The different scattering behaviors between Dirac fermion and Majorana fermion by the metric field and torsion field

薛迅

华东师范大学

With 赖俊辉，郭家明，林威

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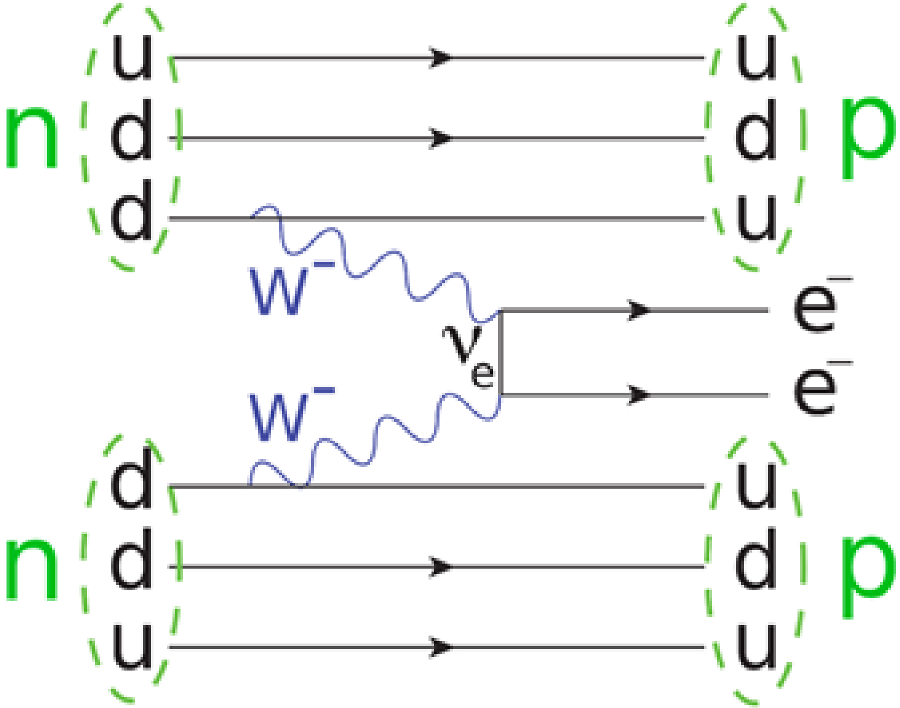
Outline

- Scattering by pure metric field（赖俊辉，郭家明）
- The Riemann-Cartan case（赖俊辉）
- Teleparallele gravity treatment（林威）

Neutrinoless double beta decay



Can Gravity Distinguish between Dirac and Majorana Neutrinos?

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Can Gravity Distinguish between Dirac and Majorana Neutrinos?

Dinesh Singh,[1,*] Nader Mobed,[1,†] and Giorgio Papini[1,2,3,‡]

[1]*Department of Physics, University of Regina, Regina, Saskatchewan, S4S 0A2, Canada*
[2]*Prairie Particle Physics Institute, Regina, Saskatchewan, S4S 0A2, Canada*
[3]*International Institute for Advanced Scientific Studies, 89019 Vietri sul Mare (SA), Italy*

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We show that spin-gravity interaction can distinguish between Dirac and Majorana neutrino wave packets propagating in a Lense-Thirring background. Using time-independent perturbation theory and the gravitational phase to generate a perturbation Hamiltonian with spin-gravity coupling, we show that the associated matrix element for the Majorana neutrino differs significantly from its Dirac counterpart. This difference can be demonstrated through significant gravitational corrections to the neutrino oscillation length for a two-flavor system, as shown explicitly for SN 1987A.

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Comment on "Can Gravity Distinguish between Dirac and Majorana Neutrinos?"

Here we point out that the treatment of Majorana neutrinos in [1] is not valid, due to an incorrect definition and construction of the spinor that corresponds to a Majorana particle. As a consequence, the results of this Letter concerning Majorana neutrinos are not valid, and, in particular, the major claim stated above does not follow.

The short coming of Papini's 2006 paper

- The gravitation phase →effective Hamiltonian →The spin-flip scattering matrix element → distinguish the Dirac from Majorana fermion----indirect gravity effect
- In the wavefunction framework, only wave packet can be self charge conjugate invariant, just like the real scalar field and the real valued wave function state

The direct scattering of Dirac and Majorana fermion by pure metric field

- The weak field approximation: $S = S_0 + S_{int} = \int d^4x\sqrt{-g}\mathcal{L} = \int d^4x\sqrt{-g}\bar{\psi}\big(i\gamma^a {e_a}^{\mu} D_\mu - m\big)\psi = \int d^4x\sqrt{-g}\bar{\psi}\left(i\gamma^a {n_a}^{\mu}\partial_\mu - m + \frac{1}{2}\gamma^a {n_a}^{\mu}\tilde{A}^{bc}{}_{\mu}S_{bc} - \frac{i}{2}\gamma^a {h_a}^{\mu}\partial_\mu - \gamma^a \frac{1}{2}{h_a}^{\mu}\frac{1}{2}\tilde{A}^{bc}{}_{\mu}S_{bc}\right)\psi$
- Where ${e_a}^{\mu} = {n_a}^{\mu} - \frac{1}{2}{h_a}^{\mu}$
  ${n_a}^{\mu}$ is the tetrad for Cartesian coordinates of Mikowskii spacetime, simplist form is ${\delta_a}^{\mu}$

- It can be obtained: $S_{int} = \int d^4x\sqrt{-g}\bar{\psi}\left(\frac{1}{2}\gamma^a \tilde{A}^{bc}{}_a S_{bc} - \frac{i}{2}\gamma^a h_a{}^\mu \partial_\mu\right)\psi = \int d^4x\sqrt{-g}\bar{\psi}\left[\frac{i}{8}\tilde{A}_{abc}(4\eta^{ca}\gamma^b - 2i\epsilon^{dcab}\gamma_d\gamma^5) - \frac{i}{2}\gamma^a h_a{}^\mu \partial_\mu\right]\psi$
- The connection part can be rewritten as: $A_{abc}(4\eta^{ca}\gamma^b - 2i\epsilon^{dcab}\gamma_d\gamma^5) = [8e_a{}^\mu e_b{}^\nu(\partial_\nu e^a{}_\mu - \partial_\mu e^a{}_\nu)]\gamma^b - (4\eta_{bp}e_a{}^\mu e_c{}^\nu \partial_\nu e^p{}_\mu \epsilon^{dcab})i\gamma_d\gamma^5$
- Finally, $S_{int} = \int d^4x\sqrt{-g}\bar{\psi}\left(K_a\gamma^a - \frac{i}{2}\Lambda_a\gamma^a\gamma^5 - \frac{i}{2}\gamma^a h_a{}^\mu \partial_\mu\right)\psi$

Where $K_a = e_b{}^\mu e_a{}^\nu\left(\partial_\nu e^b{}_\mu - \partial_\mu e^b{}_\nu\right)$ and

$$\Lambda_a = e_d{}^\mu e_c{}^\nu \partial_\nu e^b{}_\mu \varepsilon_a{}^{cd}{}_b$$

- The $|i\rangle \rightarrow |f\rangle$ scattering amplitude is

$$\langle f|S|i\rangle = \left\langle f\middle|T\exp\left[-i\int d^4x\mathcal{H}_I(x)\right]\middle|i\right\rangle =$$
$$\left\langle f\middle|T\exp\left[i\int d^4x\mathcal{L}_{int}(x)\right]\middle|i\right\rangle,$$

where $|i\rangle = \sqrt{2E_{p_i}}a^\dagger_{p_i,s_i}|0\rangle, |f\rangle = \sqrt{2E_{p_f}}a^\dagger_{q_f,m_f}|0\rangle$

- The lowest order perturbation of the scattering amplitude is $i\langle f|S_{int}|i\rangle$

- The Dirac and Majorana field expansion are respectively

$$\psi_D(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\sum_{s=1,2}\left[a_{p,s}u^s(p)e^{-ipx} + b^\dagger_{p,s}v^s(p)e^{ipx}\right]$$
$$\psi_M(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\sum_{s=1,2}\left[a_{p,s}u^s(p)e^{-ipx} + a^\dagger_{p,s}v^s(p)e^{ipx}\right]$$

- For Dirac fermion
- $M_D = i\langle f|S_{int}|i\rangle = i\int d^4x\sqrt{-g}\bar{u}^{m_f}(q_f)\left(K_a\gamma^a - \frac{i}{2}\Lambda_a\gamma^a\gamma^5 - \frac{1}{2}\gamma^a h_a{}^{\mu} p_{i_{\mu}}\right)u^{s_i}(p_i)e^{i(q_f-p_i)x}$
- For Majorana fermion
- $M_M = i\int d^4x\sqrt{-g}\bar{u}^{m_f}(q_f)\left(K_a\gamma^a - \frac{i}{2}\Lambda_a\gamma^a\gamma^5 - \frac{1}{2}\gamma^a h_a{}^{\mu} p_{i_{\mu}}\right)u^{s_i}(p_i)e^{i(q_f-p_i)x} - i\int d^4x\sqrt{-g}\bar{v}^{m_f}(q_i)\left(K_a\gamma^a - \frac{i}{2}\Lambda_a\gamma^a\gamma^5 + \frac{1}{2}\gamma^a h_a{}^{\mu} p_{f_{\mu}}\right)v^{s_i}(p_f)e^{i(q_f-p_i)x}$

- Under charge conjugation $v_s(k) = u_s^c(k) = C\bar{u}^T(k)$ for Majorana fermion and $C\gamma^\mu C^{-1} = -\gamma^{\mu T}$
- $\bar{v}^{s_i}(q_i)\left(K_a\gamma^a - \frac{i}{2}\Lambda_a\gamma^a\gamma^5 + \frac{1}{2}\gamma^a h_a{}^\mu p_{f_\mu}\right)v^{m_f}(p_f) = \bar{u}^{m_f}(p_f)\left(K_a\gamma^a + \frac{i}{2}\Lambda_a\gamma^a\gamma^5 + \frac{1}{2}\gamma^a h_a{}^\mu p_{f_\mu}\right)u^{s_i}(p_i)$
- $M_M = -\frac{i}{2}\int d^4x\sqrt{-g}\bar{u}^{m_f}(q_f)\left(i\Lambda_a\gamma^a\gamma^5 + \gamma^a h_a{}^\mu p_{i_\mu} + \gamma^a h_a{}^\mu p_{f_\mu}\right)u^{s_i}(p_i)e^{i(q_f-p_i)x}$
- $M_D = M_M + i\int d^4x\sqrt{-g}\bar{u}^{m_f}(q_f)\left(K_a\gamma^a - \frac{1}{2}\gamma^a h_a{}^\mu q_{f_\mu}\right)u^{s_i}(p_i)\, e^{i(q_f-p_i)x}$

The scattering of fermion by Kerr field

- $g_{00} = 1 - \frac{r_s r}{\rho^2}, g_{11} = -\frac{\rho^2}{\Delta}, g_{22} = -\rho^2, g_{33} = -\left(r^2 + a^2 + \frac{r_s r a^2}{\rho^2} sin^2\theta\right) sin^2\theta, g_{03} = g_{30} = \frac{r_s r a}{\rho^2} sin^2\theta, r_s = 2GM, \rho^2 = r^2 + a^2 cos^2\theta, \Delta = r^2 - r_s r + a^2$

- $g_{\mu\nu} =$

$$\begin{pmatrix} \left(1 - \frac{2Mr}{r^2+a^2\cos^2\theta}\right) & & & \frac{2aMr}{r^2+a^2\cos^2\theta}\sin^2\theta \\ & -\frac{r^2+a^2\cos^2\theta}{r^2-2Mr+a^2} & & \\ & & -(r^2+a^2\cos^2\theta) & \\ \frac{2aMr}{r^2+a^2\cos^2\theta}\sin^2\theta & & & -\left(r^2+a^2+\frac{2Mra^2}{r^2+a^2\cos^2\theta}\sin^2\theta\right)\sin^2\theta \end{pmatrix}$$

In the far end limit, the tetrads are

- $h^a{}_\mu \equiv \begin{pmatrix} \gamma_{00} & 0 & 0 & \eta \\ 0 & \gamma_{11} s\theta c\varphi & \gamma_{22} c\theta s\varphi & -\beta s\varphi \\ 0 & \gamma_{11} s\theta s\varphi & \gamma_{22} c\theta s\varphi & \beta c\varphi \\ 0 & \gamma_{11} c\theta & -\gamma_{22} s\theta & 0 \end{pmatrix}$
- $\gamma_{00} = \sqrt{g_{00}} \simeq 1 - \frac{r_s}{2r} - \frac{{r_s}^2}{8r^2} + \frac{-{r_s}^3 + 8a^2 r_s cos^2\theta}{16r^3} + O(r^{-4})$
  $\gamma_{11} = \sqrt{-g_{11}}$
  $\simeq 1 + \frac{r_s}{2r} + \frac{-4a^2 + 3{r_s}^2 + 4a^2 cos^2\theta}{8r^2} + \frac{-12a^2 r_s + 5{r_s}^3 + 4a^2 r_s cos^2\theta}{16r^3} + O(r^{-4})$
  $\gamma_{22} = \sqrt{-g_{22}} \simeq r + \frac{a^2 cos^2\theta}{2r} - \frac{a^4 cos^4\theta}{8r^3} + O(r^{-4})$
  $\gamma_{33} = \sqrt{-g_{33}} \simeq r\, sin\,\theta + \frac{a^2\, sin\,\theta}{2r} + \frac{a^2 r_s sin^3\theta}{2r^2} - \frac{a^4\, sin\,\theta}{8r^3} + O(r^{-4})$
  $\eta = \frac{g_{03}}{\gamma_{00}} \simeq \frac{a r_s sin^2\theta}{r} + \frac{a {r_s}^2 sin^2\theta}{2r^2} + \frac{a r_s \left(\frac{3{r_s}^2}{8} - a^2 cos^2\theta\right) sin^2\theta}{r^3} + O(r^{-4})$
  $\beta = \sqrt{\eta^2 - g_{33}}$
  $\simeq r\, sin\,\theta + \frac{a^2\, sin\,\theta}{2r} + \frac{a^2 r_s sin^3\theta}{2r^2} + \frac{a^2\, sin\,\theta\,(-a + 2r_s\, sin\,\theta)(a + 2r_s\, sin\,\theta)}{8r^3} + O(r^{-4})$

The difference between Dirac and Majorana is proportional to $\boldsymbol{a^2}$

- $M_D = i\langle f|S_{int}|i\rangle = 2\pi i\delta\left(E_f - E_i\right)G^2M^2a^2\bar{u}^{m_f}\left(q_f\right)\left(-i\frac{\pi^2}{2}k_zk\gamma^3\right)u^{s_i}(p_i) - 2\pi i\delta\left(E_f - E_i\right)GMa\frac{4\pi}{k^2}k_zk_\mu\bar{u}^{m_f}\left(q_f\right)(i\gamma^\mu\gamma^5)u^{s_i}(p_i) - 2\pi i\delta\left(E_f - E_i\right)\frac{1}{2}\bar{u}^{m_f}\left(q_f\right)\left(2GM\frac{4\pi}{k^2}p_{i_\mu}\gamma^\mu - 4aGM\frac{4\pi}{k^2}k_yp_{i_0}\gamma^1 + 4aGM\frac{4\pi}{k^2}k_xp_{i_0}\gamma^2\right)u^{s_i}(p_i)$
- $M_M = i\langle f|S_{int}|i\rangle = -2\pi i\delta\left(E_f - E_i\right)GMa\frac{4\pi}{k^2}k_zk_\mu\bar{u}^{m_f}\left(q_f\right)i\gamma^\mu\gamma^5u^{s_i}(p_i) - 2\pi i\delta\left(E_f - E_i\right)\frac{1}{2}\bar{u}^{m_f}\left(q_f\right)\left(2GM\frac{4\pi}{k^2}p_{i_\mu}\gamma^\mu - 4aGM\frac{4\pi}{k^2}k_yp_{i_0}\gamma^1 + 4aGM\frac{4\pi}{k^2}k_xp_{i_0}\gamma^2\right)u^{s_i}(p_i)$
- $M_D - M_M = -2\pi i\delta\left(E_f - E_i\right)G^2M^2a^2\bar{u}^{m_f}\left(q_f\right)\gamma^3i\frac{\pi^2}{2}k_zku^{s_i}(p_i)$
- In the case $a = 0$ the Schwarzschild field can not tell the difference in the lowest order expansion of perturbation

The scattering of non-polarized fermion by Schwarzschild field

- the far end of Schwarzschild field in the isotr spatial coordinate, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} = \eta_{\mu\nu} + 2\phi(r)\delta_{\nu}^{\mu}$

- $i\langle f|S_{int}|i\rangle = \frac{2\pi i}{VE}\delta(E - E')\bar{u}_s(k')\left(\frac{1}{4}\gamma^a\Gamma_a(q) - \frac{1}{2}h_a^\mu(q)\gamma^a k_\mu\right)u_r(k) = -\frac{2\pi i}{VE}\delta(E - E')\phi(q)\bar{u}_s(k')\gamma^\mu \tilde{k}_\mu u_r(k)$
- Where $\phi(q) = \frac{4\pi GM}{|\vec{q}|^2} = \frac{\pi GM}{|\vec{k}|^2 sin^2(\theta/2)}$ and $\tilde{k}^\mu = \left(E, -\vec{k}\right)$ utilizing the vector current conservation $\bar{u}_s(k')\gamma^\mu q_\mu u_r(k) = 0$
- $\Gamma_a(q)$, $h_a^\mu(q)$, $\phi(q)$ are the 3-d Fourier components

The non-polarised scattering cross section

- $d\sigma = \int \frac{V d^3 k'}{(2\pi)^3} 2\pi\delta(E - E') \frac{EV}{|\vec{k}|} \left(\frac{\phi(q)}{EV}\right)^2 \frac{1}{2} \sum_{spins} \left|\bar{u}_s(k')\gamma^\mu \tilde{k}_\mu u_r(k)\right|^2$
- For unpolarized spinor field, summation over final states and average on the initial ones
- $\frac{1}{2} \sum_{spins} \left|\bar{u}_s(k')\gamma^\mu \tilde{k}_\mu u_r(k)\right|^2 = E^4 \left[(1+v^2)^2 - v^2(3+v^2) sin^2 \frac{\theta}{2}\right]$
- Where v is the velocity of the particle

- Finally , $\frac{d\sigma}{d\Omega} = \frac{G^2M^2}{4v^4 sin^4(\theta/2)}\left[(1+v^2)^2 - v^2(3+v^2)sin^2\frac{\theta}{2}\right]$, independent of m
- In the non-relativistic limit, $\lim_{v\to 0}\frac{d\sigma}{d\Omega} = \frac{G^2M^2m^2}{16E_k^2 sin^4(\theta/2)}$, where $E_k = mv^2/2$ just the Rutherford cross section
- In the relativistic limit, $\lim_{v\to 1}\frac{d\sigma}{d\Omega} = \frac{16G^2M^2}{\theta^4}$, for small angle scattering, $\theta \sim sin\theta$, with the relation $\theta = \frac{4GM}{b}$ just the light ray deflection in GR

The scattering of fermion by torsion

- $\gamma^\mu D_\mu = \gamma^\mu \partial_\mu - \frac{i}{4}\gamma^\mu \tilde{\Gamma}_{\mu ab}\sigma^{ab} + \frac{1}{2}\gamma^\mu V_\mu + \frac{3i}{4}\gamma^\mu A_\mu \gamma_5$, where $V^\mu = T^{\mu\nu}{}_\nu$ $A^\mu = \frac{1}{3!}\varepsilon^{\mu\nu\rho\lambda}T_{\nu\rho\lambda}$, and $F^{\mu\nu\rho} = T^{\mu(\nu\rho)} + \frac{1}{3}g^{\mu(\nu}V^{\rho)} - \frac{1}{3}g^{\nu\rho}V^\mu$—No contribution from the pure tensor part of torsion
  - The amplitude for Dirac fermion: $M^T = \frac{2\pi i}{VE}\delta(E - E')\bar{u}_s(k')\left(\frac{1}{4}\gamma^a\tilde{\Gamma}_a(q) + \frac{i}{2}\gamma^\mu V_\mu(q) - \frac{3}{4}\gamma^\mu A_\mu(q)\gamma_5 - \frac{1}{2}h_a^\mu(q)\gamma^a k_\mu\right)u_r(k)$

- The amplitude for Majorana , $M^{T}{}_{M} = \frac{\pi i}{VE}\delta(E-E')\bar{u}_s(k')\left(\frac{1}{4}\gamma^a\tilde{\Gamma}_a(q)+\frac{i}{2}\gamma^\mu V_\mu(q)-\frac{3}{4}\gamma^\mu A_\mu(q)\gamma_5-\frac{1}{2}h_a^\mu(q)\gamma^a k_\mu\right)u_r(k) - \frac{\pi i}{VE}\delta(E-E')\bar{v}_r(k)\left(\frac{1}{4}\gamma^a\tilde{\Gamma}_a(q)+\frac{i}{2}\gamma^\mu V_\mu(q)-\frac{3}{4}\gamma^\mu A_\mu(q)\gamma_5+\frac{1}{2}h_a^\mu(q)\gamma^a k'_\mu\right)v_s(k')$
- The amplitude for Majorana can be shown only depending on the axial vector torsion,
- $M^{T}{}_{M} = \frac{\pi i}{VE}\delta(E-E')\bar{u}_s(k')\left(\frac{1}{4}\gamma^a\tilde{\Gamma}_a(q)-\frac{1}{2}h_a^\mu(q)\gamma^a k_\mu\right)u_r(k) - \frac{\pi i}{VE}\delta(E-E')\bar{v}_r(k)\left(\frac{1}{4}\gamma^a\tilde{\Gamma}_a(q)+\frac{1}{2}h_a^\mu(q)\gamma^a k'_\mu\right)v_s(k') + \frac{2\pi i}{VE}\delta(E-E')\bar{u}_s(k')\left(-\frac{3}{4}\gamma^\mu A_\mu(q)\gamma_5\right)u_r(k)$

Treatment in Teleparallel Gravity Framework

- $\mathrm{S_D} = \int \mathrm{d^4x\, h}\left[\mathrm{i}\left(\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - \frac{1}{2}\bar{\psi}\gamma^{\mu}\mathcal{V}_{\mu}\psi - \frac{3\mathrm{i}}{4}\bar{\psi}\gamma^{\mu}\mathcal{A}_{\mu}\gamma_5\psi\right) - \mathrm{m}\bar{\psi}\psi\right]$
- $h = det(h^{a}{}_{\mu}),\ \mathcal{V}_{\mu} = T^{\nu}{}_{\nu\mu},\ \mathcal{A}^{\mu} = \frac{1}{6}\varepsilon^{\mu\nu\rho\sigma}T_{\nu\rho\sigma}$
- $S_D = S_0 + S_{int}, h^{a}{}_{\mu} = e^{a}{}_{\mu} + B^{a}{}_{\mu}, |B^{a}{}_{\mu}| << |e^{a}{}_{\mu}|$
- $S_{int} = \int d^4 x e\left[i\left(\bar{\psi}\gamma^{a}B_{a}{}^{\mu}\partial_{\mu}\psi - \frac{1}{2}\bar{\psi}\gamma^{a}\mathcal{V}_{a}\psi - \frac{3i}{4}\bar{\psi}\gamma^{a}\mathcal{A}_{a}\gamma_5\psi\right) + \Delta e\left(i\bar{\psi}\gamma^{a}e_{a}{}^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi\right)\right]$
- $\Delta e = e e_{a}{}^{\mu}B^{a}{}_{\mu}$
- $[T_D]_{fi} = 2\pi\delta(E' - E)\bar{u}_r(k')\left(\gamma^{a}k_b B_{a}{}^{\mu}h^{b}{}_{\mu}(\vec{q}) - \frac{i}{2}\gamma^{a}\mathcal{V}_{a}(\vec{q}) + \frac{3}{4}\gamma^{a}\mathcal{A}_{a}(\vec{q})\gamma_5\right)u_s(k)$

  $$[T_M]_{fi} = 2\pi\delta(E' - E)\bar{u}_r(k')\left(\gamma^{a}\left(k_b + \frac{1}{2}q_b\right)B_{a}{}^{\mu}h^{b}{}_{\mu}(\vec{q}) + \frac{3}{4}\gamma^{a}\mathcal{A}_{a}(\vec{q})\gamma_5\right)u_s(k)$$

Difference between Dirac and Majorana particle

- $[T_D - T_M]_{fi} = -\pi\delta(E' - E)\bar{u}_r(k')[i\gamma^a \mathcal{V}_a(\vec{q}) + \gamma^a q_b B_a{}^\mu h^b{}_\mu(\vec{q})]u_s(k) = a^2\pi^3\delta(E' - E)\bar{u}_r(k')[(\frac{q_z^3}{|\vec{q}|^3} + 2\frac{q_z}{|\vec{q}|})\gamma^3]u_s(k)$
- $\frac{d\sigma^D}{d\Omega} - \frac{d\sigma^M}{d\Omega} = \frac{a^2 r_s \pi E_k}{512 v sin^5\frac{\theta_A}{2}}(1 + v^2)\left(cos\, 2\theta_f - cos\, 2\theta_i\right)\alpha\left(\theta_f, \theta_i, \theta_A\right) + \frac{a^2 {r_s}^2 \pi^2 {E_k}^2}{4096\, sin^4\frac{\theta_A}{2}}(6 + v^2)\left(cos\, 2\theta_f - cos\, 2\theta_i\right)\alpha\left(\theta_f, \theta_i, \theta_A\right) + \frac{a^4 \pi^2 {E_k}^2}{4096 sin^4\frac{\theta_A}{2}} v^2\left(cos\,\theta_f - cos\,\theta_i\right)^2\left(\alpha\left(\theta_f, \theta_i, \theta_A\right)\right)^2\left(1 + cos\,\theta_f\, cos\,\theta_i\, csc^2\frac{\theta_A}{2}\right)$
- $\alpha\left(\theta_f, \theta_i, \theta_A\right) = 14 - 12\, cos\,\theta_A + cos\, 2\,\theta_f - 4\, cos\,\theta_f\, cos\,\theta_i + cos\, 2\,\theta_i$

Summary

- The angular momentum of the gravitational source can tell the difference between Dirac and Majorana
- The scattering by vector torsion can distinguish Majorana from Dirac.
- In the teleparallel gravity treatment, the Kerr scattering can also be given by Kerr torsion equivalently

Outlook

- Prediction for the deviation of Dirac and Majorana particle from the gravitational lensing
- Distinguish fermion type of neutrino by the different gravitational lensing effects of Dirac and Majorana

- Thanks