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# Sound velocity effects on the phase transition gravitational wave spectrum

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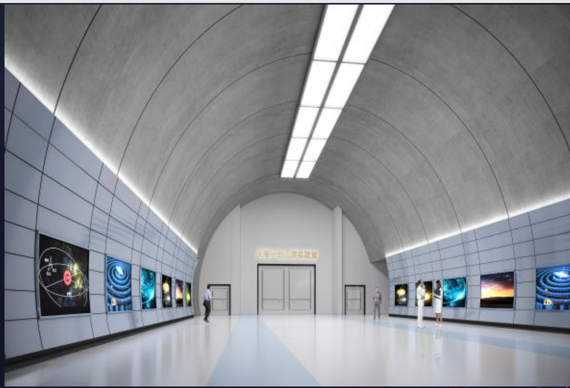
In collaboration with Fa Peng Huang, Yongping Li



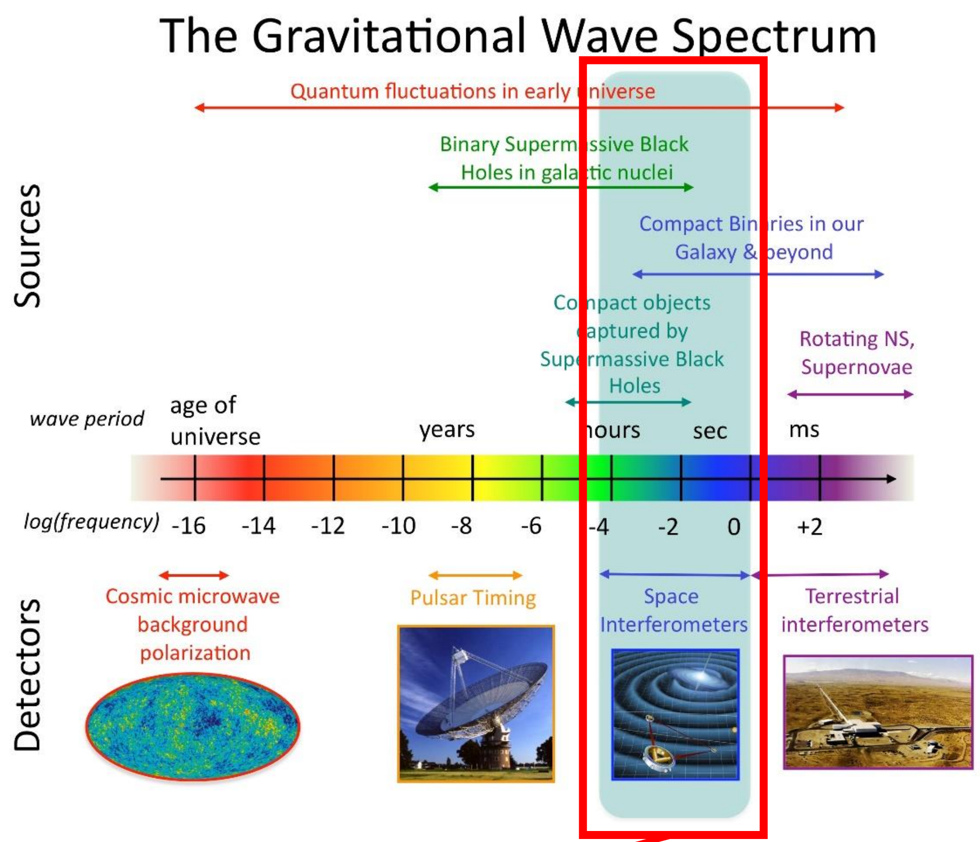
# Contents:

# Introduction

After the observation of the gravitational wave by LIGO, the GW detector provides a new technique to study the fundamental physics.

**Sources of GWs:**

- *Astrophysical origin* : black hole, neutron star, etc.
- *Cosmological origin*: inflation, **FOPT, etc.**




**EW FOPT**

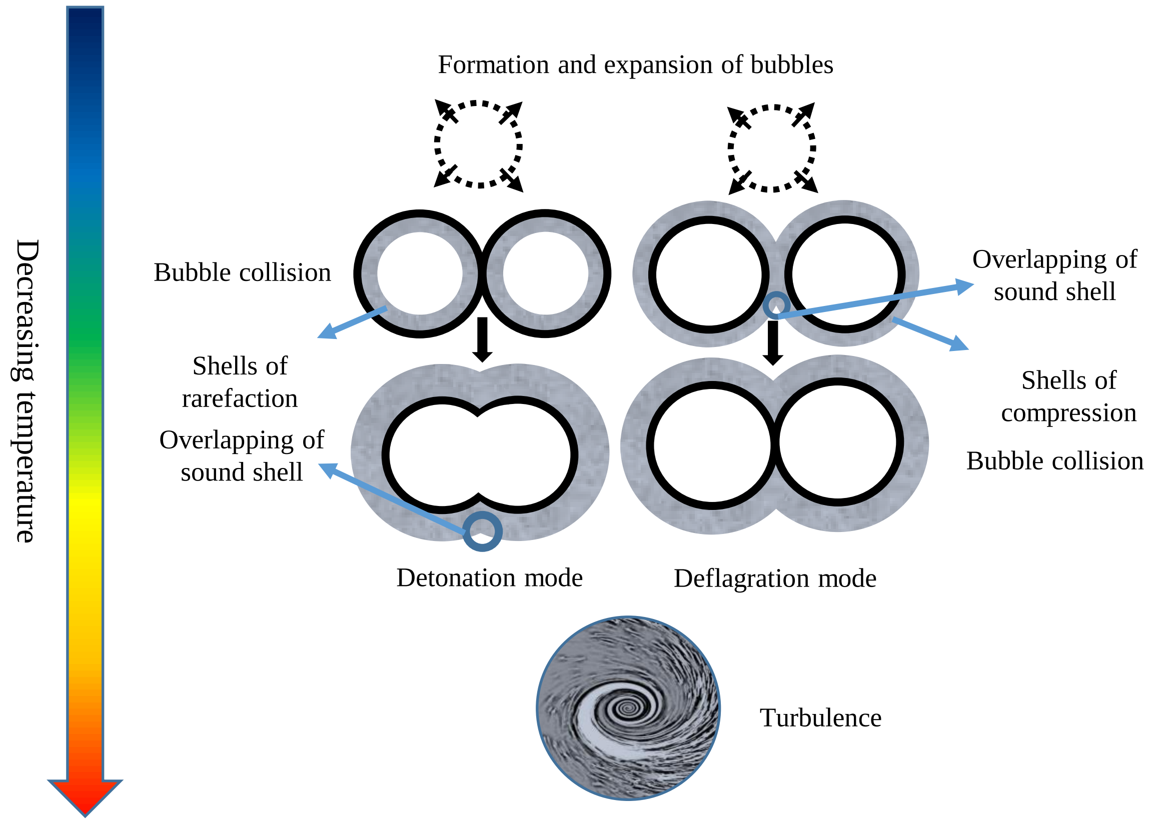
# Introduction

**GWs sources of FOPT:**

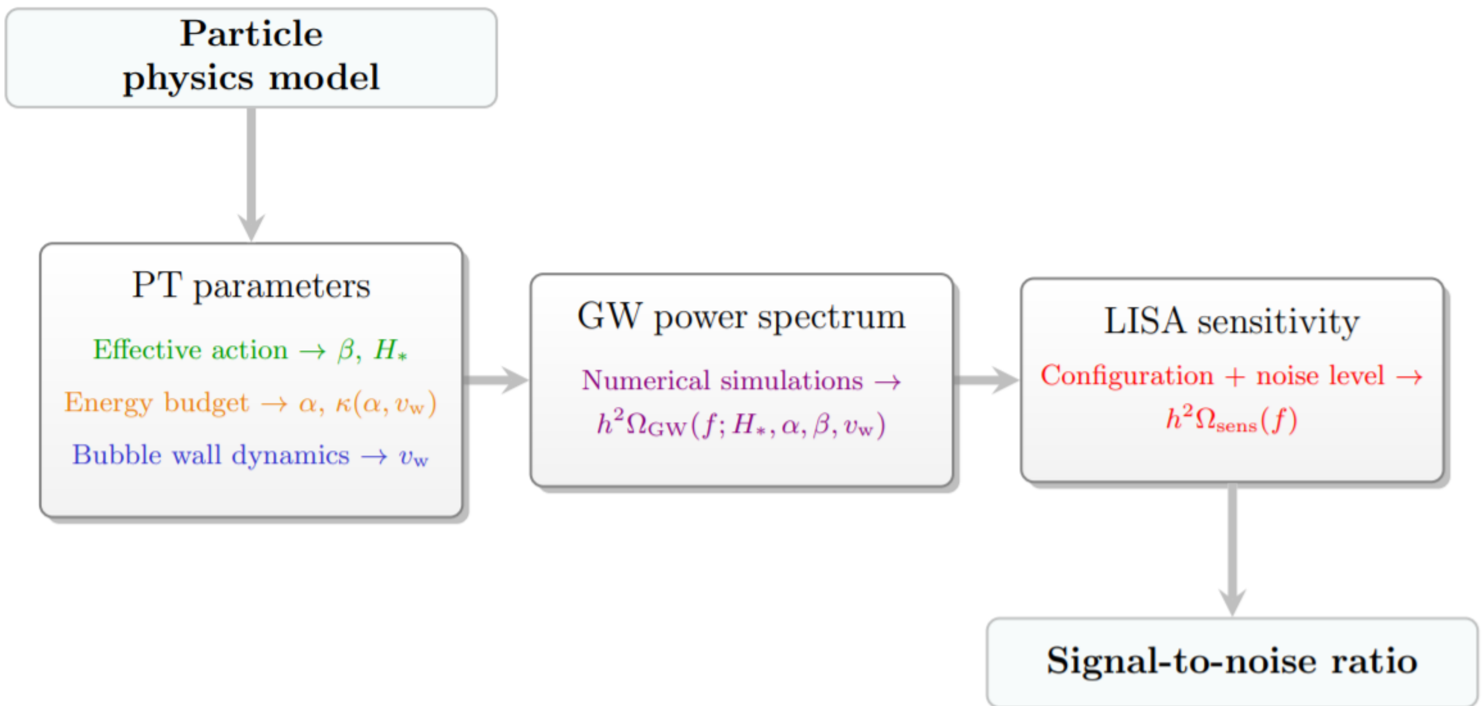
- Bubble collisions
- Sound wave
- Turbulence

*JCAP 0809, 022 (2008); PRL112, 041301 (2014); PRD92, no. 12,123009 (2015); PRD96, no. 10,103520 (2017); Phys. Rev. D 66, 024030 (2002), Phys. Rev. D 76 (2007) 083002, JCAP 0912, 024 (2009)*

The amplitude and shape of GW spectrum are strongly related to PT dynamics.



# Introduction




Chiara Caprini *et al*, JCAP03(2020)024

***To extract the information of physics BSM from phase transition gravitational wave, we need improve the accuracy of theoretical calculation of PTGW:***

# Introduction

**Hydrodynamical simulation:**

- ✓ Require a simulation volume large enough to fit a sizable number of bubbles
- ✓ The grid spacing must be small enough to resolve the Higgs bubble wall thickness

Advantages:

- Less priori assumptions
- More robust predictions

Shortcomings:

- numerically expensive / time consuming

**Analytical (semi-analytical) approach:**
(***Sound shell model.***)

- ✓ Modeling of the relevant sources
- ✓ Some basic assumptions

Advantages:

- Better physics insight
- Less cost

Shortcomings:

- Robustness of assumptions and modeling

# Introduction

**Common features of both Numerical and analytical method**

Initial conditions (input for the calculation of PTGW):

- The fluid velocity and energy distribution inside the fluid shell---fluid profile

**Numerical method**

- Directly solve and evolve the partial differential equations to derive the steady fluid profile for the fluid-order-parameter-field coupled system.

$$T^{\mu\nu}_{;\mu}(\text{field}) = \phi^{;\mu}_{;\mu}\phi^{,\nu} - \frac{\partial V}{\partial \phi}\phi^{,\nu} = \eta u^{\mu}\phi_{,\mu}\phi^{,\nu},$$

$$T^{\mu\nu}_{;\mu}(\text{fluid}) = (w u^{\mu} u^{\nu})_{;\mu} + p^{,\nu} + \frac{\partial V}{\partial \phi}\phi^{,\nu} = -\eta u^{\mu}\phi_{,\mu}\phi^{,\nu},$$

**Sound shell model**

- Construct the steady fluid profile or the self-similar solution just with a specific EoS and fluid equation.

$$(\xi - v)\frac{\partial_{\xi} e}{w} = j\frac{v}{\xi} + \gamma^2(1 - v\xi)\partial_{\xi} v,$$

$$(1 - v\xi)\frac{\partial_{\xi} p}{w} = \gamma^2(\xi - v)\partial_{\xi} v.$$

Sound velocity effect

# Sound Shell Model

## General Remarks of GW power spectrum

GW equations

$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G \Pi_{ij}.$$

$$h_{ij}(\mathbf{k}, t) = (16\pi G)\Lambda_{ij,kl}(\mathbf{k}) \int_0^t dt' \frac{\sin[k(t-t')]}{k} T_{kl}(\mathbf{k}, t')$$

Spectral density

$$\langle \dot{h}_{ij}(\mathbf{k}, t) \dot{h}^*_{ij}(\mathbf{k}', t) \rangle = P_{\dot{h}}(\mathbf{k}) \delta^3(\mathbf{k} - \mathbf{k}').$$

GW power spectrum

$$\mathcal{P}_{\rm gw}(k) \equiv \frac{d\Omega_{\rm gw}}{d\ln(k)} = \frac{1}{\rho_{\rm crit}} \frac{1}{32\pi G} \mathcal{P}_{\dot{h}}(k) = \frac{1}{12H^2} \mathcal{P}_{\dot{h}}(k) \qquad \mathcal{P}_{\dot{h}} = \frac{k^3}{2\pi^2} P_{\dot{h}}(k);$$

$$P_{\dot{h}}(k, t) = (16\pi G)^2 \frac{1}{2} \int_0^t dt_1 dt_2 \cos[k(t_1 - t_2)] U_\Pi(k, t_1, t_2).$$

$$\lambda_{ij,kl}(\mathbf{k}) \langle \tau^{ij}(\mathbf{k}, t_1) \tau^{kl}(\mathbf{k}', t_2) \rangle = U_\Pi(k, t_1, t_2) \delta^3(\mathbf{k} + \mathbf{k}') \longleftarrow \text{UETC}$$

## Sound shell model in nutshell

1. *Derive the self-similar velocity and enthalpy profiles of a single expanding bubble based on the specific equation of sate (EoS).*

2. *Obtain the single-bubble plane wave amplitude from single-bubble self-similar profile. Then, derive the plane wave amplitude correlation function for the velocity field generated by N randomly placed bubbles in a given volume.*

3. *Estimate the bubble lifetime distribution for a specific nucleation history, and derive the velocity spectral density and power spectrum.*

4. *Finally, derive the GW power spectrum using the relation between the shear stress UETC and the velocity spectral density*

# Sound Shell Model

For the sound shell model, the dominant source of shear stress

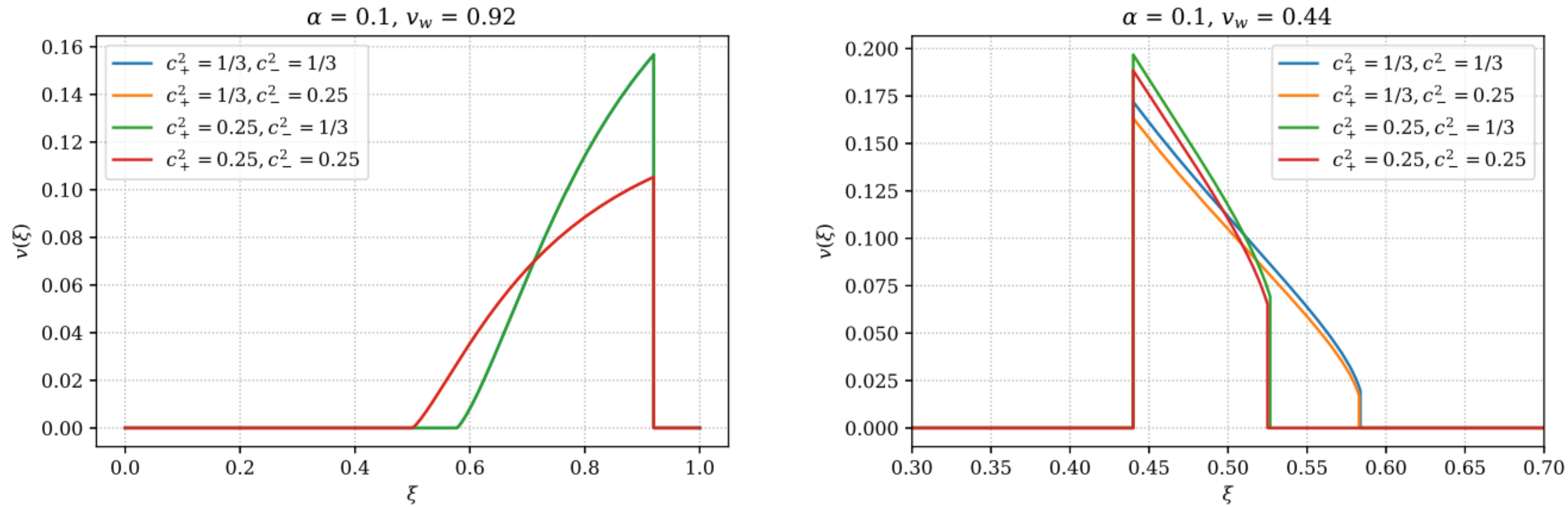
$$\tau_{ij} \simeq \bar{w} v_i v_j .$$

$$\boxed{P_{\dot{h}} \sim U_{\Pi} \sim \langle \tau\tau \rangle \sim \langle \tilde{v}\,\tilde{v}\,\tilde{v}\,\tilde{v} \rangle = \sum \langle \tilde{v}\,\tilde{v} \rangle \langle \tilde{v}\,\tilde{v} \rangle \sim \sum P_v P_v .}$$

the velocity field is the superposition of self-similar velocity profiles of N randomly placed bubbles

$$v_i(\mathbf{x}, t) = \sum_{n=1}^{N} v_i^n(\mathbf{x}, t), \qquad v_i^n(\mathbf{x}, t) = \frac{R_i^n}{R^n} v(\xi)$$

# Sound Shell Model




*The velocity profiles of the deflagration mode and detonation mode for the DSVM of EoS. The left column is detonation, and the right column is deflagration. Different colors represent different combinations of sound velocities in symmetric and broken phases.*

# Sound Shell Model

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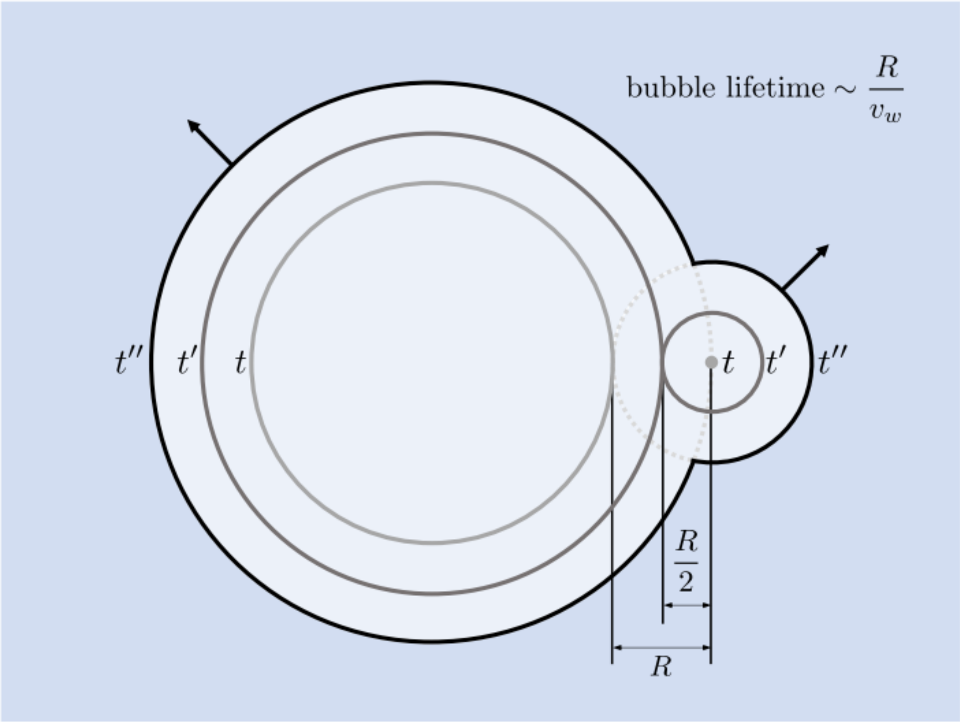
$$P_v(q) = \frac{1}{\beta^6 R_*^3} \int d\tilde{T} f_{\rm col}(\tilde{T}) \tilde{T}^6 \left| A\left(\frac{\tilde{T} q}{\beta}\right) \right|^2 ,$$

Velocity spectral density of SSM

$$\mathcal{P}_{\rm gw}(k) = 3(\tilde{\Gamma}\bar{U}_{\rm f}^2)^2 (H\tau_v)(HL_{\rm f}) \frac{(kL_{\rm f})^3}{2\pi^2} \tilde{P}_{\rm gw}(kL_f),$$

GW spectrum of SSM

# Sound Shell Model




Bubble life time distribution:

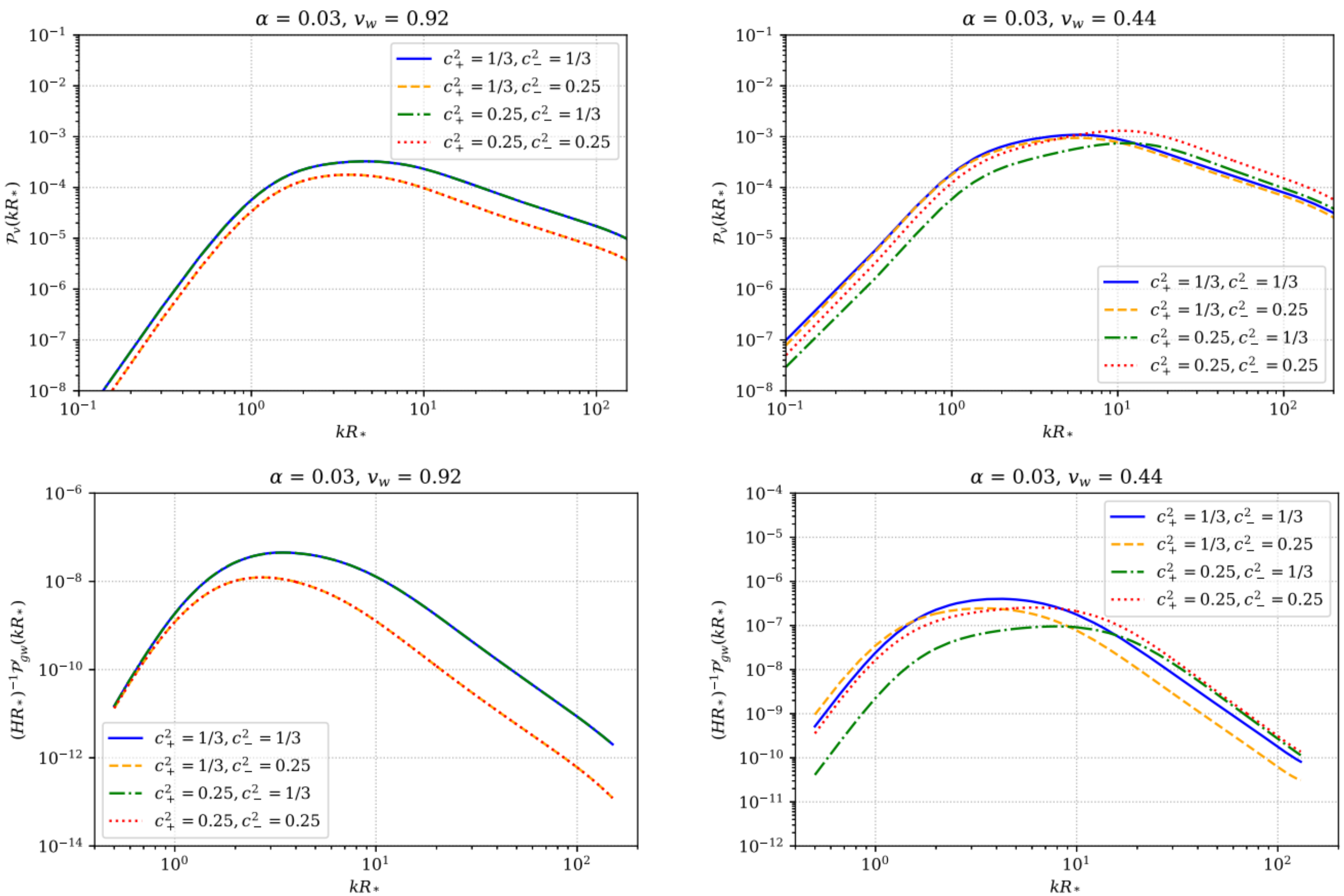
- nucleation rate
- bubble wall velocity
- the fractional volume remaining in the metastable phase
- the area per unit volume of the phase boundary

$$\frac{dn_b}{dR} = \int_{t_c}^{t'} \mathcal{A}(t + R/v_w)\Gamma(t)dt,$$

$$f_{\rm col}(\tilde{T}) = v_w \frac{R_*^3}{\beta}\frac{dn_b}{dR} \qquad f_{\rm col}(\tilde{T}) = e^{-\tilde{T}}.$$

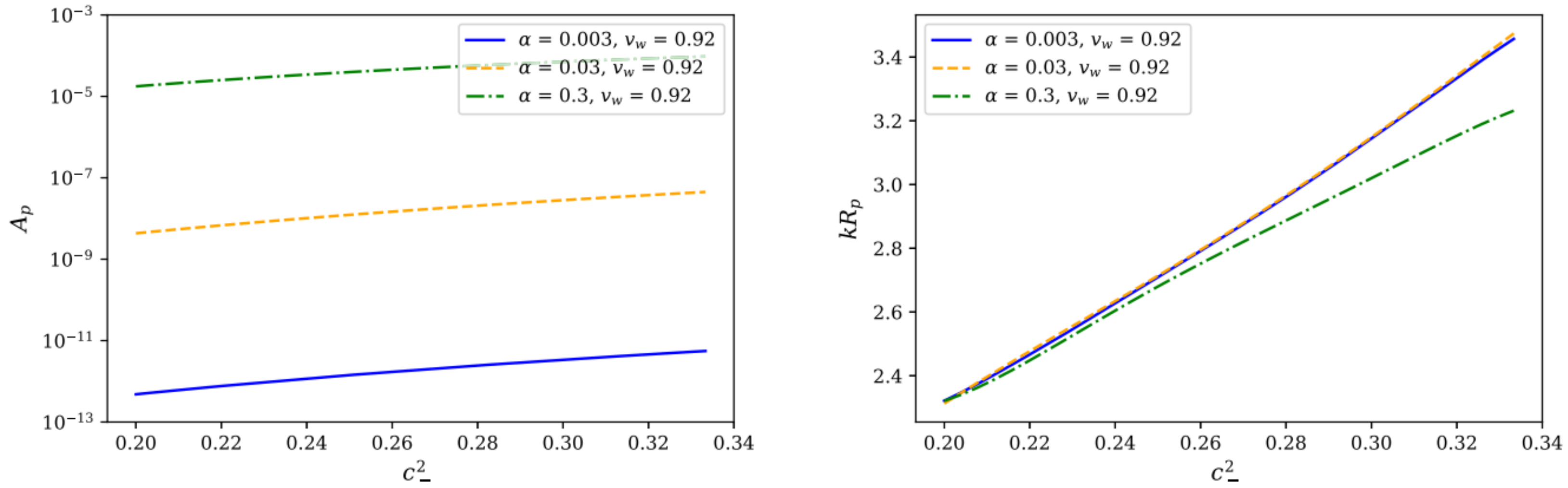
*Schematic diagram for the bubble lifetime. At time $t$, there is a bubble just formed at the light gray dot and an expanding bubble depicted by the light gray circle. The distance between the light gray dot and the wall of expanding bubble is $R$. At $t'$, the two bubbles firstly collide and start to merge, and $t' - t = R/(2v_w)$. At $t''$, half of the small bubble has merged with the large bubble, and we define that the small bubble disappears. Hence, the lifetime of the small bubble is $R/v_w$.*

# Gravitational wave



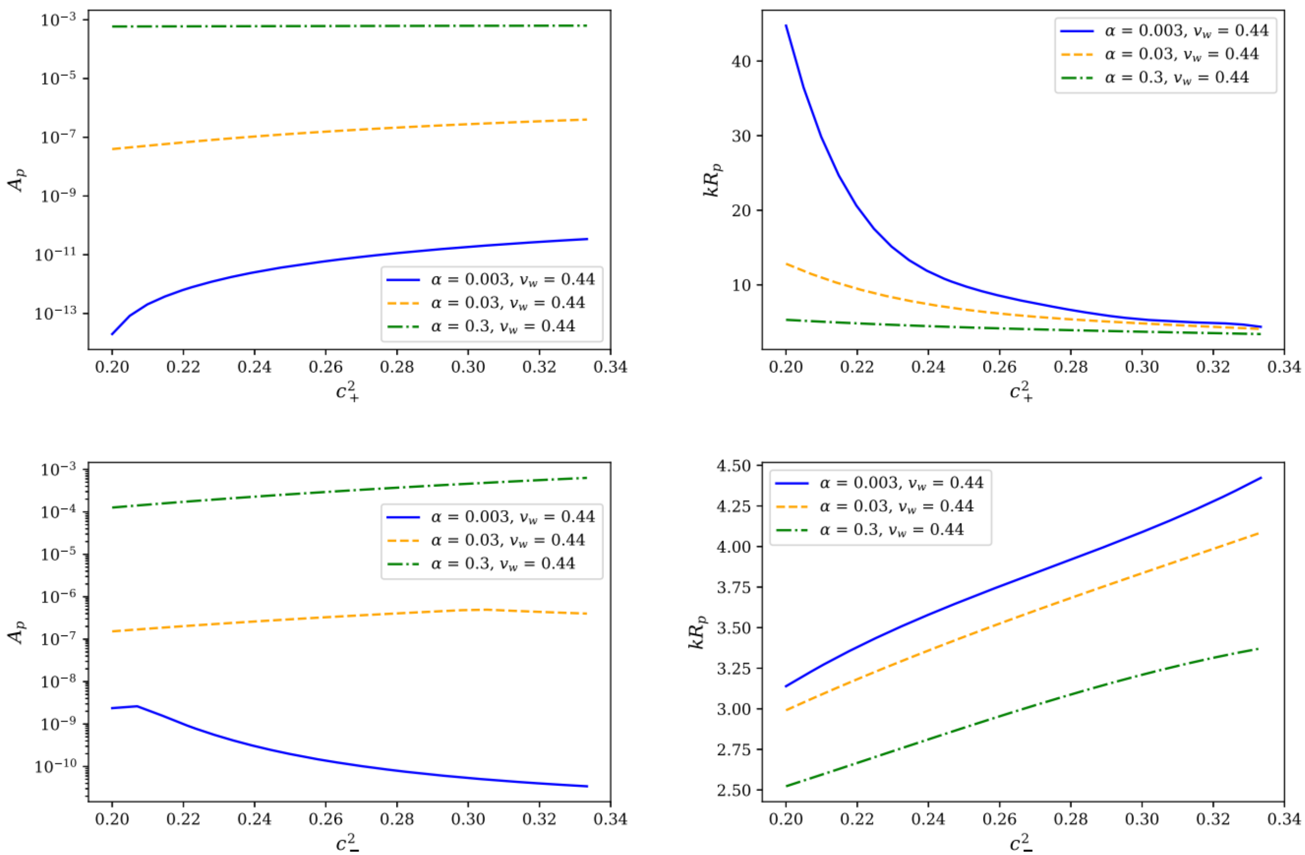

*The velocity and GW power spectrum at the production time*

# Gravitational wave




*Peak amplitude (left panel) and peak angular frequency (right panel) of scaled GW spectrum as a function of sound velocities for detonation.*

# Gravitational wave




*Peak amplitude (left column) and peak angular frequency (right column) as a function of sound velocities for deflagration*

# Conclusions

- The different sound velocity could obviously modify the peak frequency and peak amplitude of the gravitational wave power spectra; The sound velocity can give at most 63% correction to the peak amplitude and 15% correction to the peak frequency;
- Modeling of bubble life time deserve a further study;
- The validity of linear superposition of velocity field should be checked carefully, the coherent case is in progress.

Thank you!!!