Two-loop amplitudes for tW production at hadron colliders

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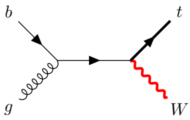
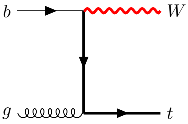
Motivation

The top quark is the heaviest elementary particle in the Standard Model.

Three major modes for single top productions, s-channel t-channel and tW production.

tW production can be used to probe the the CKM matrix element V_{tb}.





The uncertainty of the measured cross section is about 11 % [A. S. Rodríguez, 2022].

To match experiments, the theoretical predictions must include higher-order corrections.

Motivation

NLO correction [S. Zhu, 2002, Q.-H. Cao, 2008] with top and W decay [J. M. Campbell et al, 2005]

Approximate higher order corrections [N. Kidonakis, 2006, 2010, 2017, 2021]

Effect of the parton shower [S. Frixione et al, 2008, E. Re, 2011, T. Ježo, 2016]

To match experiments, the complete NNLO QCD corrections are important.

Factorization formula

The N-jettiness subtraction is based on the soft-collinear effective theory (SCET).

$$\frac{d\sigma}{d\tau_N} \propto \int H \otimes B_1 \otimes B_2 \otimes S \otimes \left(\prod_{n=1}^{N} J_n \right) . \tag{1}$$

NNLO Beam functions B_i [I.W. Stewart et al, 2010, C.F. Berger et al, 2011, J.R. Gaunt et al, 2014]

NNLO Jet function J [T. Becher et al, 2006, 2011]

NNLO Soft function S [H. T. Li et al, 2016, 2018]

The missing part is NNLO hard function, which demands one-loop squared amplitudes and the interference between two-loop and tree-level amplitudes.

Kinematics and notations

$$g(k_1) + b(k_2) \to W(k_3) + t(k_4),$$
$$k_1^2 = k_2^2 = 0,\ k_3^2 = m_W^2,\ k_4^2 = (k_1 + k_2 - k_3)^2 = m_t^2. \qquad (2)$$

The Mandelstam variables

$$s = (k_1 + k_2)^2, t = (k_1 - k_3)^2,\ u = (k_2 - k_3)^2,$$
$$s + t + u = m_W^2 + m_t^2. \qquad (3)$$

The polarization summation

$$\sum_i \epsilon_i^{*\mu}(k_3)\epsilon_i^{\nu}(k_3) = -g^{\mu\nu} + \frac{k_3^\mu k_3^\nu}{m_W^2}$$
$$\sum_i \epsilon_i^{\mu}(k_1)\epsilon_i^{*\nu}(k_1) = -g^{\mu\nu} + \frac{k_1^\mu n^\nu + k_1^\nu n^\mu}{k_1 \cdot n} \text{ (can be neglected here)}. \qquad (4)$$

Kinematics and notations

The anticommuting γ_5 scheme is implemented.

The tW amplitude can be written as

$$\mathcal{M} = \mathcal{M}^{(0)} + \frac{\alpha_s}{4\pi}\mathcal{M}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathcal{M}^{(2)} + \cdots \tag{5}$$

We do not consider the decay of the top quark and the W boson at the moment.

Do not keep the polarization information, focus on amplitude squared,

$$|\mathcal{M}^{(1)}|^2, \quad |\mathcal{M}^{(0)*}\mathcal{M}^{(2)}|. \tag{6}$$

Then all the Lorentz indices are contracted.

Color structures

According to color structures, we have

$$\mathcal{A}^{(2)} = \sum_{\text{spins}} |\mathcal{M}^{(0)*}\mathcal{M}^{(2)}| = N_c^4 A + N_c^2 B + C + \frac{1}{N_c^2} D + n_l (N_c^3 E_l + N_c F_l + \frac{1}{N_c} G_l)$$
$$+ n_h (N_c^3 E_h + N_c F_h + \frac{1}{N_c} G_h), \quad (7)$$

n_l (n_h) is the number of light (heavy) quark flavors. N_c is the color factor.

Leading contribution of the two-loop amplitudes

$$\mathcal{A}^{(2)}_{\text{L.C.}} \equiv N_c^4 A + n_l (N_c^3 E_l + N_c F_l + \frac{1}{N_c} G_l). \quad (8)$$

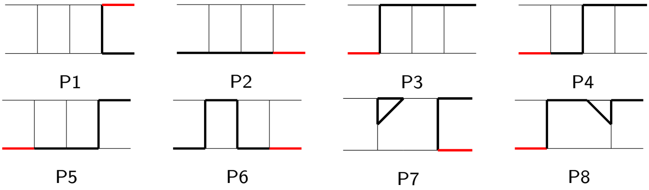
Amplitude calculation

We use FeynArts to generate 199 two-loop diagrams. 73 diagrams contribute to the leading color, 20 diagrams contribute to light fermion loop.

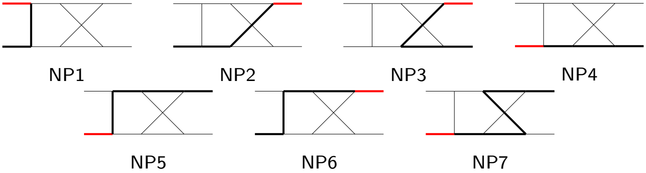
After IBP reduction of FIRE [A. V. Smirnov et al, 2020], $\mathcal{A}^{(2)} = \sum_{\text{spins}} |\mathcal{M}^{(0)*}\mathcal{M}^{(2)}|$ can be reduced to several families of master intergrals.

Calculation of master integrals

All master integrals can be expressed in 8 planar and 7 non-planar topologies.



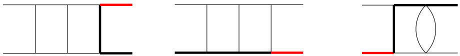
Red lines are W boson, thick lines are top quarks, others are massless particles.



Master integrals of leading contribution

$$\mathcal{A}^{(2)}_{\mathrm{L.C.}} \equiv N_c^4 A + n_l (N_c^3 E_l + N_c F_l + \frac{1}{N_c} G_l). \qquad (9)$$

Only P1 and P2 and a sub-diagram of P3 relate to leading contribution



P1 P2 sub-diagram of P3

31 and 38 master integrals in P1, P2. Their analytical results have already been obtained [L.-B. Chen et.al, 2021, M.-M. Long et.al, 2021] by the the method of differential equations.

We also calculate the P2 topology independently to do the cross check.

Brief review of differential equations

By constructing the canonical basis, the differential equations can be transforming to ϵ form [J. M. Henn, 2013].

$$d\,\mathbf{F}(s,t,m_W^2;\epsilon) = \epsilon\,(d\,\tilde{A})\,\mathbf{F}(s,t,m_W^2;\epsilon), \tag{10}$$

The square roots may appear. Combining the boundary conditions, the solutions can be expressed in multiple polylogarithms (MPLs) [A. B. Goncharov, 1998] or GPLs.

$$G_{a_1,a_2,\ldots,a_n}(x) \equiv \int_0^x \frac{\mathrm{d}t}{t-a_1} G_{a_2,\ldots,a_n}(t)\,, \tag{11}$$

$$G_{\vec{0}_n}(x) \equiv \frac{1}{n!}\ln^n x\,. \tag{12}$$

Use PolyLogTools [D. Claude, 2019] to obtain numerical results of GPLs.

Renormalization

Renormalized QCD amplitude is

$$\mathcal{M}_{\text{ren}} = Z_g^{1/2} Z_b^{1/2} Z_t^{1/2} \left(\mathcal{M}_{\text{bare}}\Big|_{\alpha_s^{\text{bare}} \to Z_{\alpha_s}\alpha_s;\ m_{t,\text{bare}} \to Z_m m_t} \right). \tag{13}$$

$Z_{g,b,t}$ is the wave function renormalization factor. α_s and m_t are renormalized by the factor Z_{α_s} and Z_m.

$$\begin{aligned} \mathcal{M}_{\text{ren}} &= \mathcal{M}_{\text{ren}}^{(0)} + \frac{\alpha_s}{4\pi}(\mathcal{M}_{\text{bare}}^{(1)} + \mathcal{M}_{\text{C.T.}}^{(1)}) + \left(\frac{\alpha_s}{4\pi}\right)^2 (\mathcal{M}_{\text{bare}}^{(2)} + \mathcal{M}_{\text{C.T.}}^{(2)}) \\ &= \mathcal{M}_{\text{ren}}^{(0)} + \frac{\alpha_s}{4\pi}\mathcal{M}_{\text{ren}}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathcal{M}_{\text{ren}}^{(2)}. \end{aligned} \tag{14}$$

The on-shell renormalization scheme for wave functions and top-quark mass is adopted. The renormalized strong coupling α_s is calculated in $\overline{\text{MS}}$ scheme.

IR divergences

IR divergences can be subtracted with a factor $\mathbf{Z}$.

$$\mathcal{M}_{\text{fin}} = \mathbf{Z}^{-1}\mathcal{M}_{\text{ren}} \,. \tag{15}$$

In the framework of SCET, $\mathbf{Z}$ can be investigated through the anomalous-dimensions of the effective operators.

$$\mathbf{Z} = 1 + \frac{\alpha_s}{4\pi}\mathbf{Z}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathbf{Z}^{(2)} \; + \mathcal{O}(\alpha_s^3). \tag{16}$$

For example,

$$\begin{aligned}\mathbf{Z}^{(1)} =& -(C_A + C_F)\frac{\gamma_{\text{cusp}}^{(0)}}{4\epsilon^2} + \frac{\gamma_g^{(0)} + \gamma_b^{(0)} + \gamma_t^{(0)}}{2\epsilon} \\ &+ \frac{\gamma_{\text{cusp}}^{(0)}}{4\epsilon}\left(-C_A \ln\frac{\mu^2}{-s} - C_A \ln\frac{\mu m_t}{m_t^2 - u} + (C_A - 2C_F)\ln\frac{\mu m_t}{m_t^2 - t}\right).\end{aligned} \tag{17}$$

where γ_{cusp}, γ_g, γ_b and γ_t are anomalous dimensions.

NNLO hard function

$$H^{(2)} = \mathcal{M}_{\rm fin}^{(2)}\mathcal{M}_{\rm fin}^{(0)*} + \mathcal{M}_{\rm fin}^{(0)}\mathcal{M}_{\rm fin}^{(2)*} + \left|\mathcal{M}_{\rm fin}^{(1)}\right|^2 \tag{18}$$

According to color structures

$$\begin{aligned} H^{(2)} =& N_c^4 H_A + N_c^2 H_B + H_C + \frac{1}{N_c^2} H_D + n_l \left(N_c^3 H_{El} + N_c H_{Fl} + \frac{1}{N_c} H_{Gl} \right) \\ &+ n_h \left(N_c^3 H_{Eh} + N_c H_{Fh} + \frac{1}{N_c} H_{Gh} \right) . \end{aligned} \tag{19}$$

Leading contribution of hard function

$$H_{\rm L.C.}^{(2)} \equiv N_c^4 H_A + n_l \left(N_c^3 H_{El} + N_c H_{Fl} + \frac{1}{N_c} H_{Gl} \right) . \tag{20}$$

Numerical results

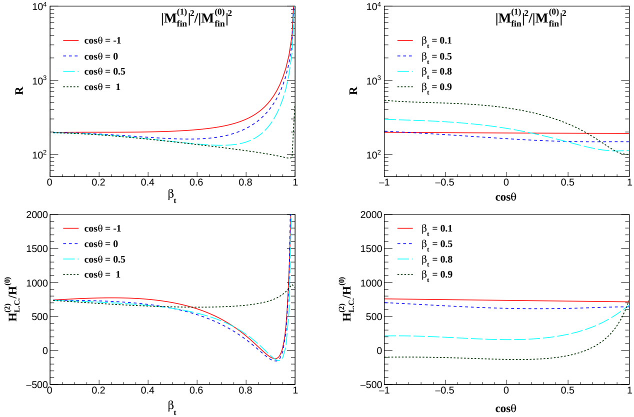
$\beta_t = \sqrt{1 - m_t^2/E_t^2}$ measures the velocity of the top quark and θ is the angle between gluon and top quark.

The divergence of ϵ^{-4} and ϵ^{-3} have been canceled analytically.

Other divergence have been checked numerically in high precision.

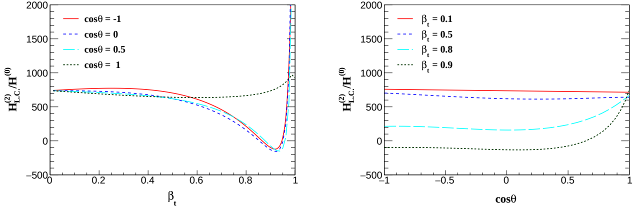
Numerical results

$$H = H^{(0)} + \frac{\alpha_s}{4\pi} H^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 H^{(2)}$$



NNLO hard function contribute a few percent $(2\% \sim 6\%)$ compared with LO hard function.

Numerical results



In the limit of $\beta_t \to 0$, no $\cos\theta$ dependence.

Divergence in the limit of $\beta_t \to 1$.

m_W expansion

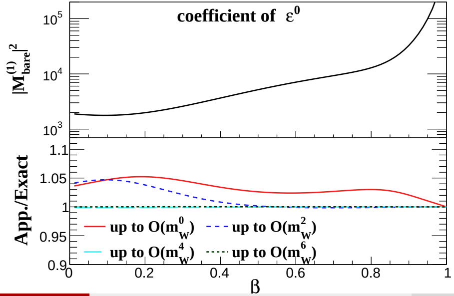
$m_W = 0$ does not bring new IR divergences due to the massive top quark propagator.

$$I^i_{n_1,n_2,n_3,n_4}\left(s,u,m_W^2,m_t^2\right) = \sum_{n=0}^{\infty} \frac{(m_W^2)^n}{n!} \left. \frac{\partial^n I^i_{n_1,n_2,n_3,n_4}}{\partial(m_W^2)^n} \right|_{m_W^2=0} . \tag{21}$$

1.The number of these master integrals is less.

2.The analytic computation is easier to perform.

m_W expansion in one-loop squared.



Conclusion

We obtain the analytical one-loop square and two-loop leading contribution amplitudes for tW production by using differential equations.

The renormalized amplitude squared has up to ϵ^{-4} poles, which have been checked against the general infrared structures predicted by anomalous dimensions.

The finite part gives rise to about a few percent corrections compared to the corresponding LO results.

We investigate how to obtain approximated results by the expansion in m_W.

We will calculate complete two-loop amplitude in the future.

Backup

In the differential equations of P1, there is square root

$$r_1 = \sqrt{(s-(m_t+m_W)^2)(s-(m_t-m_W)^2)},$$
$$s = m_t^2\,\frac{(x+z)(1+x\,z)}{x}, \quad m_W^2 = m_t^2\,z^2. \tag{22}$$

In the differential equations of P2, besides r_1, there is another square root

$$r_2 = \sqrt{s(m_t^2-u)(m_t^2\,(-4m_W^2+s+4u+4)-4m_t^4-s\,u)} \tag{23}$$

The problem is to rationalized more than one square roots simultaneously.

One choice is to use package **RationalizeRoots**. [M. Besier et.al, 2020]. For example,

$$m_W^2 = \frac{b_1 b_2 m_t^2}{1-b_2}, \quad s = -\frac{(b_1+1)m_t^2}{b_2-1}, \quad u = \frac{b_2(b_3(b_1+1)(b_3 b_2-2)+4b_1)m_t^2}{b_2(b_3(b_1+1)(b_3 b_2-2)-4)+4}, \tag{24}$$

Backup

$$
\begin{aligned}
\mathcal{M}_{\text{fin}} &= \mathcal{M}^{(0)}_{\text{fin}} + \frac{\alpha_s}{4\pi}\mathcal{M}^{(1)}_{\text{fin}} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathcal{M}^{(2)}_{\text{fin}}, \\
\mathcal{M}^{(0)}_{\text{fin}} &= \mathcal{M}^{(0)}_{\text{ren}}\,, \\
\mathcal{M}^{(1)}_{\text{fin}} &= \mathcal{M}^{(1)}_{\text{ren}} - \mathbf{Z}^{(1)}\mathcal{M}^{(0)}_{\text{ren}}\,, \\
\mathcal{M}^{(2)}_{\text{fin}} &= \mathcal{M}^{(2)}_{\text{ren}} + ((\mathbf{Z}^{(1)})^2 - \mathbf{Z}^{(2)})\mathcal{M}^{(0)}_{\text{ren}} - \mathbf{Z}^{(1)}\mathcal{M}^{(1)}_{\text{ren}}\,.
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
H &= H^{(0)} + \frac{\alpha_s}{4\pi}H^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 H^{(2)}\,, \\
H^{(0)} &= \left|\mathcal{M}^{(0)}_{\text{fin}}\right|^2\,, \\
H^{(1)} &= \mathcal{M}^{(1)}_{\text{fin}}\mathcal{M}^{(0)*}_{\text{fin}} + \mathcal{M}^{(0)}_{\text{fin}}\mathcal{M}^{(1)*}_{\text{fin}}\,, \\
H^{(2)} &= \mathcal{M}^{(2)}_{\text{fin}}\mathcal{M}^{(0)*}_{\text{fin}} + \mathcal{M}^{(0)}_{\text{fin}}\mathcal{M}^{(2)*}_{\text{fin}} + \left|\mathcal{M}^{(1)}_{\text{fin}}\right|^2\,.
\end{aligned} \tag{26}
$$