

# GWs from FOPTs: Recent progress on bubble expansion

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Based on

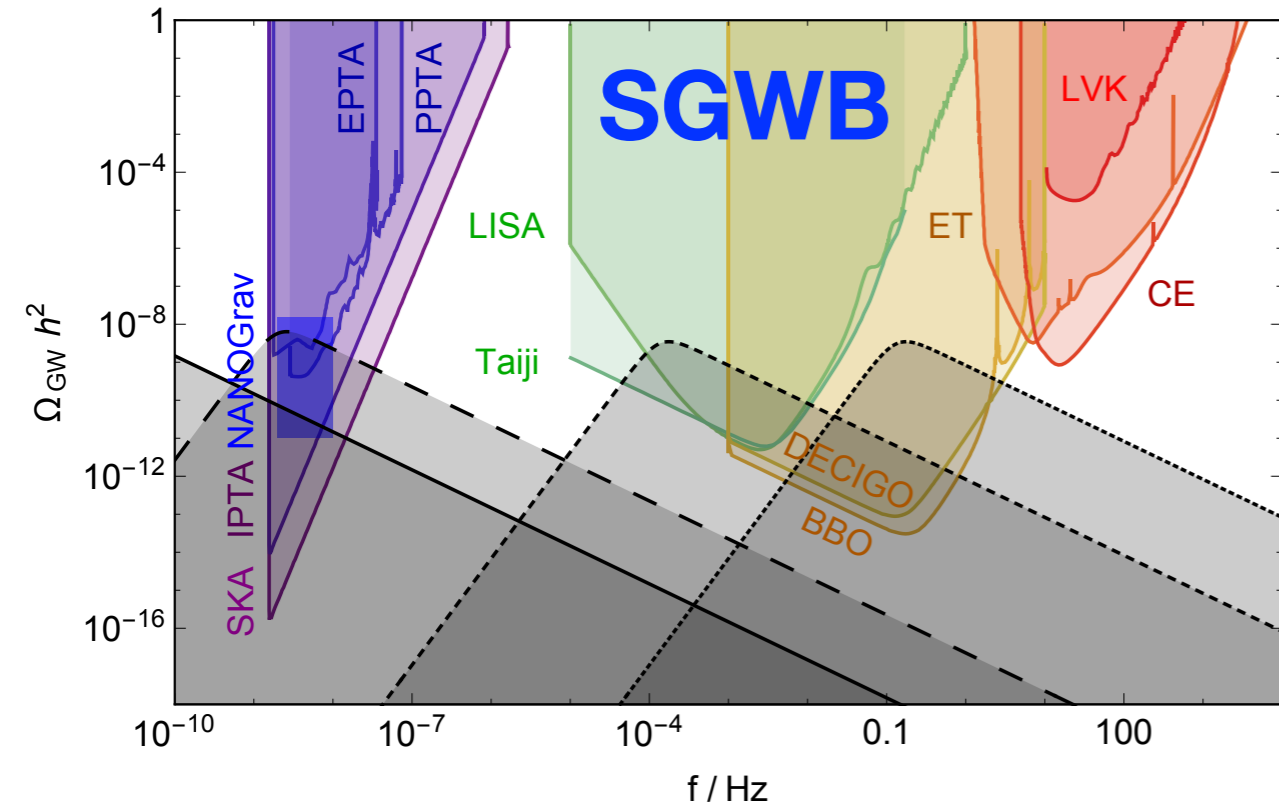
**2011.11451** Rong-Gen Cai, SJW, **Effective picture of bubble expansion**

**JCAP03 (2021)096**

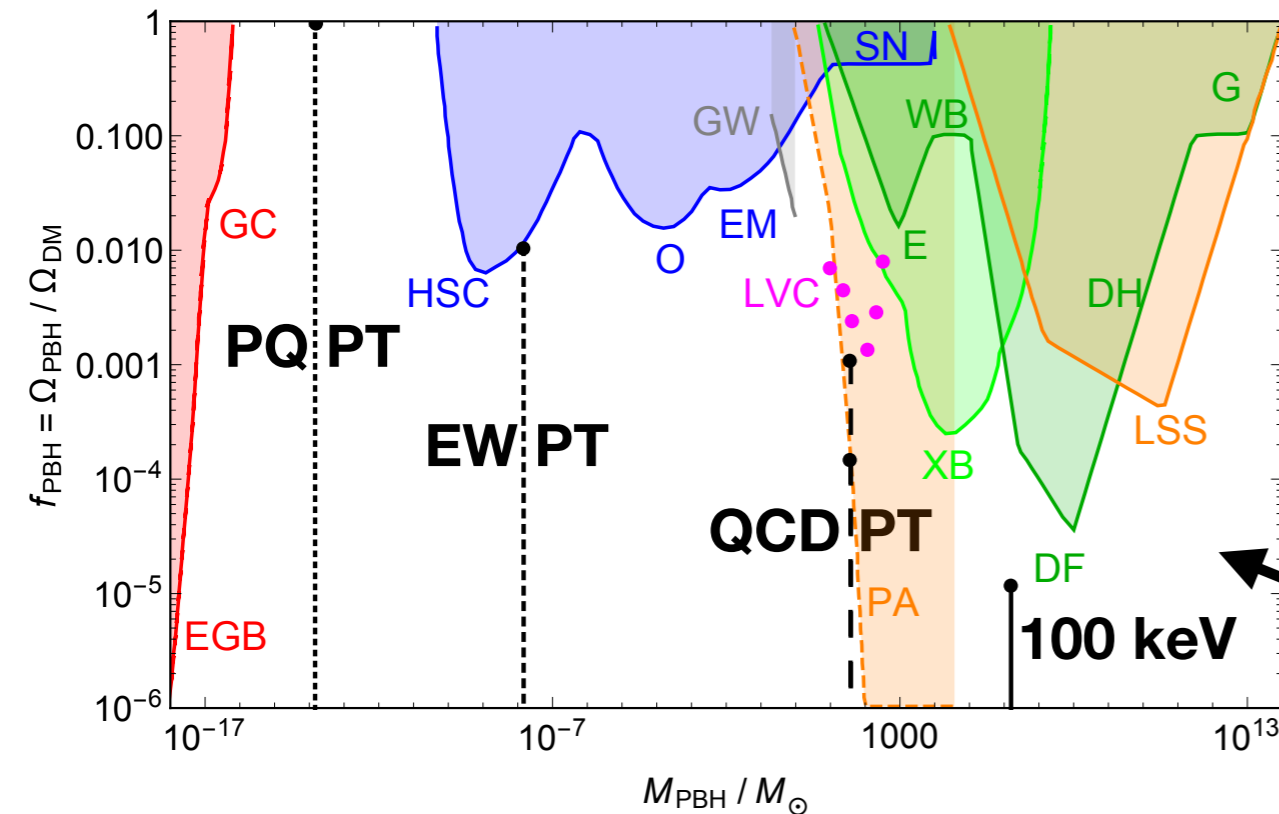
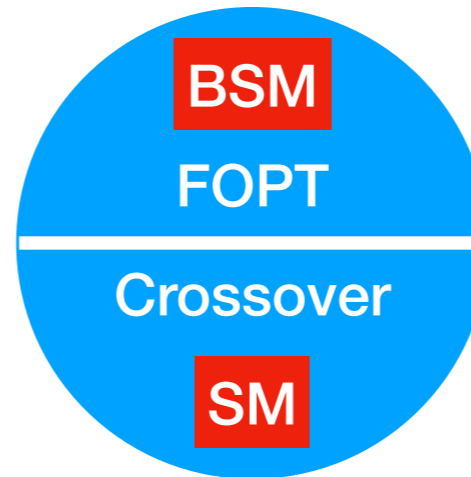
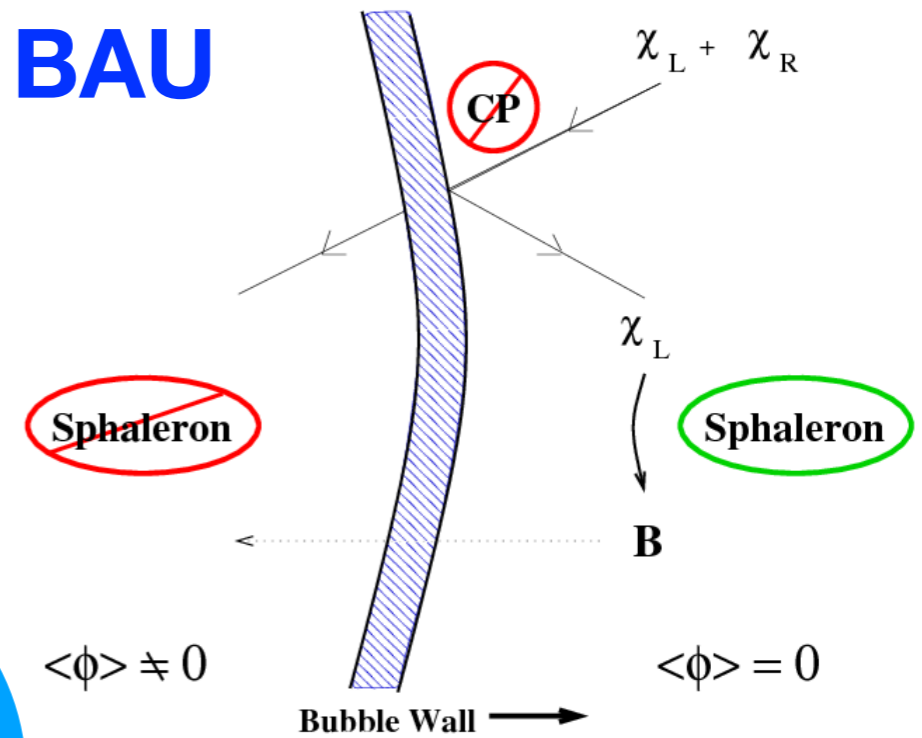
**2205.02492** SJW, Zi-Yan Yuwen **Hydrodynamic backreaction force of cosmological bubble expansion**

**2206.01148** SJW, Zi-Yan Yuwen **The energy budget of cosmological first-order phase transitions beyond the bag equation of state**

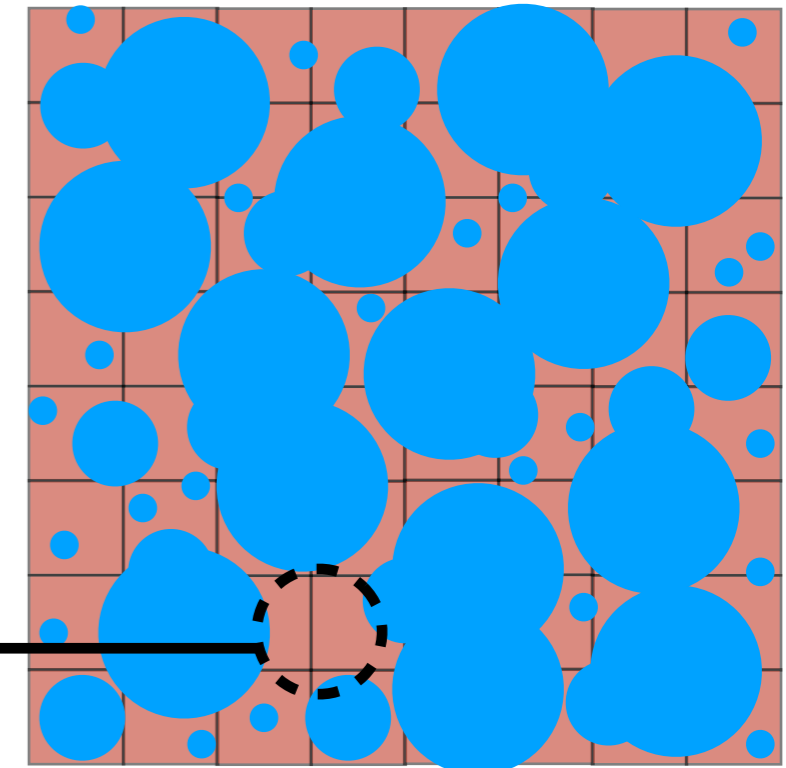
# FOPT Probe for BSM New Physics



**BAU**

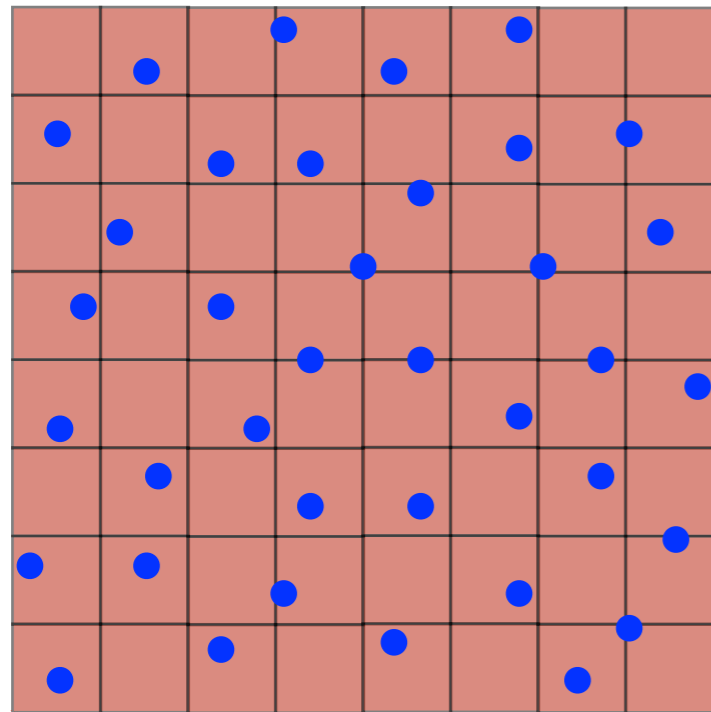


**PBH**



**2106.05637** Primordial black hole production during first-order phase transitions (PRD Letter)

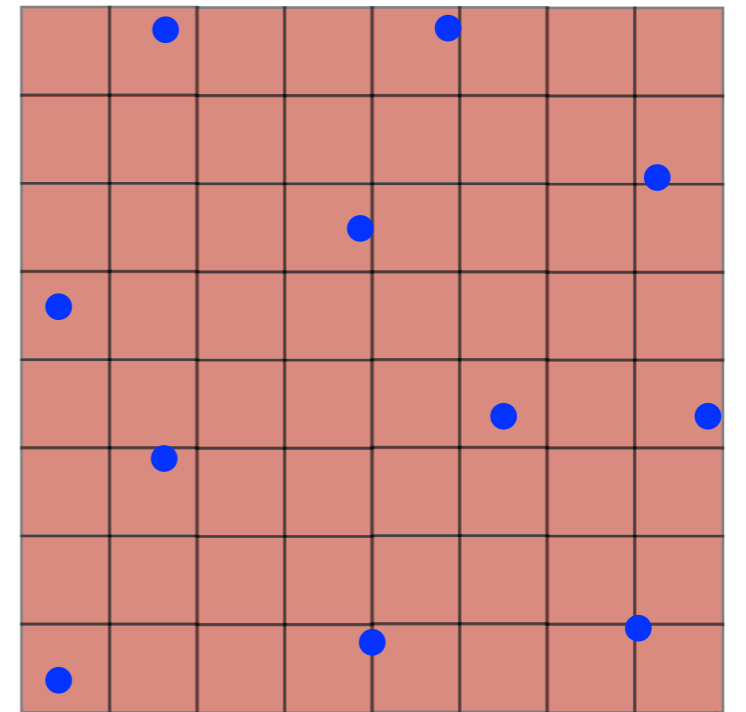
# GWs from FOPT



**Bubble nucleation**

← **large  $\frac{\beta}{H}$  small  $R_*$**

**large  $R_*$  small  $\frac{\beta}{H}$**  →



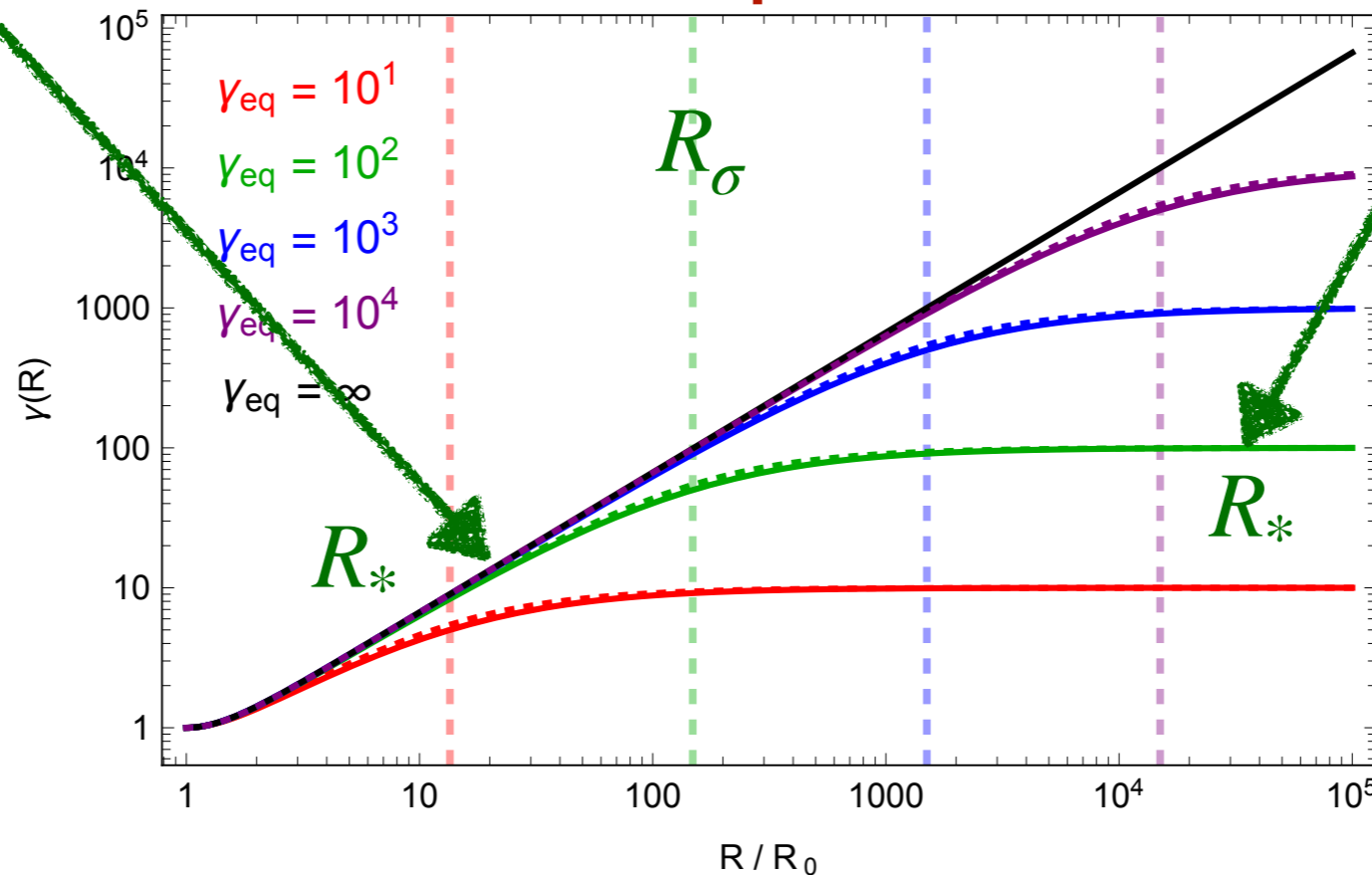
**Bubble expansion**

+ **Small Friction**

$$R_* \lesssim R_\sigma$$

**GWs dominated By wall collisions**

$$\kappa_\phi = \frac{K_{\text{wall}}}{\Delta V_{\text{eff}}}$$



+ **Large Friction**

$$R_\sigma \ll R_*$$

**GWs dominated By sound waves**

$$\kappa_\nu = \frac{K_{\text{fluid}}}{\Delta V_{\text{eff}}}$$

**2011.11451 Rong-Gen Cai, SJW,**  
**Effective picture of bubble**  
**expansion JCAP03 (2021)096**

# Effective picture of bubble expansion

Bodeker & Moore 17

Hoche et al 2007.10343

Gouttenoire et al 2112.07686

$$p_{\text{dr}} = \Delta V_{\text{eff}} \quad \text{vs} \quad \Delta p_{\text{fr}} = \Delta p_{\text{LO}} + h(\gamma) \Delta p_{\text{NLO}}$$

$\gamma$ -independent friction force

Bodeker & Moore 09

$\gamma$ -dependent friction force

$$\sigma \gamma^3 \ddot{R} + \frac{2\sigma\gamma}{R} = \Delta p_{\text{dr}} - \Delta p_{\text{fr}}$$



Kinetic energy of bubble wall

Potential energy of bubble

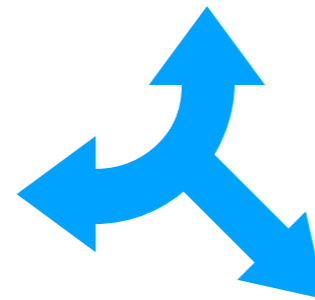
Work done by friction force on bubble wall



Conservation Law

$$E = 4\pi\sigma R^2\gamma - \frac{4}{3}\pi R^3(\Delta p_{\text{dr}} - \Delta p_{\text{fr}})$$

$$\left( \sigma + \frac{R}{3} \frac{d\Delta p_{\text{fr}}}{d\gamma} \right) \gamma^3 \ddot{R} + \frac{2\sigma\gamma}{R} = \Delta p_{\text{dr}} - \Delta p_{\text{fr}}$$



2011.11451

Effective picture of bubble expansion  
JCAP03 (2021)096

$$h(\gamma_{\text{eq}}) \equiv \frac{\Delta p_{\text{dr}} - \Delta p_{\text{LO}}}{\Delta p_{\text{NLO}}} \quad \frac{h(\gamma) - h(1)}{h(\gamma_{\text{eq}}) - h(1)} + \frac{3\gamma}{2R} = 1 + \frac{1}{2R^3}$$

General solution

# Efficiency factor for wall collisions

**Example**  $\Delta p_{\text{fr}} = \Delta p_{\text{LO}} + h(\gamma) \Delta p_{\text{NLO}}$

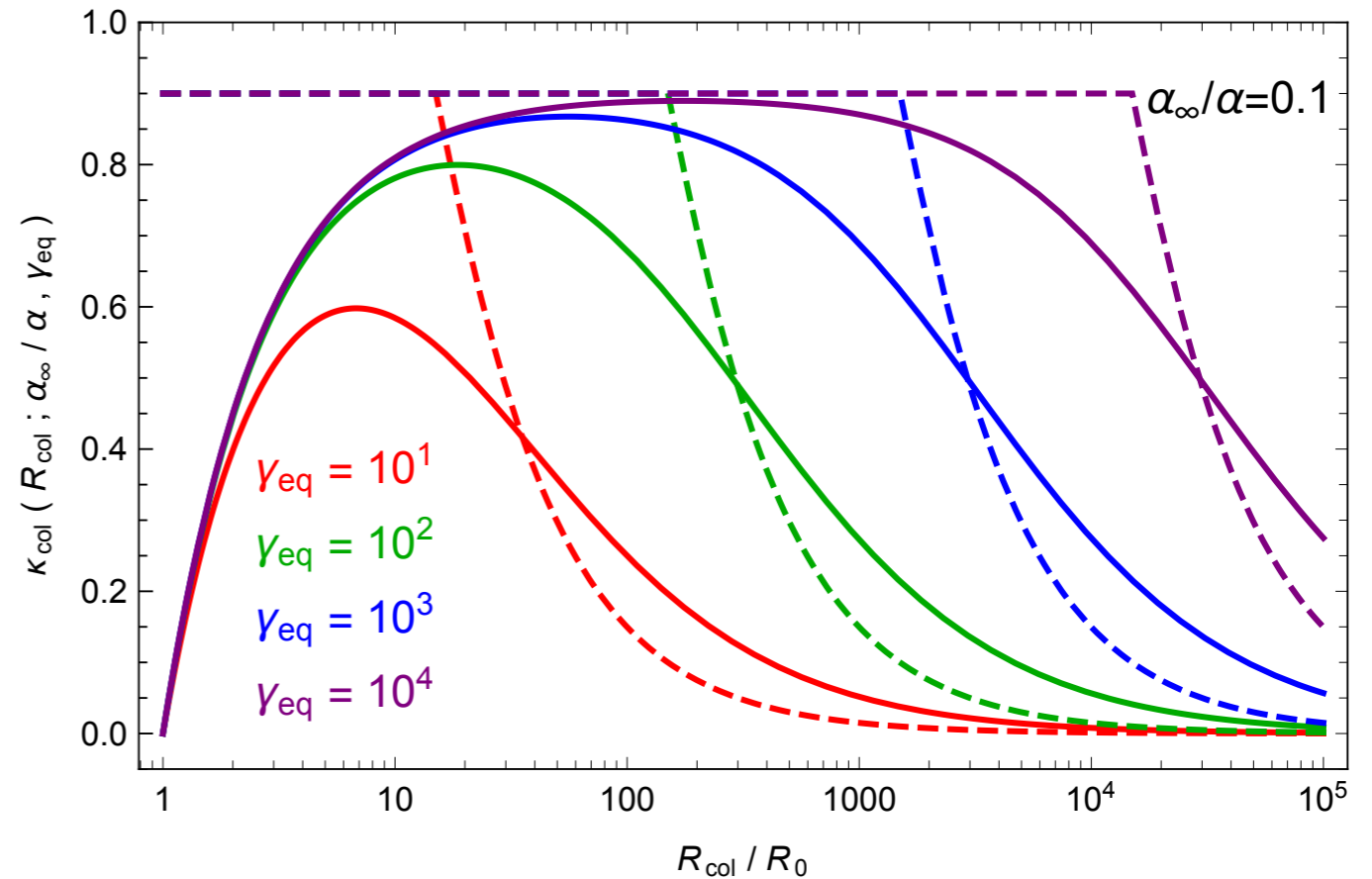
$$\alpha = \frac{\Delta V_{\text{eff}}}{\rho R}, \quad \alpha_{\infty} = \frac{\Delta p_{\text{LO}}}{\rho R}, \quad h(\gamma) = \gamma$$

$$\kappa_{\phi} = \frac{E_{\text{wall}}(R_*)}{\frac{4}{3}\pi R_*^3 \Delta V_{\text{eff}}} = \int_{R_0}^{R_*} \frac{dR}{R_*} \frac{\Delta p_{\text{dr}} - \Delta p_{\text{fr}}}{\Delta V_{\text{eff}}}$$

$$= \left(1 - \frac{\alpha_{\infty}}{\alpha}\right) \int_{R_0}^{R_*} \frac{dR}{R_*} \left[1 - \frac{h(\gamma(R))}{h(\gamma_{\text{eq}})}\right]$$

$$= \frac{1 - \alpha_{\infty}/\alpha}{27\gamma_*^2 \gamma_{\text{eq}} (\gamma_{\text{eq}} - 1)} \left[ \gamma_* (3\gamma_{\text{eq}} - 4)(3\gamma_{\text{eq}} - 1)^2 \log \frac{3(\gamma_{\text{eq}} + \gamma_* - 1)}{3\gamma_{\text{eq}} - 1} \right.$$

$$\left. \gamma_* \approx \frac{2R_*}{3R_0} \quad -4\gamma_* \log \frac{2}{3\gamma_*} - 2(\gamma_{\text{eq}} - 1)(3\gamma_* - 2) \right]$$



$$\gamma_{\text{eq}} \equiv \frac{\Delta V_{\text{eff}} - \Delta p_{\text{LO}}}{\Delta p_{\text{NLO}}} = \left(1 - \frac{\alpha_{\infty}}{\alpha}\right) \bigg/ \frac{\Delta p_{\text{NLO}}}{\Delta V_{\text{eff}}}$$

**2011.11451**  
**Effective picture of**  
**bubble expansion**  
**JCAP03 (2021)096**

**2206.01148 SJW, Zi-Yan Yuwen** **The energy budget of cosmological first-order phase transitions beyond the bag equation of state**

# Efficiency factor for sound waves

## Bag equation of state

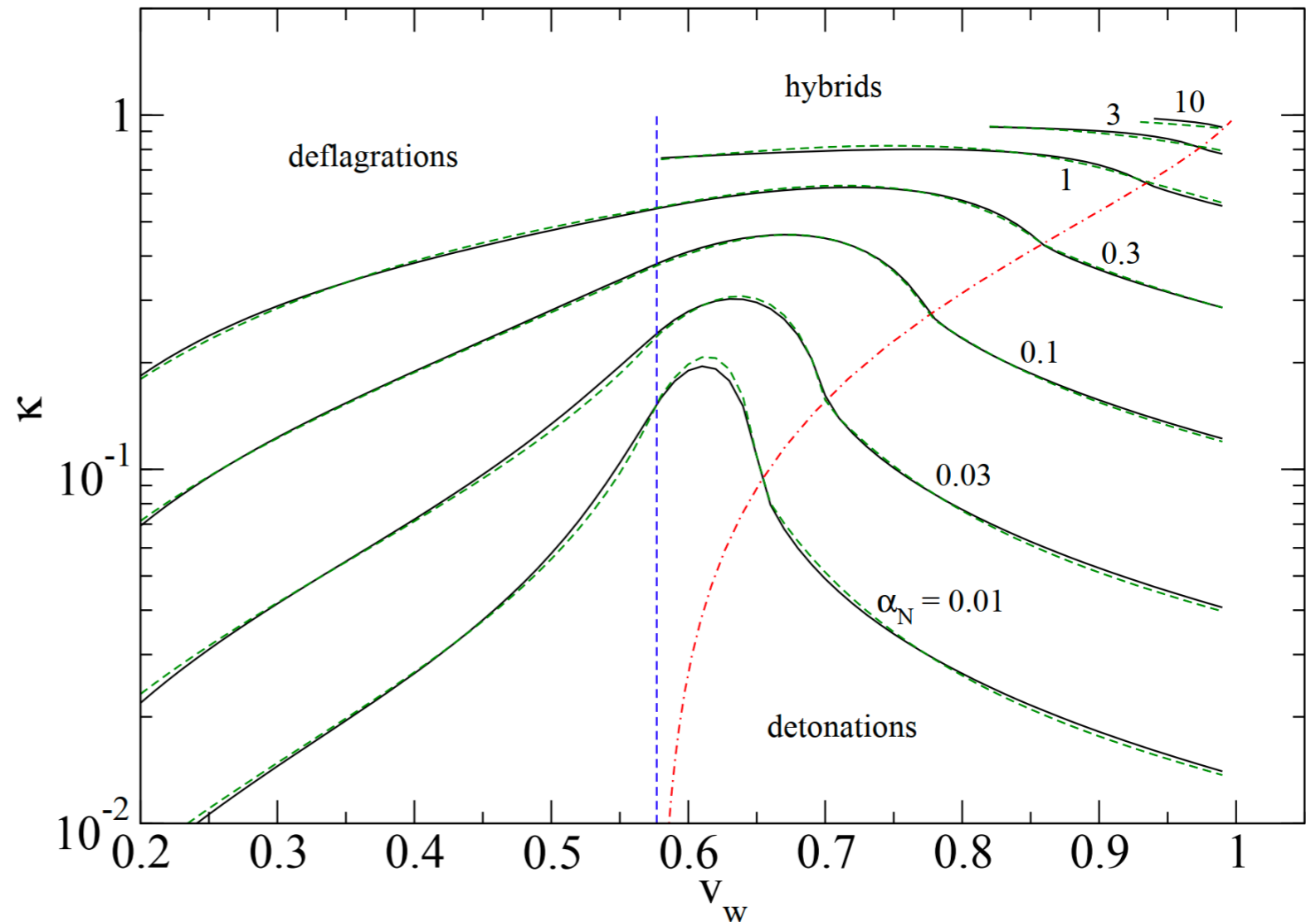
$$p_{\pm} = c_s^2 a_{\pm} T^4 - \epsilon_{\pm}$$

$$\rho_{\pm} = a_{\pm} T^4 + \epsilon_{\pm}$$

$$c_s^2 = \frac{\partial p}{\partial \rho} = \frac{1}{3}, \quad \epsilon = V_0(\phi_{\pm})$$

$$a = \frac{\pi^2}{30} \left( \sum_{\text{light B}} g_i + \frac{7}{8} \sum_{\text{light F}} g_j \right)$$

Espinosa et al. 1004.4187



## Beyond bag equation of state — constant sound velocity : $\nu$ – model

$$p_{\pm} = c_{s,\pm}^2 a_{\pm} T^{\nu_{\pm}} - \epsilon_{\pm}$$

$$\rho_{\pm} = a_{\pm} T^{\nu_{\pm}} + \epsilon_{\pm}$$

$$c_{s,\pm}^2 = \frac{\partial p_{\pm}}{\partial \rho_{\pm}} = \frac{1}{\nu_{\pm} - 1}$$

Leitao & Megevand 1410.3875

Giese et al. 2004.06995, 2010.09744

Wang et al. 2010.13770, 2112.14650



# Beyond constant sound velocity

**Effective potential**  $V_T(\phi, T) = \sum_{i=B,F} \pm g_i T \int \frac{d^3 \vec{k}}{(2\pi)^3} \ln \left[ 1 \mp e^{-\frac{\sqrt{\vec{k}^2 + m_i^2}}{T}} \right] \equiv \frac{T^4}{2\pi^2} \sum_{i=B,F} g_i J_i \left( \frac{m_i^2}{T^2} \right)$

**Heavy particles**  $J_{B/F} \left( \frac{m_i^2}{T^2} \right) = - \left( \frac{m_i}{2\pi T} \right)^{\frac{3}{2}} e^{-m_i/T} \left[ 1 + \mathcal{O} \left( \frac{T}{m_i} \right) \right]$

**Light particles**

$$J_B \left( \frac{m_i^2}{T^2} \right) = -\frac{\pi^2}{90} + \frac{1}{24} \left( \frac{m_i}{T} \right)^2 - \frac{1}{12\pi} \left( \frac{m_i}{T} \right)^3 - \frac{1}{32\pi^2} \left( \frac{m_i}{T} \right)^4 \ln \frac{m_i e^{\gamma_E - 3/4}}{4\pi T} - \frac{1}{16\pi^{5/2}} \left( \frac{m_i}{T} \right)^4 \sum_{l=1}^{\infty} (-1)^l \frac{\zeta(2l+1)}{(l+1)!} \Gamma \left( l + \frac{1}{2} \right) \left( \frac{m_i^2}{4\pi^2 T^2} \right)^l$$

$$J_F \left( \frac{m_i^2}{T^2} \right) = -\frac{7}{8} \frac{\pi^2}{90} + \frac{1}{48} \left( \frac{m_i}{T} \right)^2 + \frac{1}{32\pi^2} \left( \frac{m_i}{T} \right)^4 \ln \frac{m_i e^{\gamma_E - 3/4}}{\pi T} + \frac{1}{8\pi^{5/2}} \left( \frac{m_i}{T} \right)^4 \sum_{l=1}^{\infty} (-1)^l \frac{\zeta(2l+1)}{(l+1)!} \left( 1 - \frac{1}{2^{2l+1}} \right) \Gamma \left( l + \frac{1}{2} \right) \left( \frac{m_i^2}{\pi^2 T^2} \right)^l$$

**EoS**  $p = -\mathcal{F} = -V_{\text{eff}} = -V_0(\phi) + \frac{1}{3} a T^4 - b T^2 + c T$

$$a = \sum_{i=B} g_i + \frac{7}{8} \sum_{i=F} g_i$$

$$\rho = T \partial_T p - p = V_{\text{eff}} - T \partial_T V_{\text{eff}} = V_0(\phi) + a T^4 - b T^2$$

$$b = \frac{1}{24} \left( \sum_{i=B} g_i m_i^2 + \frac{1}{2} \sum_{i=F} g_i m_i^2 \right)$$

$$c_s^2(T) = \frac{1}{3} \left( 1 - \frac{4bT - 3c}{12aT^3 - 6bT} \right)$$

$$c = \frac{1}{12\pi} \sum_{i=B} g_i m_i^3$$

# Solving fluid EoM iteratively

## Fluid equations of motion

$$2\frac{v}{\xi} = \gamma(v)^2(1 - \xi v) \left( \frac{\mu(\xi, v)^2}{c_s^2(T(\xi))} - 1 \right) \frac{dv}{d\xi}$$

$$\frac{dw}{d\xi} = w\gamma(v)^2\mu(\xi, v) \left( \frac{1}{c_s^2(T(\xi))} + 1 \right) \frac{dv}{d\xi}$$

$$T(\xi) = T(\xi_0) \exp \left[ \int_{v(\xi_0)}^{v(\xi)} \gamma(v)^2 \mu(\xi, v) dv \right]$$

## Iteration method

$$c_s^{(0)}(\xi) = 1/3 \Rightarrow v^{(0)}(\xi), \quad w^{(0)}(\xi), \quad T^{(0)}(\xi)$$

$$c_s^{(1)}(\xi) = c_s(T^{(0)}(\xi)) \Rightarrow v^{(1)}(\xi), \quad w^{(1)}(\xi), \quad T^{(1)}(\xi)$$

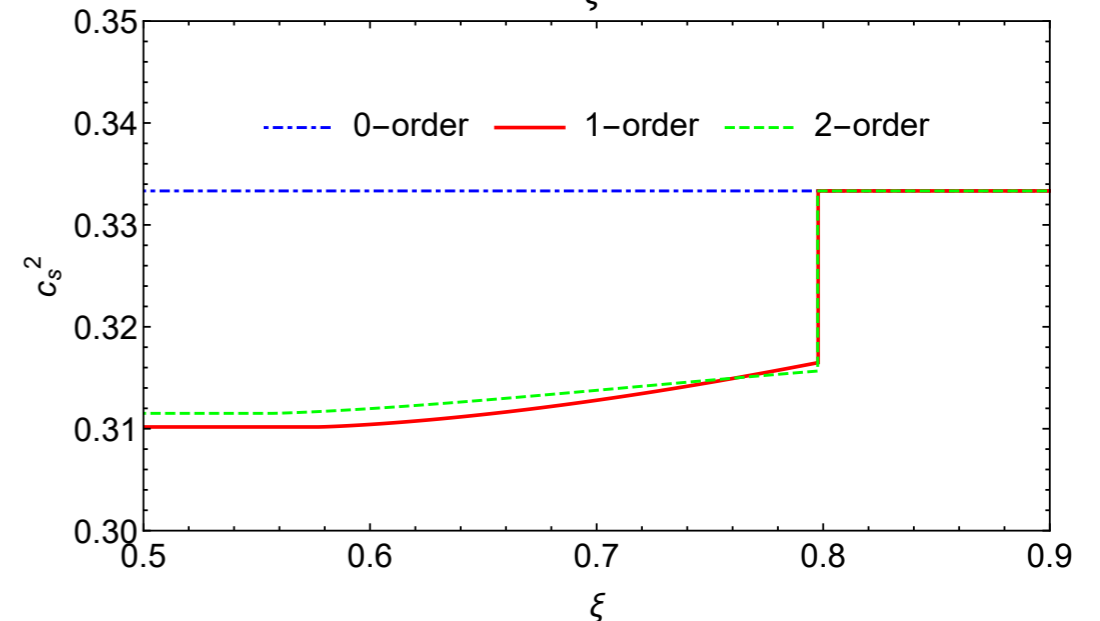
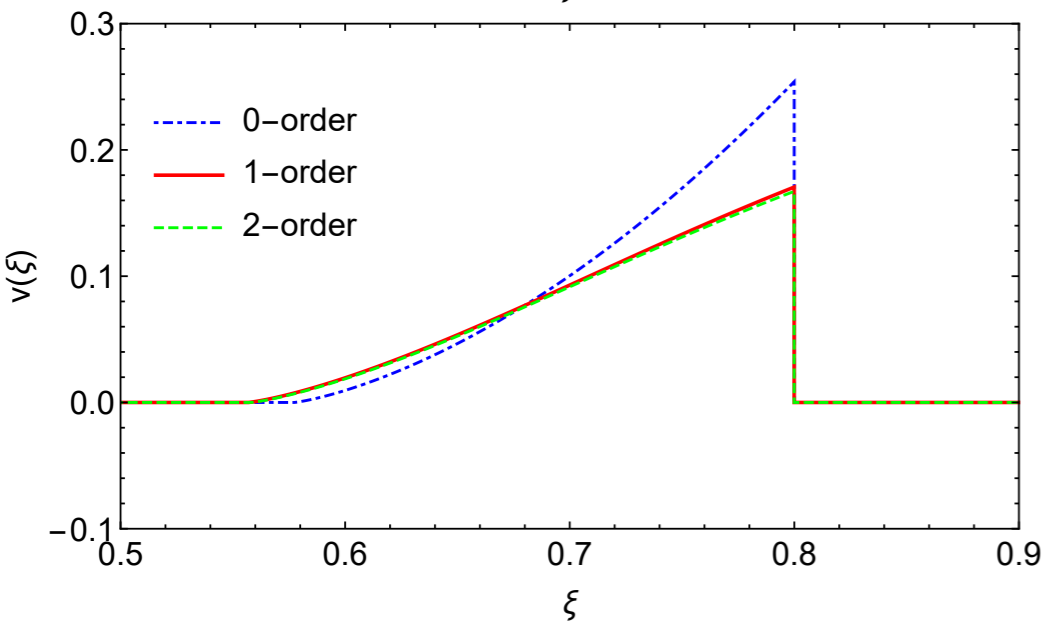
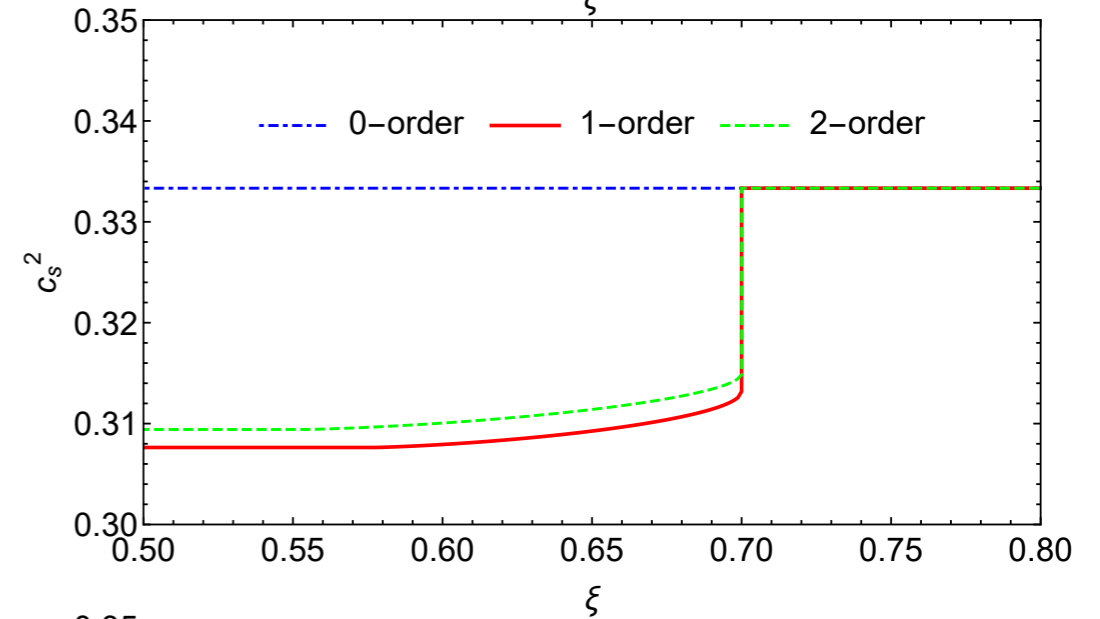
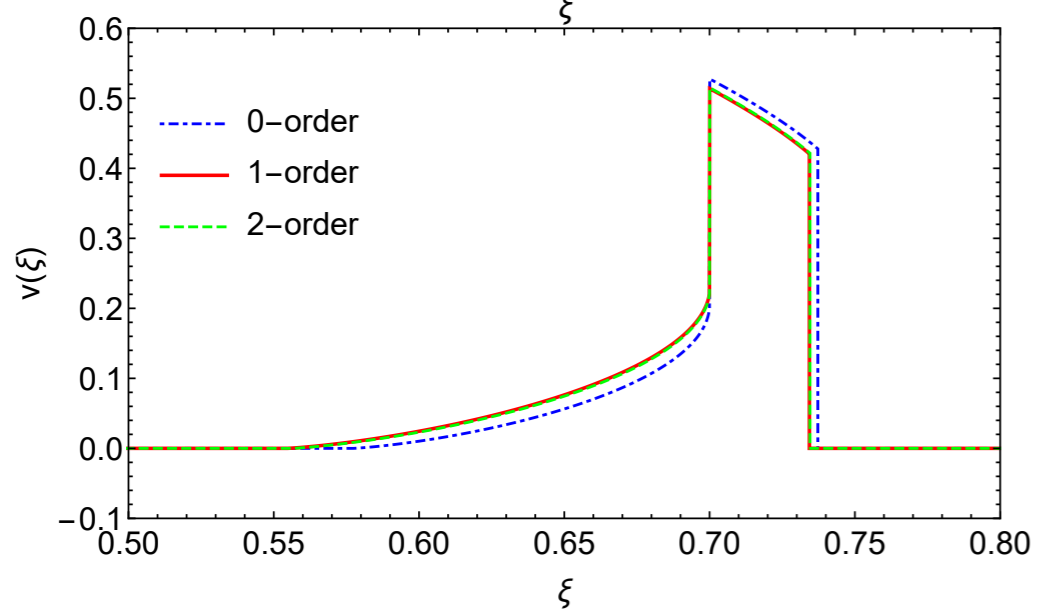
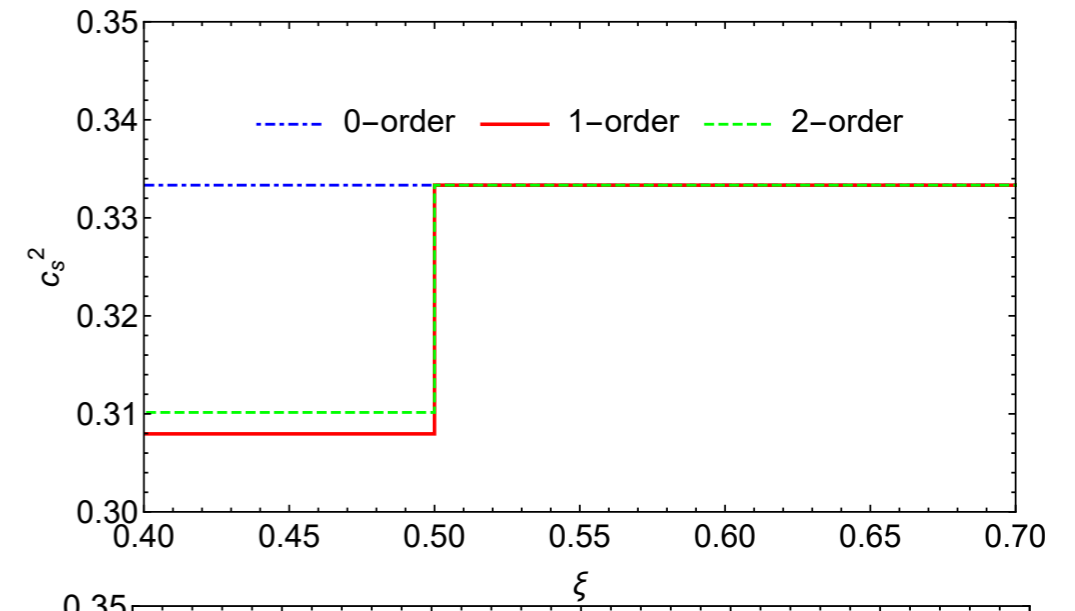
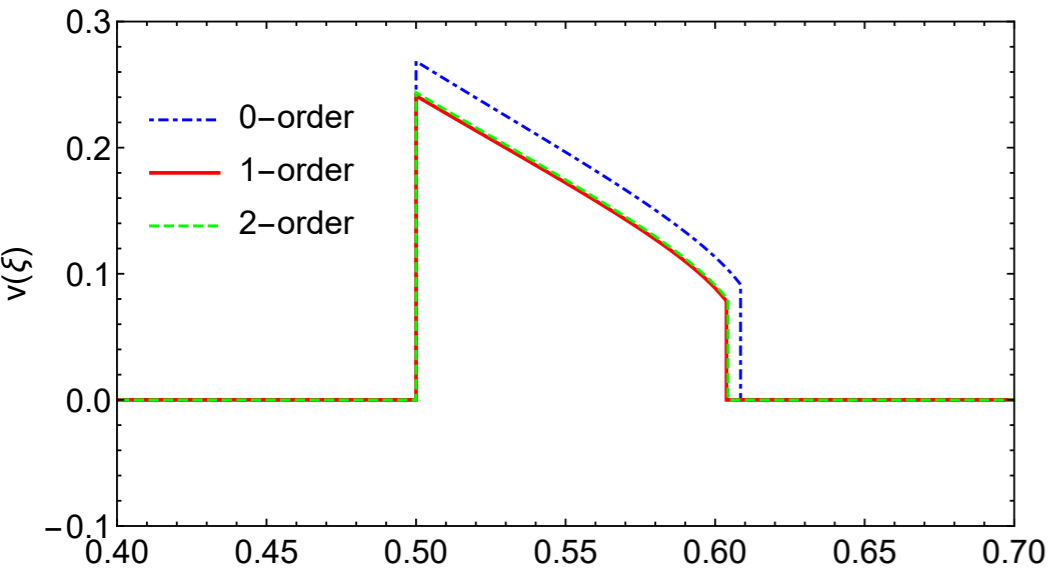
$$c_s^{(2)}(\xi) = c_s(T^{(1)}(\xi)) \Rightarrow v^{(2)}(\xi), \quad w^{(2)}(\xi), \quad T^{(2)}(\xi)$$

# Iterative solutions

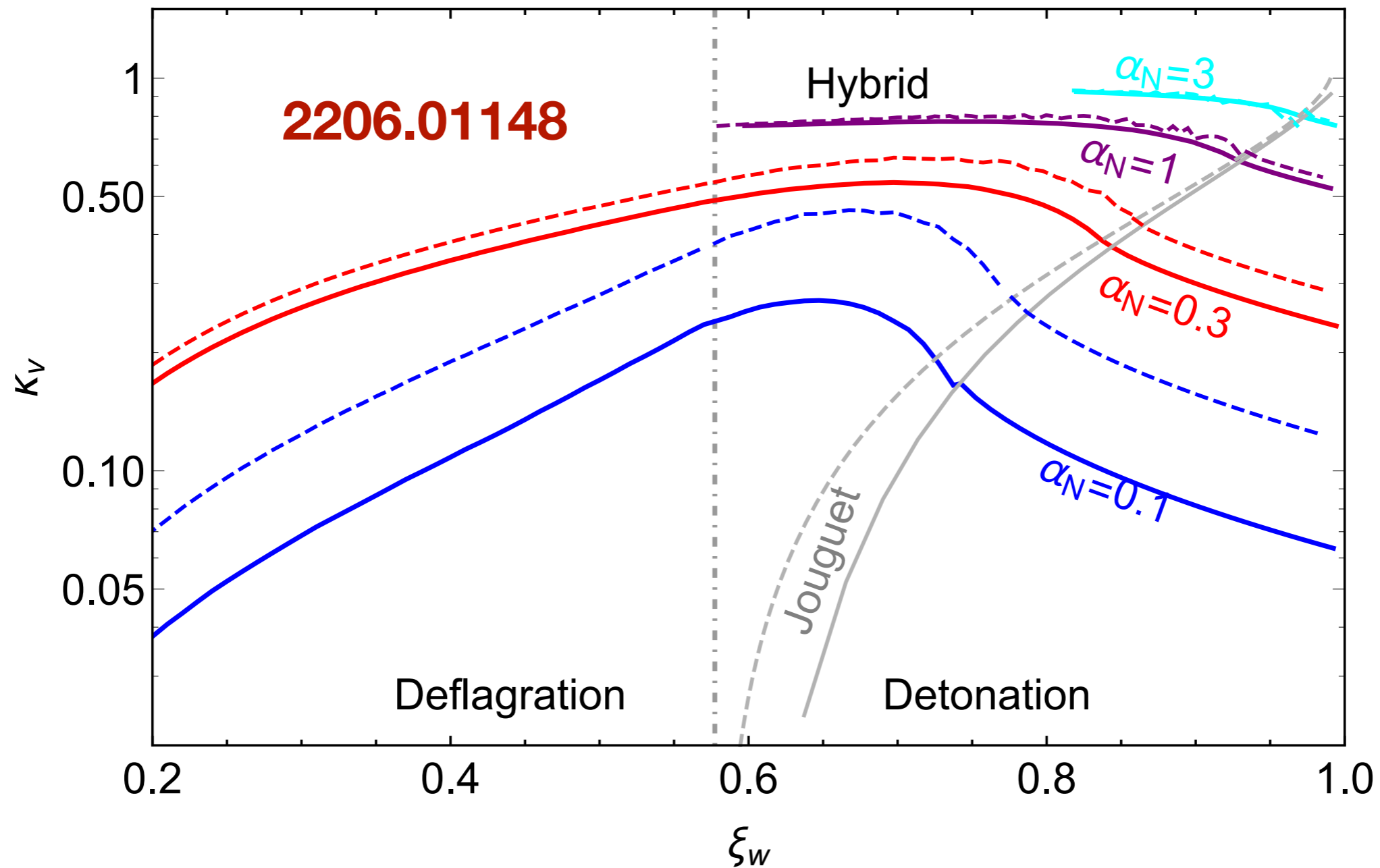
**Deflagration**

**Hybrid**

**Detonation**



# Efficiency factor for sound waves



1. The efficiency factor beyond the constant sound velocity is suppressed with respect to the case with a bag EoS
2. The suppression effect is less announced for a strong first-order phase transition with a large strength factor  $\alpha_N$

**2205.02492 SJW, Zi-Yan Yuwen**  
**Hydrodynamic backreaction force  
of cosmological bubble expansion**

# Boltzmann equations of motion

$$T_{\phi}^{\mu\nu} = \nabla^{\mu} \phi \nabla^{\nu} \phi + g^{\mu\nu} \left[ -\frac{1}{2} (\nabla \phi)^2 - V_0(\phi) \right]$$

$$T_f^{\mu\nu} = \sum_{i=B,F} g_i \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{k^{\mu} k^{\nu}}{k^0} \Big|_{k^0=E_i(\mathbf{k})} f_i(\mathbf{x}, \mathbf{k})$$

## Scalar EoM

$$\nabla_{\mu} T_{\phi}^{\mu\nu} \equiv [\nabla_{\mu} \nabla^{\mu} \phi - V'_0(\phi)] \nabla^{\nu} \phi = +f^{\nu}$$

## Plasma EoM

$$\nabla_{\mu} T_f^{\mu\nu} \equiv \sum_{i=B,F} g_i \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{k^{\mu} k^{\nu}}{E_i(\mathbf{k})} \nabla_{\mu} f_i = -f^{\nu}$$

$$f^{\nu} = \nabla^{\nu} \phi \sum_{i=B,F} g_i \frac{dm_i^2}{d\phi} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{f_i}{2E_i}$$

$$\frac{\partial V_T^{1\text{-loop}}}{\partial \phi} = \sum_{i=B,F} g_i \frac{dm_i^2}{d\phi} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{f_i^{\text{eq}}}{2E_i}$$

$$f_i = f_i^{\text{eq}} + \Delta f_i^{\text{eq}} + \delta f_i \quad f_i^{\text{eq}} = \frac{1}{e^{E_i/T} \mp 1}$$

$$f^{\nu} = \nabla^{\nu} \phi \left( \frac{\partial V_T^{1\text{-loop}}}{\partial \phi} + \frac{\partial \Delta V_T}{\partial \phi} - \frac{\partial p_{\delta f}}{\partial \phi} \right)$$

**Out-of-equilibrium effect**

$$\nabla_{\mu} \nabla^{\mu} \phi - \frac{\partial V_{\text{eff}}}{\partial \phi} = - \frac{\partial p_{\delta f}}{\partial \phi}$$

$$\nabla_{\mu} T_f^{\mu\nu} + \nabla^{\nu} \phi \frac{\partial V_T}{\partial \phi} = \nabla^{\nu} \phi \frac{\partial p_{\delta f}}{\partial \phi}$$

# New picture of bubble expansion

**Scalar EoM**

$$\int d\xi \frac{d\phi}{d\xi} \left( \nabla^2 \phi - \frac{\partial V_{\text{eff}}}{\partial \phi} + \frac{\partial p_{\delta f}}{\partial \phi} \right) = 0$$

$$\phi'(\xi \neq \xi_w)^2 = 0 \quad \downarrow \quad \downarrow \quad \frac{\partial V_{\text{eff}}}{\partial \phi} \frac{d\phi}{d\xi} = \frac{dV_{\text{eff}}}{d\xi} - \frac{\partial V_{\text{eff}}}{\partial T} \frac{dT}{d\xi}$$

$$\frac{F_{\text{driving}}}{A} \equiv \Delta V_{\text{eff}} = \int d\xi \frac{dT}{d\xi} \frac{\partial V_{\text{eff}}}{\partial T} + \int d\xi \frac{d\phi}{d\xi} \frac{\partial p_{\delta f}}{\partial \phi} \equiv \frac{F_{\text{thermal}}}{A} + \frac{F_{\text{friction}}}{A} \equiv \frac{F_{\text{backreaction}}}{A}$$

**Driving force from vacuum difference**

**=**

**Thermal force from Temperature gradient**

**+**

**Friction force from Out-of-equilibrium**

**Plasma EoM**

$$u_\nu \nabla_\mu (w u^\mu u^\nu + p_f \eta^{\mu\nu}) + u_\nu \nabla^\nu \phi \frac{\partial V_T}{\partial \phi} = u_\nu \nabla^\nu \phi \frac{\partial p_{\delta f}}{\partial \phi}$$

$$u_\nu u^\nu = -1, \quad u_\nu \nabla_\mu u^\nu = 0 \quad \downarrow \quad \nabla_\mu p_f = -\nabla_\mu V_T = -\nabla_\mu T \frac{\partial V_T}{\partial T} - \nabla_\mu \phi \frac{\partial V_T}{\partial \phi} = -\nabla_\mu T \frac{\partial V_{\text{eff}}}{\partial T} - \nabla_\mu \phi \frac{\partial V_T}{\partial \phi}$$

**Entropy flow**  $T \nabla_\mu (s u^\mu) = -u^\mu \nabla_\mu \phi \frac{\partial p_{\delta f}}{\partial \phi}$       **Enthalpy flow**  $-\nabla_\mu (w u^\mu) = u^\mu \nabla_\mu T \frac{\partial V_{\text{eff}}}{\partial T} + u^\mu \nabla_\mu \phi \frac{\partial p_{\delta f}}{\partial \phi}$

# Hydrodynamic evaluation

**Driving force**  $\frac{F_{\text{drive}}}{A} \equiv \Delta \left( -\frac{1}{3}aT^4 + V_0 \right) = -\frac{1}{4}\Delta w + \frac{3}{4}\alpha_N w_N$

**Thermal force**  $\frac{F_{\text{themo}}}{A} \equiv \int d\xi \frac{dT}{d\xi} \frac{\partial V_{\text{eff}}}{\partial T} = \int d\xi \frac{dT}{d\xi} \left( -\frac{4}{3}aT^3 \right)$  →  $s(v)$  unknown from junction condition

**Friction force**  $\frac{F_{\text{fric}}^{\text{hydro}}}{A} = \int d\xi \frac{d\phi}{d\xi} \frac{\partial p_{\delta f}}{\partial \phi} = \int dr \frac{T \nabla_{\mu}(su^{\mu})}{\gamma(\xi - v)} = \int_0^1 d\xi \left( -T \frac{ds}{d\xi} + \frac{2wv}{\xi(\xi - v)} + \frac{w\gamma^2}{\mu} \frac{dv}{d\xi} \right)$

**Backreaction force**  $\frac{F_{\text{back}}^{\text{hydro}}}{A} = \int dr \frac{\nabla_{\mu}(wu^{\mu})}{\gamma(\xi - v)} = \int_0^1 d\xi \left( -\frac{dw}{d\xi} + \frac{2wv}{\xi(\xi - v)} + \frac{w\gamma^2}{\mu} \frac{dv}{d\xi} \right)$

e.g. Detonation:

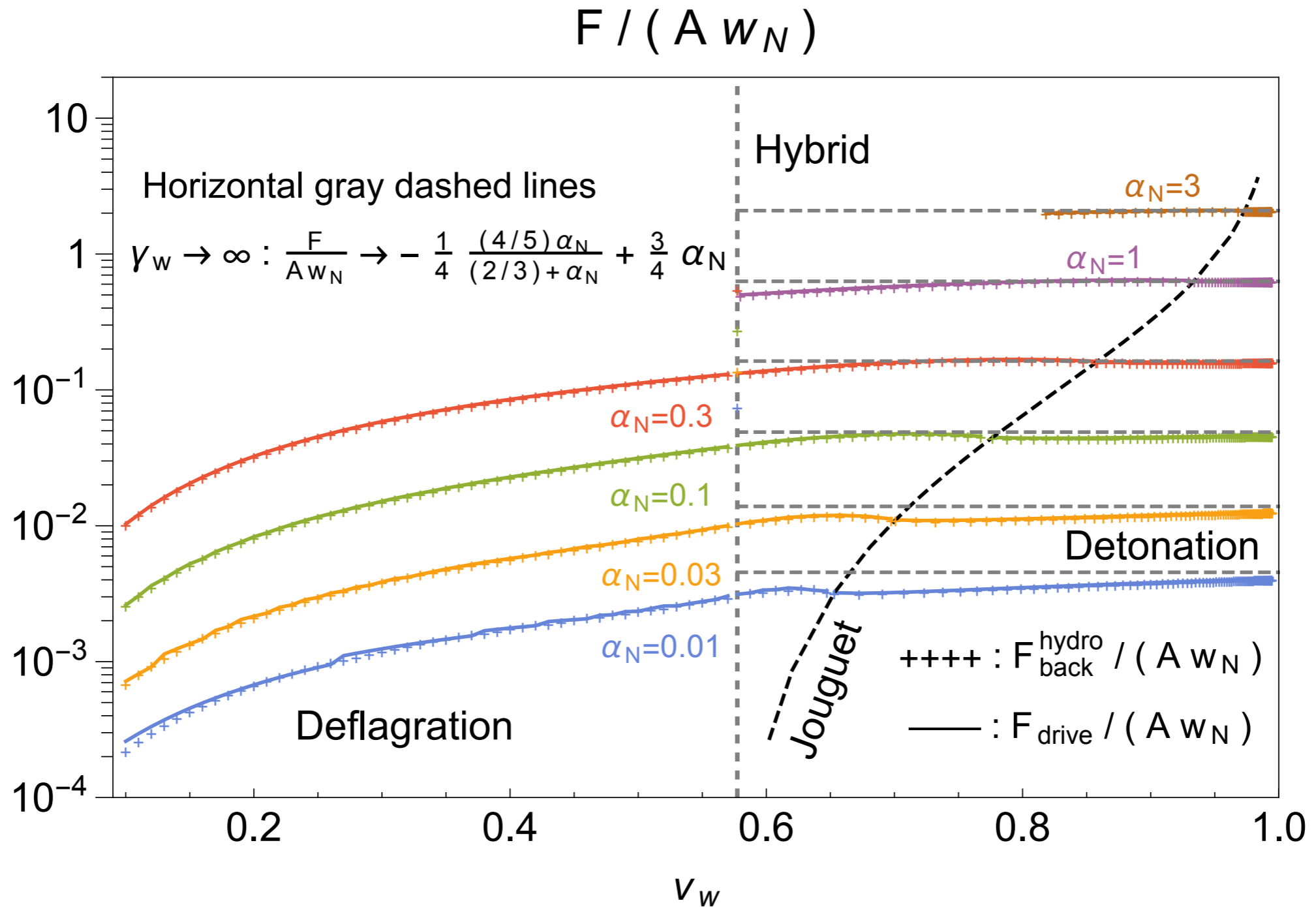
Junction condition ↓

$$\frac{F_{\text{back}}^{\text{hydro}}}{A} = -\frac{1}{4} [w(\xi = v_w) - w(\xi = c_s)] + \frac{F_{\text{back}}^{\text{hydro}}}{A} \Bigg|_{\text{wall}} \quad w(v) = w_+ \frac{\bar{\gamma}_+^2 \bar{v}_+}{\bar{\gamma}^2 \bar{v}} = w_N \frac{v_w}{1 - v_w^2} \frac{1 - \mu(v_w, v)^2}{\mu(v_w, v)}$$

$$\frac{F_{\text{back}}^{\text{hydro}}}{A} \Bigg|_{\text{wall}} = -(w_+ - w_-) + \int_{v_-}^0 dv \frac{w(v)\gamma(v)^2}{\mu(v_w, v)} = -(w_+ - w_-) + \frac{v_-}{v_- - v_w} w_+$$



# Total back-reaction force



$$\frac{F_{\text{drive}}}{A w_N} = \frac{F_{\text{back}}^{\text{hydro}}}{A w_N} \rightarrow -\frac{1}{4} \frac{(4/5)\alpha_N}{(2/3) + \alpha_N} + \frac{3}{4} \alpha_N$$

# Backreactoin force on the wall

Barroso Mancha et al., “Field theoretic derivation of bubble wall force,” 2005.10875

$$\Delta_{\text{wall}}(p) \equiv p_+ - p_- = w_- \bar{\gamma}_-^2 \bar{v}_-^2 - w_+ \bar{\gamma}_+^2 \bar{v}_+^2 \equiv - \Delta_{\text{wall}}(w \bar{\gamma}^2 \bar{v}^2)$$

$$\Delta_{\text{wall}}(V_{\text{eff}}) = \Delta_{\text{wall}}(w \bar{\gamma}^2 \bar{v}^2) \equiv \frac{F_{\text{back}}^{\text{wall}}}{A}$$

There is a misleading deduction that this evaluation implies  $\gamma_w^2$  – scaling friction force

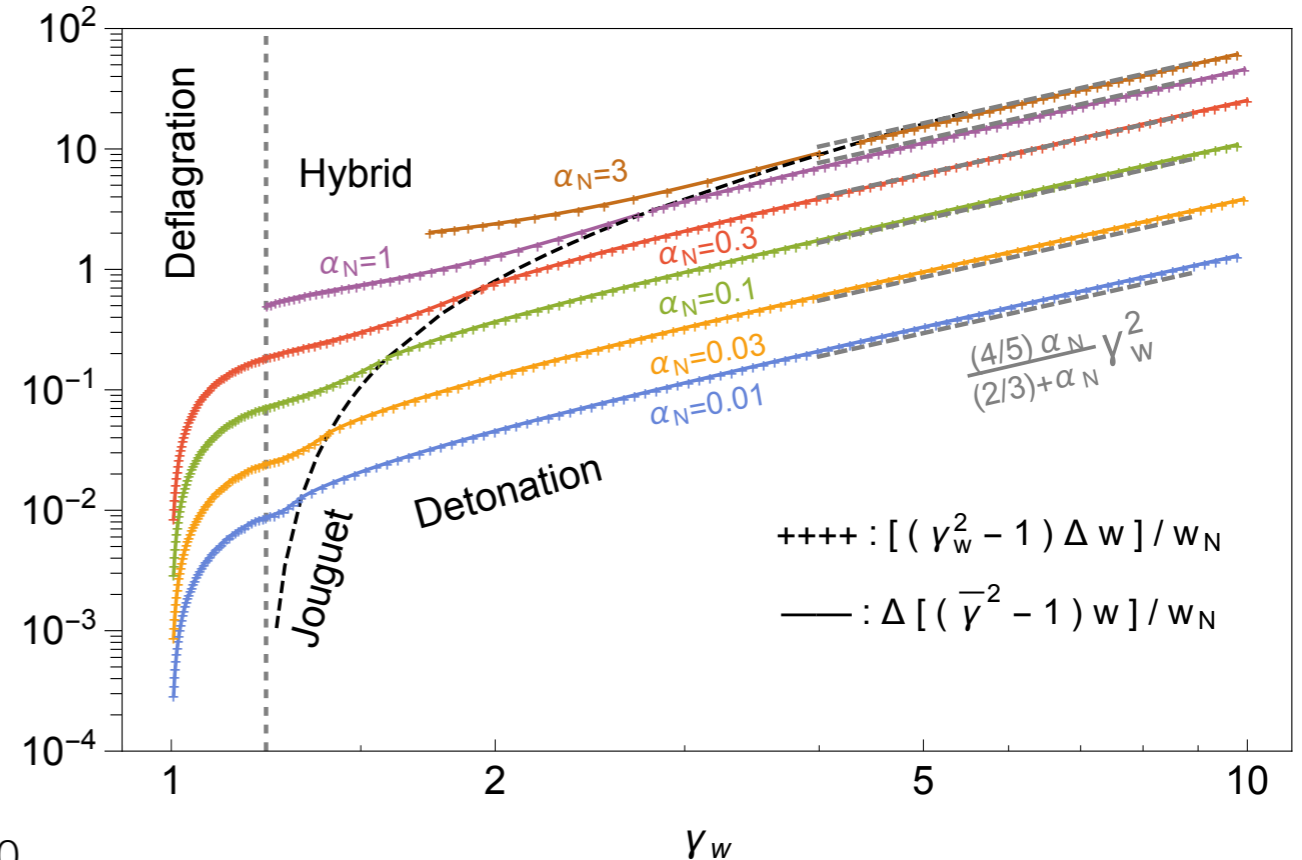
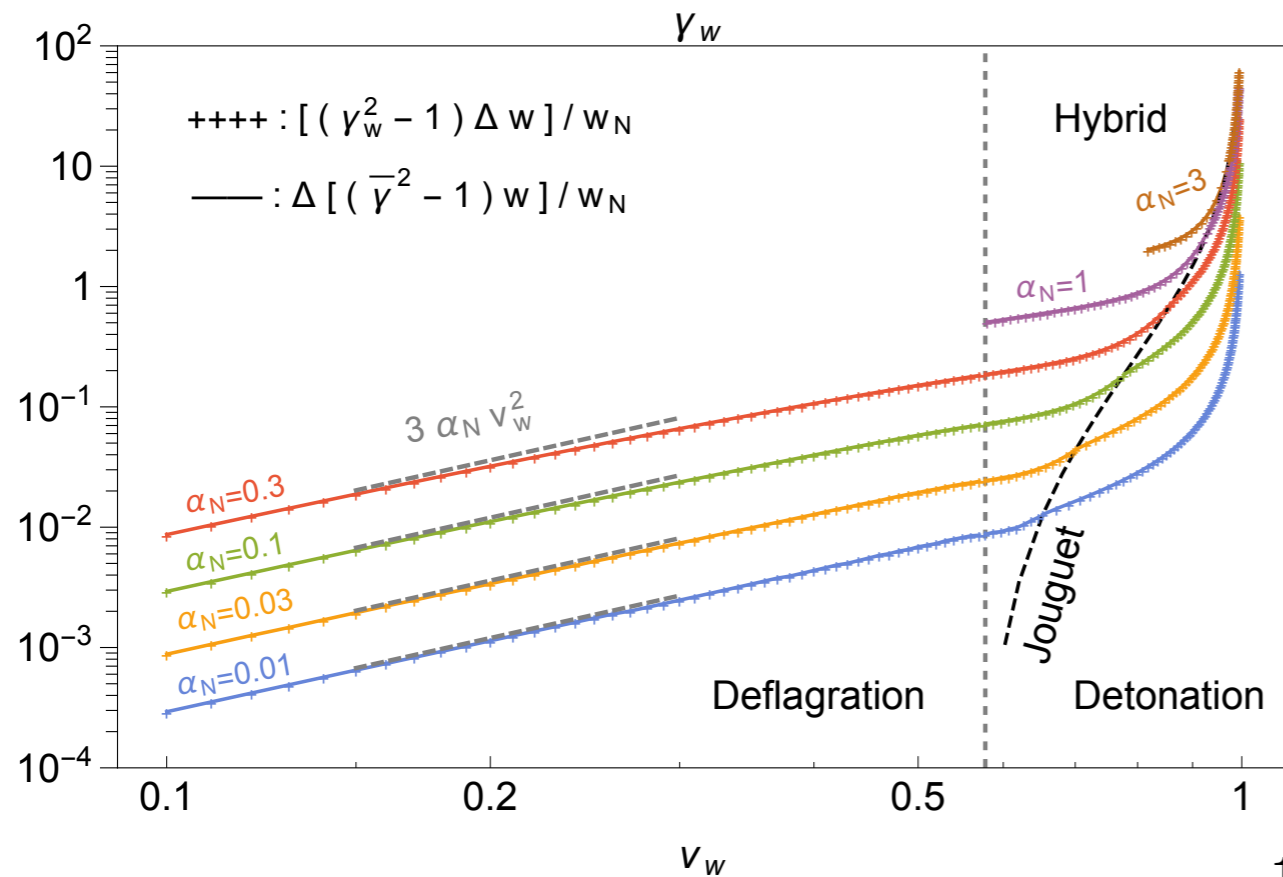
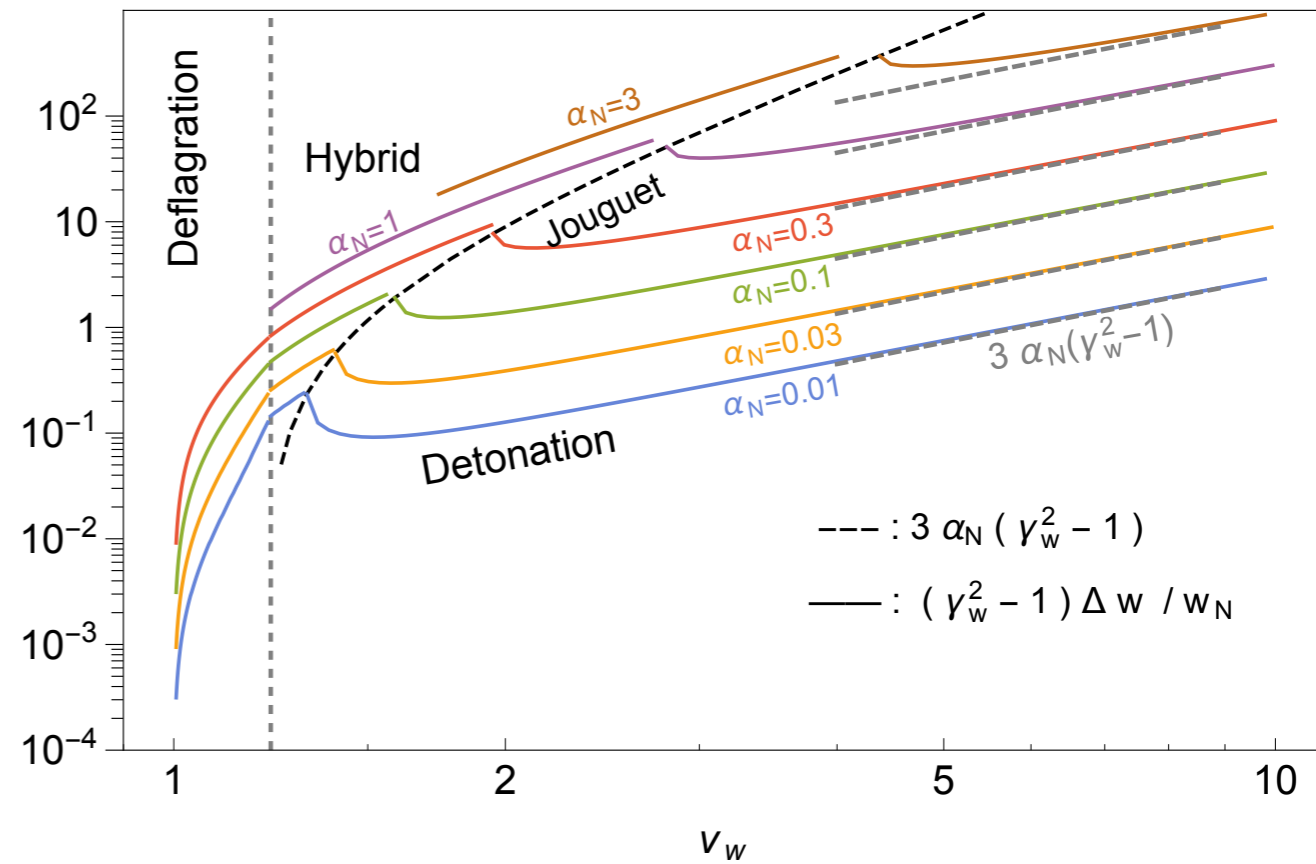
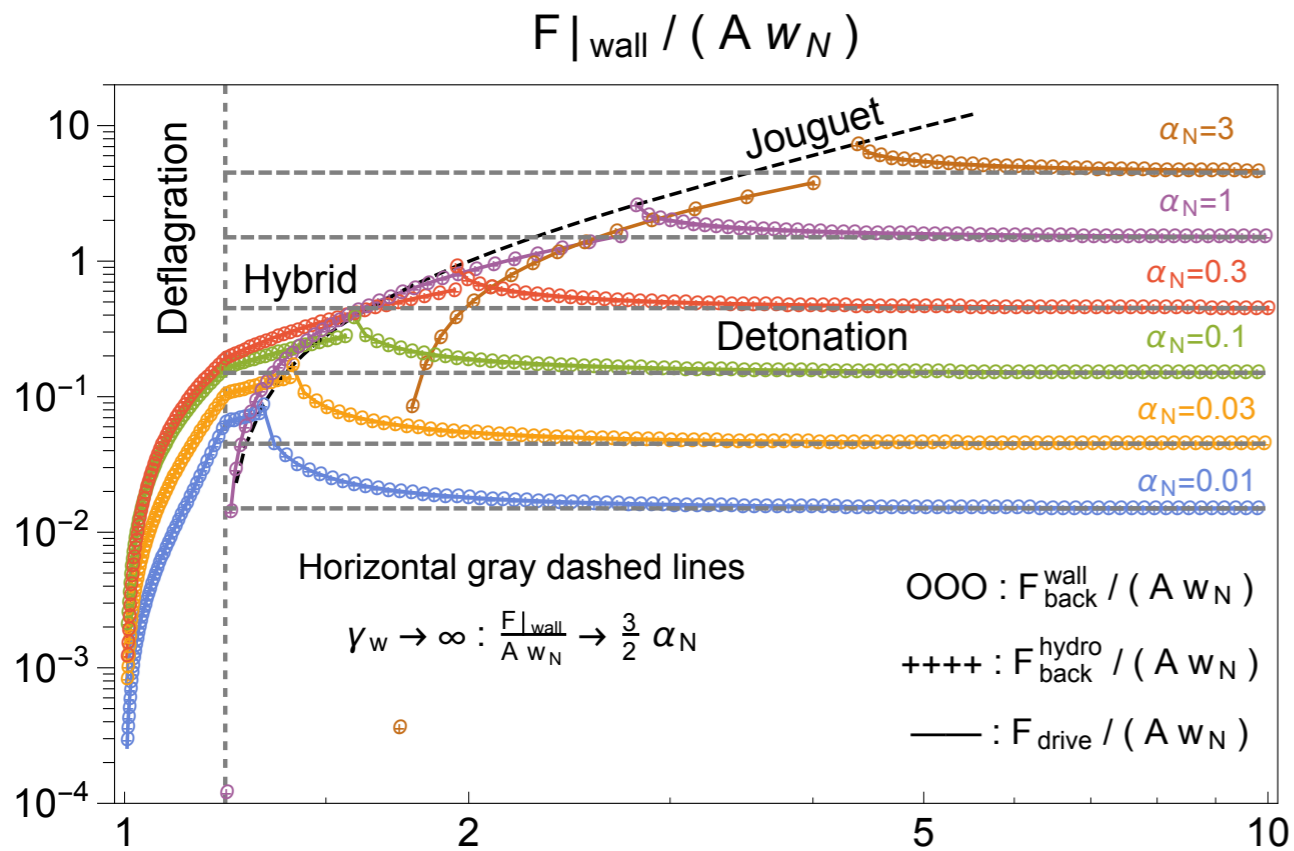
$$\Delta_{\text{wall}}(w \bar{\gamma}^2 \bar{v}^2) \equiv \Delta_{\text{wall}}[(\bar{\gamma}^2 - 1)w] = (\gamma_w^2 - 1)\Delta_{\text{wall}}(w) = (\gamma_w^2 - 1)T\Delta_{\text{wall}}(s) \quad \times$$

However, we can analytically prove that  $(w_+ \bar{\gamma}_+^2 \bar{v}_+^2 - w_- \bar{\gamma}_-^2 \bar{v}_-^2) \rightarrow \frac{3}{2}\alpha_N w_N$ ,  $\gamma_w \rightarrow \infty$

In fact, we can numerically show that

$$\Delta_{\text{wall}}(V_{\text{eff}}) = \Delta_{\text{wall}}[(\bar{\gamma}^2 - 1)w] \neq \begin{cases} (\gamma_w^2 - 1)\Delta w = \Delta[(\bar{\gamma}^2 - 1)w] \rightarrow \frac{(4/5)\alpha_N w_N}{(2/3) + \alpha_N} \gamma_w^2, & \gamma_w \rightarrow \infty \\ (\gamma_w^2 - 1)\Delta_{\text{wall}} w \rightarrow 3\alpha_N w_N (\gamma_w^2 - 1), & \gamma_w \rightarrow \infty \end{cases}$$

# Backreaction force on the wall



# Conclusions and discussions



**We have proposed an analytical evaluation method for the efficiency factor of bubble wall collisions**



**We have proposed an iteration method for the efficiency factor of bulk fluid motions beyond a constant sound velocity**



**We have proposed a hydrodynamical evaluation for the total backreaction force along with its wall contribution**



**We want to propose a hybrid method from hydrodynamics and microphysics to compute the friction force evolution**

*Thank you*  
20/20