格点规范场论

模拟

早期宇宙相变

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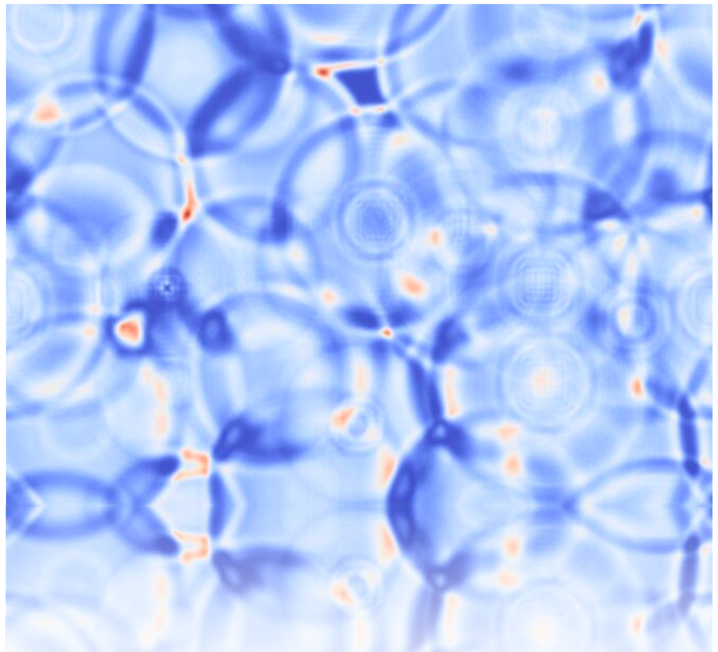
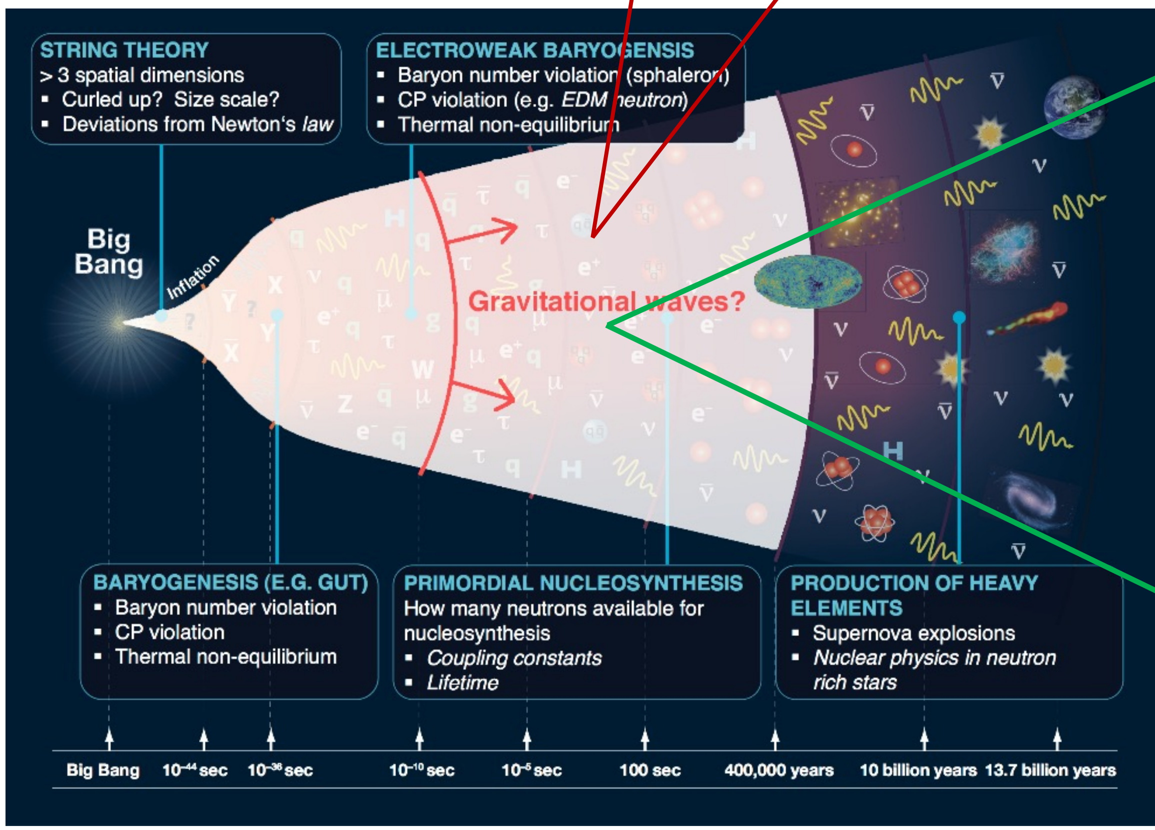
第三届粒子物理前沿研讨会@中山大学

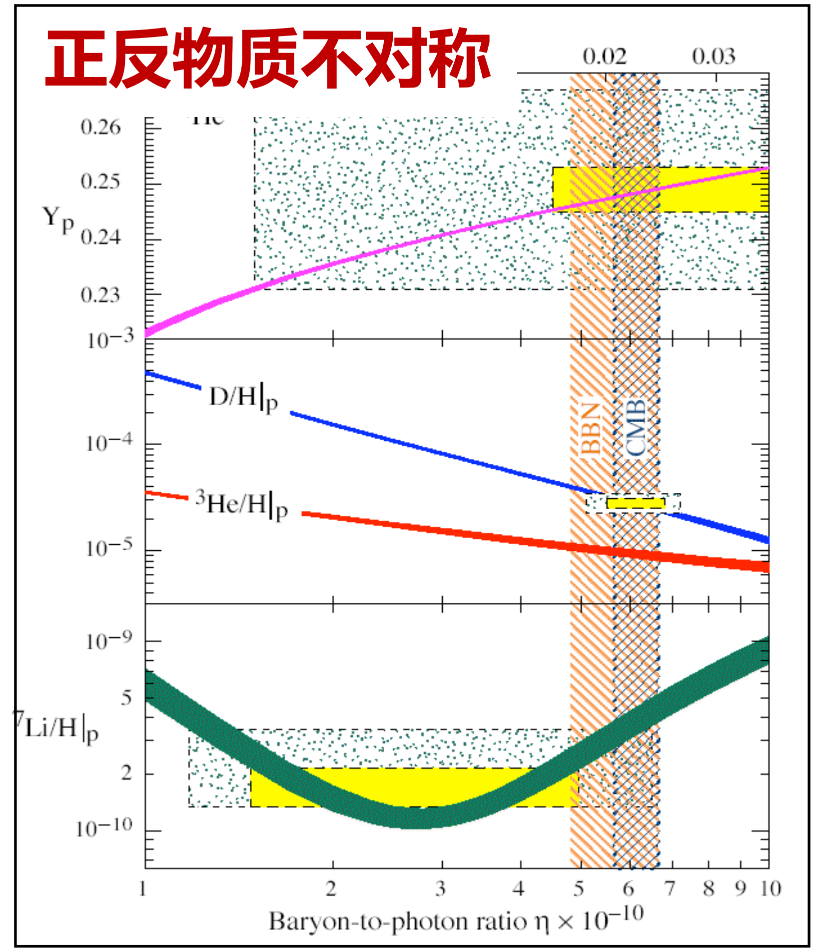
Contents

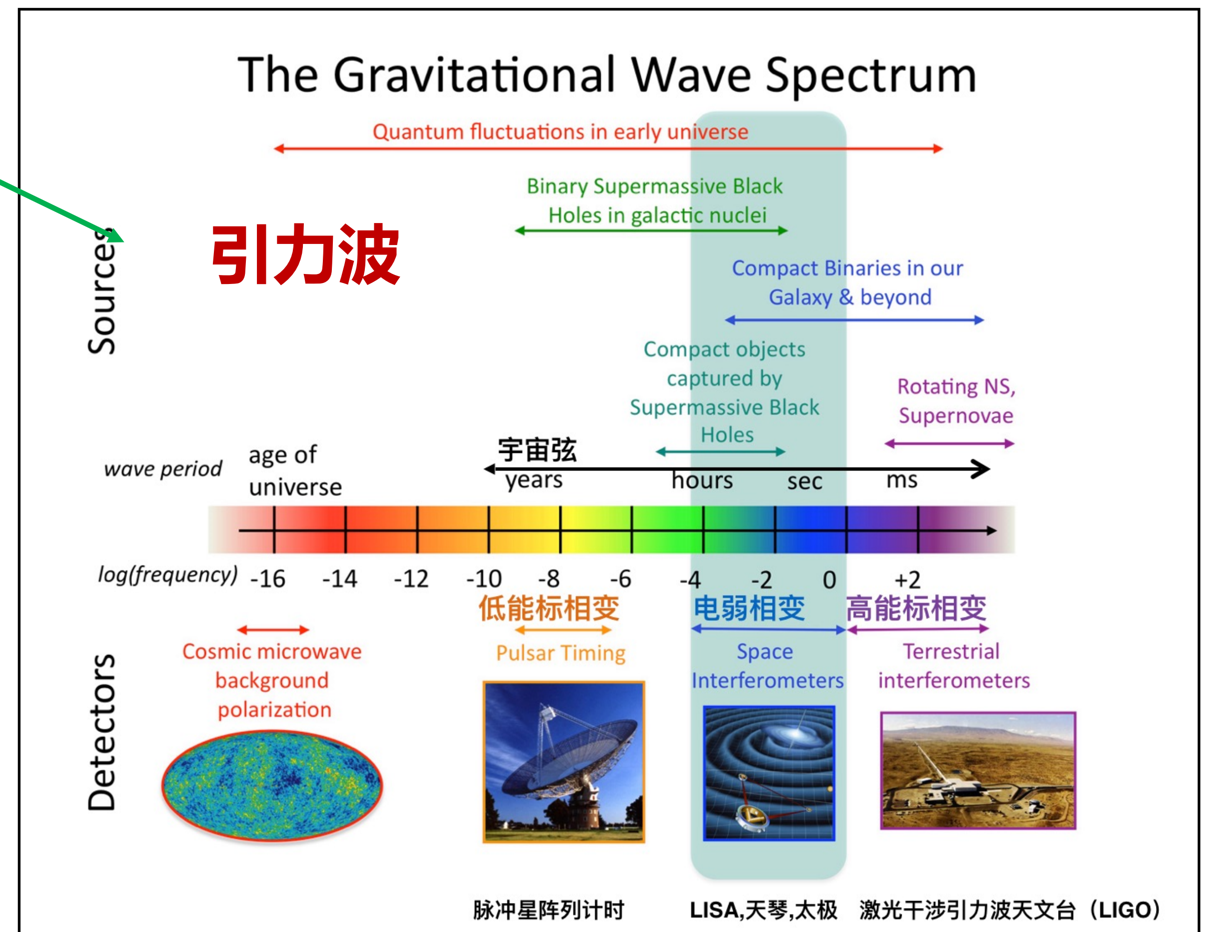
- Motivation
- First-order phase transition & Gravitational wave
- Cosmic string & First-order Phase Transition
- Future



超出粒子物理标准模型的新物理

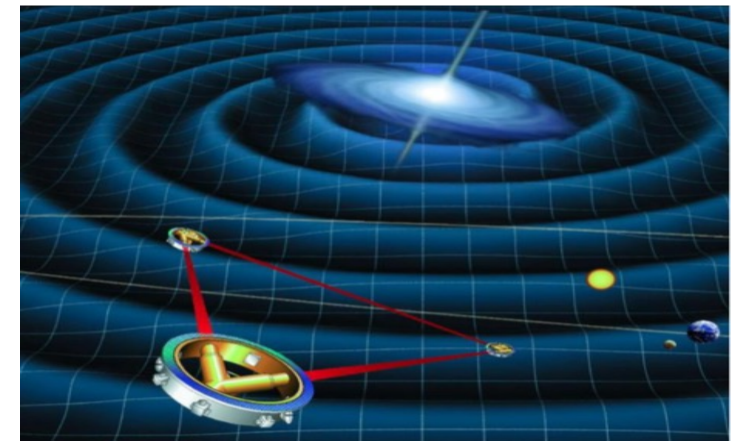




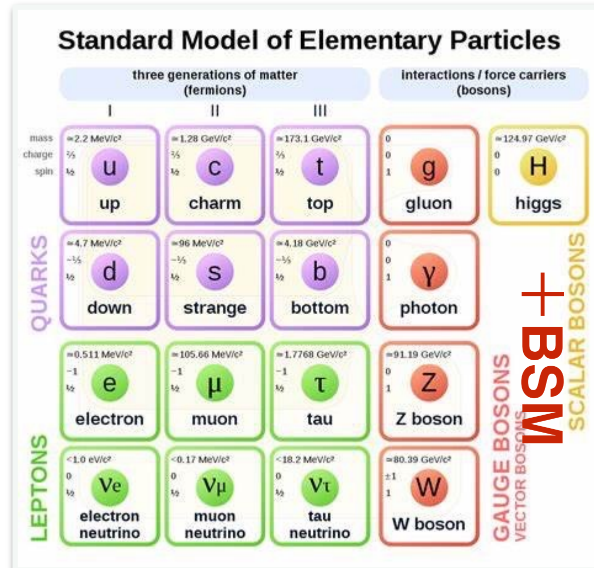


BSM for FOPT GW

PTA,LIGO,LISA,TianQin,Taiji,...



Particle physics model



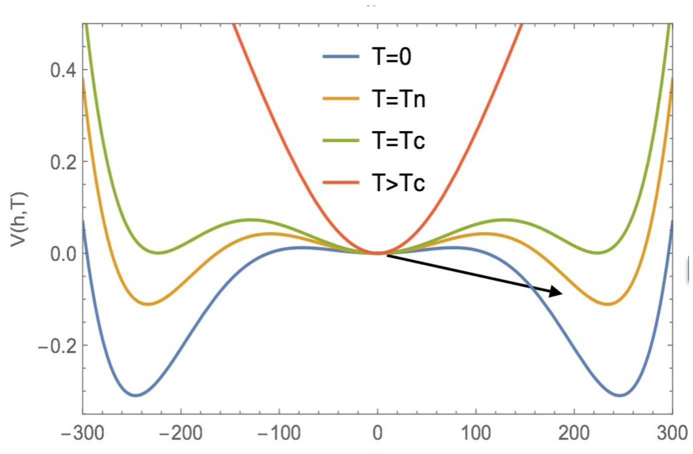
PT parameters

Effective action $\rightarrow \beta,\ H_*$

Energy budget $\rightarrow \alpha,\ \kappa(\alpha, v_{\rm w})$

Bubble wall dynamics $\rightarrow v_{\rm w}$

Thermal field theory



GW power spectrum

Numerical simulations $\rightarrow$

$h^2\Omega_{\rm GW}(f; H_*, \alpha, \beta, v_{\rm w})$

Lattice simulation

LISA sensitivity

Configuration + noise level $\rightarrow$

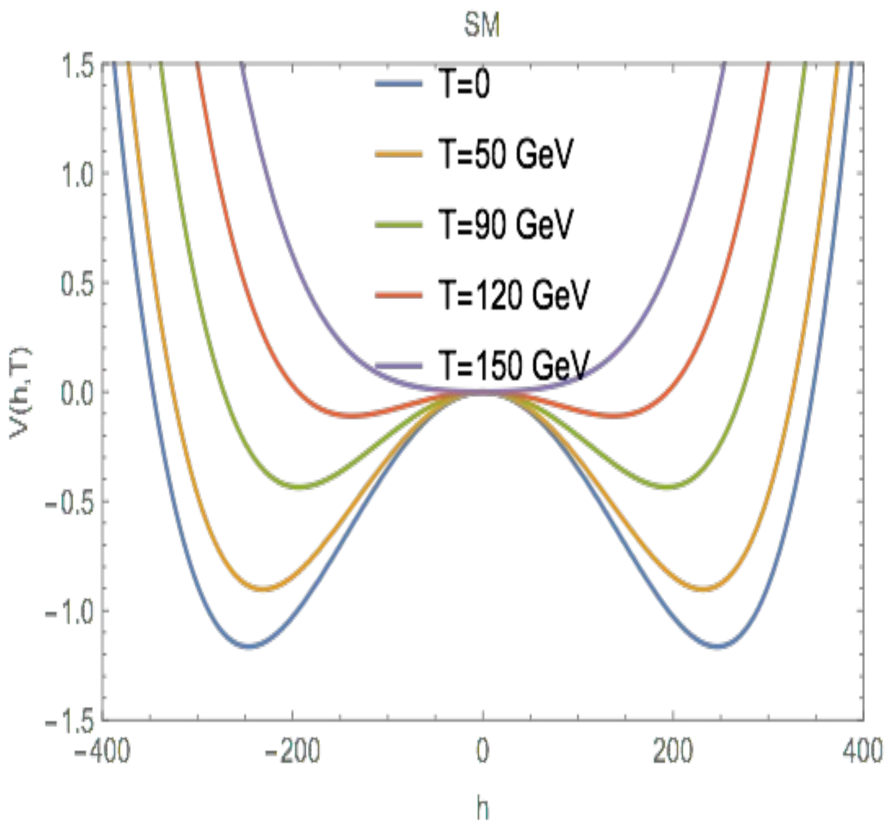
$h^2\Omega_{\rm sens}(f)$

Signal-to-noise ratio

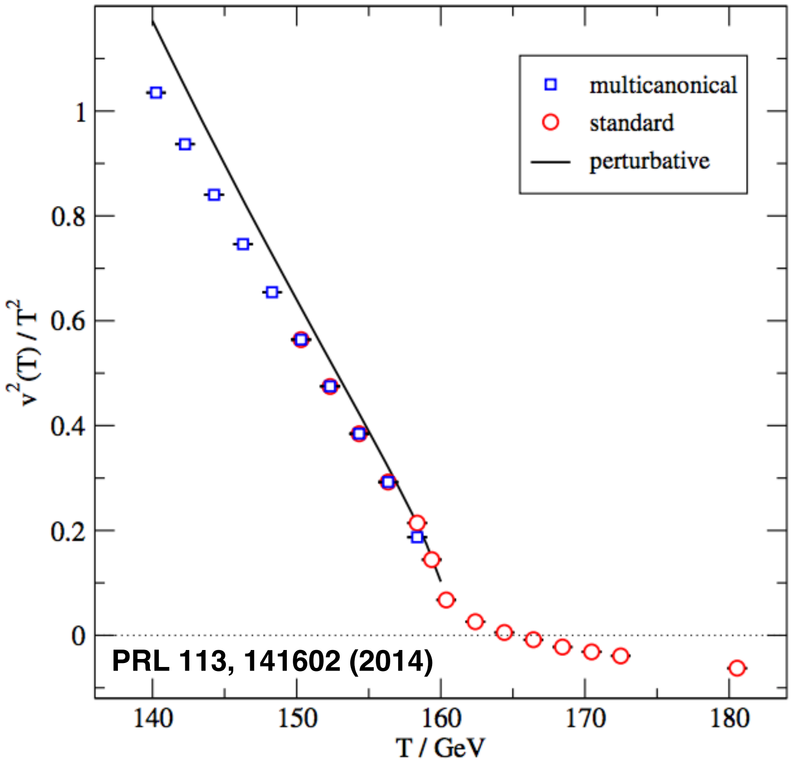
SNR

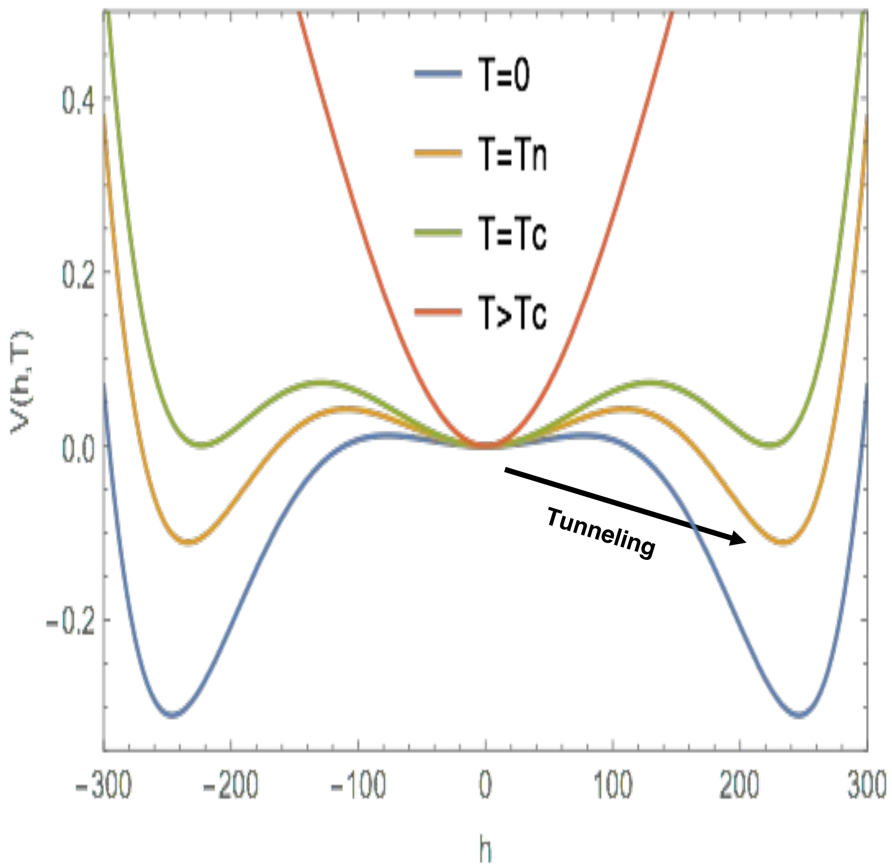
Chiara Caprini *et al* JCAP03(2020)024

FOEWPT&Higgs physics

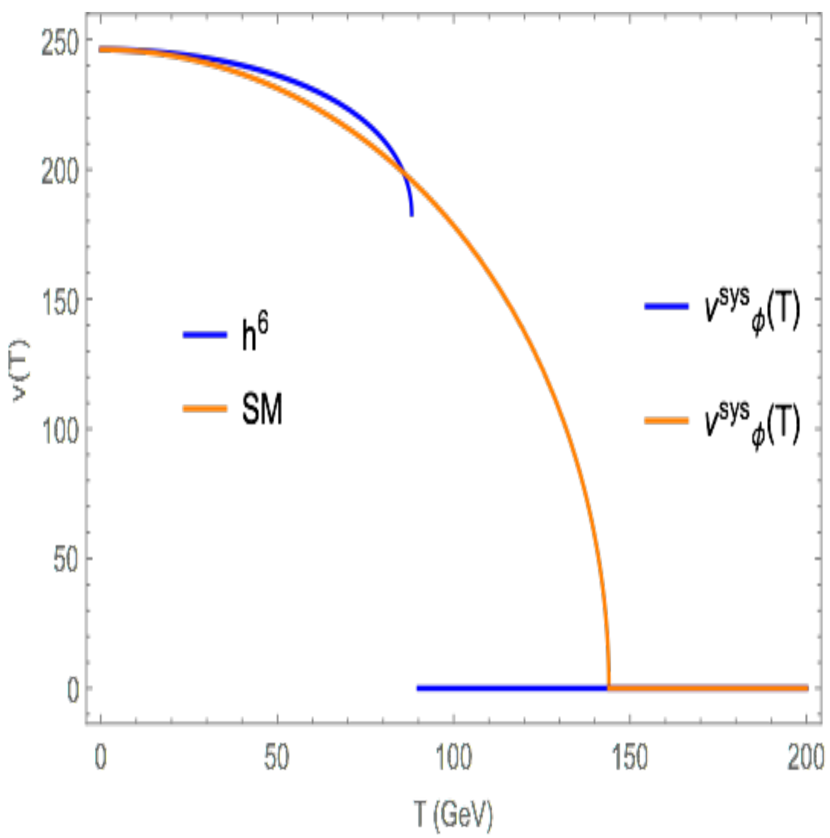


Higgs announcement seminar on 4 July 2012





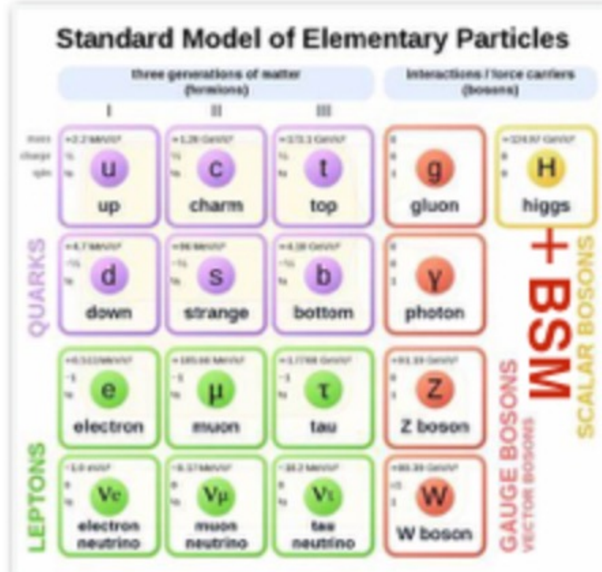
Grojean, Servant, Wells 05, Huang, Jokelar, Li, Wagner 15, Cao, Huang, Xie, & Zhang 17, Zhou, Bian, Guo 19



Beyond SM models for FOPT

Higgs&GWs

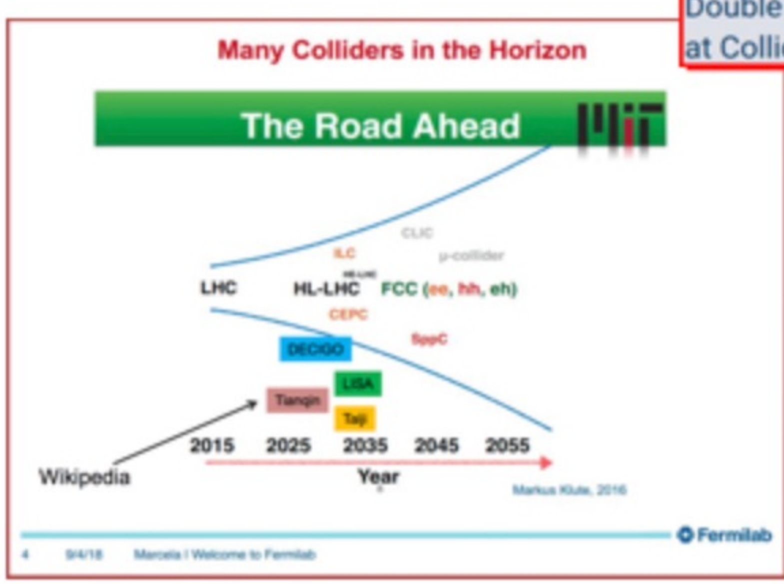
- **SM+Scalar Singlet**
  Bian, Huang, Shu 15， Cheng, Bian 17, Bian, Tang 18,Chen, Li, Wu, Bian,19
- **SM+Scalar Doublet**
  Bernon, Bian, Jiang 17, Bian, Liu 18
- **SM + Scalar Triplet**
  Zhou, Cheng, Deng, Bian, Wu 18,Zhou, Bian, Guo,Wu 19，Zhou, Bian,Du,22



- **Composite Higgs**
  Bian,Wu,Xie 19， Bian,Wu,Xie 20
- **NMSSM**
  Bi, Bian, Huang, Shu, Yin 15, Bian, Guo, Shu 17
- **SMEFT**
  Zhou, Bian, Guo 19

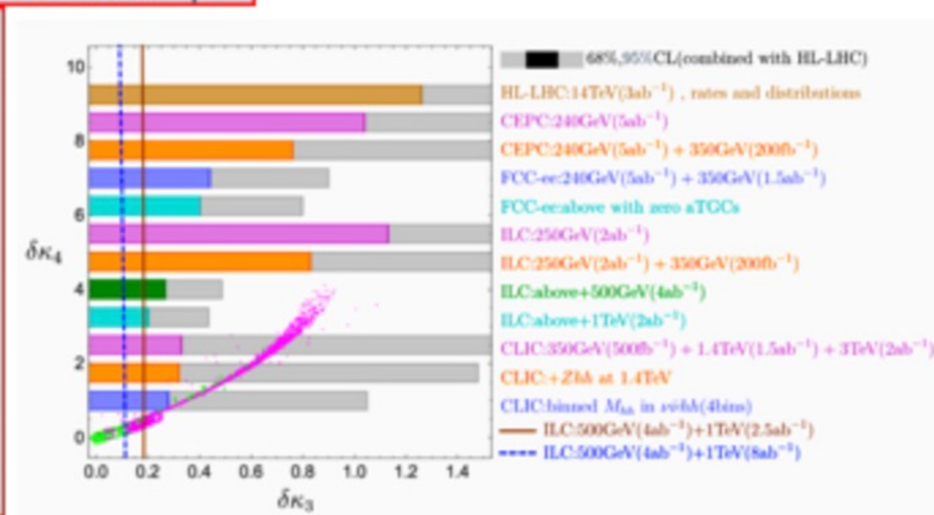
Symmetry breakdown

Symmetry breaking process

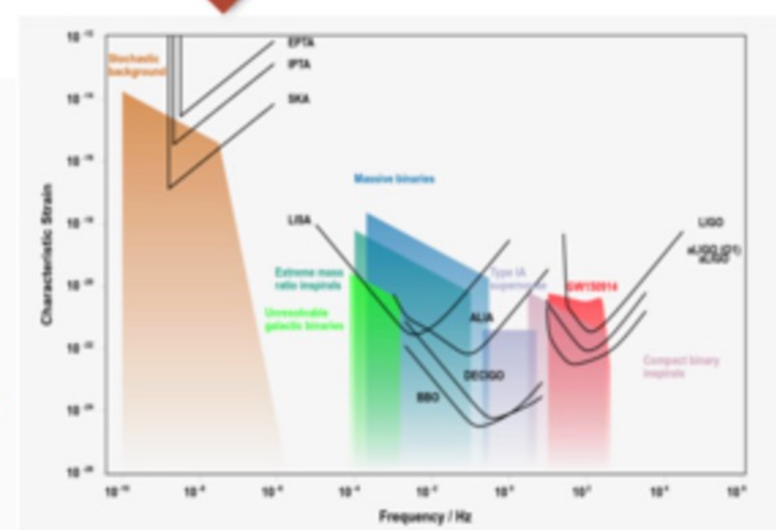


Double Higgs Production at Colliders Workshop

$$\Delta\mathscr{L} = -\frac{1}{2}\frac{m_h^2}{v}(1+\delta\kappa_3)h^3 - \frac{1}{8}\frac{m_h^2}{v^2}(1+\delta\kappa_4)h^4$$



SNR > 10 for two-step and one-step SFOEWPT



C J Moore et al. Class. Quantum Grav. 32 (2015) 015014.

PTA,LISA,TianQin,Taiji,LIGO,...

One-step FOPT

PHYSICAL REVIEW LETTERS **126**, 251102 (2021)

Magnetic Field and Gravitational Waves from the First-Order Phase Transition

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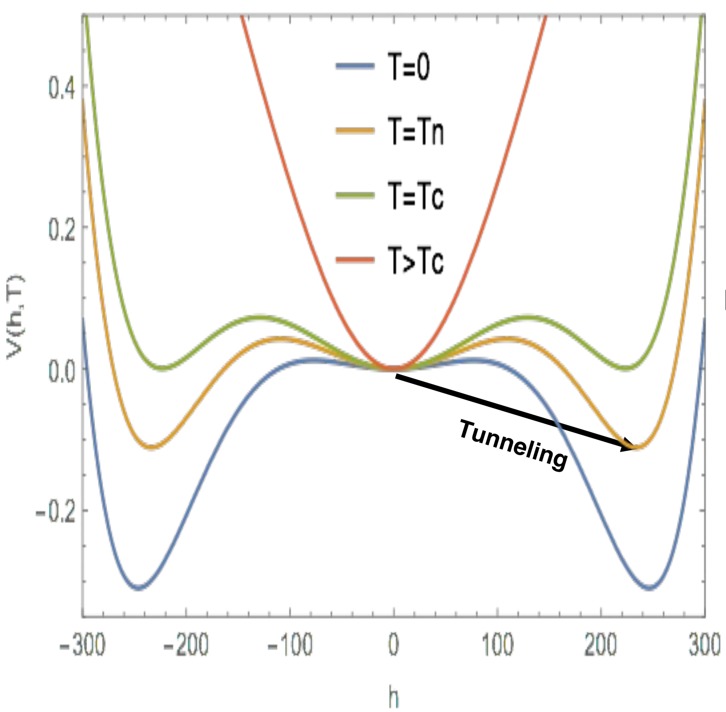
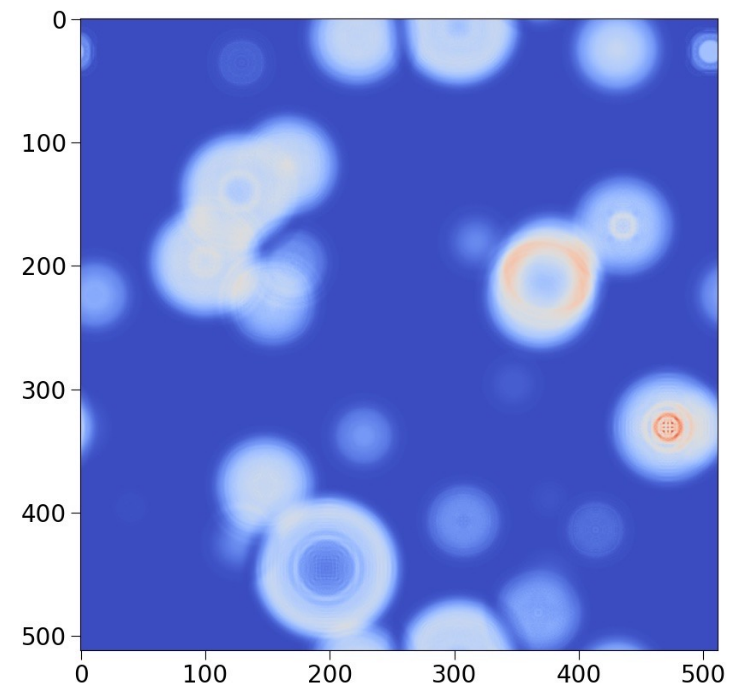
Finite-T Veff

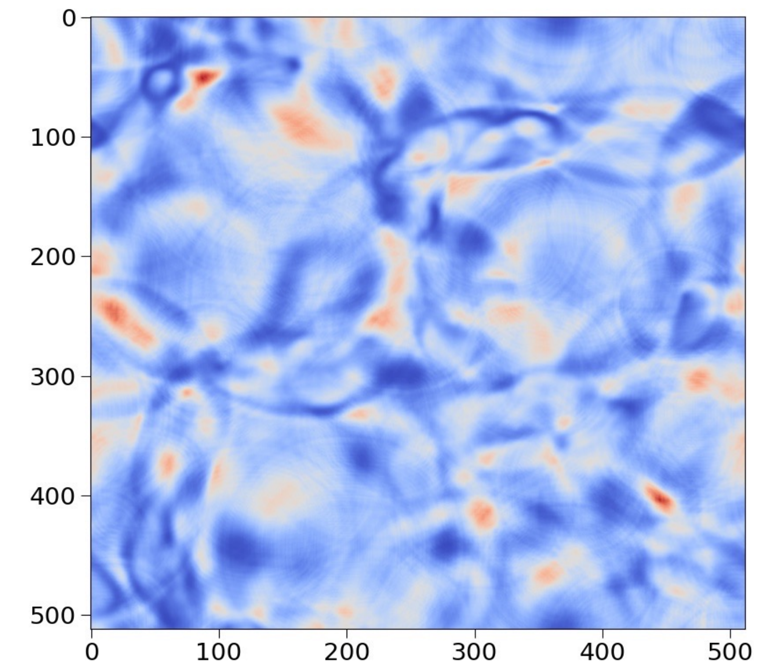


PT temperature (T_*)
duration (β)
strength (α)

Nucleation



Expansion&Percolation



Finite-T calculation

Lattice Simulation

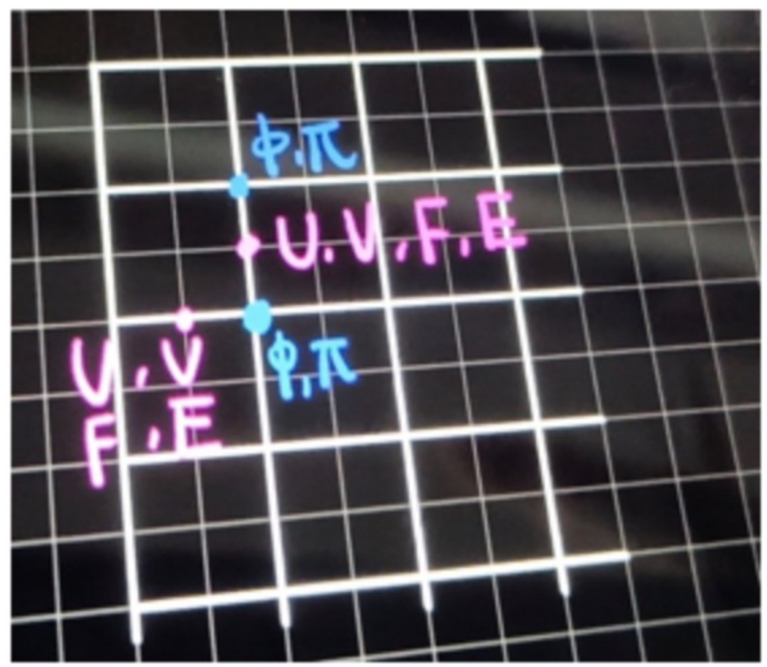
Lattice EW field foundation

$\Phi(t, x)$: **Higgs field doublet defined on sites;**

$U_i(t, x)$ and $V_i(t, x)$: SU(2) and U(1) link fields, defined on the link between the neighboring sites x and x + i , $\Phi(t, x)$, $U_i(t, x)$ and $V_i(t, x)$ are defined at time steps $t + \Delta t, t + 2\Delta t, \ldots$;

Conjugate momentum fields: $\Pi(t+\Delta t/2, x)$, $F(t+\Delta t/2, x)$ and $E(t+\Delta t/2, x)$, are defined at time steps $t + \Delta t/2, t + 3\Delta t/2$.

$$U_i(t,x) = \exp\left(-\frac{i}{2} g \Delta x \sigma^a W_i^a\right)$$
$$U_0(t,x) = \exp\left(-\frac{i}{2} g \Delta t \sigma^a W_0^a\right)$$
$$V_i(t,x) = \exp\left(-\frac{i}{2} g \Delta x B_i\right)$$
$$V_0(t,x) = \exp\left(-\frac{i}{2} g \Delta t B_0\right).$$



$$D_i\Phi = \frac{1}{\Delta x}\left[U_i(t,x)V_i(t,x)\Phi(t,x+i) - \Phi(t,x)\right]$$
$$D_0\Phi = \frac{1}{\Delta t}\left[U_0(t,x)V_0(t,x)\Phi(t+\Delta t,x) - \Phi(t,x)\right].$$
$$\Phi(t+\Delta t,x) = \Phi(t,x) + \Delta t \Pi(t+\Delta t/2,x)$$
$$V_i(t+\Delta t,x) = \frac{1}{2} g' \Delta x \Delta t E_i(t+\Delta t/2,x) V_i(t,x)$$
$$U_i(t+\Delta t,x) = g \Delta x \Delta t F_i(t+\Delta t/2,x) U_i(t,x),$$

Temporal gauge
$U_0(t, x) = I_2$, $V_0(t, x) = 1$

leapfrog

Di, Wang, Zhou, Bian*, Cai*, Liu*, Phys.Rev.Lett. 126 (2021) 251102

PT process simulation

Field basis

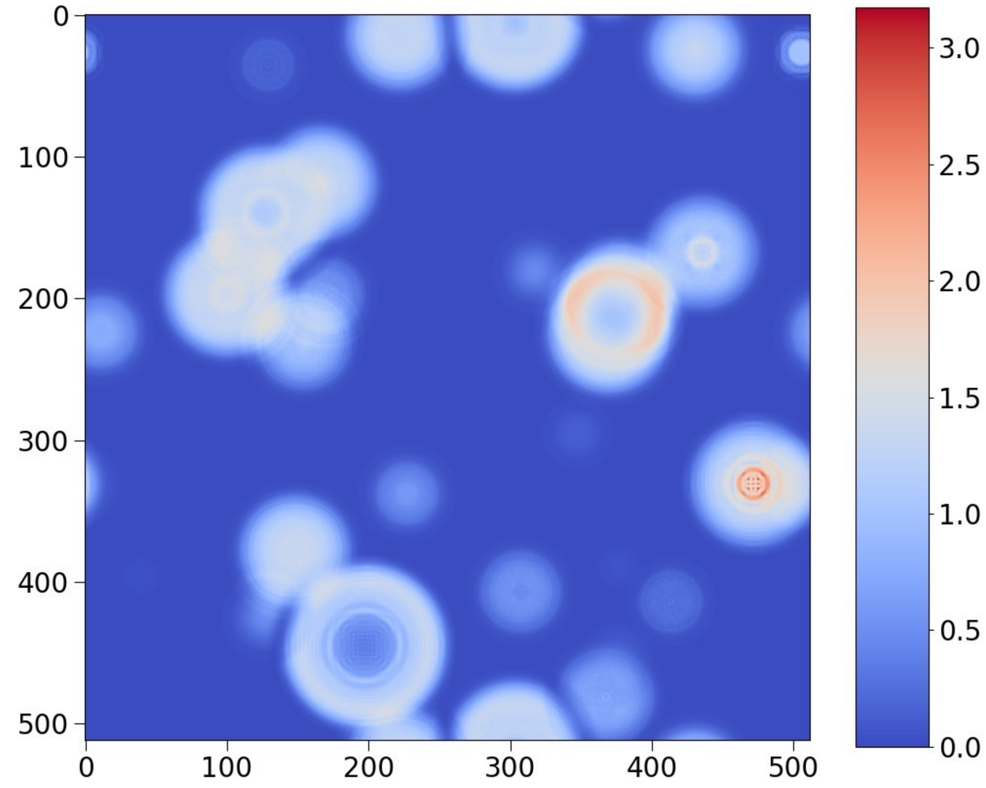
$$
\begin{aligned}
\partial_0^2\Phi &= D_iD_i\Phi - 2\lambda(|\Phi|^2-\eta^2)\Phi - 3(\Phi^\dagger\Phi)^2\Phi/\Lambda^2,\\
\partial_0^2 B_i &= -\partial_j B_{ij} + g'\,\mathrm{Im}[\Phi^\dagger D_i\Phi],\\
\partial_0^2 W_i^a &= -\partial_k W_{ik}^a - g\,\epsilon^{abc}W_k^bW_{ik}^c + g\,\mathrm{Im}[\Phi^\dagger\sigma^a D_i\Phi].
\end{aligned}
$$

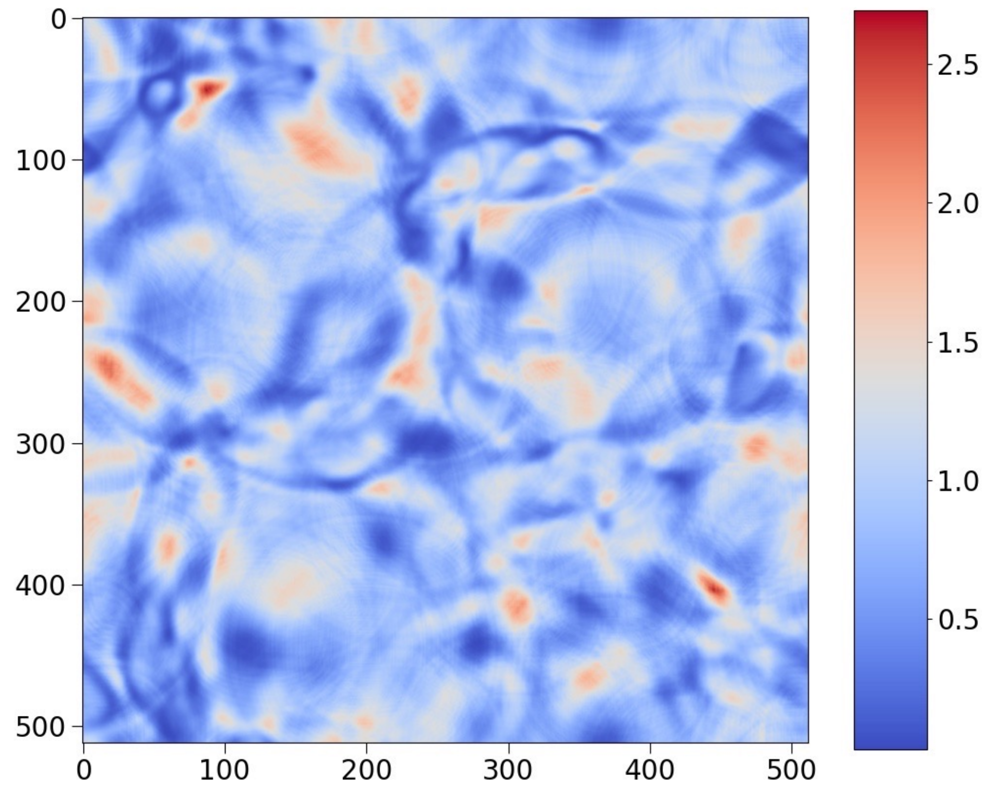
$$
\partial_0\partial_j B_j - g'\,\mathrm{Im}[\Phi^\dagger\partial_0\Phi] = 0,
$$

$$
\partial_0\partial_j W_j^a + g\,\epsilon^{abc}W_j^b\partial_0 W_j^c - g\,\mathrm{Im}[\Phi^\dagger\sigma^a\partial_0\Phi] = 0.
$$

Lattice implementation

$$
\begin{aligned}
\Pi(t+\Delta t/2,x) =& \Pi(t-\Delta t/2,x) + \Delta t\Big\{\frac{1}{\Delta x^2}\sum_i\big[U_i(t,x)V_i(t,x)\Phi(t,x+i)\\
&- 2\Phi(t,x) + U_i^\dagger(t,x-i)V_i^\dagger(t,x-i)\Phi(t,x-i)\big] - \frac{\partial U}{\partial\Phi^\dagger}\Big\}\\
\mathrm{Im}[E_k(t+\Delta t/2,x)] =& \mathrm{Im}[E_k(t-\Delta t/2,x)] + \Delta t\Big\{\frac{g'}{\Delta x}\mathrm{Im}[\Phi^\dagger(t,x+k)U_k^\dagger(t,x)V_k^\dagger(t,x)\Phi(t,x)]\\
&- \frac{2}{g'\Delta x^3}\sum_i \mathrm{Im}[V_k(t,x)V_i(t,x+k)V_k^\dagger(t,x+i)V_i^\dagger(t,x)\\
&+ V_i(t,x-i)V_k(t,x)V_i^\dagger(t,x+k-i)V_k^\dagger(t,x-i)]\Big\}\\
\mathrm{Tr}[i\sigma^m F_k(t+\Delta t/2,x)] =& \mathrm{Tr}[i\sigma^m F_k(t-\Delta t/2,x)] + \Delta t\Big\{\frac{g}{\Delta x}\mathrm{Re}[\Phi^\dagger(t,x+k)U_k^\dagger(t,x)V_k^\dagger(t,x)i\sigma^m\Phi(t,x)]\\
&- \frac{1}{g\Delta x^3}\sum_i \mathrm{Tr}[i\sigma^m U_k(t,x)U_i(t,x+k)U_k^\dagger(t,x+i)U_i^\dagger(t,x)\\
&+ i\sigma^m U_k(t,x)U_i^\dagger(t,x+k-i)U_k^\dagger(t,x-i)U_i(t,x-i)]\Big\},
\end{aligned}
$$





Di, Wang, Zhou, **Bian***, Cai*,Liu*, Phys.Rev.Lett. 126 (2021) 251102

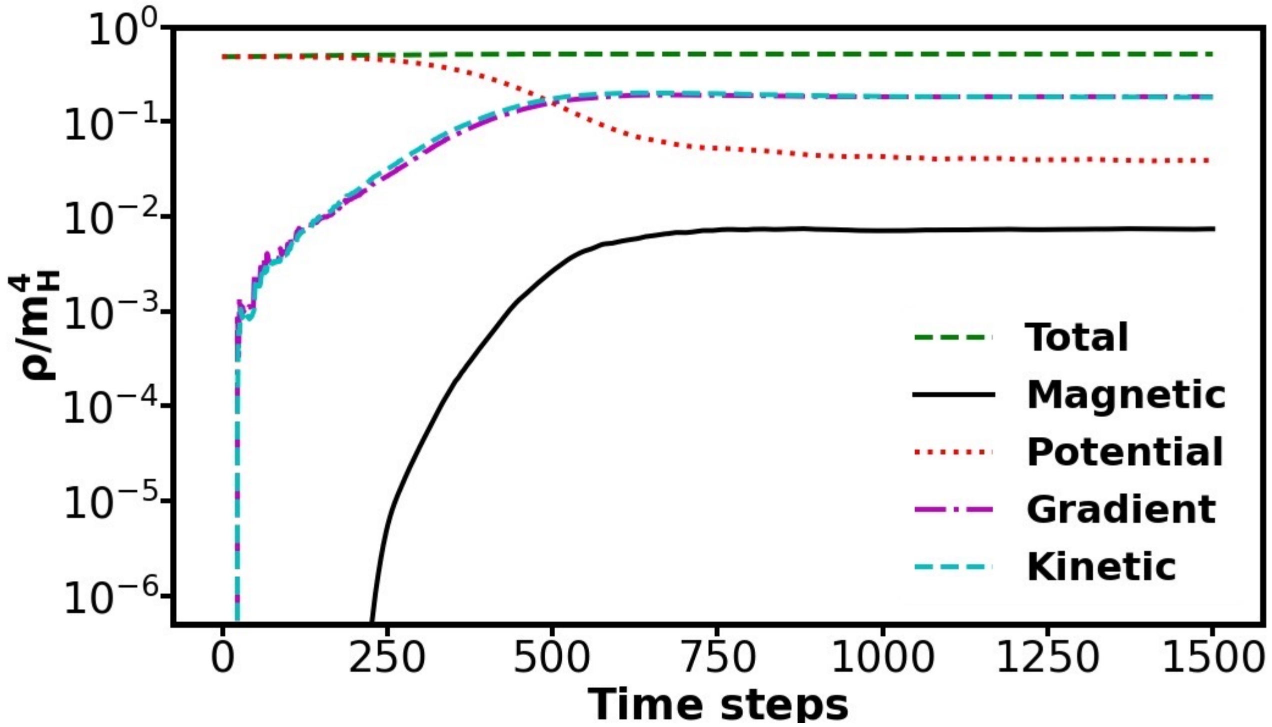
GW from Bubble collisions

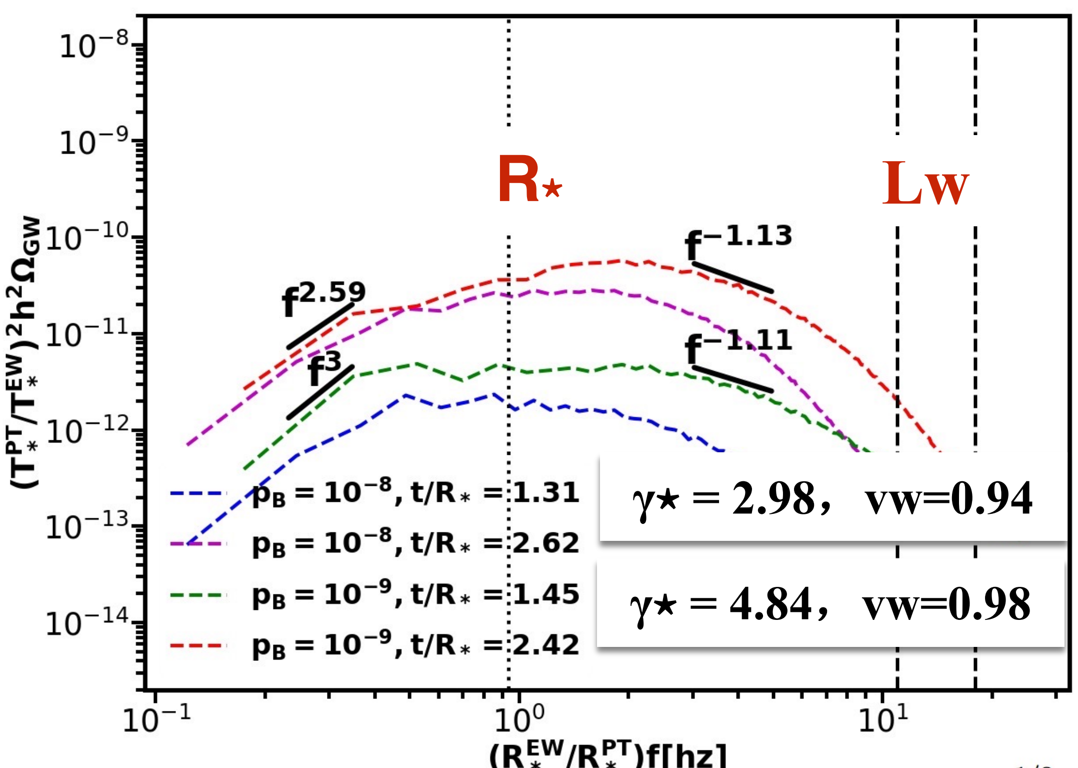
$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\mathrm{TT}}$$

$$T_{\mu\nu} = \partial_\mu \Phi^\dagger \partial_\nu \Phi - g_{\mu\nu} \frac{1}{2} \mathrm{Re}[(\partial_i \Phi^\dagger \partial^i \Phi)^2]$$

$$\langle \dot{h}_{ij}^{TT}(\mathbf{k}, t) \dot{h}_{ij}^{TT}(\mathbf{k}', t) \rangle = P_{\dot{h}}(\mathbf{k}, t)(2\pi)^3 \delta(\mathbf{k} + \mathbf{k}')$$

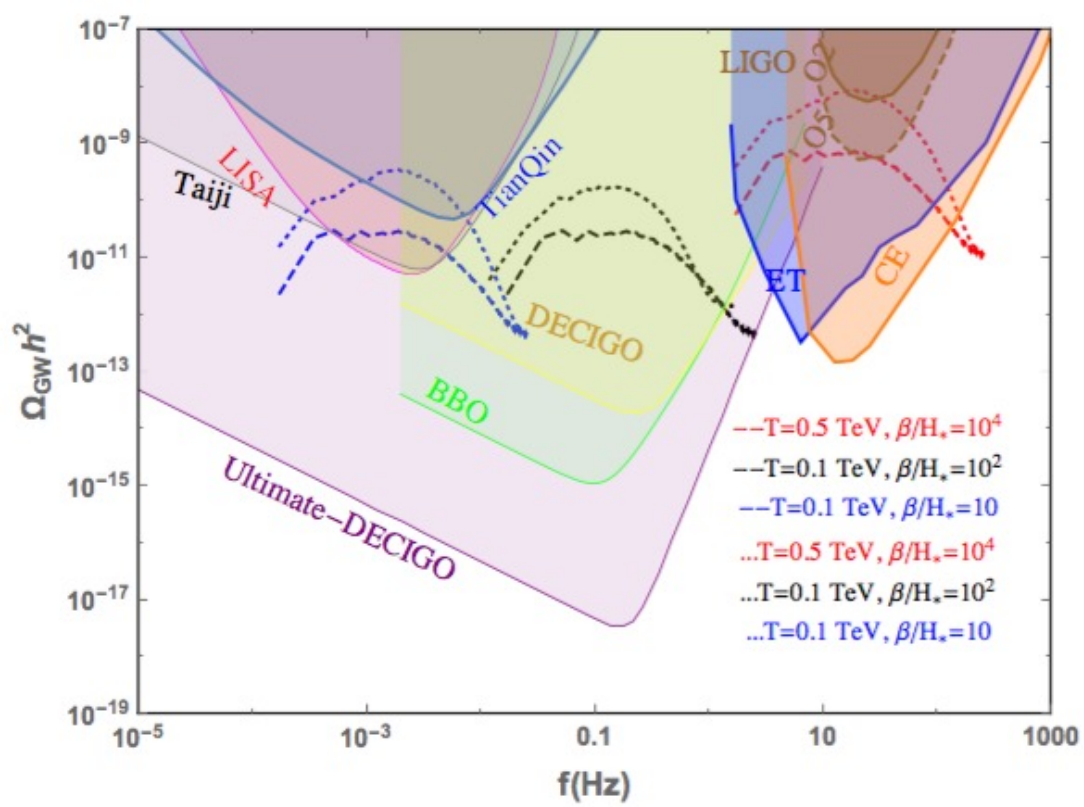
$$\frac{d\Omega_{\mathrm{gw}}}{d\ln(k)} = \frac{1}{32\pi G \rho_c} \frac{k^3}{2\pi^2} P_{\dot{h}}(\mathbf{k}, t)$$





γ⋆ = R⋆/(2Rc)

$$R_* = \left(\frac{\mathcal{V}}{N_b}\right)^{1/3}$$



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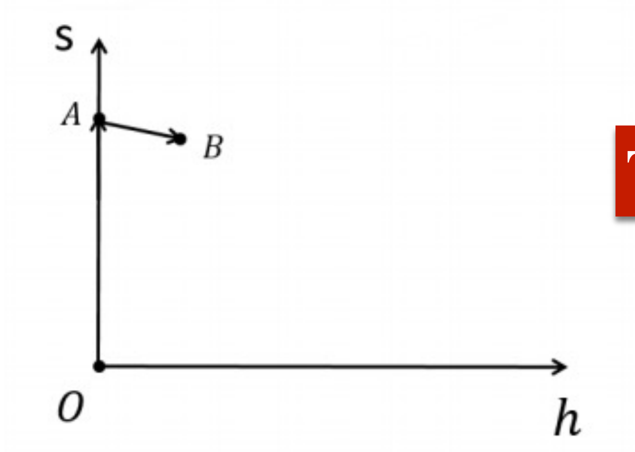
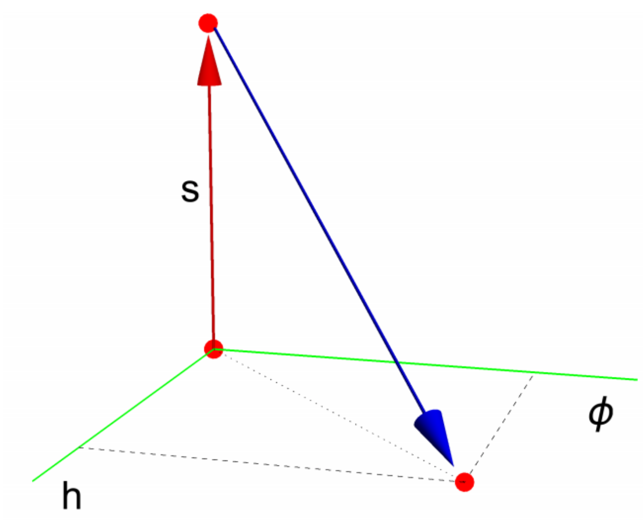
2-step FOPT DM & EWBG

WIMP DM+EWBG

CxSM Jiang, **Bian***,Huang,Shu 16, Chiang, Ramsey-Musolf, Senaha 18,…

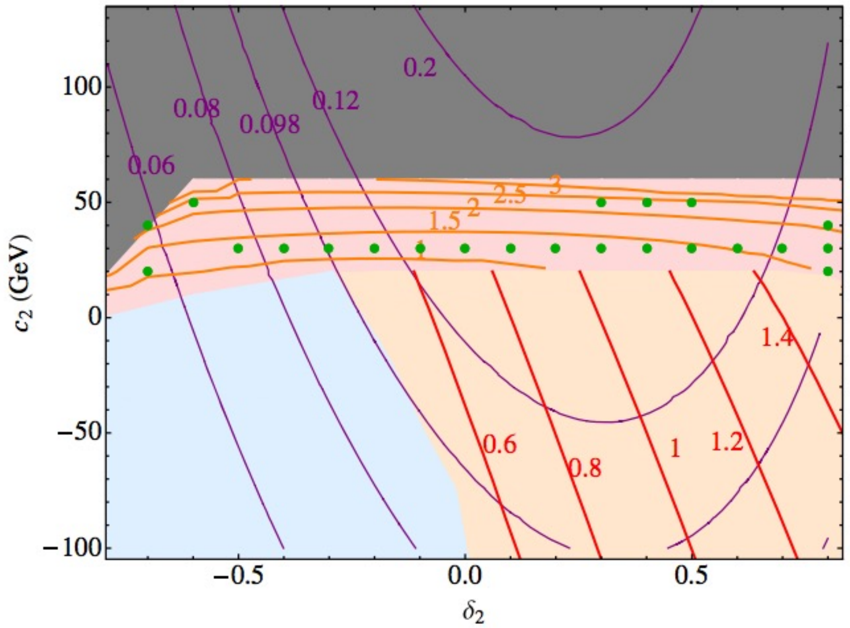


WIMP DM+FOPT

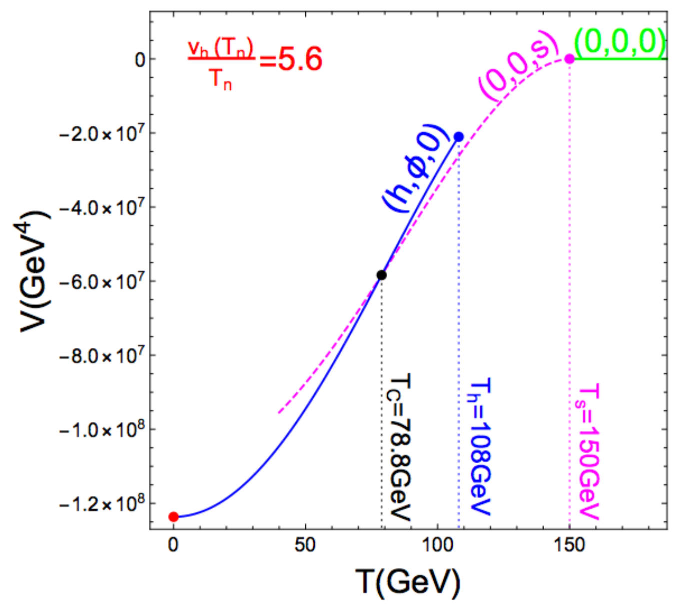


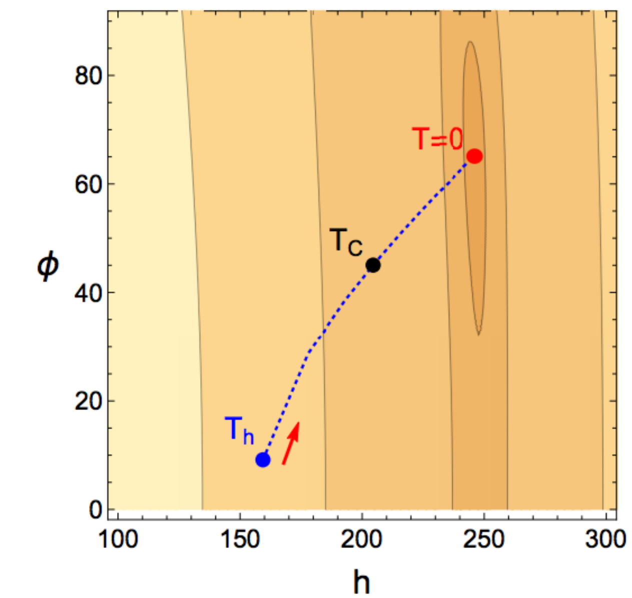
Two-step FOPT

DM
one-step
two-step
BAU



Jiang, **Bian***,Huang,Shu 16

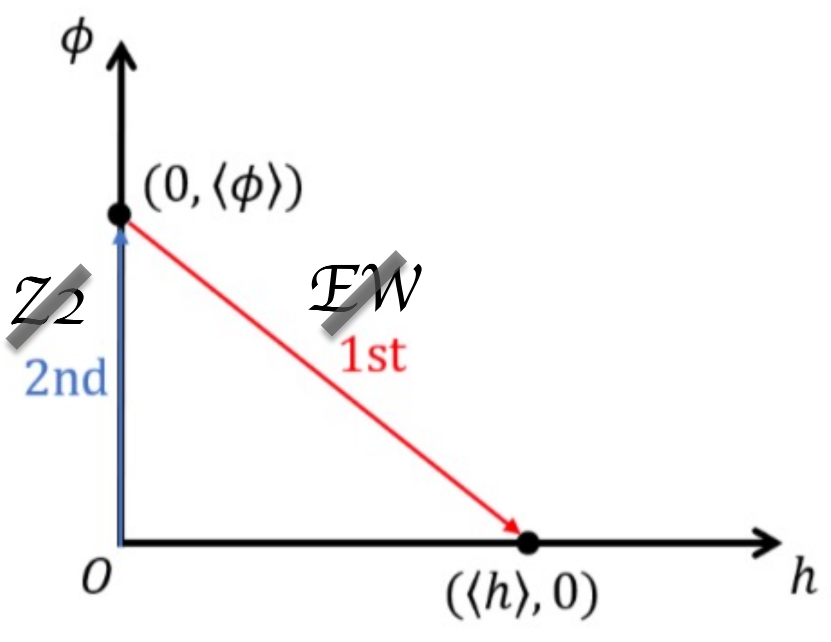




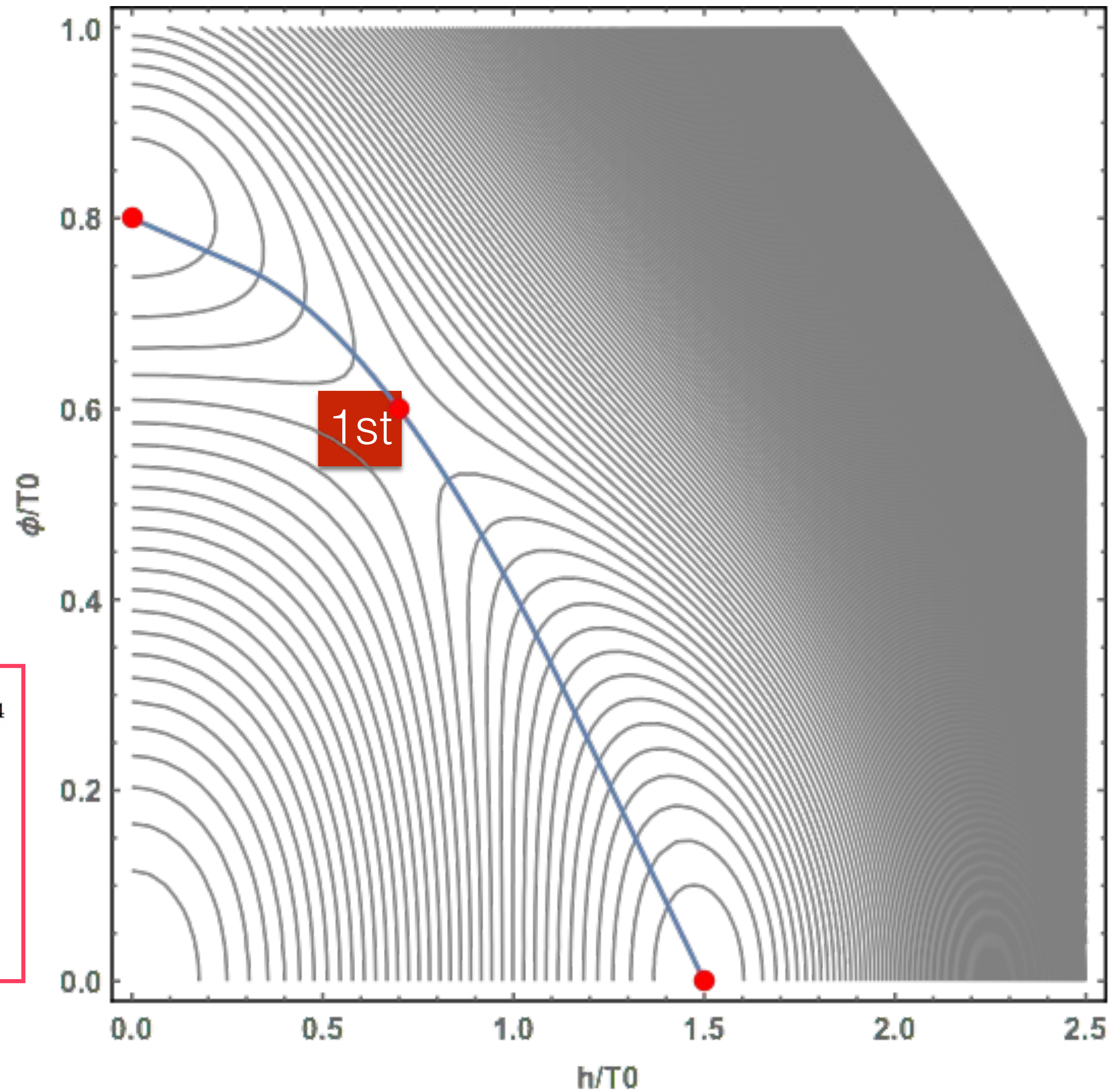
SM+2 real scalars **Chao, Guo, Shu 17,…**

Two-step FOPT potential

Type-a



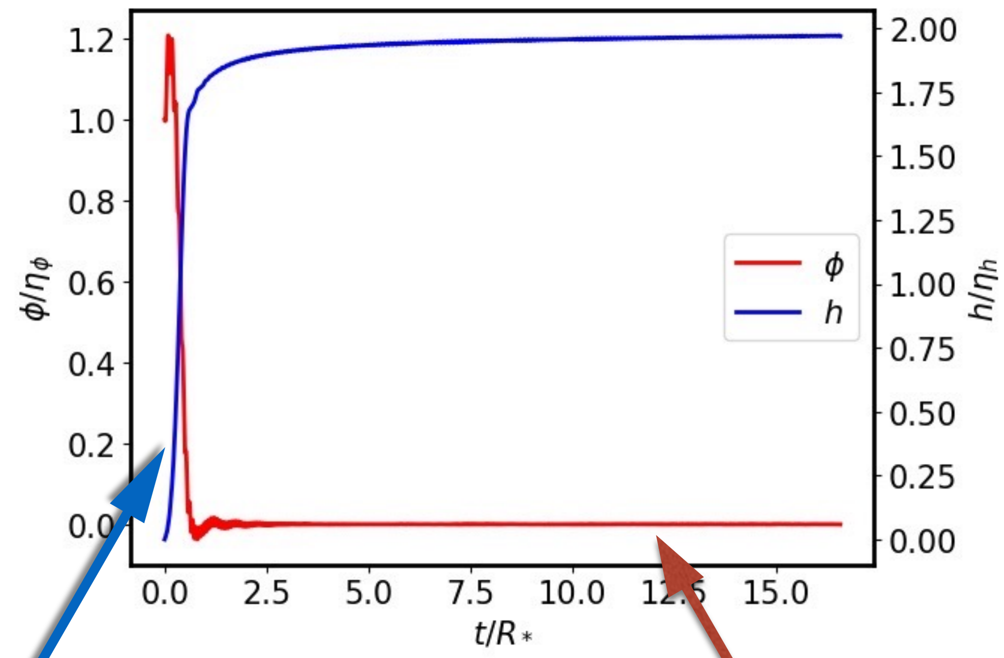
$$V_a(\phi,h,T)=\frac{1}{2}(\mu_\phi^2+c_\phi T^2)\phi^2+\frac{1}{2}\lambda_{h\phi}h^2\phi^2+\frac{1}{4}\lambda_\phi\phi^4$$
$$+\frac{1}{2}(-\mu_h^2+c_hT^2)h^2+\frac{1}{4}\lambda_h h^4$$
$$c_\phi=\lambda_\phi/4+\lambda_{h\phi}/3$$
$$c_h=(2m_W^2+m_Z^2+2m_t^2)/(4v^2)+\lambda_h/2+\lambda_{h\phi}/12$$



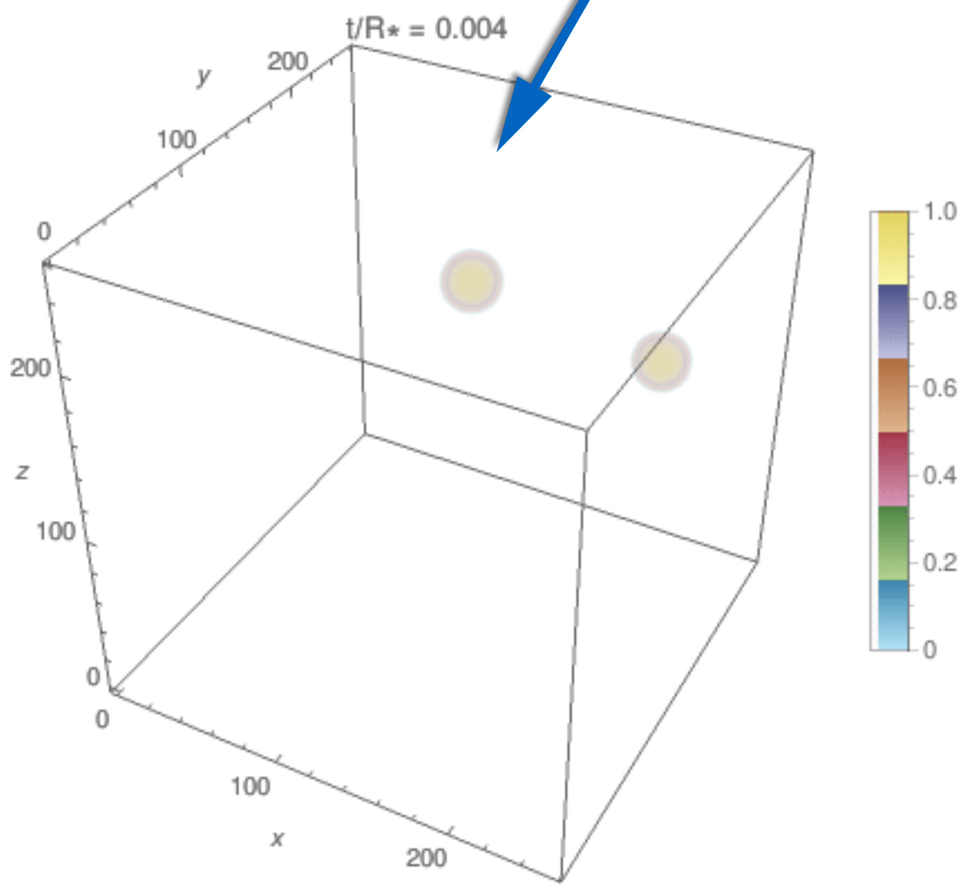
Motivated for DM&EWBG, see:1804.06813,1702.06124,1609.07143, 1605.08663, 1605.08663,etc

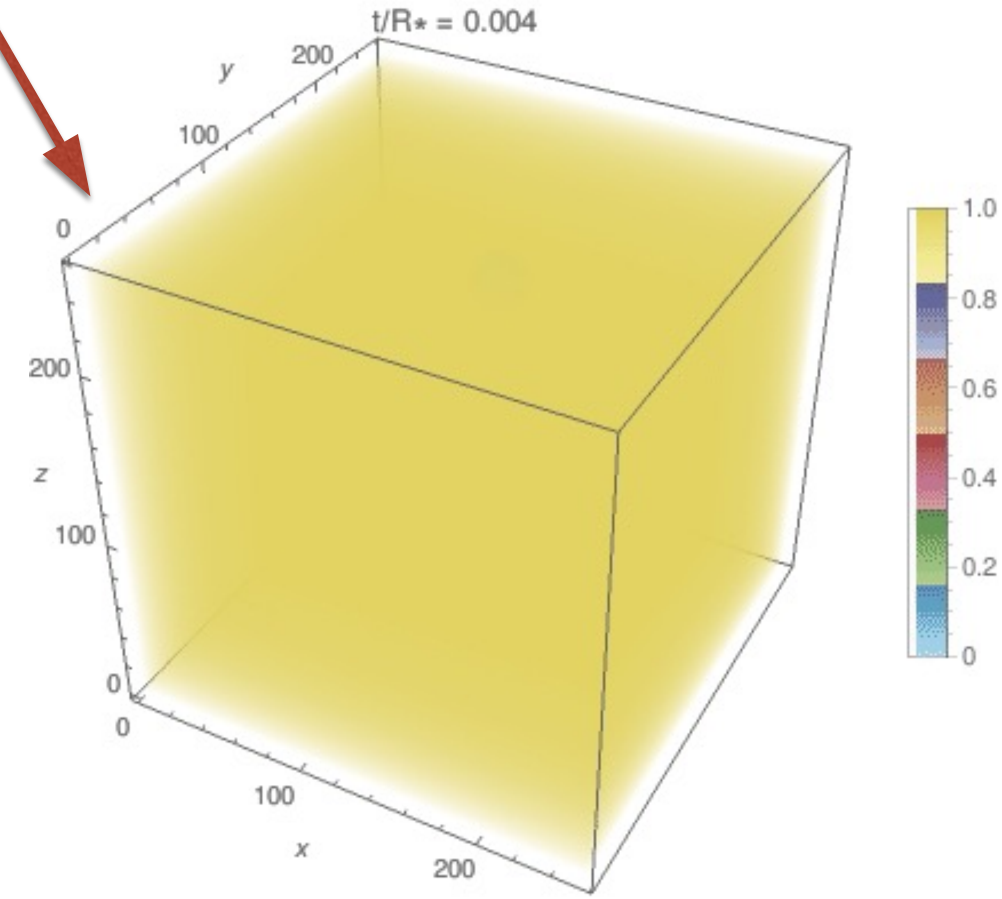
Two-step PT with the second-step being FOPT

Type-a



$$h(t=0,r) = \eta_h/2\left[1-\tanh\left(\frac{r-R_0}{L_w}\right)\right]$$
$$\phi(t=0,r) = \eta_\phi/2\left[1+\tanh\left(\frac{r-R_0}{L_w}\right)\right]$$

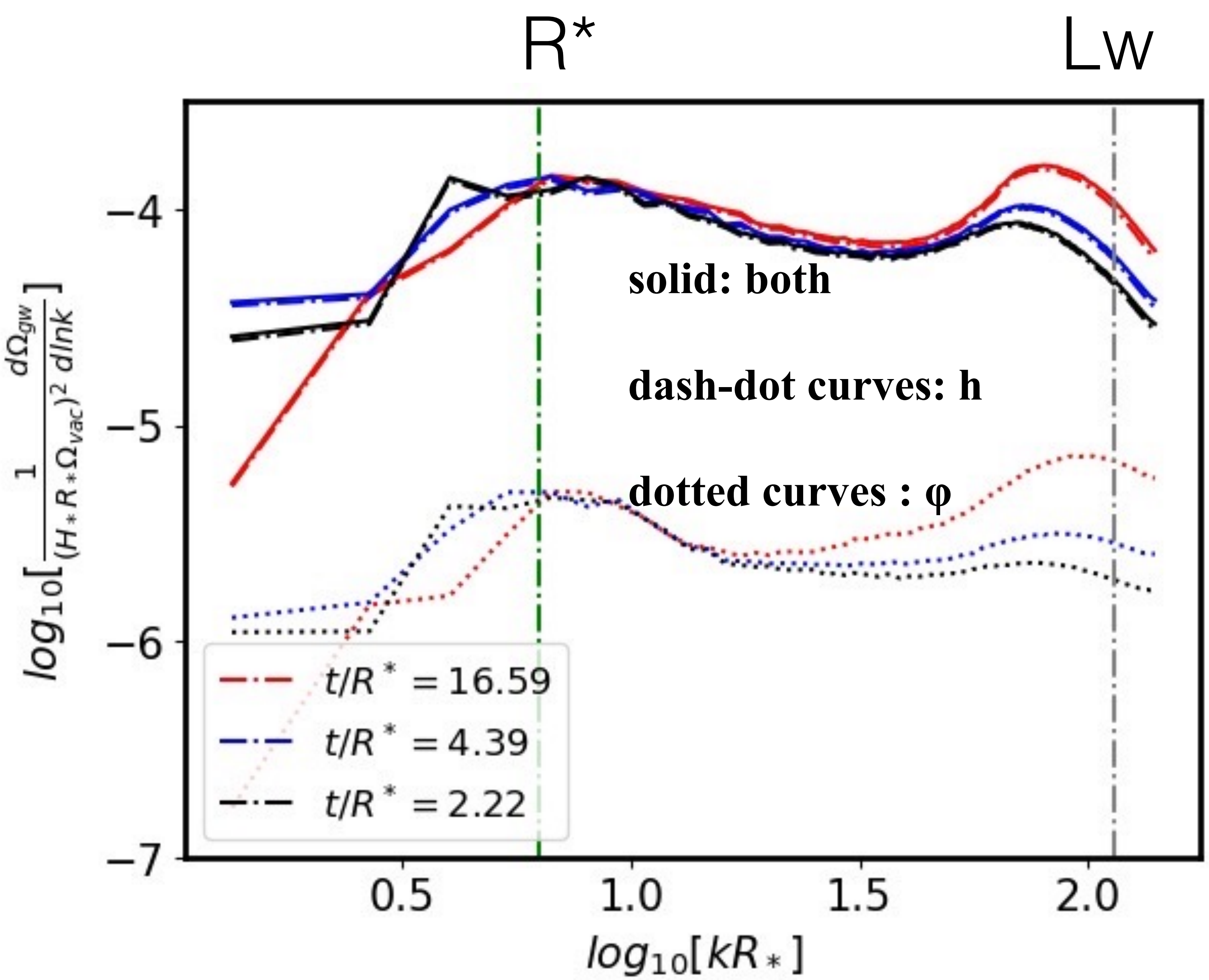




Zhao, Di, **Bian***, Cai*,2204.04427 （PRL under review）

Two-step PT with the second-step being FOPT

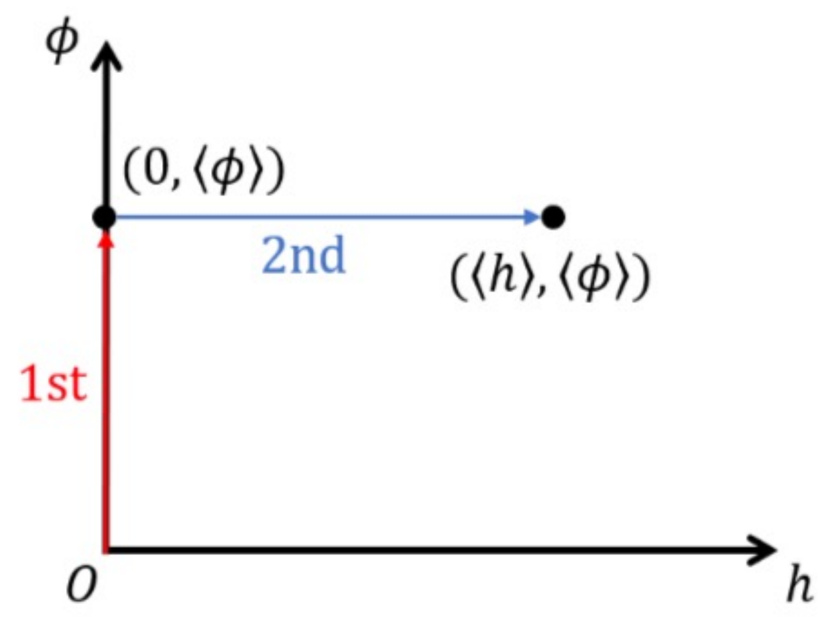
Type-a



Zhao, Di, **Bian***, Cai*,2204.04427 （PRL under review）

Two-step PT with first-step being FOPT

Type-b Without Global U(1)



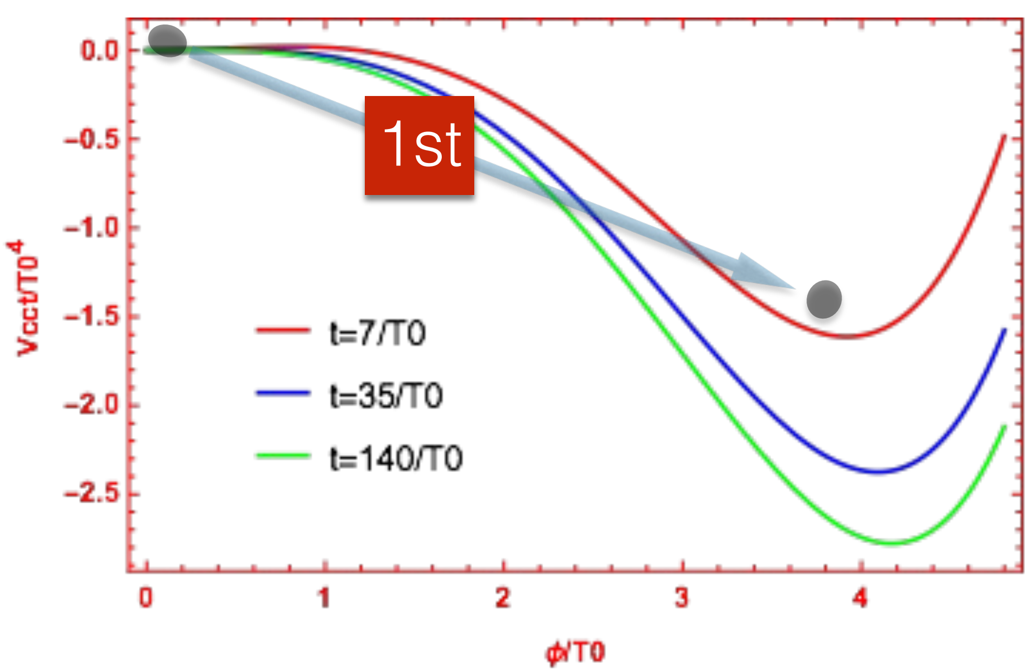
$$V_{cct}(\phi,T) = a\phi^4(\log[|\phi|^2/v_\phi^2] - 1/4) + bT^2|\phi|^2$$

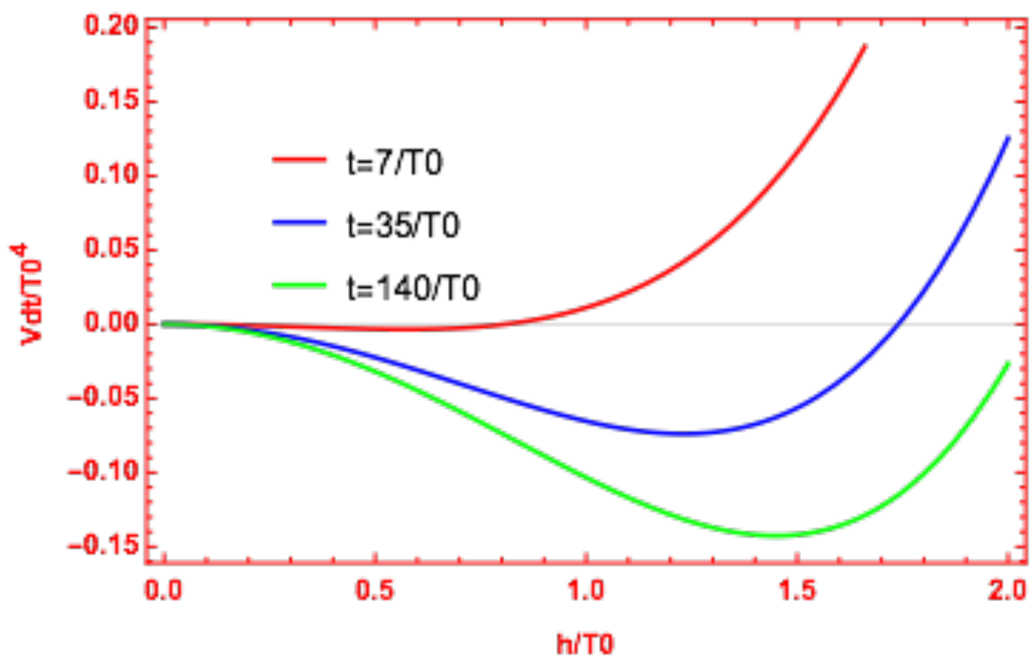
$$V_{dt}(\phi,h,T) = \frac{1}{2}c_h' T^2 h^2 + \frac{1}{4}\lambda_h h^4 - \frac{\lambda_p}{4}h^2\phi^2$$

$$c_h' = (2m_W^2 + m_Z^2 + 2m_t^2)/(4v^2) + \lambda_h/2 + \lambda_p/24$$

$$\langle h\rangle = \sqrt{(\lambda_p\eta^2 - 2c_h' T^2)/(2\lambda_h)}$$

Classical conformal + Dimensional transmutation



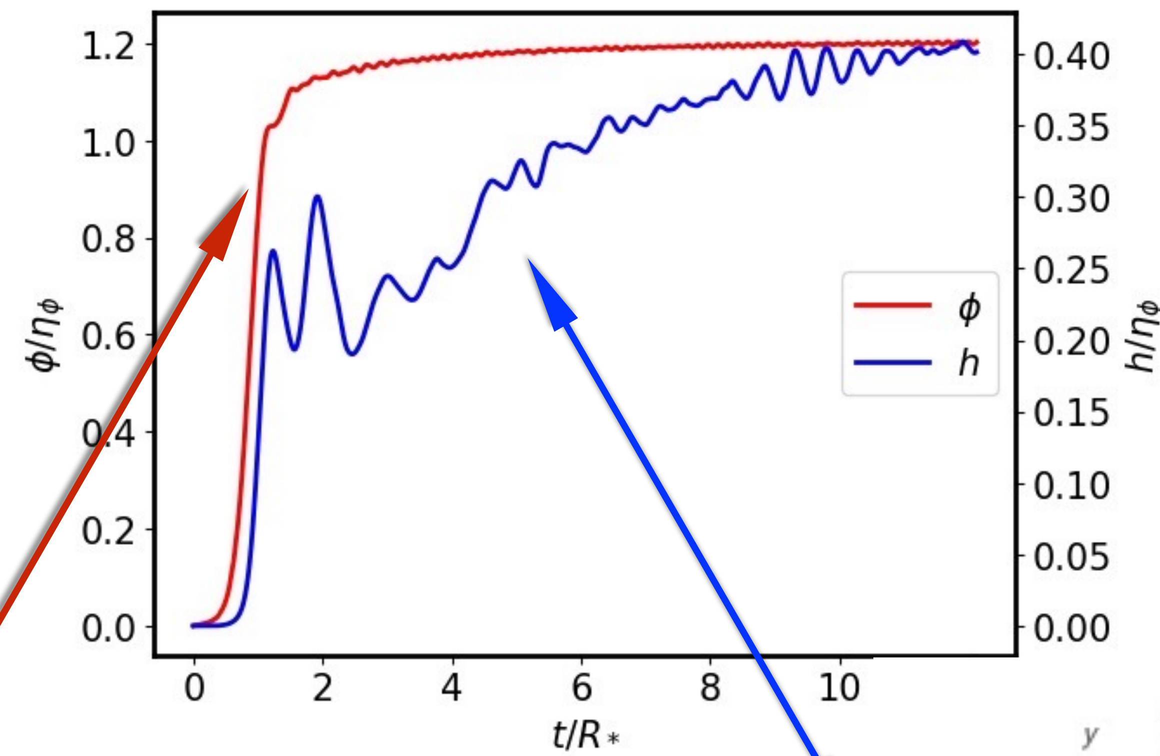


Zhao, Di, **Bian***, Cai*,2204.04427 （PRL under review）

Two-step PT with first-step being FOPT

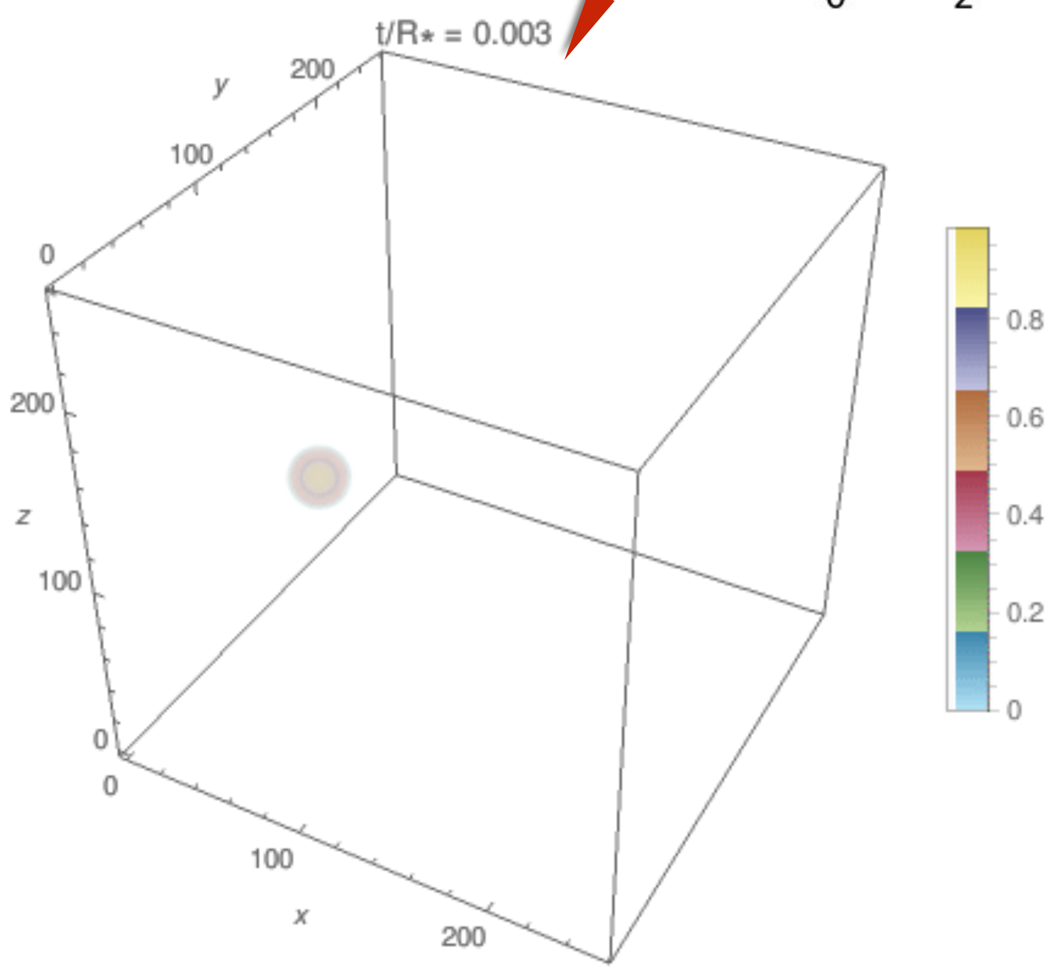
Type-b

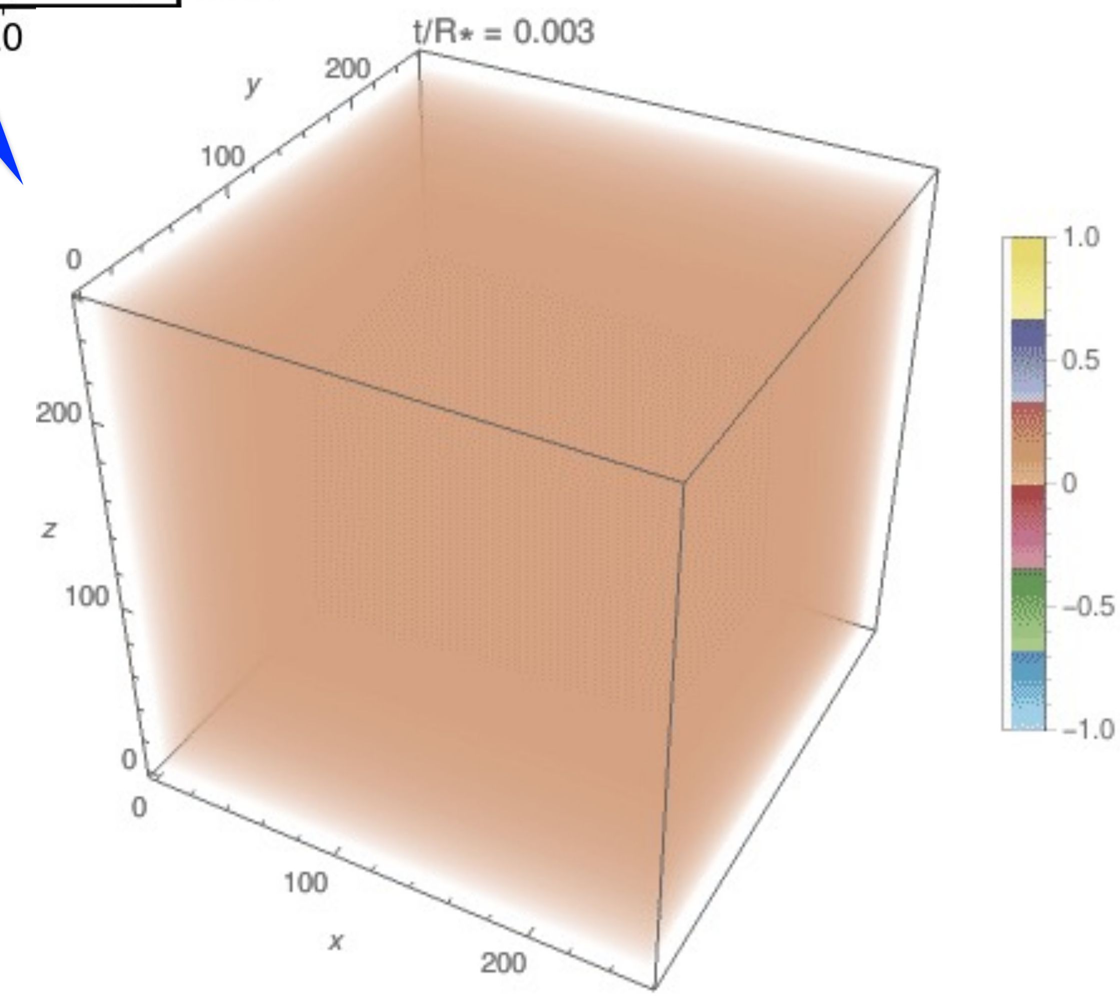
Without Global U(1)



$$\phi(t=0,r) = \eta_\phi/2\left[1 - \tanh\left(\frac{r-R_0}{L_w}\right)\right]$$

$$\langle h\rangle = \sqrt{(\lambda_p\eta^2 - 2c_h' T^2)/(2\lambda_h)}$$



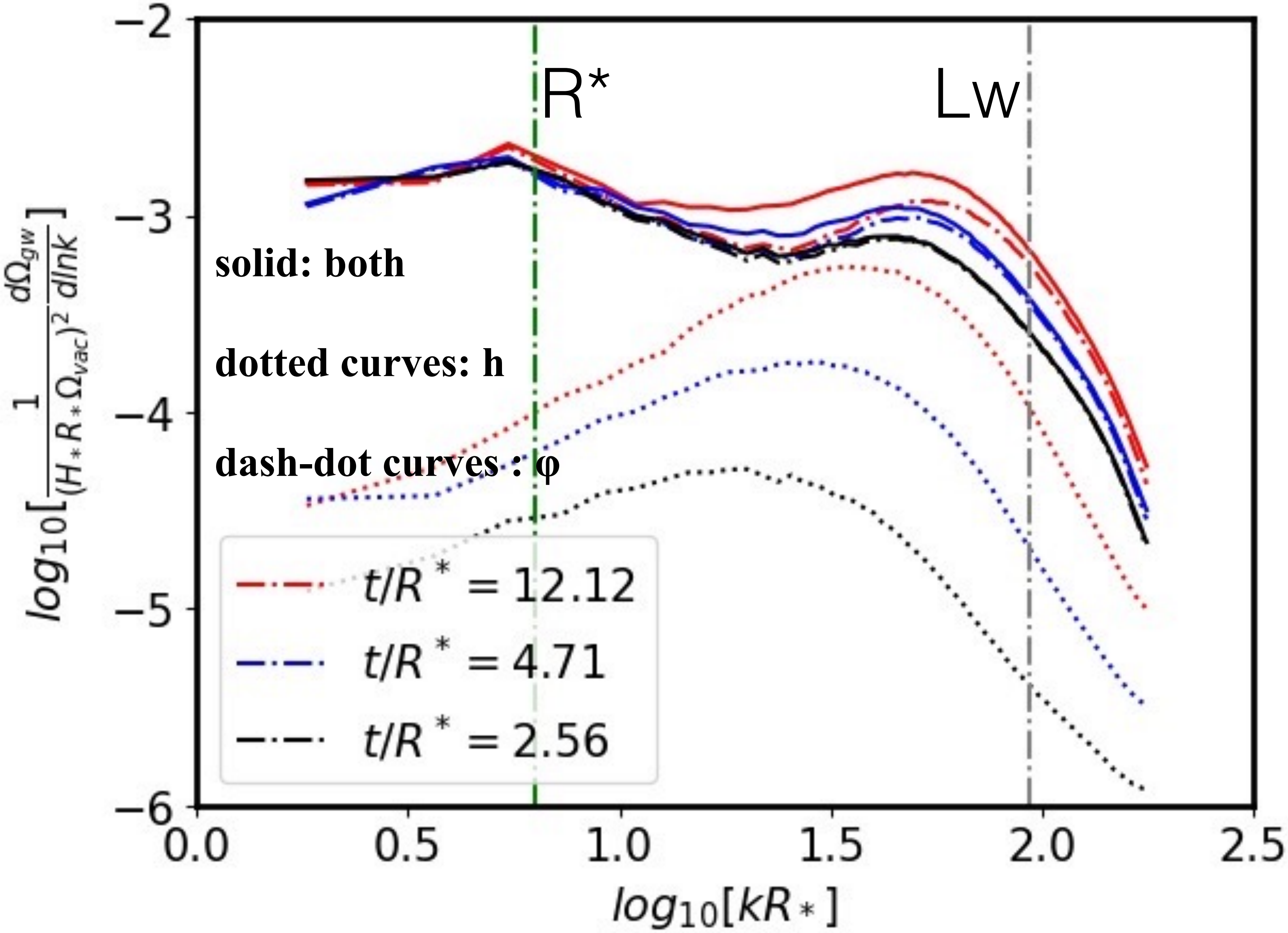


Zhao, Di, **Bian***, Cai*,2204.04427 （PRL under review）

Two-step PT with first-step being FOPT

Type-b

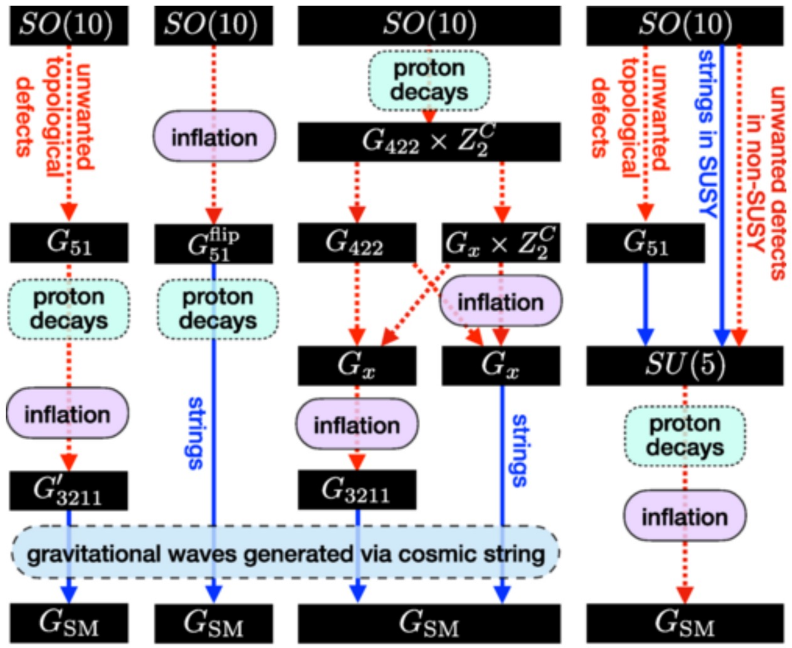
Without Global U(1)



Zhao, Di, **Bian***, Cai*,2204.04427 （PRL under review）

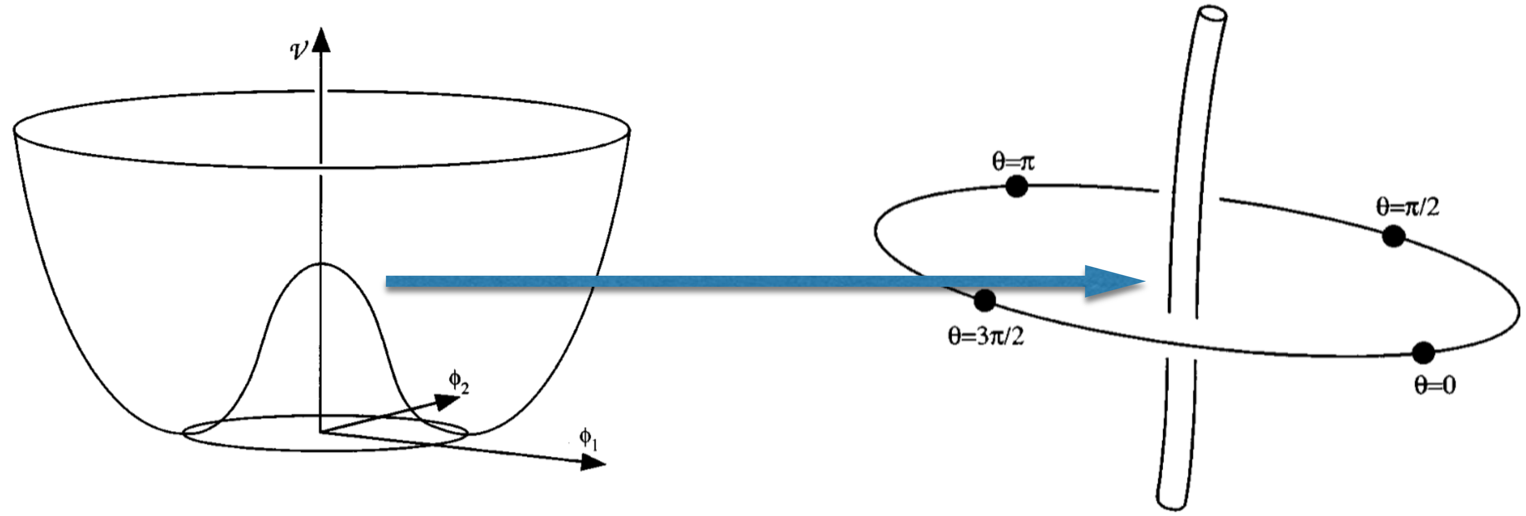
Cosmic string

通常形成于GUTs



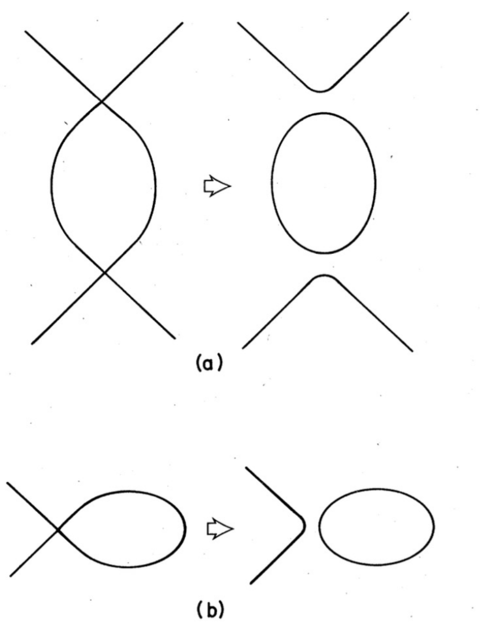
King, Pascoli, Turner, and Zhou, Phys. Rev.Lett. 126, 021802 (2021)

相变后U(1) 对称性自发破缺　　宇宙弦



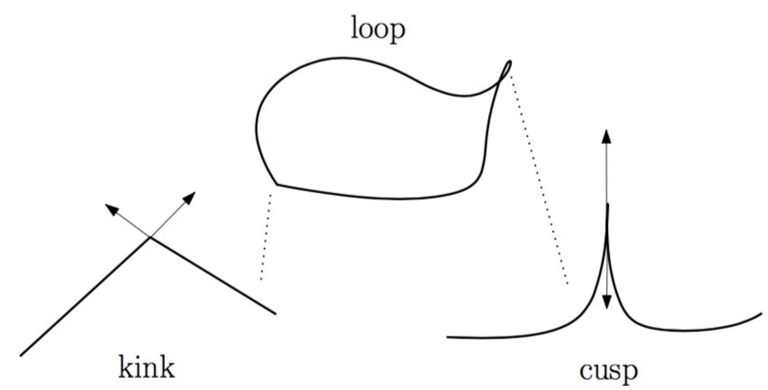
T. W. B. Kibble

宇宙弦成圈



Phys.Rev.D 30 (1984) 2036

引力波辐射

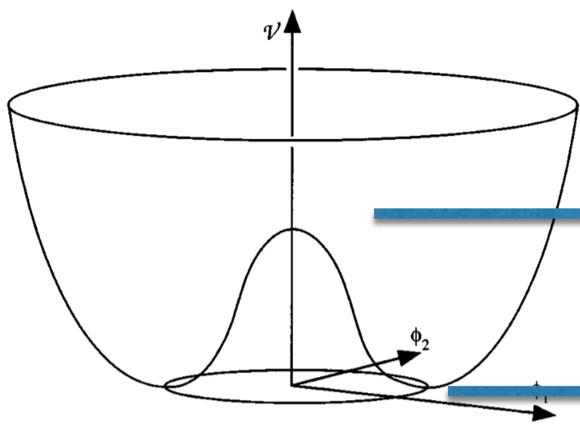


Yann Gouttenoire *et al* JCAP07(2020)032

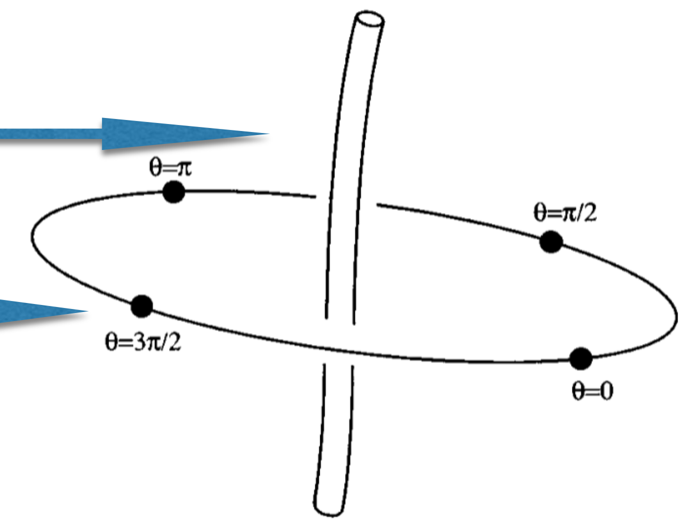
Cosmic string simulation FOPT

CS: SSB of U(1) symmetry

The one-dimension topological defects: cosmic string



Potential



T. W. B. Kibble

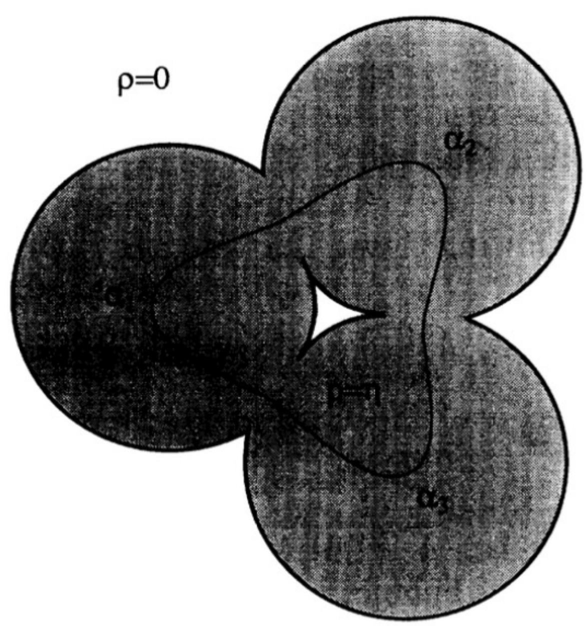


FIG. 1. Three bubbles of the broken symmetry phase ($\rho = \eta$) colliding. If the phase change of the scalar field around the loop γ is $\pm 2\pi$, a string (or antistring) is formed. If the phases α_i are ordered, then the requirement for a string is $\alpha_1 + \pi < \alpha_3 < \alpha_2 + \pi$.

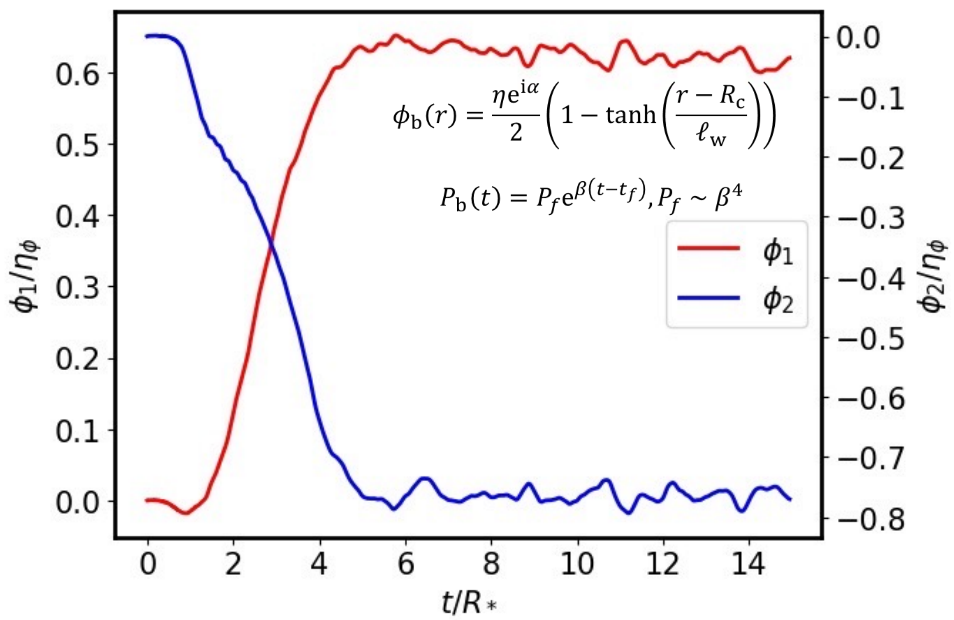
ξ_{str} of the string network is essentially the typical bubble diameter for SFOPT???

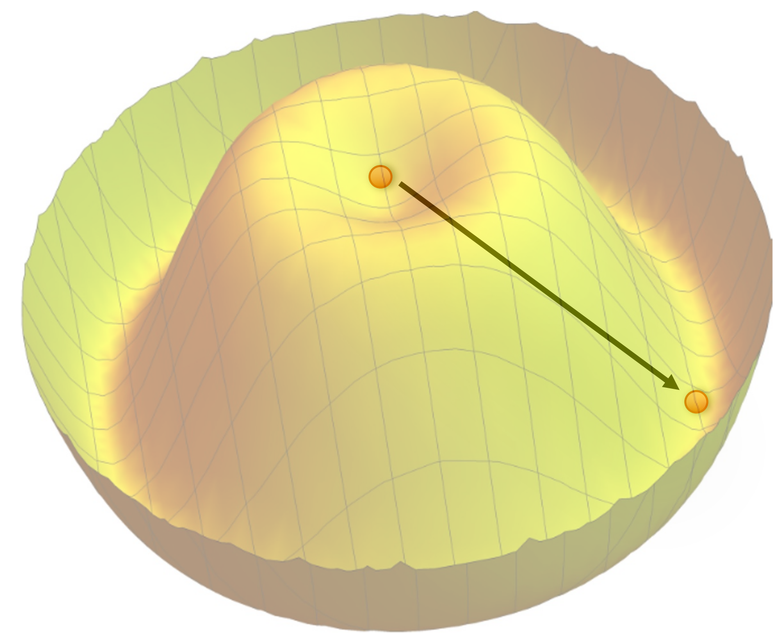
Aust. J. Phys., 1997, 50, 697–722
Phys.Rev.D 49 (1994) 1944-1950

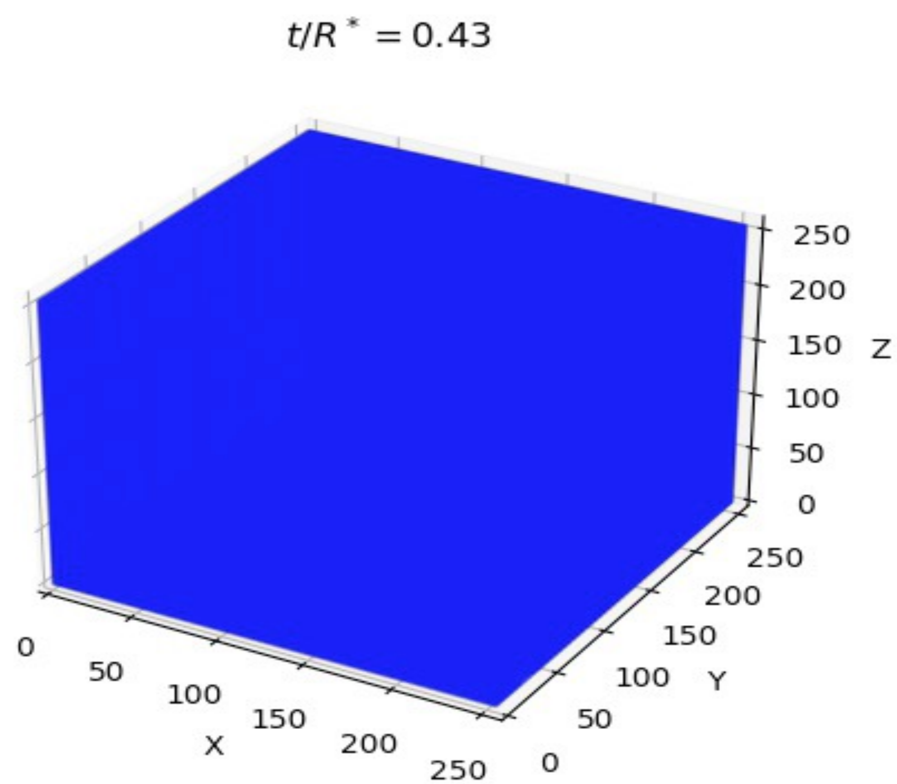
Cosmic string simulation FOPT

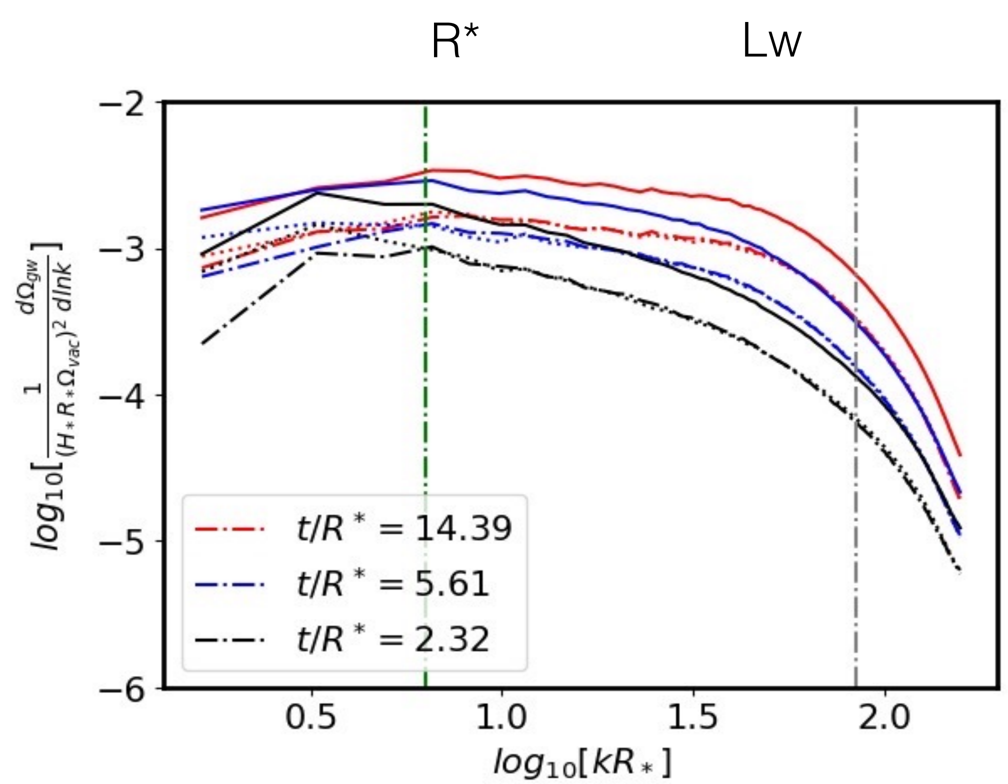
With Global U(1)

Motivated for strong CP and axion DM





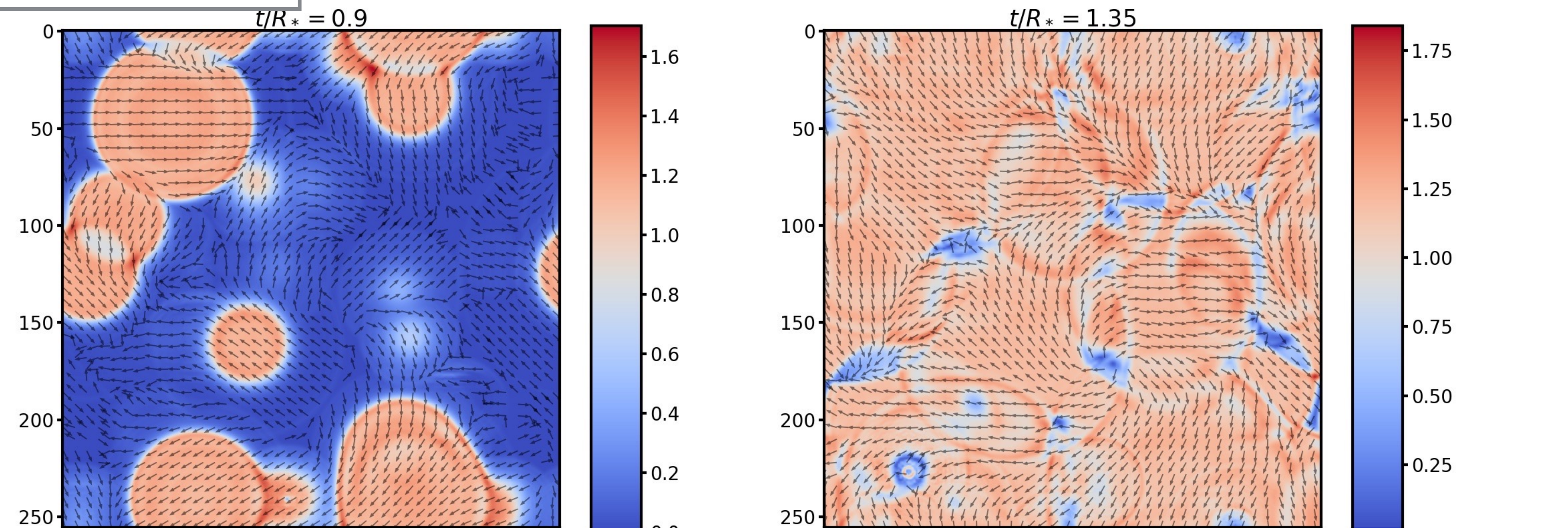




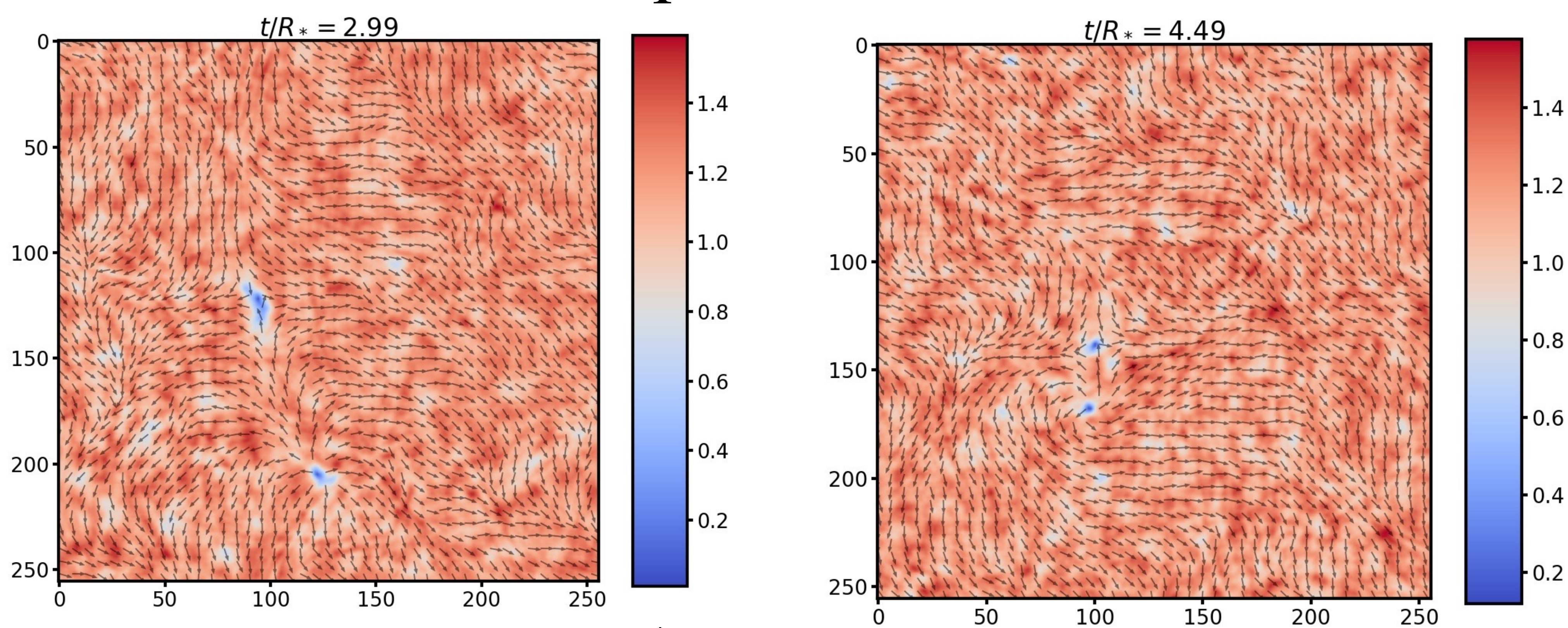
Zhao, Di, **Bian***, Cai*,2204.04427 （PRL under review）

Bubbles and vortex&anti-vortex

Type-b **With Global U(1)**



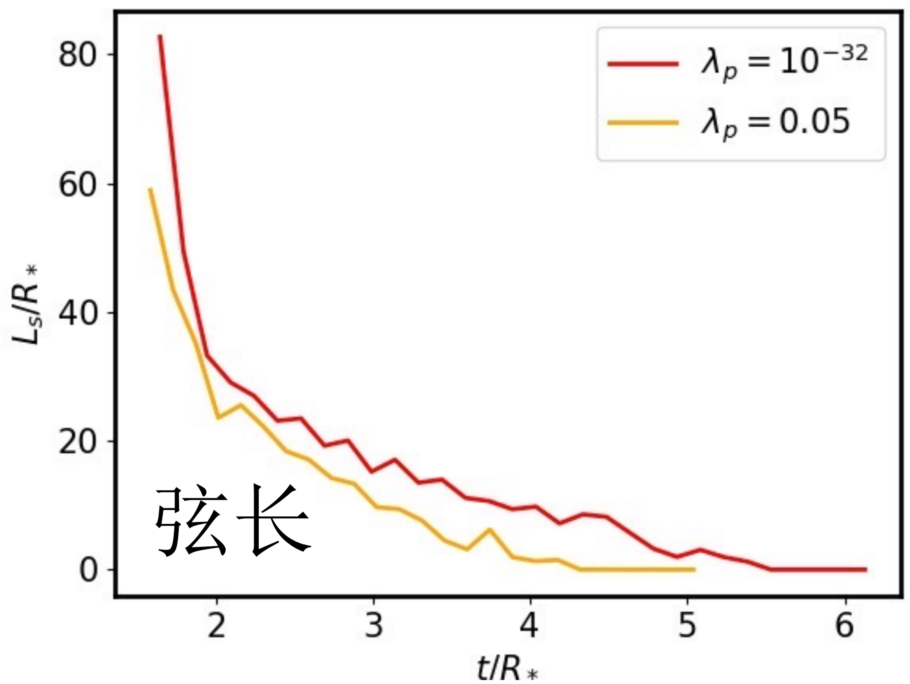
Arrows: phase distribution

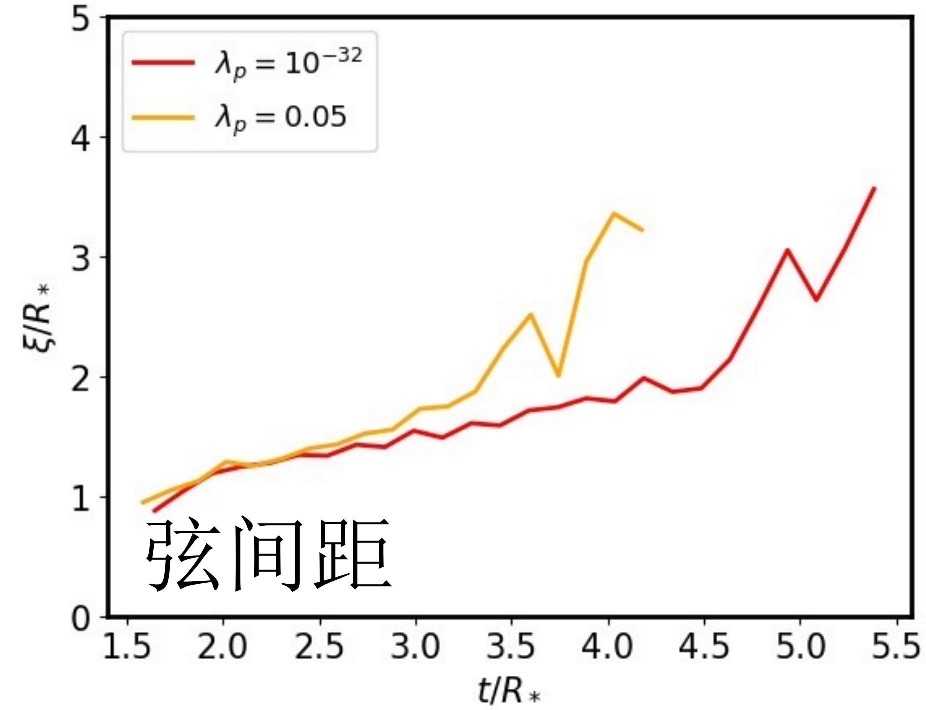


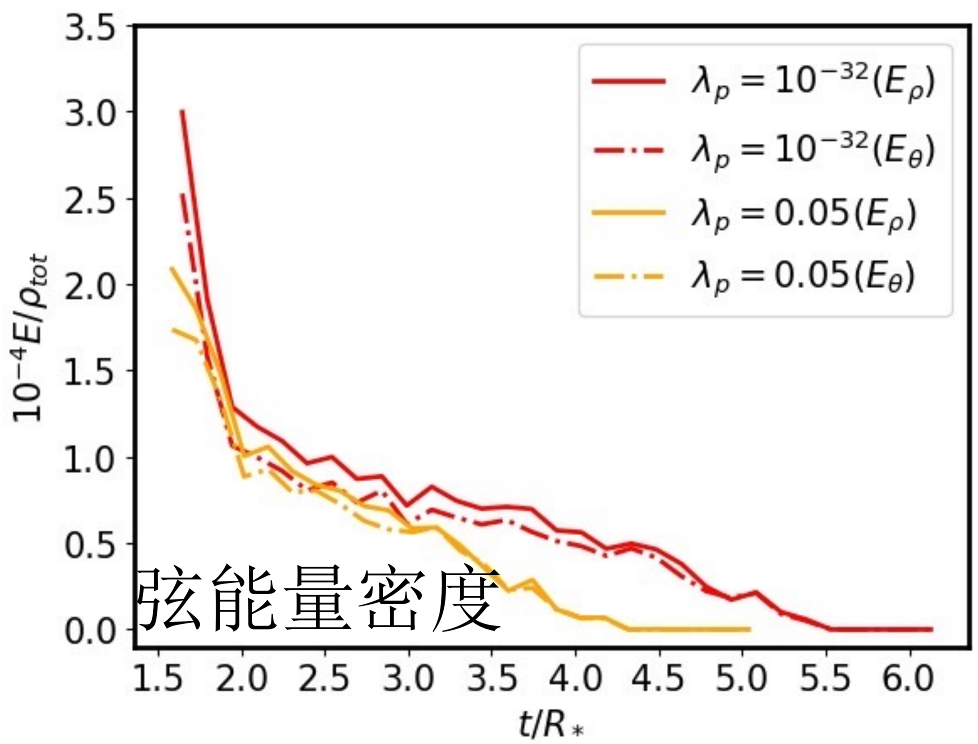
Global string from FOPT

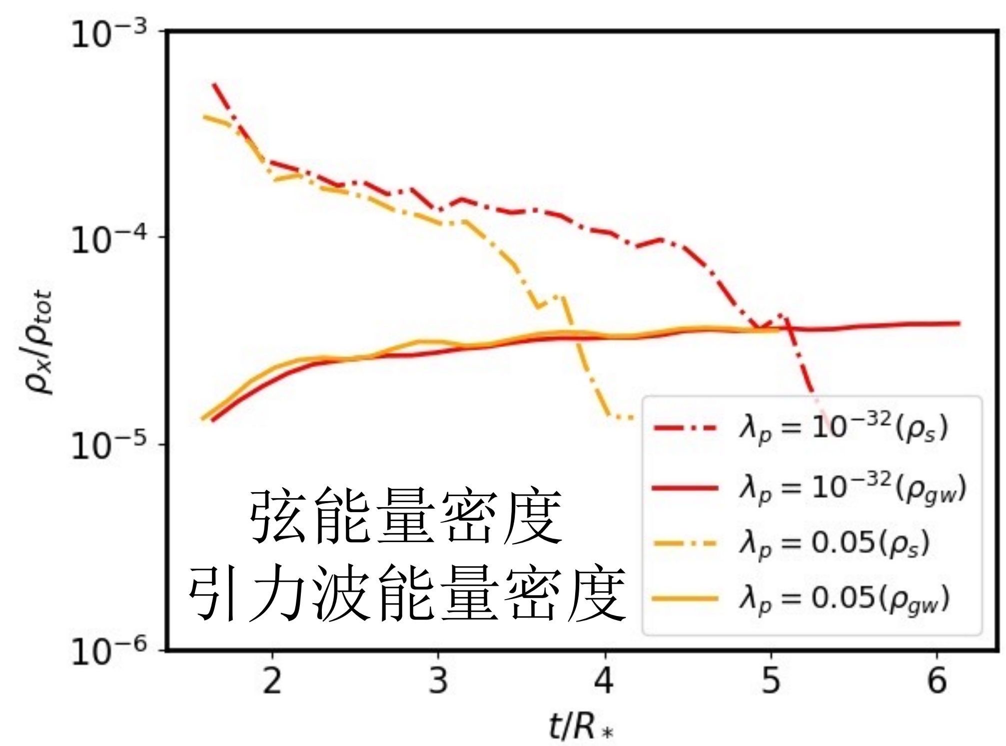
Type-b

With Global U(1)









Related topics

❖Lattice simulation

- **PT GW simulation with holography models**
- **Topological defects: Magnetic monopoles, cosmic strings, domain walls**

❖Pheno

1. EWSB and GW from FOPT

- **Probing the Higgs Potential shape and EWPT patterns with GW production and Colliders complementarily**

2. Baryon Asymmetry of the Universe and GW from FOPT

- **Sphaleron process, bubble velocity**

3. DM and GW from FOPT

- **DM and high/low-scale PT, DM out-of-equilibrium & FOPT, PBH DM&FOPT**

谢谢！