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Triangle Singularity in the Production of T_{cc}^{+} and a Soft Pion

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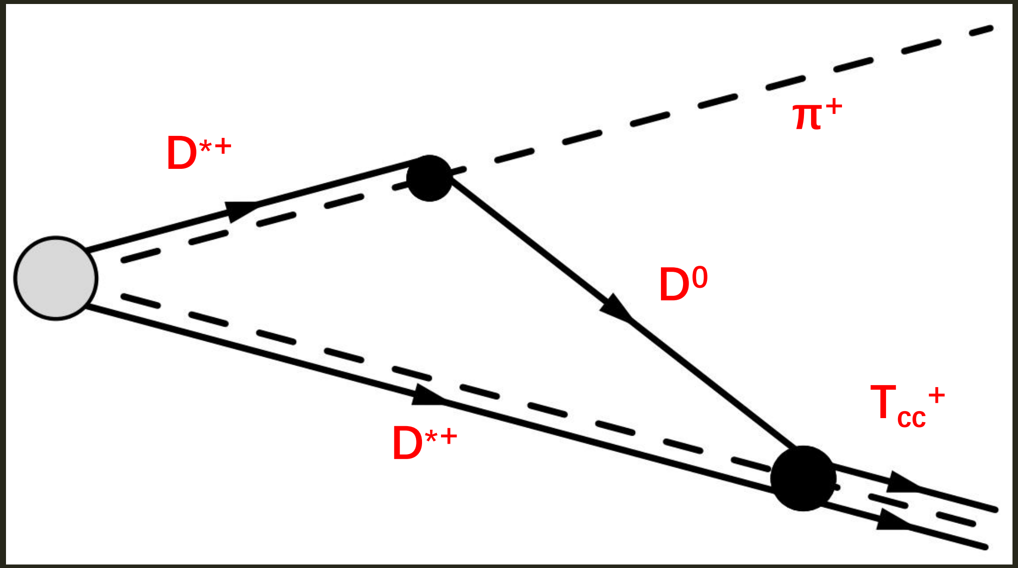
CONTENTS

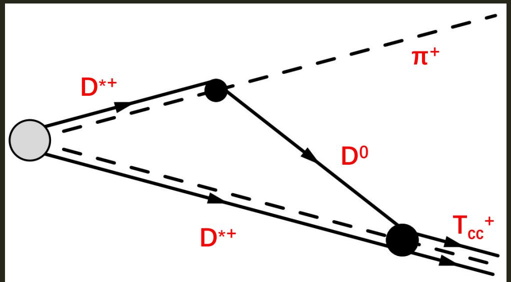
Triangle Singularity

- Kinematic singularity if three charm mesons in the loop are on their mass shells simultaneously.



1 Introduction

Triangle Singularity



$$T_+(q^2,\gamma^2) = \left(1 + \frac{mb}{2M_T c}\right)\frac{1}{\sqrt{c}}\log\frac{\sqrt{a}+\sqrt{c}+\sqrt{a+b+c}}{\sqrt{a}-\sqrt{c}+\sqrt{a+b+c}} + \frac{m}{M_T c}\left(\sqrt{a}-\sqrt{a+b+c}\right)$$

$$E_{\triangle+} = \frac{M_*}{4\mu^2}\left(\sqrt{2\mu E_+ - \gamma^2} - i\sqrt{m/M_T}\,\gamma\right)^2$$ where $E_+ = \delta_{0+} - \varepsilon_T - i\,\Gamma_{*+}$

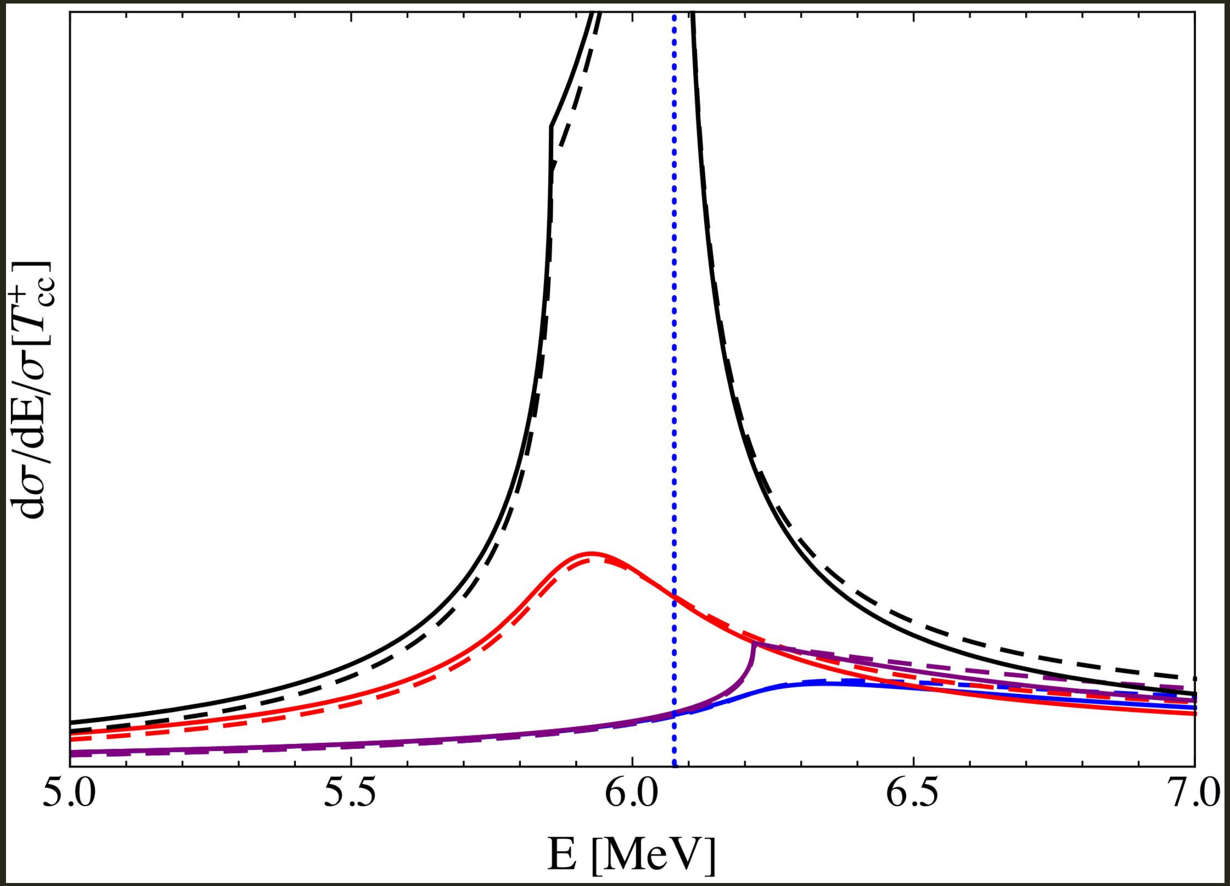
$$E_{\triangle+} \longrightarrow (M_T/2M)\delta_{0+} = 6.1\ \text{MeV}$$

- Log divergence in the limit of both binding energy and decay width go to zero.
- Square-root branch point at $E = E_+$ from the $\sqrt{a}$ term.
- Limiting behaviour of $T_+(q^2,\gamma^2)$ determined by the interplay of above two items.

1 Introduction

Triangle Singularity

- $\boldsymbol{Black\ (|\varepsilon_T|, \Gamma_{*+}) = (0, 0)}$
- $\boldsymbol{Red\ (|\varepsilon_T|, \Gamma_{*+}) = (0, 83\ keV)}$
- $\boldsymbol{Purple\ (|\varepsilon_T|, \Gamma_{*+}) = (360\ keV, 0)}$
- $\boldsymbol{Blue\ (|\varepsilon_T|, \Gamma_{*+}) = (360\ keV, 83keV)}$

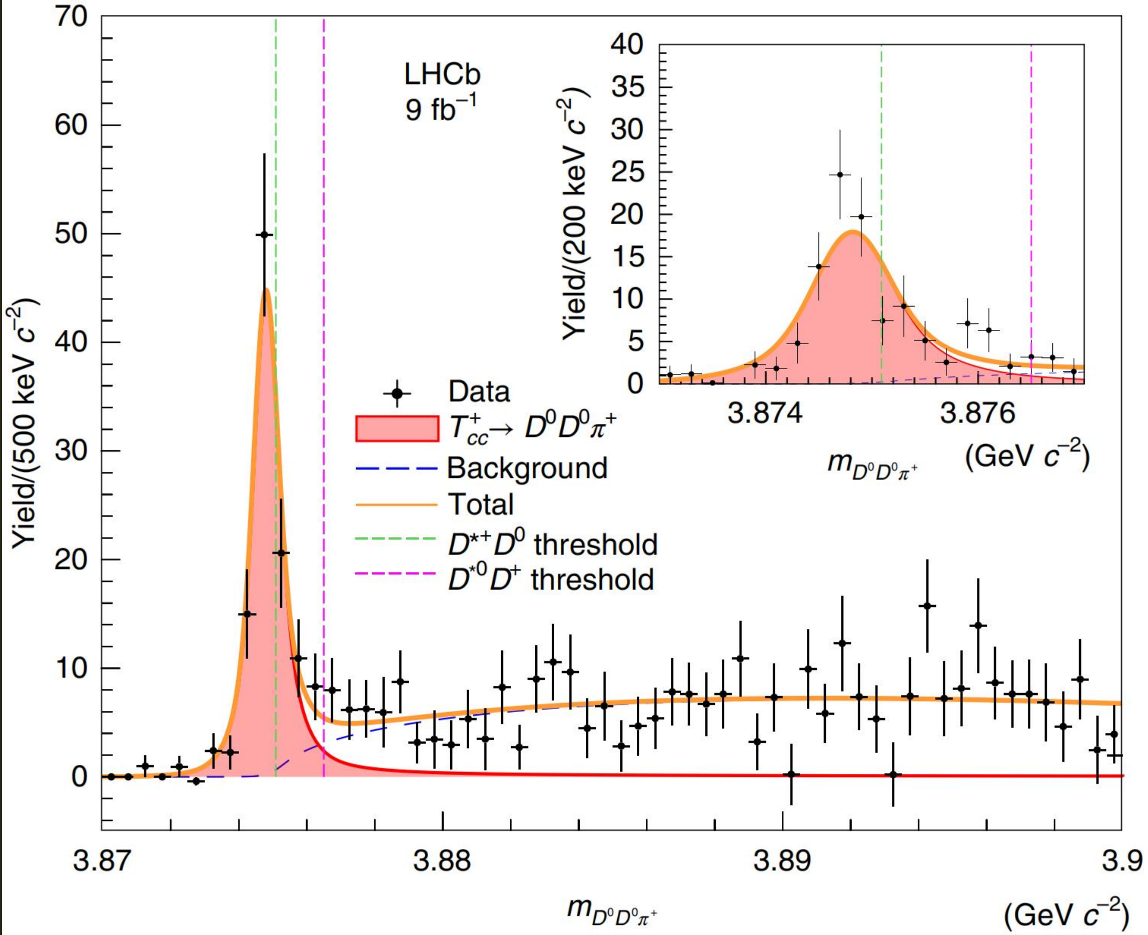


Solid: complete amplitude

Dashed: logarithmic approximation

1 Introduction

$T_{cc}^+(3875)$



LHCb, Nature Physics 18, 751 (2022)

LHCb, Nature Commun. 13, 3351 (2022)

Binding Enegy & Width

$$\delta m_{\rm BW} = -273 \pm 61 \pm 5\,^{+11}_{-14}\,\text{keV}\,c^{-2},$$

$$\Gamma_{\rm BW} = 410 \pm 165 \pm 43\,^{+18}_{-38}\,\text{keV},$$

$$\delta m_{\rm pole} = -360 \pm 40^{+4}_{-0}\,\text{keV}/c^2,$$

$$\Gamma_{\rm pole} = 48 \pm 2^{+0}_{-14}\,\text{keV},$$

Characteristic Size

$$R_a \equiv -\Re\, a = 7.16 \pm 0.51\,\text{fm}$$

$$R_{\Delta E} \equiv \frac{1}{\gamma} = 7.5 \pm 0.4\,\text{fm}$$

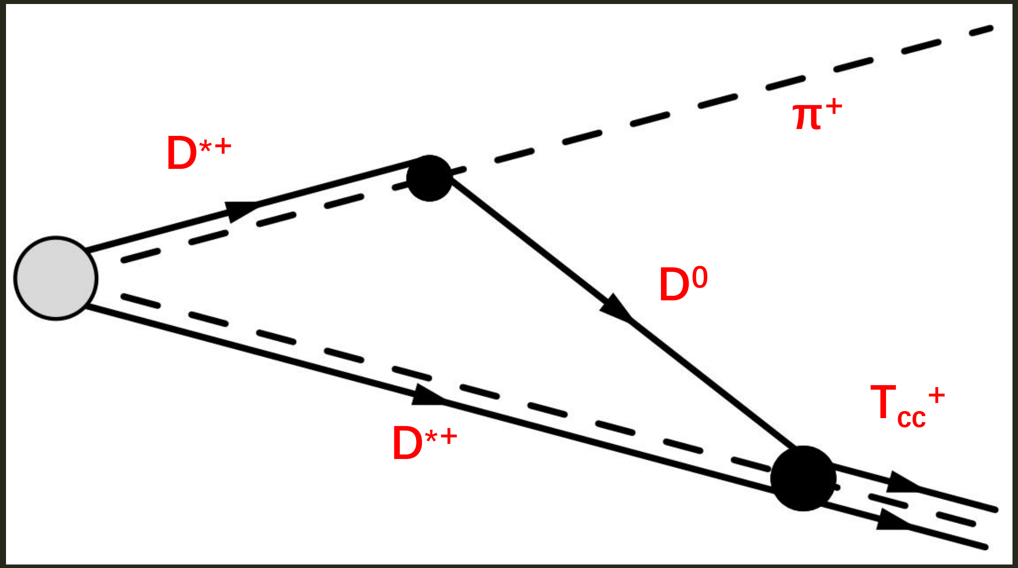
J^P & I

$$J^P = 1^+$$

$$I = 0$$

Motivation

Along with above mesearments, observation of the narrow peak from triangle singularity would support $D^{*}D$ molecule picture of T_{cc}^{+}.



2 Coupled-channel Model

XEFT

- Effective field theory for charm mesons and pions
- Validity: kinetic energy of pions $\sim m_\pi$, kinetic energy of charm mesons $\sim m_\pi^2/M \sim 10MeV$

Fleming, Kusunoki, Mehen and Van Kolck, PRD 76, 034006 (2007)

XEFT

- Effective field theory for charm mesons and pions
- Validity: kinetic energy of pions $\sim m_\pi$, kinetic energy of charm mesons $\sim m_\pi^2/M \sim 10MeV$
- Galilean-invariant formulation of XEFT: conservation of pion and (anti)charm numbers, conservation of kinetic masses

  Pion number: sum of the numbers of $\pi, D^*, \bar{D}^*$ mesons

Fleming, Kusunoki, Mehen and Van Kolck, PRD 76, 034006 (2007)
Braaten, PRD 91, 114007 (2015)
Braaten, He, Jiang, PRD 103, 036014 (2021)

Wavefunction
for loosely bound S-wave molecule

Sharp UV cutoff

Spatial wavefunction:

$$\psi(r) = \frac{\sqrt{\gamma/2\pi}}{r}\exp(-\gamma r)$$

Momentum-space:

$$\psi(k) = \frac{\sqrt{8\pi\gamma}}{k^2+\gamma^2}$$

UV divergent integral:

$$\psi(r=0) = \int d^3k\, \psi(k)/(2\pi)^3$$

Sharp momentum cutoff:

$$|\boldsymbol{k}| < (\pi/2)\Lambda \text{ with } \Lambda \gg \gamma$$

At the origin:

$$\psi(r=0) = (\Lambda - \gamma)\sqrt{\gamma/2\pi}.$$

Binding momentum: $\gamma = \sqrt{2\mu|\varepsilon|}$.

Braaten, Hammer, Phys. Rept. 428, 259 (2006)

Wavefunction
for loosely bound S-wave molecule

Smooth UV cutoff

Smooth momentum cutoff:

$$\psi^{(\Lambda)}(k) = \frac{\sqrt{8\pi(\Lambda+\gamma)\Lambda\gamma}}{\Lambda-\gamma}\left(\frac{1}{k^2+\gamma^2} - \frac{1}{k^2+\Lambda^2}\right)$$

At the origin:

$$\psi^{(\Lambda)}(r=0) = \sqrt{(\Lambda+\gamma)\Lambda\gamma/2\pi}$$

At large k:

$$\psi^{(\Lambda)}(k) \longrightarrow \sqrt{8\pi(\Lambda+\gamma)^3\Lambda\gamma}/k^4$$

Why?

Same momentum dependence at small k

More physical qualitative behavior at large k

Binding momentum: $\gamma = \sqrt{2\mu|\varepsilon|}$

Suzuki, PRD 72, 114013 (2005)

Isoscalar T_{cc}^{+}: $(D^{*+}D^{0} - D^{*0}D^{+})/\sqrt{2}$

Coupled-channel wavefunction

- $$\psi_{\mathrm{cc}}^{(\Lambda)}(k) = \frac{\sqrt{8\pi(\Lambda+\gamma)\Lambda\gamma}}{\Lambda-\gamma_{\mathrm{cc}}}\left(\frac{1}{k^2+\gamma_{\mathrm{cc}}^2} - \frac{1}{k^2+\Lambda^2}\right)$$

  $$\gamma_{\mathrm{cc}} = \sqrt{2\mu(\delta+|\varepsilon|)}$$

- Coupled-channel model for a loosely bound molecule with two channels related by symmetry

  $$\psi_{\mathrm{cc}}^{(\Lambda)}(r=0) = \psi^{(\Lambda)}(r=0)$$

- Relative probability for the coupled-channel wavefunction

  $$Z_{\mathrm{cc}} \equiv \int \frac{d^3k}{(2\pi)^3}\left|\psi_{\mathrm{cc}}^{(\Lambda)}(k)\right|^2 = \frac{(\Lambda+\gamma)\gamma}{(\Lambda+\gamma_{\mathrm{cc}})\gamma_{\mathrm{cc}}}$$

  $\Lambda = m_\pi/2, m_\pi, 2m_\pi$, $Z_{0+} = 0.34, 0.38, 0.41$, respectively.

2 Coupled-channel Model

Isoscalar T_{cc}^{+}: $(D^{*+}D^{0} - D^{*0}D^{+})/\sqrt{2}$

"Feynman rules"

- $D^{*+}D^{0}$ channel, D^{0} propagator replacement

$$\frac{1}{k^2+\gamma^2} \longrightarrow \frac{1}{\sqrt{1+Z_{0+}}} \frac{\sqrt{(\Lambda+\gamma)\Lambda}}{\Lambda-\gamma} \left(\frac{1}{k^2+\gamma^2} - \frac{1}{k^2+\Lambda^2} \right)$$

- $D^{*0}D^{+}$ channel, D^{+} propagator replacement

$$\frac{1}{k^2+\gamma_{0+}^2} \longrightarrow -\frac{1}{\sqrt{1+Z_{0+}}} \frac{\sqrt{(\Lambda+\gamma)\Lambda}}{\Lambda-\gamma_{0+}} \left(\frac{1}{k^2+\gamma_{0+}^2} - \frac{1}{k^2+\Lambda^2} \right)$$

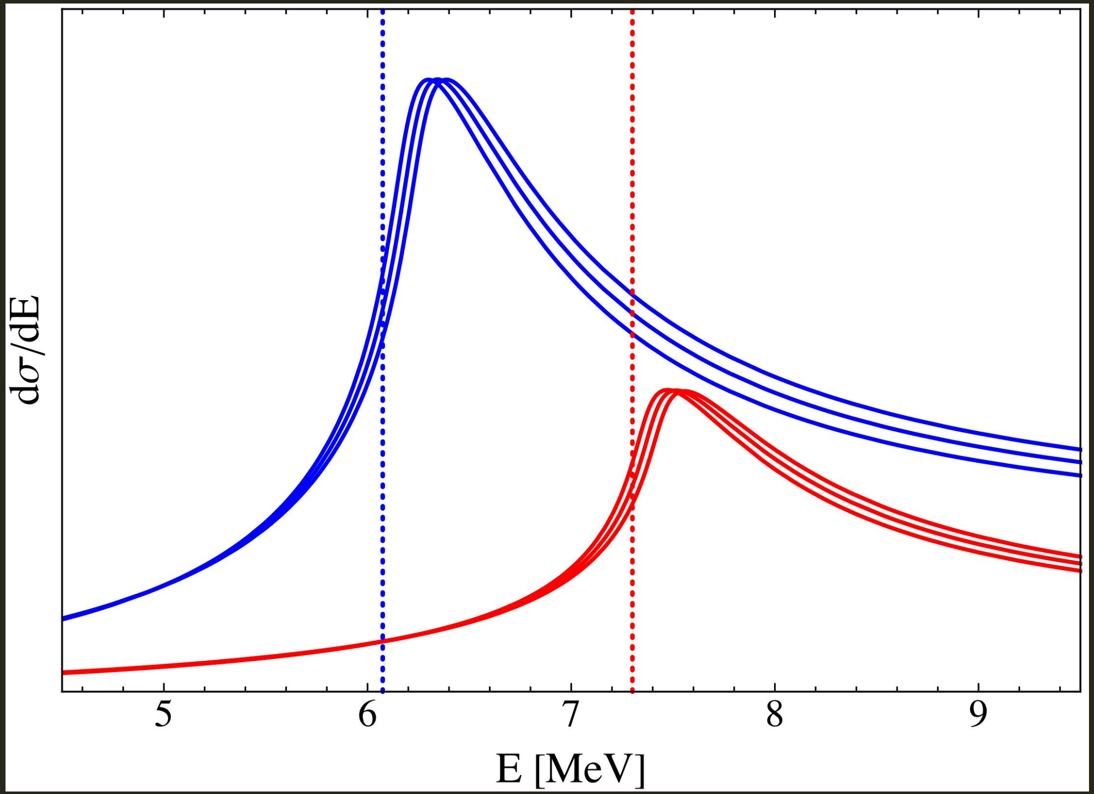
- Same vertices for $D^{*+}D^{0} - T_{cc}^{+}$ and $D^{*0}D^{+} - T_{cc}^{+}$

$D^{*+}D^0$ channel only

Triangle Singularity

$$\frac{d\sigma}{dE}[T_{cc}^+\pi^+] = \left\langle \mathcal{A}_{D^+D^0}\left(\mathcal{A}_{D^+D^0}\right)^*\right\rangle \frac{G_\pi^2 M_T m \gamma_T}{4\pi^2}(2\mu_{\pi T}E)^{3/2}\left|T_+(2\mu_{\pi T}E,\gamma^2)\right|^2,$$
$$\frac{d\sigma}{dE}[T_{cc}^+\pi^0] = \left\langle \mathcal{A}_{D^+D^0}\left(\mathcal{A}_{D^+D^0}\right)^*\right\rangle \frac{3G_\pi^2 M_T m \gamma_T}{32\pi^2}(2\mu_{\pi T}E)^{3/2}\left|T_0(2\mu_{\pi T}E,\gamma^2)\right|^2,$$

where $$\left\langle \mathcal{A}_{D^+D^0}\left(\mathcal{A}_{D^+D^0}\right)^*\right\rangle \equiv \frac{1}{\text{flux}}\sum_y \int d\Phi_{(DD)+y}\mathcal{A}_{D^+D^0+y}\left(\mathcal{A}_{D^+D^0+y}\right)^*$$



Binding energies are 320, 360, and 400 keV in order of increasing energy at the peak

3 Production of T_{cc}^{+} & Soft Pion

$D^{*+}D^0$ channel only

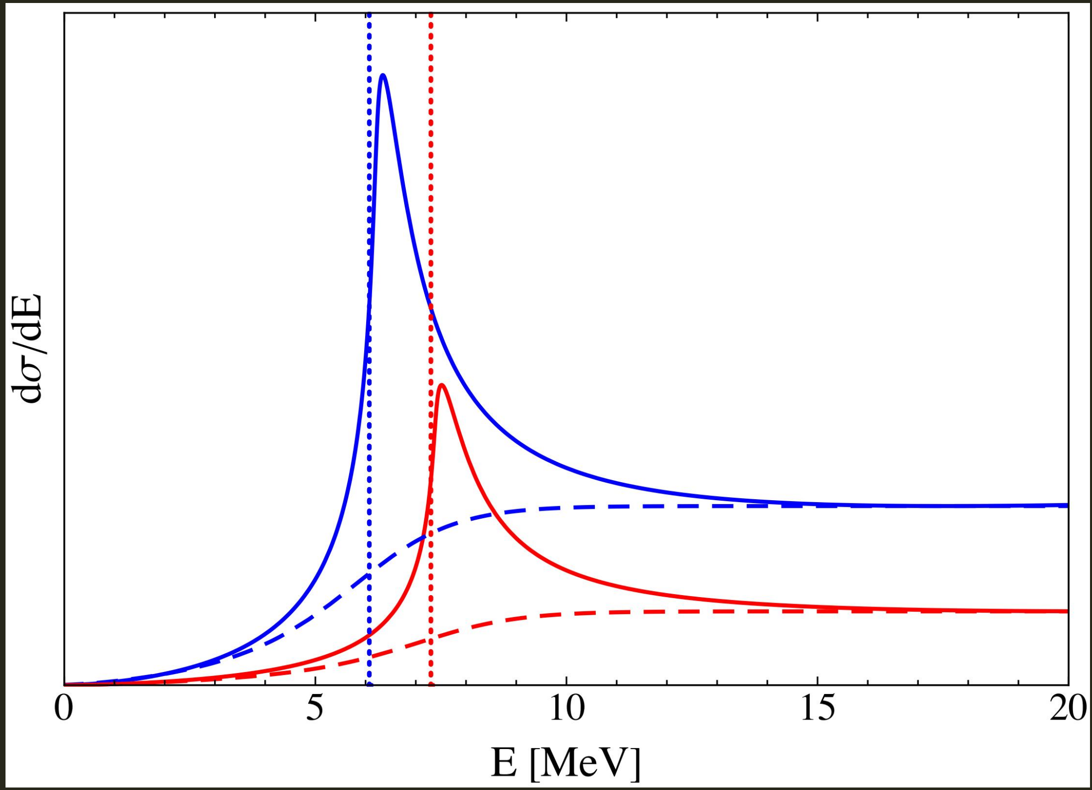
Peaks in the cross sections above background

$$\sigma\left[(T_{cc}^{+}\,\pi^{+})_{\triangle}\right] \approx (8.6 \pm 0.5)\,10^{-3}\,\left(\frac{m_\pi}{\Lambda}\right)^2 \sigma[T_{cc}^{+}, \mathrm{no}\,\pi]$$

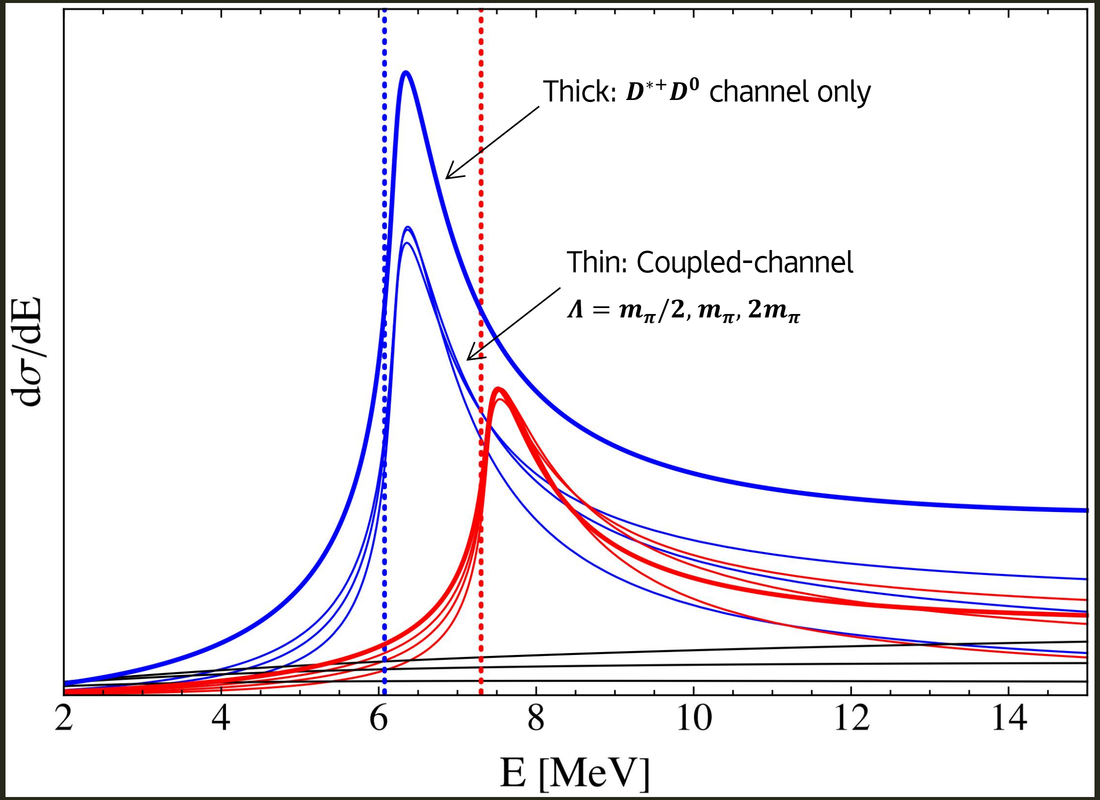
$$\sigma\left[(T_{cc}^{+}\,\pi^{0})_{\triangle}\right] \approx (4.8 \pm 0.2)\,10^{-3}\,\left(\frac{m_\pi}{\Lambda}\right)^2 \sigma[T_{cc}^{+}, \mathrm{no}\,\pi]$$

Where $\sigma[T_{cc}^{+}, \mathrm{no}\,\pi] = \left\langle \mathcal{A}_{D^+D^0}(\mathcal{A}_{D^+D^0})^* \right\rangle \frac{3}{2\mu} \left|\psi_T(r=0)\right|^2$

is cross section for T_{cc}^{+} without any pion with relative momentum smaller than a ultraviolet cutoff.



Coupled-channel model



Blue, red and black for $T_{cc}^{+}\pi^{+}$, $T_{cc}^{+}\pi^{0}$ and $T_{cc}^{+}\pi^{-}$, respectively.

Triangle Singularity

$$\frac{d\sigma}{dE}[T_{cc}^{+}\pi^{+}] = \left\langle \mathcal{A}_{D^+D^0}(\mathcal{A}_{D^+D^0})^* \right\rangle \frac{G_\pi^2 M_T m \gamma_T}{4\pi^2}(2\mu_{\pi T}E)^{3/2}\left|T_+^{(\Lambda)}(2\mu_{\pi T}E,\gamma^2)\right|^2,$$

$$\frac{d\sigma}{dE}[T_{cc}^{+}\pi^{0}] = \left\langle \mathcal{A}_{D^+D^0}(\mathcal{A}_{D^+D^0})^* \right\rangle \frac{3G_\pi^2 M_T m \gamma_T}{32\pi^2}(2\mu_{\pi T}E)^{3/2} \times \left(\left|T_0^{(\Lambda)}(2\mu_{\pi T}E,\gamma^2)\right|^2 + \left|T_0'^{(\Lambda)}(2\mu_{\pi T}E,\gamma_{0+}^2)\right|^2\right),$$

$$\frac{d\sigma}{dE}[T_{cc}^{+}\pi^{-}] = \left\langle \mathcal{A}_{D^+D^0}(\mathcal{A}_{D^+D^0})^* \right\rangle \frac{G_\pi^2 M_T m \gamma_T}{4\pi^2}(2\mu_{\pi T}E)^{3/2}\left|T_-^{(\Lambda)}(2\mu_{\pi T}E,\gamma_{0+}^2)\right|^2.$$

√ No triangle singularity in the production of $T_{cc}^{+}\pi^{-}$ channel, because the mass of D^{*0} is 2.4 MeV below $D^{+}\pi^{-}$ threshold which prevents the D^{*0} and D^{+} from being simultaneously on shell.

3 Production of T_{cc}^+ & Soft Pion

Coupled-channel model

Asymptotic behaviors at large *E*

$$\frac{d\sigma}{dE}[T_{cc}^+\pi^+] \longrightarrow \sigma^{(\Lambda)}[T_{cc}^+, \mathrm{no}\,\pi]\frac{8G_\pi^2\mu_{\pi T}^2\mu_\pi}{3\pi}(2\mu_{\pi T}E)^{-1/2},$$
$$\frac{d\sigma}{dE}[T_{cc}^+\pi^0] \longrightarrow \sigma^{(\Lambda)}[T_{cc}^+, \mathrm{no}\,\pi]\frac{2G_\pi^2\mu_{\pi T}^2\mu_\pi}{\pi}(2\mu_{\pi T}E)^{-1/2},$$
$$\frac{d\sigma}{dE}[T_{cc}^+\pi^-] \longrightarrow \sigma^{(\Lambda)}[T_{cc}^+, \mathrm{no}\,\pi]\frac{8G_\pi^2\mu_{\pi T}^2\mu_\pi}{3\pi}(2\mu_{\pi T}E)^{-1/2}.$$

Where $\sigma^{(\Lambda)}[T_{cc}^+, \mathrm{no}\,\pi]$ is the cross section for T_{cc}^+ without any pion with relative momentum smaller than a ultraviolet cutoff.

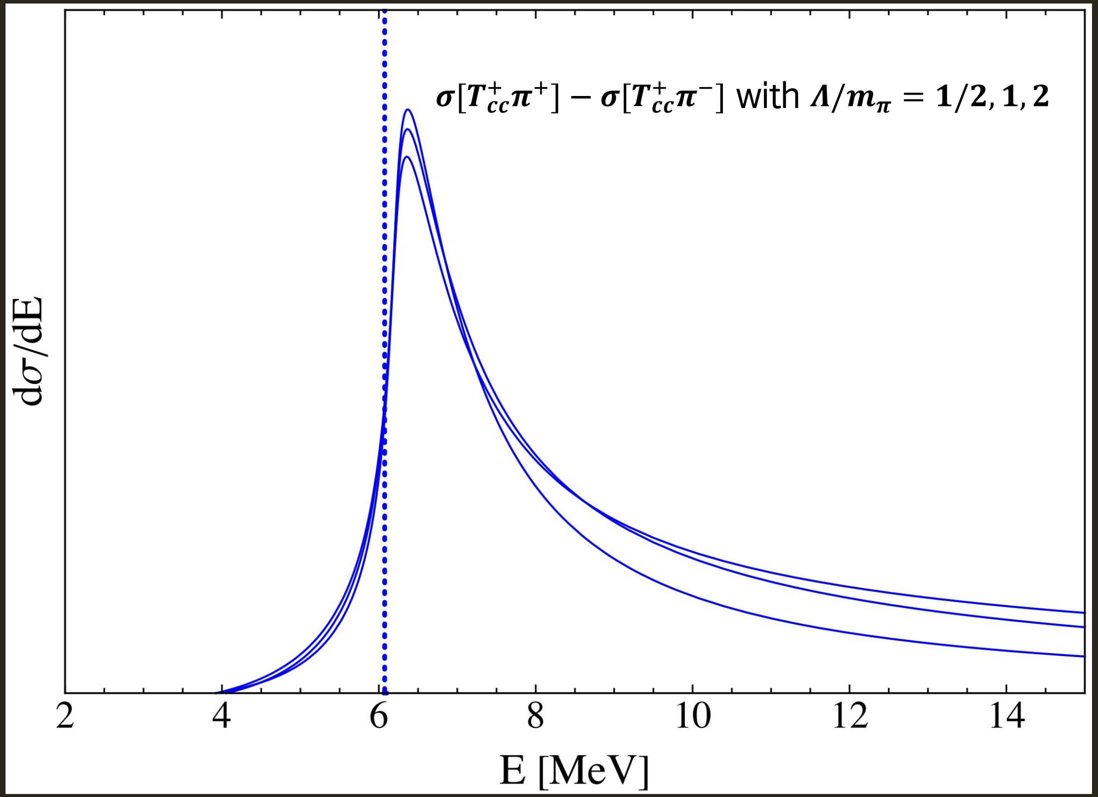
- Cross sections integrated up to E_{max} much larger than triangle-singularity energy

$$\sigma\left[T_{cc}^+\,\pi^+\right] \approx \left(3.2\sqrt{\frac{E_{\max}}{m_\pi}} - 0.0^{+1.8}_{-1.3}\right) \times 10^{-2}\,\sigma^{(\Lambda)}\left[T_{cc}^+, \mathrm{no}\,\pi\right],$$
$$\sigma\left[T_{cc}^+\,\pi^0\right] \approx \left(2.4\sqrt{\frac{E_{\max}}{m_\pi}} - 0.0^{+1.3}_{-1.0}\right) \times 10^{-2}\,\sigma^{(\Lambda)}\left[T_{cc}^+, \mathrm{no}\,\pi\right],$$
$$\sigma\left[T_{cc}^+\,\pi^-\right] \approx \left(3.2\sqrt{\frac{E_{\max}}{m_\pi}} - 1.3^{+0.3}_{-0.5}\right) \times 10^{-2}\,\sigma^{(\Lambda)}\left[T_{cc}^+, \mathrm{no}\,\pi\right].$$

Errors from $\Lambda = 2^{0\pm1}m_\pi$

3 Production of T_{cc}^{+} & Soft Pion

Coupled-channel model



Peaks in the cross sections above background

$$\sigma\left[T_{cc}^{+}\,\pi^{+}\right] - \sigma\left[T_{cc}^{+}\,\pi^{-}\right] \approx \left(1.3^{+1.5}_{-0.8}\right) \times 10^{-2}\,\sigma^{(\Lambda)}\left[T_{cc}^{+}, \mathrm{no}\,\pi\right]$$

Coupled-channel model

LHCb data

- $\mathbf{117 \pm 16}$ events
- Inclusive production: $\boldsymbol{\sigma}^{\Lambda}[\boldsymbol{T}_{cc}^{+}, no\ \pi] + \boldsymbol{\sigma}[\boldsymbol{T}_{cc}^{+}\boldsymbol{\pi}^{+}] + \boldsymbol{\sigma}[\boldsymbol{T}_{cc}^{+}\boldsymbol{\pi}^{0}] + \boldsymbol{\sigma}[\boldsymbol{T}_{cc}^{+}\boldsymbol{\pi}^{-}]$ with $E < E_{max} = q_{max}^{2}/(2\mu_{\pi T})$
- Fractions of events for $\boldsymbol{T}_{cc}^{+}\boldsymbol{\pi}^{+}$ and $\boldsymbol{T}_{cc}^{+}\boldsymbol{\pi}^{-}$: $(3.0_{-1.2}^{+1.5})\%$ and $(1.8_{-0.4}^{+0.2})\%$, respectively

LHCb, Nature Physics 18, 751 (2022)

Coupled-channel model

LHCb data

- $\mathbf{117 \pm 16}$ events
- Inclusive production: $\boldsymbol{\sigma}^{\Lambda}[T_{cc}^{+}, no\ \pi] + \boldsymbol{\sigma}[T_{cc}^{+}\pi^{+}] + \boldsymbol{\sigma}[T_{cc}^{+}\pi^{0}] + \boldsymbol{\sigma}[T_{cc}^{+}\pi^{-}]$ with $E < E_{max} = q_{max}^{2}/(2\mu_{\pi T})$
- Fractions of events for $T_{cc}^{+}\pi^{+}$ and $T_{cc}^{+}\pi^{-}$: $(3.0^{+1.5}_{-1.2})\%$ and $(1.8^{+0.2}_{-0.4})\%$, respectively
- Fractions of events in the peak from triangle singularity for $T_{cc}^{+}\pi^{+}$: $(1.2^{+1.3}_{-0.7})\%$

  Small but all with energy within 1 MeV of peak (6.1 MeV above threshold)

LHCb, Nature Physics 18, 751 (2022)

Summary

- We used the coupled-channel model to calculate the cross sections for $\boldsymbol{T_{cc}^{+}\pi^{+}}$, $\boldsymbol{T_{cc}^{+}\pi^{0}}$ and $\boldsymbol{T_{cc}^{+}\pi^{-}}$ at energies near the triangle-singularity peaks and at higher energies.
- Fraction of $\boldsymbol{T_{cc}^{+}\pi^{+}}$ events in the narrow peak is $(1.2^{+1.3}_{-0.7})\%$, all within $1\ MeV$ of the peak at $6.1\ MeV$.
- Backgrounds can be determined experimentally by measuring $\boldsymbol{T_{cc}^{+}\pi^{-}}$ events.
- Obervation of the narrow peak of triangle singularity supports molecule picture of $T_{cc}{}^{+}$.

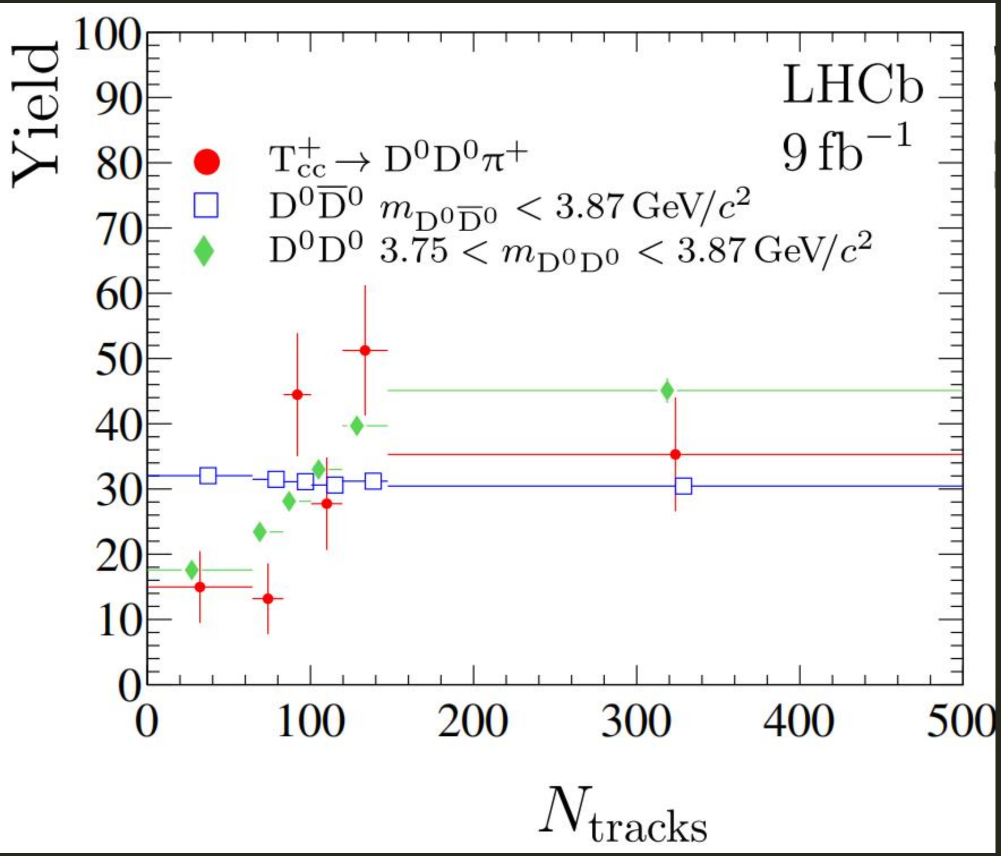
Discussion

SPS *vs* DPS

Calculation based on assumption that charm mesons are created at short distances much smaller than the mean radius ($3.7 \pm 0.2\ fm$) of T_{cc}^{+}.

In double-parton scattering (DPS), the charm mesons maybe created at distances comparable to the radius of a proton ($\sim 1\ fm$).

Single-parton scattering (SPS) makes the triangle-singularity peak stand out more clearly above the background.



LHCb, Nature Commun. 13, 3351 (2022)

Discussion

Molecule *vs* Commpact tetraquark

- Well above the triangle singularity energy E_{Δ}, $d\sigma/dE$ for $\boldsymbol{T_{cc}^{+}\pi^{+}}$ decreases as $E^{-1/2}$.

- A compact tetraquark $\boldsymbol{T_{cc}^{+}}$ would have to have a suppressed coupling to $D^{*+}D^{0}$. Goldstone nature of the pion requires the production amplitude of $\boldsymbol{T_{cc}^{+}}\pi$ to be proportional to the relative momentum of the pion. Therefore $d\sigma/dE$ should increase like $E^{3/2}$.

- Measurements on the differential cross sections above the triangle singularity energy E_{Δ} provide important clues to the nature of $\boldsymbol{T_{cc}^{+}}$.

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THANK YOU

Triangle Singularity

in the Production of T_{cc}^{+} and a Soft Pion

汇报人：蒋军

Backup slides

Triangle amplitudes

$$T_+(q^2,\gamma^2) = \left(1+\frac{mb}{2M_T c}\right)\frac{1}{\sqrt{c}}\log\frac{\sqrt{a}+\sqrt{c}+\sqrt{a+b+c}}{\sqrt{a}-\sqrt{c}+\sqrt{a+b+c}} + \frac{m}{M_T c}\left(\sqrt{a}-\sqrt{a+b+c}\right)$$

$$\begin{aligned}
a &= (\mu/\mu_\pi)q^2 - M_* E_+,\\
b &= -2(\mu/\mu_\pi)(\mu/M)q^2 + M_* E_+ - \gamma^2,\\
c &= (\mu/M)^2 q^2.
\end{aligned}$$

$T_0(q^2,\gamma^2)$ can be obtained by replacing E_+ by E_0: $E_0 = \delta_{00} - \varepsilon_T - i\,(\Gamma_{*0}+\Gamma_{*+})/2$

Logarithmic approximationcan

$$T_+^{(\log)}(q^2,\gamma^2) = \sqrt{\frac{M/M_T}{\mu_{\pi T}\delta_{0+}}}\left(\frac{2M}{M_*}\log\frac{\sqrt{a}+(\mu/M)q+i\,\gamma}{\sqrt{a}-(\mu/M)q+i\,\gamma} + \frac{m}{M_*}\right)$$

Triangle amplitudes in coupled-channel model

$$T_+^{(\Lambda)}(q^2,\gamma^2) = \frac{\sqrt{(\Lambda+\gamma)\Lambda}}{\sqrt{1+Z_{0+}}\,(\Lambda-\gamma)}\left[T_+(q^2,\gamma^2) - T_+(q^2,\Lambda^2)\right]$$

$$T_0^{(\Lambda)}(q^2,\gamma^2) = \frac{\sqrt{(\Lambda+\gamma)\Lambda}}{\sqrt{1+Z_{0+}}\,(\Lambda-\gamma)}\left[T_0(q^2,\gamma^2) - T_0(q^2,\Lambda^2)\right]$$

$$T_0^{\prime(\Lambda)}(q^2,\gamma_{0+}^2) = -\frac{\sqrt{(\Lambda+\gamma)\Lambda}}{\sqrt{1+Z_{0+}}\,(\Lambda-\gamma_{0+})}\left[T_0(q^2,\gamma_{0+}^2) - T_0(q^2,\Lambda^2)\right]$$

$$T_-^{(\Lambda)}(q^2,\gamma_{0+}^2) = -\frac{\sqrt{(\Lambda+\gamma)\Lambda}}{\sqrt{1+Z_{0+}}\,(\Lambda-\gamma_{0+})}\left[T_-(q^2,\gamma_{0+}^2) - T_-(q^2,\Lambda^2)\right]$$

where $T_-(q^2,\gamma_{0+}^2)$ can be obtained by the right side of $T_+(q^2,\gamma^2)$ with the coefficients

$$a = (\mu/\mu_\pi)q^2 - M_* E_-,$$
$$b = -2(\mu/\mu_\pi)(\mu/M)q^2 + M_* E_- - \gamma_{0+}^2,$$
$$c = (\mu/M)^2 q^2.$$

with $E_- = \delta + \delta_{+-} - \varepsilon_T - i\,\Gamma_{*0}$ and $\delta_{+-} = M_{*0} - M_+ - m_- = -2.38\ \text{MeV}$

Backup slides

Asymptotic behavior of triangle amplitudes at large E

$$T_+(q^2,\gamma^2) \longrightarrow \left(\frac{M}{M_*}\log\frac{\sqrt{M_T/m}+1}{\sqrt{M_T/m}-1}+\frac{\sqrt{M_T m}}{M_*}\right)\frac{1}{q}-\frac{2iM_T m\gamma}{M_*^2 q^2}$$

Subtraction cancels the terms that decrease as $1/q$, so the triangle amplitude decreases as $1/q^2$:

$$T_+^{(\Lambda)}(q^2,\gamma^2) \longrightarrow i\,\frac{2\sqrt{(\Lambda+\gamma)\Lambda}\,M_T m}{\sqrt{1+Z_{0+}}\,M_*^2\,q^2}.$$

or

$$T_+^{(\Lambda)}(q^2,\gamma^2) \longrightarrow i\,\frac{4\mu_{\pi T}}{M_*\sqrt{\gamma_T/2\pi}}\,\frac{\psi_T^{(\Lambda)}(r=0)}{\sqrt{1+Z_{0+}}}\,\frac{1}{q^2}.$$