{n}_\pi \right] \Gamma_{D^{*0} \to D^0 \pi},$$
$$\langle v\sigma_{\pi D^+ \to D^{*0}} \rangle = 0.$$

π D(*) reaction rates in pion gas

- $\pi D^{*a} \leftrightarrow \pi D^{b}$: increase/decrease D* density, but decrease/increase D density.

$$\langle v\sigma_{\pi*0,\pi0}\rangle = \langle v\sigma_{\pi*+,\pi+}\rangle = 0.2446\, g_\pi^4\, m_\pi^2/f_\pi^4,$$
$$\langle v\sigma_{\pi*0,\pi+}\rangle = \langle v\sigma_{\pi*+,\pi0}\rangle = 0.0056\, g_\pi^4\, m_\pi^2/f_\pi^4,$$
$$\langle v\sigma_{\pi0,\pi*0}\rangle = \langle v\sigma_{\pi+,\pi*+}\rangle = 0.2181\, g_\pi^4\, m_\pi^2/f_\pi^4,$$
$$\langle v\sigma_{\pi0,\pi*+}\rangle = \langle v\sigma_{\pi+,\pi*0}\rangle = 0.0049\, g_\pi^4\, m_\pi^2/f_\pi^4.$$

π D(*) reaction rates in pion gas

- $\pi D^a \to \pi D^b$: change the D^0 and D^+ densities

$$\langle v\sigma_{\pi D^0\to\pi D^0}\rangle = (0.5004 + 0.1900\, g_\pi^4)\frac{m_\pi^2}{f_\pi^4} + \frac{\mathfrak{f}_\pi(\Delta)}{\mathfrak{n}_\pi}\left(\frac{\Gamma^2_{D^{*0}\to D^0\pi}}{\Gamma_{*0}} + \frac{\Gamma^2_{D^{*+}\to D^0\pi}}{\Gamma_{*+}}\right),$$

$$\langle v\sigma_{\pi D^0\to\pi D^+}\rangle = (1.0007 + 0.3336\, g_\pi^4)\frac{m_\pi^2}{f_\pi^4} + \frac{\mathfrak{f}_\pi(\Delta)}{\mathfrak{n}_\pi}\frac{\Gamma_{D^{*+}\to D^0\pi}\,\Gamma_{D^{*+}\to D^+\pi}}{\Gamma_{*+}},$$

s-resonance

$$\langle v\sigma_{\pi D^+\to\pi D^0}\rangle = (1.0007 + 0.3336\, g_\pi^4)\frac{m_\pi^2}{f_\pi^4} + \frac{\mathfrak{f}_\pi(\Delta)}{\mathfrak{n}_\pi}\frac{\Gamma_{D^{*+}\to D^0\pi}\,\Gamma_{D^{*+}\to D^+\pi}}{\Gamma_{*+}},$$

$$\langle v\sigma_{\pi D^+\to\pi D^+}\rangle = (0.5004 + 0.1900\, g_\pi^4)\frac{m_\pi^2}{f_\pi^4} + \frac{\mathfrak{f}_\pi(\Delta)}{\mathfrak{n}_\pi}\frac{\Gamma^2_{D^{*+}\to D^+\pi}}{\Gamma_{*+}}.$$

The resonance term is about three orders of magnitude smaller than the nonresonant term for $n_\pi < n^{(eq)}_\pi$.

π D$^{(*)}$ reaction rates in pion gas

- $\pi D^{*a} \to \pi D^{*b}$: change the D^{*0} and D^{*+} densities

$$\langle v\sigma_{\pi D^{*0}\to\pi D^{*0}}\rangle = (0.5004 + 0.4739\, g_\pi^4)\,\frac{m_\pi^2}{f_\pi^4} + \frac{\mathfrak{f}_\pi(\Delta)}{\mathfrak{n}_\pi}\,\frac{\Gamma^2_{D^{*0}\to D^0\pi}}{\Gamma_0},$$

$$\langle v\sigma_{\pi D^{*0}\to\pi D^{*+}}\rangle = (1.0007 + 0.3086\, g_\pi^4)\,\frac{m_\pi^2}{f_\pi^4} + \frac{\mathfrak{f}_\pi(\Delta)}{\mathfrak{n}_\pi}\,\frac{\Gamma_{D^{*0}\to D^0\pi}\,\Gamma_{D^{*+}\to D^0\pi}}{\Gamma_0},$$

$$\langle v\sigma_{\pi D^{*+}\to\pi D^{*0}}\rangle = (1.0007 + 0.3086\, g_\pi^4)\,\frac{m_\pi^2}{f_\pi^4} + \frac{\mathfrak{f}_\pi(\Delta)}{\mathfrak{n}_\pi}\,\frac{\Gamma_{D^{*0}\to D^0\pi}\,\Gamma_{D^{*+}\to D^0\pi}}{\Gamma_0},$$

t-singularity

$$\langle v\sigma_{\pi D^{*+}\to\pi D^{*+}}\rangle = (0.5004 + 0.4739\, g_\pi^4)\,\frac{m_\pi^2}{f_\pi^4} + \frac{\mathfrak{f}_\pi(\Delta)}{\mathfrak{n}_\pi}\left(\frac{\Gamma^2_{D^{*+}\to D^0\pi}}{\Gamma_0} + \frac{\Gamma^2_{D^{*+}\to D^+\pi}}{\Gamma_+}\right)$$

The t-channel singularity term is larger than the nonsingular term when $n_\pi < 10^{-3}\, n^{(eq)}_\pi$ (τ>230 fm/c).

3 Evolution of charm-meson abundance

Evolution equations

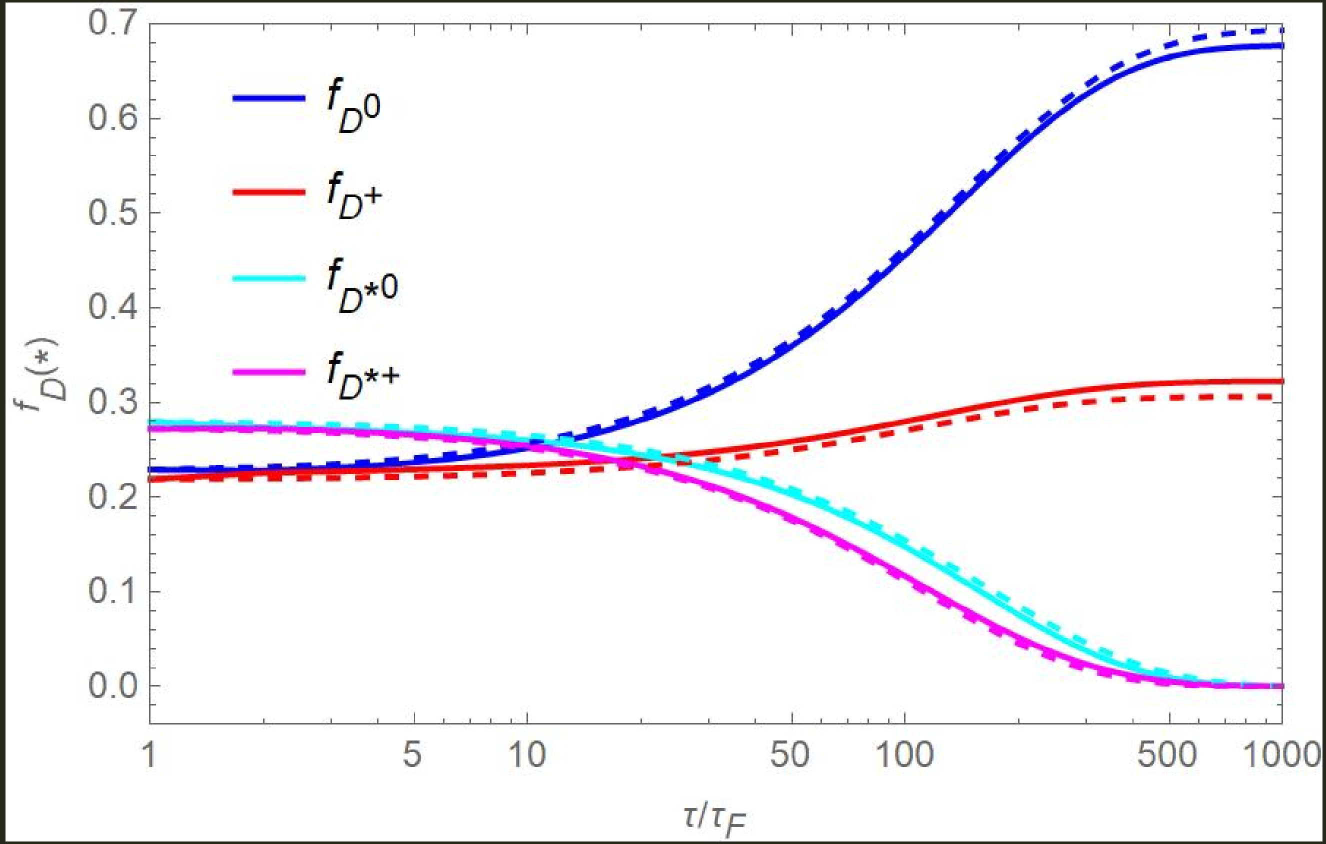
$$\mathfrak{n}_\pi \frac{d}{d\tau}\left(\frac{\mathfrak{n}_{D^a}}{\mathfrak{n}_\pi}\right) = [1+\mathfrak{f}_\pi(\Delta)]\sum_b \Gamma_{*b,a}\,\mathfrak{n}_{D^{*b}} + \Gamma_{*a,\gamma}\,\mathfrak{n}_{D^{*a}} - 3\sum_b \langle v\sigma_{\pi a,*b}\rangle\,\mathfrak{n}_{D^a}\,\mathfrak{n}_\pi$$
$$+ 3\sum_{b\neq a}\langle v\sigma_{\pi b,\pi a}\rangle\,(\mathfrak{n}_{D^b}-\mathfrak{n}_{D^a})\mathfrak{n}_\pi + 3\sum_b \left(\langle v\sigma_{\pi *b,\pi a}\rangle\,\mathfrak{n}_{D^{*b}} - \langle v\sigma_{\pi a,\pi *b}\rangle\,\mathfrak{n}_{D^a}\right)\mathfrak{n}_\pi + \ldots,$$

$$\mathfrak{n}_\pi \frac{d}{d\tau}\left(\frac{\mathfrak{n}_{D^{*a}}}{\mathfrak{n}_\pi}\right) = 3\sum_b \langle v\sigma_{\pi b\to *a}\rangle\,\mathfrak{n}_{D^b}\,\mathfrak{n}_\pi - \left([1+\mathfrak{f}_\pi(\Delta)]\sum_b \Gamma_{*a,b} + \Gamma_{*a,\gamma}\right)\mathfrak{n}_{D^{*a}}$$
$$+ 3\sum_b \left(\langle v\sigma_{\pi b,\pi *a}\rangle\,\mathfrak{n}_{D^b} - \langle v\sigma_{\pi *a,\pi b}\rangle\,\mathfrak{n}_{D^{*a}}\right)\mathfrak{n}_\pi + 3\sum_{b\neq a}\langle v\sigma_{\pi *b,\pi *a}\rangle\,(\mathfrak{n}_{D^{*b}}-\mathfrak{n}_{D^{*a}})\mathfrak{n}_\pi + \ldots.$$

Note: $$\mathfrak{n}_\pi \frac{d}{d\tau}\left(\frac{\mathfrak{n}_{D^0}+\mathfrak{n}_{D^+}+\mathfrak{n}_{D^{*0}}+\mathfrak{n}_{D^{*+}}}{\mathfrak{n}_\pi}\right) = 0$$

Evolution of the charm-meson fractions

- charm-meson fractions $f_{D^{(*)}} = \mathfrak{n}_{D^{(*)}}/(\mathfrak{n}_{D^0} + \mathfrak{n}_{D^+} + \mathfrak{n}_{D^{*0}} + \mathfrak{n}_{D^{*+}})$ (sum is 1)



solid: solving complete evolution equations

dashed: solving evolution equations with only D* decay terms

Analytical solution to evolution equations

- At times τ large enough, the only terms in evolution equations that survive are 1-body terms: decay terms and t-channel singularities.

Analytical solution to evolution function

- At times τ large enough, the only terms in evoluation function that survive are 1-body terms: decay terms and t-channel singularities.
- If we only keep the 1-body terms with the vacuum values of $\Gamma_{*a,b}$, the evolution equations can be solved analytically.

$$\frac{d}{d\tau}R(\tau) = \begin{pmatrix} 0 & 0 & \Gamma_{*0} & B_{+0}\Gamma_{*+} \\ 0 & 0 & 0 & (1-B_{+0})\Gamma_{*+} \\ 0 & 0 & -(\Gamma_{*0}+\gamma) & \gamma \\ 0 & 0 & \gamma & -(\Gamma_{*+}+\gamma) \end{pmatrix} R(\tau),\ R(\tau) = \begin{pmatrix} n_{D^0}/n_\pi \\ n_{D^+}/n_\pi \\ n_{D^{*0}}/n_\pi \\ n_{D^{*+}}/n_\pi \end{pmatrix}$$

$$\frac{1}{\gamma} = \frac{1}{B_{00}\Gamma_{*0}} + \frac{1}{B_{+0}\Gamma_{*+}}$$

B_{00}: fraction of $D^{*0} \to D^0\pi^0$; B_{+0}: fraction of $D^{*+} \to D^0\pi^+$

Analytical solution to evolution function

- The resulting predictions for the numbers of D^0 and D^+ are

$$N_0 = (N_0)_0 + \left(1 - \frac{(1-B_{+0})\Gamma_{*+}\,\gamma}{\Gamma_{*+}\Gamma_{*0} + (\Gamma_{*+} + \Gamma_{*0})\,\gamma}\right)(N_{*0})_0 + \left(B_{+0} + \frac{(1-B_{+0})\Gamma_{*0}\,\gamma}{\Gamma_{*+}\Gamma_{*0} + (\Gamma_{*+} + \Gamma_{*0})\,\gamma}\right)(N_{*+})_0\,,$$

$$N_+ = (N_+)_0 + \frac{(1-B_{+0})\Gamma_{*+}\,\gamma}{\Gamma_{*+}\Gamma_{*0} + (\Gamma_{*+} + \Gamma_{*0})\,\gamma}(N_{*0})_0 + \left(1 - B_{+0} - \frac{(1-B_{+0})\Gamma_{*0}\,\gamma}{\Gamma_{*+}\Gamma_{*0} + (\Gamma_{*+} + \Gamma_{*0})\,\gamma}\right)(N_{*+})_0\,,$$

in comparison with the naive predictions (consider D* decays only)

$$N_0 = (N_0)_0 + (N_{*0})_0 + B_{+0}\,(N_{*+})_0\,,$$

$$N_+ = (N_+)_0 + 0 + (1 - B_{+0})\,(N_{*+})_0\,,$$

Numerical comparison

- initial: SHMc prediction before D* decays
- numerical: solve the complete evolution equations
- naive: consider D* decays only
- analytical: consider 1-body terms (D* decays + t-channel singularity)

	initial	numerical	naive	analytic
N_0/N_+	1.044	2.100	2.256 ± 0.014	2.177 ± 0.016

Errors are from B_{00}, B_{+0}, Γ_{*0}, Γ_{*0+}.

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N_0/N_+	1.044	2.100	2.256 ± 0.014	2.177 ± 0.016

$$\left(\frac{N_0}{N_+}\right)_{\text{naive}} - \left(\frac{N_0}{N_+}\right)_{\text{analytic}} = 0.079 \pm 0.006$$

This difference (with or without t-channel singularity) differs from 0 by about 13 standard deviations.

Errors are from B_{00}, B_{+0}, Γ_{*0}, Γ_{*0+}.

Summary

- The evolution of charm-meson abundances after the kinetic freeze-out of an expanding hadron gas produced by a central heavy-ion collision is studied.
- We have shown that the t-channel singularities in charm-meson reactions can have observable consequences in charm-meson ratio, which have been completely overlooked in studies of the charm mesons in a thermal hadronic medium.
- It might be worthwhile to look for other aspects of the thermal physics of charm mesons in which the effects of t-channel singularities are significant, for example the production of the exotic heavy hadrons like X(3872) and $T_{cc}(3875)$.

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THANK YOU

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全国第十九届重味物理和 CP 破坏研讨会，2022/12/9-11，南京师范大学

Pion mass shift and thermal width

- The pion mass shift and thermal width after kinetic freeze-out can be calculated using χEFT at LO:

$$\delta m_\pi = (m_\pi / 2 f_\pi^2)\, \mathfrak{n}_\pi \, \langle 1/\omega_q \rangle \,,$$

The pion thermal width is 0 at this order.

D(*) mass shift and thermal width

- The charm-meson mass shift and thermal width can be calculated using HHχEFT at LO:

$$\delta M = (3g_\pi^2/2f_\pi^2)\,\mathfrak{n}_\pi\,\Delta\,\langle 1/\omega_q\rangle\,,\;\; \delta M_* = -\delta M/3,$$

$$\Gamma_a = 3\,\mathfrak{f}_\pi(\Delta) \sum_c \Gamma_{*c,a},$$

$$\Gamma_{*a} = [1 + \mathfrak{f}_\pi(\Delta)] \sum \Gamma_{*a,c} + \Gamma_{*a,\gamma},$$

with $\mathfrak{f}_\pi(\Delta) = 0.414\dfrac{\mathfrak{n}_\pi}{\mathfrak{n}_\pi^{(eq)}}$

where decay rates in the vacuum:

$$\Gamma_{*+,+} = \frac{g_\pi^2}{12\pi\, f_\pi^2}\left[(M_{*+} - M_+)^2 - m_{\pi 0}^2\right]^{3/2},$$

$$\Gamma_{*+,0} = \frac{g_\pi^2}{6\pi\, f_\pi^2}\left[(M_{*+} - M_0)^2 - m_{\pi +}^2\right]^{3/2},$$

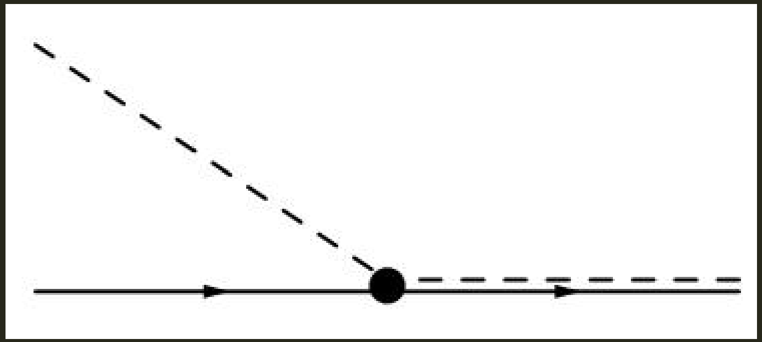
$$\Gamma_{*0,0} = \frac{g_\pi^2}{12\pi\, f_\pi^2}\left[(M_{*0} - M_0)^2 - m_{\pi 0}^2\right]^{3/2},$$

$$\Gamma_{*0,+} = 0.$$

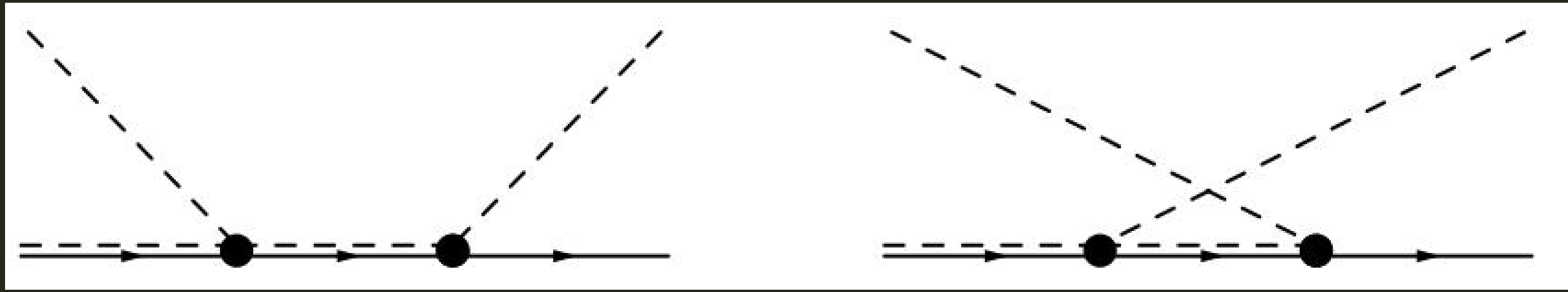
$\Gamma_{*a,\gamma}$ is the radiative decay rate

Feynman diagrams

$$D^a\pi \to D^{*b}$$



$$\pi D^{*a} \leftrightarrow \pi D^b$$



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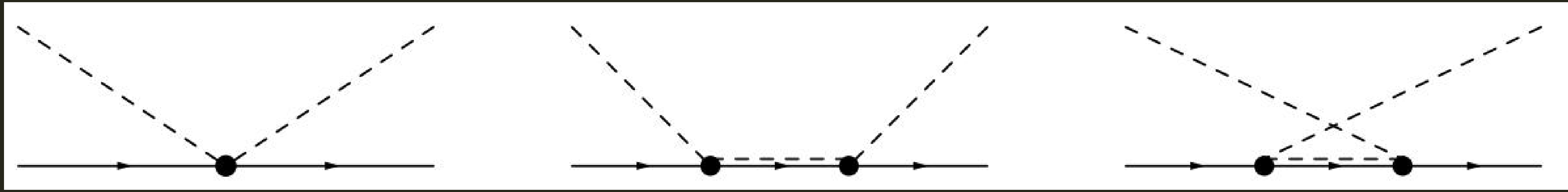

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