宇宙原初量子涨落在相变引力波各向异性中的遗迹

黄发朋(FPH)

Phys.Rev.D 105 (2022) 083527 and work in progress

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宇宙原初种子的形成机制和性质？

ΛCDM

如果探测？

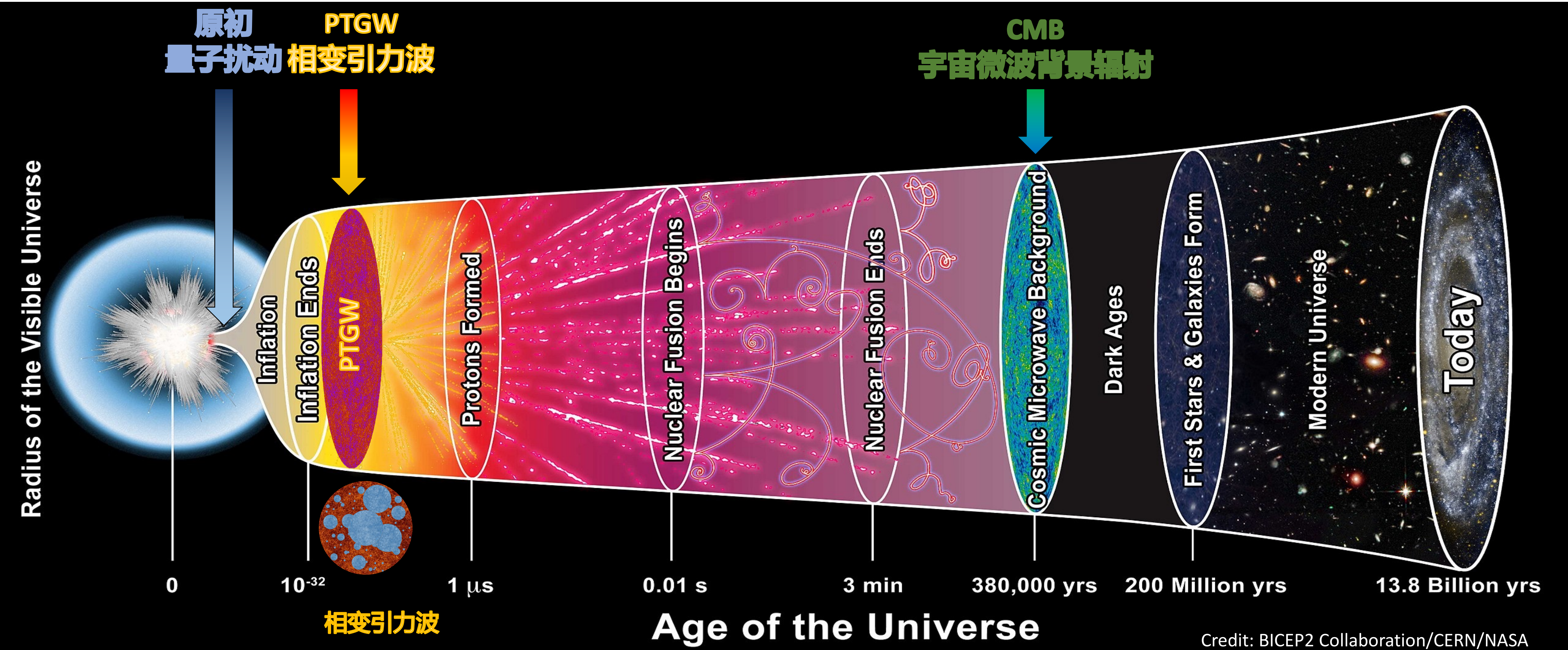
光；

引力波

(new)

宇宙大尺度结构形成的原初种子

primordial quantum fluctuations from inflation or alternatives



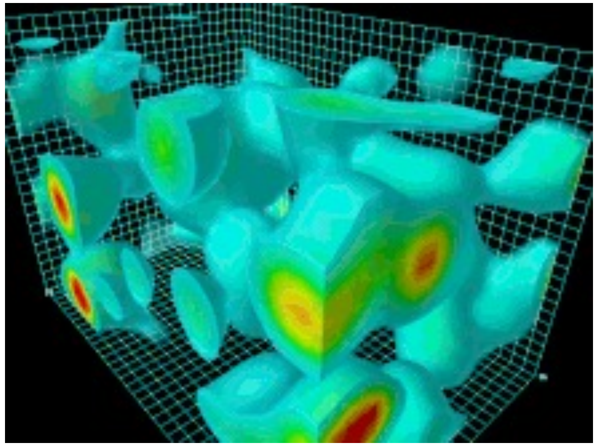
Credit: BICEP2 Collaboration/CERN/NASA

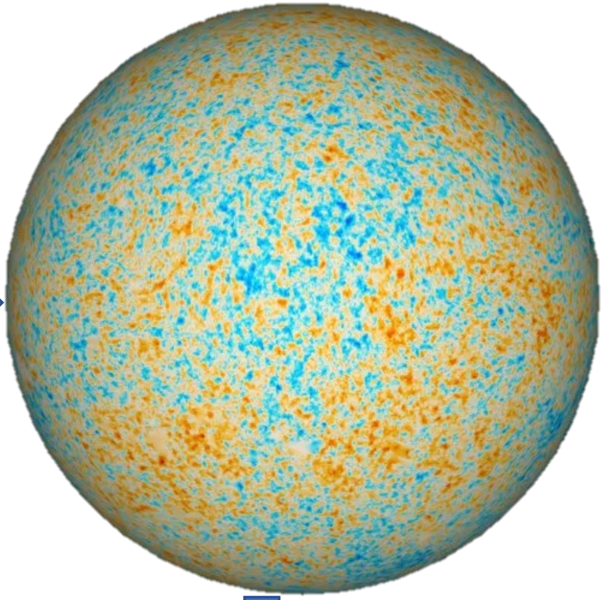
宇宙大尺度结构形成的原初种子

primordial quantum fluctuations from inflation or alternatives

- 密度扰动与原初功率谱

$$P_{\mathcal{R}}(k)=\frac{k^3}{2\pi^2}\left\langle|\mathcal{R}_k|^2\right\rangle\Big|_{aH=k}=\frac{H^2}{\pi\varepsilon_{\rm sr}m_{\rm pl}^2}\Big|_{aH=k}=A_s\left(\frac{k}{k_*}\right)^{n_s-1}$$



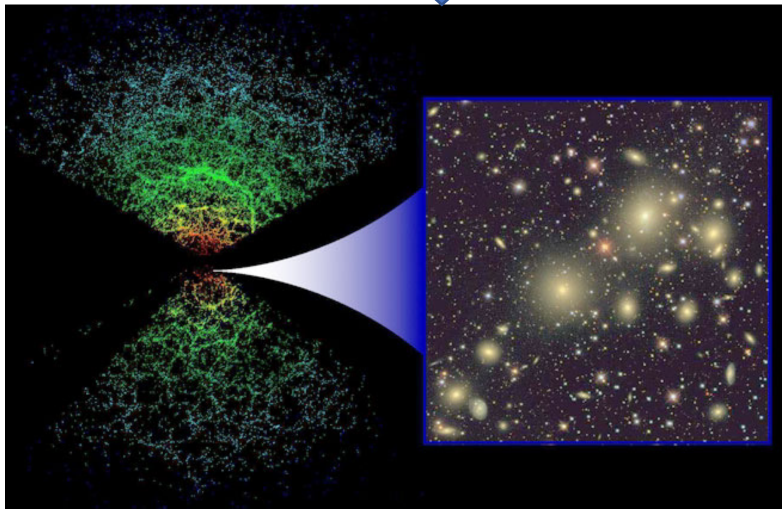


- 各向异性的角功率谱

$$C_l=\frac{2}{\pi}\int_0^\infty dk k^2 P_{\mathcal{R}}(k)\,|\mathcal{T}_l(k)|^2$$

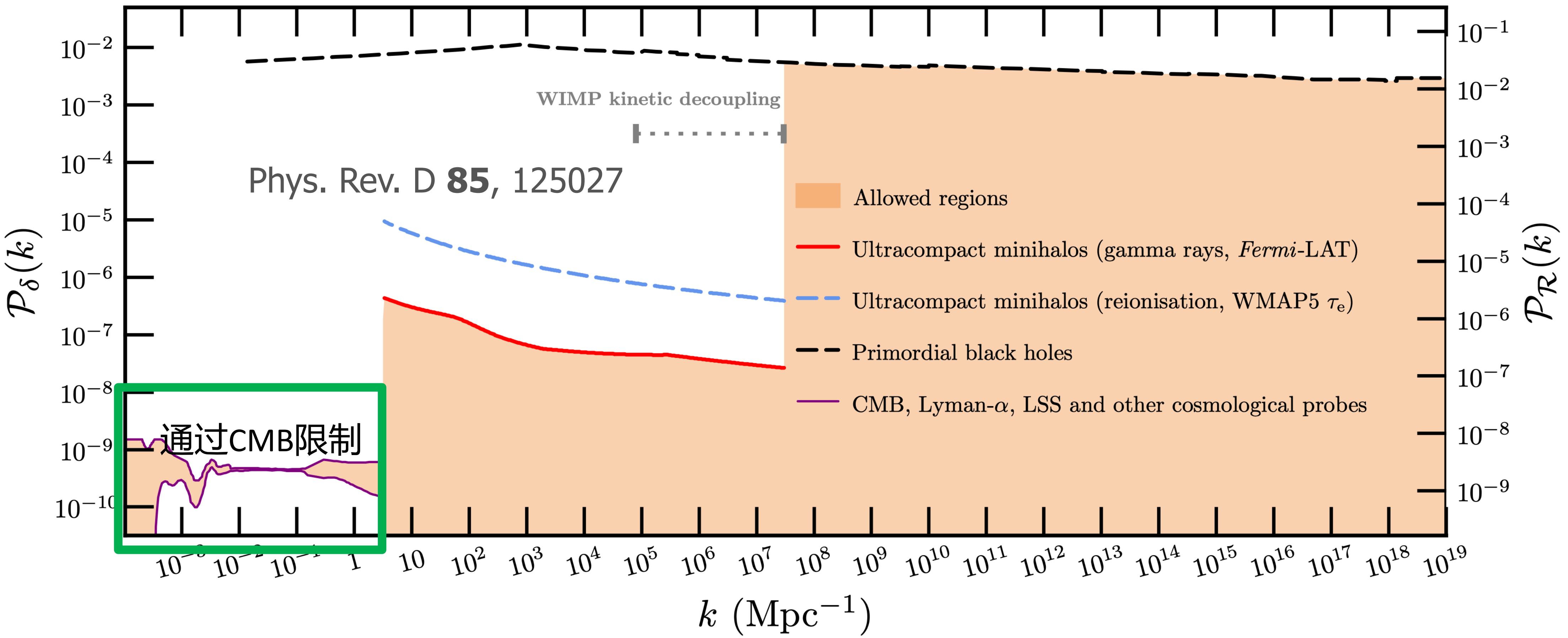
$$C_\ell^{\rm obs}=\frac{1}{2\ell+1}\sum_{\ell m}a_{\ell m}a_{\ell m}^*$$

The Primordial Density Perturbation
from inflation or alternative as the origin of structure



宇宙原初量子涨落对引力波各向异性中的遗迹

The quest for small-scale power spectrum——The road less traveled



- ✓ A complete model of inflation requires a solid understanding of the small-scale primordial power spectrum;
- ✓ however, it is hard!

宇宙原初量子涨落在引力波各向异性中的遗迹

The quest for small-scale power spectrum——The road less traveled

Yongping Li, **FPH**, Xiao Wang, Xinmin Zhang, Phys.Rev.D 105 (2022) 083527
Yongping Li, **FPH**, Xiao Wang, work in progress

1. ΛCDM:宇宙大尺度结构形成的原初种子是inflation时期的原初量子(密度)涨落——原初功率谱

2. Silk damping导致CMB难以探测小尺度的密度涨落

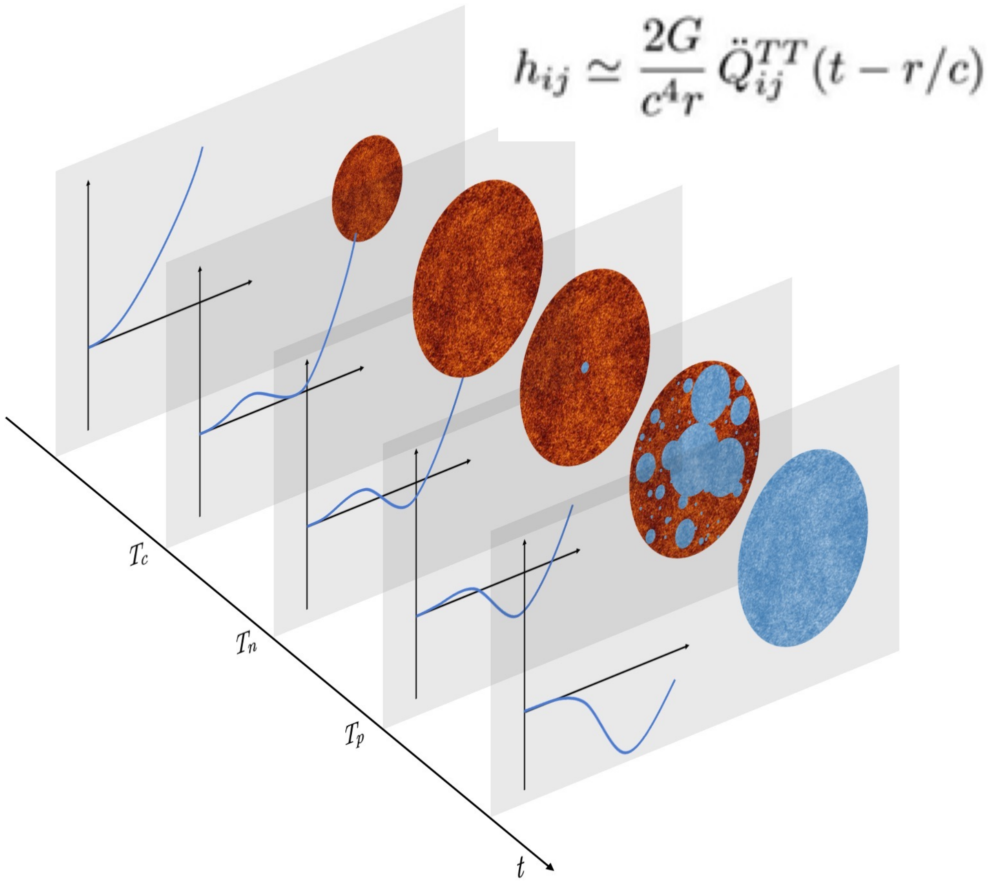
3. 我们考虑不受Silk damping影响的相变引力波探针，来了解inflation产生的原初密度涨落的小尺度信息

引力波各向异性探索早期宇宙的研究：

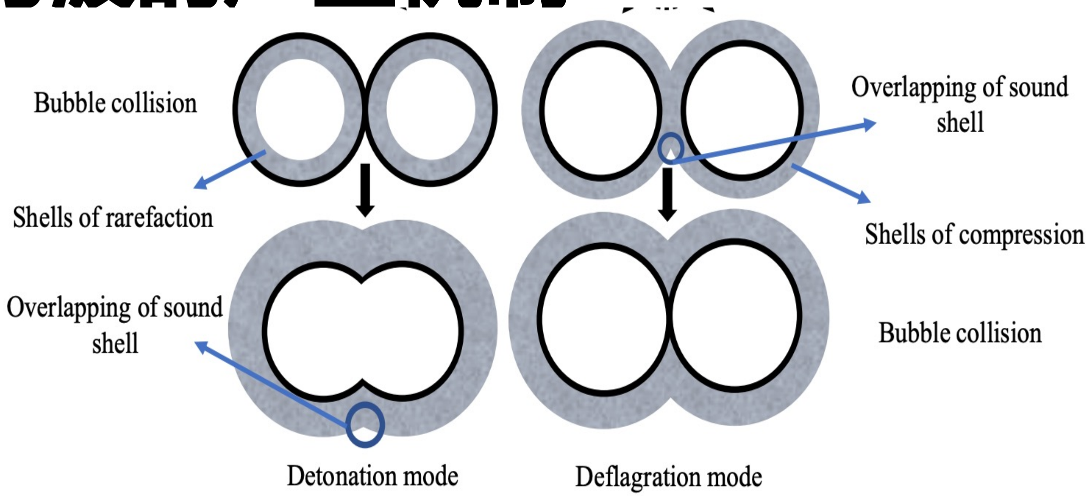
Liu J, Cai RG, Guo ZK. Phys. Rev. Lett., 2021, 126(14): 141303.

Geller M, Hook A, Sundrum R, et al, Phys. Rev. Lett., 2018, 121(20): 201303.

早期宇宙相变引力波的产生机制

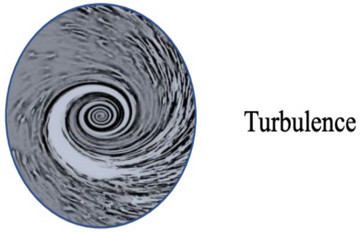


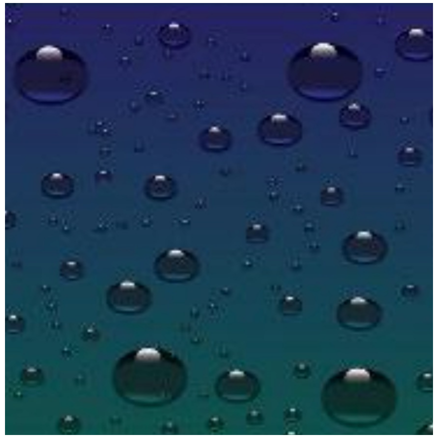
Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045



E. Witten, Phys. Rev. D 30, 272 (1984)
C. J. Hogan, Phys. Lett. B 133, 172 (1983);
M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D 49, 2837 (1994))

EW phase transition GW becomes more interesting and realistic after the discovery of Higgs by LHC and GW by LIGO.





早期宇宙相变引力波的产生机制

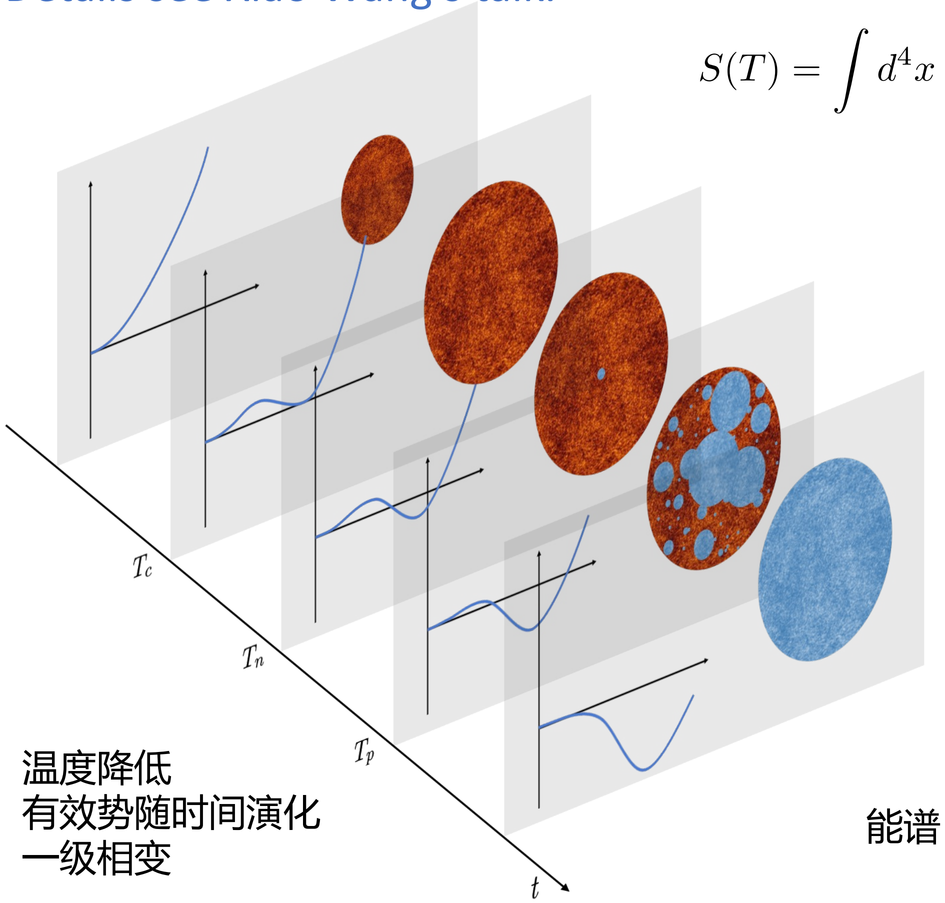
Xiao Wang, **FPH**, Yongping Li, Sound velocity effects on the phase transition gravitational wave spectrum in the Sound Shell Model, *Phys.Rev.D* 105 (2022) 103513

Details see Xiao Wang's talk.

$$S(T)=\int d^4x\left[\frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2+V_{\text{eff}}(\phi,T)\right]$$

相变发生概率

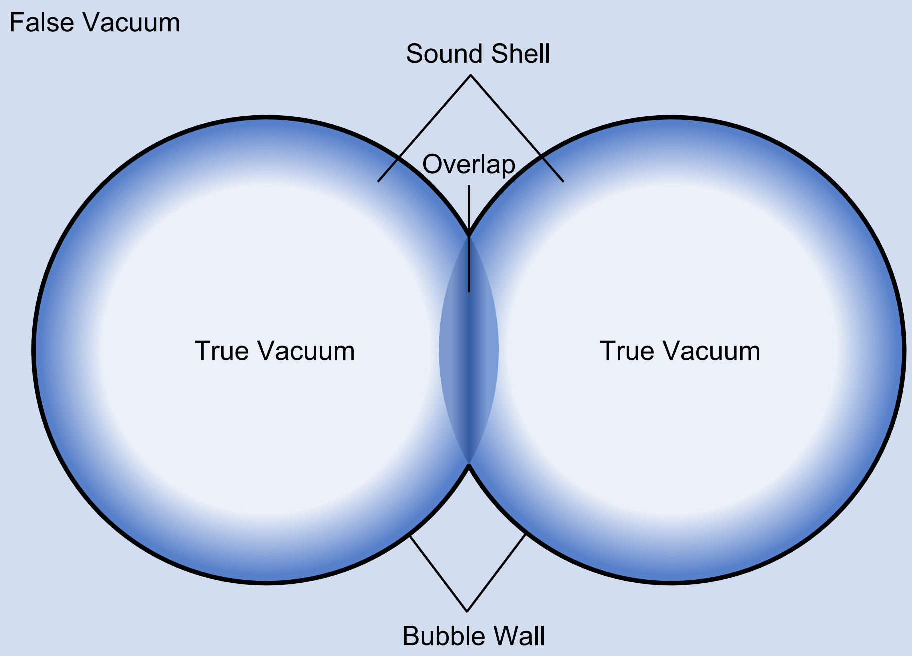
$$\Gamma=\Gamma_0 e^{-S(T)}$$



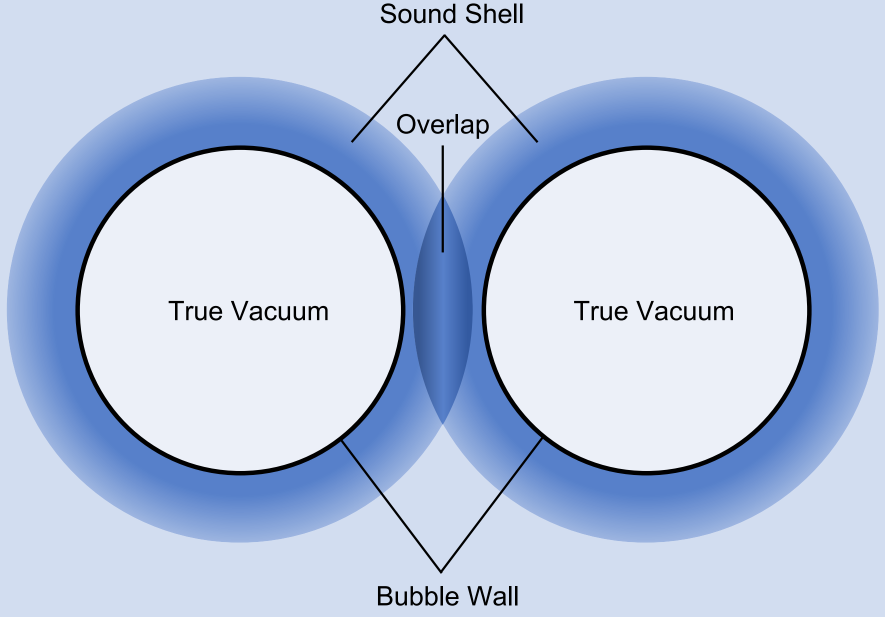
温度降低
有效势随时间演化
一级相变

产生机制：
- 泡泡碰撞
- **声波机制**
- 湍流机制

能谱 $\Omega_{\text{GW}}(f)=\frac{f}{\rho_c}\frac{d\rho_{\text{GW}}}{df}$



Detonation 爆轰



Deflagration 爆燃

早期宇宙引力波产生的一般机制

$$\ddot{h}_{ij}(\mathbf{x},t) + 3\,H\,\dot{h}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2}h_{ij}(\mathbf{x},t) = 16\pi G\,\Pi_{ij}(\mathbf{x},t)$$

Possible sources of tensor anisotropic stress in the early universe

- Scalar field gradients $\Pi_{ij} \sim [\partial_i\phi\partial_j\phi]^{TT}$ eg. Collisions of bubble walls
- Bulk fluid motion $\Pi_{ij} \sim [\gamma^2(\rho+p)v_iv_j]^{TT}$ eg. Sound waves and turbulence in the fluid
- Gauge fields $\Pi_{ij} \sim [-E_iE_j - B_iB_j]^{TT}$ eg. Primordial magnetic fields (MHD turbulence)
- Second order scalar perturbations, Π_{ij} from a combination of $\partial_i\Psi, \partial_i\Phi$
- …

arXiv:1801.04268

相变引力波声波起源的物理分析

重点是计算宇宙早期的高温粒子汤中的形成的声波的速度分布和能量分布，进而可以得到剪切应力张量。

$$P_{\dot{h}} \sim U_{\Pi} \sim \langle \tau\tau \rangle \sim \langle \tilde{v}\tilde{v}\tilde{v}\tilde{v} \rangle = \sum \langle \tilde{v}\tilde{v} \rangle \langle \tilde{v}\tilde{v} \rangle \sim \sum P_v P_v$$

Xiao Wang, **FPH**, Yongping Li, Sound velocity effects on the phase transition gravitational wave spectrum in the Sound Shell Model, *Phys.Rev.D* 105 (2022) 103513

Details see Xiao Wang's talk.

相变引力波的可能迹象

The amplitude and shape of GW spectrum are strongly related to phase transition dynamics.

JCAP 0809, 022 (2008); PRL112, 041301 (2014); PRD92, no. 12,123009 (2015); PRD96, no. 10,103520 (2017); Phys. Rev. D 66, 024030 (2002), Phys. Rev. D 76 (2007) 083002, JCAP 0912, 024 (2009)

$$h^2\Omega_{\rm co}(f) \simeq 1.67\times 10^{-5}\left(\frac{H_* R_*}{(8\pi)^{1/3}}\right)^2 \left(\frac{\kappa_\phi \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{1/3} \frac{0.11 v_w}{0.42+v_w^2} \frac{3.8(f/f_{\rm co})^{2.8}}{1+2.8(f/f_{\rm co})^{3.8}}$$

Bubble collision

$$h^2\Omega_{\rm sw}(f) \simeq 1.64\times 10^{-6}(H_*\tau_{\rm sw})(H_* R_*)\left(\frac{\kappa_v \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{1/3} (f/f_{\rm sw})^3 \left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}$$

Sound wave

$$h^2\Omega_{\rm turb}(f) \simeq 1.14\times 10^{-4} H_* R_* \left(\frac{\kappa_{\rm turb}\alpha}{1+\alpha}\right)^{3/2} \left(\frac{100}{g_*}\right)^{1/3} \frac{(f/f_{\rm turb})^3}{(1+f/f_{\rm turb})^{11/3}(1+8\pi f/H_*)}$$

Turbulence

Searching For Gravitational Waves From Cosmological Phase Transitions With The NANOGrav 12.5-year dataset (The NANOGrav Collaboration) arXiv:2104.13930 .
"We find that the data can be explained in terms of a strong first order phase transition taking place at temperatures below the electroweak scale."

相变引力波各向异性

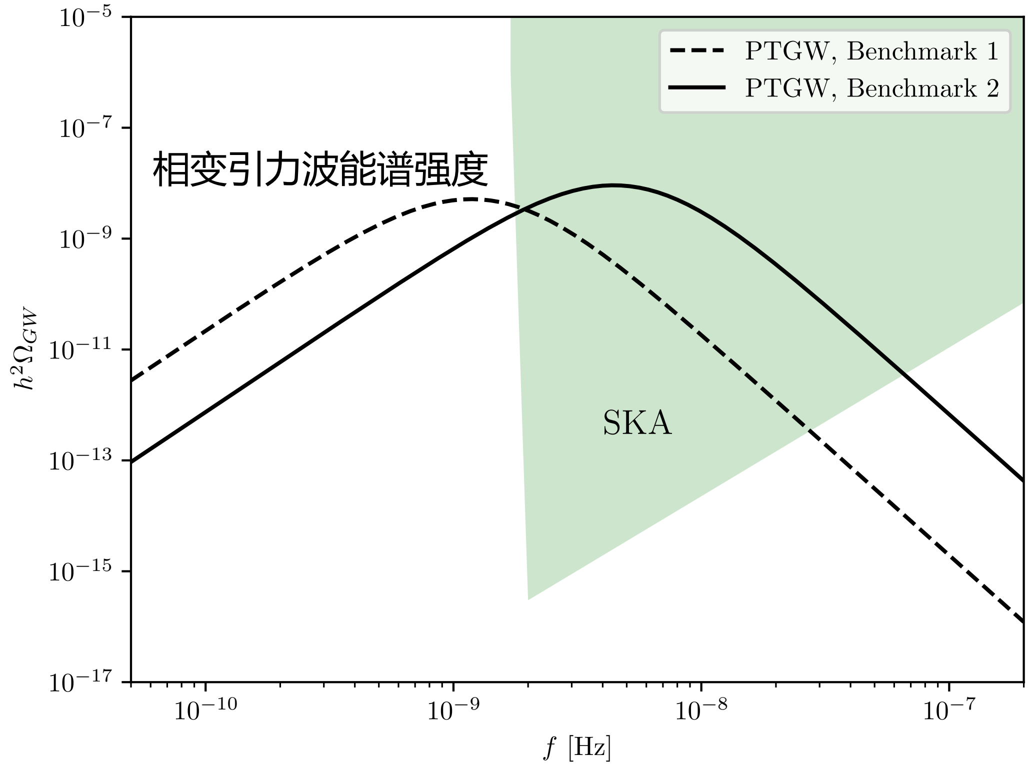
- 声波机制的相变引力波 $H_*^2 = \rho/3M_{\rm pl}^2$

$$h^2\Omega_{\rm GW}(f) \simeq 1.64\times10^{-6}\left(\frac{4}{3}\right)^{\frac{1}{2}} \boxed{(H_*R_*)^2} \left(\frac{\kappa_v\alpha}{1+\alpha}\right)^{\frac{3}{2}} \times \left(\frac{100}{g_*}\right)^{\frac{1}{3}} (f/f_{\rm sw})^3 \left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{\frac{7}{2}}$$

$$f_{\rm sw} \simeq 2.6\times10^{-5}\ {\rm Hz}\frac{1}{H_*R_*}\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$ 峰值频率

- 模型参数选取 类QCD相变的暗物质模型

相变强度	$\alpha = 0.5$	
总自由度	$g_* = 10$	
效率参数	$\kappa_v \approx 0.44$	
泡壁速度	$v_b = 0.95$	
相变温度	$T_* = 1$ MeV	$T_* = 5$ MeV
泡泡距离	$H_*R_* = 0.15$	$H_*R_* = 0.2$
	Benchmark 1	Benchmark 2



相变引力波各向异性

- 涨落的描述

Phys. Lett. B, 2017, 771: 9-12 ; Phys. Rev. D, 2019, 100(12): 121501 ; Phys. Rev. D, 2021, 103 (2): 023522 ; Phys. Rev. Lett., 2021, 127(27): 271301

分布函数 $$f(\eta, \boldsymbol{x}, \boldsymbol{p}) = \bar{f}(\eta, p) - p\frac{\partial \bar{f}(\eta, p)}{\partial p}\boxed{\mathcal{G}(\eta, \boldsymbol{x}, \hat{\boldsymbol{p}})}$$

$\mathcal{G}(\eta, \boldsymbol{x}, \hat{\boldsymbol{p}})$

表征引力子分布的不均匀性

- 涨落的演化方程

共形牛顿规范 $$ds^2 = -(1+2\Psi)dt^2 + a^2(1-2\Phi)\delta_{ij}dx^i dx^j$$

Boltzmann方程 $$\frac{df}{dt} = C[f] \Rightarrow \boxed{\mathcal{G}' + ik\mu\mathcal{G} = \Phi' - ik\mu\Psi}$$

$$\begin{aligned}
\mathcal{G}(\eta_0, k, \mu) =& \mathcal{G}(\eta_{\rm pt}, k, \mu)e^{ik\mu(\eta_{\rm pt}-\eta_0)} + \int_{\eta_{\rm pt}}^{\eta_0} d\eta\left[\Phi'(\eta, k) - ik\mu\Psi(\eta, k)\right] e^{ik\mu(\eta-\eta_0)} \\
=& \mathcal{G}(\eta_{\rm pt}, k, \mu)e^{ik\mu(\eta_{\rm pt}-\eta_0)} + \int_{\eta_{\rm pt}}^{\eta_0} d\eta\left[\Phi'(\eta, k)e^{ik\mu(\eta-\eta_0)} - \frac{d}{d\eta}\left(\Psi(\eta, k)e^{ik\mu(\eta-\eta_0)}\right) + \Psi'(\eta, k)e^{ik\mu(\eta-\eta_0)}\right] \\
=& \boxed{\underbrace{\left[\mathcal{G}(\eta_{\rm pt}, k) + \Psi(\eta_{\rm pt}, k)\right] e^{ik\mu(\eta_{\rm pt}-\eta_0)}}_{\rm SW}} + \boxed{\underbrace{\int_{\eta_{\rm pt}}^{\eta_0} d\eta\left[\Phi'(\eta, k) + \Psi'(\eta, k)\right] e^{ik\mu(\eta-\eta_0)}}_{\rm ISW}}
\end{aligned}$$

相变引力波各向异性

- 各向异性的角功率谱
  - SW效应

$$\left(\mathcal{G}+\Psi\right)\left(\eta_{\rm pt},k\right)=-\frac{1}{3}\mathcal{R}(k)$$ 辐射主导时期

$$\mathcal{G}_\ell^{\rm SW}(\eta_0,k)=\left(\mathcal{G}+\Psi\right)\left(\eta_{\rm pt},k\right)j_\ell[k(\eta_{\rm pt}-\eta_0)]$$

$$C_\ell^{\mathcal{G},{\rm SW}}=\frac{4\pi}{9}\int_0^\infty\frac{dk}{k}P_{\mathcal{R}}(k)j_\ell^2\left[k\left(\eta_0-\eta_{\rm pt}\right)\right]$$

  - ISW效应

$$\int_{\eta_{\rm pt}}^{\eta_0}d\eta\left[\Phi'(\eta,k)+\Psi'(\eta,k)\right]e^{ik\mu(\eta-\eta_0)}$$

$$\mathcal{G}_\ell^{\rm ISW}\left(\eta_0,k\right)=\int_{\eta_{\rm pt}}^{\eta_0}d\eta\left(\Phi'+\Psi'\right)(\eta,k)j_\ell\left[k\left(\eta-\eta_0\right)\right]$$

小尺度下 $$\left(\Phi'+\Psi'\right)(\eta,k)\approx-\left(\Phi+\Psi\right)\left(\eta_{\rm pt},k\right)\delta\left(\eta-\eta_k\right)$$

$$\left(\Phi+\Psi\right)\left(\eta_{\rm pt},k\right)=-\frac{4}{3}\mathcal{R}(k)$$

$$C_\ell^{\mathcal{G},{\rm ISW}}=\frac{64\pi}{9}\int_0^\infty\frac{dk}{k}P_{\mathcal{R}}(k)j_\ell^2\left[k\left(\eta_0-\eta_{\rm pt}\right)\right]$$

相变引力波各向异性

- 各向异性的角功率谱
  - 大尺度，SW效应主导

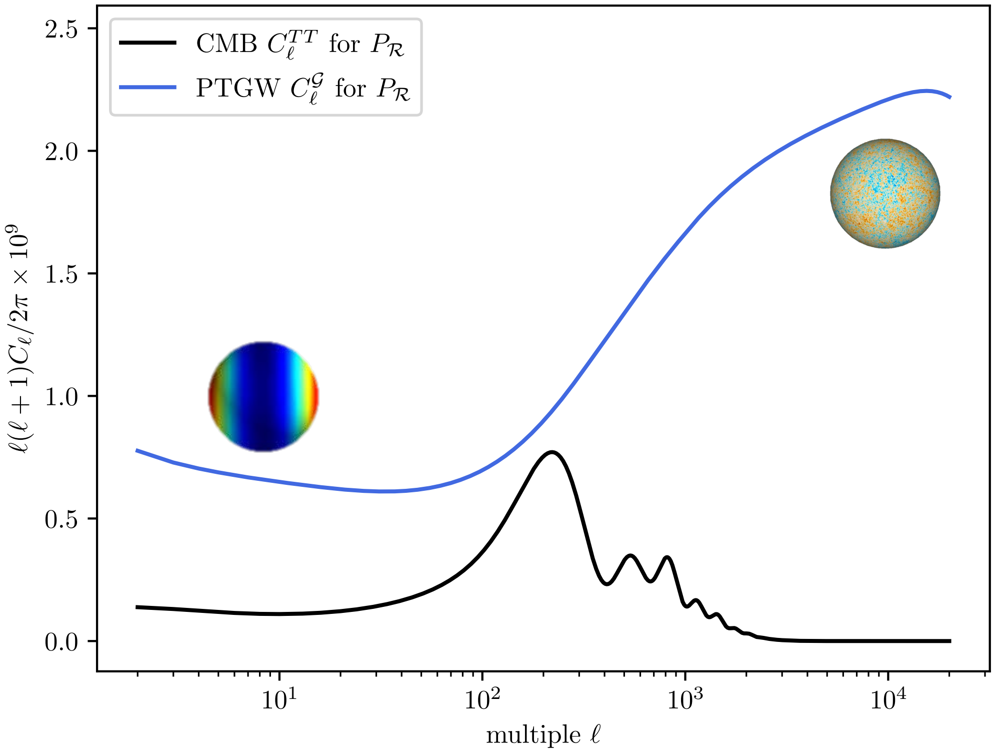
$$C_\ell^{\mathcal{G}} \approx C_\ell^{\mathcal{G},\mathrm{SW}} = \frac{4\pi}{9}\int_0^\infty \frac{dk}{k} P_{\mathcal{R}}(k) j_\ell^2\left[k\left(\eta_0-\eta_{\mathrm{pt}}\right)\right]$$

  - 小尺度，SW+ISW效应，ISW效应主导

$$\mathcal{G}_\ell\left(\eta_0, k\right) = \left(\mathcal{G}_\ell^{\mathrm{SW}}+\mathcal{G}_\ell^{\mathrm{ISW}}\right)\left(\eta_0, k\right) \approx \left[(\mathcal{G}+\Psi)-(\Phi+\Psi)\right]\left(\eta_{pt}, k\right) j_\ell\left[k\left(\eta_{\mathrm{pt}}-\eta_0\right)\right]$$

$$C_\ell^{\mathcal{G}} = 4\pi\int_0^\infty \frac{dk}{k} P_{\mathcal{R}}(k) j_\ell^2\left[k\left(\eta_0-\eta_{\mathrm{pt}}\right)\right]$$

Planck: $\ln(10^{10}A_s) = 3.040 \pm 0.016 \qquad n_s = 0.9626 \pm 0.0057$



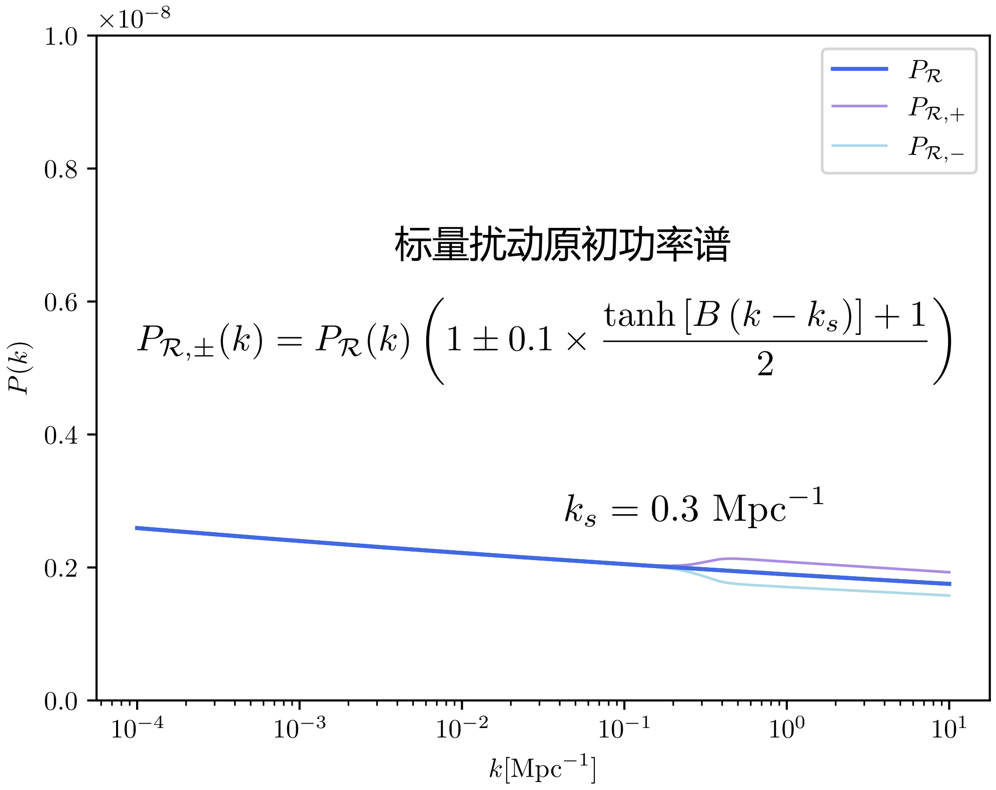
角功率谱

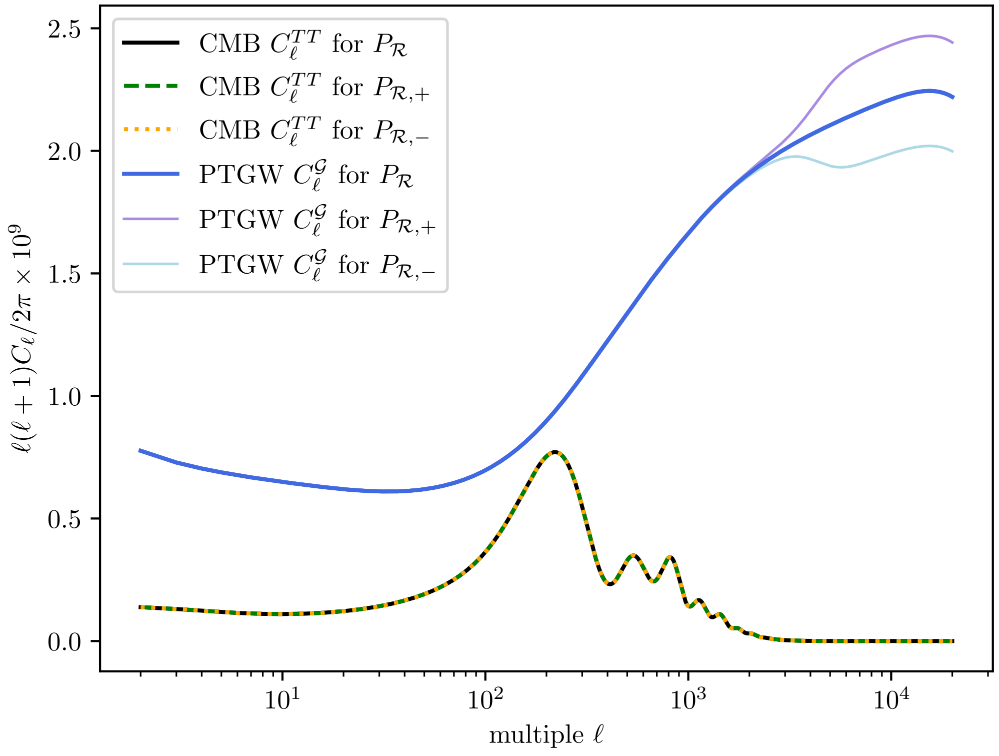
✓ PTGW在各尺度均呈现比CMB温度更强的各向异性

✓ PTGW各向异性的角功率谱在小尺度有抬升

相变引力波各向异性

- 各向异性的角功率谱





✓ PTGW的各向异性保留了更多的小尺度原初扰动的信息

相变引力波各向异性

- 相变引力波能谱各向异性

$$\rho_{\rm GW}(\eta,\boldsymbol{x}) = \int d^3\boldsymbol{p}\, p f(\eta,\boldsymbol{x},\boldsymbol{p}) = \int dp d\hat{\boldsymbol{p}} p^3 f(\eta,\boldsymbol{x},p,\hat{\boldsymbol{p}})$$

$$\Omega_{\rm GW}(\eta,\boldsymbol{x},p) = \int \frac{d\hat{\boldsymbol{p}}}{4\pi}\bar{\Omega}_{\rm GW}(\eta,p)\left[1+\delta_{\rm GW}(\eta,\boldsymbol{x},p,\hat{\boldsymbol{p}})\right]$$

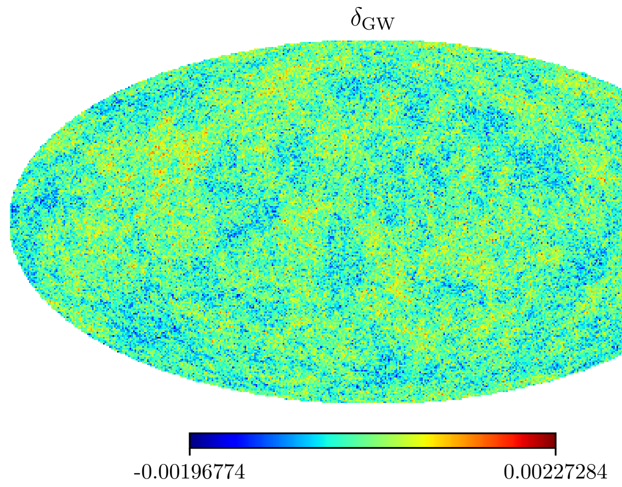
$$= \int d\hat{\boldsymbol{p}}\frac{p^4}{\rho_c}\left[\bar{f}(\eta,p) - p\frac{\partial \bar{f}(\eta,p)}{\partial p}\mathcal{G}(\eta,\boldsymbol{x},\hat{\boldsymbol{p}})\right]$$

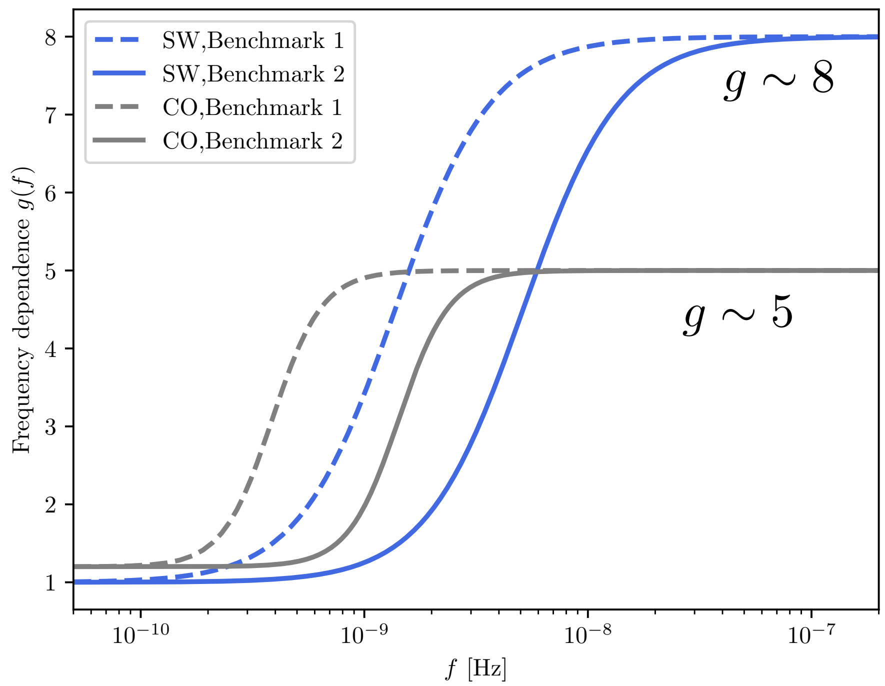
能谱各向异性 $\delta_{\rm GW} = \dfrac{\delta\Omega_{\rm GW}(\eta,\boldsymbol{x},p,\hat{p})}{\bar{\Omega}_{\rm GW}(\eta,p)}$

$$\delta_{\rm GW}(\eta,\boldsymbol{x},p,\hat{p}) = \left[4 - \frac{\partial \ln \bar{\Omega}_{\rm GW}(\eta,p)}{\partial \ln p}\right]\mathcal{G}(\eta,\boldsymbol{x},\hat{p})$$

$$C_l^{\delta_{\rm GW}}(p) = g^2(p) C_l^{\mathcal{G}}$$

不同机制的相变引力波的各向异性具有不同的频率依赖





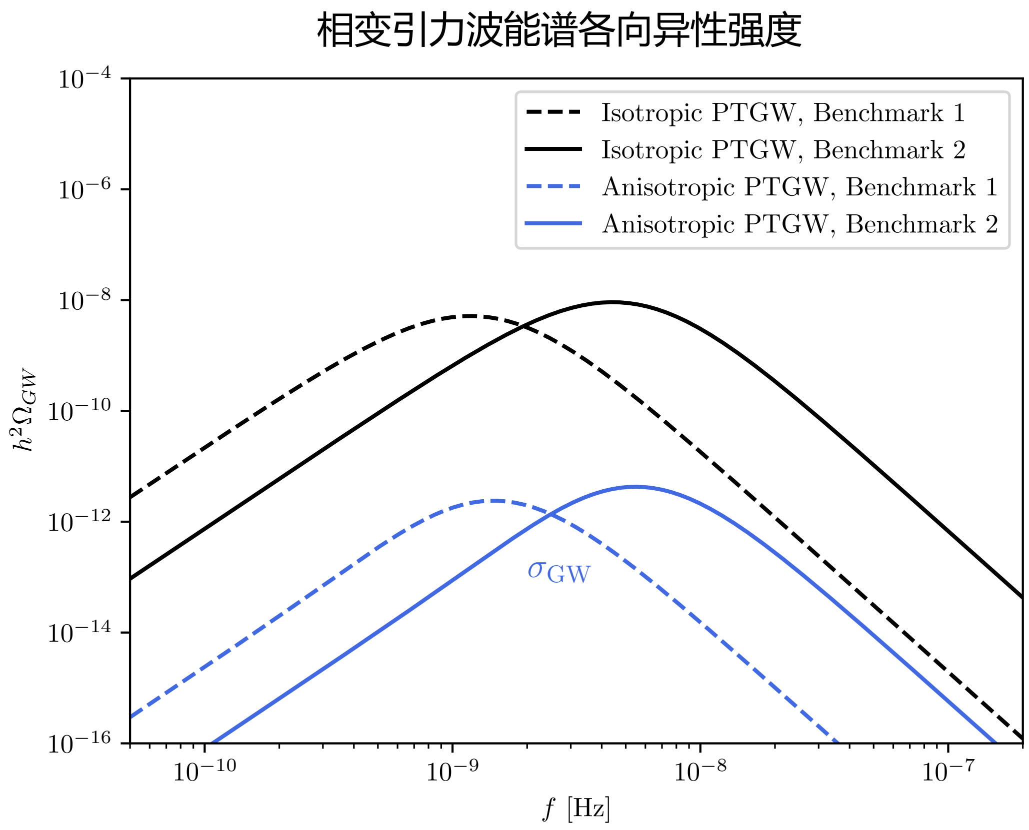
相变引力波各向异性

- 相变引力波能谱各向异性

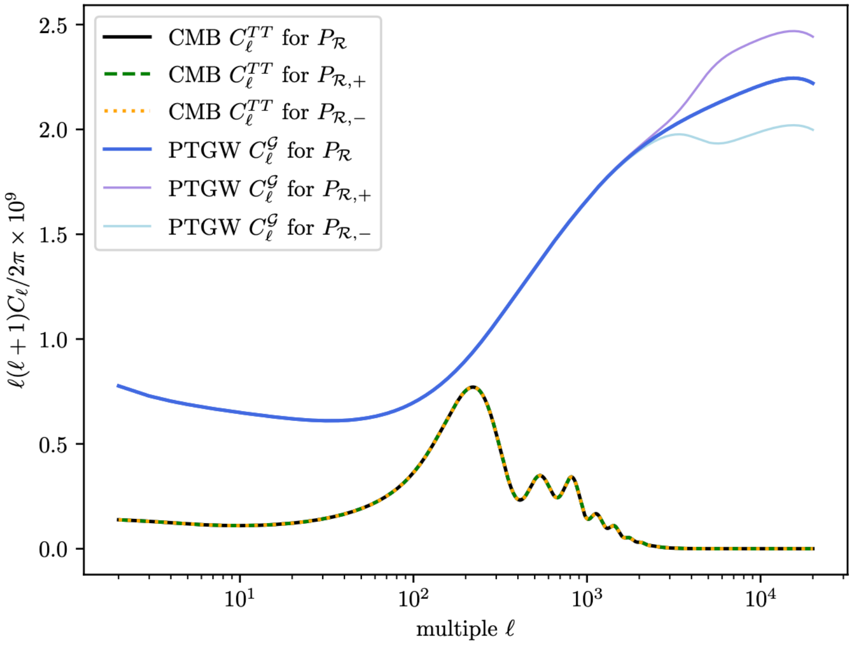
$$\mathrm{Var}^{\mathcal{G}} = \frac{1}{4\pi}\sum_{\ell}(2\ell+1)C_{\ell}^{\mathcal{G}}$$

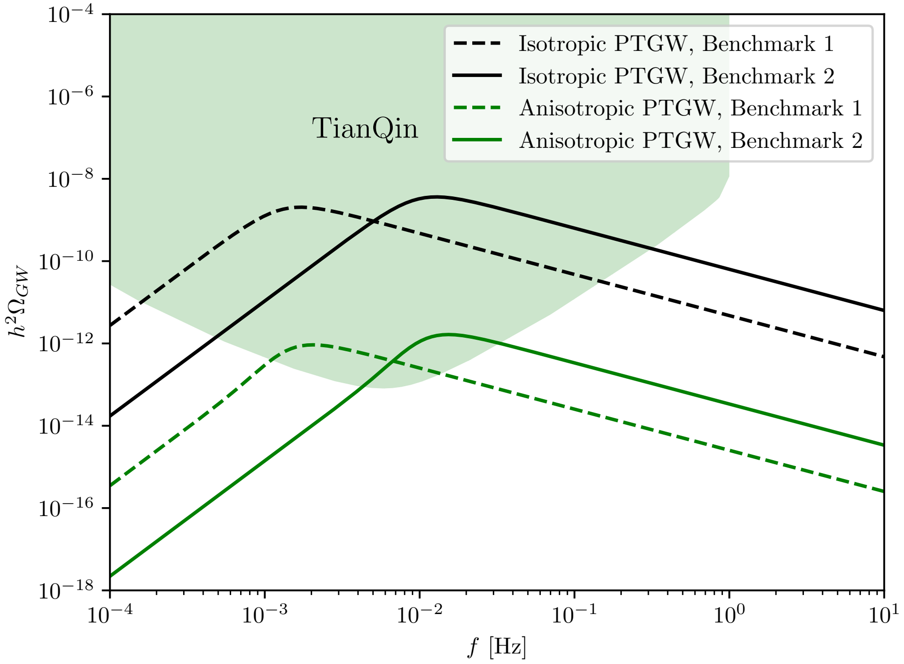
$$\sigma_{\mathrm{GW}}(p) \equiv h^2\Omega_{\mathrm{GW}}(p)\sqrt{\mathrm{Var}^{\delta_{\mathrm{GW}}}(p)}$$

CMB温度各向异性	4×10^{-5}
PTGW各向异性	1×10^{-4}
PTGW能谱的各向异性	8×10^{-4} ($> f_{\mathrm{sw}}$)



PTGW anisotropy and its implication for primordial seeds of our universe





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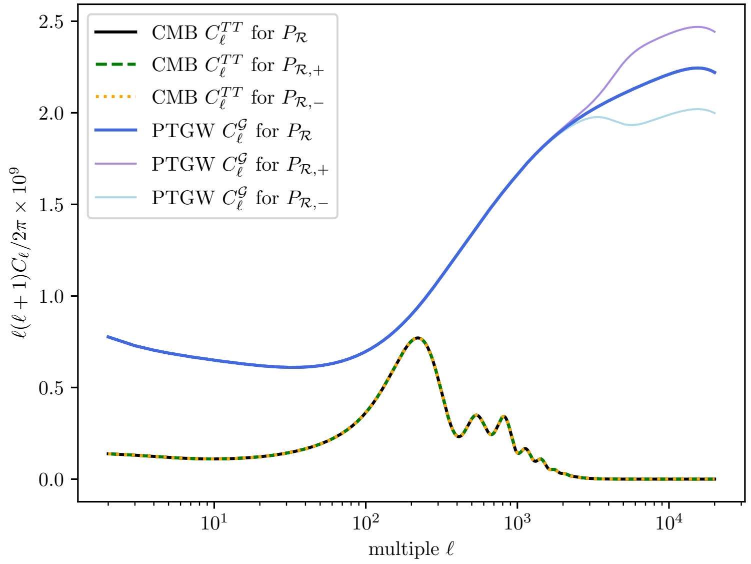
Yongping Li, **FPH**, Xiao Wang, work in progress

总结



PTGW anisotropy and its implication
for primordial seeds of our universe

- 相变引力波在各尺度均呈现出比CMB温度更强的各向异性
- 相变引力波各向异性可能保留了更多的原初密度扰动的小尺度信息
- 将更加深入的讨论引力波各向异性以及实验观测能力

THnaks!

Comments and collaborations are welcome!

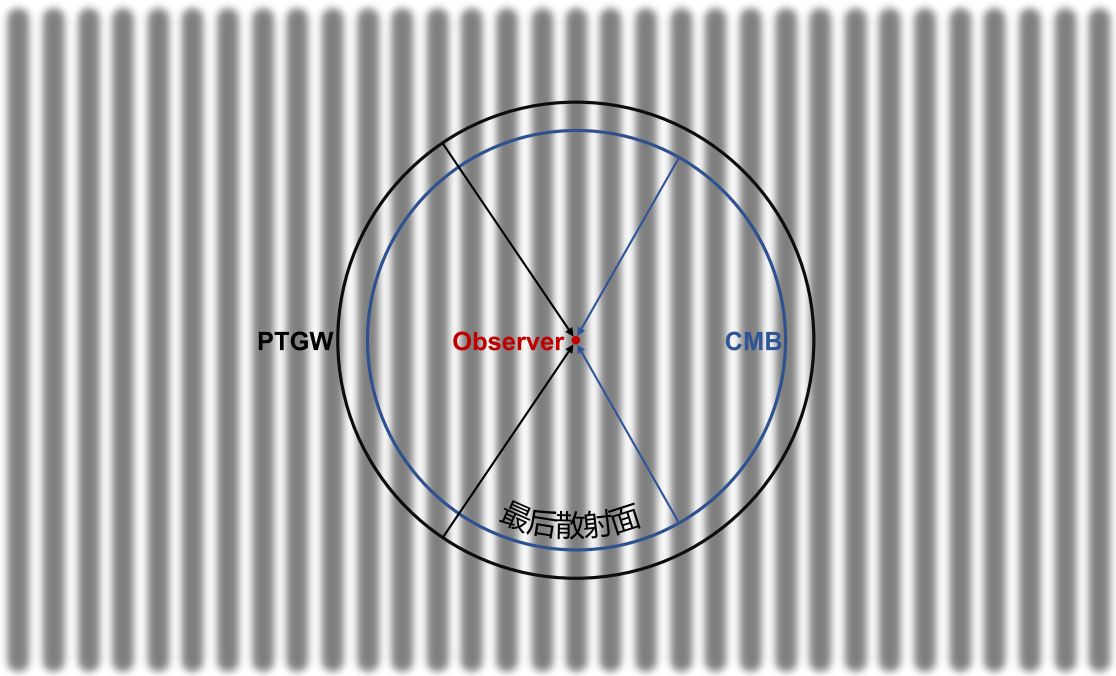
中山大学·天琴中心 http://tianqin.sysu.edu.cn/

Email:huangfp8@sysu.edu.cn

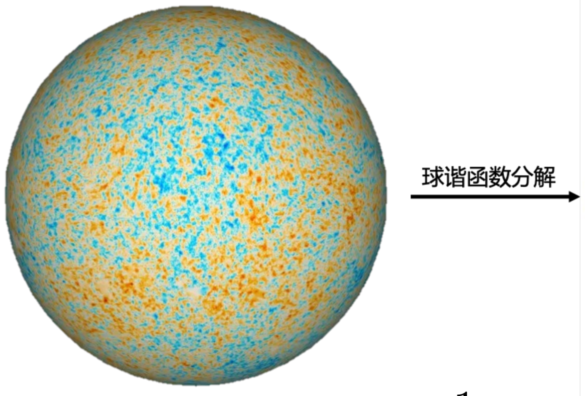
Backup Slides

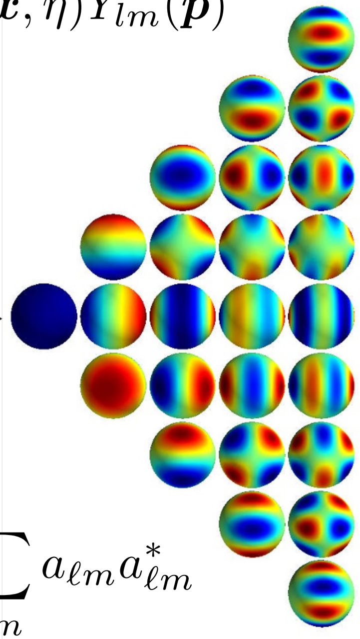
宇宙演化历史

- 原初扰动与各向异性



$$\Theta(\boldsymbol{x}, \hat{\boldsymbol{p}}, \eta) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm}(\boldsymbol{x}, \eta) Y_{lm}(\hat{\boldsymbol{p}})$$

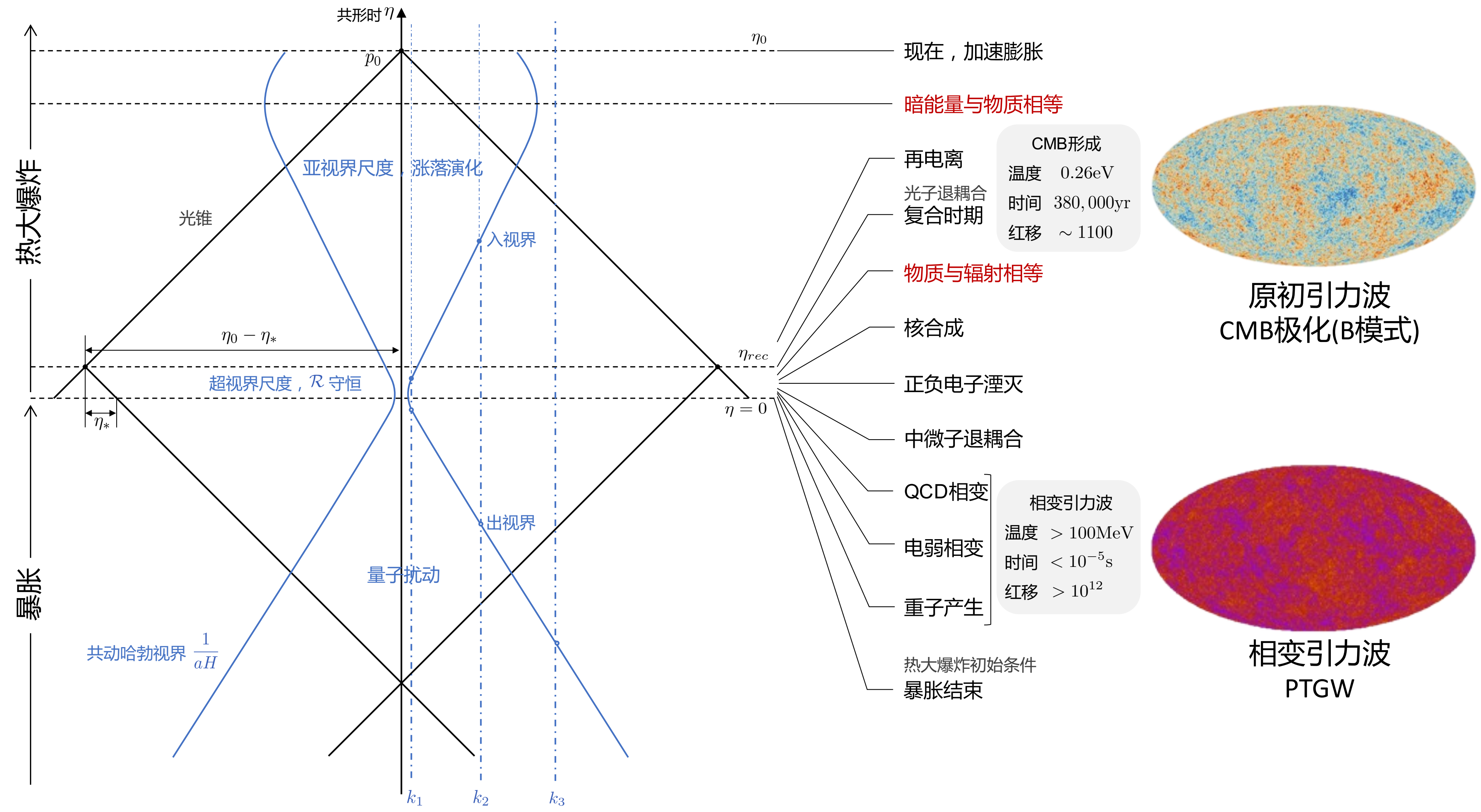




$$C_\ell^{\mathrm{obs}} = \frac{1}{2\ell+1} \sum_{\ell m} a_{\ell m} a_{\ell m}^*$$

角功率谱 $$C_l = \frac{2}{\pi} \int_0^\infty dk k^2 P_{\mathcal{R}}(k) \left|\mathcal{T}_l(k)\right|^2$$

宇宙演化历史



共形时 η
η0
p0
现在，加速膨胀
暗能量与物质相等
亚视界尺度，涨落演化
光锥
入视界
η0 − η*
ηrec
超视界尺度，R 守恒
η = 0
η*
出视界
量子扰动
共动哈勃视界 1/aH
k1
k2
k3
热大爆炸
暴胀
再电离
光子退耦合
复合时期
CMB形成
温度 0.26eV
时间 380,000yr
红移 ~ 1100
物质与辐射相等
核合成
正负电子湮灭
中微子退耦合
QCD相变
电弱相变
重子产生
相变引力波
温度 > 100MeV
时间 < 10^−5 s
红移 > 10^12
热大爆炸初始条件
暴胀结束
原初引力波
CMB极化(B模式)
相变引力波
PTGW

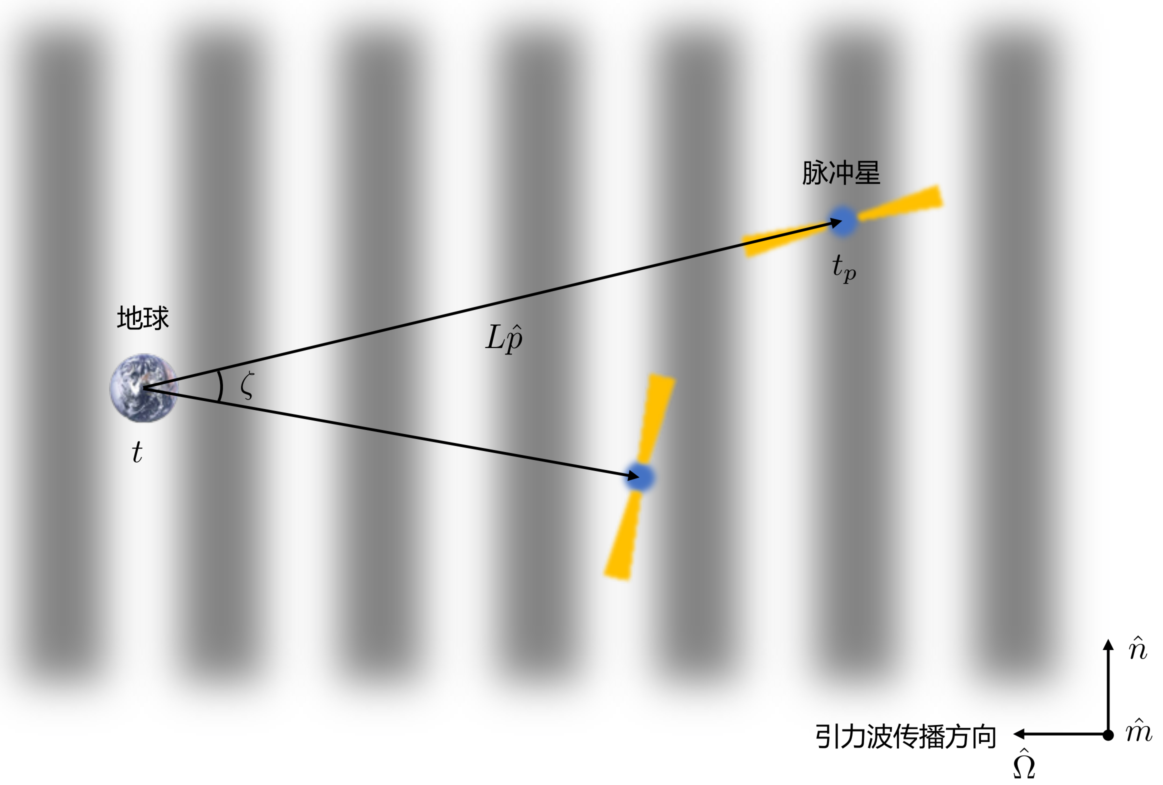
相变引力波各向异性

- 脉冲星计时阵列探测引力波背景的各向异性
  - 频率红移

$$z(t,\hat{\Omega}) \equiv \frac{\nu(t)-\nu_0}{\nu_0} = \frac{1}{2}\frac{\hat{p}^i\hat{p}^j}{1+\hat{\Omega}\cdot\hat{p}}\Delta h_{ij}(t,\hat{\Omega})$$
$$= \sum_{A=+,\times} \Delta h_A(t,\hat{\Omega})F^A(\hat{\Omega})$$

$$\Delta h_A(t,\hat{\Omega}) \equiv h_A(t,\hat{\Omega}) - h_A\left(t_p,\hat{\Omega}\right)$$
$$= \sum_{A=+,\times}\int_{-\infty}^{\infty} df\ e^{i2\pi ft}\left[1-e^{-i2\pi fL(1+\hat{\Omega}\cdot\hat{p})}\right]\ h_A(f)$$

引力波方向束 $F^A(\hat{\Omega}) = \frac{1}{2}\frac{\hat{p}^i\hat{p}^j}{1+\hat{\Omega}\cdot\hat{p}}e^A_{ij}(\hat{\Omega})$



相变引力波各向异性

- 脉冲星计时阵列探测引力波背景的各向异性
  - 计时残差

$$r(t)=\int^{t}dt'\frac{d\hat{\Omega}}{4\pi}\,z\left(t',\hat{\Omega}\right)$$

高斯随机噪声

脉冲星对关联函数 $\langle r_a^*(t_1)\,r_b(t_2)\rangle$

$I=(a,b)$

$$\begin{aligned}\langle r_a^*(t_1)\,r_b(t_2)\rangle &=\int^{t_1}dt\int^{t_2}dt'\int\frac{d\hat{\Omega}}{4\pi}\int\frac{d\hat{\Omega}'}{4\pi}\left\langle z_a^*\left(t,\hat{\Omega}\right)z_b\left(t',\hat{\Omega}'\right)\right\rangle\\ &=\int^{t_1}dt\int^{t_2}dt'\int\frac{d\hat{\Omega}}{4\pi}\int\frac{d\hat{\Omega}'}{4\pi}\int_{-\infty}^{+\infty}df\int_{-\infty}^{+\infty}df'\\ &\quad\times e^{-i2\pi t(f-f')}\left[1-e^{i2\pi fL_a\left(1+\hat{\Omega}\cdot\hat{p}_a\right)}\right]\left[1-e^{-i2\pi f'L_b\left(1+\hat{\Omega}'\cdot\hat{p}_b\right)}\right]\\ &\quad\times\sum_{A,A'=+,\times}\left[F_a^A(\hat{\Omega})F_b^{A'}(\hat{\Omega}')\right]\left\langle h_A^*(f,\hat{\Omega})h_{A'}\left(f',\hat{\Omega}'\right)\right\rangle\\ &=\int^{t_1}dt\int^{t_2}dt'\int_{-\infty}^{+\infty}df\,e^{-i2\pi f\left(t-t'\right)}H(f)\Gamma_{ab}(f)\\ &=\int_{-\infty}^{+\infty}df\,e^{-i2\pi f(t)}\frac{H(f)}{f^2}\Gamma_{ab}(f)\end{aligned}$$

引力波的特征信号：重叠减弱函数

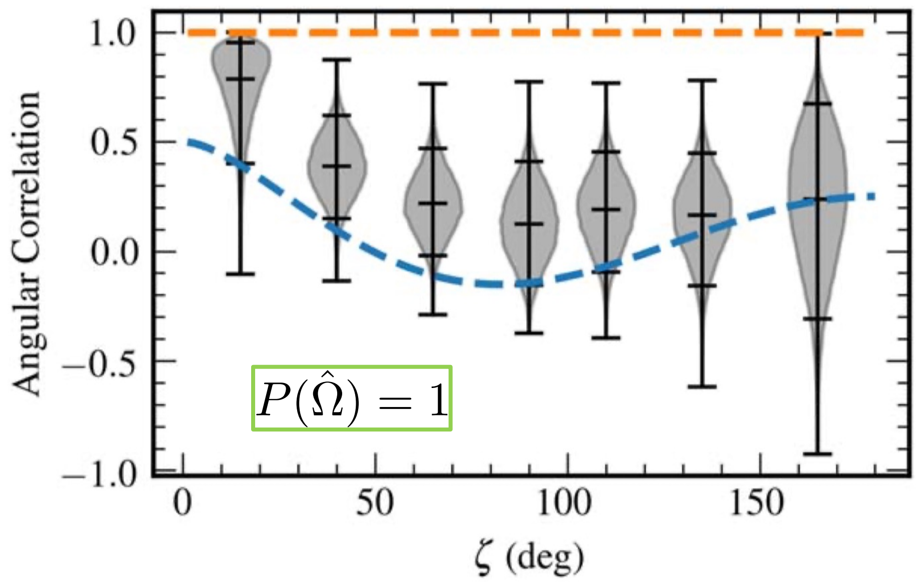
假设：同偏振、同频率、同方向的引力波存在关联，且方向依赖可以分离

$$\left\langle h_A^*(f,\hat{\Omega})h_{A'}\left(f',\hat{\Omega}'\right)\right\rangle=\delta_{AA'}\delta^2(\hat{\Omega},\hat{\Omega}')\delta(f-f')H(f)P(\hat{\Omega})$$

相变引力波各向异性

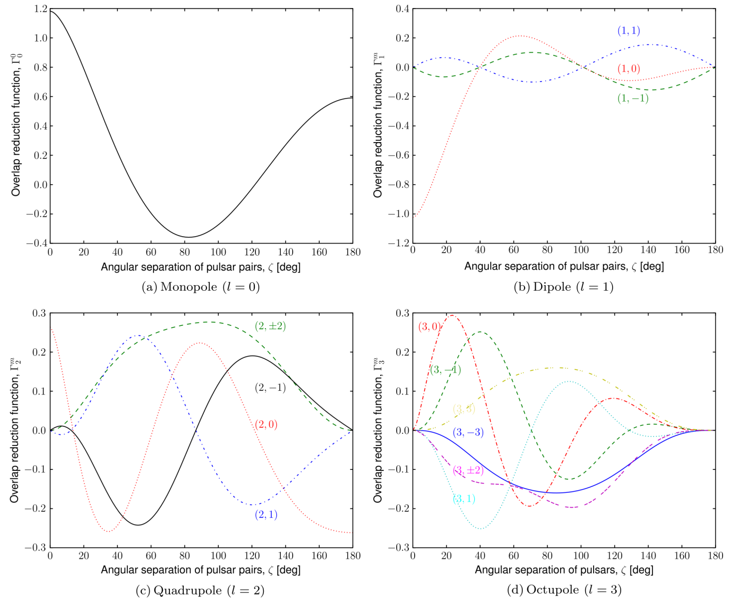
- 脉冲星计时阵列探测引力波背景的各向异性
  - 重叠减弱函数

$$\Gamma_{ab}(f) = \int \frac{d\hat{\Omega}}{4\pi}\, \kappa_{ab}(f,\hat{\Omega}) P(\hat{\Omega}) \sum_{A=+,\times} \left[F_a^A(\hat{\Omega}) F_b^A(\hat{\Omega})\right]$$



HD曲线

NANOGrav: Astrophys. J. Lett., 2020, 905(2): L34



广义叠加减弱函数

Phys. Rev. D, 2013, 88(6): 062005

相变引力波各向异性

- 脉冲星计时阵列探测引力波背景的各向异性
  - 计时残差交叉关联

$$r(t) \equiv \int df e^{2\pi i f t} r(f)$$

$$\langle r_a(f) r_b^*(f') \rangle = \delta(f' - f) \mathcal{R}_{ab}(f)$$

频域空间的关联函数 $$\mathcal{R}_{ab}(f) = \frac{1}{(4\pi f)^2} \int \frac{d\hat{\Omega}}{4\pi} \frac{\hat{p}_a^i \hat{p}_a^j \hat{p}_b^m \hat{p}_b^n \langle h_{ij}(f,\hat{\Omega}) h_{mn}(f,\hat{\Omega}') \rangle}{(1+\hat{\Omega}\cdot\hat{p}_a)(1+\hat{\Omega}\cdot\hat{p}_b)} \kappa_{ab}(f,\hat{\Omega})$$

$$\approx \frac{1+\delta_{ab}}{(4\pi f)^2} \int \frac{d\hat{\Omega}}{4\pi} \gamma_{ab}(\hat{\Omega}) \mathcal{I}(f,\hat{\Omega})$$

构造估计量

计时反馈函数

仅与脉冲星计时阵列自身相关　　仅与引力波背景相关

$$\gamma_{ab}(\hat{\Omega}) = 4 \sum_{A=+,\times} F_a^A(\hat{\Omega}) F_b^A(\hat{\Omega}) = 2 \frac{\left[\hat{p}_a \cdot \hat{p}_b - (\hat{\Omega}\cdot\hat{p}_a)(\hat{\Omega}\cdot\hat{p}_b)\right]^2}{(1+\hat{\Omega}\cdot\hat{p}_a)(1+\hat{\Omega}\cdot\hat{p}_b)} - (1-\hat{\Omega}\cdot\hat{p}_a)(1-\hat{\Omega}\cdot\hat{p}_b)$$

$$\Gamma_{ab} = \frac{1+\delta_{ab}}{4} \gamma_{ab}(\hat{\Omega}) \cdot P(\hat{\Omega})$$

$$\mathcal{I}(f,\hat{\Omega}) = \frac{H(f)}{f} P(\hat{\Omega})$$

$$\mathcal{R}_{ab}(f) = \frac{1}{4\pi^2 f^3} \Gamma_{ab} H(f)$$

相变引力波各向异性

- 脉冲星计时阵列探测引力波背景的各向异性
  - 通过计时残差的互关联估计引力波及其各向异性的强度

估计量 $\widehat{\mathcal{I}}_f(\hat{\Omega}) \equiv (4\pi f)^2 \sum_I \widehat{\mathcal{R}}_{I,f}\gamma_I^*(\hat{\Omega}), \qquad \gamma_I^* \cdot \gamma_J = \delta_{IJ}$　　$I = (a,b)$ 脉冲星对

  - 似然函数与信噪比

$$\mathcal{L}(\mathcal{I}) \propto \exp\left[-\frac{1}{2}\int df(\mathcal{I}-\hat{\mathcal{I}})(f,\hat{\Omega})\cdot\mathcal{C}^{-1}(f,\hat{\Omega},\hat{\Omega}')\cdot(\mathcal{I}-\hat{\mathcal{I}})(f,\hat{\Omega}')\right]$$

$$\begin{aligned}\mathrm{SNR}^2[\mathcal{I}(f,\hat{\Omega})] =& \mathcal{I}(f,\hat{\Omega})\cdot\mathcal{C}^{-1}(f,\hat{\Omega},\hat{\Omega}')\cdot\mathcal{I}(f,\hat{\Omega}') \\ =& \sum_{a\neq b} 2T_{ab}\left[\frac{\gamma_{ab}(\hat{\Omega})\cdot\mathcal{I}(f,\hat{\Omega})}{(4\pi f)^2\sigma_a(f)\sigma_b(f)}\right]^2\end{aligned}$$

$$\mathrm{SNR}^2[\mathcal{I}_{lm,f}(\hat{\Omega})] = \int_{f-\Delta f/2}^{f+\Delta f/2} df \sum_{a\neq b} 2T_{ab}\mathcal{I}_0^2(f)\,|a_{lm}|^2\left[\frac{\gamma_{ab}(\hat{\Omega})\cdot Y_{lm}(\hat{\Omega})}{(4\pi f)^2\sigma_a(f)\sigma_b(f)}\right]^2$$

相变引力波各向异性

- 脉冲星计时阵列探测引力波背景的各向异性
  - 信噪比

$$\mathrm{SNR}^2[\mathcal{I}_{lm,f}(\hat{\Omega})] = \int_{f-\Delta f/2}^{f+\Delta f/2} df \, \frac{N_{\mathrm{pair}}}{(4\pi f)^4} \frac{2T}{\sigma^4(f)} \mathcal{I}(f,\hat{\Omega}) \cdot F\left(\hat{\Omega}\cdot\hat{\Omega}'\right) \cdot \mathcal{I}(f,\hat{\Omega}')$$

假设脉冲星均匀分布

$$= 8\pi T \left|a_{lm}\right|^2 F_l N_{\mathrm{pair}} \int_{f-\Delta f/2}^{f+\Delta f/2} df \, \frac{\mathcal{I}_0^2(f)}{[4\pi f \sigma(f)]^4}$$

$$N_{\mathrm{pair}} = \frac{N_{\mathrm{pulsar}}(N_{\mathrm{pulsar}}-1)}{2}$$

$$N_{\mathrm{multiple}} = (l_{\mathrm{max}}+1)^2$$

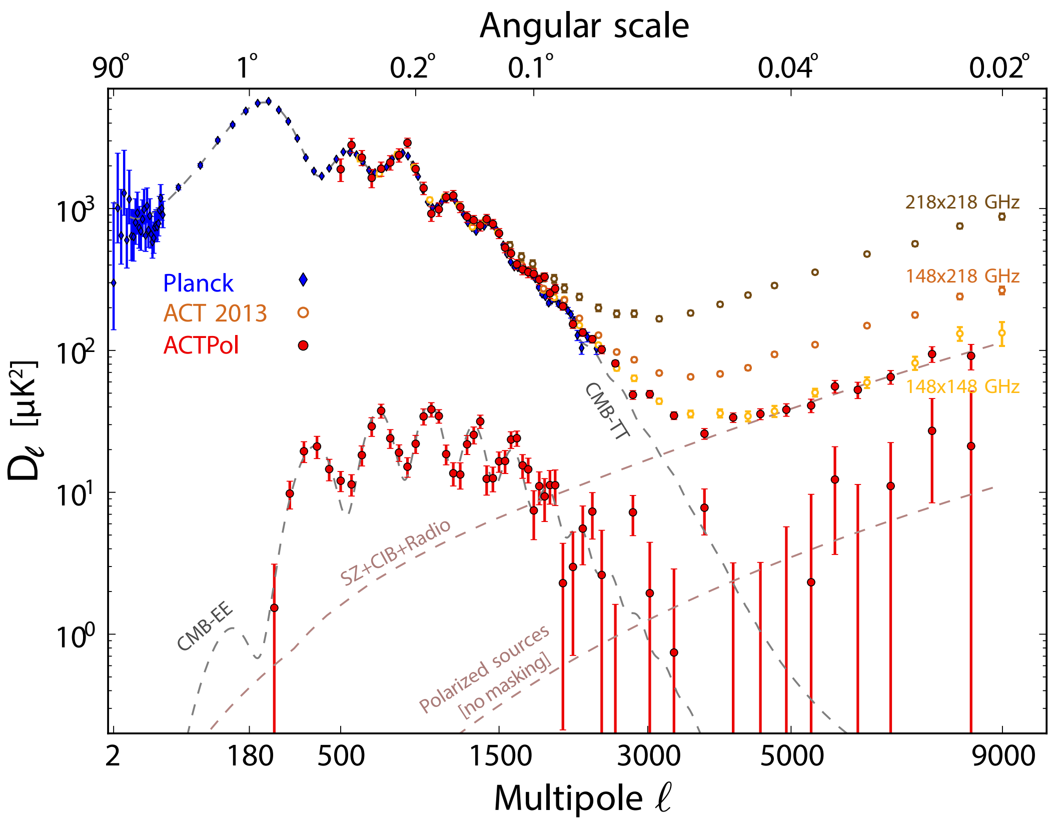
$$l_{\mathrm{max}} \lesssim \frac{N_{\mathrm{pulsar}}}{\sqrt{2}}$$

银河系内可观测毫秒脉冲星的数量： ~ 30,000

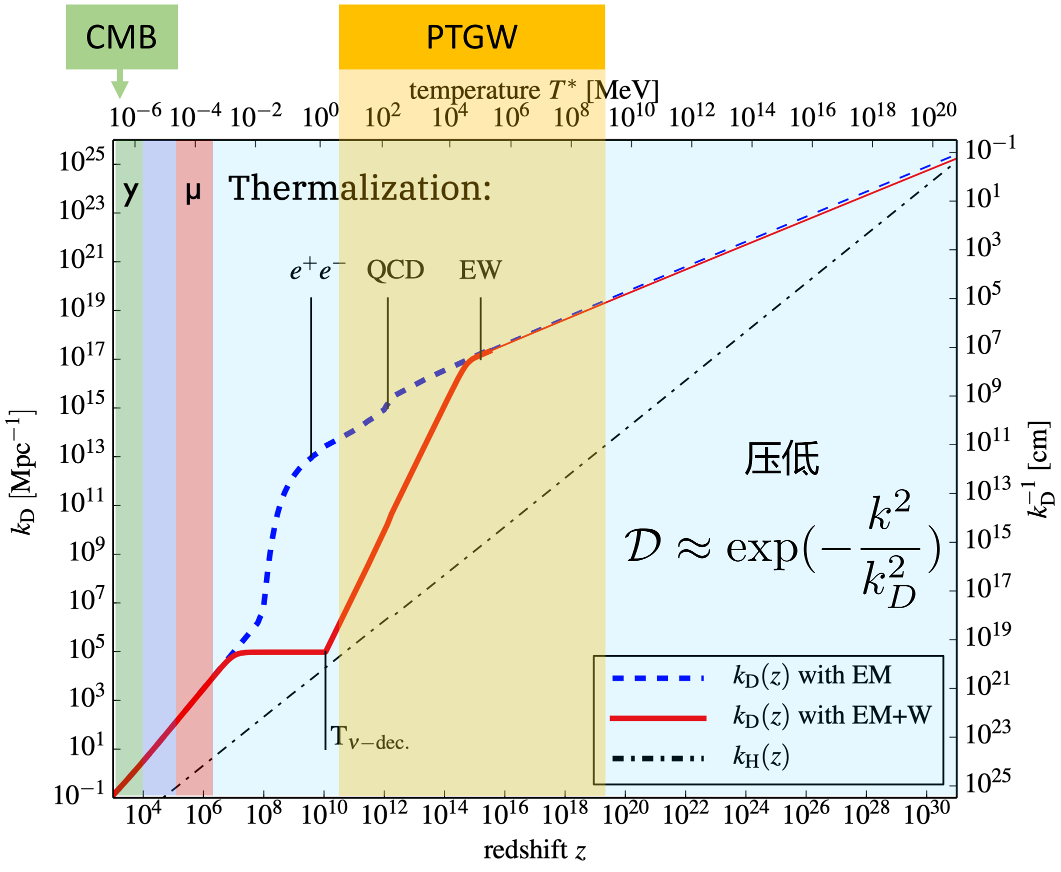
下一步：利用以上方法结合相变引力波的各向异性做定量分析（正在开展）

宇宙演化历史

- 原初标量扰动



其他影响：Sunyaev–Zel'dovich 效应、CIB等



Damping scale

Phys. Rev. Lett., 2014, 113: 061301