Domain walls beyond Z_2 — classification and preliminary study on gravitational wave signals

周也铃 (杭高院) 2022-07-23

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Symmetries in fundamental physics

- Noether 1915, symmetry $\rightarrow$ conservation
- Spacetime $R_{1,3} \rtimes O(1,3)$
- Standard Model $SU(3)_c \times SU(2)_L \times U(1)_Y \quad \supset U(1)_{\rm em}$
- Discrete symm: C, P, T, CP, CPT
- New symm: $U(1)_{B-L}$; $U(1)_{\rm PQ}$; GUTs ($SU(5)$, $SO(10)$, Pati-Salam, etc)
- New discrete symm: Z_N, $U(1)_{\rm FN}$, A_4, $S_4 \subset SU(3)_{\rm flavour}$; Modular symm Γ; Z_2^C in GUTs; R_p, Z_3 in SUSY

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New Physics $\begin{cases} \text{new particles} & \to \text{colliders, intensity experiments, …} \\ \text{symmetry breaking} & \to \text{cosmological tests (GWs, …)} \end{cases}$

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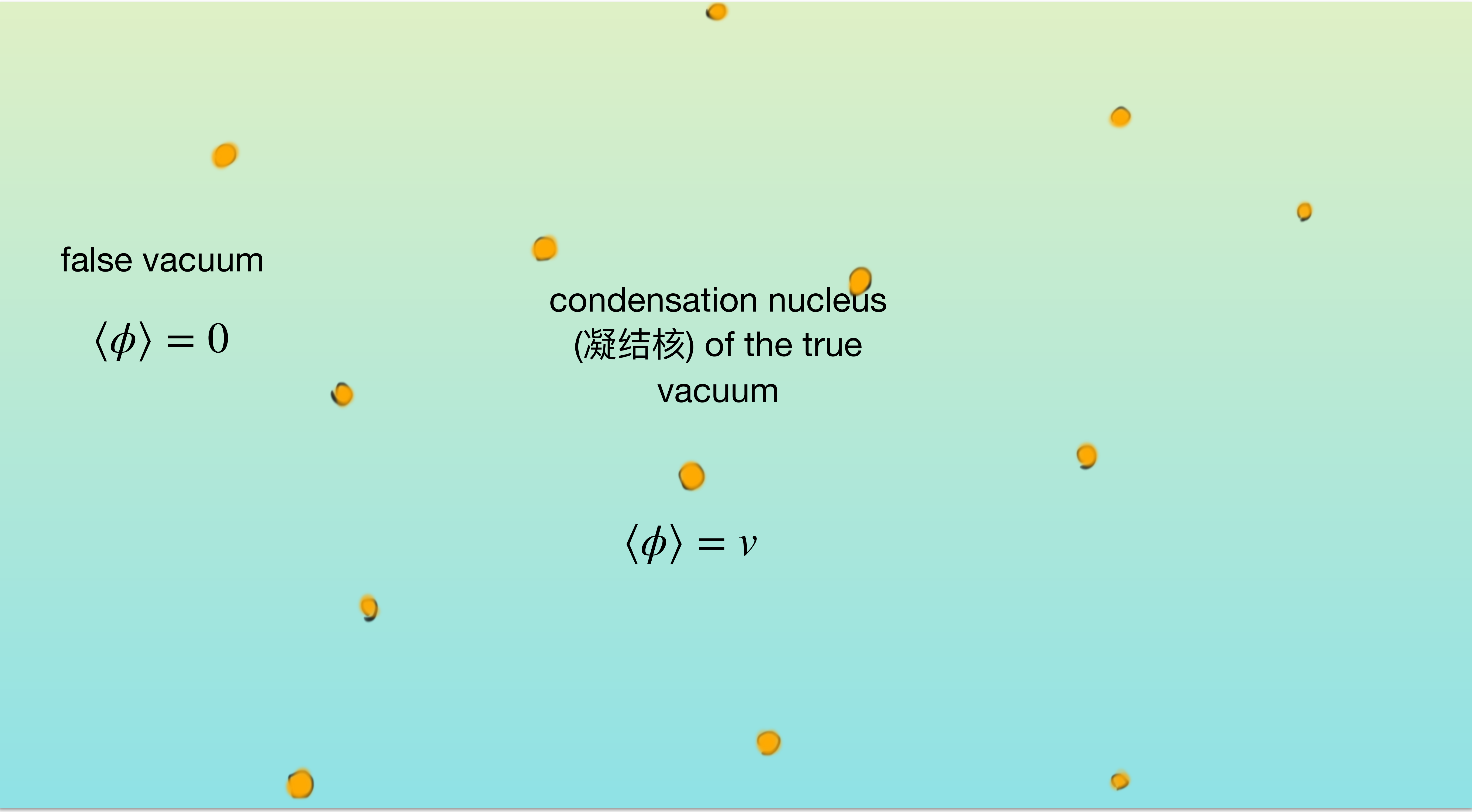
本报告 $\Longrightarrow$ 初步探讨Z_N畴壁的性质和宇宙学效应

吴永成、谢柯盼、周也铃，2204.04374; 2205.11529

A general picture of phase transition



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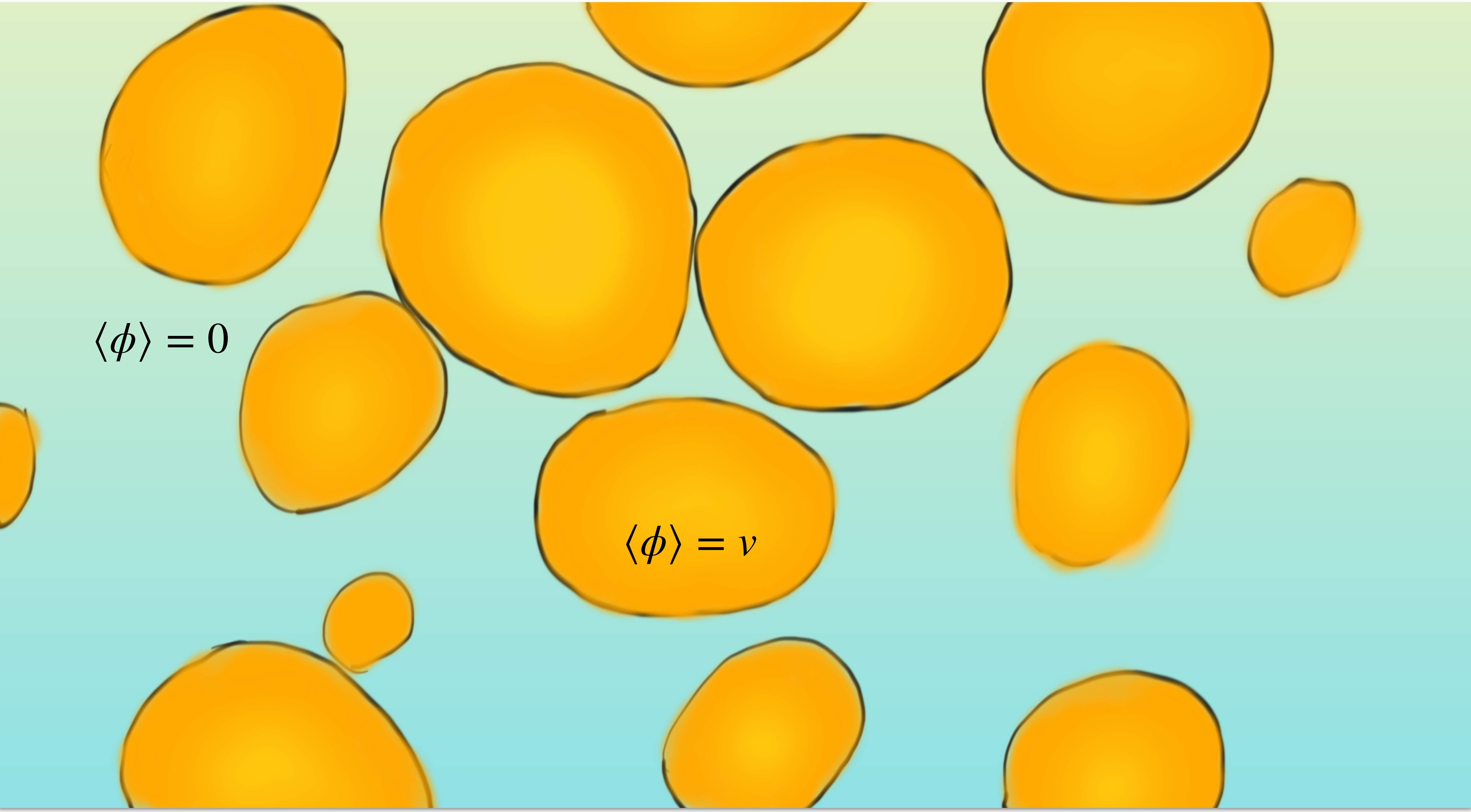

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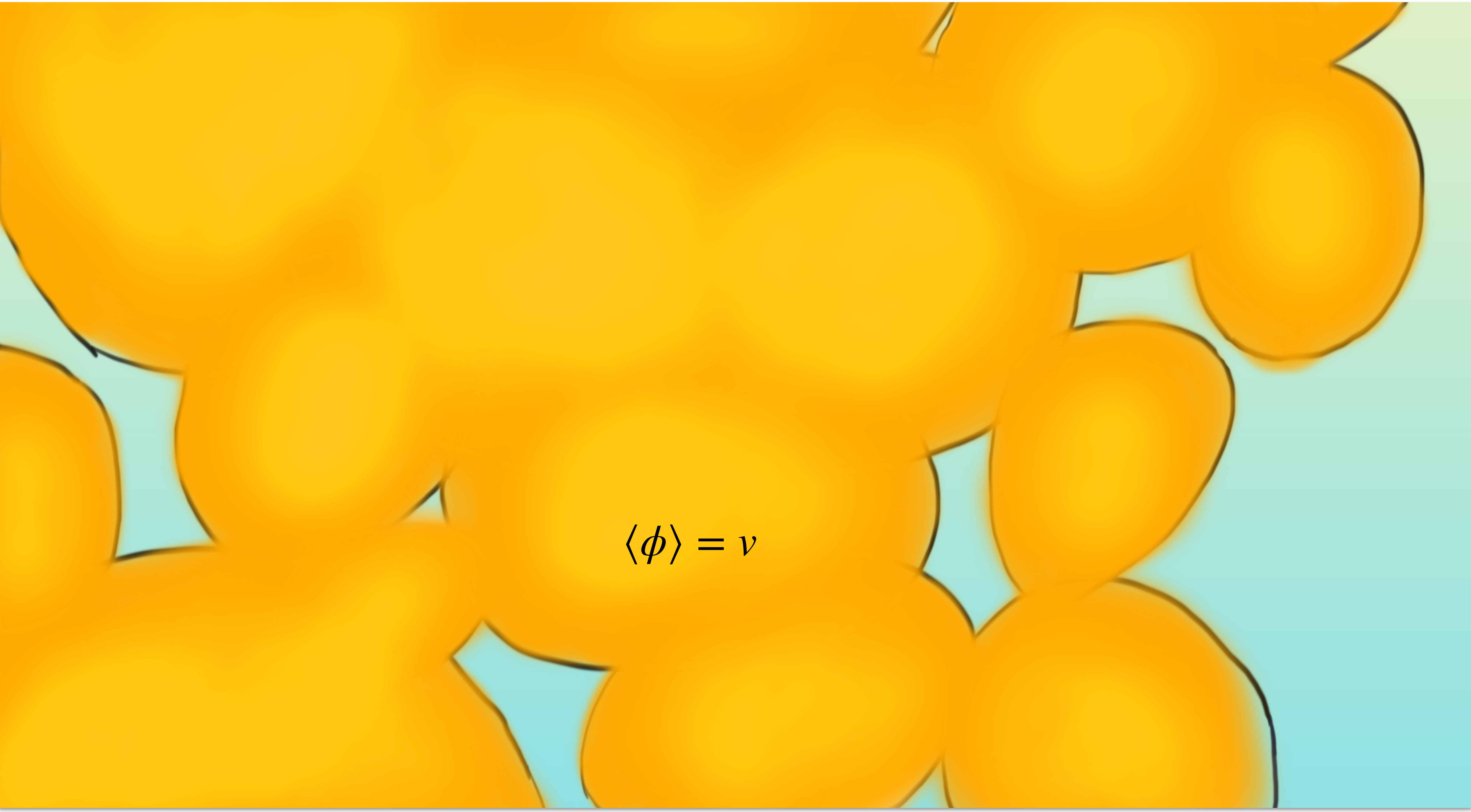
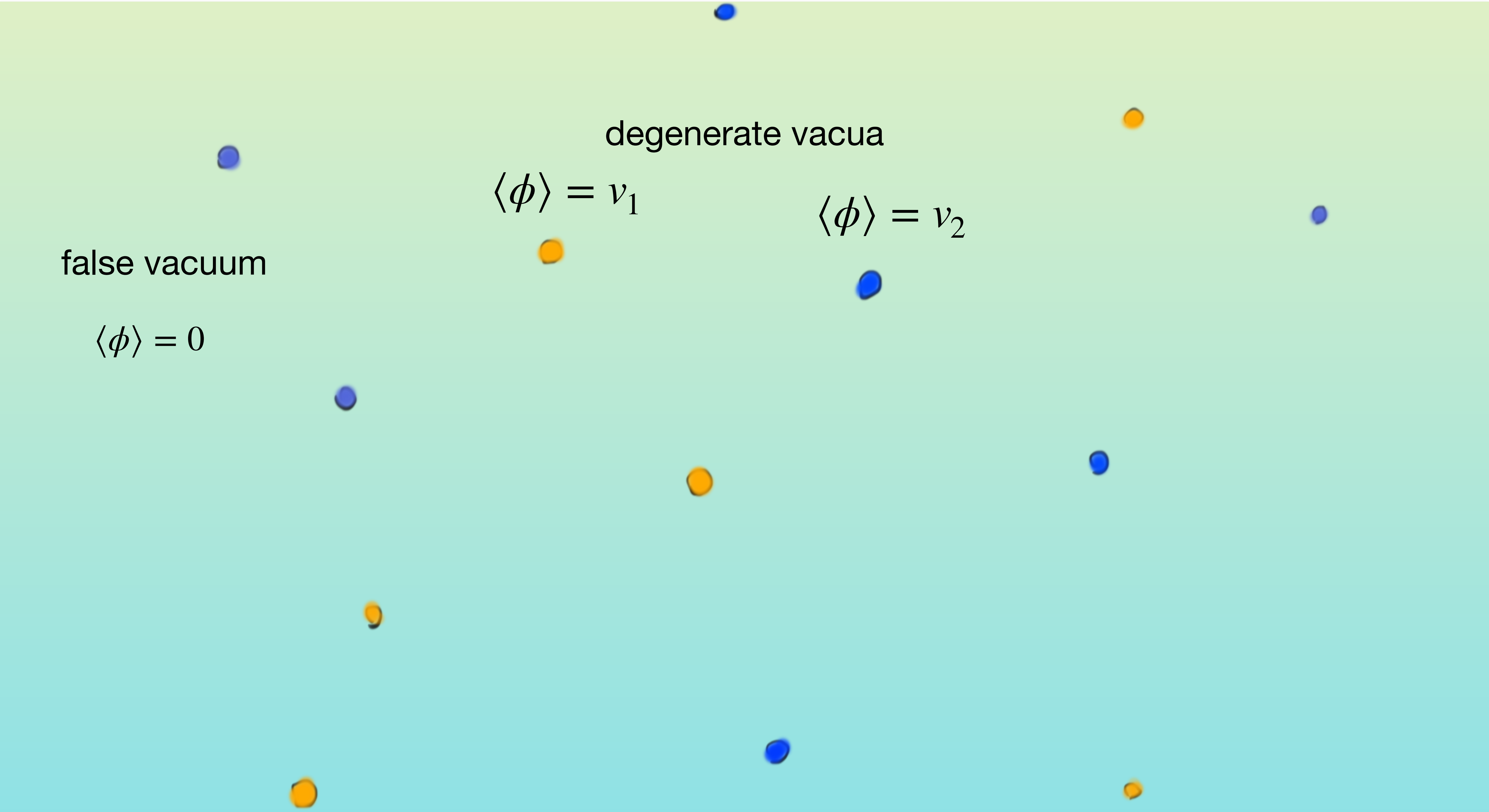


Phase transition with degenerate vacua



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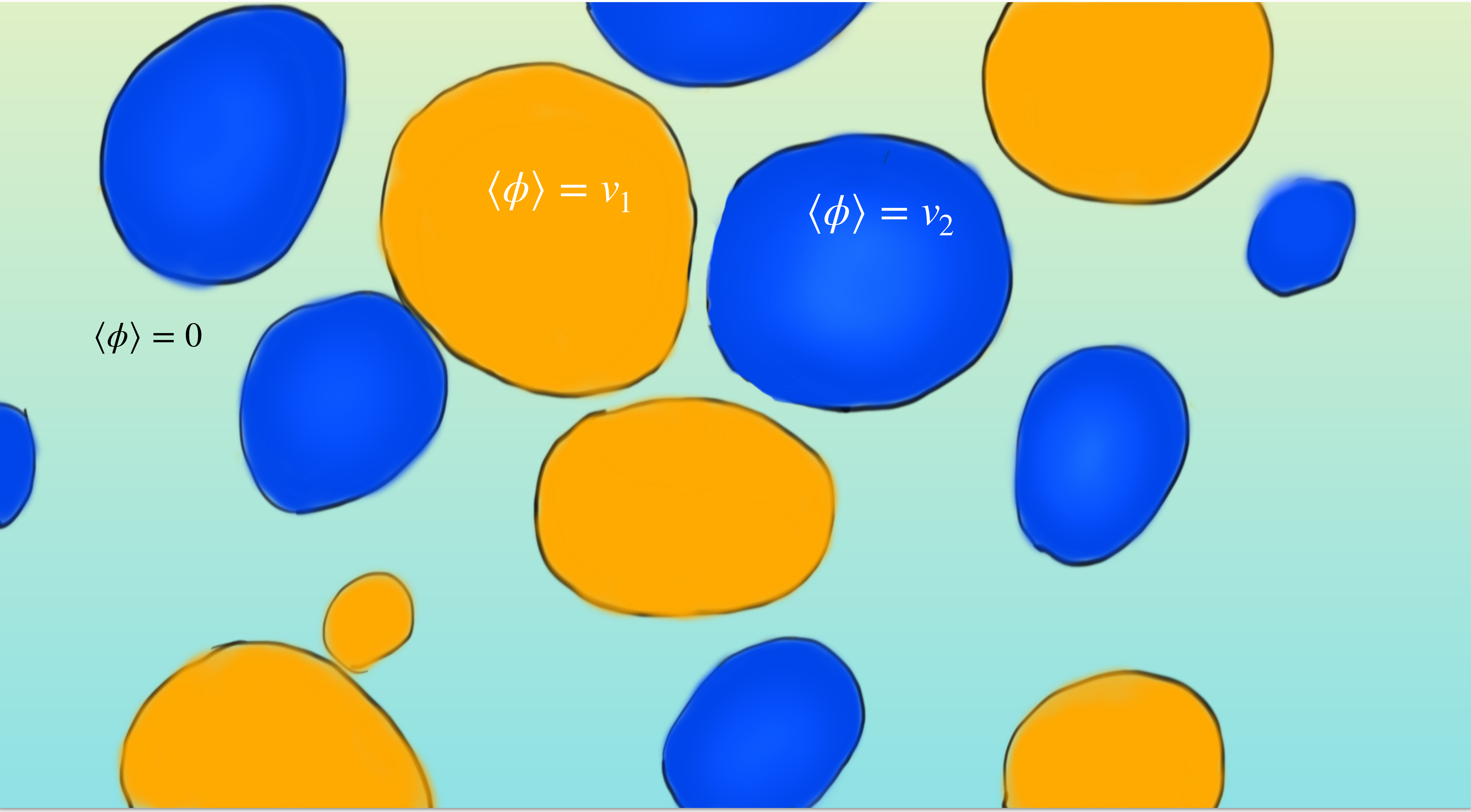

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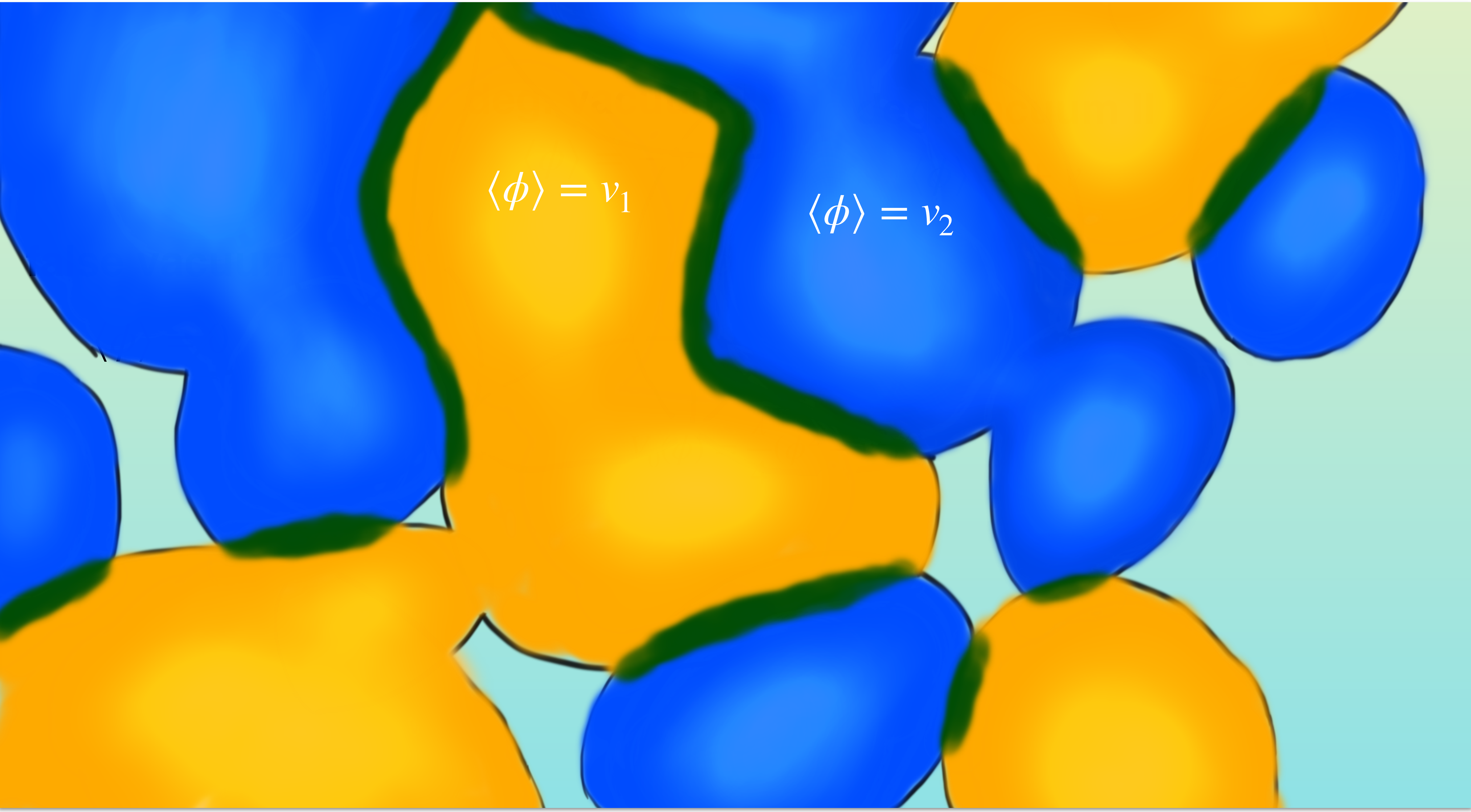
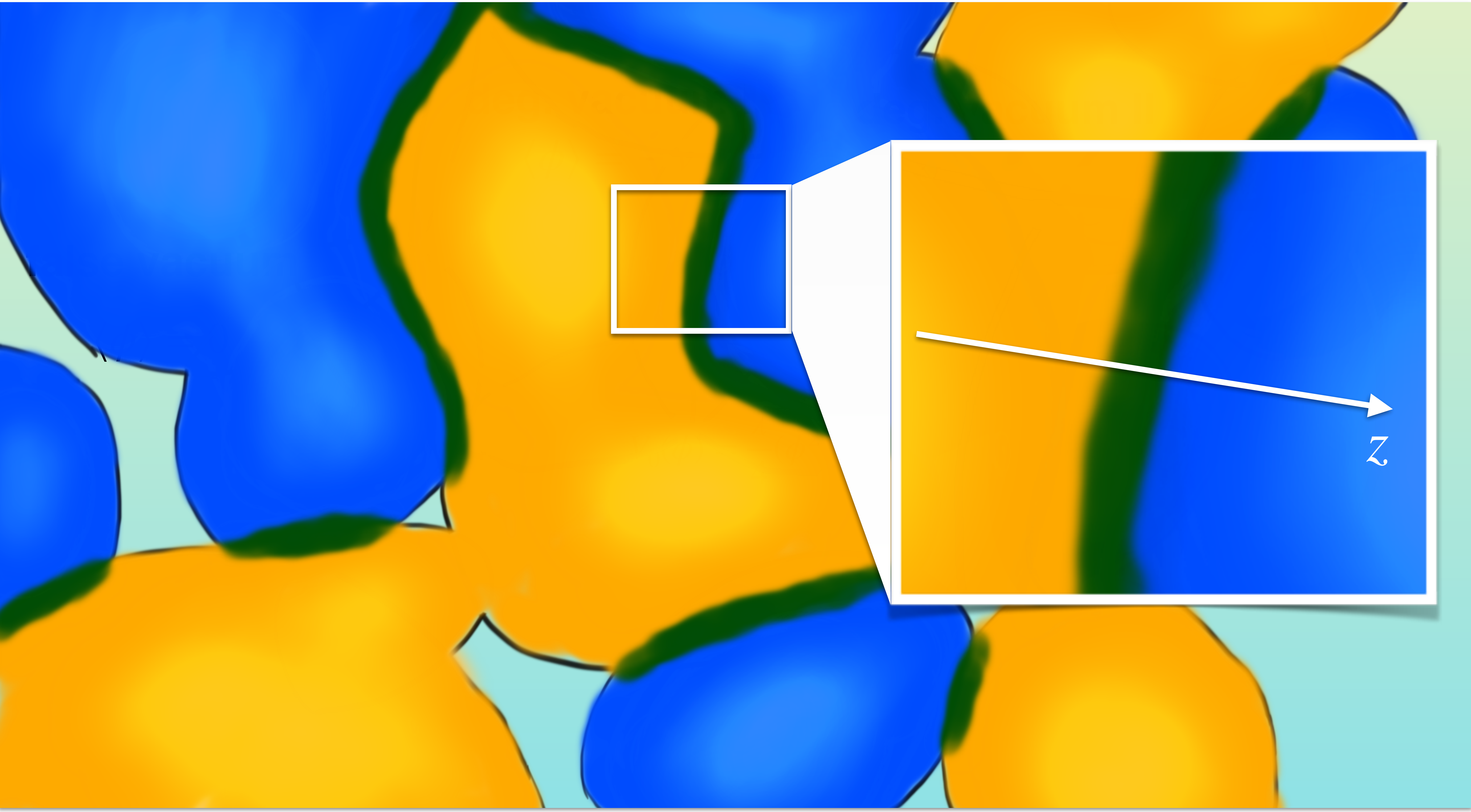


Phase transition with degenerate vacua

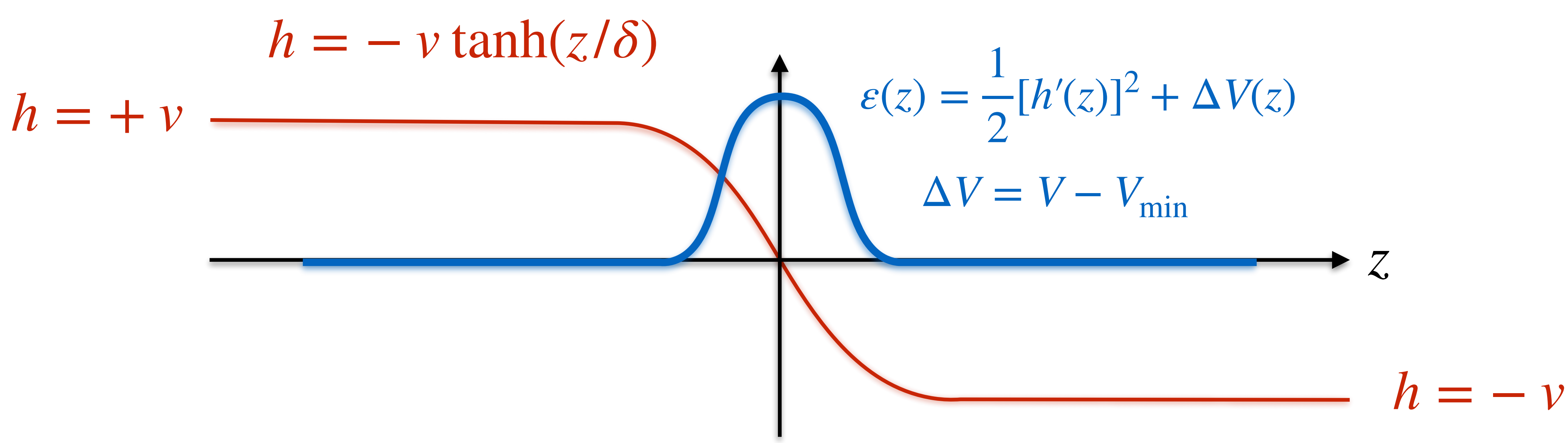


The simplest case —— Z_2 walls

Given a toy potential for a real scalar in Z_2 $$V = -\frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4}h^4$$

EOM $$h''(z) = \lambda h(h^2 - v^2)$$ $$v = \sqrt{\mu^2/\lambda}$$

Soliton solution



Vilenkin, Phys. Rept.121 (1985) 263

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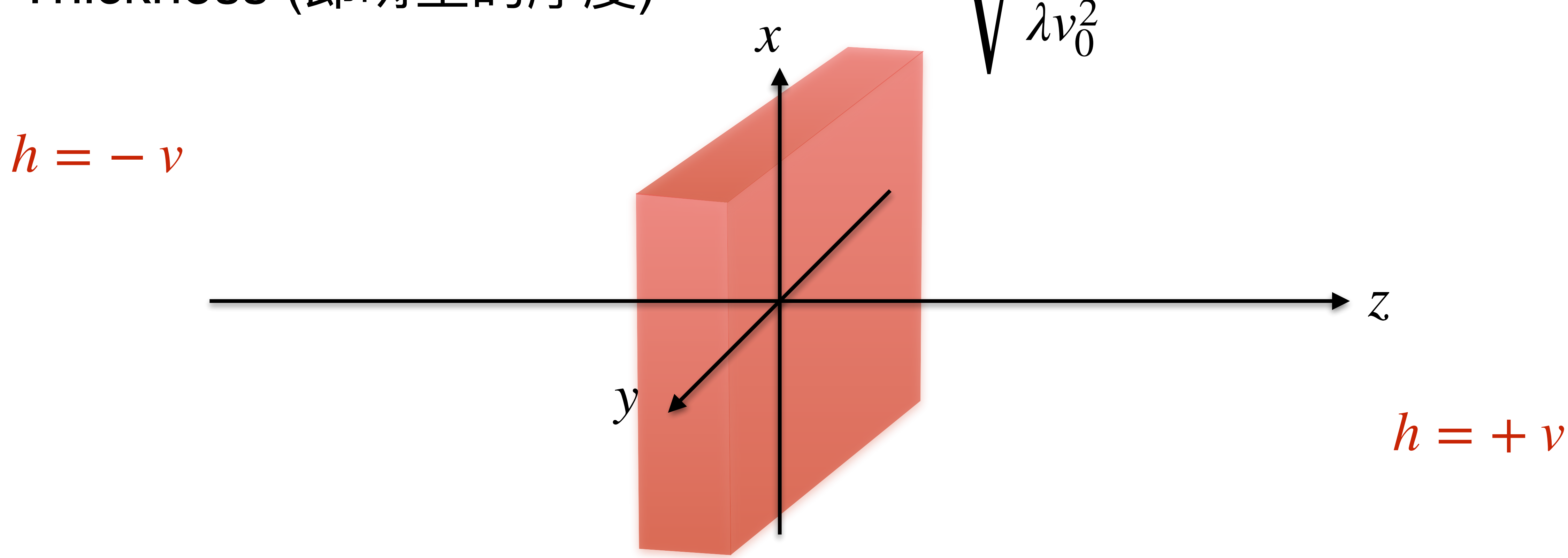
Tension (即表面能量密度) $$\sigma = \frac{4}{3}\sqrt{\frac{\lambda}{2}} v_0^3$$ $$\sigma = \int_{-\infty}^{+\infty} \varepsilon(z) dz$$

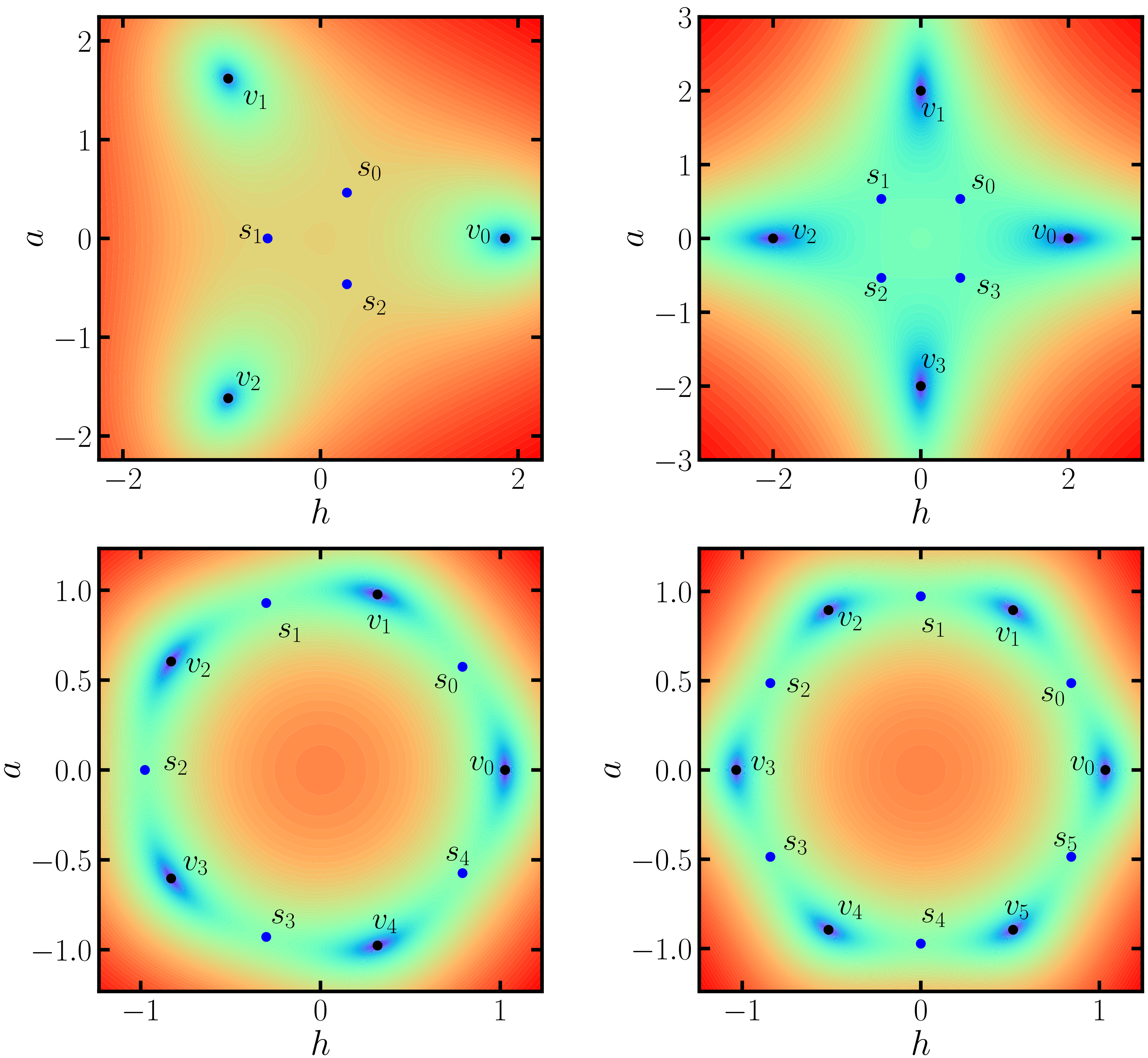
Thickness (即畴壁的厚度) $$\delta = \sqrt{\frac{2}{\lambda v_0^2}}$$



Vilenkin, Phys. Rept.121 (1985) 263

How about walls beyond Z_2?

Z_N-invariant potential $V = -\mu^2|\phi|^2 + \lambda_1|\phi|^4 - \lambda_2\mu^{4-N}(\phi^N + \phi^{*N})$



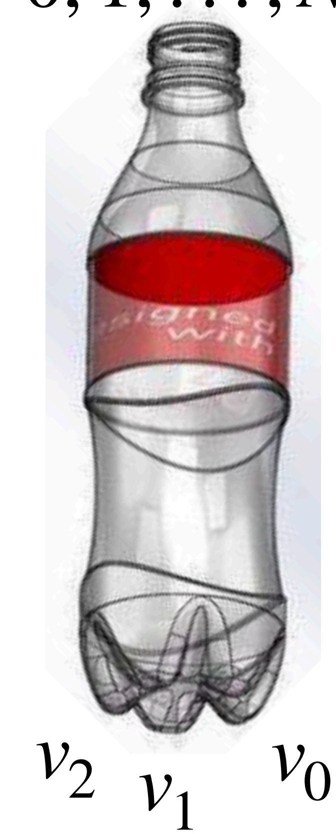
$$\phi = \frac{1}{\sqrt{2}}(h + ia)$$

(assuming CP conservation, simplest form)

N degenerate vacua:

$$v_k = ve^{i2\pi k/N}$$

$$k = 0, 1, \ldots, N-1$$

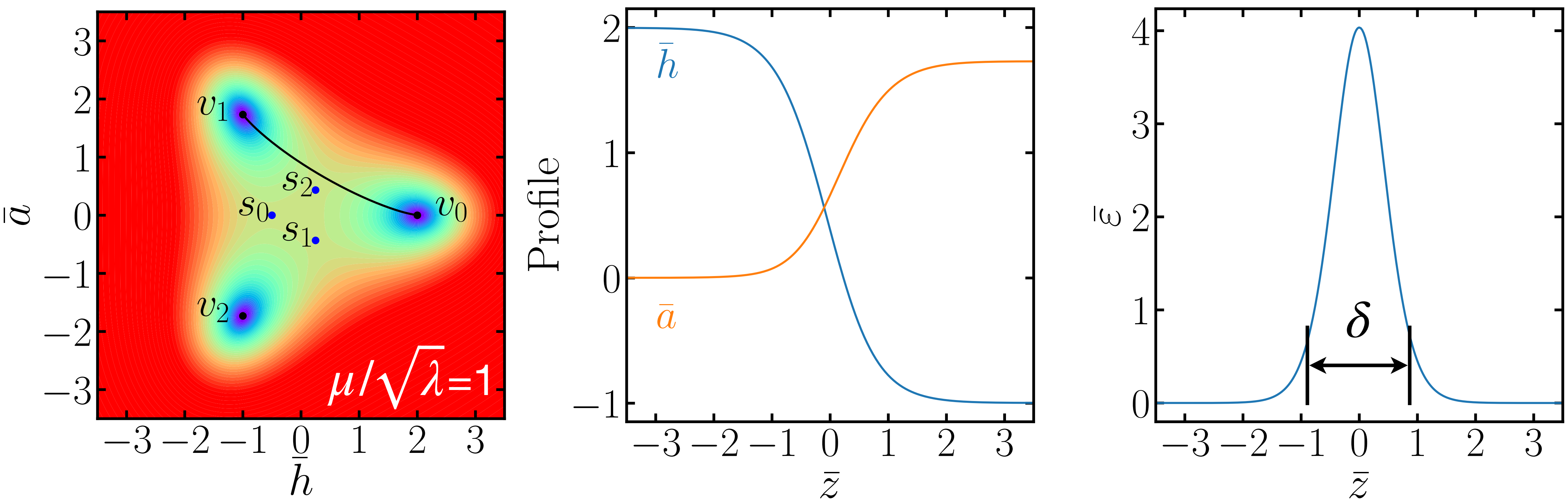


Z_3 walls

Z_3-invariant potential $\quad V = -\mu^2|\phi|^2 + \lambda_1|\phi|^4 - \lambda_2\mu(\phi^3 + \phi^{*3})$

3 vacua $\quad v_k = \frac{\mu}{\sqrt{2\lambda_1}}(\beta + \sqrt{1+\beta^2})\, e^{i2\pi k/3} \qquad \beta = 3\lambda_2/\sqrt{8\lambda_1} > 0$

$\beta = 3/4$



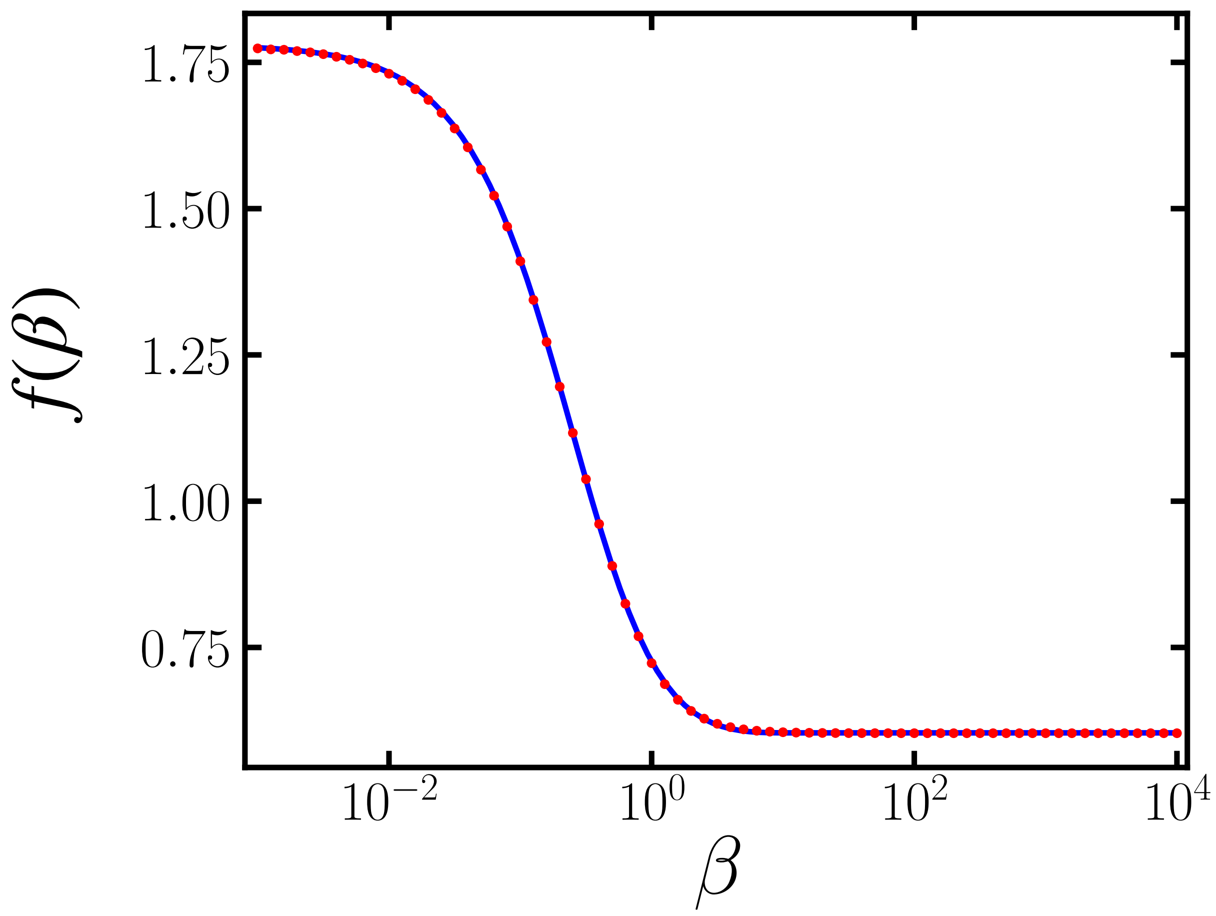
$$\varepsilon(z) = \frac{1}{2}\{[h'(z)]^2 + [a'(z)]^2\} + \Delta V(z)$$

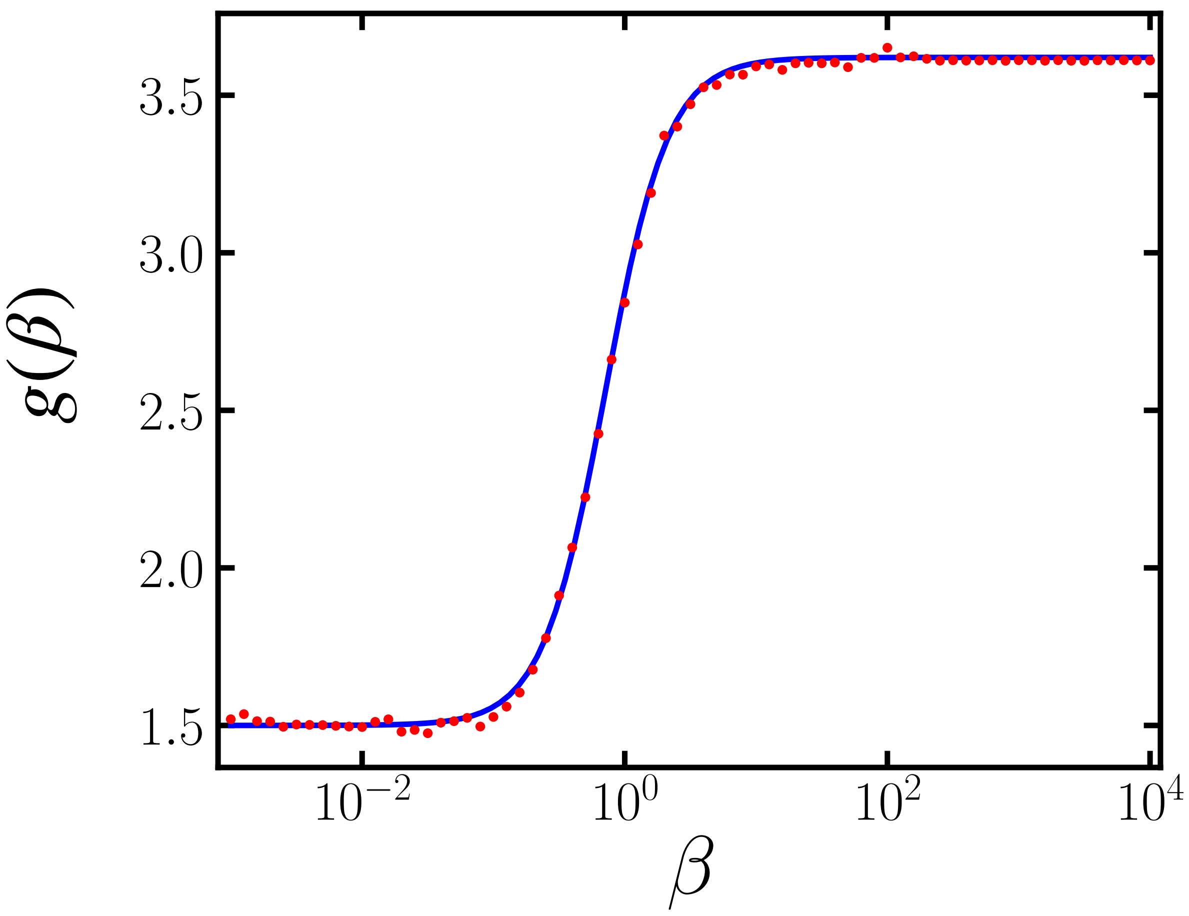
$$\Longrightarrow \qquad \sigma = \int_{-\infty}^{+\infty} \varepsilon(z)dz \qquad \int_{-\delta/2}^{\delta/2} dz\varepsilon(z) = 64\% \times \sigma\,.$$

Z_3 walls

畴壁的表面能 $\sigma = m_a v_0^2 f(\beta)$

畴壁的厚度 $\delta = m_a^{-1} g(\beta)$





$$f(\beta) = 0.604 + \frac{0.234}{e^{0.826\beta} + 0.435\beta^2 - 0.801}$$

$$g(\beta) = 3.62 - \frac{2.12}{1 + 1.85\beta^{1.81}}$$

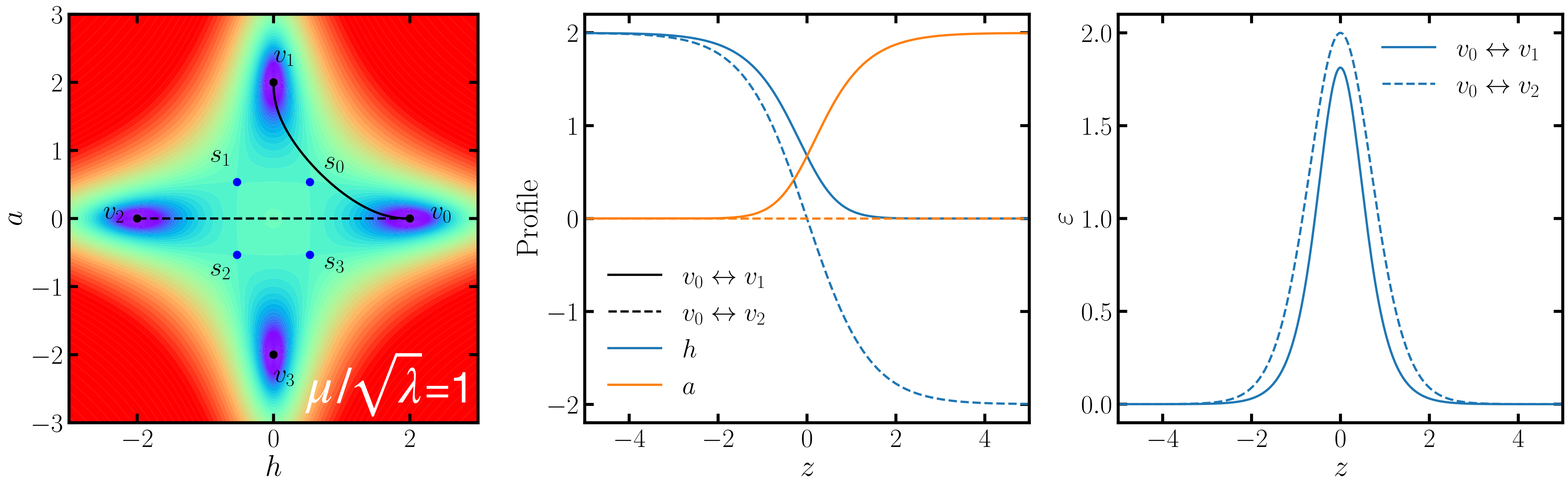
m_a 是 PGB 的质量

Z_4 walls

Z_4-invariant potential $V = -\mu^2|\phi|^2 + \lambda_1|\phi|^4 - \lambda_2(\phi^4 + \phi^{*4})$

$$v_k = \frac{\mu}{\sqrt{2\lambda_1(1-\beta)}} e^{i\frac{2\pi}{4}k} \qquad \beta \equiv 2\lambda_2/\lambda_1$$

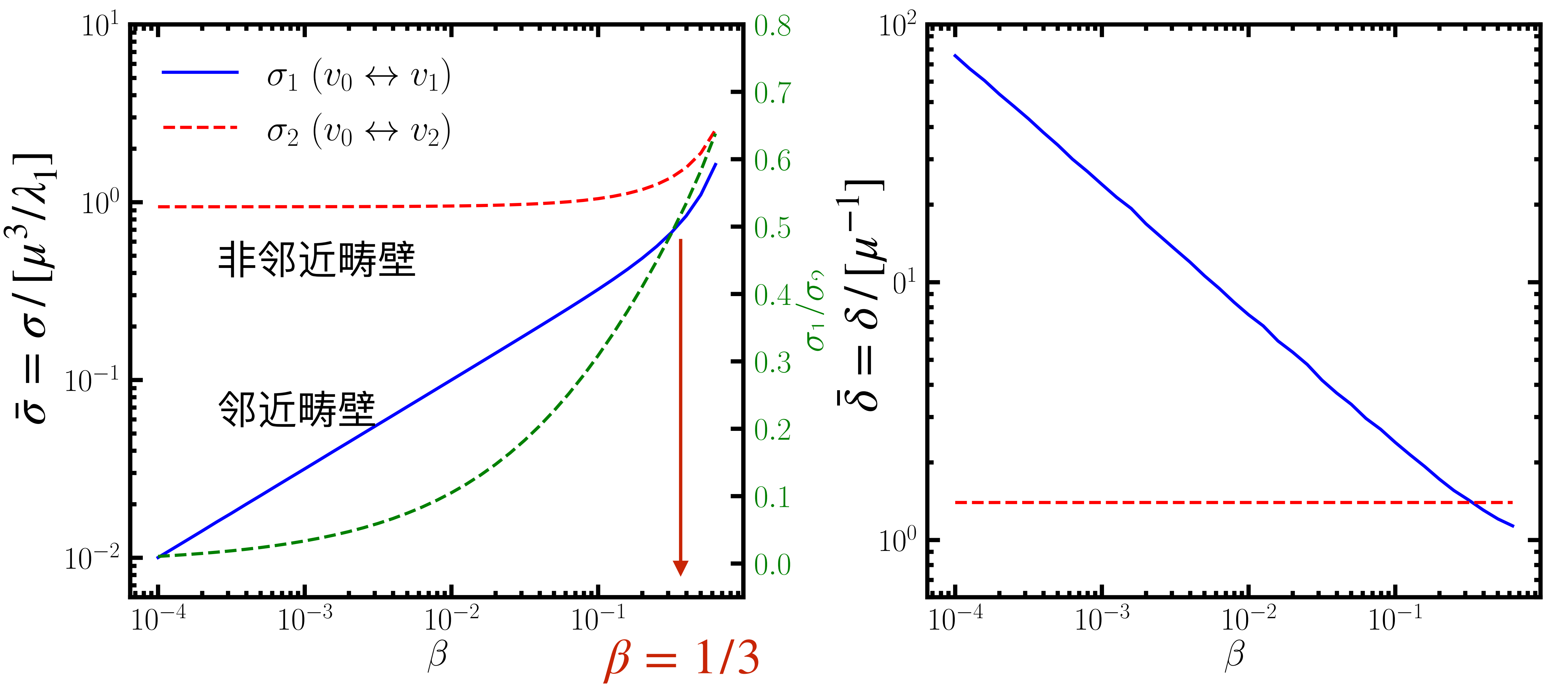
$\beta = 3/4$



Adjacent walls (邻近畴壁): 分隔邻近真空(比如 v_0 和 v_1)的畴壁

Non-adjacent walls (非邻近畴壁): 分隔非邻近真空(比如 v_0 和 v_2)的畴壁

Z_4 walls

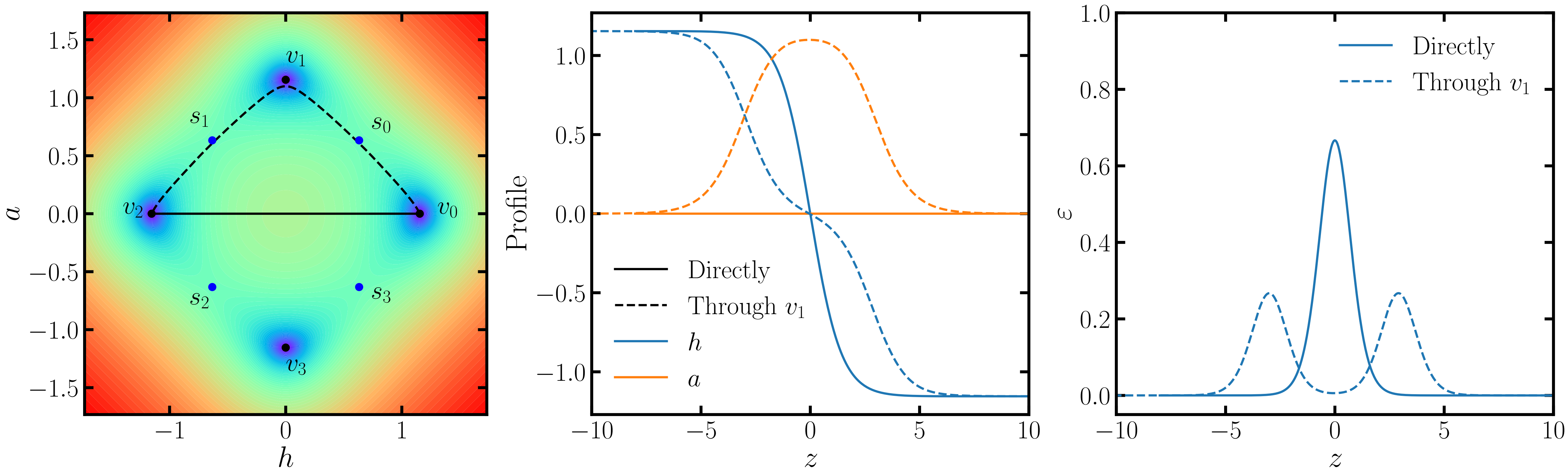


$\beta = 1/3$

$$\bar{\sigma}_{\text{non-adj}} = 2\,\bar{\sigma}_{\text{adj}} = \sqrt{2}$$

Z_4 walls

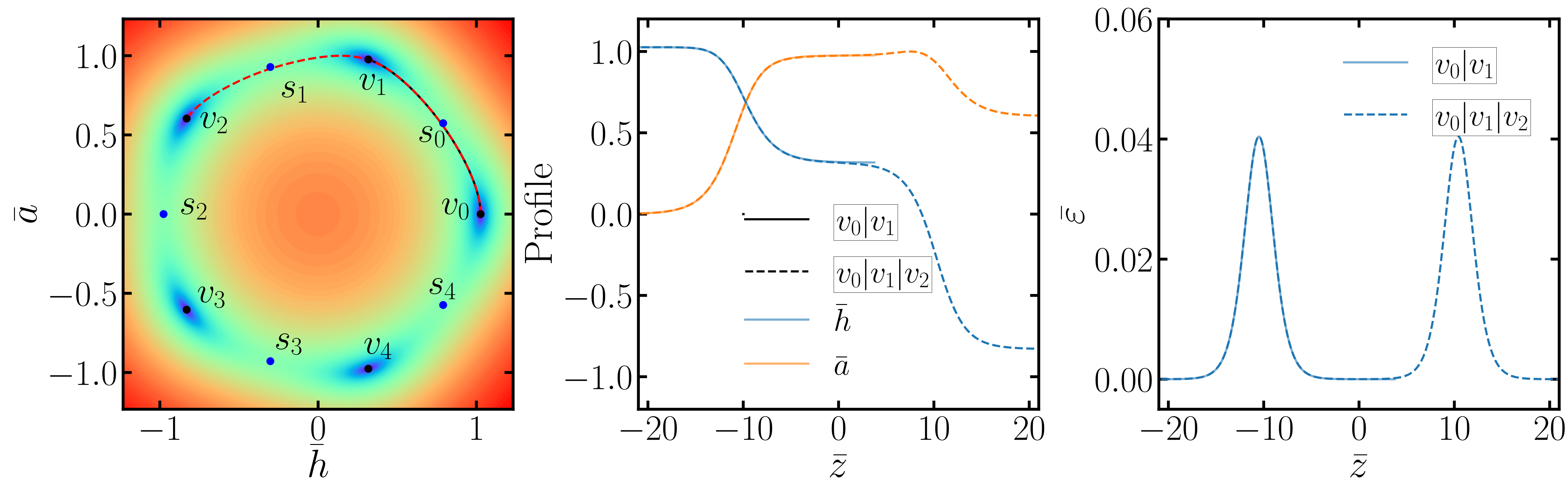
$\beta = 1/4$



当 $\beta < 1/3$ 时，$\sigma_2 > 2\sigma_1$，非邻近畴壁不稳定，会衰变为两个邻近畴壁

$$\boxed{v_0|v_2} \to \boxed{v_0|v_1|v_2} \to \boxed{v_0|v_1} + \boxed{v_1|v_2}$$

Z_N walls with small Z_N effects



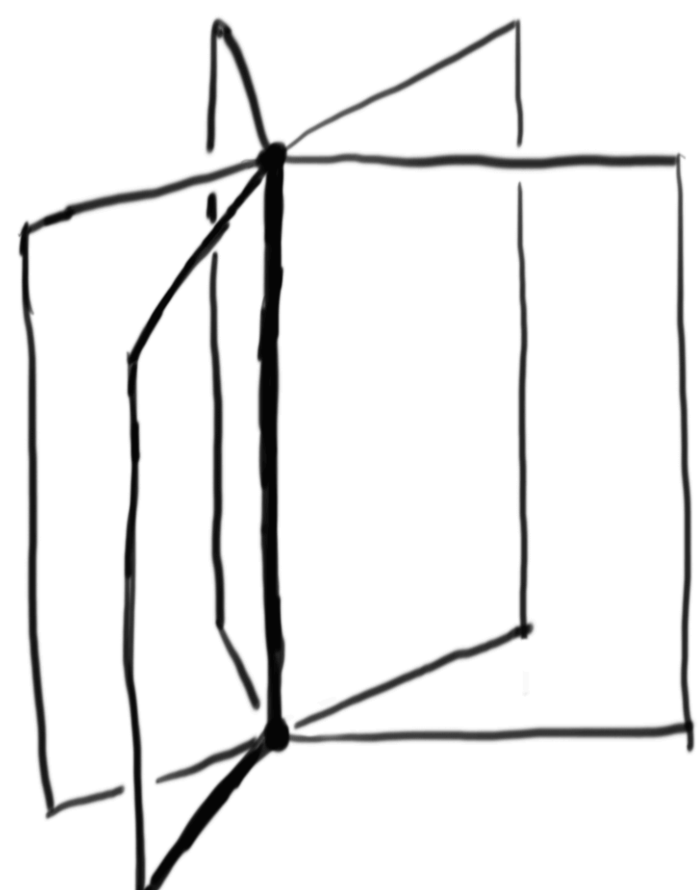
Approx U(1) $\phi = |\phi| e^{i\theta}$

$\downarrow$ $\langle|\phi|\rangle \approx v$

Z_N

$\downarrow$ $\langle\theta\rangle = 2\pi k/N$

1



String bounded by walls

其性质与类轴子模型中出现的畴壁相似。动力学演化可以直接参考后者的数值模拟结果，比如 Hiramatsu, Kawasaki, Saikawa, Sekiguchi, 1207.3166; 1412.0789

Z_N walls with multiscalars

e.g., Z_6-invariant potential with two scalars

$$V = -\mu^2 |\phi|^2 + \lambda_1 |\phi|^4$$
$$-\frac{1}{2}\mu_\xi^2 \xi^2 + \frac{1}{4}\lambda_\xi \xi^4$$
$$-\lambda_{\phi\xi}(\phi^3 + \phi^{*3})\xi$$

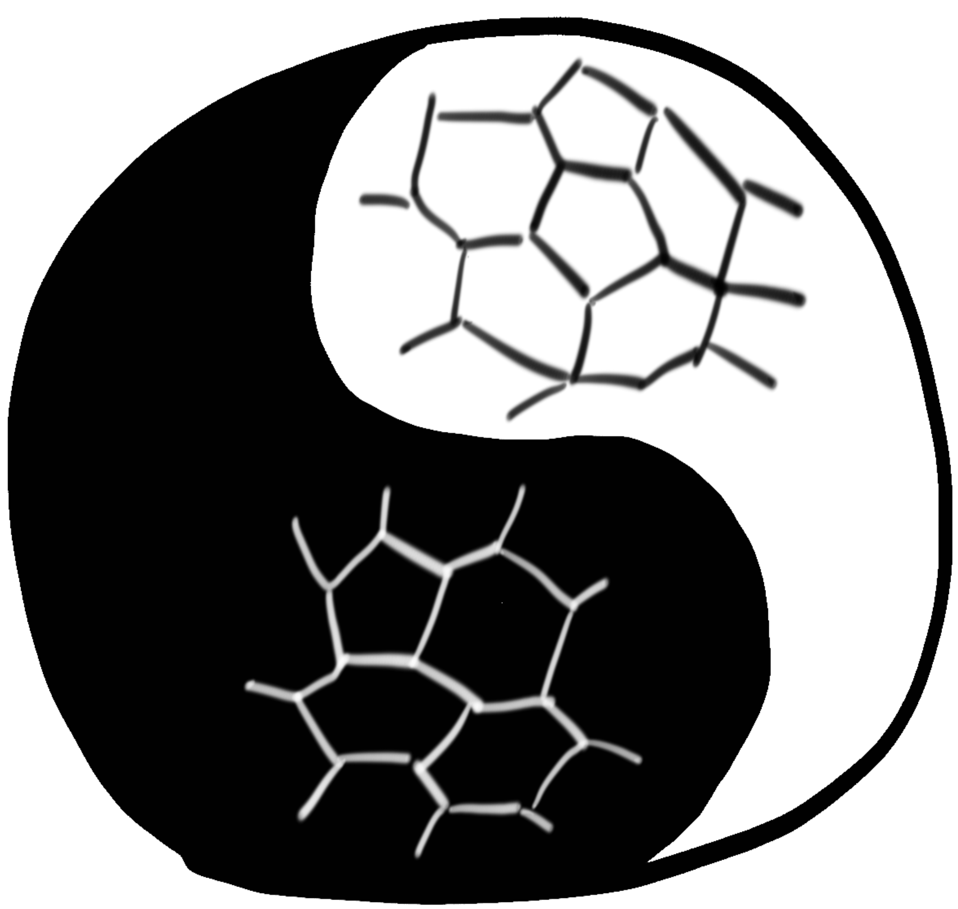
Z_6

↓ $\langle \xi \rangle = \pm \sqrt{\mu_\xi^2 / 2\lambda_\xi}$

Z_3

↓ $\langle \phi \rangle = \frac{\mu(\beta + \sqrt{1+\beta^2})}{\sqrt{2\lambda_1}} e^{\pm i 2\pi k/3}$

1



Scalar	ϕ	ξ
Z_6 charge	1	3

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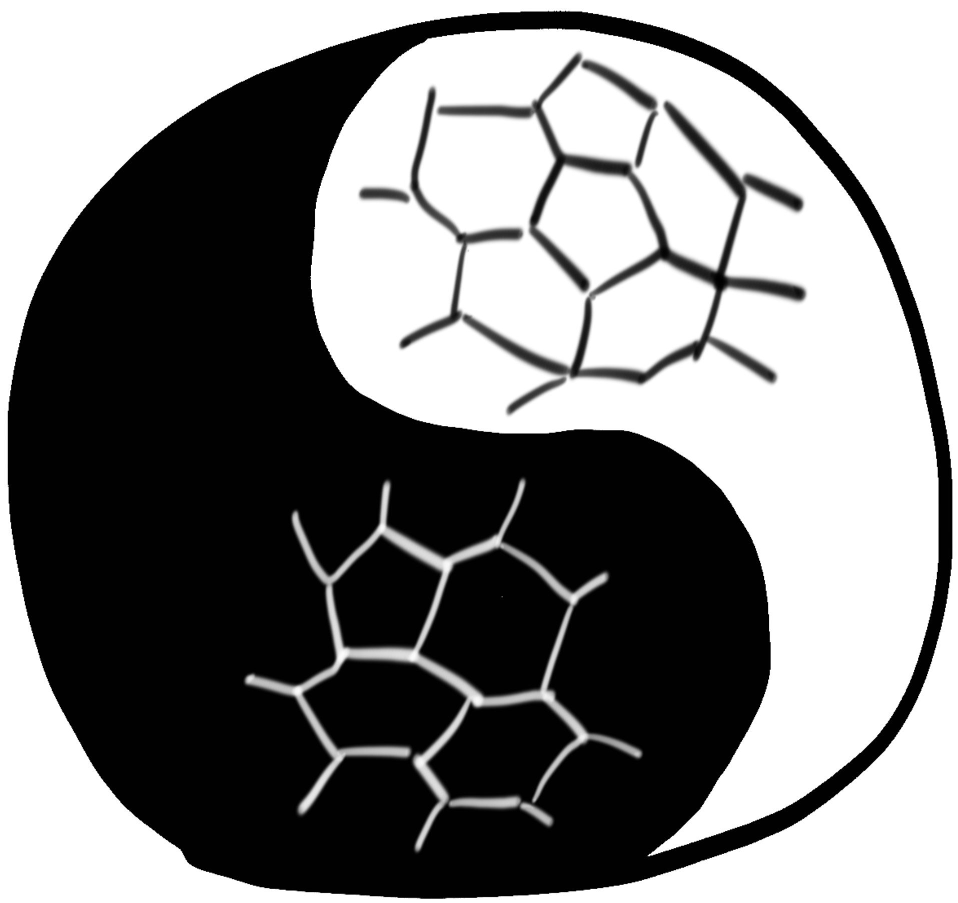

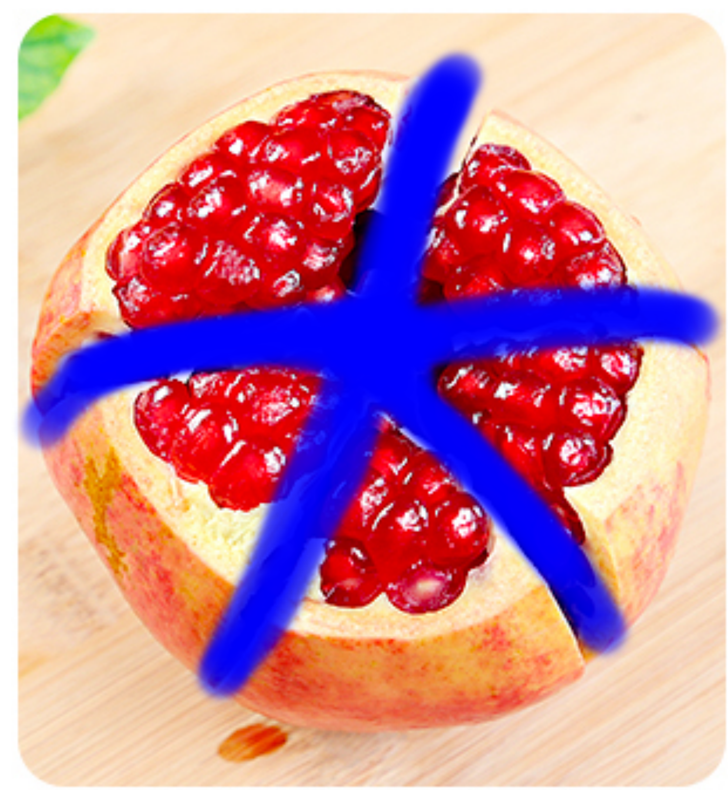

Z_N

$\downarrow$

Z_n

$\downarrow$

1

关于 Z_N domain wall结构的汇总，请参考[2205.11529]中的表一

GW sources in particle theories

- Directly from first-order phase transition during spontaneous symmetry breaking (SSB) of underlying symmetries

宇宙原初量子涨落在相变引力波各向异性中的遗迹-黄发朋(中山大学，天琴中心)
15:20 - 15:40

Dynamics of electroweak phase transitions: from nucleation to percolation-蒋赟(中山大学，天琴中心)
16:15 - 16:35

GWs from FOPTs: Recent progress on bubble expansion-Shaojiang Wang(中科院理论物理研究所)
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Sound velocity effects on the phase transition gravitational wave spectrum-Wang Xiao(中山大学)
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- Oscillation of cosmic strings formed after SSB of symmetries

格点规范场论模拟早期宇宙相变-边立功(重庆大学)
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- Domain walls can also generates GWs

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Sound velocity effects on the phase transition gravit

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格点规范场论模拟早期宇宙相变-边立功(重庆大学)

- Domain walls can also gener

How?

Domain wall-induced cosmological problem

- Scaling solution: $\rho_{\rm DW} \sim \sigma H$

Vilenkin, Phys. Rept.121 (1985) 263

$$\Longrightarrow \quad \frac{\rho_{\rm DW}}{\rho_c} \sim \frac{\sigma G}{H} \sim \frac{\lambda^{1/2} v^3}{M_{\rm pl} T^2} \qquad \rho_c = \frac{3H^2}{8\pi G}$$

在辐射为主时期，温度降低会迅速导致$\rho_{\rm DW} \gg \rho_c$

- 解决办法：引入Explicit breaking
- Z_2: $\delta V = \epsilon v h \left(\frac{1}{3}h^2 - v^2\right)$

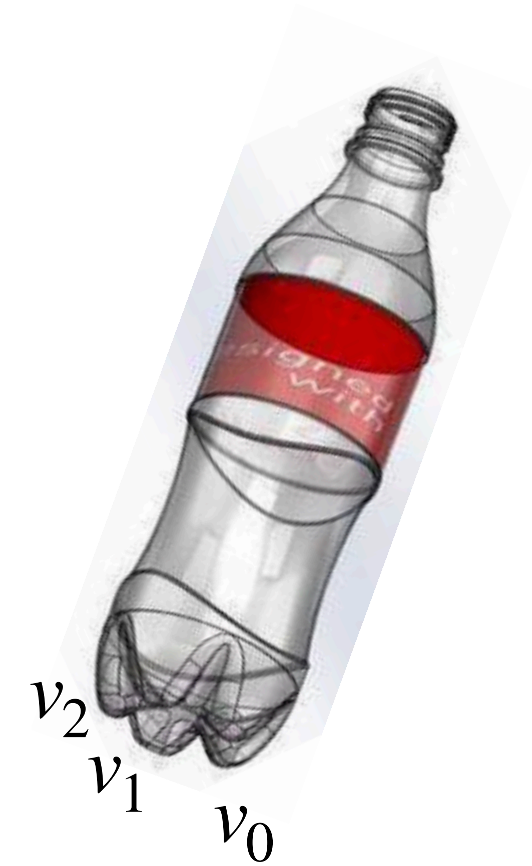
Hiramatsu, Kawasaki, Saikawa, 1002.1555

- Z_3: $\delta V = \frac{2e^{i\alpha}}{3\sqrt{3}} \epsilon \phi \left(\frac{1}{4}\phi^3 - v_0^3\right) + {\rm h.c.}$

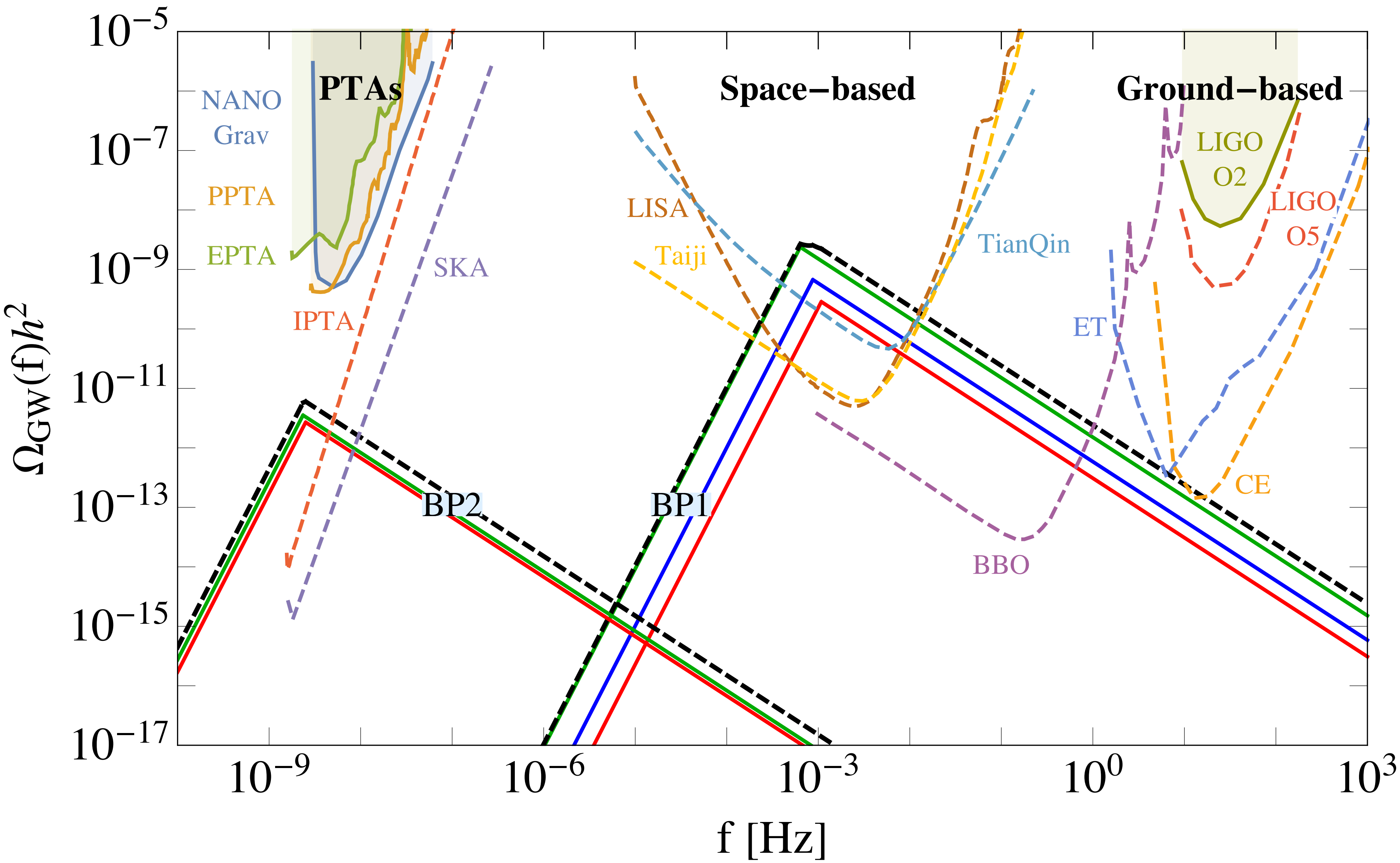
$$(V_{\rm bias})_{10} = V|_{v_1} - V|_{v_0} = \epsilon v_0^4 \cos\left(\alpha + \frac{\pi}{6}\right)$$

$$(V_{\rm bias})_{20} = V|_{v_2} - V|_{v_0} = \epsilon v_0^4 \cos\left(\alpha - \frac{\pi}{6}\right)$$

Wu, Xie, YLZ，2204.04374



随机引力波的预言

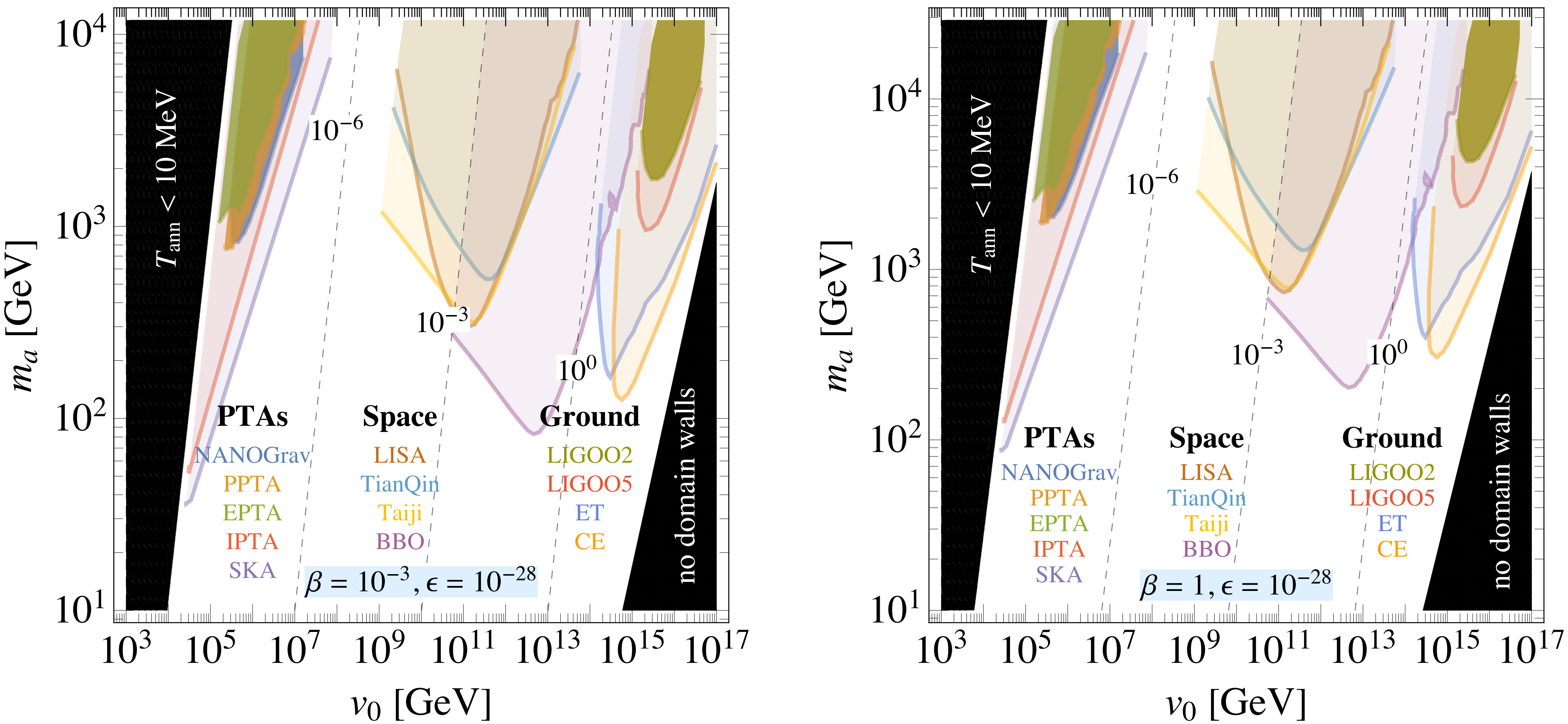


$$\text{BP1}: v_0 = 10^{11}\ \text{GeV}, \quad m_a = 2\ \text{TeV}, \quad \alpha = \frac{2\pi}{9};$$

$$\text{BP2}: v_0 = 10^{5}\ \text{GeV}, \quad m_a = 500\ \text{GeV}, \quad \alpha = \frac{\pi}{27}.$$

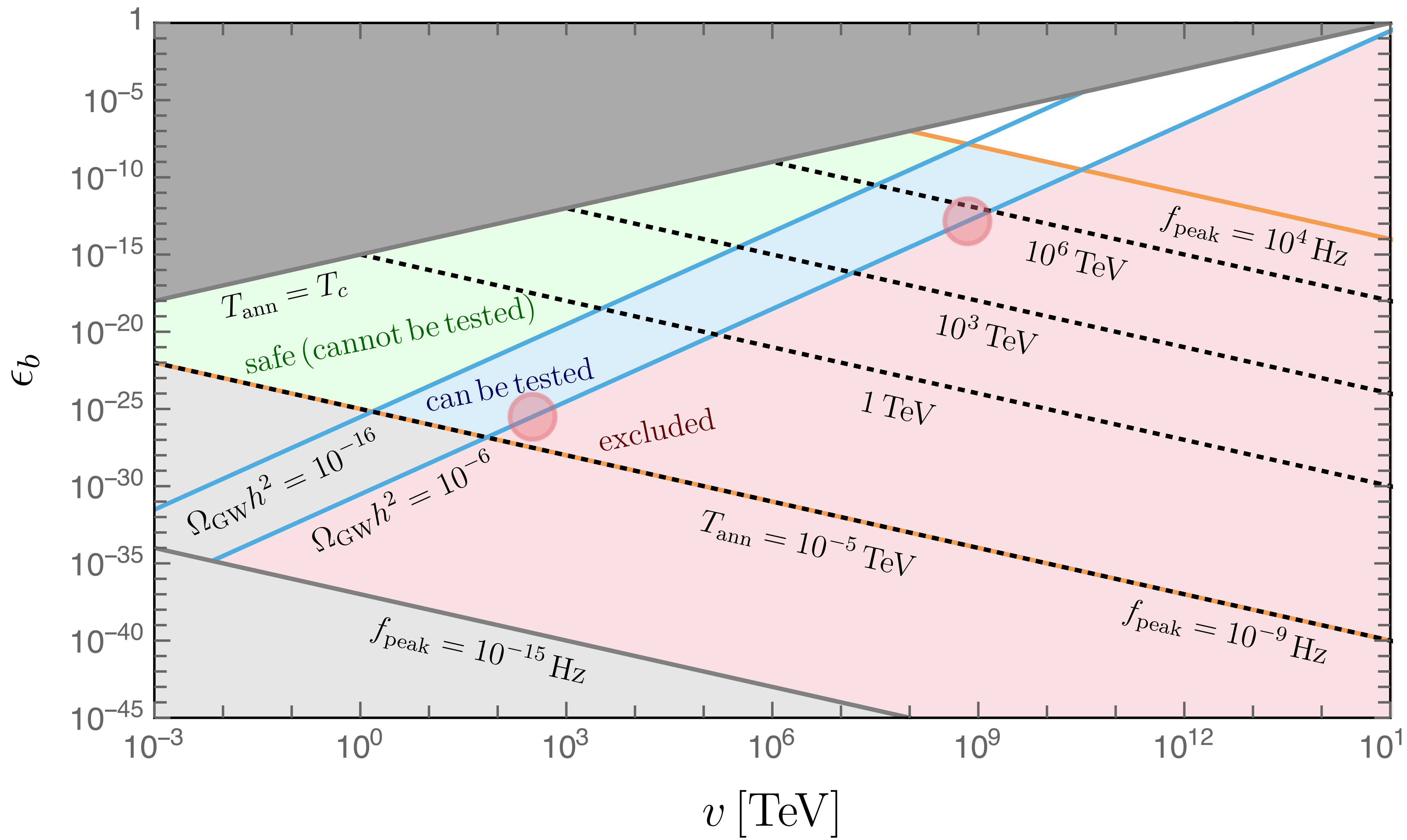
畴壁引力波并没有很好的模拟结果可以用，我们采用Saikawa [1703.02576]中所论述的幂律谱

引力波观测对“Z_3对称性自发破缺”的检验能力



- 定量方面，Z_3导致的畴壁引力波与Z_2引力波不同。
- $\beta \ll 1$时，等效于类轴子模型中的畴壁引力波的结果；但$\beta \gtrsim 1$时不等效。
- 上述结论还很粗略，忽略了畴壁的动力学演化。

拓展讨论：对非Abel分立对称性的检验



畴壁引力波对A_4味对称性的检验：Gelmini, Pascoli, Vitagliano, YLZ, 2009.01903


感谢倾听！