Two-loop QCD amplitudes for tW production at hadron colliders

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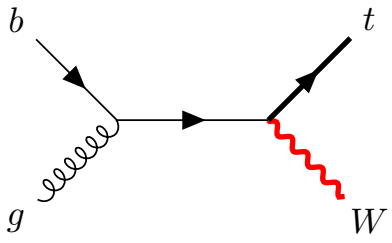
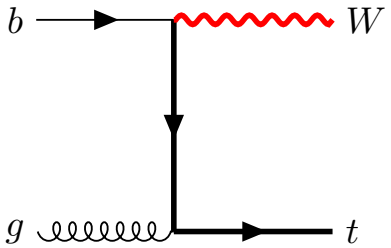
Motivation

The top quark is the heaviest elementary particle in the Standard Model.

Three major modes for single top productions, s-channel, t-channel and tW production.

tW production can be used to probe the the CKM matrix element V_{tb}.





The uncertainty of the measured cross section is about 10 %. [CMS, 2022].

To match experiments, the theoretical predictions must include higher-order corrections.

Motivation

NLO correction [S. Zhu 2002, Q.-H. Cao 2008] with top and W decay [Campbell, Tramontano 2005]

Approximate higher order corrections [N. Kidonakis 2006, 2010, 2017, 2021]

Effect of the parton shower [Frixione, Laenen, Motylinski, Webber, White 2008, E. Re 2011, Ježo, Lindert, Nason, Oleari, Pozzorini 2016]

NNLL soft-gluon resummation [Li, Li, Shao, Wang 2019]

To match experiments, the complete NNLO QCD corrections are important.

Factorization formula

The N-jettiness subtraction is based on the soft-collinear effective theory (SCET).

$$\frac{d\sigma}{d\tau_N} \propto \int H \otimes B_1 \otimes B_2 \otimes S \otimes \left(\prod_{n=1}^{N} J_n \right) . \tag{1}$$

NNLO Beam functions B_i [Stewart, Tackmann, Waalewijn 2010, Berger, Marcantonini, Stewart, Tackmann, Waalewijn 2011, Gaunt, Stahlhofen, Tackmann, 2014]

NNLO Jet function J [Becher, Bell 2006, Becher, Neubert 2011]

NNLO Soft function S [Li, Wang 2016, 2018]

The missing part is NNLO hard function, which demands one-loop squared amplitudes and the interference between two-loop and tree-level amplitudes.

Kinematics and notations

$$g(k_1) + b(k_2) \to W(k_3) + t(k_4),$$
$$k_1^2 = k_2^2 = 0,\ k_3^2 = m_W^2,\ k_4^2 = (k_1 + k_2 - k_3)^2 = m_t^2. \tag{2}$$

The polarization summation

$$\sum_i \epsilon_i^{*\mu}(k_3)\epsilon_i^{\nu}(k_3) = -g^{\mu\nu} + \frac{k_3^\mu k_3^\nu}{m_W^2}$$
$$\sum_i \epsilon_i^{\mu}(k_1)\epsilon_i^{*\nu}(k_1) = -g^{\mu\nu} + \frac{k_1^\mu n^\nu + k_1^\nu n^\mu}{k_1 \cdot n} \text{ (can be neglected here)}. \tag{3}$$

Kinematics and notations

The anticommuting γ_5 scheme is implemented.

The tW amplitude can be written as

$$\mathcal{M} = \mathcal{M}^{(0)} + \frac{\alpha_s}{4\pi}\mathcal{M}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathcal{M}^{(2)} + \cdots \tag{4}$$

We do not consider the decay of the top quark and the W boson at the moment.

Do not keep the polarization information, focus on amplitude squared,

$$|\mathcal{M}^{(1)}|^2, \quad |\mathcal{M}^{(0)*}\mathcal{M}^{(2)}|. \tag{5}$$

Then all the Lorentz indices are contracted.

Color structures

According to color structures, we have

$$\begin{aligned}
&\mathcal{M}^{(2)}\mathcal{M}^{(0)*} + \mathcal{M}^{(0)}\mathcal{M}^{(2)*} + \left|\mathcal{M}^{(1)}\right|^2 \\
&= N_c^4 A + N_c^2 B + C + \frac{1}{N_c^2} D + n_l (N_c^3 E_l + N_c F_l + \frac{1}{N_c} G_l) \\
&\quad + n_h (N_c^3 E_h + N_c F_h + \frac{1}{N_c} G_h),
\end{aligned} \tag{6}$$

n_l (n_h) is the number of light (heavy) quark flavors. N_c is the color factor.

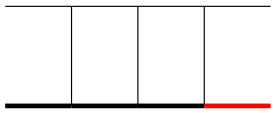
Leading contribution

$$\mathcal{A}^{(2)}_{\mathrm{L.C.+n_l}} \equiv N_c^4 A + n_l (N_c^3 E_l + N_c F_l + \frac{1}{N_c} G_l). \tag{7}$$

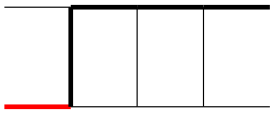
Calculation of master integrals

After IBP reduction of FIRE [Smirnov, Chuharev 2019], $\mathcal{A}^{(2)} = \sum_{\text{spins}} |\mathcal{M}^{(0)*}\mathcal{M}^{(2)}|$ can be reduced to several families of master integrals.

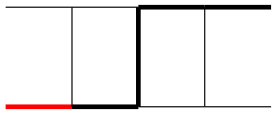
P1



P2



P3



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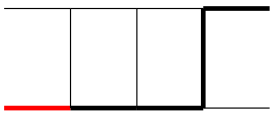

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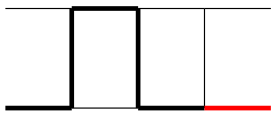
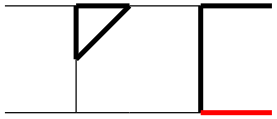
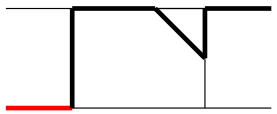


P6

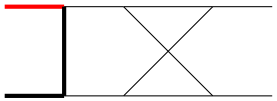


P7

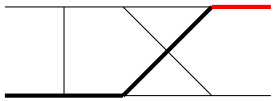


P8

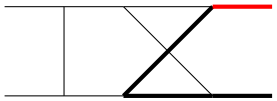
Red lines are W boson, thick lines are top quarks, others are massless particles.



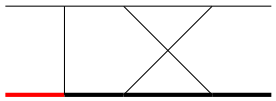
NP1



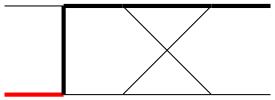
NP2



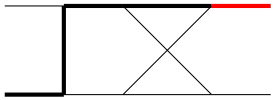
NP3



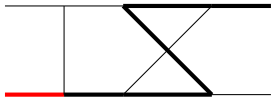
NP4



NP5



NP6

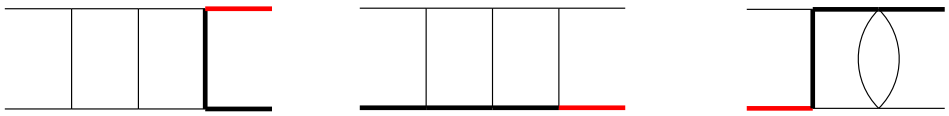


NP7

Master integrals of leading contribution

$$\mathcal{A}^{(2)}_{\mathrm{L.C.+n_l}} \equiv N_c^4 A + n_l(N_c^3 E_l + N_c F_l + \frac{1}{N_c} G_l). \tag{8}$$

Only P1 and P2 and a sub-diagram of P3 relate to $\mathcal{A}^{(2)}_{\mathrm{L.C.+n_l}}$.



P1 P2 sub-diagram of P3

Their analytical results have been obtained [Chen, Wang 2021, Long, Zhang, Ma, Jiang, Han, Li, Wang 2021] by the differential equations.

IR divergences

IR divergences can be subtracted with a factor $\mathbf{Z}$.

$$\mathcal{M}_{\text{fin}} = \mathbf{Z}^{-1}\mathcal{M}_{\text{ren}} \,. \tag{9}$$

In the framework of SCET, $\mathbf{Z}$ can be investigated through the anomalous-dimensions of the effective operators.

$$\mathbf{Z} = 1 + \frac{\alpha_s}{4\pi}\mathbf{Z}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathbf{Z}^{(2)} \; + \mathcal{O}(\alpha_s^3). \tag{10}$$

For example,

$$\begin{aligned}\mathbf{Z}^{(1)} = &-(C_A + C_F)\frac{\gamma_{\text{cusp}}^{(0)}}{4\epsilon^2} + \frac{\gamma_g^{(0)} + \gamma_b^{(0)} + \gamma_t^{(0)}}{2\epsilon} \\ &+ \frac{\gamma_{\text{cusp}}^{(0)}}{4\epsilon}\left(-C_A \ln\frac{\mu^2}{-s} - C_A \ln\frac{\mu m_t}{m_t^2 - u} + (C_A - 2C_F)\ln\frac{\mu m_t}{m_t^2 - t}\right).\end{aligned} \tag{11}$$

where γ_{cusp}, γ_g, γ_b and γ_t are anomalous dimensions.

NNLO hard function

$$H^{(2)} = \mathcal{M}^{(2)}_{\text{fin}}\mathcal{M}^{(0)*}_{\text{fin}} + \mathcal{M}^{(0)}_{\text{fin}}\mathcal{M}^{(2)*}_{\text{fin}} + \left|\mathcal{M}^{(1)}_{\text{fin}}\right|^2 \tag{12}$$

According to color structures

$$\begin{aligned} H^{(2)} =& N_c^4 H_A + N_c^2 H_B + H_C + \frac{1}{N_c^2} H_D + n_l \left(N_c^3 H_{El} + N_c H_{Fl} + \frac{1}{N_c} H_{Gl} \right) \\ &+ n_h \left(N_c^3 H_{Eh} + N_c H_{Fh} + \frac{1}{N_c} H_{Gh} \right) . \end{aligned} \tag{13}$$

Leading contribution of hard function

$$H^{(2)}_{\text{L.C.+n}_\text{l}} \equiv N_c^4 H_A + n_l \left(N_c^3 H_{El} + N_c H_{Fl} + \frac{1}{N_c} H_{Gl} \right) . \tag{14}$$

Numerical results

$\beta_t = \sqrt{1 - m_t^2/E_t^2}$ measures the velocity of the top quark and θ is the angle between gluon and top quark.

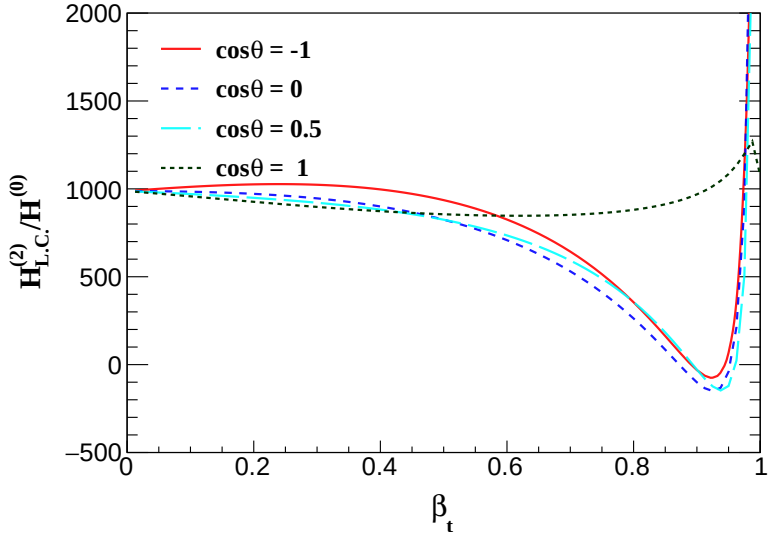
The divergence of ϵ^{-4} and ϵ^{-3} have been canceled analytically.

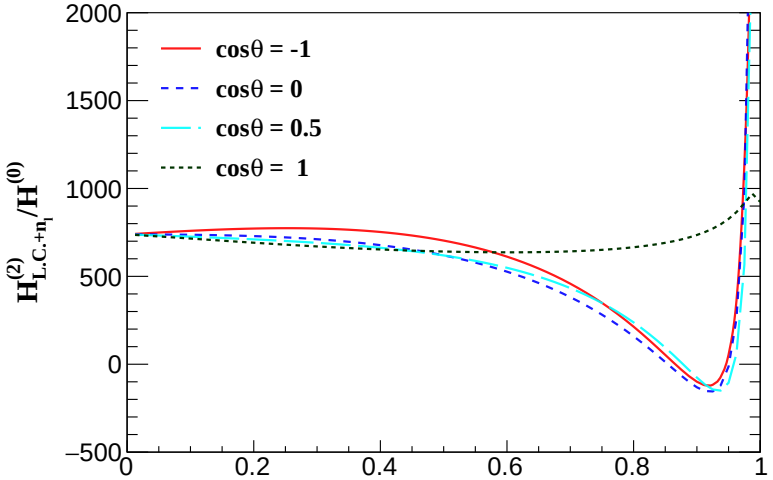
Other divergence have been checked numerically in high precision.

Numerical results

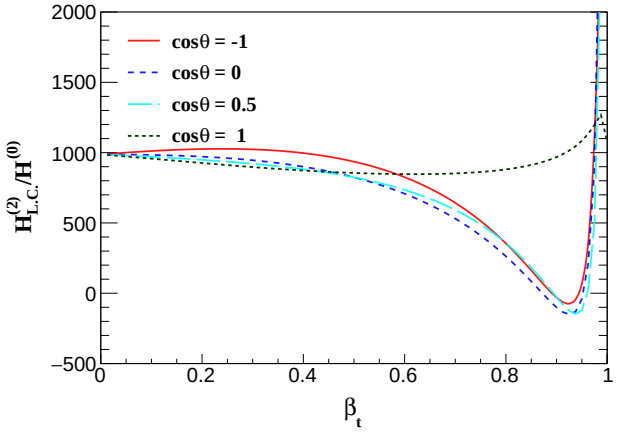
$$H = H^{(0)} + \frac{\alpha_s}{4\pi} H^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 H^{(2)}$$

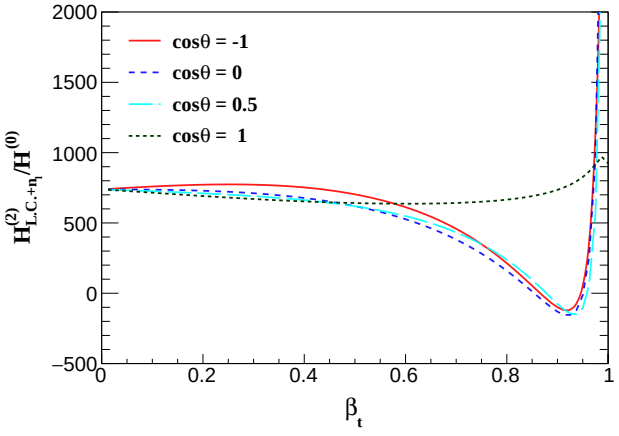
$$H^{(2)}_{\mathrm{L.C.+n_l}} \equiv N_c^4 H_A + n_l \left(N_c^3 H_{El} + N_c H_{Fl} + \frac{1}{N_c} H_{Gl}\right)$$





Numerical results





Divergence in the limit of $\beta_t \to 1$.

Fermion-loop diagrams provide negative contributions, decrease L.C. result by about 30%.

Integrate over all phase space, the L.C. and nl dependent NNLO hard function contribute 5.4% and -1.4% to the LO cross section at the 13 TeV LHC.

m_W expansion

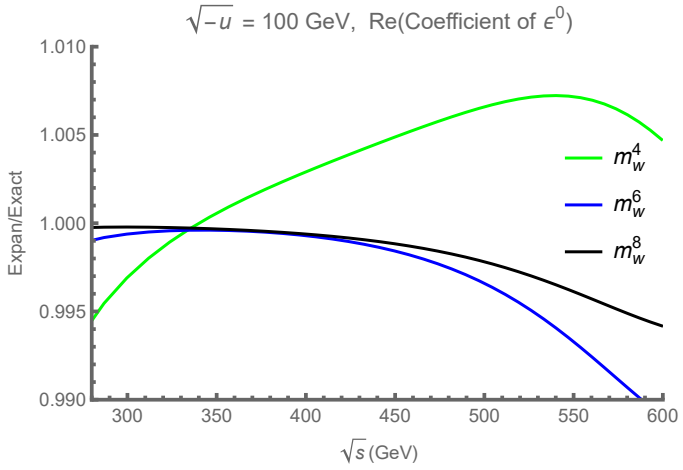
$m_W = 0$ does not bring new IR divergences due to the massive top quark propagator.

The method is used in the $gg \to HH$ calculations [Wang, Wang, Xu, Xu, Yang 2020]

$$I^i_{n_1,n_2,n_3,n_4}(s,u,m_W^2,m_t^2) = \sum_{n=0}^{\infty} \frac{(m_W^2)^n}{n!} \left. \frac{\partial^n I^i_{n_1,n_2,n_3,n_4}}{\partial (m_W^2)^n} \right|_{m_W^2=0} . \tag{15}$$

1. The number of these master integrals is less.
2. The analytic computation is easier to perform.

One two-loop master integral in NP3.



Summary and Overlook

We obtain the analytical one-loop square and two-loop leading contribution amplitudes for tW production.

The poles of renormalized amplitude squared have been checked against the infrared structures predicted by anomalous dimensions.

The finite part gives rise to about a few percent corrections compared to the corresponding LO results.

We investigate how to obtain approximated results by the expansion in m_W.

We will calculate complete two-loop amplitude.

$$\left.\frac{H^{(2)}}{H^{(0)}}\right|_{\beta_t=0.4,\,\cos\theta=-1,\,n_f=0} = \underbrace{997}_{H_A N_C^4}\ \underbrace{-162}_{H_B N_C^2}\ \underbrace{-6.60}_{H_C}\ \underbrace{-0.098}_{H_D/N_C^2}, \tag{16}$$