



Light double-gluon hybrid states from QCD sum rules

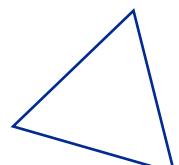
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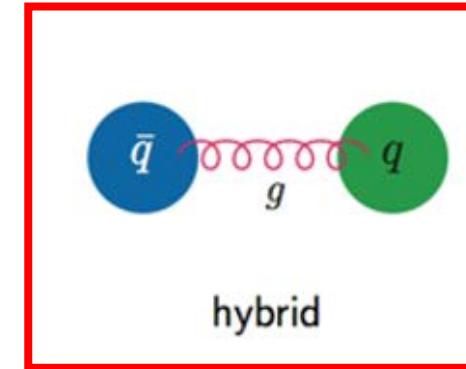
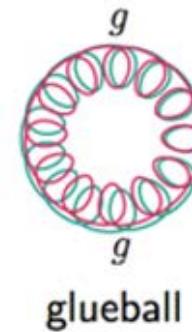
Contents



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- Double-gluon hybrid currents
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Introduction

- Exotic hadron: multiquark, glueball, **hybrid state**, etc



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- Some of the hybrid states can have the **exotic quantum numbers**

$$J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{--} \dots$$

Introduction



- There are **four candidates** observed in experiments with the exotic quantum number $J^{PC} = 1^{-+}$
 - $I^G J^{PC} = 1^- 1^{-+} : \pi_1(1400), \pi_1(1600), \pi_1(2015)$
 - D. Alde, et al, Phys.Lett. B 205 (1988)
 - E852 Collaboration, Phys. Rev. Lett. 81 (1998)
 - COMPASS Collaboration, Phys. Rev. Lett. 104 (2010)
 - E852 Collaboration, Phys. Lett. B 595 (2004)
 - $I^G J^{PC} = 0^+ 1^{-+} : \eta_1(1855)$
 - BESIII Collaboration, Phys.Rev.Lett. 129 (2022)
 - BESIII Collaboration, Phys.Rev.D 106 (2022)

Introduction



- These candidates are possible **single-gluon hybrid states** and have been studies using **various** theoretical methods and models

- MIT bag model
- Flux-tube model
- Constituent gluon model
- Ads/QCD model
- Dyson-Schwinger equation
- Lattice QCD
- QCD sum rules

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M. S. Chanowitz, S. R. Sharpe, Nucl. Phys. B 228, 588 (1983)

P. R. Page, E. S. Swanson, A. P. Szczepaniak, Phys. Rev. D 59 (1999)

P. Guo, A. P. Szczepaniak, G. Galata, A. Vassallo, E. Santopinto, Phys. Rev. D 77 (2008)

S.-S. Xu, Z.-F. Cui, L. Chang, J. Papavassiliou, C. D. Roberts, H.-S. Zong, Eur. Phys. J. A 55 (7) (2019)

K. J. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90 (2003)

H. X. Chen, N. Su, S. L. Zhu, Chin. Phys. Lett. 39 (2022)

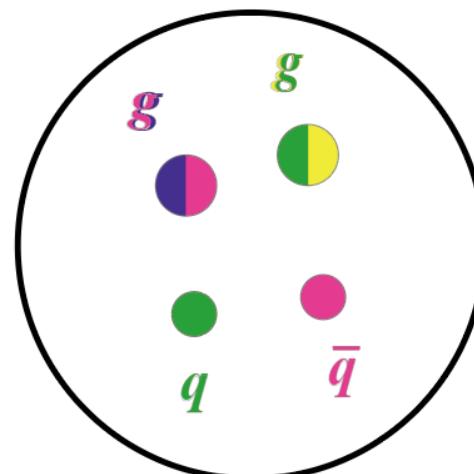
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Introduction

- Recently TOTEM and D0 experiments announce the discovery of **C-odd three-gluon glueball**

TOTEM and D0 Collaborations, Phys. Rev. Lett. 127, 062003 (2021)

- Motivated by it, we want to study **the double-gluon hybrid states** composed of one valence quark , one valence antiquark and two valence gluons



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Double-gluon hybrid currents



- Color-octet quark-antiquark fields

$$\begin{aligned} & \bar{q}_a \lambda_n^{ab} q_b, \boxed{\bar{q}_a \lambda_n^{ab} \gamma_5 q_b}, \\ & \bar{q}_a \lambda_n^{ab} \gamma_\mu q_b, \bar{q}_a \lambda_n^{ab} \gamma_\mu \gamma_5 q_b, \\ & \bar{q}_a \lambda_n^{ab} \sigma_{\mu\nu} q_b, \end{aligned}$$

- Color-octet double-gluon fields

$$d^{npq} G_p^{\alpha\beta} G_q^{\gamma\delta}, f^{npq} G_p^{\alpha\beta} G_q^{\gamma\delta},$$

- Color structure

$$\begin{aligned} \mathbf{3}_q \otimes \bar{\mathbf{3}}_{\bar{q}} \otimes \mathbf{8}_g \otimes \mathbf{8}_g & \rightarrow \mathbf{8}_{\bar{q}q} \otimes \mathbf{8}_g \otimes \mathbf{8}_g \\ & \rightarrow \mathbf{1}_{\bar{q}qgg}^S \oplus \mathbf{1}_{\bar{q}qgg}^A \end{aligned}$$

Double-gluon hybrid currents



- **Twelve** double-gluon hybrids currents
- These currents may couple to **the lowest-lying** double-gluon hybrid states
- $\bar{q}_a \gamma_5 \lambda_n^{ab} q_b$ has **S-wave** spin-parity quantum number $J^P = 0^-$

$$J_{0++} = \bar{q}_a \gamma_5 \lambda_n^{ab} q_b d^{npq} g_s^2 G_p^{\mu\nu} \tilde{G}_{q,\mu\nu}, \quad (1)$$

$$J_{0+-} = \bar{q}_a \gamma_5 \lambda_n^{ab} q_b f^{npq} g_s^2 G_p^{\mu\nu} \tilde{G}_{q,\mu\nu}, \quad (2)$$

$$J_{0-+} = \bar{q}_a \gamma_5 \lambda_n^{ab} q_b d^{npq} g_s^2 G_p^{\mu\nu} G_{q,\mu\nu}, \quad (3)$$

$$J_{0--} = \bar{q}_a \gamma_5 \lambda_n^{ab} q_b f^{npq} g_s^2 G_p^{\mu\nu} G_{q,\mu\nu}, \quad (4)$$

$$J_{1++}^{\alpha\beta} = \bar{q}_a \gamma_5 \lambda_n^{ab} q_b d^{npq} g_s^2 G_p^{\alpha\mu} \tilde{G}_{q,\mu}^{\beta} - \{\alpha \leftrightarrow \beta\}, \quad (5)$$

$$J_{1+-}^{\alpha\beta} = \bar{q}_a \gamma_5 \lambda_n^{ab} q_b f^{npq} g_s^2 G_p^{\alpha\mu} \tilde{G}_{q,\mu}^{\beta} - \{\alpha \leftrightarrow \beta\}, \quad (6)$$

$$J_{1-+}^{\alpha\beta} = \bar{q}_a \gamma_5 \lambda_n^{ab} q_b d^{npq} g_s^2 G_p^{\alpha\mu} G_{q,\mu}^{\beta} - \{\alpha \leftrightarrow \beta\}, \quad (7)$$

$$J_{1--}^{\alpha\beta} = \bar{q}_a \gamma_5 \lambda_n^{ab} q_b f^{npq} g_s^2 G_p^{\alpha\mu} G_{q,\mu}^{\beta} - \{\alpha \leftrightarrow \beta\}, \quad (8)$$

$$J_{2++}^{\alpha_1\beta_1, \alpha_2\beta_2} = \bar{q}_a \gamma_5 \lambda_n^{ab} q_b d^{npq} \mathcal{S}[g_s^2 G_p^{\alpha_1\beta_1} \tilde{G}_q^{\alpha_2\beta_2}], \quad (9)$$

$$J_{2+-}^{\alpha_1\beta_1, \alpha_2\beta_2} = \bar{q}_a \gamma_5 \lambda_n^{ab} q_b f^{npq} \mathcal{S}[g_s^2 G_p^{\alpha_1\beta_1} \tilde{G}_q^{\alpha_2\beta_2}], \quad (10)$$

$$J_{2-+}^{\alpha_1\beta_1, \alpha_2\beta_2} = \bar{q}_a \gamma_5 \lambda_n^{ab} q_b d^{npq} \mathcal{S}[g_s^2 G_p^{\alpha_1\beta_1} G_q^{\alpha_2\beta_2}], \quad (11)$$

$$J_{2--}^{\alpha_1\beta_1, \alpha_2\beta_2} = \bar{q}_a \gamma_5 \lambda_n^{ab} q_b f^{npq} \mathcal{S}[g_s^2 G_p^{\alpha_1\beta_1} G_q^{\alpha_2\beta_2}], \quad (12)$$



Double-gluon hybrid currents

- We find five currents to be zero
- Some internal symmetries between the two gluon fields

$$\begin{aligned} J_{0--} &= \dots \times f^{npq} G_p^{\mu\nu} G_{q,\mu\nu} \\ &= \dots \times f^{nqp} G_q^{\mu\nu} G_{p,\mu\nu} \\ &= -\dots \times f^{npq} G_p^{\mu\nu} G_{q,\mu\nu} \\ &= -J_{0--}. \end{aligned}$$

$$J_{0++} = \bar{q}_a \gamma_5 \lambda_n^{ab} q_b d^{npq} g_s^2 G_p^{\mu\nu} \tilde{G}_{q,\mu\nu}, \quad (1)$$

$$J_{0+-} = \bar{q}_a \gamma_5 \lambda_n^{ab} q_b f^{npq} g_s^2 G_p^{\mu\nu} \tilde{G}_{q,\mu\nu}, \quad (2)$$

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$$J_{2++}^{\alpha_1\beta_1, \alpha_2\beta_2} = \bar{q}_a \gamma_5 \lambda_n^{ab} q_b d^{npq} \mathcal{S}[g_s^2 G_p^{\alpha_1\beta_1} \tilde{G}_q^{\alpha_2\beta_2}], \quad (9)$$

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$$J_{2--}^{\alpha_1\beta_1, \alpha_2\beta_2} = \bar{q}_a \gamma_5 \lambda_n^{ab} q_b f^{npq} \mathcal{S}[g_s^2 G_p^{\alpha_1\beta_1} G_q^{\alpha_2\beta_2}], \quad (12)$$

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QCD sum rule analyses



- In QCD sum rule analyses, we consider **two-point correlation functions**

$$\begin{aligned} & \Pi^{\alpha_1\beta_1,\alpha_2\beta_2;\alpha'_1\beta'_1,\alpha'_2\beta'_2}(q^2) \\ & \equiv i \int d^4x e^{iqx} \langle 0 | \mathbf{T}[J_{2^{+-}}^{\alpha_1\beta_1,\alpha_2\beta_2}(x) J_{2^{+-}}^{\alpha'_1\beta'_1,\alpha'_2\beta'_2\dagger}(0)] | 0 \rangle \\ & = \mathcal{S}'[g^{\alpha_1\alpha'_1} g^{\beta_1\beta'_1} g^{\alpha_2\alpha'_2} g^{\beta_2\beta'_2}] \Pi(q^2), \end{aligned}$$

- At the **hadron level**: described by the **dispersion relation**

$$\Pi(q^2) = \int_{s<}^{\infty} \frac{\rho(s)}{s - q^2 - i\varepsilon} ds \quad \rho(s) \equiv \text{Im}\Pi(s)/\pi$$

$$\begin{aligned} \rho_{\text{phen}}(s) & \equiv \sum_n \delta(s - M_n^2) \langle 0 | J | n \rangle \langle n | J^\dagger | 0 \rangle \\ & = f_X^2 \delta(s - M_X^2) + \text{continuum}. \end{aligned}$$

QCD sum rule analyses



- At the **quark-gluon level**: evaluate via operator product expansion(OPE)

$$\begin{aligned}\rho_{\text{OPE}}(s) = & \frac{\alpha_s^2 s^5}{80640\pi^4} + \left(\frac{\alpha_s \langle g_s^2 GG \rangle}{3840\pi^3} + \frac{7\alpha_s^2 \langle g_s^2 GG \rangle}{61440\pi^4} \right) s^3 \\ & + \left(\frac{\alpha_s^2 \langle \bar{q}q \rangle^2}{9} - \frac{\alpha_s \langle g_s^3 G^3 \rangle}{1536\pi^3} \right) s^2 \\ & + \left(-\frac{2\alpha_s^2 \langle \bar{q}q \rangle \langle \bar{g}_s q \sigma G q \rangle}{9} - \frac{\alpha_s \langle g_s^2 GG \rangle^2}{18432\pi^3} \right) s,\end{aligned}$$

- Perform the **Borel transform**, utilize **quark hadron duality**, finally arrive at the sum rule equation

$$\Pi(s_0, M_B^2) \equiv f_X^2 e^{-M_X^2/M_B^2} = \int_{s_<}^{s_0} e^{-s/M_B^2} \rho_{\text{OPE}}(s) ds$$

QCD sum rule analyses



- The mass equation:

$$M_X^2(s_0, M_B) = \frac{\int_{s_<}^{s_0} e^{-s/M_B^2} s \rho_{\text{OPE}}(s) ds}{\int_{s_<}^{s_0} e^{-s/M_B^2} \rho_{\text{OPE}}(s) ds}$$

- Two parameters: s_0, M_B
- Criteria:
 1. The convergence of OPE
 2. The pole contribution
 3. The mass dependence on these two parameters

QCD sum rule analyses

- The convergence of OPE

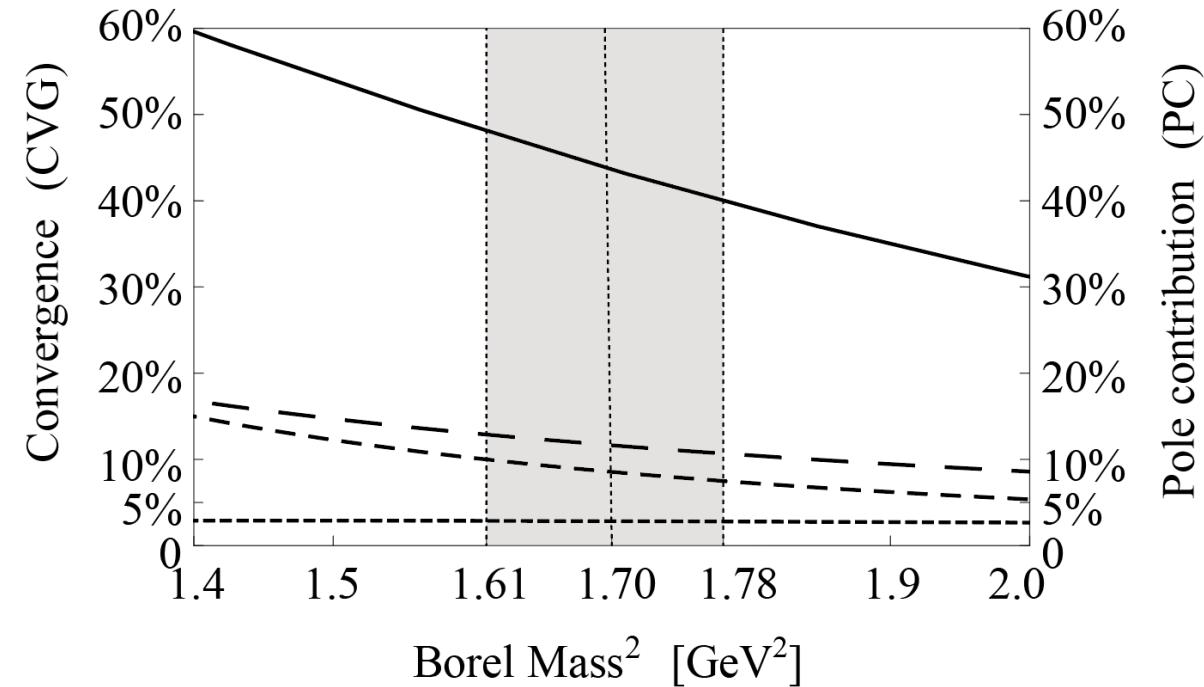
$$\text{CVG} \equiv \left| \frac{\Pi^{g_s^{n=6}}(\infty, M_B^2)}{\Pi(\infty, M_B^2)} \right| \leq 5\%,$$

$$\text{CVG}' \equiv \left| \frac{\Pi^{D=8}(\infty, M_B^2)}{\Pi(\infty, M_B^2)} \right| \leq 10\%,$$

$$\text{CVG}'' \equiv \left| \frac{\Pi_{11}^{D=6}(\infty, M_B^2)}{\Pi_{11}(\infty, M_B^2)} \right| \leq 20\%.$$

- The pole contribution

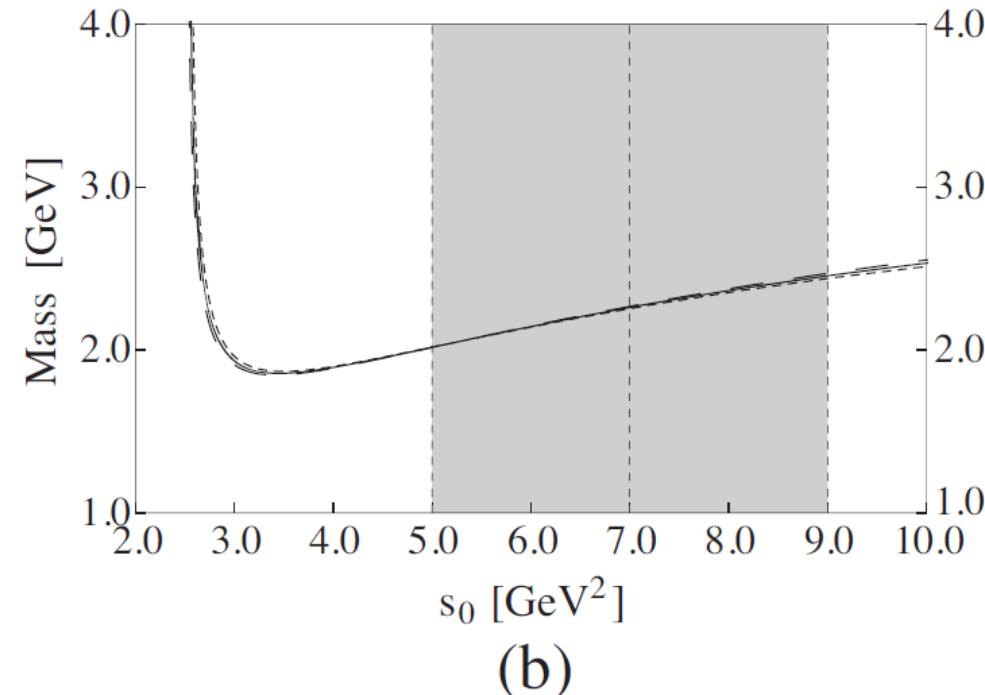
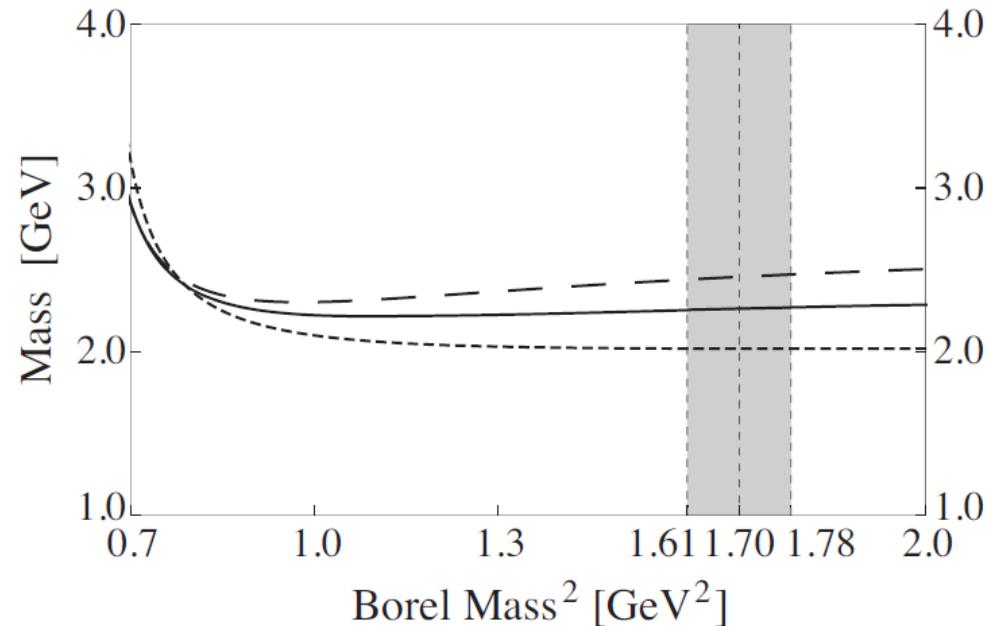
$$\text{PC} \equiv \left| \frac{\Pi(s_0, M_B^2)}{\Pi(\infty, M_B^2)} \right| \geq 40\%.$$



$1.61 \text{ GeV}^2 \leq M_B^2 \leq 1.78 \text{ GeV}^2$

QCD sum rule analyses

- The mass dependence on these two parameters



$$\begin{aligned} M_{|X;2^{+-}\rangle} &= 2.26_{-0.25}^{+0.19} \pm 0.07 \pm 0.03 \text{ GeV} \\ &= 2.26_{-0.25}^{+0.20} \text{ GeV}, \end{aligned}$$

QCD sum rule analyses



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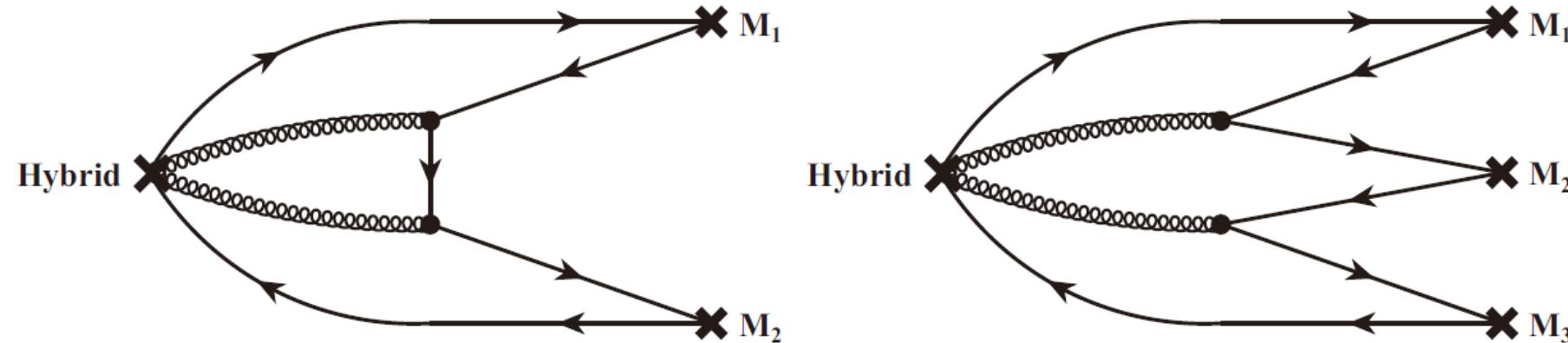
- The QCD sum rules **results**

State [J^{PC}]	Current	s_0^{\min} [GeV 2]	Working Regions		Pole [%]	Mass [GeV]
			M_B^2 [GeV 2]	s_0 [GeV 2]		
$ \bar{q}qgg; 0^{++}\rangle$	J_{0++}	34.9	6.12–6.92	38 ± 8.0	40–50	$5.61_{-0.27}^{+0.29}$
$ \bar{q}qgg; 0^{-+}\rangle$	J_{0-+}	24.4	5.34–5.78	27 ± 5.0	40–48	$4.25_{-0.39}^{+0.32}$
$ \bar{q}qgg; 1^{+-}\rangle$	$J_{1+-}^{\alpha\beta}$	32.1	5.51–6.31	35 ± 7.0	40–50	$5.46_{-0.18}^{+0.25}$
$ \bar{q}qgg; 1^{--}\rangle$	$J_{1--}^{\alpha\beta}$	20.0	4.60–4.91	22 ± 4.0	40–47	$3.74_{-0.35}^{+0.30}$
$ \bar{q}qgg; 2^{++}\rangle$	$J_{2++}^{\alpha_1\beta_1,\alpha_2\beta_2}$	20.0	5.39–5.76	22 ± 4.0	40–46	$3.74_{-0.32}^{+0.27}$
$ \bar{q}qgg; 2^{+-}\rangle$	$J_{2+-}^{\alpha_1\beta_1,\alpha_2\beta_2}$	6.4	1.61–1.78	7 ± 2.0	40–48	$2.26_{-0.25}^{+0.20}$
$ \bar{q}qgg; 2^{-+}\rangle$	$J_{2-+}^{\alpha_1\beta_1,\alpha_2\beta_2}$	16.8	4.39–4.81	19 ± 4.0	40–49	$3.51_{-0.35}^{+0.29}$
$ \bar{s}sgg; 0^{++}\rangle$	J_{0++}	35.3	6.22–7.61	41 ± 8.0	40–57	$5.72_{-0.32}^{+0.29}$
$ \bar{s}sgg; 0^{-+}\rangle$	J_{0-+}	24.5	5.36–5.95	28 ± 6.0	40–50	$4.34_{-0.46}^{+0.36}$
$ \bar{s}sgg; 1^{+-}\rangle$	$J_{1+-}^{\alpha\beta}$	32.5	5.60–6.79	37 ± 8.0	40–55	$5.52_{-0.27}^{+0.29}$
$ \bar{s}sgg; 1^{--}\rangle$	$J_{1--}^{\alpha\beta}$	20.2	4.62–5.07	23 ± 5.0	40–50	$3.84_{-0.44}^{+0.35}$
$ \bar{s}sgg; 2^{++}\rangle$	$J_{2++}^{\alpha_1\beta_1,\alpha_2\beta_2}$	20.4	5.45–6.11	24 ± 5.0	40–51	$3.91_{-0.39}^{+0.32}$
$ \bar{s}sgg; 2^{+-}\rangle$	$J_{2+-}^{\alpha_1\beta_1,\alpha_2\beta_2}$	7.1	1.79–2.01	8 ± 2.0	40–50	$2.38_{-0.25}^{+0.19}$
$ \bar{s}sgg; 2^{-+}\rangle$	$J_{2-+}^{\alpha_1\beta_1,\alpha_2\beta_2}$	17.1	4.44–5.00	20 ± 4.0	40–51	$3.61_{-0.34}^{+0.28}$

Decay analyses



- Exciting two $q\bar{q}$ / $s\bar{s}$ pairs from two gluons, followed by recombining three $q\bar{q}$ / $s\bar{s}$ pairs into two- and three-mesons



Decay analyses



- Possible two- and three-meson decay patterns

Two-Meson	$ \bar{q}qgg; 1^+2^{+-}\rangle$	$ \bar{q}qgg; 0^-2^{+-}\rangle$	$ \bar{s}sgg; 0^-2^{+-}\rangle$
<i>S-wave</i>		$K_2^* \bar{K}_0^*$	
<i>P-wave</i>	$h_1\pi, a_1\pi, a_2\pi, b_1\eta, b_1\eta', \rho f_0, \omega a_0$	$b_1\pi, h_1\eta, h_1\eta', \rho a_0, \omega f_0$	$h'_1\eta, h'_1\eta'$
		$K_1 \bar{K}, K_1 \bar{K}^*, K_2^* \bar{K}, K^* \bar{K}_0^*$	
<i>D-wave</i>	$\rho^+ \rho^-, \omega \pi, \rho \eta, \rho \eta'$	$\rho \pi, \omega \eta, \omega \eta'$	$\phi \eta, \phi \eta'$
		$K^* \bar{K}, K^* \bar{K}^*, K_1 \bar{K}_0^*$	
Three-Meson	$ \bar{q}qgg; 1^+2^{+-}\rangle$	$ \bar{q}qgg; 0^-2^{+-}\rangle$	$ \bar{s}sgg; 0^-2^{+-}\rangle$
<i>S-wave</i>	$f_1 \omega \pi, a_1 \rho \pi$	$f_1 \rho \pi, a_1 \omega \pi$	—
<i>P-wave</i>	$\rho \pi \pi, \omega \eta \pi, \omega \eta' \pi, \rho \eta \eta$	$\omega \pi \pi, \rho \eta \pi, \rho \eta' \pi, \omega \eta \eta$	$\phi \eta \eta$
		$\pi K^* \bar{K}, \rho K \bar{K}, \omega K \bar{K}, \phi K \bar{K}, \pi K^* \bar{K}^*, \rho K^* \bar{K}, \omega K^* \bar{K}, \eta K^* \bar{K}$	

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Summary



- We systematically contract twelve **double-gluon hybrid currents** using the color-octet quark-antiquark fields $\bar{q}_a \lambda_n^{ab} \gamma_5 q_b$ and the color-octet double-gluon fields $d^{npq} G_p^{\alpha\beta} G_q^{\gamma\delta}$, $f^{npq} G_p^{\alpha\beta} G_q^{\gamma\delta}$
- We apply **QCD sum rule** to analyse these currents and extract the masses of $J^{PC} = 2^{+-}$ double-gluon hybrid states are

$$M_{|\bar{q}qgg;2^{+-}\rangle} = 2.26^{+0.20}_{-0.25} \text{ GeV}$$

$$M_{|\bar{s}sgg;2^{+-}\rangle} = 2.38^{+0.19}_{-0.25} \text{ GeV}$$

- We propose to **search for** $|\bar{q}qgg; 1^+ 2^{+-}\rangle$ in the $\omega\pi/\rho\pi\pi\cdots$
search for $|\bar{q}qgg; 0^- 2^{+-}\rangle$ in the $\rho\pi/\omega\pi\pi\cdots$
search for $|\bar{s}sgg; 0^- 2^{+-}\rangle$ in the $\phi\eta/\rho K\bar{K}\cdots$

Thank You for your attention !

